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K. Balasubramaniyan, A. Gobikrishnan and A. Vadivel

Some Topological Operations and $N_{nc}\ Z^*$ Continuity in N_{nc} Topological Spaces

K. Balasubramaniyan[1,b)], A. Gobikrishnan[2,c)] and A. Vadivel[3,a)]

[1] *PG Department of Mathematics, Arignar Anna Government Arts College, Attur, Tamil Nadu, India.*
[2] *Department of Mathematics, Thiruvalluvar Government Arts College, Rasipuram, Tamil Nadu, India.*
[3] *Department of Mathematics, Government Arts College (Autonomous), Karur, Tamil Nadu, India.*

[a)] Corresponding author : avmaths@gmail.com
[b)] kgbalumaths@gmail.com
[c)] vainugobi@gmail.com

Abstract. The aim of this paper is to introduce and study the notion of $N_{nc}Z^*$-continuity. Some characterizations of these notions are presented. Also, some topological operations such as: $N_{nc}Z^*$-boundary, $N_{nc}Z^*$-border, $N_{nc}Z^*$-exterior, $N_{nc}Z^*$-limit point are introduced.

Keywords and Phrases: $N_{nc}Z^*o$-sets, $N_{nc}Z^*$-boundary, $N_{nc}Z^*$-border, $N_{nc}Z^*$-exterior, $N_{nc}Z^*$-limit point, $N_{nc}Z^*$-neighbourhood and $N_{nc}Z^*$-continuity.

AMS 2000 Subject Classification: 54D10, 54C05, 54C08.

INTRODUCTION

Smarandache's neutrosophic framework have wide scope of constant applications for the fields of Computer Science, Information Systems, Applied Mathematics, Artificial Intelligence, Mechanics, dynamic, Medicine, Electrical & Electronic, and Management Science and so forth [1, 2, 3, 4, 20, 21]. Topology is an classical subject, as a generalization topological spaces numerous kinds of topological spaces presented throughout the year. Smarandache [14] characterized the Neutrosophic set on three segment Neutrosophic sets (T-Truth, F-Falsehood, I-Indeterminacy).

Neutrosophic topological spaces (nts's) presented by Salama and Alblowi [11]. Lellies Thivagar et al. [9] was given the geometric existence of N topology, which is a non-empty set equipped with N arbitrary topologies. Lellis Thivagar et al. [10] introduced the notion of N_n-open (closed) sets in N neutrosophic crisp topological spaces. Al-Hamido et al. [7] investigate the chance of extending the idea of neutrosophic crisp topological spaces into N-neutrosophic crisp topological spaces and examine a portion of their essential properties. In 2008, Ekici [8] introduced the notion of e-open sets in topology. In 2020, Vadivel and John Sundar [17] introduced N-neutrosophic δ-open, N-neutrosophic δ-semiopen and N-neutrosophic δ-preopen sets are introduced.

The purpose of this paper is to introduce and study the notion of $N_{nc}Z^*$-continuity. Some topological operations such as: $N_{nc}Z^*$ limit point, $N_{nc}Z^*$-boundary and $N_{nc}Z^*$-exterior etc. are introduced. Also, some characterizations of these notions are presented.

