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Cite as: AIP Conference Proceedings **2364**, 020022 (2021); https://doi.org/10.1063/5.0062886
Published Online: 23 September 2021

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Abstract. A new class of functions, called strongly faint $N_{nc}e$-continuous function, has been defined and studied. Relationships among strongly faint $N_{nc}e$-continuous functions and $N_{nc}e$-connected spaces, $N_{nc}e$-normal spaces and $N_{nc}e$-compact spaces are investigated. Furthermore, the relationships between strongly faint $N_{nc}e$-continuous functions and graphs are also investigated.

Keywords and phrases: $N_{nc}\theta$-open sets, $N_{nc}e$-open sets, strongly $N_{nc}\theta$-continuity, strongly faint $N_{nc}e$-continuity.
AMS (2000) subject classification: 54B05, 54C08, 54D10

INTRODUCTION

Smarandache's neutrosophic system have wide range of real time applications for the fields of Computer Science, Information Systems, Applied Matheamatics, Artifical Intelligence, Mechanics, decision making, Medicine, Electrical & Electronic, and Management Science etc [1, 2, 3, 4, 27, 28]. Smarandache [20] defined the Neutrosophic set on three component Neutrosophic sets (T-Truth, F-Falsehood, I-Indeterminacy). Salama and Alblowi [17] introduced Neutrosophic topological spaces (nts's) . Lellis Thivagar et.al. [12] was given the geometric existence of N topology, which is a non-empty set equipped with N arbitrary topologies. Also in [13] the notion of N_n-open (closed) sets and N_n continuous in N neutrosophic crisp topological spaces are introduced. Al-Hamido et al. [5] explore the possibility of expanding the concept of neutrosophic crisp topological spaces into N-neutrosophic crisp topological spaces and investigate some of their basic properties. In 2020, Vadivel and co-author [23, 24] the concept of N-neutrosophic δ-open, N-neutrosophic δ-semiopen, N-neutrosophic δ-preopen and $N_{nc}e$ open sets are introduced. The first step of generalizing closed set was done by Levine [14] in 1970. Ekici [6, 7, 8, 9, 10, 11] introduced and studied a generalization of closed sets, e-closed sets and the related sets. Nasef and Noiri [16] introduce three classes of strong forms of faintly continuity namely: strongly faint semicontinuity, strongly faint precontinuity and strongly faint β-continuity. In 2009, Nasef [15] defined strong forms of faint continuity under the terminologies strongly faint α-continuity and strongly faint γ-continuity. In this paper using $N_{nc}e$-open sets, strongly faint $N_{nc}e$-continuity is introduced and studied with basic properties and theorems. Also a relationships between strongly faint $N_{nc}e$-continuous functions and graphs are investigated.

PRELIMINARIES

The definitions of neutrosophic crisp set (in short, ncs) are studied in [18, 19]. In [5], N_{nc}-topological space (briefly, $N_{nc}ts$), N_{nc}-open sets ($N_{nc}os$), N_{nc}-closed sets ($N_{nc}cs$), N_{nc} interior of H (briefly, $N_{nc}int(H)$) and N_{nc} closure of H (briefly, $N_{nc}cl(H)$) are introduced. Also, N_{nc}-regular open [21] set (briefly, $N_{nc}ros$), N_{nc}-pre open set (briefly, $N_{nc}\mathcal{P}os$), N_{nc}-semi open set (briefly, $N_{nc}\mathcal{S}os$), N_{nc}-α-open set (briefly, $N_{nc}\alpha os$), N_{nc}-β-open set [22] (briefly, $N_{nc}\beta os$), N_{nc}-γ-open set[21] (briefly, $N_{nc}\gamma os$) are described. The complement of an $N_{nc}ros$ (resp. $N_{nc}\mathcal{S}os$, $N_{nc}\mathcal{P}os$, $N_{nc}\alpha os$, $N_{nc}\beta os$

International Conference on Recent Trends in Applied Mathematical Sciences (ICRTAMS-2020)
AIP Conf. Proc. 2364, 020022-1–020022-7; https://doi.org/10.1063/5.0062886
Published by AIP Publishing. 978-0-7354-4122-4/$30.00

& $N_{nc}\gamma os$) is called an N_{nc}-regular (resp. N_{nc}-semi, N_{nc}-pre, N_{nc}-α, N_{nc}-β & N_{nc}-γ) closed set (briefly, $N_{nc}rcs$ (resp. $N_{nc}\mathcal{S}cs$, $N_{nc}\mathcal{P}cs$, $N_{nc}\alpha cs$, $N_{nc}\beta cs$ & $N_{nc}\gamma cs$)). $N_{nc}\theta$-cluster (resp. $N_{nc}\delta$-cluster) point, $N_{nc}\theta$-closure (resp. $N_{nc}\delta$-closure) denoted by $N_{nc}\theta cl(H)$ (resp. $N_{nc}\delta cl(H)$), $N_{nc}\theta int(H)$ and also $N_{nc}\theta c$ (resp. $N_{nc}\delta c$), $N_{nc}\theta o$ and $N_{nc}\delta o$ [23]. In [23]$N_{nc}\delta$ interior of H (briefly, $N_{nc}\delta int(H)$), $N_{nc}\delta$ closure of H (briefly, $N_{nc}\delta cl(H)$), $N_{nc}\delta$- open set (briefly, $N_{nc}\delta os$), $N_{nc}\delta$-pre open set (briefly, $N_{nc}\delta\mathcal{P}os$), $N_{nc}\delta$-semi open set (briefly, $N_{nc}\delta\mathcal{S}os$) and $N_{nc}e$-open set (briefly, $N_{nc}eo$). Also, $N_{nc}\delta$ (resp. $N_{nc}\delta$-pre, $N_{nc}\delta$-semi & $N_{nc}e$) closed set (briefly, $N_{nc}\delta cs$ (resp. $N_{nc}\delta\mathcal{P}cs$, $N_{nc}\delta\mathcal{S}cs$ & $N_{nc}ecs$)). A $N_{nc}e$-continuous [25] (briefly. $N_{nc}eCts$) and $N_{nc}e$-irresolute [26] (briefly. $N_{nc}eIrr$) are also used in this paper.

