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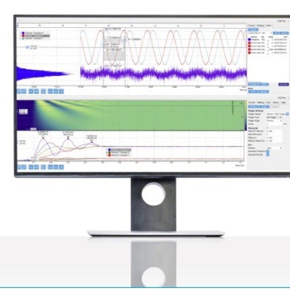

Neutrosophic e-Open Maps, Neutrosophic e-Closed Maps and Neutrosophic e-Homeomorphisms in Neutrosophic Topological Spaces

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Abstract. In this article, we introduce the concept of neutrosophic e-open and neutrosophic e-closed mappings in neutrosophic topological spaces and studied some of their related properties. Further the work is extended to neutrosophic e-homeomorphism, neutrosophic e-Completely homeomorphism and neutrosophic $eT_{\frac{1}{2}}$-space in neutrosophic topological spaces and establishes some of their related attributes.
Keywords and phrases: neutrosophic e-open map, neutrosophic e-closed map, $NeT_{\frac{1}{2}}$-space, neutrosophic e-homeomorphism, neutrosophic e-C homeomorphism.
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INTRODUCTION

The concept of fuzzy set (briefly, fs) was introduced by Lotfi Zadeh in 1965 [20], then Chang depended the fuzzy set to introduce the concept of fuzzy topological space (briefly, fts) in 1968 [5]. After that the concept of fuzzy set was developed into the concept of intutionistic fuzzy set (briefly, Ifs) by Atanassov in 1983 [2, 3, 4], the intutionistic fuzzy set gives a degree of membership and a degree of non-membership functions. Cokor in 1997 [5] relied on intutionistic fuzzy set to introduced the concept of intutionistic fuzzy topological space (briefly, $Ifts$). In 2005 Smaradache [13] study the concept of neutrosophic set (briefly, N_ss). After that and as developed the term of neutrosophic set, Salama has studied neutrosophic topological space (briefly, N_sts) and many of its applications [8, 9, 10, 11]. In 2012 Salama and Alblowi defined neutrosophic topological space [8]. The neutrosophic closed sets and neutrosophic continuous functions were introduced by Salama et al. [10] in 2014. Saha [14] defined δ-open sets in topological spaces. Vadivel et al. in [18] introduced δ-open sets in a neutrosophic topological space. In 2008, Ekici [6] introduced the notion of e-open sets in a general topology. In 2014, Seenivasan et al. [12] introduced fuzzy e-open sets in a fuzzy topological space along with fuzzy e-continuity. Vadivel et al. [19] studied fuzzy e-open sets in intuitionistic fuzzy topological space. The focus of this article is to introduce the idea of N_se-open and N_se-closed mappings in neutrosophic topological spaces and also the work is extended to N_se-homeomorphism, N_se-C homeomorphism and $N_seT_{\frac{1}{2}}$-space in neutrosophic topological spaces and obtain some of its basic properties.

PRELIMINARIES

The needful basic definitions & properties of neutrosophic topological spaces are discussed in this section.

Definition 2.1 [8] Let Y be a non-empty set. A neutrosophic set (briefly, N_ss) L is an object having the form $L = \{\langle y, \mu_L(y), \sigma_L(y), \nu_L(y)\rangle : y \in Y\}$ where $\mu_L \to [0,1]$ denote the degree of membership function, $\sigma_L \to [0,1]$ denote the degree of indeterminacy function and $\nu_L \to [0,1]$ denote the degree of non-membership function respectively of each element $y \in Y$ to the set L and $0 \le \mu_L(y) + \sigma_L(y) + \nu_L(y) \le 3$ for each $y \in Y$.

Remark 2.1 [8] A N_ss $L = \{\langle y, \mu_L(y), \sigma_L(y), \nu_L(y)\rangle : y \in Y\}$ can be identified to an ordered triple $\langle y, \mu_L(y), \sigma_L(y), \nu_L(y)\rangle$ in $[0,1]$ on Y.

Definition 2.2 [8] Let Y be a non-empty set & the N_ss's L & M in the form $L = \{\langle y, \mu_L(y), \sigma_L(y), \nu_L(y)\rangle : y \in Y\}$, $M = \{\langle y, \mu_M(y), \sigma_M(y), \nu_M(y)\rangle : y \in Y\}$, then

(i) $0_N = \langle y, 0, 0, 1\rangle$ and $1_N = \langle y, 1, 1, 0\rangle$,

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(ii) $L \subseteq M$ iff $\mu_L(y) \leq \mu_M(y)$, $\sigma_L(y) \leq \sigma_M(y)$ & $\nu_L(y) \geq \nu_M(y) : y \in Y$,

(iii) $L = M$ iff $L \subseteq M$ and $M \subseteq L$,

(iv) $1_N - L = \{\langle y, \nu_L(y), 1-\sigma_L(y), \mu_L(y)\rangle : y \in Y\} = L^c$,

(v) $L \cup M = \{\langle y, \max(\mu_L(y), \mu_M(y)), \max(\sigma_L(y), \sigma_M(y)), \min(\nu_L(y), \nu_M(y))\rangle : y \in Y\}$,

(vi) $L \cap M = \{\langle y, \min(\mu_L(y), \mu_M(y)), \min(\sigma_L(y), \sigma_M(y)), \max(\nu_L(y), \nu_M(y))\rangle : y \in Y\}$.

Definition 2.3 [8] A neutrosophic topology (briefly, N_st) on a non-empty set Y is a family Ψ_N of neutrosophic subsets of Y satisfying

(i) 0_N, $1_N \in \Psi_N$.

(ii) $L_1 \cap L_2 \in \Psi_N$ for any $L_1, L_2 \in \Psi_N$.

(iii) $\bigcup L_x \in \Psi_N, \forall\ L_x : x \in X \subseteq \Psi_N$.

Then (Y, Ψ_N) is called a neutrosophic topological space (briefly, N_sts) in Y. The Ψ_N elements are called neutrosophic open sets (briefly, N_sos) in Y. A N_ss C is called a neutrosophic closed sets (briefly, N_scs) iff its complement C^c is N_sos.

