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Research Article

The Approach of Induced Generalized Neutrosophic Cubic Shapley Choquet Integral Aggregation Operators via the CODAS Method to Solve Distance-Based Multicriteria Decision-Making Problems

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This study aims to define a conjecture that can handle complex frames of work more efficiently that occurs in daily life problems. In decision-making theory inter-relation of criteria, weights and choice decision-making method subject to the given circumstances which are an important component for appropriate decisions. For this, we define neutrosophic cubic Shapley–Choquet integral (NCSCI) measure; combinative distance-based assessment selection (CODAS) is accomplished over NCSCI and is implemented over a numerical example of a company foreign investment model as an application in decision-making (DM) theory. The neutrosophic cubic set (NCS) is a hybrid of the neutrosophic set (NS) and interval neutrosophic set (INS), which provides a better plate form to handle inconsistent and vague data more conveniently. The novel CODAS method is based on Shapley–Choquet integral and Minkowski distance which contain more information measures than usual criteria weights and distances. The weights of criteria are measured by Shapley–Choquet integral and distance is evaluated by Minkowski distance. The Choquet integral considers the interaction among the criteria, and Shapley considers the overall weight criteria. Motivated by these characteristics NCSCI, we defined two aggregation operators' induced-generalized neutrosophic cubic Shapley–Choquet integral arithmetic (IGNCSCIA) and operators' induced-generalized neutrosophic cubic Shapley–Choquet integral geometric (IGNCSCIG) operators. To find the distance between two NC values, Minkowski distance is defined to evaluate neutrosophic cubic combinative distance-based assessment selection (NCCODAS). To examine the feasibility of the proposed method, an example of company investment in a foreign country is considered. To check, the validity of the method, the comparative analysis of the proposed method with other methods is conducted.

1. Introduction

Increasing uncertainty and complexity in decision-making (DM) theory, the representation of data is no longer the real number. The researcher developed different theories that can handle such data appropriately. Among these, Zadeh initiated the fuzzy set (FS) [1] to deal with the uncertainty. A fuzzy set consists of a crisp value from [0,1] referred to as a membership degree. FS was further extended into an interval-valued fuzzy set (IVFS) [2, 3], in which the membership degree is a subinterval of [0,1]. Atnassove instigated a nonmembership function to FS and named it as an intuitionistic fuzzy set (IFS) [4]. Both membership and nonmembership are dependent. IFS was generalized into an interval-valued intuitionistic fuzzy set (IVIFS) [5]. Jun combined FS and IVFS to form a cubic set (CS) [6]. These generalizations of FS handle vague and inconsistent data in the form of membership, and nonmembership degrees can be assigned crisp and interval values. In a complex frame of work, the situation often arises in which one is unable to completely specify the data by assigning an argument membership grade and nonmembership grades only. This limitation can be overcome by Smrandache neutrosophic set (NS) [7]. An NS consists of three independent components, truth, indeterminancy, and falsity grades. The NS provides a wide range of choosing so that the data can easily be associated according to the complex frame of the environment. NS was further extended into the interval neutrosophic set (INS) [8]. INS provides the choice of choosing in the form of interval values. The problem arises that whether these components can be assigned with both the interval value and the crisp value at the same time. This problem can be tackled by neutrosophic cubic set (NCS) [9] and the hybrid of NS and INS. NCS provides the plate form to choose the value in the form of a crisp value along with the interval value at the same time. This makes NCS a useful tool to represent the fuzziness of acceptance, neutral, and rejection in the complex frame of the environment more conveniently. These characteristics attract the researcher to apply in the field of DM theory. Majid et al. defined novel operational laws on NCS [10].

The aggregation operator is an important component of DM theory. MCDM problem involves conflicting criteria and aggregation operators are used to aggregate the conflicting criteria to conclude problems [11–16]. Most of the aggregation operators deal with the criteria independently; interaction among the criteria and overall criteria is not considered by such aggregation operators. These limitations can be overcome by Coquet integral [17, 18] that considers the interaction among two adjacent criteria. To consider the overall interaction of criteria, Sugano defined Shapley fuzzy measure [19–23] that considers the overall interaction and importance of criteria. It can also be used to establish the weights and distribution of criteria [24]. Shapley measure is more flexible than probability by its additive property [25]. Combining the idea of Shapley measure and Choquet integral will tackle the overall and partial information of input argument [26, 27].

1.1. Motivation. The motivation of this research is to generalize Shapley measure and Choquet integral operator in the NCS plate form. That is aggregation operators that tackle the interaction among the criteria and overall interaction of criteria become a handy tool to handle complex frames of the environment. The Shapley measure will handle the overall interaction of criteria and weightage of criteria. Choquet integral will look after the interaction of amongst criteria, and NCS will provide a platform for data to handle complex frames of the environment.

1.2. Contribution. This study contributes the following work:

(i) The induced generalized Shapley–Choquet integral is defined
(ii) The IGNCSCIA operator is defined
(iii) The IGNCSCIG operator is defined
(iv) Some significant properties are investigated
(v) Minkowski distance is defined
(vi) NCCODAS method is defined to handle distance-based DM problems

To check the validity of the proposed method, the comparative analysis is investigated with some existing methods.

1.3. Organization. The organization process of research is shown in Figure 1.

The research paper has been divided into four sections. Section 1 comprises of introduction. Section 2 comprises of preliminaries, definition, and results. The section will help to work out the proposed research. Section 3 consists of IGNCSCI, IGNCSCIA, and IGNCSCIG aggregation operators along with some important properties and neutrosophic cubic Minkowski distance. Section 4 consists of NCCODAS method, numerical example as an application, and comparative analysis.

2. Preliminaries

This section consists of two section developments in NCS to neutrosophic cubic set and fuzzy preferences.

2.1. Development of NCS

Definition 1 (see [1]). A mapping $\psi: U \longrightarrow [0,1]$ is called a fuzzy set, and $\psi(u)$ is called a membership function, simply denoted by ψ.

Definition 2 (see [2]). A mapping $\widetilde{\psi}: U \longrightarrow D[0,1]$, where $D[0,1]$ is the interval value of $[0,1]$, called the interval-valued fuzzy set(IVF). For all $u \in U$, $\widetilde{\psi}(u) = \{[\psi^L(u), \psi^U(u)] | \psi^L(u), \psi^U(u) \in [0,1] \text{ and } \psi^L(u) \leq \psi^U(u)\}$ is membership degree of u in $\widetilde{\psi}$. This is simply denoted by $\widetilde{\psi} = [\psi^L, \psi^U]$.

Definition 3 (see [6]). A structure $C = \{(u, \widetilde{\psi}(u), \psi(u)) | u \in U\}$ is a cubic set in U in which $\widetilde{\psi}(u)$ is IVFS in U, that is, $\widetilde{\psi} = [\psi^L, \psi^U]$ and $\widetilde{\psi}$ is a fuzzy set in U. This can be simply denoted by $C = (\widetilde{\psi}, \psi)$.

Definition 4 (see [7]). The neutrosophic set is defined as

$$N = \{(T_N(\ddot{\gamma}), I_N(\ddot{\gamma}), F_N(\ddot{\gamma})) | \ddot{\gamma} \in \Gamma\}, \tag{1}$$

where $T_N(\ddot{\gamma}), I_N(\ddot{\gamma})$, and $F_N(\ddot{\gamma})$ are truth, indeterminacy, and falsity fuzzy functions.

Definition 5 (see [8]). An INS is an extension of NS defined by

$$\widetilde{N} = \{(\widetilde{T}_N(\ddot{\gamma}), \widetilde{I}_N(\ddot{\gamma}), \widetilde{F}_N(\ddot{\gamma})) | \ddot{\gamma} \in \Gamma\}, \tag{2}$$

where $\widetilde{T}_N(\ddot{\gamma}), \widetilde{I}_N(\ddot{\gamma})$, and $\widetilde{F}_N(\ddot{\gamma})$ are interval-valued fuzzy truth, indeterminacy, and falsity function.

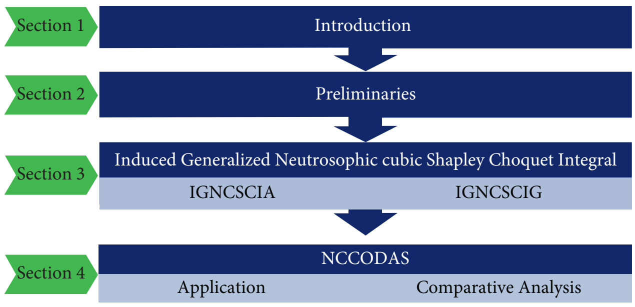

Figure 1: Research organization process.

Definition 6 (see [9]). A NCS is hybrid of NS and INS and defined as

$$N = \left\{ \left(\widetilde{T}_N(\ddot{\gamma}), \widetilde{I}_N(\ddot{\gamma}), \widetilde{F}_N(\ddot{\gamma}), T_N(\ddot{\gamma}), I_N(\ddot{\gamma}), F_N(\ddot{\gamma}) \right) / \ddot{\gamma} \in \Gamma, \tag{3}$$

where $(\widetilde{T}_N(\ddot{\gamma}) = [T_N^L(\ddot{\gamma}), T_N^U(\ddot{\gamma})], \widetilde{I}_N(\ddot{\gamma}) = [I_N^L(\ddot{\gamma}), I_N^U(\ddot{\gamma})], \widetilde{F}_N(\ddot{\gamma}) = [F_N^L(\ddot{\gamma}), F_N^U(\ddot{\gamma})])$ is an INS, and $(T_N(\ddot{\gamma}), I_N(\ddot{\gamma}), F_N(\ddot{\gamma}))$ is a NS, where $\widetilde{0} \leq \widetilde{T}_N + \widetilde{I}_N + \widetilde{F}_N \leq \widetilde{3}, 0 \leq T_N + I_N + F_N \leq 3$.