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PRELIMINARIES

The definitions of neutrosophic crisp set (in short, *ncs*) are studied in [12, 13]. In [7], N_{nc}-topological space (briefly, $N_{nc}ts$), N_{nc}-open sets ($N_{nc}os$), N_{nc}-closed sets ($N_{nc}cs$), N_{nc} interior of H (briefly, $N_{nc}int(H)$) and N_{nc} closure of H (briefly, $N_{nc}cl(H)$) are introduced. Also, N_{nc}-regular open [15] set (briefly, $N_{nc}ros$), N_{nc}-pre open set (briefly, $N_{nc}\mathcal{P}os$), N_{nc}-semi open set (briefly, $N_{nc}\mathcal{S}os$), N_{nc}-α-open set (briefly, $N_{nc}\alpha os$), N_{nc}-β-open set [16] (briefly, $N_{nc}\beta os$), N_{nc}-γ-open set[15] (briefly, $N_{nc}\gamma os$) are described. The complement of an $N_{nc}ros$ (resp. $N_{nc}\mathcal{S}os$, $N_{nc}\mathcal{P}os$, $N_{nc}\alpha os$, $N_{nc}\beta os$ & $N_{nc}\gamma os$) is called an N_{nc}-regular (resp. N_{nc}-semi, N_{nc}-pre, N_{nc}-α, N_{nc}-β & N_{nc}-γ) closed set (briefly, $N_{nc}rcs$ (resp. $N_{nc}\mathcal{S}cs$, $N_{nc}\mathcal{P}cs$, $N_{nc}\alpha cs$, $N_{nc}\beta cs$ & $N_{nc}\gamma cs$)). In [17], $N_{nc}\delta$ interior of H (briefly, $N_{nc}\delta int(H)$), $N_{nc}\delta$ closure of H (briefly, $N_{nc}\delta cl(H)$), $N_{nc}\delta$- open set (briefly, $N_{nc}\delta os$), $N_{nc}\delta$-pre open set (briefly, $N_{nc}\delta\mathcal{P}os$), $N_{nc}\delta$-semi open set (briefly, $N_{nc}\delta\mathcal{S}os$), $N_{nc}a$-open set (briefly, $N_{nc}aos$), $N_{nc}e^*$-open set (briefly, $N_{nc}e^*os$) and $N_{nc}e$-open set [18] (briefly, $N_{nc}eo$). Also, $N_{nc}\delta$ (resp. $N_{nc}\delta$-pre, $N_{nc}\delta$-semi, $N_{nc}a$, $N_{nc}e^*$ & $N_{nc}e$) closed set (briefly, $N_{nc}\delta cs$ (resp. $N_{nc}\delta\mathcal{P}cs$, $N_{nc}\delta\mathcal{S}cs$, $N_{nc}acs$, $N_{nc}e^*cs$ & $N_{nc}ecs$)). In [5], $N_{nc}Z^*$-open (briefly, $N_{nc}Z^*o$), $N_{nc}Z^*$-closed (briefly, $N_{nc}Z^*c$), $N_{nc}Z^*int(A)$, $N_{nc}Z^*cl(A)$ are also defined. Also, $N_{nc}\alpha$-continuous [19] (resp. N_{nc} pre-continuous [19], $N_{nc}\delta$-almost continuous [19], $N_{nc}\gamma$-continuous [15], $N_{nc}e$-continuous [18] & $N_{nc}e^*$-continuous [17]) used in this paper.

SOME TOPOLOGICAL OPERATIONS

Definition 1: Let $(X, N_{nc}\tau)$ be a $N_{nc}ts$ and L be a N_{nc} set of X. Then the $N_{nc}Z^*$-boundary of L (briefly, $N_{nc}Z^*b(L)$) is defined by $N_{nc}Z^*b(L) = N_{nc}Z^*cl(L) \cap N_{nc}Z^*cl(X\backslash L)$.
Theorem 2: If L is a N_{nc} sets of a $N_{nc}ts$ $(X, N_{nc}\tau)$, then the statements

(i) $N_{nc}Z^*b(L)$ is $N_{nc}Z^*c$,
(ii) $N_{nc}Z^*b(L) = N_{nc}Z^*b(X\backslash L)$,
(iii) $N_{nc}Z^*b(L) = N_{nc}Z^*cl(L)\backslash N_{nc}Z^*int(L)$,
(iv) $N_{nc}Z^*b(L) \cap N_{nc}Z^*int(L) = \phi$,
(v) $N_{nc}Z^*b(L) \cap N_{nc}Z^*int(L) = N_{nc}Z^*cl(L)$,
(vi) $N_{nc}Z^*b(N_{nc}Z^*b(L)) \subseteq N_{nc}Z^*b(L)$,
(vii) $N_{nc}Z^*b(N_{nc}Z^*int(L)) \subseteq N_{nc}Z^*b(L)$,
(viii) $N_{nc}Z^*b(N_{nc}Z^*cl(L)) \subseteq N_{nc}Z^*b(L)$,
(ix) $N_{nc}Z^*int(L) = L\backslash N_{nc}Z^*b(L)$,
(x) $N_{nc}Z^*b(L \cap M) \subseteq N_{nc}Z^*b(L) \cup N_{nc}Z^*b(M)$

are hold.
Proof. (i) It is clear. Others are also similar.
Theorem 3: If L is a N_{nc} set of a $N_{nc}ts$ X, then the statements