Strongly Faint $N_{nc}e$-continuous Functions

Definition 3.1 A function $h : (U, N_{nc}\tau) \to (V, N_{nc}\sigma)$ is said to be

(i) strongly faint $N_{nc}e$ (resp. N_{nc} semi, N_{nc} pre, $N_{nc}\beta$, $N_{nc}\alpha$ and $N_{nc}\gamma$)-continuous (briefly, $sfN_{nc}eCts$, $sfN_{nc}\mathcal{S}Cts$, $sfN_{nc}\mathcal{P}Cts$, $sfN_{nc}\beta Cts$, $sfN_{nc}\alpha Cts$ and $sfN_{nc}\gamma Cts$)) if $\forall\ x \in U$ and each $N_{nc}eo$ (resp. $N_{nc}\mathcal{S}o$, $N_{nc}\mathcal{P}o$, $N_{nc}\beta o$, $N_{nc}\alpha o$ and $N_{nc}\gamma o$) set M of V containing $h(x)$, there exists a $N_{nc}\theta o$ set L of U containing x such that $h(L) \subseteq M$.

(ii) strongly $N_{nc}e$ (resp. $N_{nc}\theta$)-continuous (briefly, $sN_{nc}eCts$ (resp. $sN_{nc}\theta Cts$)) if $h^{-1}(M)$ is $N_{nc}o$ in U for every $N_{nc}eo$ (resp. $N_{nc}\theta o$) set M of V.

(iii) quasi $N_{nc}\theta$-continuous (briefly. $qN_{nc}\theta Cts$), if $h^{-1}(M)$ is $N_{nc}\theta o$ in U for every $N_{nc}\theta o$ set M of V.

Theorem 3.1 For a function $h : (U, N_{nc}\tau) \to (V, N_{nc}\sigma)$, the statements

(i) h is $sfN_{nc}eCts$,
(ii) $h : (U, N_{nc}\tau) \to (V, N_{nc}\sigma)$ is $sN_{nc}eCts$,
(iii) $h^{-1}(M)$ is $N_{nc}\theta o$ in U $\forall$ $N_{nc}eo$ set M of V,
(iv) $h^{-1}(M)$ is $N_{nc}\theta c$ in U $\forall$ $N_{nc}ec$ set M of V

are equivalent.

Proof. (i) $\Rightarrow$ (iii): Let M be an $N_{nc}eo$ set of V and $x \in h^{-1}(M)$. Since $h(x) \in M$ and h is $sfN_{nc}eCts$, there exists a $N_{nc}\theta o$ set L of U containing $x \ni h(L) \subseteq M$. It follows that $x \in L \in h^{-1}(M)$. Hence $h^{-1}(M)$ is $N_{nc}\theta o$ in U.

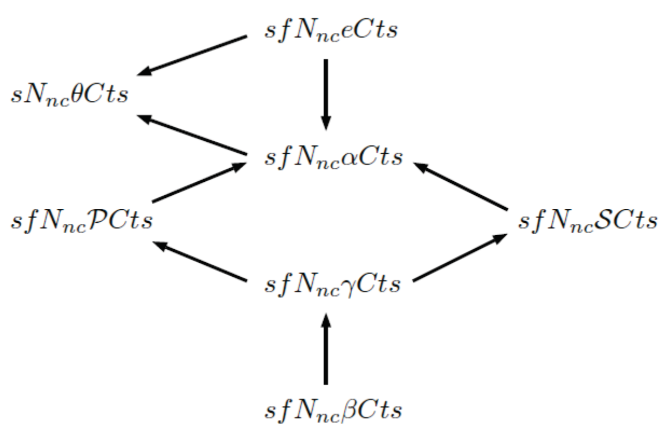
(iii) $\Rightarrow$ (i): Let $x \in X$ and M be an $N_{nc}eo$ set of V containing $h(x)$. By (iii), $h^{-1}(M)$ is a $N_{nc}\theta o$ set containing x. Take $L = h^{-1}(M)$. Then $h(L) \subseteq M$. Thus h is $sfN_{nc}eCts$.

(iii) $\Rightarrow$ (iv): Let M be any $N_{nc}ec$ set of V. Since $Y\backslash M$ is an $N_{nc}eo$ set, by (iii), it follows that $h^{-1}(Y\backslash M) = X\backslash h^{-1}(M)$ is $N_{nc}\theta o$. Thus $h^{-1}(M)$ is $N_{nc}\theta c$ in X.

(iv) $\Rightarrow$ (iii): Let M be an $N_{nc}eo$ set of V. Then $Y\backslash M$ is $N_{nc}ec$ in V. By (iv), $h^{-1}(Y\backslash M) = X\backslash h^{-1}(M)$ is $N_{nc}\theta c$ and thus $h^{-1}(M)$ is $N_{nc}\theta o$.