For the sake of convenience, the NCS are written as $N = ([T_N^L, T_N^U], [I_N^L, I_N^U], [F_N^L, F_N^U], T_N, I_N, F_N)$.

Definition 7 (see [10]). The sum of two NCS, $N = ([T_N^L, T_N^U], [I_N^L, I_N^U], [F_N^L, F_N^U], T_N, I_N, F_N)$ and $M = ([T_M^L, T_M^U], [I_M^L, I_M^U], [F_M^L, F_M^U], T_M, I_M, F_M)$, is defined as

$$N \oplus M = \begin{pmatrix} [T_N^L + T_M^L - T_N^L T_M^L, T_N^U + T_M^U - T_N^U T_M^U], [I_N^L + I_M^L - I_N^L I_M^L, I_N^U + I_M^U - I_N^U I_M^U], \\ [F_N^L F_M^L, F_N^U F_M^U], T_N T_M, I_N I_M, F_N + F_M - F_N F_M \end{pmatrix}. \tag{4}$$

Definition 8 (see [10]). The product of two NCS, $N = ([T_N^L, T_N^U], [I_N^L, I_N^U], [F_N^L, F_N^U], T_N, I_N, F_N)$ and $M = ([T_M^L, T_M^U], [I_M^L, I_M^U], [F_M^L, F_M^U], T_M, I_M, F_M)$, is defined as

$$N \otimes M = \begin{pmatrix} [T_N^L T_M^L, T_N^U T_M^U], [I_N^L I_M^L, I_N^U I_M^U], [F_N^L + F_M^L - F_N^L F_M^L, F_N^U + F_M^U - F_N^U F_M^U], \\ T_N + T_M - T_N T_M, I_N + I_M - I_N I_M, F_N F_M \end{pmatrix}, \tag{5}$$

Definition 9 (see [10]). The scalar multiplication on a NCS $N = ([T_N^L, T_N^U], [I_N^L, I_N^U], [F_N^L, F_N^U], T_N, I_N, F_N)$ and a scalar η is defined:

$$\eta N = \begin{pmatrix} \left[1 - \left(1 - T_N^L\right)^\eta, 1 - \left(1 - T_N^U\right)^\eta\right], \left[1 - \left(1 - I_N^L\right)^\eta, 1 - \left(1 - I_N^U\right)^\eta\right], \\ \left[\left(F_N^L\right)^\eta, \left(F_N^U\right)^\eta\right], (T_N)^\eta, (I_N)^\eta, 1 - (1 - F_N)^\eta \end{pmatrix}. \tag{6}$$

2.2. Developments in Fuzzy Measure. In decision-making (DM) process, the value $\widehat{l}$ is weighted by w_i weight and then aggregated using weighted averaging and weighted geometric aggregation operators, where $w_i \in [0, 1]$ such that $\sum_{i=1}^{\ddot{n}} w_i = 1$. In real-life problems, there exist interactive phenomena amongst the elements. The overall significance of an element not only specified by itself, but by all the other elements in process.

Sangeno [20] established the notion of fuzzy measure, which not only determines weight of an element and each combination of elements as well, and sum of weights need not to be equal to one. Murofushi and Saneno [21] proposed Choquet integral as an extension of Lebesgue integral. It is a significant aggregation operator for MCDM by considering significance of element by fuzzy measure.

Definition 10 (see [23]). Let $\widehat{T} = \left\{\widehat{l}_1, \widehat{l}_2, \widehat{l}_3, \ldots, \widehat{l}_{\ddot{n}}\right\}$ be a set. A fuzzy measure θ on $\widehat{T}$ is defined as a function θ: $\widehat{T} \longrightarrow [0, 1]$ fulfilling the following properties:

(i) $\theta(\varphi) = 0$ and $\theta(\widehat{T}) = 1$

(ii) For $U, V \in P(\widehat{T}), U \subseteq V \Longrightarrow \theta(U) \leq \theta(V)$, where $P(\widehat{T})$ is power set of $\widehat{T}$

Definition 11 (see [23]). In MCDM, for $U, V \in P(\widehat{T})$ such that $U \cap V = \varphi$, three types of interactive relation are possible, that is,

(i) **Additive measure:** if U and V are independent (no interaction), then

$$\theta(U \cup V) = \theta(U) + \theta(V). \tag{7}$$

(ii) **Super additive measure:** if U and V are positive synergetic interaction, then

$$\theta(U\cup V) > \theta(U) + \theta(V), \tag{8}$$

(iii) **Subadditive measure:** if U and V are negative synergetic interaction, then

$$\theta(U\cup V) < \theta(U) + \theta(V). \tag{9}$$

Definition 12 (see [23]). Let k be a function on $\widehat{T}$ and θ be a fuzzy measure on $\widehat{T}$. Then, discrete Choquet with respect to θ is defined by

$$C_\theta\left(k\left(\widehat{l}_{\rho(i)}\right)\right) = \sum_{i=1}^{\ddot{n}} k\left(\widehat{l}_{\rho(i)}\right)\left(\theta\left(\widehat{T}_{\rho(i)}\right) - \theta\left(\widehat{T}_{\rho(i+1)}\right)\right), \tag{10}$$

for $\rho(i)$ as $(\rho(1),\rho(2),\ldots,\rho(i),\ldots,\rho(\ddot{n}))$ present the permutations of $(1,2,\ldots,i,\ldots,\ddot{n})$ such that $k(\widehat{l}_{(1)}) \le k(\widehat{l}_{(2)}) \le k(\widehat{l}_{(3)}) \le \cdots \le k(\widehat{l}_{(\ddot{n})})$ and $\widehat{T} = \left\{\widehat{l}_1, \widehat{l}_2, \widehat{l}_3, \ldots, \widehat{l}_{\ddot{n}}\right\}$ with $\widehat{T}_{\rho(\ddot{n}+1)} = \varphi$. From definition, it is observed that the Choquet integral handles the interaction between two consecutive values; it is unable to handle the overall all interaction. This limitation is overcome by the Shapley index [18]:

$$\varpi_S^{Sh}(\theta, N) = \sum_{T\subseteq N\backslash S} \frac{(\ddot{n}-u-v)!t!}{(\ddot{n}-u+1)!t!}\,(\theta(U\cup V) - \theta(V)), \forall U\subseteq N, \tag{11}$$

where θ is a fuzzy function of fuzzy measure λ, on N, and the cardinality of U, N, and V is, respectively, u, $\ddot{n}$, and v.

Meng [25] generalized the Shapley index to generalized Shapley index as $\lambda-$ fuzzy measure θ_λ on N by

$$\varpi_S^{Sh}(\theta_\lambda, N) = \sum_{T\subseteq N\backslash S} \frac{(\ddot{n}-u-v)!v!}{(\ddot{n}-u+1)!v!}\,(\theta_\lambda(U\cup V) - \theta_\lambda(V)), \forall U\subseteq N, \tag{12}$$

where $\theta_\lambda$is fuzzy measure expressed as

$$\theta_\lambda(A) = \begin{cases} \dfrac{1}{\lambda}\left(\prod_{k\in\widehat{T}}((\lambda\theta_\lambda(i)+1)-1)\right) & \text{if } \lambda \ne 0 \\ \sum_{k\in\widehat{T}} \lambda\theta_\lambda(k) & \text{if } \lambda = 0 \end{cases}, \tag{13}$$

where $\prod_{c\in N}(1+\theta_\lambda\{c_k\}) = 1+\theta_\lambda$ is used to measure λ.Thus, if in $S=\{k\}$, then

$$\varpi_k^{Sh}(\theta_\lambda, N) = \sum_{T\subseteq N\backslash S} \frac{t!(\ddot{n}-1-t)!}{t!(\ddot{n})!}\left(\theta_\lambda(k)\left(\prod_{j\in T}(1+\lambda\theta_\lambda(j))\right)\right), \forall k\subseteq N. \tag{14}$$

Definition 13. Based on these definitions, Meng [25] defined arithmetic $\lambda-$Shapley–Choquet integral operator as

$$C_{\varpi_S^{Sh}(\theta_\lambda,N)}\left(k\left(\widehat{l}_{\rho(i)}\right)\right) = \sum_{i=1}^{\ddot{n}} k\left(\widehat{l}_{\rho(i)}\right)\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right), \tag{15}$$

where $\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right) = \left(\varpi^{Sh}_{\rho(\widehat{T})_{(i)}}(\theta_\lambda, N) - \varpi^{Sh}_{\rho(\widehat{T})_{(i+1)}}(\theta_\lambda, N)\right)$,

for $\rho(i)$ as $(\rho(1),\rho(2),\ldots,\rho(i),\ldots,\rho(\ddot{n}))$ present the permutations of $(1,2,\ldots,i,\ldots,\ddot{n})$ such that $k(\widehat{l}_{(1)}) \le k(\widehat{l}_{(2)}) \le k(\widehat{l}_{(3)}) \le \cdots \le k(\widehat{l}_{(\ddot{n})})$ and $\widehat{T} = \left\{\widehat{l}_1, \widehat{l}_2, \widehat{l}_3, \ldots, \widehat{l}_{\ddot{n}}\right\}$ with $\widehat{T}_{\rho(\ddot{n}+1)} = \varphi$.

