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Characterizations of Quasi N_{nc} e-open (closed) Functions in N_{nc} Topological Spaces

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Abstract. The purpose of this paper is to introduce and investigate several new classes of functions called, quasi $N_{nc}e$-open and quasi $N_{nc}e$-closed functions in topological spaces by using the concept of $N_{nc}e$-open sets. Also discuss their characterizations and properties in $N_{nc}ts$.

Keywords and phrases: $N_{nc}e$-open sets, quasi $N_{nc}e$-open functions and quasi $N_{nc}e$-closed functions.
AMS (2000) subject classification: 54A05, 54A10, 54C08

INTRODUCTION

Smarandache's neutrosophic system have wide range of real time applications for the fields of Computer Science, Information Systems, Applied Matheamatics, Artifical Intelligence, Mechanics, decision making, Medicine, Electrical & Electronic, and Management Science etc [1, 2, 3, 4, 16, 17]. Smarandache [12] defined the Neutrosophic set on three component Neutrosophic sets (T-Truth, F-Falsehood, I-Indeterminacy). Neutrosophic topological spaces (nts's) introduced by Salama and Alblowi [9]. Lellies Thivagar et.al. [7] was given the geometric existence of N topology, which is a non-empty set equipped with N arbitrary topologies. Lellis Thivagar et al. [8] introduced the notion of N_n-open (closed) sets and N_n topological spaces. Al-Hamido [5] explore the possibility of expanding the concept of neutrosophic crisp topological spaces into N-neutrosophic crisp topological spaces and investigate some of their basic properties. In 2008, Erdal Ekici [6] introduced a new class of generalized open sets called e-open sets and studied several fundamental and interesting properties of e-open sets and introduced a new class of continuous functions called e-continuous functions into the field of topology. In 2020, Vadivel and co-authors [14, 15] the concept of N-neutrosophic δ-open, N-neutrosophic δ-semiopen, N-neutrosophic δ-preopen and N-neutrosophic e-open sets are introduced. In this paper, we will continue the study of related functions by involving $N_{nc}e$-open sets. The aim of this paper is to introduce and investigate several new types of quasi N_{nc}-open and quasi N_{nc}-closed functions in topological spaces via $N_{nc}e$-open sets. Some characterizations and several interesting properties of these functions are discussed. Additionally, these kinds of functions have strong application in the area of Image Processing and have very important applications in quantum particle physics, high energy physics and superstring theory.

PRELIMINARIES

The definitions of neutrosophic crisp set (in short, ncs) are studied in [10, 11]. In [5], N_{nc}-topological space (briefly, $N_{nc}ts$), N_{nc}-open sets ($N_{nc}os$), N_{nc}-closed sets ($N_{nc}cs$), N_{nc} interior of H (briefly, $N_{nc}int(H)$) and N_{nc} closure of H (briefly, $N_{nc}cl(H)$) are introduced. In paper [13], a N_{nc}-regular open set (briefly, $N_{nc}ros$) is introduced. In Paper [14], a $N_{nc}\delta$ interior of H (briefly, $N_{nc}\delta int(H)$) and $N_{nc}\delta$ closure of H (briefly, $N_{nc}\delta cl(H)$), $N_{nc}\delta$- open (briefly, $N_{nc}\delta o$) set and in [15] a $N_{nc}e$-open (briefly, $N_{nc}eo$) sets are introduced. Also, a $N_{nc}\delta$ (resp. $N_{nc}e$) closed set (briefly, $N_{nc}\delta cs$ (resp. $N_{nc}ecs$)) are defined.

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Characterizations of Quasi $N_{nc}e$-open Functions

Definition 3.1 A function $h : (U, N_{nc}\tau) \to (V, N_{nc}\tau^*)$ is said to be quasi $N_{nc}e$-open (briefly, quasi $N_{nc}eO$) if the image of every $N_{nc}eo$ set in U is $N_{nc}o$ in V.

Remark 3.1

(i) It is clear that, the concepts quasi $N_{nc}e$-openness and $N_{nc}e$-continuity coincide if the function is a bijection.
(ii) It is obvious that, every quasi $N_{nc}eO$ function is $N_{nc}O$ as well as $N_{nc}eO$.

But inverse implications are not true in general as shown in the following example.