(i) $\Rightarrow$ (ii): Clear.

The relationships between this new class of functions are shown in the following diagram but reversible is not true.



Example 3.1 Let $X = \{a_1, b_1, c_1\} = Y$, ${}_{nc}\tau_1 = \{\phi_N, X_N, A, B, C\}$, ${}_{nc}\tau_2 = \{\phi_N, X_N, D\}$. $A = \langle\{a_1\}, \{\phi\}, \{b_1, c_1\}\rangle$, $B = \langle\{b_1\}, \{\phi\}, \{a_1, c_1\}\rangle$, $C = \langle\{a_1, b_1\}, \{\phi\}, \{c_1\}\rangle$, $D = \langle\{a_1, c_1\}, \{\phi\}, \{b_1\}\rangle$, then we have $2_{nc}\tau = \{\phi_N, X_N, A, B, C, D\}$ and ${}_{nc}\sigma_1 = \{\phi_N, Y_N, E\}$, ${}_{nc}\tau_2 = \{\phi_N, Y_N\}$, $E = \langle\{a_1, c_1\}, \{\phi\}, \{b_1\}\rangle$, then we have $2_{nc}\sigma = \{\phi_N, X_N, E\}$. Define $h : (X, 2_{nc}\tau) \to (Y, 2_{nc}\sigma)$ as an identity map, then h is $sf2_{nc}\mathcal{S}Cts$ (resp. $sf2_{nc}\alpha Cts$) but not $sf2_{nc}\gamma Cts$ (resp. $sf2_{nc}\mathcal{P}Cts$).

Example 3.2 Let $X = \{a_1, b_1, c_1, d_1\} = Y$, ${}_{nc}\tau_1 = \{\phi_N, X_N, A, B, C\}$, ${}_{nc}\tau_2 = \{\phi_N, X_N, D, E, F\}$. $A = \langle\{a_1\}, \{\phi\}, \{b_1, c_1, d_1\}\rangle$, $B = \langle\{c_1\}, \{\phi\}, \{a_1, b_1, d_1\}\rangle$, $C = \langle\{a_1, c_1\}, \{\phi\}, \{b_1, d_1\}\rangle$, $D = \langle\{b_1, d_1\}, \{\phi\}, \{a_1, c_1\}\rangle$, $E = \langle\{a_1, b_1, d_1\}, \{\phi\}, \{c_1\}\rangle$, $F = \langle\{b_1, c_1, d_1\}, \{\phi\}, \{a_1\}\rangle$, then we have $2_{nc}\tau = \{\phi_N, X_N, D, E, F, A, B, C\}$ and ${}_{nc}\sigma_1 = \{\phi_N, Y_N, G, H, I\}$, ${}_{nc}\tau_2 = \{\phi_N, Y_N\}$, $G = \langle\{a_1\}, \{\phi\}, \{b_1, c_1, d_1\}\rangle$, $H = \langle\{b_1, d_1\}, \{\phi\}, \{a_1, c_1\}\rangle$, $I = \langle\{a_1, b_1, d_1\}, \{\phi\}, \{c_1\}\rangle$, then we have $2_{nc}\sigma = \{\phi_N, X_N, G, H, I\}$. Define $h : (X, 2_{nc}\tau) \to (Y, 2_{nc}\sigma)$ as an identity map, then h is $sf2_{nc}\alpha Cts$ (resp. $s2_{nc}\theta Cts$) but not $sf2_{nc}eCts$ (resp. $sf2_{nc}eCts$).

Example 3.3 Let $X = \{a_1, b_1, c_1\}$, ${}_{nc}\tau_1 = \{\phi_N, X_N, A, B, C\}$, ${}_{nc}\tau_2 = \{\phi_N, X_N, D\}$. $A = \langle\{a_1\}, \{\phi\}, \{b_1, c_1\}\rangle$, $B = \langle\{b_1\}, \{\phi\}, \{a_1, c_1\}\rangle$, $C = \langle\{a_1, b_1\}, \{\phi\}, \{c_1\}\rangle$, $D = \langle\{a_1, c_1\}, \{\phi\}, \{b_1\}\rangle$, then we have $2_{nc}\tau = \{\phi_N, X_N, D, C, B, A\}$. Define $h : (X, 2_{nc}\tau) \to (X, 2_{nc}\tau)$ as an identity map, then h is $s2_{nc}\theta Cts$ but not $sf2_{nc}\alpha Cts$.