(i) L is a $N_{nc}Z^*o$ set iff $A \cap N_{nc}Z^*b(L) = \phi$,
(ii) L is a $N_{nc}Z^*c$ set iff $N_{nc}Z^*b(L) \subset L$,
(iii) L is a $N_{nc}Z^*clo$ set iff $N_{nc}Z^*b(L) = \phi$

are hold.
Proof. (i) It follows from Theorem 2.
Definition 4: $N_{nc}Z^*Bd(L) = L\backslash N_{nc}Z^*int(L)$ is said to be $N_{nc}Z^*$-border of L.
Theorem 5: For a N_{nc} set L of a $N_{nc}ts$ X, then the statements

(i) $N_{nc}Z^*Bd(L) \subseteq L$, for any N_{nc} set L of X,
(ii) $L = N_{nc}Z^*int(L) \cup N_{nc}Z^*Bd(L)$,
(iii) $N_{nc}Z^*int(L) \cap N_{nc}Z^*Bd(L) = \phi$,
(iv) L is $N_{nc}Z^*o$ iff $N_{nc}Z^*Bd(L) = \phi$,
(v) $N_{nc}Z^*Bd(N_{nc}Z^*int(L)) = \phi$,
(vi) $N_{nc}Z^*int(N_{nc}Z^*Bd(L)) = \phi$,

(vii) $N_{nc}Z^{*}Bd(N_{nc}Z^{*}Bd(L)) = N_{nc}Z^{*}Bd(L)$,

(viii) $N_{nc}Z^{*}Bd(L) = L \cap N_{nc}Z^{*}cl(X\backslash L)$

are hold.

Proof. (vi) Let $x \in N_{nc}Z^{*}int(N_{nc}Z^{*}Bd(L))$. Then $x \in N_{nc}Z^{*}Bd(L)$. Since, $N_{nc}Z^{*}Bd(L) \subseteq L$, then $x \in N_{nc}Z^{*}int(N_{nc}Z^{*} Bd(L)) \subseteq N_{nc}Z^{*}int(L)$. Hence, $x \in N_{nc}Z^{*}int(L) \cap N_{nc}Z^{*}Bd(L)$, which contradicts (iii). Thus, $N_{nc}Z^{*}int(N_{nc}Z^{*}Bd(L)) = \phi$. (viii) $N_{nc}Z^{*}Bd(L) = L\backslash N_{nc}Z^{*}int(L) = L\backslash(X\backslash N_{nc}Z^{*}cl(X\backslash L)) = L \cap N_{nc}Z^{*}cl(X\backslash L)$.

Definition 6: Let $(X, N_{nc}\tau)$ be a $N_{nc}ts$ and L be N_{nc} set of X. Then the N_{nc} set $X\backslash(N_{nc}Z^{*}cl(L))$ is called the $N_{nc}Z^{*}$-exterior of L and is denoted by $N_{nc}Z^{*}ext(L)$. A point $p \in X$ is called a $N_{nc}Z^{*}$-exterior point of L, if it is a $N_{nc}Z^{*}$-interior point of $X\backslash L$.

Theorem 7: If L and M are two N_{nc} sets of a $N_{nc}ts$ $(X, N_{nc}\tau)$, then the statement

(i) $N_{nc}Z^{*}ext(L)$ is $N_{nc}Z^{*}o$,

(ii) $N_{nc}Z^{*}ext(L) = N_{nc}Z^{*}int(X\backslash L)$,

(iii) $N_{nc}Z^{*}ext(N_{nc}Z^{*}ext(L)) = N_{nc}Z^{*}int(N_{nc}Z^{*}cl(L))$,

(iv) $N_{nc}Z^{*}ext(X\backslash N_{nc}Z^{*}ext(L)) = N_{nc}Z^{*}ext(L)$,

(v) $N_{nc}Z^{*}int(L) \subseteq N_{nc}Z^{*}ext(N_{nc}Z^{*}ext(L))$,

(vi) $N_{nc}Z^{*}ext(L) \cap N_{nc}Z^{*}b(L) = \phi$,

(vii) $N_{nc}Z^{*}ext(L) \cup N_{nc}Z^{*}b(L) = N_{nc}Z^{*}cl(X\backslash L)$,

(viii) $\{N_{nc}Z^{*}int(L)$, $N_{nc}Z^{*}b(L)$ and $N_{nc}Z^{*}ext(L)\}$ form a partition of X,

(ix) If $A \subseteq B$, then $N_{nc}Z^{*}ext(M) \subseteq N_{nc}Z^{*}ext(L)$,

(x) $N_{nc}Z^{*}ext(\phi) = X$ and $N_{nc}Z^{*}ext(X) = \phi$,

(xi) $N_{nc}Z^{*}ext(L \cup M) \subseteq N_{nc}Z^{*}ext(L) \cup N_{nc}Z^{*}ext(M)$,

(xii) $N_{nc}Z^{*}ext(L \cap M) \supseteq N_{nc}Z^{*}ext(L) \cap N_{nc}Z^{*}ext(M)$,

(xiii) $N_{nc}Z^{*}ext(L \cup M) \subseteq N_{nc}Z^{*}ext(L \cap M)$

are hold.