Example 3.1 Let $U = \{a_1, b_1, c_1\} = V$, ${}_{nc}\tau_1 = \{\phi_N, U_N, A, B\}$, ${}_{nc}\tau_2 = \{\phi_N, U_N\}$. $A = \langle\{a_1\}, \{\phi\}, \{b_1, c_1\}\rangle$, $B = \langle\{b_1, c_1\}, \{\phi\}, \{a_1\}\rangle$, then we have $2_{nc}\tau = \{\phi_N, U_N, A, B\}$. ${}_{nc}\sigma_1 = \{\phi_N, V_N, A\}$, ${}_{nc}\sigma_2 = \{\phi_N, V_N\}$. $A = \langle\{a_1\}, \{\phi\}, \{b_1, c_1\}\rangle$, then we have $2_{nc}\sigma = \{\phi_N, V_N, A\}$. Define $h : (U, N_{nc}\tau) \to (V, N_{nc}\tau^*)$ be an identity function. Then h is a $2_{nc}eO$ as well as $2_{nc}O$ but not quasi $2_{nc}eO$ function.

Definition 3.2 A subset A is called an $N_{nc}e$-neighborhood of a point x in U if there exists an $N_{nc}eo$ set L such that $x \in L \subseteq A$.

Theorem 3.1 *For a functions $h : (U, N_{nc}\tau) \to (V, N_{nc}\tau^*)$ then the properties*

(i) h is quasi $N_{nc}eO$,
(ii) For each subset P of U, $h(N_{nc}eint(P)) \subseteq N_{nc}int(h(P))$,
(iii) For each $x \in X$ and each $N_{nc}e$-neighborhood L of x in U, there exists a neighborhood M of $h(x)$ in V such that $M \subseteq h(L)$.

are equivalent.

Proof. *(i) $\Rightarrow$ (ii): Let h be quasi $N_{nc}eO$ and $P \subseteq X$. Now we have $N_{nc}int(P) \subseteq P$ and $N_{nc}eint(P) \in N_{nc}eOS(X)$. Hence we obtain that $h(N_{nc}eint(P)) \subseteq h(P)$. Since $h(N_{nc}eint(P))$ is $N_{nc}o$, then $h(N_{nc}eint(P)) \subseteq N_{nc}int(h(P))$.*

(ii) $\Rightarrow$ (iii): Let $x \in X$ and L be an $N_{nc}e$-neighborhood of x in U. Then there exists $M \in N_{nc}eOS(X) \ni x \in M \subseteq L$. Then by (ii), we have, $h(M) = h(N_{nc}eint(M) \subseteq N_{nc}int(h(M)))$ and hence $h(M) = N_{nc}int(h(M))$. Therefore, it is follow that $h(M)$ is $N_{nc}o$ in $V \ni h(x) \in h(M) \subseteq h(L)$.

(iii) $\Rightarrow$ (i): Let $L \in N_{nc}eOS(X)$. Then $\forall\ y \in h(L)$, there exists a neighborhood M_y of y in $V \ni M_y \subseteq h(L)$. Since M_y is a neighborhood of y, there exists an $N_{nc}o$ set N_y in $V \ni y \in N_y \subseteq M_y$. Thus, $h(L) = \bigcup\{N_y : y \in h(L)\}$ which is an $N_{nc}o$ set in V. This implies that h is quasi $N_{nc}eO$ function.

Theorem 3.2 *A function $h : (U, N_{nc}\tau) \to (V, N_{nc}\tau^*)$ is quasi $N_{nc}eO$ iff $N_{nc}eint(h^{-1}(Q)) \subseteq h^{-1}(N_{nc}int(Q))$ for every subset Q of V.*

Proof. *Let Q be any subset of V. Then, $N_{nc}eint(h^{-1}(Q)) \in N_{nc}eOS(X)$ and h is quasi $N_{nc}eO$, then $h(N_{nc}eint(h^{-1}(Q))) \subseteq N_{nc}int(h(h^{-1}(Q))) \subseteq N_{nc}int(Q)$. Thus, $N_{nc}eint(h^{-1}(Q)) \subseteq h^{-1}(N_{nc}int(Q))$.*

Conversely, let $L \in N_{nc}eOS(X)$. Then by assumption $N_{nc}eint(h^{-1}(h(L))) \subseteq h^{-1}(N_{nc}int(h(L)))$ then, $N_{nc}eint(L) \subseteq h^{-1}(N_{nc}int(h(L)))$, but $N_{nc}eint(L) = L$ so $L \subseteq h^{-1}(N_{nc}int(h(L)))$ and hence $h(L) \subseteq N_{nc}int(h(L))$ so h is quasi $N_{nc}eO$.

