Neutrosophic *Z*-continuous maps and *Z*-irresolute maps

Cite as: AIP Conference Proceedings **2364**, 020020 (2021); https://doi.org/10.1063/5.0062905
Published Online: 23 September 2021

N. Moogambigai, A. Vadivel and S. Tamilselvan

Neutrosophic Z-Continuous Maps and Z-Irresolute Maps

N.Moogambigai[1,b)], A.Vadivel[2,a)] and S.Tamilselvan[3,c)]

[1]*Department of Mathematics, Thiruvalluvar Government Arts College, Tamil Nadu, India.*
[2]*Department of Mathematics, Government Arts College (Autonomous) Karur, Tamil Nadu, India.*
[3]*Mathematics Section (FEAT), Annamalai University, Annamalainagar, Tamil Nadu, India.*

[a)]Corresponding author: avmaths@gmail.com
[b)]toyesmohan@gmail.com
[c)]tamil_au@yahoo.com

Abstract. Aim of this present paper is, we introduce and investigate neutrosophic Z-continuous maps which is a new kind of neutrosophic continuous in neutrosophic topological spaces. Also discussed about some properties and characterization of neutrosophic Z-irresolute maps.

Keywords and Phrases. Neutrosophic Z-closed sets, neutrosophic Z-continuous maps and neutrosophic Z-irresolute maps.

AMS 2000 Subject Classification: 03E72, 54A10, 54A40, 54C05.

INTRODUCTION

The concept of fuzzy set (briefly, fs) was introduced by Lotfi Zadeh in 1965 [20], then Chang depended the fuzzy set to introduce the concept of fuzzy topological space (briefly, fts) in 1968 [6]. After that the concept of fuzzy set was developed into the concept of intutionistic fuzzy set (briefly, Ifs) by Atanassov in 1983[2,3,4], which gives a degree of membership and a degree of non-membership functions.

In 2005 Smaradache [14] study the concept of neutrosophic set (briefly, $N_s s$). After that and as developed the term of neutrosophic set, Salama has studied neutrosophic topological space (briefly, $N_s ts$) and many of its applications [9,10,11,12]. In 2012 Salama and Alblowi defined neutrosophic topological space [9].

Saha [15] defined δ-open sets in topological spaces. Vadivel et.al. [17] introduced δ-open sets in a neutrosophic topological space. In 2008, Ekici [7] introduced the notion of e-open sets in a general topology. In 2014, Seenivasan et.al. [13] introduced e-open sets in a fuzzy topological space along with fuzzy e-continuity. Vadivel et.al. [5] studied fuzzy e-open sets in intuitionistic fuzzy topological space.

In this paper, we develop the concept of neutrosophic Z continuity in a topological spaces and also specialized some of their basic properties with examples. Also, we discuss about properties and characterization of neutrosophic Z-irresolute maps.

PRELIMINARIES

In Paper [9], a neutrosophic set (briefly, $N_s s$) is defined & properties such as $0_N, I_N, L \subseteq M, L = M, I_N - L, L \cup M$ & $L \cap M$. Also, a neutrosophic topological space (briefly, $N_s ts$), neutrosophic open sets (briefly, $N_s os$), neutrosophic closed sets (briefly, $N_s cs$), neutrosophic interior of L (briefly, $N_s int(L)$) and neutrosophic closure of L (briefly, $N_s cl(L)$) are defined. In [1], neutrosophic regular (resp. pre) open set (briefly, $N_s ros$ (resp. $N_s \mathcal{P}os$)) and neutrosophic regular (resp. pre) closed set (briefly, $N_s rsc$ (resp. $N_s \mathcal{P}cs$)) are described. In [17], neutrosophic δ interior of G (briefly, $N_s \delta\, int(K)$), neutrosophic δ closure of K (briefly, $N_s \delta cl(K)$), neutrosophic δ-open set (briefly, $N_s \delta os$), neutrosophic δ-semi open set (briefly, $N_s \delta \mathcal{S} os$) and their respective closed sets. A neutrosophic e-

International Conference on Recent Trends in Applied Mathematical Sciences (ICRTAMS-2020)
AIP Conf. Proc. 2364, 020020-1–020020-6; https://doi.org/10.1063/5.0062905
Published by AIP Publishing. 978-0-7354-4122-4/$30.00

open set (briefly, $N_s eos$) [16], neutrosophic Z-open set (briefly, $N_s Zos$) [8] and their respective closed sets. A neutrosophic (resp. $\delta, \delta\mathcal{S}, \mathcal{P}\&e$) continuous (briefly, $N_s Cts$ [11] (resp. $N_s\delta Cts, N_s\delta\mathcal{S}Cts, N_s\mathcal{P}Cts$ [1] & $N_s eCts$ [18])) are also used in this paper.

NEUTROSOPHIC *Z*-CONTINUOUS MAPS

Definition 1: A map $h:(X,\tau_N)\to(Y,\sigma_N)$ is called neutrosophic Z-continuous ($N_s Z\ Cts$ in short) if $h^{-1}(\psi)$ is a $N_s Zos$ in (X,τ_N) for every $N_s os\psi$ in (Y,σ_N).