Definition 2.4 [8] Let (Y, Ψ_N) be N_sts on Y and L be an N_ss on Y, then the neutrosophic interior of L (briefly, $N_sint(L)$) and the neutrosophic closure of L (briefly, $N_scl(L)$) are defined as

$$N_sint(L) = \bigcup\{I : I \subseteq L \text{ \& } I \text{ is a } N_sos \text{ in } Y\}$$

$$N_scl(L) = \bigcap\{I : L \subseteq I \text{ \& } I \text{ is a } N_scs \text{ in } Y\}.$$

Definition 2.5 [1] Let (Y, Ψ_N) be N_sts on Y and L be an N_ss on Y. Then L is said to be a neutrosophic regular open set (briefly, N_sros) if $L = N_sint(N_scl(L))$.

The complement of a N_sros is called a neutrosophic regular closed set (briefly, N_srcs) in Y.

Definition 2.6 [18] A set K is said to be a neutrosophic

(i) δ interior of G (briefly, $N_s\delta int(K)$) is defined by $N_s\delta int(K) = \bigcup\{B : B \subseteq K \text{ \& } B \text{ is a } N_sros \text{ in } Y\}$.

(ii) δ closure of K (briefly, $N_s\delta cl(K)$) is defined by $N_s\delta cl(K) = \bigcap\{A : K \subseteq A \text{ \& } A \text{ is a } N_srcs \text{ in } Y\}$.

Definition 2.7 [18] A set L is said to be a neutrosophic

(i) δ-open set (briefly, $N_s\delta os$) if $L = N_s\delta int(L)$.

(ii) δ-pre open set (briefly, $N_s\delta\mathcal{P}os$) if $L \subseteq N_sint(N_s\delta cl(L))$.

(iii) δ-semi open set (briefly, $N_s\delta\mathcal{S}os$) if $L \subseteq N_scl(N_s\delta int(L))$.

(iv) e-open set (briefly, N_seos) [16] if $L \subseteq N_scl(N_s\delta int(L)) \cup N_sint(N_s\delta cl(L))$.

(v) e^*-open set (briefly, N_se^*os) if $L \subseteq N_scl(N_sint(N_s\delta cl(L)))$.

The complement of an $N_s\delta os$ (resp. $N_s\delta\mathcal{P}os$, $N_s\delta\mathcal{S}os$, N_seos & N_se^*os) is called a neutrosophic δ (resp. δ-pre, δ-semi, e & e^*) closed set (briefly, $N_s\delta cs$ (resp. $N_s\delta\mathcal{P}cs$, $N_s\delta\mathcal{S}cs$, N_secs & N_se^*cs)) in Y.

Definition 2.8 [18] Let (X, τ_N) and (Y, σ_N) be any two Nts's. A map $h : (X, \tau_N) \to (Y, \sigma_N)$ is said to be neutrosophic (resp. δ, $\delta\mathcal{S}$, $\delta\mathcal{P}$, e & e^*) continuous (briefly, N_sCts [10] (resp. $N_s\delta Cts$, $N_s\delta\mathcal{S}Cts$, $N_s\delta\mathcal{P}Cts$, N_seCts [17] & N_se^*Cts)) if the inverse image of every N_sos in (Y, σ_N) is a N_sos (resp. $N_s\delta os$, $N_s\delta\mathcal{S}os$, $N_s\delta\mathcal{P}os$, N_seos & N_se^*os) in (X, τ_N).

Definition 2.9 Let (X, τ_N) and (Y, σ_N) be any two N_sts's. A map $h : (X, \tau_N) \to (Y, \sigma_N)$ is said to be neutrosophic

(i) e-irresolute (briefly, N_seIrr) [17] if the inverse image of every N_seos in (Y, σ_N) is a N_seos in (X, τ_N).

(ii) homeomorphism (briefly, N_sHom) [7] if h and h^{-1} are N_sCts mappings.

Definition 2.10 [15] Let (X, τ_N) and (Y, σ_N) be any two Nts's. A map $h : (X, \tau_N) \to (Y, \sigma_N)$ is said to be neutrosophic (resp. δ, $\delta\mathcal{S}$, $\delta\mathcal{P}$ & e^*) open map (briefly, N_sO (resp. $N_s\delta O$, $N_s\delta\mathcal{S}O$, $N_s\delta\mathcal{P}O$ & N_se^*O)) if the image of every N_sos in (X, τ_N) is a N_sos (resp. $N_s\delta os$, $N_s\delta\mathcal{S}os$, $N_s\delta\mathcal{P}os$ & N_se^*os) in (Y, σ_N).

NEUTROSOPHIC e-OPEN MAPPING

Definition 3.1 A mapping $h:(X,\tau_N)\to(Y,\sigma_N)$ is neutrosophic e-open (briefly, N_seO) if image of every neutrosophic open set of (X,τ_N) is N_seo set in (Y,σ_N).

Theorem 3.1 The statements are hold but the converse does not true.

(i) Every $N_s\delta O$ mapping is a N_sO mapping.

(ii) Every N_sO mapping is a $N_s\delta\mathcal{S}O$ mapping.

(iii) Every N_sO mapping is a $N_s\delta\mathcal{P}O$ mapping.

(iv) Every $N_s\delta\mathcal{S}O$ mapping is a N_seO mapping.

(v) Every $N_s\delta\mathcal{P}O$ mapping is a N_seO mapping.

(vi) Every N_seO mapping is a N_se^*O mapping.

Proof. We prove only (v), the others are similar.

(v) Let λ be a N_sos in X. Since h is $N_s\delta\mathcal{P}O$ mapping, $h(\lambda)$ is a $N_s\delta\mathcal{P}os$ in Y. Since every $N_s\delta\mathcal{P}os$ is a N_seos [16], $h(\lambda)$ is a N_seos in Y. Hence h is a N_seO mapping.

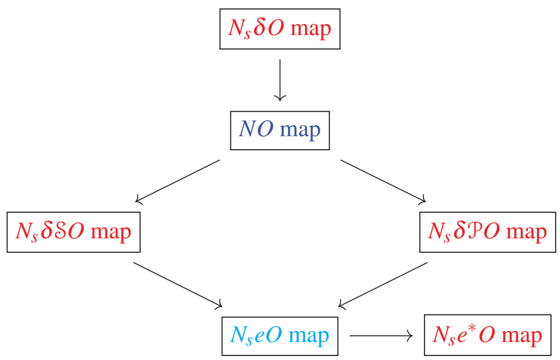

FIGURE 1. N_seO map's in N_sts.