Example 3.4 Let $X = \{a_1, b_1, c_1\}$, ${}_{nc}\tau_1 = \{\phi_N, X_N, A, B, C\}$, ${}_{nc}\tau_2 = \{\phi_N, X_N, D\}$. $A = \langle\{a_1\}, \{\phi\}, \{b_1, c_1\}\rangle$, $B = \langle\{b_1\}, \{\phi\}, \{a_1, c_1\}\rangle$, $C = \langle\{a_1, b_1\}, \{\phi\}, \{c_1\}\rangle$, $D = \langle\{a_1, c_1\}, \{\phi\}, \{b_1\}\rangle$, then we have $2_{nc}\tau = \{\phi_N, X_N, C, D, A, B\}$ and $Y = \{a_1, b_1, c_1, d_1\}$, ${}_{nc}\sigma_1 = \{\phi_N, Y_N, G, H, I\}$, ${}_{nc}\tau_2 = \{\phi_N, Y_N\}$, $G = \langle\{a_1\}, \{\phi\}, \{b_1, c_1, d_1\}\rangle$, $H = \langle\{b_1, c_1\}, \{\phi\}, \{a_1, d_1\}\rangle$, $I = \langle\{a_1, b_1, c_1\}, \{\phi\}, \{d_1\}\rangle$, then we have $2_{nc}\sigma = \{\phi_N, X_N, G, H, I\}$. Define $h : (X, 2_{nc}\tau) \to (Y, 2_{nc}\sigma)$ as $h(a_1) = b_1$, $h(b_1) = a_1$, $h(c_1) = c_1$ & $h(d_1) = d_1$, then h is $sf2_{nc}\alpha Cts$ (resp. $sf2_{nc}\mathcal{P}Cts$) but not $sf2_{nc}\mathcal{S}Cts$ (resp. $sf2_{nc}\gamma Cts$).

Example 3.5 Let $X = \{a_1, b_1, c_1\} = Y$, ${}_{nc}\tau_1 = \{\phi_N, X_N, A, B, C\}$, ${}_{nc}\tau_2 = \{\phi_N, X_N, D, E, F\}$. $A = \langle\{a_1\}, \{\phi\}, \{b_1, c_1\}\rangle$, $B = \langle\{b_1\}, \{\phi\}, \{a_1, c_1\}\rangle$, $C = \langle\{a_1, b_1\}, \{\phi\}, \{c_1\}\rangle$, $D = \langle\{c_1\}, \{\phi\}, \{a_1, b_1\}\rangle$, $E = \langle\{a_1, c_1\}, \{\phi\}, \{b_1\}\rangle$, $F = \langle\{b_1, c_1\}, \{\phi\}, \{a_1\}\rangle$, then we have $2_{nc}\tau = \{\phi_N, X_N, D, E, F, A, B, C\}$ and ${}_{nc}\sigma_1 = \{\phi_N, Y_N, E\}$, ${}_{nc}\tau_2 = \{\phi_N, Y_N\}$, $E = \langle\{a_1, c_1\}, \{\phi\}, \{b_1\}\rangle$, then we have $2_{nc}\sigma = \{\phi_N, X_N, E\}$. Define $h : (X, 2_{nc}\tau) \to (Y, 2_{nc}\sigma)$ as an identity map, then h is $sf2_{nc}\gamma Cts$ but not $sf2_{nc}\beta Cts$.

Theorem 3.2 If a function $h : (U, N_{nc}\tau) \to (V, N_{nc}\sigma)$ is $sfN_{nc}eCts$ function, then it is $sN_{nc}eCts$.

If $(U, N_{nc}\tau)$ is a regular space, we have $N_{nc}\tau = N_{nc}\tau_\theta$ and the next theorem follows immediately.

Theorem 3.3 Let $(U, N_{nc}\tau)$ be a regular space. Then for a function $h : (U, N_{nc}\tau) \to (V, N_{nc}\sigma)$ the following properties are equivalent:

(i) h is $sN_{nc}eCts$.
(ii) h is $sfN_{nc}eCts$.

Definition 3.2 A $N_{nc}ts$ $(V, N_{nc}\sigma)$ is said to be a $N_{nc}T_e$-space if every $N_{nc}eo$ set of $(V, N_{nc}\sigma)$ is $N_{nc}o$.

Definition 3.3 A $N_{nc}ts$ X is said to be N_{nc} submaximal if each N_{nc} dense set of X is $N_{nc}o$ in X and N_{nc} extremely disconnected (briefly $N_{nc}ed$) if the N_{nc} closure of each $N_{nc}o$ set of X is $N_{nc}o$ in X.

Theorem 3.4 Let $h : (U, N_{nc}\tau) \to (V, N_{nc}\sigma)$ be a function and let $(V, N_{nc}\sigma)$ be a N_{nc} submaximal, $N_{nc}ed$ and $N_{nc}T_e$-space. Then the following

(i) h is $sfN_{nc}\alpha Cts$,
(ii) h is $sfN_{nc}\gamma Cts$,
(iii) h is $sfN_{nc}\mathcal{S}Cts$.
(iv) h is $sfN_{nc}\ \mathcal{P}Cts$.
(v) h is $sfN_{nc}\beta Cts$.
(vi) h is $sN_{nc}\theta Cts$.
(vii) h is $sfN_{nc}eCts$

are equivalent.

Proof. We have (i) $\Rightarrow$ (ii) $\Rightarrow$ (iii) $\Rightarrow$ (iv) $\Rightarrow$ (v) $\Rightarrow$ (vi) $\Rightarrow$ (i) (since U is N_{nc} submaximal and $N_{nc}ed$, then $N_{nc}\sigma = N_{nc}\alpha OS(Y) = N_{nc}\gamma OS(Y) = N_{nc}\mathcal{S}OS(Y) = N_{nc}\mathcal{P}OS(Y) = N_{nc}\beta OS(Y)$.

(vi) $\Leftrightarrow$ (vii): This follows from the fact that if $(V, N_{nc}\sigma)$ is $N_{nc}T_e$-space, then $N_{nc}\sigma = N_{nc}eOS(Y)$.