Proof. It follows from Theorems 2 in [6] and 3.

Remark 8: The inclusion relation in parts (xi) and (xii) of the above theorem cannot be replaced by equality is shown in the following example.

Example 9: Let $X = \{a_1, b_1, c_1, d_1, e_1\}$, ${}_{nc}\tau_1 = \{\phi_N, X_N, A, B, C\}$, ${}_{nc}\tau_2 = \{\phi_N, X_N\}$. $A = \langle\{c_1\}, \{\phi\}, \{a_1, b_1, d_1, e_1\}\rangle$, $B = \langle\{a_1, b_1\}, \{\phi\}, \{c_1, d_1, e_1\}\rangle$, $C = \langle\{a_1, b_1, c_1\}, \{\phi\}, \{d_1, e_1\}\rangle$, then we have $2_{nc}\tau = \{\phi_N, X_N, A, B, C\}$, the sets $L = \langle\{c_1, d_1\}, \{\phi\}, \{a_1, b_1, e_1\}\rangle$ and $M = \langle\{c_1, e_1\}, \{\phi\}, \{a_1, b_1, d_1\}\rangle$. $2_{nc}Z^{*}ext(L) = \langle\{a_1, b_1, e_1\}, \{\phi\}, \{c_1, d_1\}\rangle$, $2_{nc}Z^{*}ext(M) = \langle\{a_1, b_1, d_1\}, \{\phi\}, \{c_1, e_1\}\rangle$, $2_{nc}Z^{*}ext(L \cup M) = \langle\{a_1, b_1\}, \{\phi\}, \{c_1, d_1, e_1\}\rangle$ and $2_{nc}Z^{*}ext(L \cap M) = \langle\{a_1, b_1, d_1, e_1\}, \{\phi\}, \{c_1\}\rangle$. Then

(i) $2_{nc}Z^{*}ext(L) \cup 2_{nc}Z^{*}ext(M) \not\subset 2_{nc}Z^{*}ext(L \cup M)$.

(ii) $2_{nc}Z^{*}ext(L \cap M) \not\subset 2_{nc}Z^{*}ext(L) \cap 2_{nc}Z^{*}ext(M)$.

Definition 10: Let L be a N_{nc} set of a $N_{nc}ts$ $(X, N_{nc}\tau)$. Then a point $p \in X$ is called a $N_{nc}Z^{*}$-limit point of a set $L \subseteq X$ if every $N_{nc}Z^{*}o$ set $O \subseteq X$ containing p contains a point of L other than p. The set of all $N_{nc}Z^{*}$-limit points of L is called a $N_{nc}Z^{*}$-derived set of L and is denoted by $N_{nc}Z^{*}d(L)$.

Theorem 11: If L and M are two N_{nc} sets of a $N_{nc}ts$ X, then the statement

(i) If $L \subseteq M$, then $N_{nc}Z^{*}d(L) \subseteq N_{nc}Z^{*}d(M)$,

(ii) $N_{nc}Z^{*}d(L) \cup N_{nc}Z^{*}d(M) \subseteq N_{nc}Z^{*}d(L \cup M)$,

(iii) $N_{nc}Z^{*}d(L \cap M) \subseteq N_{nc}Z^{*}d(L) \cap N_{nc}Z^{*}d(M)$,

(iv) L is a $N_{nc}Z^{*}c$ set iff it contains each of its $N_{nc}Z^{*}$-limit points,