Definition 14 (see [25]). Geometric $\lambda-$Shapley–Choquet integral operator is as

$$C_{\varpi_S^{Sh}(\eta_\lambda,N)}\left(k\left(\widehat{l}_{\rho(i)}\right)\right) = \prod_{i=1}^{\ddot{n}} \left(k\left(\widehat{l}_{\rho(i)}\right)\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}$$

$$\text{where } \Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right) = \left(\varpi^{Sh}_{\rho(\widehat{T})_{(i)}}(\theta_\lambda, N) - \varpi^{Sh}_{\rho(\widehat{T})_{(i+1)}}(\theta_\lambda, N)\right) \tag{16}$$

for $\rho(i)$ as $(\rho(1),\rho(2),\ldots,\rho(i),\ldots,\rho(\ddot{n}))$ present the permutations of $(1,2,\ldots,i,\ldots,\ddot{n})$ such that $k(\widehat{l}_{(1)}) \le k(\widehat{l}_{(2)}) \le k(\widehat{l}_{(3)}) \le \cdots \le k(\widehat{l}_{(\ddot{n})})$ and $\widehat{T} = \left\{\widehat{l}_1, \widehat{l}_2, \widehat{l}_3, \ldots, \widehat{l}_{\ddot{n}}\right\}$ with $\widehat{T}_{\rho(\ddot{n}+1)} = \varphi$.

2.3. Induced Generalized Shapley–Choquet Integral. Aggregation operator is an important component of DM theory. The suitable aggregation operator may reduce the challenges that are present in vague and inconsistent data.

Different operators are defined to meet these challenges. IGNCSCIA and IGNCSCIG aggregation operators will be defined to meet the challenges of criterion weights and interaction.

Definition 15. Let $\widehat{l}_i = ([(T^L)_{\widehat{l}_i}, (T^U)_{\widehat{l}_i}], [(I^L)_{\widehat{l}_i}, (I^U)_{\widehat{l}_i}], [(F^L)_{\widehat{l}_i}, (F^U)_{\widehat{l}_i}], (T)_{\widehat{l}_i}, (I)_{\widehat{l}_i}, (F)_{\widehat{l}_i}),$ where $(i = 1, 2, \ldots, \ddot{n})$ be a collection of NC values, and θ be a fuzzy measure on $\widehat{T} = \{\widehat{l}_1, \widehat{l}_2, \widehat{l}_3, \ldots, \widehat{l}_n\}$ such that $\theta(\widehat{l}_i) = \widehat{u}_i$; then, the IGNCSCIA operator is defined as

$$IGNCSCIA_{\theta_\lambda}\left(\langle\widehat{u}_1, \widehat{l}_1\rangle, \langle\widehat{u}_2, \widehat{l}_2\rangle, \ldots, \langle\widehat{u}_n, \widehat{l}_{\ddot{n}}\rangle\right) = \left(\oplus_{i=1}^{\ddot{n}} k\left(\widehat{l}_i\right)^q \Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)^{1/q}\right), \quad (17)$$

where $q \in (0, \infty)$ and $\rho(i)$ as $(\rho(1), \rho(2), \ldots, \rho(i), \ldots, \rho(\ddot{n}))$ present the permutations of $(1, 2, \ldots, i, \ldots, \ddot{n})$ such that $k(\widehat{l}_{(1)}) \leq k(\widehat{l}_{(2)}) \leq k(\widehat{l}_{(3)}) \leq \cdots \leq k(\widehat{l}_{(\ddot{n})})$ and $\widehat{T} = \{\widehat{l}_1, \widehat{l}_2, \widehat{l}_3, \ldots, \widehat{l}_{\ddot{n}}\}$ with $\widehat{T}_{\rho(\ddot{n}+1)} = \varphi$.

Theorem 1. *Let $\widehat{l}_i = ([(T^L)_{\widehat{l}_i}, (T^U)_{\widehat{l}_i}], [(I^L)_{\widehat{l}_i}, (I^U)_{\widehat{l}_i}], [(F^L)_{\widehat{l}_i}, (F^U)_{\widehat{l}_i}], (T)_{\widehat{l}_i}, (I)_{\widehat{l}_i}, (F)_{\widehat{l}_i})$, where $(1, 2, \ldots, i, \ldots, \ddot{n})$ be a collection of NC values, and θ be a fuzzy measure on $\widehat{T} = \{\widehat{l}_1, \widehat{l}_2, \widehat{l}_3, \ldots, \widehat{l}_{\ddot{n}}\}$ such that $\theta(\widehat{l}_i) = \widehat{u}_i$; then, the IGNCSCIA operator is an NC value:*

$$\begin{aligned}
&\mathrm{IGNCSCIA}_{\theta_\lambda}\left(\langle\widehat{u}_1, \widehat{l}_1\rangle, \langle\widehat{u}_2, \widehat{l}_2\rangle, \ldots, \langle\widehat{u}_n, \widehat{l}_{\ddot{n}}\rangle\right) = \left(\oplus_{i=1}^{\ddot{n}} k\left(\widehat{l}_i\right)^q \Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)^{1/q}\right) \\
&= \left(\left[\left(1 - \prod_{i=1}^{\ddot{n}}\left(1 - \left(T^L_{\widehat{l}_{\rho(i)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q}, \left(1 - \prod_{i=1}^{\ddot{n}}\left(1 - \left(T^U_{\widehat{l}_{\rho(i)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q}\right],\right. \\
&\left[\left(1 - \prod_{i=1}^{\ddot{n}}\left(1 - \left(T^L_{\widehat{l}_{\rho(i)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q}, \left(1 - \prod_{i=1}^{\ddot{n}}\left(1 - \left(T^U_{\widehat{l}_{\rho(i)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q}\right], \\
&\left[1 - \left(1 - \prod_{i=1}^{\ddot{n}}\left(1 - \left(1 - \left(F^L_{\widehat{l}_{\rho(i)}}\right)\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q}, 1 - \left(1 - \prod_{i=1}^{\ddot{n}}\left(1 - \left(1 - \left(F^U_{\widehat{l}_{\rho(i)}}\right)\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q}\right], 1 \\
&- \left(1 - \prod_{i=1}^{\ddot{n}}\left(1 - \left(1 - \left(T_{\widehat{l}_{\rho(i)}}\right)\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q}, 1 \\
&- \left(1 - \prod_{i=1}^{\ddot{n}}\left(1 - \left(1 - \left(I_{\widehat{l}_{\rho(i)}}\right)\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q}, \left(1 - \prod_{i=1}^{\ddot{n}}\left(\left(1 - \left(F_{\widehat{l}_{\rho(i)}}\right)^q\right)\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q}\Bigg),
\end{aligned} \quad (18)$$

where $q \in (0, \infty)$ and $(\rho(1), \rho(2), \ldots, \rho(i), \ldots, \rho(\ddot{n}))$ present the permutations of $(1, 2, \ldots, i, \ldots, \ddot{n})$ such that $k(\widehat{l}_{(1)}) \leq k(\widehat{l}_{(2)}) \leq k(\widehat{l}_{(3)}) \leq \cdots \leq k(\widehat{l}_{(\ddot{n})})$ and $\widehat{T} = \{\widehat{l}_1, \widehat{l}_2, \widehat{l}_3, \ldots, \widehat{l}_{\ddot{n}}\}$ with $\widehat{T}_{\rho(\ddot{n}+1)} = \varphi$.

Proof. For $\ddot{n} = 1$, (17) reduces to the NC value by operational laws and equations (15). For $\ddot{n} = 2$,

$$\text{IGNCSCIA}_{\theta_\lambda}\left(\langle\widehat{u}_1,\widehat{l}_1\rangle,\langle\widehat{u}_2,\widehat{l}_2\rangle\right)=\left(\underset{i=1}{\overset{2}{\oplus}}k\left(\widehat{l}_{\rho(i)}\right)^q\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)^{1/q}\right)$$

$$=\left(\left[\left(1-\prod_{i=1}^{2}\left(1-\left(T^L_{\widehat{l}_{\rho(i)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q},\left(1-\prod_{i=1}^{2}\left(1-\left(T^U_{\widehat{l}_{\rho(i)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q}\right],\right.$$

$$\left[\left(1-\prod_{i=1}^{2}\left(1-\left(T^L_{\widehat{l}_{\rho(i)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q},\left(1-\prod_{i=1}^{2}\left(1-\left(T^U_{\widehat{l}_{\rho(i)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q}\right],$$

$$\left[1-\left(1-\prod_{i=1}^{2}\left(1-\left(1-\left(F^L_{\widehat{l}_{\rho(i)}}\right)\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q},1-\left(1-\prod_{i=1}^{2}\left(1-\left(1-\left(F^U_{\widehat{l}_{\rho(i)}}\right)\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q}\right],1$$

$$-\left(1-\prod_{i=1}^{2}\left(1-\left(1-\left(T_{\widehat{l}_{\rho(i)}}\right)\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q},1$$

$$\left.-\left(1-\prod_{i=1}^{2}\left(1-\left(1-\left(I_{\widehat{l}_{\rho(i)}}\right)\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q},\left(1-\prod_{i=1}^{2}\left(\left(1-\left(F_{\widehat{l}_{\rho(i)}}\right)^q\right)\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q}\right), \tag{19}$$

$$\text{IGNCSCIA}_{\theta_\lambda}\left(\langle\widehat{u}_1,\widehat{l}_1\rangle,\langle\widehat{u}_2,\widehat{l}_2\rangle\right)=\left(\underset{i=1}{\overset{2}{\oplus}}k\left(\widehat{l}_{\rho(i)}\right)^q\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)^{1/q}\right)$$

$$=\left(\left[\left(1-\left(\left(1-\left(T^L_{\widehat{l}_{\rho(1)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(1)}}\right)}\right)\left(\left(1-\left(T^L_{\widehat{l}_{\rho(2)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(2)}}\right)}\right)\right)^{1/q},\right.\right.$$