Example 3.1 Let $X=\{a\}=Y$ and define N_ss's X_1 in X and Y_1 & Y_2 in Y are

$$X_1=\langle X,(\frac{\mu_a}{0.2},\frac{\sigma_a}{0.5},\frac{\nu_a}{0.8})\rangle,\ Y_1=\langle Y,(\frac{\mu_a}{0.2},\frac{\sigma_a}{0.5},\frac{\nu_a}{0.8})\rangle,\ Y_2=\langle Y,(\frac{\mu_a}{0.5},\frac{\sigma_a}{0.5},\frac{\nu_a}{0.5})\rangle.$$

Then we have $\tau_N=\{0_N,X_1,1_N\}$ and $\sigma_N=\{0_N,Y_1,Y_2,1_N\}$. Let $h:(X,\tau_N)\to(Y,\sigma_N)$ be an identity mapping, then h is N_sO map but not $N_s\delta O$ map.

Example 3.2 Let $X=\{a,b,c\}=Y$ and define N_ss's X_1 in X and Y_1,Y_2 & Y_3 in Y are

$$X_1=\langle X,(\frac{\mu_a}{0.2},\frac{\mu_b}{0.4},\frac{\mu_c}{0.4}),(\frac{\sigma_a}{0.5},\frac{\sigma_b}{0.5},\frac{\sigma_c}{0.5}),(\frac{\nu_a}{0.8},\frac{\nu_b}{0.6},\frac{\nu_c}{0.6})\rangle,$$
$$Y_1=\langle Y,(\frac{\mu_a}{0.2},\frac{\mu_b}{0.3},\frac{\mu_c}{0.4}),(\frac{\sigma_a}{0.5},\frac{\sigma_b}{0.5},\frac{\sigma_c}{0.5}),(\frac{\nu_a}{0.8},\frac{\nu_b}{0.7},\frac{\nu_c}{0.6})\rangle,$$
$$Y_2=\langle Y,(\frac{\mu_a}{0.1},\frac{\mu_b}{0.1},\frac{\mu_c}{0.4}),(\frac{\sigma_a}{0.5},\frac{\sigma_b}{0.5},\frac{\sigma_c}{0.5}),(\frac{\nu_a}{0.9},\frac{\nu_b}{0.9},\frac{\nu_c}{0.6})\rangle,$$
$$Y_3=\langle Y,(\frac{\mu_a}{0.2},\frac{\mu_b}{0.4},\frac{\mu_c}{0.4}),(\frac{\sigma_a}{0.5},\frac{\sigma_b}{0.5},\frac{\sigma_c}{0.5}),(\frac{\nu_a}{0.8},\frac{\nu_b}{0.6},\frac{\nu_c}{0.6})\rangle.$$

Then we have $\tau_N=\{0_N,X_1,1_N\}$ and $\sigma_N=\{0_N,Y_1,Y_2,1_N\}$. Let $h:(X,\tau_N)\to(Y,\sigma_N)$ be an identity mapping, then h is a $N_s\delta\mathcal{S}O$ map but not N_sO map.

Example 3.3 Let $X = \{a,b,c\} = Y$ and define $N_s s$'s X_1 in X and Y_1, Y_2, Y_3 & Y_4 in Y are

$$X_1 = \langle X, (\frac{\mu_a}{0.3}, \frac{\mu_b}{0.5}, \frac{\mu_c}{0.4}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\nu_a}{0.7}, \frac{\nu_b}{0.5}, \frac{\nu_c}{0.6}) \rangle,$$
$$Y_1 = \langle Y, (\frac{\mu_a}{0.3}, \frac{\mu_b}{0.5}, \frac{\mu_c}{0.5}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\nu_a}{0.7}, \frac{\nu_b}{0.5}, \frac{\nu_c}{0.5}) \rangle,$$
$$Y_2 = \langle Y, (\frac{\mu_a}{0.4}, \frac{\mu_b}{0.2}, \frac{\mu_c}{0.6}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\nu_a}{0.6}, \frac{\nu_b}{0.8}, \frac{\nu_c}{0.4}) \rangle,$$
$$Y_3 = \langle Y, (\frac{\mu_a}{0.4}, \frac{\mu_b}{0.5}, \frac{\mu_c}{0.6}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\nu_a}{0.6}, \frac{\nu_b}{0.5}, \frac{\nu_c}{0.4}) \rangle,$$
$$Y_4 = \langle Y, (\frac{\mu_a}{0.3}, \frac{\mu_b}{0.5}, \frac{\mu_c}{0.4}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\nu_a}{0.7}, \frac{\nu_b}{0.5}, \frac{\nu_c}{0.6}) \rangle.$$

Then we have $\tau_N = \{0_N, X_1, 1_N\}$ and $\sigma_N = \{0_N, Y_1, Y_2, Y_3, Y_1 \cap Y_2, 1_N\}$. Let $h : (X, \tau_N) \to (Y, \sigma_N)$ be an identity mapping, then h is a $N_s\delta\mathcal{P}O$ map but not N_sO map.

Example 3.4 Let $X = \{a,b,c\} = Y$ and define $N_s s$'s X_1 in X and Y_1, Y_2 & Y_3 in Y are

$$X_1 = \langle X, (\frac{\mu_a}{0.2}, \frac{\mu_b}{0.4}, \frac{\mu_c}{0.4}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\nu_a}{0.8}, \frac{\nu_b}{0.6}, \frac{\nu_c}{0.6}) \rangle$$
$$Y_1 = \langle Y, (\frac{\mu_a}{0.2}, \frac{\mu_b}{0.3}, \frac{\mu_c}{0.4}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\nu_a}{0.8}, \frac{\nu_b}{0.7}, \frac{\nu_c}{0.6}) \rangle,$$
$$Y_2 = \langle Y, (\frac{\mu_a}{0.1}, \frac{\mu_b}{0.1}, \frac{\mu_c}{0.4}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\nu_a}{0.9}, \frac{\nu_b}{0.9}, \frac{\nu_c}{0.6}) \rangle,$$
$$Y_3 = \langle Y, (\frac{\mu_a}{0.2}, \frac{\mu_b}{0.4}, \frac{\mu_c}{0.4}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\nu_a}{0.8}, \frac{\nu_b}{0.6}, \frac{\nu_c}{0.6}) \rangle.$$

Then we have $\tau_N = \{0_N, X_1, 1_N\}$ and $\sigma_N = \{0_N, Y_1, Y_2, 1_N\}$. Let $h : (X, \tau_N) \to (Y, \sigma_N)$ be an identity mapping, then h is a N_seO map but not $N_s\delta\mathcal{P}O$ map.