(v) $N_{nc}Z^{*}cl(L) = L \cup N_{nc}Z^{*}d(L)$

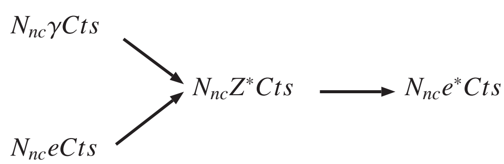
are hold.
Proof: It is clear.
Definition 12: A N_{nc} set P of a $N_{nc}ts$ $(X, N_{nc}\tau)$ is called a $N_{nc}Z^*$-neighbourhood (briefly, $N_{nc}Z^*nbd$) of a point $p \in X$ if there exists a $N_{nc}Z^*o$ set $L \ni p \in L \subseteq P$. The class of all $N_{nc}Z^*nbd$'s of $p \in X$ is called the $N_{nc}Z^*$-neighbourhood system of p and denoted by $N_{nc}Z^*P_p$.
Theorem 13: A N_{nc} set P of a $N_{nc}ts$ X is $N_{nc}Z^*o$ iff it is $N_{nc}Z^*nbd$, $\forall$ point $p \in P$.
Proof. It is clear.
Theorem 14: In a $N_{nc}ts$ $(X, N_{nc}\tau)$. Let $N_{nc}Z * P_p$ be the $N_{nc}Z^*nbd$ System of a point $p \in X$. Then the statement

(i) $N_{nc}Z^*P_p$ is not empty and p belongs to each member of $N_{nc}Z^*P_p$,
(ii) Each superset of members of P_p belongs to $N_{nc}Z^*P_p$,
(iii) Each member $P \in N_{nc}Z^*P_p$ is a superset of a member $L \in N_{nc}Z^*P_p$, where L is $N_{nc}Z^*nbd$ of each point $p \in L$

are hold.
Proof. Obvious.

$N_{nc}Z^*$-CONTINUOUS MAPPINGS

Definition 15: A function $h : (X, N_{nc}\tau) \to (Y, N_{nc}\sigma)$ is called $N_{nc}Z^*$-continuous (briefly, $N_{nc}Z^*Cts$) if $h^{-1}(V)$ is $N_{nc}Z^*o$ in X for each $V \in N_{nc}\sigma$.
Remark 16: Let $h : (X, N_{nc}\tau) \to (Y, N_{nc}\sigma)$ be mapping from a space $(X, N_{nc}\tau)$ into a space $(Y, N_{nc}\sigma)$, The following hold:



Now, the following examples show that these implication are not reversible.
Example 17: Let $X = \{a_1, b_1, c_1, d_1\} = Y$, ${}_{nc}\tau_1 = \{\phi_N, X_N, A, B, C, D\}$, ${}_{nc}\tau_2 = \{\phi_N, X_N, E, F\}$. $A = \langle\{a_1\}, \{\phi\}, \{b_1, c_1, d_1\}\rangle$, $B = \langle\{c_1\}, \{\phi\}, \{a_1, b_1, d_1\}\rangle$, $C = \langle\{a_1, c_1\}, \{\phi\}, \{b_1, d_1\}\rangle$, $D = \langle\{a_1, c_1, d_1\}, \{\phi\}, \{b_1\}\rangle$, $E = \langle\{a_1, b_1\}, \{\phi\}, \{c_1, d_1\}\rangle$, $F = \langle\{a_1, b_1, c_1\}, \{\phi\}, \{d_1\}\rangle$, then we have $2_{nc}\tau = \{\phi_N, X_N, A, B, C, D, E, F\}$. ${}_{nc}\sigma_1 = \{\phi_N, Y_N, G, H\}$, ${}_{nc}\sigma_2 = \{\phi_N, Y_N\}$. $G = \langle\{a_1, d_1\}, \{\phi\}, \{b_1, c_1\}\rangle$, $H = \langle\{b_1, c_1\}, \{\phi\}, \{a_1, d_1\}\rangle$, then we have $2_{nc}\sigma = \{\phi_N, Y_N, G, H\}$. Define $h : (X, 2_{nc}\tau) \to (Y, 2_{nc}\sigma)$ as identity map, then

(i) $2_{nc}Z^*Cts$ but not $2_{nc}\gamma Cts$, the set $h^{-1}(\langle\{b_1, c_1\}, \{\phi\}, \{a_1, d_1\}\rangle) = \langle\{b_1, c_1\}, \{\phi\}, \{a_1, d_1\}\rangle$ is a $2_{nc}Z^*os$ but not $2_{nc}\gamma os$.
(ii) $2_{nc}Z^*Cts$ but not $2_{nc}eCts$, the set $h^{-1}(\langle\{a_1, d_1\}, \{\phi\}, \{b_1, c_1\}\rangle) = \langle\{a_1, d_1\}, \{\phi\}, \{b_1, c_1\}\rangle$ is a $2_{nc}Z^*os$ but not $2_{nc}eos$.