Example 2: Let $X=\{l,m,n\}=Y$ and define $N_s s' s X_1, X_2 \& X_3$ in X and Y_1 in Y are

$X_1=\langle X,\left(\frac{\mu_l}{0.2},\frac{\mu_m}{0.3},\frac{\mu_n}{0.4}\right),\left(\frac{\sigma_l}{0.5},\frac{\sigma_m}{0.5},\frac{\sigma_n}{0.5}\right),\left(\frac{\nu_l}{0.8},\frac{\nu_m}{0.7},\frac{\nu_n}{0.6}\right)\rangle,$

$X_2=\langle X,\left(\frac{\mu_l}{0.1},\frac{\mu_m}{0.1},\frac{\mu_n}{0.4}\right),\left(\frac{\sigma_l}{0.5},\frac{\sigma_m}{0.5},\frac{\sigma_n}{0.5}\right),\left(\frac{\nu_l}{0.9},\frac{\nu_m}{0.9},\frac{\nu_n}{0.6}\right)\rangle,$

$X_3=\langle X,\left(\frac{\mu_l}{0.2},\frac{\mu_m}{0.4},\frac{\mu_n}{0.4}\right),\left(\frac{\sigma_l}{0.5},\frac{\sigma_m}{0.5},\frac{\sigma_n}{0.5}\right),\left(\frac{\nu_l}{0.8},\frac{\nu_m}{0.6},\frac{\nu_n}{0.6}\right)\rangle,$

$Y_1=\langle Y,\left(\frac{\mu_l}{0.2},\frac{\mu_m}{0.4},\frac{\mu_n}{0.4}\right),\left(\frac{\sigma_l}{0.5},\frac{\sigma_m}{0.5},\frac{\sigma_n}{0.5}\right),\left(\frac{\nu_l}{0.8},\frac{\nu_m}{0.6},\frac{\nu_n}{0.6}\right)\rangle.$

Then we have $\tau_N=\{0_N,X_1,X_2,1_N\}$ and $\sigma_N=\{0_N,Y_1,1_N\}$. Let $h:(X,\tau_N)\to(Y,\sigma_N)$ be an identity mapping, then h is $N_s ZCts$ function.

Proposition 3: The statements are hold but the converse does not true.

(i) Every $N_s\delta Cts$ is a $N_s Cts$.
(ii) Every $N_s Cts$ is a $N_s\delta\mathcal{S}Cts$.
(iii) Every $N_s Cts$ is a $N_s\mathcal{P}Cts$.
(iv) Every $N_s\delta\mathcal{S}Cts$ is a $N_s ZCts$.
(v) Every $N_s\mathcal{P}Cts$ is a $N_s ZCts$.
(vi) Every $N_s ZCts$ is a $N_s eCts$.

Proof: The proof of (i), (ii) & (iii) are studied in [19] & [18].

(iv) Let ψ be a $\$N_s os$ in Y. Since h is $N_s\delta\mathcal{S}Cts, h^{-1}(\psi)$ is a $N_s\delta\mathcal{S}os$ in X. Since every $N_s\delta os$ is a $N_s Zos$ [16], $h^{-1}(\psi)$ is a $N_s Zos$ in X. Hence h is a $N_s ZCts$.
(v) Let ψ be a $\$N_s os$ in Y. Since h is $N_s\mathcal{P}Cts, h^{-1}(\psi)$ is a $N_s\mathcal{P}os$ in X. Since every $N_s\mathcal{P}os$ is a $N_s Zos$ [16], $h^{-1}(\psi)$ is a $N_s Zos$ in X. Hence h is a $N_s ZCts$.
(vi) Let ψ be a $\$N_s os$ in Y. Since h is $N_s ZCts, h^{-1}(\psi)$ is a $N_s Zos$ in X. Since every $N_s Zos$ is a $N_s eos$ [16], $h^{-1}(\psi)$ is a $N_s eos$ in X. Hence h is a $N_s eCts$.

Example 4: Let $X=\{l,m,n\}=Y$ and define $N_s s' s X_1, X_2 \& X_3$ in X are

$X_1=\langle X,\left(\frac{\mu_l}{0.2},\frac{\mu_m}{0.3},\frac{\mu_n}{0.4}\right),\left(\frac{\sigma_l}{0.5},\frac{\sigma_m}{0.5},\frac{\sigma_n}{0.5}\right),\left(\frac{\nu_l}{0.8},\frac{\nu_m}{0.7},\frac{\nu_n}{0.6}\right)\rangle,$

$X_2=\langle X,\left(\frac{\mu_l}{0.1},\frac{\mu_m}{0.1},\frac{\mu_n}{0.4}\right),\left(\frac{\sigma_l}{0.5},\frac{\sigma_m}{0.5},\frac{\sigma_n}{0.5}\right),\left(\frac{\nu_l}{0.9},\frac{\nu_m}{0.9},\frac{\nu_n}{0.6}\right)\rangle,$

$X_3=\langle Y,\left(\frac{\mu_l}{0.2},\frac{\mu_m}{0.2},\frac{\mu_n}{0.3}\right),\left(\frac{\sigma_l}{0.5},\frac{\sigma_m}{0.5},\frac{\sigma_n}{0.5}\right),\left(\frac{\nu_l}{0.8},\frac{\nu_m}{0.8},\frac{\nu_n}{0.7}\right)\rangle.$

Then we have $\tau_N=\{0_N,X_1,X_2,1_N\}$. Let $h:(X,\tau_N)\to(X,\sigma_N)$ be an identity mapping, then h is $N_s\mathcal{P}Cts$ but not $N_s Cts$, the set $h^{-1}(X_3)=X_3$ is a $N_s\mathcal{P}os$ but not $N_s os$.

Example 5: In Example 2, h is $N_s ZCts$ but not $N_s\mathcal{P}Cts$, the set $h^{-1}(Y_1)=X_3$ is a $N_s Zos$ but not $N_s\mathcal{P}os$.