Theorem 3.3 *A function $h : (U, N_{nc}\tau) \to (V, N_{nc}\tau^*)$ is quasi $N_{nc}eO$ iff for any subset Q of V and for any set $M \in N_{nc}eCS(X)$ containing $h^{-1}(Q)$, there exists a $N_{nc}c$ set F of V containing $Q \ni h^{-1}(F) \subseteq M$.*

Proof. *Let h be quasi $N_{nc}eO$ and $Q \subseteq Y$. Let $M \in N_{nc}eCS(X)$ with $h^{-1}(Q) \subseteq M$. Now, put $F = Y - h(X - M)$. It is clear that since $h^{-1}(Q) \subseteq M$, $Q \subseteq F$. Since h is quasi $N_{nc}eO$, F is a $N_{nc}c$ set of V. Also, we have $h^{-1}(F) \subseteq M$.*

Conversely, let $L \in N_{nc}eOS(X)$ and put $Q = Y - h(L)$. Then $X - L \in N_{nc}eCS(X)$ with $h^{-1}(Q) \subseteq X - L$. By assumption, there exists a $N_{nc}c$ set F of $V \ni Q \subseteq F$ and $h^{-1}(F) \subseteq X - L$. Hence, we obtain $h(L) \subseteq Y - F$. On the other hand, it follows that $Q \subseteq F$, $Y - F \subseteq Y - Q = h(L)$. Thus, we have $h(L) = Y - F$ which is $N_{nc}o$ and hence h is a quasi $N_{nc}eO$ function.

Theorem 3.4 *A function $h : (U, N_{nc}\tau) \to (V, N_{nc}\tau^*)$ is quasi $N_{nc}eO$ iff $h^{-1}(N_{nc}cl(Q)) \subseteq N_{nc}ecl(h^{-1}(Q))$ for every subset Q of V.*

Proof. *Suppose that h is quasi $N_{nc}eO$ function. For any subset Q of V, $h^{-1}(Q) \subseteq N_{nc}ecl(h^{-1}(Q))$. Therefore by Theorem 2, there exists a $N_{nc}c$ set F in $V \ni Q \subseteq F$ & $h^{-1}(F) \subseteq N_{nc}ecl(h^{-1}(Q))$. Therefore, we obtain, $h^{-1}(N_{nc}cl(Q)) \subseteq h^{-1}(F) \subseteq N_{nc}ecl(h^{-1}(Q))$.*

Conversely, let Q be any subset of V and $M \in N_{nc}eCS(X)$ with $h^{-1}(Q) \subseteq M$. Put $F = cl(Q)$, then we have $Q \subseteq F$ & F is $N_{nc}c$ and $h^{-1}(F) \subseteq N_{nc}ecl(h^{-1}(Q)) \subseteq M$. Then by Theorem 2, the function h is quasi $N_{nc}eO$.

Lemma 1.1 *Let $h : (U, N_{nc}\tau) \to (V, N_{nc}\tau^*)$ and $g : (V, N_{nc}\tau^*) \to (W, N_{nc}\tau^{**})$ be two functions and $g \circ h : (U, N_{nc}\tau) \to (W, N_{nc}\tau^{**})$ is quasi $N_{nc}eO$. If g is $N_{nc}Cts$ injective, then h is quasi $N_{nc}eO$.*

Proof. *Let L be a $N_{nc}eo$ set in U, then $(g \circ h)(L)$ is $N_{nc}o$ in Z since $g \circ h$ is quasi $N_{nc}eO$. Again g is an injective $N_{nc}Cts$ function, $h(L) = g^{-1}(g \circ h(L))$ is $N_{nc}o$ in V. This shows that h is quasi $N_{nc}eO$.*

Theorem 3.4 *If $h : (U, N_{nc}\tau) \to (V, N_{nc}\tau^*)$ is quasi $N_{nc}eO$ bijective. Then the properties*

(i) If $(U, N_{nc}\tau)$ is $N_{nc}e$-T_1 then $(V, N_{nc}\tau^*)$ is $N_{nc}T_1$.
(ii) If $(U, N_{nc}\tau)$ is $N_{nc}e$-T_2 then $(V, N_{nc}\tau^*)$ is $N_{nc}T_2$.

are hold.