Theorem 3.5 If $h : (U, N_{nc}\tau) \to (V, N_{nc}\sigma)$ is $sfN_{nc}eCts$ and $g : (V, N_{nc}\sigma) \to (W, N_{nc}\psi)$ is $N_{nc}eIrr$, then $g \circ h : (U, N_{nc}\tau) \to (W, N_{nc}\psi)$ is $sfN_{nc}eCts$.

Proof. Let $G \in N_{nc}eOS(Z)$. Then $g^{-1}(G) \in N_{nc}eOS(Y)$ and hence $(g \circ h)^{-1}(G) = h^{-1}(g^{-1}(G))$ is $N_{nc}\theta o$ in U. Therefore $g \circ h$ is $sfN_{nc}eCts$.

Theorem 3.6 The functions $h : (U, N_{nc}\tau) \to (V, N_{nc}\sigma)$ and $g : (V, N_{nc}\sigma) \to (W, N_{nc}\psi)$ are hold for

(i) If both h and g are $sfN_{nc}eCts$, then the composition $g \circ h : (U, N_{nc}\tau) \to (W, N_{nc}\psi)$ is $sfN_{nc}eCts$.

(ii) If h $sfN_{nc}eCts$ and g is an $N_{nc}eIrr$, then $g \circ h$ is $sN_{nc}eCts$.
(iii) If h $sfN_{nc}eCts$ and g is a $N_{nc}eCts$, then $g \circ h$ is $sN_{nc}\theta Cts$.
(iv) If h is quasi $N_{nc}\theta Cts$ and g is $sfN_{nc}eCts$, then $g \circ h$ is $sfN_{nc}eCts$.
(v) If h is $sN_{nc}\theta Cts$ and g is $sfN_{nc}eCts$, then $g \circ h$ is $sfN_{nc}eCts$.

Definition 3.4 A $N_{nc}\theta$-frontier of a subset A of $(U, N_{nc}\tau)$ is $N_{nc}\theta Fr(A) = N_{nc}\theta cl(A) \cap N_{nc}\theta cl(X\backslash A)$.

Theorem 3.7 Let $(U, N_{nc}\tau)$ be a N_{nc} regular space. Then the set of all points $x \in X$ in which a function $h : (U, N_{nc}\tau) \to (V, N_{nc}\sigma)$ is not $sfN_{nc}eCts$ at x is identical with the union of the $N_{nc}\theta$-frontier of the inverse images of $N_{nc}eo$ sets of V containing $h(x)$.

Proof. Necessity. Suppose that h is not $sfN_{nc}eCts$ at $x \in X$. Then there exists a $N_{nc}eo$ set M of V containing $h(x)$ $\ni h(L)$ is not a subset of M $\forall$ $L \in \tau_\theta$ containing x. Hence we have $L \cap (X\backslash h^{-1}(M)) \neq \phi$ $\forall$ $L \in N_{nc}\tau_\theta$ containing x. Since U is N_{nc} regular, it follows that $x \in N_{nc}\theta cl(X\backslash h^{-1}(M))$. We have that, $x \in h^{-1}(M) \subseteq N_{nc}\theta cl(h^{-1}(M))$. This means that $x \in N_{nc}\theta Fr(h^{-1}(M))$. Sufficiency. Suppose that $x \in N_{nc}\theta Fr(h^{-1}(M))$ for some $M \in eO(Y, h(x))$. Now, we assume that h is $sfN_{nc}eCts$ at $x \in X$. Then $\exists$ $L \in N_{nc}\tau_\theta$ containing $x \ni h(L) \subseteq M$. Therefore, we have $L \subseteq h^{-1}(M)$ and hence $x \in N_{nc}\theta int(h^{-1}(M)) \subseteq X\backslash N_{nc}\theta Fr(h^{-1}(M))$. This is a contradiction. This means that h is not $sfN_{nc}eCts$.

Definition 3.5

(i) A space $(U, N_{nc}\tau)$ is said to be $N_{nc}e$-connected (resp. $N_{nc}\theta$-connected & N_{nc}-connected) if U cannot be written as the union of two nonempty disjoint $N_{nc}eo$ (resp. $N_{nc}\theta o$ & $N_{nc}o$) sets.
(ii) A subset K of a $(U, N_{nc}\tau)$ space is said to be, $N_{nc}e$-compact (resp. $N_{nc}\theta$-compact) relative to $(U, N_{nc}\tau)$, if for every cover of K by $N_{nc}eo$ (resp. $N_{nc}\theta o$) sets has a finite subcover. A $N_{nc}ts$ $(U, N_{nc}\tau)$ is $N_{nc}e$-compact (resp. $N_{nc}\theta$-compact) if the set U is $N_{nc}e$-compact (resp. $N_{nc}\theta$-compact) relative to $(U, N_{nc}\tau)$.

It should be mentioned that $N_{nc}\theta$-connected is equivalent with N_{nc} connected.

Theorem 3.8 If $h : (U, N_{nc}\tau) \to (V, N_{nc}\sigma)$ is a $sfN_{nc}eCts$ surjection function and $(U, N_{nc}\tau)$ is a $N_{nc}\theta$-connected space, then V is an $N_{nc}e$-connected space.