Example 6: Let $X=\{l,m,n\}=Y$ and define $N_s s' s X_1, X_2 \& X_3$ in X and Y_1 in Y are

$X_1=\langle X,\left(\frac{\mu_l}{0.2},\frac{\mu_m}{0.3},\frac{\mu_n}{0.4}\right),\left(\frac{\sigma_l}{0.5},\frac{\sigma_m}{0.5},\frac{\sigma_n}{0.5}\right),\left(\frac{\nu_l}{0.8},\frac{\nu_m}{0.7},\frac{\nu_n}{0.6}\right)\rangle,$

$X_2=\langle X,\left(\frac{\mu_l}{0.1},\frac{\mu_m}{0.1},\frac{\mu_n}{0.4}\right),\left(\frac{\sigma_l}{0.5},\frac{\sigma_m}{0.5},\frac{\sigma_n}{0.5}\right),\left(\frac{\nu_l}{0.9},\frac{\nu_m}{0.9},\frac{\nu_n}{0.6}\right)\rangle,$

$X_3=\langle X,\left(\frac{\mu_l}{0.8},\frac{\mu_m}{0.7},\frac{\mu_n}{0.8}\right),\left(\frac{\sigma_l}{0.5},\frac{\sigma_m}{0.5},\frac{\sigma_n}{0.5}\right),\left(\frac{\nu_l}{0.2},\frac{\nu_m}{0.3},\frac{\nu_n}{0.2}\right)\rangle,$

$Y_1=\langle Y,\left(\frac{\mu_l}{0.8},\frac{\mu_m}{0.7},\frac{\mu_n}{0.8}\right),\left(\frac{\sigma_l}{0.5},\frac{\sigma_m}{0.5},\frac{\sigma_n}{0.5}\right),\left(\frac{\nu_l}{0.2},\frac{\nu_m}{0.3},\frac{\nu_n}{0.2}\right)\rangle.$

Then we have $\tau_N=\{0_N,X_1,X_2,1_N\}$ and $\sigma_N=\{0_N,Y_1,1_N\}$. Let $h:(X,\tau_N)\to(Y,\sigma_N)$ be an identity mapping, then h is $N_s eCts$ but not $N_s ZCts$, the set $h^{-1}(Y_1)=X_3$ is a $N_s eos$ but not $N_s Zos$.

Example 7: Let $X=\{l,m,n\}=Y$ and define $N_s s' s X_1, X_2 \& X_3$ in X and Y_1 in Y are

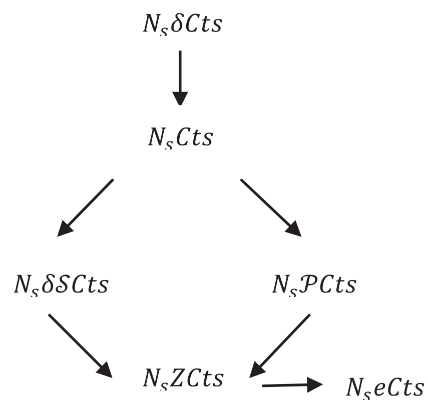
$X_1=\langle X,\left(\frac{\mu_l}{0.4},\frac{\mu_m}{0.6},\frac{\mu_n}{0.5}\right),\left(\frac{\sigma_l}{0.5},\frac{\sigma_m}{0.5},\frac{\sigma_n}{0.5}\right),\left(\frac{\nu_l}{0.6},\frac{\nu_m}{0.4},\frac{\nu_n}{0.5}\right)\rangle,$

$X_2=\langle X,\left(\frac{\mu_l}{0.6},\frac{\mu_m}{0.4},\frac{\mu_n}{0.4}\right),\left(\frac{\sigma_l}{0.5},\frac{\sigma_m}{0.5},\frac{\sigma_n}{0.5}\right),\left(\frac{\nu_l}{0.4},\frac{\nu_m}{0.6},\frac{\nu_n}{0.6}\right)\rangle,$

$X_3 = \langle X, \left(\frac{\mu_l}{0.4}, \frac{\mu_m}{0.5}, \frac{\mu_n}{0.5}\right), \left(\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}\right), \left(\frac{\nu_l}{0.6}, \frac{\nu_m}{0.5}, \frac{\nu_n}{0.5}\right) \rangle$,
$Y_1 = \langle Y, \left(\frac{\mu_l}{0.4}, \frac{\mu_m}{0.5}, \frac{\mu_n}{0.5}\right), \left(\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}\right), \left(\frac{\nu_l}{0.6}, \frac{\nu_m}{0.5}, \frac{\nu_n}{0.5}\right) \rangle$.

Then we have $\tau_N = \{0_N, X_1, X_2, X_1 \cup X_2, X_1 \cap X_2, 1_N\}$ and $\sigma_N = \{0_N, Y_1, 1_N\}$. Let $h: (X, \tau_N) \to (Y, \sigma_N)$ be an identity mapping, then h is N_sZCts but not $N_s\delta\mathcal{S}Cts$, the set $h^{-1}(Y_1) = X_3$ is a N_sZos but not $N_s\delta\mathcal{S}os$. The other implications are shown in [17].

Remark 8: We obtain the following diagram from the results we discussed above.



Theorem 9: A map $h: (X, \tau_N) \to (Y, \sigma_N)$ is $N_s\ ZCts$ iff the inverse image of each N_scs in Y is N_sZcs in X.