Proof.*(i) Let m_1 and m_2 be any distinct points in V. Then there exist l_1 & l_2 in $U \ni h(l_1) = m_1$ & $h(l_2) = m_2$. Since U is $N_{nc}eT_1$ then, there exist two $N_{nc}eo$ sets L & M in U with $l_1 \in L$, $l_2 \notin L$ & $l_2 \in M$, $l_1 \notin M$. Now $h(L)$ & $h(M)$ are $N_{nc}o$ in V with $m_1 \in h(L)$, $m_2 \notin h(L)$ & $m_2 \in h(M)$, $m_1 \notin h(M)$.*

(ii) It is similar to (i). Thus is omitted.

Theorem 3.5 If $h : (U, N_{nc}\tau) \to (V, N_{nc}\tau^*)$ is quasi $N_{nc}eO$ bijective. Then the properties

(i) If $(V, N_{nc}\tau^*)$ is compact, then $(U, N_{nc}\tau)$ is $N_{nc}e$-compact.
(ii) If $(V, N_{nc}\tau^*)$ is Lindelöf, then $(U, N_{nc}\tau)$ is $N_{nc}e$-Lindelöf.

are hold.

Proof. (i) Let $L_1 = \{L_\lambda : \lambda \in \Delta\}$ be an $N_{nc}eo$ cover of U. Then $K_1 = \{h(L_\lambda) : \lambda \in \Delta\}$ is a cover of V by $N_{nc}o$ sets in V. Since V is N_{nc}-compact, Then K_1 has a N_{nc} finite subcover $K_2 = \{h(L_{\lambda_1}), h(L_{\lambda_2}), \cdots, h(L_{\lambda_n})\}$ for V. Then $L_2 = \{L_{\lambda_1}, L_{\lambda_2}, \cdots, L_{\lambda_n}\}$ is a N_{nc} finite subcover of L for U.

(ii) It is similar to (i). Thus is omitted.

Theorem 3.6 *If $h : (U, N_{nc}\tau) \to (V, N_{nc}\tau^*)$ is quasi $N_{nc}eO$ surjective and V is N_{nc} connected. Then U is $N_{nc}e$-connected.*

Proof. *Suppose that U is not $N_{nc}e$-connected. Then there exist two non-empty disjoint $N_{nc}eo$ sets L & M in $U \ni X = L \cup M$. Then $h(L)$ & $h(M)$ are non-empty disjoint $N_{nc}o$ sets in V with $Y = h(L) \cup h(M)$ which contradicts the fact that V is $N_{nc}e$-connected.*

Definition 3.3 *A function $h : (U, N_{nc}\tau) \to (V, N_{nc}\tau^*)$ is called pre-$N_{nc}e$ open (briefly, pre-$N_{nc}eO$) if the image of each $N_{nc}eo$ set of U is an $N_{nc}eo$ set in V.*

Definition 3.4 *A $N_{nc}ts$ $(U, N_{nc}\tau)$ is said to be a $N_{nc}T_e$-space if every $N_{nc}eo$ set of $(U, N_{nc}\tau)$ is $N_{nc}o$ in $(U, N_{nc}\tau)$.*

Remark 3.2 *Let $h : (U, N_{nc}\tau) \to (V, N_{nc}\tau^*)$ be a quasi $N_{nc}eO$ function. If V is a $N_{nc}T_e$-space, then quasi $N_{nc}e$-openness coincide with pre-$N_{nc}e$-openness.*

Definition 3.5 *A function $h : (U, N_{nc}\tau) \to (V, N_{nc}\tau^*)$ is said to be $N_{nc}e$-irresolute if $h^{-1}(M)$ is $N_{nc}eo$ in U for every $N_{nc}eo$ set M of V.*

Theorem 3.7 *Let $h : (U, N_{nc}\tau) \to (V, N_{nc}\tau^*)$ and $g : (V, N_{nc}\tau^*) \to (W, N_{nc}\tau^{**})$ be two functions such that $g \circ h : (U, N_{nc}\tau) \to (W, N_{nc}\tau^{**})$ is quasi $N_{nc}eO$.*

(i) If h is $N_{nc}eIrr$ surjective, then g is $N_{nc}O$.
(ii) If g is $N_{nc}eCts$ injective, then h is pre-$N_{nc}eO$.