Example 3.5 Let $X = \{a,b,c\} = Y$ and define $N_s s$'s X_1 in X and Y_1, Y_2, Y_3 & Y_4 in Y are

$$X_1 = \langle X, (\frac{\mu_a}{0.3}, \frac{\mu_b}{0.5}, \frac{\mu_c}{0.4}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\nu_a}{0.7}, \frac{\nu_b}{0.5}, \frac{\nu_c}{0.6}) \rangle,$$
$$Y_1 = \langle Y, (\frac{\mu_a}{0.3}, \frac{\mu_b}{0.5}, \frac{\mu_c}{0.5}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\nu_a}{0.7}, \frac{\nu_b}{0.5}, \frac{\nu_c}{0.5}) \rangle,$$
$$Y_2 = \langle Y, (\frac{\mu_a}{0.4}, \frac{\mu_b}{0.2}, \frac{\mu_c}{0.6}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\nu_a}{0.6}, \frac{\nu_b}{0.8}, \frac{\nu_c}{0.4}) \rangle,$$
$$Y_3 = \langle Y, (\frac{\mu_a}{0.4}, \frac{\mu_b}{0.5}, \frac{\mu_c}{0.6}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\nu_a}{0.6}, \frac{\nu_b}{0.5}, \frac{\nu_c}{0.4}) \rangle,$$
$$Y_4 = \langle Y, (\frac{\mu_a}{0.3}, \frac{\mu_b}{0.5}, \frac{\mu_c}{0.4}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\nu_a}{0.7}, \frac{\nu_b}{0.5}, \frac{\nu_c}{0.6}) \rangle.$$

Then we have $\tau_N = \{0_N, X_1, 1_N\}$ and $\sigma_N = \{0_N, Y_1, Y_2, Y_3, Y_1 \cap Y_2, 1_N\}$. Let $h : (X, \tau_N) \to (Y, \sigma_N)$ be an identity mapping, then h is a N_seO map but not $N_s\delta\mathcal{S}O$ map.

Example 3.6 Let $X = \{a,b\} = Y$ and define $N_s s$'s X_1 in X and Y_1 & Y_2 in Y are

$$X_1 = \Big\langle Y, (\frac{\mu_a}{0.3}, \frac{\mu_b}{0.5}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}), (\frac{\nu_a}{0.7}, \frac{\nu_b}{0.6}) \Big\rangle,$$
$$Y_1 = \Big\langle Y, (\frac{\mu_a}{0.3}, \frac{\mu_b}{0.2}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}), (\frac{\nu_a}{0.5}, \frac{\nu_b}{0.5}) \Big\rangle,$$
$$Y_2 = \Big\langle Y, (\frac{\mu_a}{0.3}, \frac{\mu_b}{0.5}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}), (\frac{\nu_a}{0.7}, \frac{\nu_b}{0.6}) \Big\rangle.$$

Then we have $\tau_N = \{0_N, X_1, 1_N\}$ and $\sigma_N = \{0_N, Y_1, 1_N\}$. Let $h : (X, \tau_N) \to (Y, \sigma_N)$ be an identity mapping, then h is a N_se^*O map but not N_seO map.

Theorem 3.2 A mapping $h : (X, \tau_N) \to (Y, \sigma_N)$ is N_seO iff for every $N_s s$ λ of (X, τ_N), $h(N_s int(\lambda)) \subseteq N_s eint(h(\lambda))$.

Proof. Necessity: Let h be a N_seO mapping and λ be a $N_s os$ in (X, τ_N). Now, $N_s int(\lambda) \subseteq \lambda$ implies $h(N_s int(\lambda)) \subseteq h(\lambda)$. Since h is a N_seO mapping, $h(N_s int(\lambda))$ is $N_s eos$ in (Y, σ_N) such that $h(N_s int(\lambda)) \subseteq h(\lambda)$ therefore $h(N_s int(\lambda)) \subseteq N_s eint(h(\lambda))$.

Sufficiency: Assume λ is a N_sos of (X,τ_N). Then $h(\lambda)=h(N_sint(\lambda))\subseteq N_seint(h(\lambda))$. But $N_seint(h(\lambda))\subseteq h(\lambda)$. So $h(\lambda)=N_seint(\lambda)$ which implies $h(\lambda)$ is a N_seos of (Y,σ_N) and hence h is a N_seO.

Theorem 3.3 If $h:(X,\tau_N)\to(Y,\sigma_N)$ is a N_seO mapping then $N_sint(h^{-1}(\lambda))\subseteq h^{-1}(N_seint(\lambda))$ for every N_ss λ of (Y,σ_N).

Proof. Let λ be a N_ss of (Y,σ_N). Then $N_sint(h^{-1}(\lambda))$ is a N_sos in (X,τ_N). Since h is N_seO, $h(N_sint(h^{-1}(\lambda))$ is N_seo in (Y,σ_N) and hence $h(N_sint(h^{-1}(\lambda)))\subseteq N_seint(h(h^{-1}(\lambda)))\subseteq N_seint(\lambda)$. Thus $N_sint(h^{-1}(\lambda))\subseteq h^{-1}(N_seint(\lambda))$.

Theorem 3.4 A mapping $h:(X,\tau_N)\to(Y,\sigma_N)$ is N_seO iff for each N_ss μ of (Y,σ_N) and for each N_scs λ of (X,τ_N) containing $h^{-1}(\mu)$ there is a N_secs ψ of (Y,σ_N) such that $\mu\subseteq\lambda$ and $h^{-1}(\psi)\subseteq\lambda$.

Proof. Necessity: Assume h is a N_seO mapping. Let μ be the N_scs of (Y,σ_N) and λ is a N_scs of (X,τ_N) such that $h^{-1}(\mu)\subseteq\lambda$. Then $\psi=(h^{-1}(\lambda^c))^c$ is N_secs of (Y,σ_N) such that $h^{-1}(\psi)\subseteq\lambda$.