Proof: Let ψ be a N_scs in Y. This implies ψ^c is N_sos in Y. Since h is $N_sZCts, h^{-1}(\psi^c)$ is N_sZos in X. Since $h^{-1}(\psi^c) = \left(h^{-1}(\psi)\right)^c, h^{-1}(\psi)$ is a N_sZcs in X.

Conversely, let ψ be a N_scs in Y. Then ψ^c is a N_sos in Y. By hypothesis $h^{-1}(\psi^c)$ is N_sZos in X. Since $h^{-1}(\psi^c) = (h^{-1}(\psi))^c, (h^{-1}(\psi))^c$ is a N_sZos in X. Therefore $h^{-1}(\psi)$ is a N_sZcs in X. Hence h is N_sZCts.

Theorem 10: Let $h: (X, \tau_N) \to (Y, \sigma_N)$ be a N_sZCts where every N_sZos in X is a N_sos in X, then h is a N_sCts.

Proof: Let ψ be a N_sos in Y. Then $h^{-1}(\psi)$ is a N_sZos in X. By hypothesis, $h^{-1}(\psi)$ is a N_sos in X. Hence h is a N_sZCts.

Theorem 11: Let $h: (X, \tau_N) \to (Y, \sigma_N)$ be a N_sZCts map and $g: (Y, \sigma_N) \to (Z, \rho_N)$ be an N_sCts, then $g \circ h: (X, \tau_N) \to (Z, \rho_N)$ is a N_sZCts map.

Proof: Let ψ be a N_sos in Z. Then $g^{-1}(\psi)$ is a N_sZos in Y, by hypothesis. Since h is a N_sZCts maps, $h^{-1}(g^{-1}(\psi))$ is a N_sZos in X. Hence $g \circ h$ is a N_sZCts map.

Theorem 12: Let $h: (X, \tau_N) \to (Y, \sigma_N)$ be a N_sZCts map. Then the following conditions are hold.

(i) $h\left(N_sZcl(\psi)\right) \subseteq N_scl\left(h(\psi)\right)$, for all $N_s\psi$ in X.

(ii) $N_sZcl\left(h^{-1}(\mu)\right) \subseteq h^{-1}\left(N_scl(\mu)\right)$, for all N_ss in Y.

Proof: (i) Since $N_scl(h(\psi))$ is a N_scs in Y and h is N_sZCts, then $h^{-1}N_scl(h(\psi))$ is N_sZc in X. That is, $N_sZcl(\psi) \subseteq h^{-1}N_scl\left(h(\psi)\right)$. Therefore, $h\left(N_sZcl(\psi)\right) \subseteq N_scl\left(h(\psi)\right)$.

(ii) By replacing ψ by $h^{-1}(\mu)$ in (i), we obtain $h(N_sZcl(h^{-1}(\mu))) \subseteq N_scl(h(h^{-1}(\mu))) \subseteq N_scl(\mu)$. Hence, $N_sZcl\left(h^{-1}(\mu)\right) \subseteq h^{-1}\left(N_scl(\mu)\right)$, for every neutrosophic set μ in Y.

Remark 13: If h is N_sZCts, then

(i) $h(N_sZcl(\psi))$ is not necessarily equal to $N_scl(h(\psi))$ where $\psi \in X$.

(ii) $N_sZcl(h^{-1}(\mu))$ is not necessarily equal to $h^{-1}(N_scl(\mu))$ where $\mu \in Y$.

Example 14: In Example 2, h is a N_sZCts.

(i) Let $\psi = \langle\left(\left(\frac{\mu_a}{0.2}, \frac{\mu_b}{0.4}, \frac{\mu_c}{0.4}\right), \left(\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}\right), \left(\frac{\nu_a}{0.8}, \frac{\nu_b}{0.6}, \frac{\nu_c}{0.6}\right)\right)\rangle$. Then

$$h\left(N_sZcl(\psi)\right) = f\left(N_sZcl\left(\langle\left(\frac{\mu_a}{0.2}, \frac{\mu_b}{0.4}, \frac{\mu_c}{0.4}\right), \left(\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}\right), \left(\frac{\nu_a}{0.8}, \frac{\nu_b}{0.6}, \frac{\nu_c}{0.6}\right)\rangle\right)\right)$$
$$= f\left(\langle\left(\frac{\mu_a}{0.2}, \frac{\mu_b}{0.4}, \frac{\mu_c}{0.4}\right), \left(\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}\right), \left(\frac{\nu_a}{0.8}, \frac{\nu_b}{0.6}, \frac{\nu_c}{0.6}\right)\rangle\right)$$
$$= \langle\left(\frac{\mu_a}{0.2}, \frac{\mu_b}{0.4}, \frac{\mu_c}{0.4}\right), \left(\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}\right), \left(\frac{\nu_a}{0.8}, \frac{\nu_b}{0.6}, \frac{\nu_c}{0.6}\right)\rangle.$$

But

$$N_scl\left(h(\psi)\right) = N_scl\left(h\left(\langle\left(\frac{\mu_a}{0.2}, \frac{\mu_b}{0.4}, \frac{\mu_c}{0.4}\right), \left(\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}\right), \left(\frac{\nu_a}{0.8}, \frac{\nu_b}{0.6}, \frac{\nu_c}{0.6}\right)\rangle\right)\right)$$

$$= N_s cl\left(\left\langle\left(\frac{\mu_a}{0.2},\frac{\mu_b}{0.4},\frac{\mu_c}{0.4}\right),\left(\frac{\sigma_a}{0.5},\frac{\sigma_b}{0.5},\frac{\sigma_c}{0.5}\right),\left(\frac{\nu_a}{0.8},\frac{\nu_b}{0.6},\frac{\nu_c}{0.6}\right)\right\rangle\right)$$
$$= \left\langle\left(\frac{\mu_a}{0.8},\frac{\mu_b}{0.7},\frac{\mu_c}{0.6}\right),\left(\frac{\sigma_a}{0.5},\frac{\sigma_b}{0.5},\frac{\sigma_c}{0.5}\right),\left(\frac{\nu_a}{0.2},\frac{\nu_b}{0.3},\frac{\nu_c}{0.4}\right)\right\rangle.$$

Thus $h\big(N_s Zcl(\psi)\big) \neq N_s cl\big(h(\psi)\big)$.