$$\left.\left(1-\left(\left(1-\left(T^U_{\widehat{l}_{\rho(1)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(1)}}\right)}\right)\left(\left(1-\left(T^U_{\widehat{l}_{\rho(2)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(2)}}\right)}\right)\right)^{1/q}\right],$$

$$\left[\left(1-\left(\left(1-\left(I^L_{\widehat{l}_{\rho(1)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(1)}}\right)}\right)\left(\left(1-\left(I^L_{\widehat{l}_{\rho(2)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(2)}}\right)}\right)\right)^{1/q},\right.$$

$$\left.\left(1-\left(\left(1-\left(I^U_{\widehat{l}_{\rho(1)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(1)}}\right)}\right)\left(\left(1-\left(I^U_{\widehat{l}_{\rho(2)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(2)}}\right)}\right)\right)^{1/q}\right],$$

$$\left[1-\left(1-\left(1-\left(1-\left(F^{L}_{\widehat{l}_{\rho(1)}}\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(1)}}\right)}\right)\left(1-\left(1-\left(F^{L}_{\widehat{l}_{\rho(2)}}\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(2)}}\right)}\right)\right)^{1/q},\right.$$

$$\left.1-\left(1-\left(1-\left(1-\left(F^{U}_{\widehat{l}_{\rho(1)}}\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(1)}}\right)}\right)\left(1-\left(1-\left(F^{U}_{\widehat{l}_{\rho(2)}}\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(2)}}\right)}\right)\right)^{1/q}\right],1$$
$$-\left(1-\left(1-\left(1-\left(T_{\widehat{l}_{\rho(1)}}\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(1)}}\right)}\right)\left(1-\left(1-\left(T_{\widehat{l}_{\rho(2)}}\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(2)}}\right)}\right)\right)^{1/q},1$$
$$-\left(1-\left(1-\left(1-\left(I_{\widehat{l}_{\rho(1)}}\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(1)}}\right)}\right)\left(1-\left(1-\left(I_{\widehat{l}_{\rho(2)}}\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(2)}}\right)}\right)\right)^{1/q},$$

$$\left(1-\left(\left(1-\left(F_{\widehat{l}_{\rho(1)}}\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(1)}}\right)}\right)\left(\left(1-\left(F_{\widehat{l}_{\rho(2)}}\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(2)}}\right)}\right)\right)^{1/q}. \tag{20}$$

By operational laws equation, let $\ddot{n} = j$ result holds good:

$$IGNCSCIA_{\theta_{\lambda}}\left(\langle\widehat{u}_1,\widehat{l}_1\rangle,\langle\widehat{u}_2,\widehat{l}_2\rangle,\ldots,\langle\widehat{u}_j,\widehat{l}_j\rangle\right)=\left(\overset{j}{\underset{i=1}{\oplus}}k\left(\widehat{l}_{\rho(i)}\right)^{q}\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)^{1/q}\right)$$
$$=\left(\left[\left(1-\prod_{i=1}^{j}\left(1-\left(T^{L}_{\widehat{l}_{\rho(i)}}\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q},\left(1-\prod_{i=1}^{j}\left(1-\left(T^{U}_{\widehat{l}_{\rho(i)}}\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q}\right],\right.$$

$$\left[\left(1-\prod_{i=1}^{j}\left(1-\left(T^{L}_{\widehat{l}_{\rho(i)}}\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q},\left(1-\prod_{i=1}^{j}\left(1-\left(T^{U}_{\widehat{l}_{\rho(i)}}\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q}\right],$$

$$\left[1-\left(1-\prod_{i=1}^{j}\left(1-\left(1-\left(F^{L}_{\widehat{l}_{\rho(i)}}\right)\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q},1-\left(1-\prod_{i=1}^{j}\left(1-\left(1-\left(F^{U}_{\widehat{l}_{\rho(i)}}\right)\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q}\right],1$$
$$-\left(1-\prod_{i=1}^{j}\left(1-\left(1-\left(T_{\widehat{l}_{\rho(i)}}\right)\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q},1$$
$$-\left(1-\prod_{i=1}^{j}\left(1-\left(1-\left(I_{\widehat{l}_{\rho(i)}}\right)\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q},\left(1-\prod_{i=1}^{j}\left(\left(1-\left(F_{\widehat{l}_{\rho(i)}}\right)^{q}\right)\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q}. \tag{21}$$

For $\ddot{n}=j+1$,

$$\text{IGNCSCIA}_{\theta_\lambda}\left(\langle\widehat{u}_1,\widehat{l}_1\rangle,\langle\widehat{u}_2,\widehat{l}_2\rangle,\ldots,\langle\widehat{u}_{j+1},\widehat{l}_{j+1}\rangle\right)=\left(\overset{j+1}{\underset{i=1}{\oplus}}k\left(\widehat{l}_{\rho(i)}\right)^{q}\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)^{1/q}\right)$$
$$=\left(\left[\left(1-\prod_{i=1}^{j+1}\left(1-\left(T^{L}_{\widehat{l}_{\rho(i)}}\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q},\left(1-\prod_{i=1}^{j+1}\left(1-\left(T^{U}_{\widehat{l}_{\rho(i)}}\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q}\right],\right.$$
$$\left[\left(1-\prod_{i=1}^{j+1}\left(1-\left(T^{L}_{\widehat{l}_{\rho(i)}}\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q},\left(1-\prod_{i=1}^{j+1}\left(1-\left(T^{U}_{\widehat{l}_{\rho(i)}}\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q}\right],$$
$$\left[1-\left(1-\prod_{i=1}^{j+1}\left(1-\left(1-\left(F^{L}_{\widehat{l}_{\rho(i)}}\right)\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q},1-\left(1-\prod_{i=1}^{j+1}\left(1-\left(1-\left(F^{U}_{\widehat{l}_{\rho(i)}}\right)\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q}\right],1$$
$$-\left(1-\prod_{i=1}^{j+1}\left(1-\left(1-\left(T_{\widehat{l}_{\rho(i)}}\right)\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q},1$$
$$-\left(1-\prod_{i=1}^{j+1}\left(1-\left(1-\left(I_{\widehat{l}_{\rho(i)}}\right)\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q},\left(1-\prod_{i=1}^{j+1}\left(\left(1-\left(F_{\widehat{l}_{\rho(i)}}\right)^{q}\right)\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q}, \tag{22}$$

which in the form of

$$\mathrm{IGNCSCIA}_{\eta_\lambda}\left(\langle \widehat{u}_1, \widehat{l}_1\rangle, \langle \widehat{u}_2, \widehat{l}_2\rangle, \ldots, \langle \widehat{u}_j, \widehat{l}_j\rangle\right) \oplus \mathrm{IGNCSCIA}_{\eta}\left(\langle \widehat{u}_{j+1}, \widehat{l}_{j+1}\rangle\right)$$

$$= \left(\bigoplus_{i=1}^{j} k\left(\widehat{l}_i\right)^q \Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)^{1/q}\right) \oplus \left(k\left(\widehat{l}_{j+1}\right)^q \Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(j+1)}}\right)^{1/q}\right) = \left(\left[\left(\left(1-\prod_{i=1}^{j}\left(1-\left(T^L_{\widehat{l}_{\rho(i)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)\right.\right.\right.$$

$$\left.\left(1-\left(1-\left(T^L_{\widehat{l}_{\rho(j+1)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(j+1)}}\right)}\right)\right)^{1/q},$$

$$\left.\left(\left(1-\prod_{i=1}^{j}\left(1-\left(T^U_{\widehat{l}_{\rho(i)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)\left(1-\left(1-\left(T^U_{\widehat{l}_{\rho(j+1)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(j+1)}}\right)}\right)\right)^{1/q}\right],$$

$$\left[\left(\left(1-\prod_{i=1}^{j}\left(1-\left(I^L_{\widehat{l}_{\rho(i)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)\left(1-\left(1-\left(I^L_{\widehat{l}_{\rho(j+1)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(j+1)}}\right)}\right)\right)^{1/q},\right.$$

$$\left.\left(\left(1-\prod_{i=1}^{j}\left(1-\left(I^U_{\widehat{l}_{\rho(i)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)\left(1-\left(1-\left(I^U_{\widehat{l}_{\rho(j+1)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(j+1)}}\right)}\right)\right)^{1/q}\right],$$

$$\left[1-\left(\left(1-\prod_{i=1}^{j}\left(1-\left(1-\left(F^L_{\widehat{l}_{\rho(i)}}\right)\right)\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)\left(1-\left(1-\left(F^L_{\widehat{l}_{\rho(j+1)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(j+1)}}\right)}\right)\right)^{1/q},$$

$$\left.1-\left(\left(1-\prod_{i=1}^{j}\left(1-\left(1-\left(F^U_{\widehat{l}_{\rho(i)}}\right)\right)\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)\left(1-\left(1-\left(F^U_{\widehat{l}_{\rho(j+1)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(j+1)}}\right)}\right)\right)^{1/q}\right], 1$$

$$-\left(1-\prod_{i=1}^{j}\left(1-\left(1-\left(T_{\widehat{l}_{\rho(i)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)\left(1-\left(1-\left(T_{\widehat{l}_{\rho(j+1)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(j+1)}}\right)}\right)\right)^{1/q}, 1$$

$$-\left(1-\prod_{i=1}^{j}\left(1-\left(1-\left(T_{\widehat{l}_{\rho(i)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)\left(1-\left(1-\left(T_{\widehat{l}_{\rho(j+1)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(j+1)}}\right)}\right)\right)^{1/q}$$

$$\left.\left(1-\prod_{i=1}^{j}\left(\left(1-\left(F_{\widehat{l}_{\rho(i)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)\left(\left(1-\left(F_{\widehat{l}_{\rho(j+1)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(j+1)}}\right)}\right)\right)^{1/q}\right)$$