Sufficiency: Assume ω is a N_sos of (X,τ_N). Then $h^{-1}((h(\omega))^c\subseteq\omega^c$ and ω^c is N_scs in (X,τ_N). By hypothesis there is a N_secs ψ of (Y,σ_N) such that $(h(\omega))^c\subseteq\psi$ and $h^{-1}(\psi)\subseteq\omega^c$. Therefore $\omega\subseteq(h^{-1}(\psi))^c$. Hence $\psi^c\subseteq h(\omega)\subseteq h((h^{-1}(\psi))^c)\subseteq\psi^c$ which implies $h(\omega)=\psi^c$. Since ψ^c is N_seos of (Y,σ_N). Hence $h(\omega)$ is N_seo in (Y,σ_N) and thus h is N_seO mapping.

Theorem 3.5 A mapping $h:(X,\tau_N)\to(Y,\sigma_N)$ is N_seO iff $h^{-1}(N_secl(\lambda)\subseteq N_scl(h^{-1}(\lambda))$ for every N_ss λ of (Y,σ_N).

Proof. Necessity: Assume h is a N_seO mapping. For any N_ss λ of (Y,σ_N), $h^{-1}(\lambda)\subseteq N_scl(h^{-1}(\lambda))$. Therefore by Theorem 3.4 there exists a N_secs μ in (Y,σ_N) such that $\lambda\subseteq\mu$ and $h^{-1}(\mu)\subseteq N_scl(h^{-1}(\lambda))$. Therefore we obtain that $h^{-1}(N_secl(\lambda))\subseteq h^{-1}(\mu)\subseteq N_scl(h^{-1}(\lambda))$.

Sufficiency: Assume λ is a N_ss of (Y,σ_N) and μ is a N_scs of (X,τ_N) containing $h^{-1}(\lambda)$. Put $\zeta=cl(\lambda)$, then $\lambda\subseteq\zeta$ and ζ is N_sec and $h^{-1}(\zeta)\subsetneq cl(h^{-1}(\lambda))\subseteq\mu$. Then by Theorem 3.4, h is N_seO mapping.

Theorem 3.6 If $h:(X,\tau_N)\to(Y,\sigma_N)$ and $g:(Y,\sigma_N)\to(Z,\rho_N)$ be two neutrosophic mappings and $g\circ h:(X,\tau_N)\to(Z,\rho_N)$ is N_seO. If $g:(Y,\sigma_N)\to(Z,\rho_N)$ is N_seIrr then $h:(X,\tau_N)\to(Y,\sigma_N)$ is N_seO mapping.

Proof. Let ψ be a N_sos in (X,τ_N). Then $g\circ h(\psi)$ is N_seos of (Z,ρ_N) because $g\circ h$ is N_seO mapping. Since g is N_seIrr and $g\circ h(\psi)$ is N_seos of (Z,ρ_N) therefore $g^{-1}(g\circ h(\psi))=h(\psi)$ is N_seos in (Y,σ_N). Hence h is N_seO mapping.

Theorem 3.7 If $h:(X,\tau_N)\to(Y,\sigma_N)$ is N_sO and $g:(Y,\sigma_N)\to(Z,\rho_N)$ is N_seO mappings then $g\circ f:(X,\tau_N)\to(Z,\rho_N)$ is N_seO.

Proof. Let ψ be a N_sos in (X,τ_N). Then $h(\psi)$ is a N_sos of (Y,σ_N) because h is a N_sO mapping. Since g is N_seO, $g(h(\psi))=(g\circ h)(\psi)$ is N_seos of (Z,ρ_N). Hence $g\circ h$ is N_seO mapping.

NEUTROSOPHIC e-CLOSED MAPPING

Definition 4.1 A mapping $h:(X,\tau_N)\to(Y,\sigma_N)$ is N_se-closed (briefly, N_seC) if image of every N_scs of (X,τ_N) is N_secs in (Y,σ_N).

Theorem 4.1 The statements are hold but the converse does not true.

(i) Every $N_s\delta C$ mapping is a N_sC mapping.

(ii) Every N_sC mapping is a $N_s\delta\mathcal{S}C$ mapping.

(iii) Every N_sC mapping is a $N_s\delta\mathcal{P}C$ mapping.

(iv) Every $N_s\delta\mathcal{S}C$ mapping is a N_seC mapping.

(v) Every $N_s\delta\mathcal{P}C$ mapping is a N_seC mapping.

(vi) Every N_seC mapping is a N_se^*C mapping.

Proof. We prove only (v), the others are similar.

(v) Let λ be a N_scs in X. Since h is $N_s\delta\mathcal{P}C$ mapping, $h(\lambda)$ is a $N_s\delta\mathcal{P}cs$ in Y. Since every $N_s\delta\mathcal{P}cs$ is a N_secs [16], $h(\lambda)$ is a N_secs in Y. Hence h is a N_seC mapping.

Example 4.1 In Example 3.1, h is a N_sC map but not $N_s\delta C$ map.

Example 4.2 In Example 3.2, h is a $N_s\delta\mathcal{S}C$ map but not N_sC map.

Example 4.3 In Example 3.3, h is a $N_s\delta\mathcal{P}C$ map but not N_sC map.

Example 4.4 In Example 3.4, h is a N_seC map but not $N_s\delta\mathcal{P}C$ map.

Example 4.5 In Example 3.5, h is a N_seC map but not $N_s\delta\mathcal{S}C$ map.

Example 4.6 In Example 3.6, h is a N_se^*C map but not N_seC map.

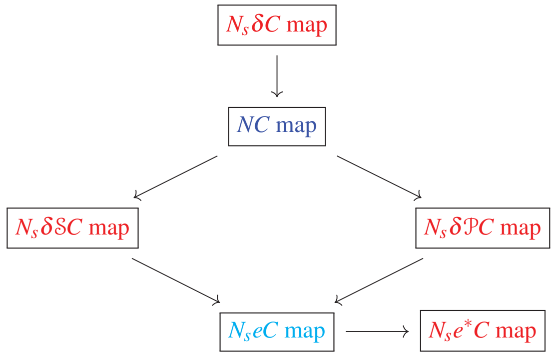

FIGURE 2. N_seC map's in N_sts.