(ii) Let $\eta = \left\langle\left(\frac{\mu_a}{0.2},\frac{\mu_b}{0.4},\frac{\mu_c}{0.4}\right),\left(\frac{\sigma_a}{0.5},\frac{\sigma_b}{0.5},\frac{\sigma_c}{0.5}\right),\left(\frac{\nu_a}{0.8},\frac{\nu_b}{0.6},\frac{\nu_c}{0.6}\right)\right\rangle$.

Then

$$N_s Zcl(h^{-1}(\eta)) \leq N_s Zcl(h^{-1}\left(\left\langle\left(\frac{\mu_a}{0.2},\frac{\mu_b}{0.4},\frac{\mu_c}{0.4}\right),\left(\frac{\sigma_a}{0.5},\frac{\sigma_b}{0.5},\frac{\sigma_c}{0.5}\right),\left(\frac{\nu_a}{0.8},\frac{\nu_b}{0.6},\frac{\nu_c}{0.6}\right)\right\rangle\right))$$
$$= N_s Zcl\left(\left\langle\left(\frac{\mu_a}{0.2},\frac{\mu_b}{0.4},\frac{\mu_c}{0.4}\right),\left(\frac{\sigma_a}{0.5},\frac{\sigma_b}{0.5},\frac{\sigma_c}{0.5}\right),\left(\frac{\nu_a}{0.8},\frac{\nu_b}{0.6},\frac{\nu_c}{0.6}\right)\right\rangle\right)$$
$$= \left\langle\left(\frac{\mu_a}{0.2},\frac{\mu_b}{0.4},\frac{\mu_c}{0.4}\right),\left(\frac{\sigma_a}{0.5},\frac{\sigma_b}{0.5},\frac{\sigma_c}{0.5}\right),\left(\frac{\nu_a}{0.8},\frac{\nu_b}{0.6},\frac{\nu_c}{0.6}\right)\right\rangle.$$

But

$$h^{-1}(N_s cl(\eta)) = h^{-1}\left(N_s cl\left(\left\langle\left(\frac{\mu_a}{0.2},\frac{\mu_b}{0.4},\frac{\mu_c}{0.4}\right),\left(\frac{\sigma_a}{0.5},\frac{\sigma_b}{0.5},\frac{\sigma_c}{0.5}\right),\left(\frac{\nu_a}{0.8},\frac{\nu_b}{0.6},\frac{\nu_c}{0.6}\right)\right\rangle\right)\right)$$
$$= h^{-1}\left(\left\langle\left(\frac{\mu_a}{0.8},\frac{\mu_b}{0.7},\frac{\mu_c}{0.6}\right),\left(\frac{\sigma_a}{0.5},\frac{\sigma_b}{0.5},\frac{\sigma_c}{0.5}\right),\left(\frac{\nu_a}{0.2},\frac{\nu_b}{0.3},\frac{\nu_c}{0.4}\right)\right\rangle\right)$$
$$= \left\langle\left(\frac{\mu_a}{0.8},\frac{\mu_b}{0.7},\frac{\mu_c}{0.6}\right),\left(\frac{\sigma_a}{0.5},\frac{\sigma_b}{0.5},\frac{\sigma_c}{0.5}\right),\left(\frac{\nu_a}{0.2},\frac{\nu_b}{0.3},\frac{\nu_c}{0.4}\right)\right\rangle.$$

Thus $N_s Zcl(h^{-1}(\eta)) \neq h^{-1}(N_s cl(\eta))$.

Theorem 15: If h is $N_s ZCts$, then $h^{-1}\big(N_s int(\mu)\big) \leq N_s Z\ int\big(h^{-1}(\mu)\big)$, for all $N_s s\mu$ in Y.

Proof: If h is $N_s ZCts$ and $\mu \in \sigma_N . N_s\ int(\mu)$ is $N_s o$ in Y and hence, $h^{-1}(N_s\ int(\mu))$ is $N_s\ Zo$ in X. Therefore $N_s\ int\left(h^{-1}\big(N_s Z\ int(\mu)\big)\right) = h^{-1}\big(N_s\ int(\mu)\big)$. Also, $N_s\ int(\mu) \leq \mu$, implies that $h^{-1}\big(N_s\ int(\mu)\big) \leq h^{-1}(\mu)$. Therefore $N_s\ Zint\left(h^{-1}\big(N_s int(\mu)\big)\right) \leq N_s Zint\big(h^{-1}(\mu)\big)$. That is $h^{-1}\big(N_s int(\mu)\big) \leq N_s Zint\ \big(h^{-1}(\mu)\big)$.