$$= \left(\left[\left(1-\prod_{i=1}^{j+1}\left(1-\left(T^L_{\widehat{l}_{\rho(i)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q}, \left(1-\prod_{i=1}^{j+1}\left(1-\left(T^U_{\widehat{l}_{\rho(i)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{(i)}}\right)}\right)^{1/q}\right],\right.$$

$$\left[\left(1-\prod_{i=1}^{j+1}\left(1-\left(T^{L}_{\hat{l}_{\rho(i)}}\right)^{q}\right)^{\Theta\left(\hat{\Upsilon}_{\rho(\hat{T})(i)}\right)}\right)^{1/q},\left(1-\prod_{i=1}^{j+1}\left(1-\left(T^{U}_{\hat{l}_{\rho(i)}}\right)^{q}\right)^{\Theta\left(\hat{\Upsilon}_{\rho(\hat{T})(i)}\right)}\right)^{1/q}\right],$$

$$\left[1-\left(1-\prod_{i=1}^{j+1}\left(1-\left(1-\left(F^{L}_{\hat{l}_{\rho(i)}}\right)\right)^{q}\right)^{\Theta\left(\hat{\Upsilon}_{\rho(\hat{T})(i)}\right)}\right)^{1/q},1-\left(1-\prod_{i=1}^{j+1}\left(1-\left(1-\left(F^{U}_{\hat{l}_{\rho(i)}}\right)\right)^{q}\right)^{\Theta\left(\hat{\Upsilon}_{\rho(\hat{T})(i)}\right)}\right)^{1/q}\right],1$$
$$-\left(1-\prod_{i=1}^{j+1}\left(1-\left(1-\left(T_{\hat{l}_{\rho(i)}}\right)\right)^{q}\right)^{\Theta\left(\hat{\Upsilon}_{\rho(\hat{T})(i)}\right)}\right)^{1/q},1$$
$$-\left(1-\prod_{i=1}^{j+1}\left(1-\left(1-\left(I_{\hat{l}_{\rho(i)}}\right)\right)^{q}\right)^{\Theta\left(\hat{\Upsilon}_{\rho(\hat{T})(i)}\right)}\right)^{1/q},\left(1-\prod_{i=1}^{j+1}\left(\left(1-\left(F_{\hat{l}_{\rho(i)}}\right)^{q}\right)\right)^{\Theta\left(\hat{\Upsilon}_{\rho(\hat{T})(i)}\right)}\right)^{1/q}. \quad (23)$$

is a NC value by assumption hypothesis $\ddot{n}=k$ and $\ddot{n}=2$. Hence, $\ddot{n}=j+1$ is NC, which completes the proof. □

Definition 16. Let $\hat{l}_i=([(T^L)_{\hat{l}_i},(T^U)_{\hat{l}_i}],[(I^L)_{\hat{l}_i},(I^U)_{\hat{l}_i}],[(F^L)_{\hat{l}_i},(F^U)_{\hat{l}_i}],(T)_{\hat{l}_i},(I)_{\hat{l}_i},(F)_{\hat{l}_i})$, where $(1,2,\ldots,i,\ldots,\ddot{n})$ be a collection of NC values, and θ be a fuzzy measure on $\hat{T}=\{\hat{l}_1,\hat{l}_2,\hat{l}_3,\ldots,\hat{l}_{\ddot{n}}\}$ such that $\theta(\hat{l}_i)=\hat{u}_i$; then, the IGNCSCIG operator is defined as

$$\text{IGNCSCIA}_{\theta_\lambda}\left(\langle\hat{u}_1,\hat{l}_1\rangle,\langle\hat{u}_2,\hat{l}_2\rangle,\ldots,\langle\hat{u}_{\ddot{n}},\hat{l}_{\ddot{n}}\rangle\right)=\frac{1}{q}\left(\bigotimes_{i=1}^{\ddot{n}}\left(q\left(k\left(\hat{l}_{\rho(i)}\right)\right)\right)^{\Theta\left(\hat{\Upsilon}_{\rho(\hat{T})_{\rho(i)}}\right)}\right), \quad (24)$$

where $q\in(0,\infty)$ and $(\rho(1),\rho(2),\ldots,\rho(i),\ldots,\rho(\ddot{n}))$ present the permutations of $(1,2,\ldots,i,\ldots,\ddot{n})$ such that $k(\hat{l}_{(1)})\le k(\hat{l}_{(2)})\le k(\hat{l}_{(3)})\le\ldots\le k(\hat{l}_{(\ddot{n})})$ and $\hat{T}=\{\hat{l}_1,\hat{l}_2,\hat{l}_3,\ldots,\hat{l}_{\ddot{n}}\}$ with $\hat{T}_{\rho(\ddot{n}+1)}=\varphi$.

Theorem 2. *Let $\hat{l}_i=([(T^L)_{\hat{l}_i},(T^U)_{\hat{l}_i}],[(I^L)_{\hat{l}_i},(I^U)_{\hat{l}_i}],[(F^L)_{\hat{l}_i},(F^U)_{\hat{l}_i}],(T)_{\hat{l}_i},(I)_{\hat{l}_i},(F)_{\hat{l}_i})$, where $(i=1,2,\ldots,\ddot{n})$ be a collection of NC values, and θ be a fuzzy measure on $\hat{T}=\{\hat{l}_1,\hat{l}_2,\hat{l}_3,\ldots,\hat{l}_{\ddot{n}}\}$ such that $\theta(\hat{l}_i)=\hat{u}_i$; then, the aggregated result obtained by the IGNCGSCIG operator is an NC value:*

$$\mathrm{IGNCSCIA}_{\theta_\lambda}\left(\langle\widehat{u}_1,\widehat{l}_1\rangle,\langle\widehat{u}_2,\widehat{l}_2\rangle,\ldots,\langle\widehat{u}_{\ddot{n}},\widehat{l}_{\ddot{n}}\rangle\right)=\frac{1}{q}\left(\underset{i=1}{\overset{\ddot{n}}{\otimes}}\left(q\left(k\left(\widehat{l}_{\rho(i)}\right)\right)\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{\rho(i)}}\right)}\right)$$

$$=\left(\begin{array}{c}\left[1-\left(1-\prod_{i=1}^{\ddot{n}}\left(1-\left(1-\left(T^{L}_{\widehat{l}_{\rho(i)}}\right)^{q}\right)\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{\rho(i)}}\right)}\right)^{1/q},1-\left(1-\prod_{i=1}^{\ddot{n}}\left(1-\left(1-\left(T^{U}_{\widehat{l}_{\rho(i)}}\right)^{q}\right)\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{\rho(i)}}\right)}\right)^{1/q}\right],\\ \left[1-\left(1-\prod_{i=1}^{\ddot{n}}\left(1-\left(1-\left(I^{L}_{\widehat{l}_{\rho(i)}}\right)^{q}\right)\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{\rho(i)}}\right)}\right)^{1/q},1-\left(1-\prod_{i=1}^{\ddot{n}}\left(1-\left(1-\left(I^{U}_{\widehat{l}_{\rho(i)}}\right)^{q}\right)\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{\rho(i)}}\right)}\right)^{1/q}\right],\\ \left[\left(1-\prod_{i=1}^{\ddot{n}}\left(1-\left(F^{L}_{\widehat{l}_{\rho(i)}}\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{\rho(i)}}\right)}\right)^{1/q},\left(1-\prod_{i=1}^{\ddot{n}}\left(1-\left(F^{U}_{\widehat{l}_{\rho(i)}}\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{\rho(i)}}\right)}\right)^{1/q}\right],\\ \left(1-\prod_{i=1}^{\ddot{n}}\left(\left(1-\left(T_{\widehat{l}_{\rho(i)}}\right)^{q}\right)\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{\rho(i)}}\right)}\right)^{1/q},\left(1-\prod_{i=1}^{\ddot{n}}\left(\left(1-\left(I_{\widehat{l}_{\rho(i)}}\right)^{q}\right)\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{\rho(i)}}\right)}\right)^{1/q},\\ 1-\left(1-\prod_{i=1}^{\ddot{n}}\left(1-\left(1-\left(F_{\widehat{l}_{\rho(i)}}\right)^{q}\right)\right)^{\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{T})_{\rho(i)}}\right)}\right)^{1/q}\end{array}\right), \tag{25}$$

where $q\in(0,\infty)$ and $(\rho(1),\rho(2),\ldots,\rho(i),\ldots,\rho(\ddot{n}))$ present the permutations of $(1,2,\ldots,i,\ldots,\ddot{n})$ such that $k(\widehat{l}_{(1)})\leq k(\widehat{l}_{(2)})\leq k(\widehat{l}_{(3)})\leq\ldots\leq k(\widehat{l}_{(\ddot{n})})$ and $\widehat{T}=\left\{\widehat{l}_1,\widehat{l}_2,\widehat{l}_3,\ldots,\widehat{l}_{\ddot{n}}\right\}$ with $\widehat{T}_{\rho(\ddot{n}+1)}=\varphi$.