Proof. *(i) Suppose that $M \in N_{nc}eOS(Y)$. Since h is $N_{nc}eIrr$, then $h^{-1}(M)$ is $N_{nc}eO$ in $(X, N_{nc}\tau)$. Since $g \circ h$ is quasi $N_{nc}eO$ and h is surjective, $(g \circ h(h^{-1}(M))) = g(M)$, which is $N_{nc}o$ in $(W, N_{nc}\tau^{**})$. This implies that g is an $N_{nc}O$ function.*

(ii) Suppose that $M \in N_{nc}eOS(X)$. Since $g \circ h$ is quasi $N_{nc}eO$, $(g \circ h)(M)$ is $N_{nc}o$ in $(W, N_{nc}\tau^{**})$. Again g is a $N_{nc}eCts$ injective function, $g^{-1}(g \circ h(M)) = h(M)$, which is $N_{nc}eo$ in $(V, N_{nc}\tau^*)$. This shows that h is pre $N_{nc}eo$.

Characterizations of Quasi $N_{nc}e$-closed Functions

Definition 4.1 *A function $h : (U, N_{nc}\tau) \to (V, N_{nc}\tau^*)$ is said to be quasi $N_{nc}e$-closed (briefly quasi $N_{nc}eC$) if the image of every $N_{nc}ec$ set in U is $N_{nc}c$ in V.*

Remark 4.1 *Clearly, every quasi $N_{nc}eC$ function is $N_{nc}C$ as well as $N_{nc}eC$, but not converse shown in Example 2.*

Theorem 4.1 *If a function $h : (U, N_{nc}\tau) \to (V, N_{nc}\tau^*)$ is quasi $N_{nc}eC$. Then, $h^{-1}(N_{nc}int(Q)) \subseteq N_{nc}eint(h^{-1}(Q))$ for every subset Q of V.*

Proof. *Similar to the proof of Theorem 2.*

Theorem 4.2 *A function $h : (U, N_{nc}\tau) \rightarrow (V, N_{nc}\tau^*)$ is quasi $N_{nc}eC$ iff for any subset Q of V and for any set $L \in N_{nc}eOS(U)$ containing $h^{-1}(Q)$, there exists an $N_{nc}o$ set M of V containing $Q \ni h^{-1}(M) \subseteq L$.*

Proof. *Similar to the proof of Theorem 2.*

Definition 4.2 *A function $h : (U, N_{nc}\tau) \rightarrow (V, N_{nc}\tau^*)$ is called pre-$N_{nc}e$-closed (briefly, pre-$N_{nc}eC$) if the image of each $N_{nc}ec$ set of $(U, N_{nc}\tau)$ is an $N_{nc}ec$ set in $(V, N_{nc}\tau^*)$.*

Definition 4.3 *A space U is said to be a $N_{nc}C_e$-space if every $N_{nc}ec$ set in U is $N_{nc}c$ in U.*

Remark 4.2 *Let $h : (U, N_{nc}\tau) \rightarrow (V, N_{nc}\tau^*)$ be a quasi $N_{nc}eC$ function. If V is $N_{nc}C_e$-space, then $N_{nc}e$-closedness coincides with pre-$N_{nc}e$-closedness.*

Theorem 4.3 *Let $h : (U, N_{nc}\tau) \rightarrow (V, N_{nc}\tau^*)$ and $g : (V, N_{nc}\tau^*) \rightarrow (W, N_{nc}\tau{**})$ be any two functions. Then:*

- *(i) If h is quasi $N_{nc}eC$ and g is quasi $N_{nc}eC$, then $g \circ h$ is quasi $N_{nc}eC$,*
- *(ii) If h is $N_{nc}eC$ and g is quasi $N_{nc}eC$, then $g \circ h$ is $N_{nc}C$,*
- *(iii) If h is quasi $N_{nc}eC$ and g is $N_{nc}eC$, then $g \circ h$ is pre-$N_{nc}eC$,*
- *(iv) If h is pre-$N_{nc}eC$ and g is quasi $N_{nc}eC$, then $g \circ h$ is quasi $N_{nc}eC$.*

Theorem 4.4 Let $(U, N_{nc}\tau)$ and $(V, N_{nc}\tau^*)$ be $N_{nc}ts$'s. Then the function $h : (U, N_{nc}\tau) \rightarrow (V, N_{nc}\tau^*)$ is a quasi $N_{nc}eC$ iff $h(X)$ is $N_{nc}c$ in V and $h(M)\backslash h(U\backslash M)$ is $N_{nc}o$ in $h(U)$ whenever M is $N_{nc}eo$ in U.