Conversely, let $h^{-1}\big(N_s int(\mu)\big) \leq N_s Zint(h^{-1}(\mu))$ for all subset μ of Y. If μ is $N_s o$ in Y, then $N_s\ int(\mu) = \mu$. By assumption, $h^{-1}\big(N_s\ int(\mu)\big) \leq N_s Zint\big(h^{-1}(\mu)\big)$. Thus $h^{-1}(\mu) \leq N_s Zint\big(h^{-1}(\mu)\big)$. But $N_s Zint\big(h^{-1}(\mu)\big) \leq h^{-1}(\mu)$. Therefore $N_s Zint\big(h^{-1}(\mu)\big) = h^{-1}(\mu)$. That is, $h^{-1}(\mu)$ is $N_s Zo$ in X, for all $N_s os\mu$ in Y. Therefore h is $N_s ZCts$ on X.

Remark 16: If h is $N_s ZCts$, then $N_s Zint(h^{-1}(\mu))$ is not necessarily equal to $h^{-1}(N_s int(\mu))$ where $\mu \in Y$.

Example 17: In Example 2, h is a $N_s ZCts$. Let $\eta = \left\langle\left(\frac{\mu_a}{0.2},\frac{\mu_b}{0.4},\frac{\mu_c}{0.4}\right),\left(\frac{\sigma_a}{0.5},\frac{\sigma_b}{0.5},\frac{\sigma_c}{0.5}\right),\left(\frac{\nu_a}{0.8},\frac{\nu_b}{0.6},\frac{\nu_c}{0.6}\right)\right\rangle$. Then

$$N_s Zint(h^{-1}(\eta)) \leq N_s Zint\left(h^{-1}\left(\left\langle\left(\frac{\mu_a}{0.2},\frac{\mu_b}{0.4},\frac{\mu_c}{0.4}\right),\left(\frac{\sigma_a}{0.5},\frac{\sigma_b}{0.5},\frac{\sigma_c}{0.5}\right),\left(\frac{\nu_a}{0.8},\frac{\nu_b}{0.6},\frac{\nu_c}{0.6}\right)\right\rangle\right)\right)$$
$$= N_s Zint\left(\left\langle\left(\frac{\mu_a}{0.2},\frac{\mu_b}{0.4},\frac{\mu_c}{0.4}\right),\left(\frac{\sigma_a}{0.5},\frac{\sigma_b}{0.5},\frac{\sigma_c}{0.5}\right),\left(\frac{\nu_a}{0.8},\frac{\nu_b}{0.6},\frac{\nu_c}{0.6}\right)\right\rangle\right)$$
$$= \left\langle\left(\frac{\mu_a}{0.2},\frac{\mu_b}{0.4},\frac{\mu_c}{0.4}\right),\left(\frac{\sigma_a}{0.5},\frac{\sigma_b}{0.5},\frac{\sigma_c}{0.5}\right),\left(\frac{\nu_a}{0.8},\frac{\nu_b}{0.6},\frac{\nu_c}{0.6}\right)\right\rangle.$$

But

$$h^{-1}\big(N_s int(\eta)\big) = h^{-1}(N_s int\ \left(\left\langle\left(\frac{\mu_a}{0.2},\frac{\mu_b}{0.4},\frac{\mu_c}{0.4}\right),\left(\frac{\sigma_a}{0.5},\frac{\sigma_b}{0.5},\frac{\sigma_c}{0.5}\right),\left(\frac{\nu_a}{0.8},\frac{\nu_b}{0.6},\frac{\nu_c}{0.6}\right)\right\rangle\right))$$
$$= h^{-1}\left(\left\langle\left(\frac{\mu_a}{0.2},\frac{\mu_b}{0.4},\frac{\mu_c}{0.4}\right),\left(\frac{\sigma_a}{0.5},\frac{\sigma_b}{0.5},\frac{\sigma_c}{0.5}\right),\left(\frac{\nu_a}{0.8},\frac{\nu_b}{0.6},\frac{\nu_c}{0.6}\right)\right\rangle\right)$$
$$= \left\langle\left(\frac{\mu_a}{0.1},\frac{\mu_b}{0.1},\frac{\mu_c}{0.4}\right),\left(\frac{\sigma_a}{0.5},\frac{\sigma_b}{0.5},\frac{\sigma_c}{0.5}\right),\left(\frac{\nu_a}{0.9},\frac{\nu_b}{0.9},\frac{\nu_c}{0.6}\right)\right\rangle.$$

Thus $N_s Zint\big(h^{-1}(\eta)\big) \neq h^{-1}\big(N_s int(\eta)\big)$.

NEUTROSOPHIC *Z*-IRRESOLUTE MAPS

Definition 18: A map $h:(X,\tau_N) \rightarrow (Y,\sigma_N)$ is called neutrosophic Z-irresolute (briefly, $N_s ZIrr$) map if $h^{-1}(\psi)$ is a $N_s Zos$ in (X,τ_N) for every $N_s Zos\psi$ in (Y,σ_N).

Theorem 19: Let $h:(X,\tau_N) \rightarrow (Y,\sigma_N)$ be a $N_s ZIrr$ then h is a $N_s ZCts$ map. But not conversely.

Proof: Let h be a $N_s Z\ Irr$ map. Let ψ be any $N_s\ os$ in Y. Since every $N_s\ os$ is a $N_s\ Zos, \psi$ is a $N_s\ Zos$ in Y. By hypothesis $h^{-1}(\psi)$ is a $N_s\ Zos$ in Y. Hence h is a $N_s\ ZCts$ map.