Proof. Assume that $(V, N_{nc}\sigma)$ is not $N_{nc}e$-connected. Then there exist nonempty $N_{nc}eo$ sets M_1 and M_2 of V such that $M_1 \cap M_2 = \phi$ and $M_1 \cup M_2 = Y$. Hence we have $h^{-1}(M_1) \cap h^{-1}(M_2) = \phi$ and $h^{-1}(M_1) \cup h^{-1}(M_2) = X$. Since h is surjective, $h^{-1}(M_1)$ and $h^{-1}(M_2)$ are nonempty subsets of U. Since h is $sfN_{nc}eCts$, $h^{-1}(M_1)$ and $h^{-1}(M_2)$ are $N_{nc}\theta o$ sets of U. Therefore $(U, N_{nc}\tau)$ is not $N_{nc}\theta$-connected. This is a contradiction and hence $(V, N_{nc}\sigma)$ is $N_{nc}e$-connected.

Theorem 3.9 If $h : (U, N_{nc}\tau) \to (V, N_{nc}\sigma)$ is a $sfN_{nc}eCts$, then $h(H)$ is $N_{nc}e$-compact relative to $(V, N_{nc}\sigma)$ for each subset H which is $N_{nc}\theta$-compact relative to $(U, N_{nc}\tau)$.

Proof. Let $\{M_i : i \in I\}$ be any cover of $h(H)$ by $N_{nc}eo$ sets. For each $x \in H$, $\exists$ $i_x \in I$, $\ni h(x) \in M_{i_x}$. Since h is $sfN_{nc}eCts$, $\exists$ $L_x \in \tau_\theta$ containing $x \ni h(L_x) \subseteq M_{i_x}$. The family $\{L_x : x \in H\}$ is a cover of H by $N_{nc}\theta o$ sets of $(U, N_{nc}\tau)$. Since H is $N_{nc}\theta$-compact relative to $(U, N_{nc}\tau)$, there exists a N_{nc} finite subset H_0 of $H \ni H \subseteq \bigcup\{L_x : x \in H_0\}$. Therefore, we obtain $h(H) \subseteq \bigcup\{h(L_x) : x \in H_0\} \subseteq \bigcup\{M_{i_x} : x \in H_0\}$. Therefore, $h(H)$ is $N_{nc}e$-compact relative to $(V, N_{nc}\sigma)$.

Theorem 3.10 The surjective $sfN_{nc}eCts$ image of a $N_{nc}\theta$-compact space is $N_{nc}e$-compact.

Proof. Let $h : (U, N_{nc}\tau) \to (V, N_{nc}\sigma)$ be a $sfN_{nc}eCts$ function from a $N_{nc}\theta$-compact space U onto a space V. Let $\{G_i : i \in I\}$ be any $N_{nc}eo$ cover of V. Since h is $sfN_{nc}eCts$, $\{h^{-1}(G_i) : i \in I\}$ is a $N_{nc}\theta o$ cover of U. Since U is $N_{nc}\theta$-compact, there exists a N_{nc} finite subcover $\{h^{-1}(G_i) : i = 1, 2, \cdots, n\}$ of U. Then it follows that $\{G_i : i = 1, 2, \cdots, n\}$ is a finite subfamily which cover V. Hence V is $N_{nc}e$-compact.

Separation Axioms

Definition 4.1 A $N_{nc}ts$ $(U, N_{nc}\tau)$ is said to be:

(i) $N_{nc}e$-T_1 (resp. $N_{nc}\theta$-T_1) if for each pair of distinct points l and m of U, there exists $N_{nc}eo$ (resp. $N_{nc}\theta o$) sets L and M containing l and m, respectively such that $m \notin L$ and $l \notin M$.
(ii) $N_{nc}e$-T_2 (resp. $N_{nc}\theta$-T_2) if for each pair of distinct points l and m in U, there exists disjoint $N_{nc}eo$ (resp. $N_{nc}\theta o$) sets L and M in U such that $l \in L$ and $m \in M$.

Remark 4.1 N_{nc} Hausdorff $\Leftrightarrow$ $N_{nc}\theta$-T_1.

Theorem 4.1 If $h : (U, N_{nc}\tau) \to (V, N_{nc}\sigma)$ is $sfN_{nc}eCts$ injection and V is an $N_{nc}e$-T_1 space, then U is a $N_{nc}\theta$-T_1 (or N_{nc} Hausdorff) space.

Proof. Suppose that V is $N_{nc}e$-T_1. For any distinct points l and m in U, then $\exists L, M \in N_{nc}eO(Y) \ni h(l) \in L$, $h(m) \notin L$, $h(l) \notin M$ & $h(m) \in M$. Since h is $sfN_{nc}eCts$, $h^{-1}(L)$ & $h^{-1}(M)$ are $N_{nc}\theta o$ subsets of $(U, N_{nc}\tau) \ni l \in h^{-1}(L)$, $m \notin h^{-1}(L)$, $l \notin h^{-1}(M)$ & $m \in h^{-1}(M)$. This shows that U is $N_{nc}\theta$-T_1 (equivalently N_{nc} Hausdorff by Remark 3).

Theorem 4.2 If $h : (U, N_{nc}\tau) \to (V, N_{nc}\sigma)$ is $sfN_{nc}eCts$ injection and V is an $N_{nc}e$-T_2 space, then U is a $N_{nc}\theta$-T_2 space.

Proof. Suppose that V is $N_{nc}e$-T_2. For any pair of distinct points l & m in U, there exist disjoint $N_{nc}eo$ sets L & M in V such that $h(l) \in L$ & $h(m) \in M$. Since h is $sfN_{nc}eCts$, $h^{-1}(L)$&$h^{-1}(M)$ are $N_{nc}\theta o$ in U containing l& m, respectively. Therefore, $h^{-1}(L) \cap h^{-1}(M) = \phi$ because $L \cap M = \phi$. This shows that U is $N_{nc}\theta$-T_2.