Proof. Suppose $h : (U, N_{nc}\tau) \rightarrow (V, N_{nc}\tau^*)$ is a quasi $N_{nc}eC$ function. Since $(U, N_{nc}\tau)$ is $N_{nc}ec$, $h(U)$ is $N_{nc}c$ in $(V, N_{nc}\tau^*)$ and $h(M)\backslash h(U\backslash M) = h(U)\backslash h(U\backslash M)$ is $N_{nc}o$ in $h(X)$ when M is $N_{nc}eo$ in $(U, N_{nc}\tau)$.

Conversely, suppose $h(U)$ is $N_{nc}c$ in $(V, N_{nc}\tau^*)$, $h(M)\backslash h(U\backslash M)$ is $N_{nc}o$ in $h(U)$ when M is $N_{nc}eo$ in U, and let K be $N_{nc}c$ in U. Then, $h(K) = h(U)\backslash(h(U\backslash K)\backslash h(K))$ is $N_{nc}c$ in $h(U)$ and hence, $N_{nc}c$ in $(V, N_{nc}\tau^*)$.

Corollary 4.1 Let $(U, N_{nc}\tau)$ and $(V, N_{nc}\tau^*)$ be $N_{nc}ts$'s. Then a surjective function $h : (U, N_{nc}\tau) \rightarrow (V, N_{nc}\tau^*)$ is quasi $N_{nc}eC$ iff $h(M)\backslash h(U\backslash M)$ is $N_{nc}o$ in $(V, N_{nc}\tau^*)$ whenever L is $N_{nc}eo$ in $(U, N_{nc}\tau)$.

Corollary 4.2 Let $(U, N_{nc}\tau)$ and $(V, N_{nc}\tau^*)$ be $N_{nc}ts$'s and let $h : (U, N_{nc}\tau) \rightarrow (V, N_{nc}\tau^*)$ be an $N_{nc}eCts$ quasi $N_{nc}eC$ surjective function. Then the topology on V is $\{h(M)\backslash h(U\backslash M) :\ M$ is $N_{nc}eo$ in $U\}$.

Proof. Let H be $N_{nc}o$ in V. Then $h^{-1}(H)$ is $N_{nc}eo$ in U, and $h(h^{-1}(H))\backslash h(U\backslash h^{-1}(H)) = H$. Hence, all $N_{nc}o$ sets in V are of the form $h(M)\backslash h(U\backslash M)$, M is $N_{nc}eo$ in U. On the other hand, all sets of the form $h(M)\backslash h(U\backslash M)$, M is $N_{nc}eo$ in U, are $N_{nc}o$ in V from Corollary 2.

Definition 4.4 A $N_{nc}ts$ $(U, N_{nc}\tau)$ is said to be $N_{nc}e$-normal (briefly, $N_{nc}eNor$) if for any pair of disjoint $N_{nc}ec$ sets K_1 & K_2 of U, there exist disjoint $N_{nc}o$ sets L & $M \ni K_1 \subseteq L$ & $K_2 \subseteq M$.

Theorem 4.5 Let $(U, N_{nc}\tau)$ and $(V, N_{nc}\tau^*)$ be $N_{nc}ts$'s with U is $N_{nc}eNor$ and let $h : (U, N_{nc}\tau) \rightarrow (V, N_{nc}\tau^*)$ be an $N_{nc}eCts$ quasi $N_{nc}eC$ surjective function. Then V is $N_{nc}Nor$.

Proof. Let K_1 and K_2 be disjoint $N_{nc}c$ sets of V. Then $h^{-1}(K_1)$, $h^{-1}(K_2)$ are disjoint $N_{nc}ec$ subsets of U. Since U is $N_{nc}eNor$, there exist disjoint $N_{nc}o$ sets M_1 & $M_2 \ni h^{-1}(K_1) \subseteq M_1$ & $h^{-1}(K_2) \subseteq M_2$, then $K_1 \subseteq h(M_1)\backslash h(X\backslash M_1)$ & $K_2 \subseteq h(M_2)\backslash h(X\backslash M_2)$. Further by Corollary 2, $h(M_1)\backslash h(X\backslash M_1)$ & $h(M_2)\backslash h(X\backslash M_2)$ are $N_{nc}o$ sets in V and clearly $(h(M_1)\backslash h(U\backslash M_1)) \cap (h(M_2)\backslash h(U\backslash M_2)) = \phi$. This shows that V is $N_{nc}Nor$.

CONCLUSION

In this paper we introduced and investigated the notions of new classes of functions several new types of quasi N_{nc}-open and quasi N_{nc}-closed functions in topological spaces via $N_{nc}e$-open sets. Some characterizations and several interesting properties of these functions are discussed.

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