Example 20: Let $X = \{a,b,c\} = Y$ and define $N_s s' s X_1, X_2 \& X_3$ in X and Y_1 & Y_2 in Y are

$$X_1 = \left\langle X, \left(\frac{\mu_a}{0.2},\frac{\mu_b}{0.3},\frac{\mu_c}{0.4}\right),\left(\frac{\sigma_a}{0.5},\frac{\sigma_b}{0.5},\frac{\sigma_c}{0.5}\right),\left(\frac{\nu_a}{0.8},\frac{\nu_b}{0.7},\frac{\nu_c}{0.6}\right)\right\rangle,$$

$X_2 = \langle X, \left(\frac{\mu_a}{0.1}, \frac{\mu_b}{0.1}, \frac{\mu_c}{0.4}\right), \left(\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}\right), \left(\frac{\nu_a}{0.9}, \frac{\nu_b}{0.9}, \frac{\nu_c}{0.6}\right) \rangle$,
$X_3 = \langle X, \left(\frac{\mu_a}{0.2}, \frac{\mu_b}{0.4}, \frac{\mu_c}{0.4}\right), \left(\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}\right), \left(\frac{\nu_a}{0.8}, \frac{\nu_b}{0.6}, \frac{\nu_c}{0.6}\right) \rangle$,
$Y_1 = \langle Y, \left(\frac{\mu_a}{0.1}, \frac{\mu_b}{0.1}, \frac{\mu_c}{0.4}\right), \left(\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}\right), \left(\frac{\nu_a}{0.9}, \frac{\nu_b}{0.9}, \frac{\nu_c}{0.6}\right) \rangle$,
$Y_2 = \langle Y, \left(\frac{\mu_a}{0.1}, \frac{\mu_b}{0.4}, \frac{\mu_c}{0.5}\right), \left(\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}\right), \left(\frac{\nu_a}{0.9}, \frac{\nu_b}{0.6}, \frac{\nu_c}{0.5}\right) \rangle$.

Then we have $\tau_N = \{0_N, X_1, X_2, 1_N\}$ and $\sigma_N = \{0_N, Y_1, 1_N\}$. Let $h: (X, \tau_N) \to (Y, \sigma_N)$ be an identity mapping, then h is $N_s ZCts$ but not $N_s ZIrr$, the set Y_2 is a $N_s\ Zos$ in Y but $h^{-1}(Y_2)$ is not $N_s\ Zos$ in X.

Theorem 21: Let $h: (X, \tau_N) \to (Y, \sigma_N)$ be a $N_s\ ZIrr$ where every $N_s\ Zos$ in X is a $N_s\ os$ in X, then h is a $N_s\ Irr$ map.

Proof: Let ψ be a $N_s\ os$ in Y. Then ψ is a $N_s\ Zos$ in Y. Therefore $h^{-1}(\psi)$ is a $N_s\ Zos$ in X. By hypothesis $h^{-1}(\psi)$ is a $N_s\ os$ in X. Hence h is a $N_s\ Irr$ map.

Theorem 22: Let $h: (X, \tau_N) \to (Y, \sigma_N)$ and $g: (Y, \sigma_N) \to (Z, \rho_N)$ be a $N_s\ ZIrr$ maps, then $g \circ h : (X, \tau_N) \to (Z, \rho_N)$ is a $N_s\ ZIrr$ map.

Proof: Let ψ be a $N_s\ Zos$ in Z. Then $g^{-1}(\psi)$ is a $N_s\ Zos$ in Y. Since h is a $N_s\ ZIrr$ map, $h^{-1}(g^{-1}(\psi))$ is a $N_s\ Zos$ in X. Hence $g \circ h$ is a $N_s\ ZIrr$ map.

Theorem 23: Let $h: (X, \tau_N) \to (Y, \sigma_N)$ be $N_s\ ZIrr$ map and $g: (Y, \sigma_N) \to (Z, \rho_N)$ be a $N_s\ ZCts$ map, then $g \circ h : (X, \tau_N) \to (Z, \rho_N)$ is a $N_s\ ZCts$ map.

Proof: Let ψ be a $N_s\ os$ in Z. Then $g^{-1}(\psi)$ is a $N_s\ Zos$ in Y. Since h is a $N_s\ ZIrr$ map, $h^{-1}(g^{-1}(\psi))$ is a $N_s\ Zos$ in X. Hence $g \circ h$ is a $N_s\ ZCts$ map.

Theorem 24: Let $h: (X, \tau_N) \to (Y, \sigma_N)$ be a map from a $N_s\ tX$ into a $N_s\ t\ Y$ where every $N_s Zos$ in X is a $N_s\ os$ in both X & Y. Then the following conditions are equivalent:

(i) h is a $N_s\ ZIrr$ map.
(ii) $h^{-1}(\mu)$ is a $N_s\ Zos$ in X for each $N_s\ Zos\ \mu$ in Y.
(iii) $N_s\ cl(h^{-1}(\mu)) \subseteq h^{-1}(N_s\ cl(\mu))$ for each $N_s\ s\ \mu$ of Y.

Proof: (i) → (ii): Let μ be any $N_s\ Zos$ in Y. Then μ^c is a $N_s\ Zcs$ in Y. Since h is $N_s\ ZIrr, h^{-1}(\mu^c)$ is a $N_s\ Zcs$ in X. But $h^{-1}(\mu^c) = (h^{-1}(\mu))^c$. Therefore $h^{-1}(\mu)$ is a $N_s Zos$ in X.