Proof. The proof is analogy of Theorem 1. □

2.4. Properties of IGNCSCIA and IGNCSCIG

2.4. Properties of IGNCSCIA and IGNCSCIG. The IGNCSCIA and IGNCSCIG satisfy the following properties.

Proposition 1 (idempotency). *Let $\widehat{l}_i=([(T^L)_{\widehat{l}_i},(T^U)_{\widehat{l}_i}],[(I^L)_{\widehat{l}_i},(I^U)_{\widehat{l}_i}],[(F^L)_{\widehat{l}_i},(F^U)_{\widehat{l}_i}],(T)_{\widehat{l}_i},(I)_{\widehat{l}_i},(F)_{\widehat{l}_i})$, where $(i=1,2,\ldots,\ddot{n})$ be a collection of NC values and θ be a fuzzy measure on $\widehat{T}=\left\{\widehat{l}_1,\widehat{l}_2,\widehat{l}_3,\ldots,\widehat{l}_{\ddot{n}}\right\}$ such that $\theta(\widehat{l}_i)=\widehat{u}_i$:*

$$\mathrm{IGNCSCIA}_{\theta_\lambda}\left(\langle\widehat{u}_1,\widehat{l}_1\rangle,\langle\widehat{u}_2,\widehat{l}_2\rangle,\ldots,\langle\widehat{u}_n,\widehat{l}_{\ddot{n}}\rangle\right)=\left(\underset{i=1}{\overset{\ddot{n}}{\oplus}}k\left(\widehat{l}_i\right)^{q}\Theta\left(\widehat{\Upsilon}_{(\widehat{T})_{\rho(i)}}\right)^{1/q}\right), \tag{26}$$

$$\mathrm{IGNCSCIA}_{\theta_\lambda}\left(\langle\widehat{u}_1,\widehat{l}_1\rangle,\langle\widehat{u}_2,\widehat{l}_2\rangle,\ldots,\langle\widehat{u}_{\ddot{n}},\widehat{l}_{\ddot{n}}\rangle\right)=\frac{1}{q}\left(\underset{i=1}{\overset{\ddot{n}}{\otimes}}\left(q\left(k\left(\widehat{l}_{\rho(i)}\right)\right)\right)^{\Theta\left(\widehat{\Upsilon}_{(\widehat{T})_{\rho(i)}}\right)}\right), \tag{27}$$

where $q\in(0,\infty)$ and $(\rho(1),\rho(2),\ldots,\rho(i),\ldots,\rho(\ddot{n}))$ present the permutations of $(1,2,\ldots,i,\ldots,\ddot{n})$ such that $k(\widehat{l}_{(1)})\leq k(\widehat{l}_{(2)})\leq k(\widehat{l}_{(3)})\leq\ldots\leq k(\widehat{l}_{(\ddot{n})})$ and $\widehat{T}=\left\{\widehat{l}_1,\widehat{l}_2,\widehat{l}_3,\ldots,\widehat{l}_{\ddot{n}}\right\}$ with $\widehat{T}_{\rho(\ddot{n}+1)}=\varphi$. For $\widehat{l}_i=\widehat{l}$,

$$\begin{aligned}&\mathrm{IGNCSCIA}_{\theta_\lambda}\left(\langle\widehat{u}_1,\widehat{l}_1\rangle,\langle\widehat{u}_2,\widehat{l}_2\rangle,\ldots,\langle\widehat{u}_{\ddot{n}},\widehat{l}_{\ddot{n}}\rangle\right)=\widehat{l},\\&\mathrm{IGNCSCIG}_{\theta_\lambda}\left(\langle\widehat{u}_1,\widehat{l}_1\rangle,\langle\widehat{u}_2,\widehat{l}_2\rangle,\ldots,\langle\widehat{u}_{\ddot{n}},\widehat{l}_{\ddot{n}}\rangle\right)=\widehat{l}.\end{aligned} \tag{28}$$

Proposition 2 (monotonicity). *Let* $\widehat{l}_i = ([(T^L)_{\widehat{l}_i}, (T^U)_{\widehat{l}_i}], [(I^L)_{\widehat{l}_i}, (I^U)_{\widehat{l}_i}], [(F^L)_{\widehat{l}_i}, (F^U)_{\widehat{l}_i}], (T)_{\widehat{l}_i}, (I)_{\widehat{l}_i}, (F)_{\widehat{l}_i})$, *where* $(i = 1, 2, \ldots, \widehat{\ddot{n}})$ *be a collection of NC values and* θ *be a fuzzy measure on* $T = \{\widehat{l}_1, \widehat{l}_2, \widehat{l}_3, \ldots, \widehat{l}_{\ddot{n}}\}$ *such that* $\theta(\widehat{l}_i) = \widehat{u}_i$, $\widehat{B} = \{\widehat{b}_1, \widehat{b}_2, \widehat{b}_3, \ldots, \widehat{b}_{\ddot{n}}\}$, *and* $\theta(\widehat{b}_i) = \widehat{v}_i$ *with* $T^L_{\widehat{l}_i} \le T^L_{\widehat{b}_i}, T^U_{\widehat{l}_i} \le T^U_{\widehat{b}_i}, I^L_{\widehat{l}_i} \le I^L_{\widehat{b}_i}, I^U_{\widehat{l}_i} \le I^U_{\widehat{b}_i}, T^L_{\widehat{l}_i} \ge T^L_{\widehat{b}_i}, T^U_{\widehat{l}_i} \ge T^U_{\widehat{b}_i}, T_{\widehat{l}_i} \ge T_{\widehat{b}_i}, I_{\widehat{l}_i} \ge I_{\widehat{b}_i}, F_{\widehat{l}_i} \le F_{\widehat{b}_i}$; *then,*

$$\begin{aligned}
&\mathrm{IGNCSCIA}_{\theta_\lambda}\left(\langle \widehat{u}_1, \widehat{l}_1\rangle, \langle \widehat{u}_2, \widehat{l}_2\rangle, \ldots, \langle \widehat{u}_{\ddot{n}}, \widehat{l}_{\ddot{n}}\rangle\right) \le \mathrm{IGNCSCIA}_{\theta_\lambda}\left(\langle \widehat{v}_1, \widehat{b}_1\rangle, \langle \widehat{v}_2, \widehat{b}_2\rangle, \ldots, \langle \widehat{v}_{\ddot{n}}, \widehat{b}_{\ddot{n}}\rangle\right),\\
&\mathrm{IGNCSCIG}_{\theta_\lambda}\left(\langle \widehat{u}_1, \widehat{l}_1\rangle, \langle \widehat{u}_2, \widehat{l}_2\rangle, \ldots, \langle \widehat{u}_{\ddot{n}}, \widehat{l}_{\ddot{n}}\rangle\right) \le \mathrm{IGNCSCIG}_{\theta_\lambda}\left(\langle \widehat{v}_1, \widehat{b}_1\rangle, \langle \widehat{v}_2, \widehat{b}_2\rangle, \ldots, \langle \widehat{v}_{\ddot{n}}, \widehat{b}_{\ddot{n}}\rangle\right).
\end{aligned} \tag{29}$$

Proof. To prove this proposition, the following inequality is proved:

$$\begin{aligned}
&x, y \in (0,1) \text{ such that } (x)^k \le (y)^k \Longrightarrow 1-(x)^k \ge 1-(y)^k \Longrightarrow 1-\left(1-(x)^k\right) \le 1-\left(1-(y)^k\right)(i),\\
&\text{for } (x)^k \ge (y)^k \Longrightarrow 1-(x)^k \le 1-(y)^k \Longrightarrow 1-\left(1-(x)^k\right) \le 1-\left(1-(y)^k\right)(ii).
\end{aligned} \tag{30}$$

For $x = y = 0$ and $x = y = 1$, the result is trivial.

Since $\Theta(\widehat{\Upsilon}_{(\widehat{T})_{\rho(i)}}) = \varpi^{Sh}_{\widehat{T}_{\rho(i)}}(\eta_\lambda, N) - \varpi_{\widehat{T}_{\rho(i+1)}}{}^{Sh}(\eta_\lambda, N) = \varpi^{Sh}_{\widehat{B}_{\rho(i)}}(\eta_\lambda, N) - \varpi^{Sh}_{\widehat{B}_{\rho(i+1)}}(\eta_\lambda, N) = \Theta(\widehat{\Upsilon}_{(\widehat{B})_{\rho(i)}})$, putting $x = T^L_{\widehat{l}_i}$, $y = T^L_{\widehat{b}_i}$, and $k = q$ and using equation (i),

$$\begin{aligned}
&\left(1-\prod_{i=1}^{\ddot{n}}\left(1-\left(T^L_{\widehat{l}_{\rho(i)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{(\widehat{T})_{\rho(i)}}\right)}\right)^{1/q} \le \left(1-\prod_{i=1}^{\ddot{n}}\left(1-\left(T^L_{\widehat{b}_{\rho(i)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{(\widehat{B})_{\rho(i)}}\right)}\right)^{1/q},\\
&\left(1-\prod_{i=1}^{\ddot{n}}\left(1-\left(T^U_{\widehat{l}_{\rho(i)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{(\widehat{T})_{\rho(i)}}\right)}\right)^{1/q} \le \left(1-\prod_{i=1}^{\ddot{n}}\left(1-\left(T^U_{\widehat{b}_{\rho(i)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{(\widehat{B})_{\rho(i)}}\right)}\right)^{1/q} \left(1-\prod_{i=1}^{\ddot{n}}\left(1-\left(I^L_{\widehat{l}_{\rho(i)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{(\widehat{T})_{\rho(i)}}\right)}\right)^{1/q}\\
&\le \left(1-\prod_{i=1}^{\ddot{n}}\left(1-\left(I^L_{\widehat{b}_{\rho(i)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{(\widehat{B})_{\rho(i)}}\right)}\right)^{1/q}, \left(1-\prod_{i=1}^{\ddot{n}}\left(1-\left(I^U_{\widehat{l}_{\rho(i)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{(\widehat{T})_{\rho(i)}}\right)}\right)^{1/q}\\
&\le \left(1-\prod_{i=1}^{\ddot{n}}\left(1-\left(I^U_{\widehat{b}_{\rho(i)}}\right)^q\right)^{\Theta\left(\widehat{\Upsilon}_{(\widehat{B})_{\rho(i)}}\right)}\right)^{1/q} \left(1-\prod_{i=1}^{\ddot{n}}\left(\left(1-\left(F_{\widehat{l}_{\rho(i)}}\right)^q\right)\right)^{\Theta\left(\widehat{\Upsilon}_{(\widehat{T})_{\rho(i)}}\right)}\right)^{1/q} \le \left(1-\prod_{i=1}^{\ddot{n}}\left(\left(1-\left(F_{\widehat{b}_{\rho(i)}}\right)^q\right)\right)^{\Theta\left(\widehat{\Upsilon}_{(\widehat{B})_{\rho(i)}}\right)}\right)^{1/q},
\end{aligned} \tag{31}$$