Example 18: In Example 17, ${}_{nc}\sigma_1 = \{\phi_N, Y_N, G\}$, ${}_{nc}\sigma_2 = \{\phi_N, Y_N\}$. $G = \langle\{b_1, d_1\}, \{\phi\}, \{a_1, c_1\}\rangle$, then we have $2_{nc}\sigma = \{\phi_N, Y_N, G\}$. Define $f : (X, 2_{nc}\tau) \to (Y, 2_{nc}\sigma)$ as identity map, then $2_{nc}e^*Cts$ but not $2_{nc}Z^*Cts$, the set $h^{-1}(\langle\{b_1, d_1\}, \{\phi\}, \{a_1, c_1\}\rangle) = \langle\{b_1, d_1\}, \{\phi\}, \{a_1, c_1\}\rangle$ is a $2_{nc}e^*os$ but not $2_{nc}Z^*os$.
Theorem 19: Let $h : (X, N_{nc}\tau) \to (Y, N_{nc}\sigma)$ be a mapping. Then the statements

(i) h is $N_{nc}Z^*Cts$,
(ii) For each $l \in X$ and $M \in N_{nc}\sigma$ containing $h(X)$, there exists $L \in N_{nc}Z^*OS(X)$ containing $l \ni h(L) \subseteq M$,
(iii) The inverse image of each $N_{nc}c$ set in Y is $N_{nc}Z^*c$ in X,
(iv) $N_{nc}int(N_{nc}cl(h^{-1}(M))) \cap N_{nc}cl(N_{nc}int_\delta(h^{-1}(M))) \subseteq h^{-1}(N_{nc}cl(M))$, for each $M \subseteq Y$,
(v) $h^{-1}(N_{nc}int(M)) \subseteq N_{nc}cl(N_{nc}int(h^{-1}(M))) \cup N_{nc}int(N_{nc}cl_\delta(h^{-1}(M)))$, for each $M \subseteq Y$,
(vi) If h is bijective, then $N_{nc}int(h(L)) \subseteq h(N_{nc}cl(N_{nc}int(L))) \cup h(N_{nc}int(N_{nc}cl_\delta(L)))$, $\forall$ $L \subseteq X$,
(vii) If h is bijective, then $h(N_{nc}int(N_{nc}cl(L))) \cap h(N_{nc}cl(N_{nc}int_\delta(L))) \subseteq N_{nc}cl(h(L))$, $\forall$ $L \subseteq X$

are equivalent.
Proof. (i)$\Leftrightarrow$ (ii) and (i)$\Leftrightarrow$ (iii) are obvious,
(iii) $\Rightarrow$ (iv).
Let $M \subseteq Y$, then by (iii) $h^{-1}(N_{nc}cl(M))$ is $N_{nc}Z^{*}c$. This means $h^{-1}(N_{nc}cl(M)) \supseteq N_{nc}int(N_{nc}cl(h^{-1}(N_{nc}\ cl(M)))) \cap N_{nc}cl(N_{nc}int_{\delta}(h^{-1}(N_{nc}cl(M)))) \supseteq N_{nc}int(N_{nc}cl(h^{-1}(M))) \cap N_{nc}cl(N_{nc}int_{\delta}(h^{-1}(M)))$.
(iv) $\Rightarrow$ (v). By replacing $Y\backslash M$ instead of M in (iv) we have $N_{nc}int(N_{nc}cl(h^{-1}(Y\backslash M))) \cap N_{nc}cl(N_{nc}int_{\delta}(h^{-1}(Y\backslash M))) \subseteq h^{-1}(N_{nc}cl(Y\backslash M))$ and therefore $h^{-1}(N_{nc}int(M)) \subseteq N_{nc}cl(N_{nc}int(h^{-1}(M))) \cup N_{nc}int(N_{nc}cl_{\delta}(h^{-1}(M)))$,
(v) $\Rightarrow$ (vi). Follows directly by replacing L instead of $h^{-1}(M)$ in (v) and applying the bijection of h,
(vi) $\Rightarrow$ (vii). By complementation of (vi) and applying the bijective of h, we have $h(N_{nc}int(N_{nc}cl(X\backslash L))) \cap h(N_{nc}cl(N_{nc}\ int_{\delta}(X\backslash L))) \subseteq N_{nc}cl(h(X\backslash L))$. We obtain the required by replacing L instead of $X\backslash L$,
(vii) $\Rightarrow$ (i). Let $L \in N_{nc}\sigma$. But $M = Y\backslash L$, by (vii) we have $h(N_{nc}int(N_{nc}cl(h^{-1}(M)))) \cap h(N_{nc}cl(N_{nc}int_{\delta}(h^{-1}(M)))) \subseteq N_{nc}cl(hh^{-1}(M)) \subseteq N_{nc}cl(M) = M$. So $N_{nc}int(N_{nc}cl(h^{-1}(M))) \cap N_{nc}cl(N_{nc}int_{\delta}(h^{-1}(M))) \subseteq h^{-1}(M)$ implies $h^{-1}(M)$ is $N_{nc}Z^{*}c$ and therefore $h^{-1}(L) \in N_{nc}Z^{*}OS(X)$.
Theorem 20: Let $h : (X, N_{nc}\tau) \to (Y, N_{nc}\sigma)$ be a mapping. Then the statements