(ii) → (iii): Let μ be any $N_s\ s$ in Y and $\mu \leq N_s\ cl(\mu)$. Then $h^{-1}(\mu) \leq h^{-1}(N_s\ cl(\mu))$. Since $N_s\ cl(\mu)$ is a $N_s\ cs$ in Y, $N_s\ cl(\mu)$ is a $N_s\ Zcs$ in Y. Therefore $(N_s\ cl(\mu))^c$ is a $N_s\ Zos$ in Y. By hypothesis, $h^{-1}((N_s cl(\mu)))^c$ is a $N_s\ Zos$ in X. Since $h^{-1}\left(\left(N_s cl(\mu)\right)^c\right) = \left(h^{-1}\left(N_s cl(\mu)\right)\right)^c$, $h^{-1}(N_s cl(\mu))$ is a $N_s\ Zcs$ in X. By hypothesis, $h^{-1}(N_s cl(\mu))$ is a $N_s\ cs$ in X. Hence $N_s\ cl\left(h^{-1}(\mu)\right) \subseteq N_s cl\left(h^{-1}\left(N_s cl(\mu)\right)\right) = h^{-1}\left(N_s cl(\mu)\right)$. That is $N_s cl\left(h^{-1}(\mu)\right) \subseteq h^{-1}\left(N_s cl(\mu)\right)$.

(iii) → (i): Let μ be any $N_s\ Zcs$ in Y. By hypothesis, μ is a $N_s\ cs$ in Y and $N_s\ cl(\mu) = \mu$. Hence $h^{-1}(\mu) = h^{-1}\left(N_s\ Zcl(\mu)\right) \supseteq N_s Zcl\left(h^{-1}(\mu)\right)$. But clearly $h^{-1}(\mu) \subseteq N_s cl\left(h^{-1}(\mu)\right)$. Therefore $N_s\ cl\left(h^{-1}(\mu)\right) = h^{-1}(\mu)$. This implies $h^{-1}(\mu)$ is a $N_s\ cs$ and hence it is a $N_s\ Zcs$ in X. Thus h is a $N_s\ ZIrr$ map.

REFERENCES

1. I. Arokiarani, R. Dhavaseelan, S. Jafari and M. Parimala, *On some new notions and functions in neutrosophic topological spaces*, Neutrosophic Sets and Systems, **16** (2017), 16-19.
2. K. Atanassov and S. Stoeva, *Intuitionistic fuzzy sets, in : polish syrup. on interval and fuzzy mathematics*, Poznan, (1983), 23-26.
3. K. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, **20** (1986), 87-96.
4. K. Atanassov, *Review and new results on intuitionistic fuzzy sets*, Preprint IM-MFAIS, Sofia, (1988), 1-88.
5. V. Chandrasekar, D. Sobana and A. Vadivel, *On Fuzzy e-open Sets, fuzzy e-continuity and fuzzy e-compactness in intuitionistic fuzzy topological spaces*, Sahand Communications in Mathematical Analysis (SCMA), **12(1)** (2018), 131-153.
6. C. L. Chang, *Fuzzy topological space*, J. Math. Anal. Appl., **24** (1968),182-190.
7. Erdal Ekici, *On e-open sets, $\mathcal{DP}^*$-sets and $\mathcal{DP}\ \epsilon^*$-sets and decomposition of continuity*, The Arabian Journal for Science and Engineering, **33 (2A)** (2008), 269-282.
8. N. Moogambigai, A. Vadivel and S. Tamilselvan, *Z-open sets in a Neutrosophic Topological Spaces*, Turkish Journal of Computer and Mathematics Education, **12(1S)** (2021), 357-362.

9. A. A. Salama and S. A. Alblowi, *Neutrosophic Set and Neutrosophic Topological Spaces*, IOSR Journal of Mathematics, **3(4)** (2012), 31-35.
10. A. A. Salama, F. Smarandache and S. A. Alblowi, *Characteristic function of neutrosophic set*, Neutrosophic Sets and Systems, **3** (2014), 14-17.
11. A. A. Salama, F. Smarandache and V. Kromov, *Neutrosophic closed set and neutrosophic continuous functions*, Neutrosophic Sets and Systems, **4** (2014), 4-8.
12. A. A. Salama, *Basic structure of some classes of neutrosophic crisp nearly open sets and Possible application to GIS topology*, Neutrosophic Sets and Systems, **7** (2015), 18-22.
13. V. Seenivasan and K. Kamala, *Fuzzy e-continuity and fuzzy e-open sets*, Annals of Fuzzy Mathematics and Informatics, **8** (2014), 141-148.
14. F. Smaradache, *Neutrosophic set: A generalization of the intuitionistic fuzzy sets*, Inter. J. Pure Appl. Math., **24** (2005), 287-297.
15. Supriti Saha, *Fuzzy δ-continuous mappings*, Journal of Mathematical Analysis and Applications, **126** (1987), 130-142.
16. A. Vadivel, C. John Sundar and P. Thangaraja, *Neutrosophic e-open sets in a neutrosophic topological spaces*, Submitted.
17. A. Vadivel, M. Seenivasan and C. John Sundar, *An introduction to $ \delta $-open sets in a neutrosophic topological spaces*, Journal of Physics: Conference Series, **1724** (2021), 012011.
18. A. Vadivel, P. Thangaraja and C. John Sundar, *Neutrosophic e-continuous maps and neutrosophic e-irresolute maps*, Turkish Journal of Computer and Mathematics Education, **12(1S)** (2021), 369-375.
19. V. Venkateswara Rao and Y. Srinivasa Rao, *Neutrosophic pre-open sets and pre-closed sets in Neutrosophic topology*, International Journal of Chem. Tech Research, **10(10)** (2017), 449-458.
20. L. A. Zadeh, *Fuzzy sets*, Information and control, **8** (1965), 338-353.