Theorem 4.3 If $h, g : (U, N_{nc}\tau) \to (V, N_{nc}\sigma)$ are $sfN_{nc}eCts$ functions and V is $N_{nc}e$-T_2, then $E = \{e \in X : h(e) = g(e)\}$ is $N_{nc}c$ in U.

Proof. Suppose that $e \notin E$. Then $h(e) \neq g(e)$. Since V is $N_{nc}e$-T_2, $\exists\, L \in N_{nc}eO(Y, h(e))$ & $M \in N_{nc}eO(Y, g(e)) \ni L \cap M = \phi$. Since h & g are $sfN_{nc}eCts$, there exist a $N_{nc}\theta o$ set N of U containing e and a $N_{nc}\theta o$ set O of U containing $e \ni h(N) \subseteq L$ & $g(O) \subseteq M$. Set $D = N \cap O$, then $D \cap E = \phi$ with D a $N_{nc}\theta o$ set and hence $N_{nc}o \ni e \in D$. Then $e \notin N_{nc}cl(E)$ and thus E is $N_{nc}c$ in U.

Definition 4.2 A space $(U, N_{nc}\tau)$ is said to be:

(i) $N_{nc}\theta$ regular (briefly. $N_{nc}\theta Reg$ (resp. $N_{nc}e$ regular (briefly $N_{nc}eReg$))) if $\forall$ $N_{nc}\theta c$ (resp. $N_{nc}ec$) set C and each point $x \notin C$, there exist disjoint $N_{nc}\theta o$ (resp. $N_{nc}eo$) sets L& $M \ni C \subseteq L$ and $x \in M$.

(ii) $N_{nc}\theta$-normal (briefly. $N_{nc}\theta Nor$ (resp. $N_{nc}e$-normal (briefly. $N_{nc}eNor$))) if for any pair of disjoint $N_{nc}\theta c$ (resp. $N_{nc}ec$) subsets C_1 & C_2 of U, there exist disjoint $N_{nc}\theta o$ (resp. $N_{nc}eo$) sets L& $M \ni C_1 \subseteq L$ & $C_2 \subseteq M$.

Definition 4.3 A function $h : (U, N_{nc}\tau) \to (V, N_{nc}\sigma)$ is called $N_{nc}\theta e$-open (briefly, $N_{nc}\theta eO$) if $h(L) \in N_{nc}eOS(Y)$ for each $L \in N_{nc}\tau_\theta$.

Theorem 4.4 If h is $sfN_{nc}eCts$ & $N_{nc}\theta eO$ injection from a $N_{nc}\theta Reg$ space $(U, N_{nc}\tau)$ onto a space $(V, N_{nc}\sigma)$, then $(V, N_{nc}\sigma)$ is $N_{nc}eReg$.

Proof. Let C be an $N_{nc}ec$ set of V and $y \notin C$. Take $y = h(x)$. Since h is $sfN_{nc}eCts$, $h^{-1}(C)$ is $N_{nc}\theta c$ in $U \ni h^{-1}(y) = x \notin h^{-1}(C)$. Take $D = h^{-1}(C)$. We have $x \notin D$. Since U is $N_{nc}\theta Reg$, then there exist disjoint $N_{nc}\theta o$ sets L & M in $U \ni D \subseteq L$ & $x \in M$. We obtain that $C = h(D) \subseteq h(L)$ & $y = h(x) \in h(L) \ni h(L)$ & $h(M)$ are disjoint $N_{nc}eo$ sets. This shows that V is $N_{nc}eReg$.

Theorem 4.5 If h is $sfN_{nc}eCts$ & $N_{nc}\theta eO$ injection from a $N_{nc}\theta Nor$ space $(U, N_{nc}\tau)$ onto a space $(V, N_{nc}\sigma)$, then V is $N_{nc}eNor$.

Proof. Let F_1 & F_2 be disjoint $N_{nc}ec$ sets of V. Since h is $sfN_{nc}eCts$, $h^{-1}(F_1)$ & $h^{-1}(F_2)$ are $N_{nc}\theta c$ sets. Take $L = h^{-1}(F_1)$ & $M = h^{-1}(F_2)$. We have $L \cap M = \phi$. Since U is $N_{nc}\theta Nor$, there exist disjoint $N_{nc}\theta o$ sets C & $D \ni L \subseteq C$ & $M \subseteq D$. We obtain that $F_1 = h(L) \subseteq h(C)$ & $F_2 = h(M) \subseteq h(D) \ni h(C)$ & $h(D)$ are disjoint $N_{nc}eo$ sets. Thus, V is $N_{nc}eNor$.

Definition 4.4 A function $h : (U, N_{nc}\tau) \to (V, N_{nc}\sigma)$, the subset $\{(g, h(g)) : g \in X\} \subseteq X \times Y$ is called the graph of h and is denoted by $N_{nc}G(h)$.

Definition 4.5 A graph $N_{nc}G(h)$ of a function $h : (U, N_{nc}\tau) \to (V, N_{nc}\sigma)$ is said to be $N_{nc}(e, \theta)$-closed if for each $(l, m) \in (X \times Y)\backslash G(h)$, there exist a $N_{nc}\theta o$ L set of U containing l and an $N_{nc}eo$ set M of V containing $m \ni (L \times M) \cap G(h) = \phi$.