Theorem 4.2 A mapping $h:(X,\tau_N)\to(Y,\sigma_N)$ is N_seC iff for each N_ss μ of (Y,σ_N) and for each N_sos λ of (X,τ_N) containing $h^{-1}(\mu)$ there is a N_seos ψ of (Y,σ_N) such that $\mu\subseteq\psi$ and $h^{-1}(\psi)\subseteq\lambda$.

Proof. Necessity: Assume h is a N_seC mapping. Let μ be the N_scs of (Y,σ_N) and λ is a N_sos of (X,τ_N) such that $h^{-1}(\mu)\subseteq\lambda$. Then $\psi=Y-h^{-1}(\lambda^c)$ is N_seos of (Y,σ_N) such that $h^{-1}(\psi)\subseteq\lambda$.

Sufficiency: Assume ψ is a N_scs of (X,τ_N). Then $(h(\psi))^c$ is a N_ss of (Y,σ_N) and ψ^c is N_sos in (X,τ_N) such that $h^{-1}((h(\psi))^c)\subseteq\psi^c$. By hypothesis there is a N_seos ψ of (Y,σ_N) such that $(h(\psi))^c\subseteq\psi$ and $h^{-1}(\psi)\subseteq\psi^c$. Therefore $\psi\subseteq(h^{-1}(\psi))^c$. Hence $\psi^c\subseteq h(\psi)\subseteq h((h^{-1}(\psi))^c)\subseteq\psi^c$ which implies $h(\psi)=\psi^c$. Since ψ^c is N_secs of (Y,σ_N). Hence $h(\psi)$ is N_sec in (Y,σ_N) and thus h is N_seC mapping.

Theorem 4.3 If $h:(X,\tau_N)\to(Y,\sigma_N)$ is N_sC and $g:(Y,\sigma_N)\to(Z,\rho_N)$ is N_seC. Then $g\circ h:(X,\tau_N)\to(Z,\rho_N)$ is N_seC.

Proof. Let ψ be a N_scs in (X,τ_N). Then $h(\psi)$ is N_scs of (Y,σ_N) because h is N_sC mapping. Now $(g\circ h)(\psi)=g(h(\psi))$ is N_secs in (Z,ρ_N) because g is N_seC mapping. Thus $g\circ h$ is N_seC mapping.

Theorem 4.4 If $h:(X,\tau_N)\to(Y,\sigma_N)$ is N_seC map, then $N_secl(h(\psi))\subsetneq h(N_scl(\psi))$.

Proof. Obvious.

Theorem 4.5 Let $h:(X,\tau_N)\to(Y,\sigma_N)$ and $g:(Y,\sigma_N)\to(Z,\rho_N)$ are N_seC mappings. If every N_secs of (Y,σ_N) is N_sc then, $g\circ h:(X,\tau_N)\to(Z,\rho_N)$ is N_seC.

Proof. Let ψ be a N_scs in (X,τ_N). Then $h(\psi)$ is N_secs of (Y,σ_N) because h is N_seC mapping. By hypothesis $h(\psi)$ is N_scs of (Y,σ_N). Now $g(h(\psi))=(g\circ h)(\psi)$ is N_secs in (Z,ρ_N) because g is N_seC mapping. Thus $g\circ h$ is N_seC mapping.

Theorem 4.6 Let $h:(X,\tau_N)\to(Y,\sigma_N)$ be a objective mapping, then the following statements are equivalent:

(i) h is a N_seO mapping.

(ii) h is a N_seC mapping.

(iii) h^{-1} is N_seCts mapping.

Proof. (i) $\Rightarrow$ (ii): Let us assume that h is a N_seO mapping. By definition, ψ is a N_sos in (X,τ_N), then $h(\psi)$ is a N_seos in (Y,σ_N). Here, ψ is N_scs in (X,τ_N), then $X-\psi$ is a N_sos in (X,τ_N). By assumption, $h(X-\psi)$ is a N_seos in (Y,σ_N). Hence, $Y-h(X-\psi)$ is a N_secs in (Y,σ_N). Therefore, h is a N_seC mapping.

(ii) $\Rightarrow$ (iii): Let ψ be a N_scs in (X,τ_N) By (ii), $h(\psi)$ is a N_secs in (Y,σ_N). Hence, $h(\psi)=(h^{-1})^{-1}(\psi)$, so h^{-1} is a N_secs in (Y,σ_N). Hence, h^{-1} is N_seCts.

(iii) $\Rightarrow$ (i): Let ψ be a N_sos in (X,τ_N) By (iii), $(h^{-1})^{-1}(\psi)=h(\psi)$ is a N_seO mapping.

NEUTROSOPHIC e-HOMEOMORPHISM

Definition 5.1 A bijection $h:(X,\tau_N)\to(Y,\sigma_N)$ is called a N_se-homeomorphism (briefly N_seHom) if h and h^{-1} are N_seCts.

Theorem 5.1 Each N_sHom is a N_seHom. But not conversely.

Proof. Let h be N_sHom, then h and h^{-1} are N_sCts. But every N_sCts function is N_seCts. Hence, h and h^{-1} is N_seCts. Therefore, h is a N_seHom.

Example 5.1 Let $X = \{a,b,c\} = Y$ and define N_ss's X_1, X_2 & X_3 in X and Y_1 in Y are

$$X_1 = \langle X, (\frac{\mu_a}{0.2}, \frac{\mu_b}{0.3}, \frac{\mu_c}{0.4}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\nu_a}{0.8}, \frac{\nu_b}{0.7}, \frac{\nu_c}{0.6})\rangle,$$
$$X_2 = \langle X, (\frac{\mu_a}{0.1}, \frac{\mu_b}{0.1}, \frac{\mu_c}{0.4}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\nu_a}{0.9}, \frac{\nu_b}{0.9}, \frac{\nu_c}{0.6})\rangle,$$
$$X_3 = \langle X, (\frac{\mu_a}{0.2}, \frac{\mu_b}{0.4}, \frac{\mu_c}{0.4}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\nu_a}{0.8}, \frac{\nu_b}{0.6}, \frac{\nu_c}{0.6})\rangle,$$
$$Y_1 = \langle Y, (\frac{\mu_a}{0.2}, \frac{\mu_b}{0.4}, \frac{\mu_c}{0.4}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\nu_a}{0.8}, \frac{\nu_b}{0.6}, \frac{\nu_c}{0.6})\rangle.$$

Then we have $\tau_N = \{0_N, X_1, X_2, 1_N\}$ and $\sigma_N = \{0_N, Y_1, 1_N\}$. Let $h : (X, \tau_N) \to (Y, \sigma_N)$ be an identity mapping, then h is N_seHom but not N_sHom.