Putting $x = F^L_{\widehat{l}_{\rho(i)}}$, $y = F^L_{\widehat{b}_{\rho(i)}}$, and $k = q$ and using equation (ii),

$$1-\left(1-\prod_{i=1}^{\ddot{n}}\left(1-\left(1-\left(F^{L}_{\widehat{l}_{\rho(i)}}\right)\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{(\widehat{T})_{\rho(i)}}\right)}\right)^{1/q}\geq 1-\left(1-\prod_{i=1}^{\ddot{n}}\left(1-\left(1-\left(F^{L}_{\widehat{b}_{\rho(i)}}\right)\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{(\widehat{B})_{\rho(i)}}\right)}\right)^{1/q},$$

$$1-\left(1-\prod_{i=1}^{\ddot{n}}\left(1-\left(1-\left(F^{U}_{\widehat{l}_{\rho(i)}}\right)\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{(\widehat{T})_{\rho(i)}}\right)}\right)^{1/q}\geq 1-\left(1-\prod_{i=1}^{\ddot{n}}\left(1-\left(1-\left(F^{U}_{\widehat{b}_{\rho(i)}}\right)\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{(\widehat{B})_{\rho(i)}}\right)}\right)^{1/q},$$

$$1-\left(1-\prod_{i=1}^{\ddot{n}}\left(1-\left(1-\left(T_{\widehat{l}_{\rho(i)}}\right)\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{(\widehat{T})_{\rho(i)}}\right)}\right)^{1/q}\geq 1-\left(1-\prod_{i=1}^{\ddot{n}}\left(1-\left(1-\left(T_{\widehat{b}_{\rho(i)}}\right)\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{(\widehat{B})_{\rho(i)}}\right)}\right)^{1/q}, \quad (32)$$

$$1-\left(1-\prod_{i=1}^{\ddot{n}}\left(1-\left(1-\left(I_{\widehat{l}_{\rho(i)}}\right)\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{(\widehat{T})_{\rho(i)}}\right)}\right)^{1/q}\geq 1-\left(1-\prod_{i=1}^{\ddot{n}}\left(1-\left(1-\left(I_{\widehat{b}_{\rho(i)}}\right)\right)^{q}\right)^{\Theta\left(\widehat{\Upsilon}_{(\widehat{B})_{\rho(i)}}\right)}\right)^{1/q},$$

$$IGNCSCIA_{\theta_\lambda}\left(\langle\widehat{u}_1,\widehat{l}_1\rangle,\langle\widehat{u}_2,\widehat{l}_2\rangle,\ldots,\langle\widehat{u}_{\ddot{n}},\widehat{l}_{\ddot{n}}\rangle\right)\leq IGNCSCIA_{\theta_\lambda}\left(\langle\widehat{v}_1,\widehat{b}_1\rangle,\langle\widehat{v}_2,\widehat{b}_2\rangle,\ldots,\langle\widehat{v}_{\ddot{n}},\widehat{b}_{\ddot{n}}\rangle\right),$$

$$IGNCSCIG_{\theta_\lambda}\left(\langle\widehat{u}_1,\widehat{l}_1\rangle,\langle\widehat{u}_2,\widehat{l}_2\rangle,\ldots,\langle\widehat{u}_{\ddot{n}},\widehat{l}_{\ddot{n}}\rangle\right)\leq IGNCSCIG_{\theta_\lambda}\left(\langle\widehat{v}_1,\widehat{b}_1\rangle,\langle\widehat{v}_2,\widehat{b}_2\rangle,\ldots,\langle\widehat{v}_{\ddot{n}},\widehat{b}_{\ddot{n}}\rangle\right).$$

□

Proposition 3 (boundedness). *Let* $\widehat{l}_i=([(T^L)_{\widehat{l}_i},(T^U)_{\widehat{l}_i}],[(I^L)_{\widehat{l}_i},(I^U)_{\widehat{l}_i}],[(F^L)_{\widehat{l}_i},(F^U)_{\widehat{l}_i}],(T)_{\widehat{l}_i},(I)_{\widehat{l}_i},(F)_{\widehat{l}_i})$, *where* $(i=1,2,\ldots,\ddot{n})$ *be a collection of NC values and* θ *be a fuzzy measure on* $\widehat{T}=\left\{\widehat{l}_1,\widehat{l}_2,\widehat{l}_3,\ldots,\widehat{l}_{\ddot{n}}\right\}$ *such that* $\theta(\widehat{l}_i)=\widehat{u}_i$, *such that*

$$(\widehat{l})^-=\left(\left[\min\left(T^L\right)_{\widehat{l}_i},\min\left(T^U\right)_{\widehat{l}_i}\right],\left[\min\left(I^L\right)_{\widehat{l}_i},\min\left(I^U\right)_{\widehat{l}_i}\right],\left[1-\max\left(F^L\right)_{\widehat{l}_i},1-\max\left(F^U\right)_{\widehat{l}_i}\right],\min(T)_{\widehat{l}_i},\min(I)_{\widehat{l}_i},1-\max(F)_{\widehat{l}_i}\right),$$
$$\left(\widehat{l}_i\right)^+=\left(\left[\max\left(T^L\right)_{\widehat{l}_i},\max\left(T^U\right)_{\widehat{l}_i}\right],\left[\max\left(I^L\right)_{\widehat{l}_i},\max\left(I^U\right)_{\widehat{l}_i}\right],\left[1-\min\left(F^L\right)_{\widehat{l}_i},1-\min\left(F^U\right)_{\widehat{l}_i}\right],\max(T)_{\widehat{l}_i},\max(I)_{\widehat{l}_i},1-\min(F)_{\widehat{l}_i}\right). \quad (33)$$

Then,

$$(\widehat{l})^-\leq \mathrm{IGNCSCIA}_{\theta_\lambda}\left(\widehat{l}_1,\widehat{l}_2,\widehat{l}_3,\ldots,\widehat{l}_{\ddot{n}}\right)\leq(\widehat{l})^+,$$
$$(\widehat{l})^-\leq \mathrm{IGNCSCIG}_{\theta_\lambda}\left(\widehat{l}_1,\widehat{l}_2,\widehat{l}_3,\ldots,\widehat{l}_{\ddot{n}}\right)\leq(\widehat{l})^+. \quad (34)$$

The proof is followed by idempotency.

Remark 1. When different values are assigned to p, IGNCSCA and IGNCSG will deduce into different neutrosophic cubic aggregation operators as follows.

Proof. When $q=1$, IGNCSCIA deduce into induced neutrosophic cubic Shapley–Choquet integral average (INCSCIA) operator, and IGNCSCIG deduced into induced neutrosophic cubic Shapley–Choquet integral geometric (INCSCIG) operator.

When $\widehat{u}_i=\widehat{l}_i$, IGNCSCIA deduce into generalized neutrosophic cubic Shapley–Choquet integral average (GNCSCIA) operator, and IGNCSCIG deduce into generalized neutrosophic cubic Shapley–Choquet integral geometric (GNCSCIG) operator.

When $\widehat{u}_i=\widehat{l}_i$ and $q=1$, IGNCSCIA deduce into neutrosophic cubic Shapley–Choquet integral average (NCSCIA) operator, and IGNCSCIG deduce into neutrosophic cubic Shapley–Choquet integral geometric (NCSCG) operator. □

3. Application

Example 1. In order to evaluate the IGNCSCIA and IGNCSCIG operator, the following example is presented. A company wants to expand its foreign country investment, to choose the best country out of five alternatives (countries) $\widehat{T}=\left\{\widehat{l}_1,\widehat{l}_2,\widehat{l}_3,\widehat{l}_4,\widehat{l}_5\right\}$ to make investment. The four main factors (criteria) E to decide are resources, policies, economy, and infrastructure, $E=\{e_1,e_2,e_3,e_4\}$. First, the data are presented in the form of NC values in Table 1.

The weights are obtained by fuzzy Shapley measures presented in Table 2.

With the help of these values, the following weight are measured:
$\varpi_1^{Sh}(\mu_\lambda, N) = 0.0892, \varpi_2^{Sh}(\mu_\lambda, N) = 0.0056, \varpi_3^{Sh}(\mu_\lambda, N) = 0.5939$, and $\varpi_4^{Sh}(\mu_\lambda, N) = 0.2624$.

Using IGNCSCIA and IGNCSCIG operator, the following values are obtained Table 3.

The alternatives are ranked, and graphical representation is given below for both IGNCSCIA and IGNCSCIG operators (Figures 2 and 3).

The ranking of alternatives in the IGNCSCIA operator for different values of q is tabulated in Table 4.

The ranking of alternatives in the IGNCSCIG operator for different values of q is tabulated in Table 5.

From graphs and tabulated data, it is observed that the value of q affects the rank of alternatives in the IGNCSCIG operator.