Lemma 4.1 A graph $N_{nc}G(h)$ of a function $h : (U, N_{nc}\tau) \to (V, N_{nc}\sigma)$ is $N_{nc}(e, \theta)$-closed in $X \times Y$ iff for each $(l, m) \in (X \times Y)\backslash N_{nc}G(h)$, there exist a $N_{nc}\theta o$ set L of U containing l and an $N_{nc}eo$ set M of V containing $m \ni h(L) \cap M = \phi$.

Theorem 4.6 If $h : (U, N_{nc}\tau) \to (V, N_{nc}\sigma)$ is $sfN_{nc}eCts$ function and $(V, N_{nc}\sigma)$ is $N_{nc}e$-T_2, then $N_{nc}G(h)$ is $N_{nc}(e, \theta)$-closed.

Proof. Let $(l, m) \in (X \times Y)\backslash N_{nc}G(h)$, then $h(l) \notin m$. Since V is $N_{nc}e$-T_2, there exist disjoint $N_{nc}eo$ sets L & M in $V \ni h(l) \in L$ & $m \in M$. Since h is $sfN_{nc}eCts$, $h^{-1}(L)$ is $N_{nc}\theta o$ in U containing l. Take $N = h^{-1}(L)$. We have $h(N) \subseteq L$. Therefore, we obtain $h(N) \cap M = \phi$. This shows that $N_{nc}G(h)$ is $N_{nc}(e, \theta)$-closed.

Theorem 4.7 Let $h : (U, N_{nc}\tau) \to (V, N_{nc}\sigma)$ has an $N_{nc}(e, \theta)$-closed graph $N_{nc}G(h)$. If h is a $sfN_{nc}eCts$ injection, then $(U, N_{nc}\tau)$ is $N_{nc}\theta$-T_2.

Proof. Let l & m be any two distinct points of U. Then since h is injective, we have $h(l) \neq h(m)$. Then, we have $(l, h(m)) \in (X \times Y)\backslash N_{nc}G(h)$. By Lemma 3, there exist a $N_{nc}\theta o$ set L of U and an $N_{nc}eo$ set M of $V \ni (l, h(m)) \in L \times M$ & $h(L) \cap M = \phi$. Hence $L \cap h^{-1}(M) = \phi$ & $m \notin L$. Since h is $sfN_{nc}eCts$, there exists a $N_{nc}\theta o$ set N of U containing m $\ni h(N) \subseteq N$. Therefore, we have $h(L) \cap h(N) = \phi$. Since h is injective, we obtain $L \cap N = \phi$. This implies that $(U, N_{nc}\tau)$ is $N_{nc}\theta$-T_2.

Definition 4.6 A $N_{nc}ts$ U is said to be $N_{nc}e$-Alexandroff if every finite intersection of $N_{nc}eo$ sets is $N_{nc}eo$.

Theorem 4.8 Let $(V, N_{nc}\sigma)$ be $N_{nc}e$-Alexandroff. If $h : (U, N_{nc}\tau) \rightarrow (V, N_{nc}\sigma)$ has the $N_{nc}(e, \theta)$-closed graph, then $h(H)$ is $N_{nc}ec$ in $(V, N_{nc}\sigma)$ for each subset H which is $N_{nc}\theta$-compact relative to U.

Proof. Suppose that $m \notin h(H)$. Then $(l, m) \backslash N_{nc}G(h)$ for each $l \in H$. Since $N_{nc}G(h)$ is $N_{nc}(e, \theta)$-closed, there exist a $N_{nc}\theta o$ L_l set of U containing l and $N_{nc}eo$ M_l set of V containing $m \ni h(L_l) \cap M_l = \phi$, by Lemma 3. The family $\{L_l : l \in H\}$ is a cover of H by $N_{nc}\theta o$ sets. Since H is $N_{nc}\theta$-compact relative to $(U, N_{nc}\tau)$, there exists a N_{nc} finite subset H_0 of $H \ni H \subseteq \bigcup\{L_l : l \in H_0\}$. Set $M = \bigcap\{M_l : l \in H_0\}$. Then M is a $N_{nc}eo$ set in V containing m. Therefore, we have $h(H) \cap M \subseteq (\bigcap_{l \in H_0} h(L_l)) \cap M \subseteq \bigcap_{l \in H_0} (h(L_l) \cap M) = \phi$. It follows that $m \notin N_{nc}ecl(h(H))$. Therefore, $h(H)$ is $N_{nc}ec$ in $(V, N_{nc}\sigma)$.

Corollary 4.1 Let $(V, N_{nc}\sigma)$ be $N_{nc}e$-Alexandroff. If $h : (U, N_{nc}\tau) \rightarrow (V, N_{nc}\sigma)$ is $sfN_{nc}eCts$ and $(V, N_{nc}\sigma)$ is $N_{nc}e$-T_2, then $h(H)$ is $N_{nc}ec$ in $(V, N_{nc}\sigma)$ for each subset H which is $N_{nc}\theta$-compact relative to (X, τ).

CONCLUSION

In this work, we have introduced some new class of functions, called strongly faint $N_{nc}e$-continuous. Also the relationships among strongly faint $N_{nc}e$-continuous function and $N_{nc}e$-connected, normal and compact spaces are investigated. Furthermore, the relationships between strongly faint $N_{nc}e$-continuous functions and graphs are also investigated.

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