Theorem 5.2 Let $h : (X, \tau_N) \to (Y, \sigma_N)$ be a bijective mapping. If h is N_seCts, then the following statements are equivalent:

(i) h is a N_seC mapping.

(ii) h is a N_seO mapping.

(iii) h^{-1} is a N_seHom.

Proof. (i) $\Rightarrow$ (ii) : Assume that h is a bijective mapping and a N_seC mapping. Hence, h^{-1} is a N_seCts mapping. We know that each N_sos in (X, τ_N) is a N_seos in (Y, σ_N). Hence, h is a N_seO mapping.

(ii) $\Rightarrow$ (iii) : Let h be a bijective and N_sO mapping. Further, h^{-1} is a N_seCts mapping. Hence, h and h^{-1} are N_seCts. Therefore, h is a N_seHom.

(iii) $\Rightarrow$ (i): Let h be a N_seHom, then h and h^{-1} are N_seCts. Since each N_scs in (X, τ_N) is a N_secs in (Y, σ_N), hence h is a N_seC mapping.

Definition 5.2 A N_sts (X, τ_N) is said to be a neutrosophic $eT_{\frac{1}{2}}$ (briefly, $N_seT_{\frac{1}{2}}$)-space if every N_secs is N_sc in (X, τ_N).

Theorem 5.3 Let $h : (X, \tau_N) \to (Y, \sigma_N)$ be a N_seHom, then h is a N_sHom if (X, τ_N) and (Y, σ_N) are $N_seT_{\frac{1}{2}}$-space.

Proof. Assume that ψ is a N_scs in (Y, σ_N), then $h^{-1}(\psi)$ is a N_secs in (X, τ_N). Since (X, τ_N) is an $N_seT_{\frac{1}{2}}$-space, $h^{-1}(\psi)$ is a N_scs in (X, τ_N). Therefore, h is N_sCts. By hypothesis, h^{-1} is N_seCts. Let ζ be a N_scs in (X, τ_N). Then, $(h^{-1})^{-1}(\zeta) = h(\zeta)$ is a N_scs in (Y, σ_N), by presumption. Since (Y, σ_N) is a $N_seT_{\frac{1}{2}}$-space, $h(\zeta)$ is a N_scs in (Y, σ_N). Hence, h^{-1} is N_sCts. Hence, h is a N_sHom.

Theorem 5.4 Let $h : (X, \tau_N) \to (Y, \sigma_N)$ be a N_sts, then the following are equivalent if (Y, σ_N) is a $N_seT_{\frac{1}{2}}$-space:

(i) h is N_seC mapping.

(ii) If ψ is a N_sos in (X, τ_N), then $h(\psi)$ is N_seos in (Y, σ_N).

(iii) $h(N_sint(\psi)) \subseteq N_scl(N_sint(h(\psi)))$ for every N_ss ψ in (X, τ_N).

Proof. (i) $\Rightarrow$ (ii): Obvious.

(ii) $\Rightarrow$ (iii): Let ψ be a N_ss in (X, τ_N). Then, $N_sint(\psi)$ is a N_sos in (X, τ_N). Then, $h(N_sint(\psi))$ is a N_seos in (Y, σ_N). Since (Y, σ_N) is a $N_seT_{\frac{1}{2}}$-space, so $h(N_sint(\psi))$ is a N_sos in (Y, σ_N). Therefore, $h(N_sint(\psi)) = N_sint(h(N_sint(\psi))) \subseteq N_scl(N_sint(h(\psi)))$.

(iii) $\Rightarrow$ (i): Let ψ be a N_scs in (X, τ_N). Then, ψ^c is a N_sos in (X, τ_N). From, $h(N_sint(\psi^c)) \subseteq N_scl(N_sint(h(\psi^c)))$. Hence, $h(\psi^c) \subseteq N_scl(N_sint(h(\psi^c)))$. Therefore, $h(\psi^c)$ is N_seos in (Y, σ_N). Therefore, $h(\psi)$ is a N_secs in (X, τ_N). Hence, h is a N_sC mapping.

Theorem 5.5 Let $h : (X, \tau_N) \to (Y, \sigma_N)$ and $g : (Y, \sigma_N) \to (Z, \rho_N)$ be N_seC, where (X, τ_N) and (Z, ρ_N) are two N_sts's and (Y, σ_N) a $N_seT_{\frac{1}{2}}$-space, then the composition $g \circ h$ is N_seC.

Proof. Let ψ be a N_scs in (X, τ_N). Since h is N_sec and $h(\psi)$ is a N_secs in (Y, σ_N), by assumption, $h(\psi)$ is a N_scs in (Y, σ_N). Since g is N_sec, then $g(h(\psi))$ is N_sec in (Z, ρ_N) and $g(h(\psi)) = (g \circ h)(\psi)$. Therefore, $g \circ h$ is N_seC.

Theorem 5.6 Let $h : (X, \tau_N) \to (Y, \sigma_N)$ and $g : (Y, \sigma_N) \to (Z, \rho_N)$ be two N_sts's, then the following hold:

(i) If $g \circ h$ is N_seO and h is N_sCts, then g is N_seO.

(ii) If $g \circ h$ is N_sO and g is N_seCts, then h is N_seO.

Proof. Obvious.

NEUTROSOPHIC *e*-C HOMEOMORPHISM

Definition 6.1 A bijection $h : (X, \tau_N) \to (Y, \sigma_N)$ is called a N_se-Completely homeomorphism (briefly, N_seCHom) if h and h^{-1} are N_seIrr mappings.

Theorem 6.1 Each N_seCHom is a N_seHom. But not conversely.