(i) h is $N_{nc}Z^{*}Cts$,
(ii) $N_{nc}Z^{*}cl(h^{-1}(M)) \subseteq h^{-1}(N_{nc}cl(M))$, $\forall$ N_{nc} set M of Y,
(iii) $h(N_{nc}Z^{*}cl(L)) \subseteq N_{nc}cl(h(L))$, $\forall$ N_{nc} set L of X,
(iv) $N_{nc}Z^{*}Bd(h^{-1}(M)) \subseteq h^{-1}(Bd(M))$, $\forall$ N_{nc} set M of Y,
(v) $h(N_{nc}Z^{*}d(L)) \subseteq N_{nc}cl(h(L))$, $\forall$ N_{nc} set L of X,
(vi) $h^{-1}(N_{nc}int(M)) \subseteq N_{nc}Z^{*}int(h^{-1}(M))$, $\forall$ N_{nc} set M of Y

are equivalent.
Proof. (i)$\Rightarrow$ (ii). Let M be N_{nc} set of Y, $h^{-1}(N_{nc}cl(M))$ is $N_{nc}Z^{*}c$ in X. Then $N_{nc}Z^{*}cl(h^{-1}(M)) \subseteq N_{nc}Z^{*}cl(h^{-1}(N_{nc}cl(M))) = h^{-1}(N_{nc}cl(M))$,
(ii) $\Rightarrow$ (iii). Let L be N_{nc} set of X then $h(L) \subseteq Y$, by (ii), $h^{-1}(N_{nc}cl(h(L))) \supseteq N_{nc}Z^{*}cl(h^{-1}(h(L))) \supseteq N_{nc}Z^{*}cl(L)$. Therefore, $N_{nc}cl(h(L)) \supseteq hh^{-1}(N_{nc}cl(h(L))) \supseteq h(N_{nc}Z^{*}cl(L))$.
(iii) $\Rightarrow$ (i). Let M be $N_{nc}o$ set of Y. Then, $L = Y\backslash M$ is N_{nc} closed in Y and $h^{-1}(L) = X\backslash h^{-1}(M)$.
Hence, by (iii), $h(N_{nc}Z^{*}cl(h^{-1}(L))) \subseteq N_{nc}cl(h(h^{-1}(L))) \subseteq N_{nc}cl(L) = L$ thus, $N_{nc}Z^{*}cl(h^{-1}(L)) \subseteq h^{-1}(L)$, So, $h^{-1}(L) = X\backslash h^{-1}(M) \in N_{nc}Z^{*}CS(X)$ and therefore $h^{-1}(M) \in N_{nc}Z^{*}OS(X)$,
(iv) $\Rightarrow$ (vi). Let M be N_{nc} set of Y. Then by (vi), $N_{nc}Z^{*}Bd(h^{-1}(M)) = h^{-1}(M)\backslash N_{nc}Z^{*}int(h^{-1}(M)) \subseteq h^{-1}(N_{nc}Bd(M)) = h^{-1}(M\backslash N_{nc}int(M)) = h^{-1}(M)\backslash h^{-1}(N_{nc}int(M))$ this implies $h^{-1}(N_{nc}int(M)) \subseteq N_{nc}Z^{*}int(h^{-1}(M))$.
(vi) $\Rightarrow$ (iv). Let $M \subseteq Y$. Then by (vi), $h^{-1}(N_{nc}int(M)) \subseteq N_{nc}Z^{*}int(h^{-1}(M))$ we have $h^{-1}(M)\backslash N_{nc}Z^{*}int(h^{-1}(M)) \subseteq h^{-1}(M)\backslash h^{-1}(N_{nc}int(M)) \to N_{nc}Z^{*}Bd(h^{-1}(M)) \subseteq h^{-1}(N_{nc}Bd(M))$.
(i) $\to$ (v). It is obvious, since h is $N_{nc}Z^{*}Cts$ and by (iii), $h(N_{nc}Z^{*}cl(L)) \subseteq N_{nc}cl(h(L))$, $\forall$ L be N_{nc} set of X. So, $h(N_{nc}Z^{*}d(L)) \subseteq h(N_{nc}Z^{*}cl(L)) \subseteq N_{nc}cl(h(L))$.