Proof. Let us assume that ψ is a N_scs in (Y, σ_N). This shows that ψ is a N_secs in (Y, σ_N). By assumption, $h^{-1}(\psi)$ is a N_secs in (X, τ_N). Hence, h is a N_seCts mapping. Hence, h and h^{-1} are N_seCts mappings. Hence h is a N_seHom.

Example 6.1 Let $X = \{a, b, c\} = Y$ and define N_ss's X_1 & X_2 in X and Y_1 in Y are

$$X_1 = \langle X, (\frac{\mu_a}{0.2}, \frac{\mu_b}{0.3}, \frac{\mu_c}{0.4}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\nu_a}{0.8}, \frac{\nu_b}{0.7}, \frac{\nu_c}{0.6}) \rangle,$$
$$X_2 = \langle X, (\frac{\mu_a}{0.1}, \frac{\mu_b}{0.1}, \frac{\mu_c}{0.4}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\nu_a}{0.9}, \frac{\nu_b}{0.9}, \frac{\nu_c}{0.6}) \rangle,$$
$$Y_1 = \langle Y, (\frac{\mu_a}{0.4}, \frac{\mu_b}{0.3}, \frac{\mu_c}{0.2}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\nu_a}{0.6}, \frac{\nu_b}{0.7}, \frac{\nu_c}{0.8}) \rangle.$$

Then we have $\tau_N = \{0_N, X_1, X_2, 1_N\}$ and $\sigma_N = \{0_N, Y_1, 1_N\}$. Let $h : (X, \tau_N) \to (Y, \sigma_N)$ be mapping defined as $h(a) = c$, $h(b) = b$ & $h(c) = a$, then h is N_seHom but not N_seCHom.

Theorem 6.2 If $h : (X, \tau_N) \to (Y, \sigma_N)$ is a N_seCHom, then $N_secl(h^{-1}(\psi)) \subseteq h^{-1}(N_scl(\psi))$ for each N_sts ψ in (Y, σ_N).

Proof. Let ψ be a N_sts in (Y, σ_N). Then, $N_scl(\psi)$ is a N_scs in (Y, σ_N), and every N_scs is a N_secs in (Y, σ_N). Assume h is N_seIrr, $h^{-1}(N_scl(\lambda))$ is a N_secs in (X, τ_N), then $N_scl(h^{-1}(N_scl(\psi))) = h^{-1}(N_scl(\psi))$. Here, $N_secl(h^{-1}(\psi)) \subseteq N_secl(h^{-1}(N_scl(\psi))) = h^{-1}(N_scl(\psi))$. Therefore, $N_secl(h^{-1}(\psi)) \subseteq h^{-1}(N_scl(\psi))$ for every N_ss ψ in (Y, σ_N).

Theorem 6.3 Let $h : (X, \tau_N) \to (Y, \sigma_N)$ be a N_seCHom, then $N_secl(h^{-1}(\psi)) = h^{-1}(N_secl(\psi))$ for each N_ss ψ in (Y, σ_N).

Proof. Since h is a N_seCHom, then h is a N_seIrr mapping. Let ψ be a N_ss in (Y, σ_N). Clearly, $N_secl(\psi)$ is a N_secs in (X, τ_N). Then $N_secl(\psi)$ is a N_secs in (X, τ_N). Since $h^{-1}(\psi) \subseteq h^{-1}(N_secl(\psi))$, then $N_secl(h^{-1}(\psi)) \subseteq N_secl(h^{-1}(N_secl(\psi))) = h^{-1}(N_secl(\psi))$. Therefore, $N_secl(h^{-1}(\psi)) \subseteq h^{-1}(N_secl(\psi))$. Let h be a N_seCHom. h^{-1} is a N_seIrr mapping. Let us consider N_ss $h^{-1}(\psi)$ in (X, τ_N), which implies $N_secl(h^{-1}(\psi))$ is a N_secs in (X, τ_N). Hence, $N_secl(h^{-1}(\psi))$ is a N_secs in (X, τ_N). This implies that $(h^{-1})^{-1}(N_secl(h^{-1}(\psi))) = h(N_secl(h^{-1}(\psi)))$ is a N_secs in (Y, σ_N). This proves $\psi = (h^{-1})^{-1}(h^{-1}(\psi)) \subseteq (h^{-1})^{-1}(N_secl(h^{-1}(\psi))) = h(N_secl(h^{-1}(\psi)))$. Therefore, $N_secl(\psi) \subseteq N_secl(h(N_secl(h^{-1}(\psi)))) = h(N_secl(h^{-1}(\psi)))$, since h^{-1} is a N_seIrr mapping. Hence, $h^{-1}(N_secl(\psi)) \subseteq h^{-1}(h(N_secl(h^{-1}(\psi)))) = N_secl(h^{-1}(\psi))$. That is, $h^{-1}(N_secl(\psi)) \subseteq N_secl(h^{-1}(\psi))$. Hence, $N_secl(h^{-1}(\psi)) = h^{-1}(N_secl(\psi))$.

Theorem 6.4 If $h : (X, \tau_N) \to (Y, \sigma_N)$ and $g : (Y, \sigma_N) \to (Z, \rho_N)$ are N_seCHom's, then $g \circ h$ is a N_seCHom.

Proof. Let h and g to be two N_seCHom's. Assume ψ is a N_secs in (Z, ρ_N). Then, $g^{-1}(\psi)$ is a N_secs in (Y, σ_N). Then, by hypothesis, $h^{-1}(g^{-1}(\psi))$ is a N_secs in (X, τ_N). Hence, $g \circ h$ is a N_seIrr mapping. Now, let ζ be a N_secs in (X, τ_N). Then, by presumption, $h(g)$ is a N_secs in (Y, σ_N). Then, by hypothesis, $g(h(\zeta))$ is a N_secs in (Z, ρ_N). This implies that $g \circ h$ is a N_seIrr mapping. Hence, $g \circ h$ is a N_seCHom.

CONCLUSIONS

In this paper, the new concept of a N_sHom and a N_seHom in N_sts was discussed. Furthermore, the work was extended as the N_seCHom, N_seO and N_seC mapping and $N_seT_{\frac{1}{2}}$-space. Further, the study demonstrated N_seCHom's and also derived some of their related attributes. In future, we can carry out the further research on neutrosophic *e*-compactness, neutrosophic *e*-connectedness and neutrosophic contra *e*-continuous functions.

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