(v) $\Rightarrow$ (i). Let M be $N_{nc}o$ set of Y. Then, $L = Y\backslash M$ is N_{nc} closed in Y and $h^{-1}(L) = X\backslash h^{-1}(M)$.
Hence, by (v), $h(N_{nc}Z^{*}d(h^{-1}(L))) \subseteq N_{nc}cl(h(h^{-1}(L))) \subseteq N_{nc}cl(L) = L$. Hence, $N_{nc}Z^{*}d(h^{-1}(L)) \subseteq h^{-1}(L)$. By Theorem 7, $h^{-1}(L) = X\backslash h^{-1}(M)$ is $N_{nc}Z^{*}c$ in X. Therefore, $h^{-1}(M)$ is $N_{nc}Z^{*}o$ in X.
(i) $\Rightarrow$(vi). Let M be N_{nc} set of Y. Then $h^{-1}(N_{nc}int(M))$ is $N_{nc}Z^{*}o$ in X. Thus, $h^{-1}(N_{nc}int(M)) = N_{nc}Z^{*}int(h^{-1}(N_{nc}\ int(M))) \subseteq N_{nc}Z^{*}int(h^{-1}(M))$. Therefore, $h^{-1}(N_{nc}int(M)) \subseteq N_{nc}Z^{*}int(h^{-1}(M))$.
(v) $\Rightarrow$ (i). Let $M \subseteq Y$ be an $N_{nc}o$ set Y. Then $h^{-1}(M) = h^{-1}(N_{nc}int(M)) \subseteq h^{-1}(N_{nc}Z^{*}int(M))$. Hence, $h^{-1}(M)$ is $N_{nc}Z^{*}o$ in X. Therefore, h is $N_{nc}Z^{*}Cts$.
Remark 21: The composition of two $N_{nc}Z^{*}Cts$ mappings need not be $N_{nc}Z^{*}Cts$.
Example 22: Let $X = Y = Z = \{a_1, b_1, c_1, d_1, e_1\}$, ${}_{nc}\tau_1 = \{\phi_N, X_N, A, B, C\}$, ${}_{nc}\tau_2 = \{\phi_N, X_N\}$. $A = \langle\{a_1, b_1\}, \{\phi\}, \{c_1, d_1, e_1\}\rangle$, $B = \langle\{c_1, d_1\}, \{\phi\}, \{a_1, b_1, e_1\}\rangle$, $C = \langle\{a_1, b_1, c_1, d_1\}, \{\phi\}, \{e_1\}\rangle$, then we have $2_{nc}\tau = \{\phi_N, X_N, A, B, C\}$. ${}_{nc}\sigma_1 = \{\phi_N, Y_N, D\}$, ${}_{nc}\sigma_2 = \{\phi_N, Y_N\}$. $D = \langle\{a_1, b_1\}, \{\phi\}, \{c_1, d_1, e_1\}\rangle$, then we have $2_{nc}\sigma = \{\phi_N, Y_N, D\}$. ${}_{nc}\mu_1 = \{\phi_N, Z_N, E\}$, ${}_{nc}\mu_2 = \{\phi_N, Z_N\}$. $E = \langle\{a_1, e_1\}, \{\phi\}, \{b_1, c_1, d_1\}\rangle$, then we have $2_{nc}\mu = \{\phi_N, Z_N, E\}$. Let $h : (X, 2_{nc}\tau) \to (Y, 2_{nc}\sigma)$ and $g : (Y, 2_{nc}\sigma) \to (Z, 2_{nc}\mu)$ are defined as identity function. It is clear that h and g are $2_{nc}Z^{*}Cts$ but $g \circ h$ is not $2_{nc}Z^{*}Cts$.

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