3.1. Sensitivity Analysis. From the data, it is observed that the ranking of alternative changes with change value of q. For $0 < q < 1$, the ranking is $\widehat{l}_1 > \widehat{l}_3 > \widehat{l}_5 > \widehat{l}_4 > \widehat{l}_2$. For $q \in [1, 2]$, the ranking changes to $\widehat{l}_1 > \widehat{l}_3 > \widehat{l}_4 > \widehat{l}_5 > \widehat{l}_2$. For $q = 3$, the ranking changes to $\widehat{l}_3 > \widehat{l}_1 > \widehat{l}_5 > \widehat{l}_4 > \widehat{l}_2$, and for $q \geq 5$, the ranking $\widehat{l}_3 > \widehat{l}_1 > \widehat{l}_5 > \widehat{l}_4 > \widehat{l}_2$. So, the decision of best alternative changes with the value q. For such situation, the distance decision-making methods are the best choices to conclude the best ranking. A number of distance-based decision-making methods are proposed by researchers. Among these, the CODAS method is one of the user-friendly methods that evaluate the data more precisely. This limitation is overcome by proposing novel distance-based decision method and CODAS method in NC environment. Since the CODAS method is the distance-based method, so first the Minkwoski distance formula is proposed for NC values.

3.2. Minkowski Distance. In this section, Minkowski distance formula is introduced to determine the distance of two NC values. Minkowski distance is the generalized distance formula which can deduced to hamming, Euclidian, and Chebyshev formula. This provides a plate form for expressing more information.

Definition 17. Let $N = ([T_N^L, T_N^U], [I_N^L, I_N^U], [F_N^L, F_N^U], T_N, I_N, F_N)$ and $M = ([T_M^L, T_M^U], [I_M^L, I_M^U], [F_M^L, F_M^U], T_M, I_M, F_M)$; the Minkowski distance is defined as

$$d(N, M) = \frac{1}{9}\begin{pmatrix} (T_N^L - T_M^L)^q + (T_N^U - T_M^U)^q + (I_N^L - I_M^L)^q + (I_N^U - I_M^U)^q + \\ (F_N^L - F_M^L)^q + (F_N^U - F_M^U)^q + (T_N - T_M)^q + (I_N - I_M)^q + (F_N - F_M)^q \end{pmatrix}^{1/q}. \quad (35)$$

Assigning different values to q Minkowski distance generates different distances:

(i) $q = 1$ will reduce it into hamming distance
(ii) $q = 2$ will reduce it into Euclidean distance
(iii) $q \longrightarrow \infty$ will reduce it into Chebyshev distance

3.3. The NCCODAS Method. This section proposes the neutrosophic cubic combinative distance-based assessment (NCCODAS) method, and the CODAS method was proposed by Keshavarz et al. [28], which is a systemized method to handle MCDM problems. The CODAS method predominantly uses the combinative form of distances for computation of alternative rating.

3.3.1. The Presentation of Methodology. The demonstration of the proposed method is shown in Figure 4.

The steps of the NCCODAS method for MCDM problems are as follows:

Step 1: construction of DM as $A = [\widehat{l}_{ij}], i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$, and $\widehat{l}_{ij} = ([(T^L)_{\widehat{l}_{ij}}, (T^U)_{\widehat{l}_{ij}}], [(I^L)_{\widehat{l}_{ij}}, (I^U)_{\widehat{l}_{ij}}], [(F^L)_{\widehat{l}_{ij}}, (F^U)_{\widehat{l}_{ij}}], (T)_{\widehat{l}_{ij}}, (I)_{\widehat{l}_{ij}}, (F)_{\widehat{l}_{ij}})$ is neutrosophic cubic values assigned to alternative $\dot{a}_i$ subject to criteria e_j by decision makers.

Step 2: the neutrosophic cubic normalized weighted matrix is accomplished by $U = [\widetilde{u}_{ij}], i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$, where

$$\widetilde{u}_{ij} = w_j \otimes \widehat{l}_{ij}, \quad (36)$$

such that $\sum_{j=1}^n w_j = 1$ and. $0 < w_j < 1$

Step 3: determine the neutrosophic cubic negative ideals by score/ accuracy function:

$$ri = \min_i \{\widetilde{u}_{ij} | j = 1, 2, \ldots, n\}. \quad (37)$$

Step 4: calculate neutrosophic cubic Minkwoski distances of alternatives from negative ideals obtained by equation (37).

Step 5: construct relative assessment matrix on neutrosophic cubic distances by

$$Ra = [h_{ik}]_{mn} by\, h_{ik} = (MD_i - MD_k) + t(MD_i + MD_k)(MD_i - MD_k), \quad (38)$$

Table 1: The data in NC form of given data.

e_1	e_2	e_3	e_4
$\dot{a}_{11} = ([0.7, 0.8], [0.5, 0.7], [0.1, 0.2], 0.9, 0.7, 0.2)$	$\dot{a}_{21} = ([0.6, 0.8], [0.4, 0.5], [0.3, 0.3], 0.9, 0.8, 0.2)$	$\dot{a}_{31} = ([0.8, 0.8], [0.4, 0.6], [0.1, 0.2], 0.8, 0.9, 0.1)$	$\dot{a}_{31} = ([0.7, 0.7], [0.3, 0.4], [0.2, 0.2], 0.8, 0.6, 0.2)$
$\dot{a}_{12} = ([0.6, 0.8], [0.4, 0.6], [0.4, 0.6], 0.2, 0.3, 0.7)$	$\dot{a}_{22} = ([0.5, 0.7], [0.4, 0.6], [0.1, 0.3], 0.6, 0.4, 0.2)$	$\dot{a}_{32} = ([0.6, 0.6], [0.2, 0.3], [0.4, 0.5], 0.5, 0.1, 0.3)$	$\dot{a}_{32} = ([0.6, 0.8], [0.4, 0.4], [0.2, 0.4], 0.7, 0.3, 0.5)$
$\dot{a}_{13} = ([0.4, 0.5], [0.5, 0.6], [0.4, 0.6], 0.3, 0.3, 0.3)$	$\dot{a}_{23} = ([0.6, 0.7], [0.4, 0.6], [0.3, 0.4], 0.8, 0.7, 0.5)$	$\dot{a}_{33} = ([0.7, 0.8], [0.6, 0.7], [0.1, 0.2], 0.5, 0.4, 0.4)$	$\dot{a}_{33} = ([0.5, 0.6], [0.5, 0.6], [0.2, 0.3], 0.6, 0.6, 0.4)$
$\dot{a}_{14} = ([0.6, 0.8], [0.5, 0.6], [0.4, 0.5], 0.5, 0.4, 0.5)$	$\dot{a}_{24} = ([0.5, 0.6], [0.3, 0.4], [0.4, 0.5], 0.4, 0.4, 0.3)$	$\dot{a}_{34} = ([0.6, 0.8], [0.5, 0.6], [0.1, 0.2], 0.5, 0.7, 0.3)$	$\dot{a}_{34} = ([0.8, 0.9], [0.3, 0.4], [0.1, 0.2], 0.7, 0.5, 0.4)$
$\dot{a}_{15} = ([0.6, 0.7], [0.4, 0.5], [0.4, 0.5], 0.5, 0.4, 0.5)$	$\dot{a}_{25} = ([0.8, 0.9], [0.3, 0.4], [0.1, 0.2], 0.7, 0.4, 0.3)$	$\dot{a}_{35} = ([0.7, 0.8], [0.5, 0.6], [0.1, 0.2], 0.5, 0.3, 0.4)$	$\dot{a}_{35} = ([0.5, 0.7], [0.5, 0.5], [0.2, 0.3], 0.6, 0.5, 0.3)$

Table 2: The fuzzy measure of elements of power set of A.

A	$\mu_\lambda(A)$	A	$\mu_\lambda(A)$	A	$\mu_\lambda(A)$
ϕ	0	$\{e_1, e_2\}$	0.1441	$\{e_1, e_2, e_3\}$	0.7378
$\{e_1\}$	0.664	$\{e_1, e_3\}$	0.6596	$\{e_1, e_2, e_4\}$	0.4050
$\{e_2\}$	0.0777	$\{e_1, e_4\}$	0.3271	$\{e_1, e_3, e_4\}$	0.9217
$\{e_3\}$	0.5929	$\{e_2, e_3\}$	0.6710	$\{e_2, e_3, e_4\}$	0.9931
$\{e_4\}$	0.2605	$\{e_2, e_4\}$	0.33840	E	1
		$\{e_3, e_4\}$	0.8584		

Table 3: The neutrosophic cubic values using IGNCSCIA and IGNCSCIG operator.

IGNCSCA

$$\tilde{u}_{31} = ([0.735, 0.862], [0.412, 0.557], [0.158, 0.204], 0.172, 0.267, 0.191)$$
$$\tilde{u}_{32} = ([0.594, 0.746], [0.383, 0.468], [0.137, 0.329], 0.531, 0.207, 0.375)$$
$$\tilde{u}_{33} = ([0.648, 0.750], [0.528, 0.670], [0.165, 0.270], 0.588, 0.516, 0.480)$$
$$\tilde{u}_{34} = ([0.662, 0.801], [0.406, 0.511], [0.156, 0.267], 0.444, 0.449, 0.358)$$
$$\tilde{u}_{35} = ([0.761, 0.872], [0.451, 0.528], [0.093, 0.188], 0.573, 0.348, 0.369)$$

IGNCSCG

$$\tilde{u}_{31} = ([0.644, 0.803], [0.339, 0.468], [0.251, 0.271], 0.888, 0.843, 0.869)$$
$$\tilde{u}_{32} = ([0.518, 0.663], [0.286, 0.372], [0.268, 0.421], 0.634, 0.343, 0.757)$$
$$\tilde{u}_{33} = ([0.560, 0.664], [0.436, 0.597], [0.261, 0.370], 0.731, 0.646, 0.598)$$
$$\tilde{u}_{34} = ([0.540, 0.664], [0.311, 0.414], [0.317, 0.415], 0.560, 0.573, 0.714)$$
$$\tilde{u}_{35} = ([0.645, 0.786], [0.350, 0.436], [0.158, 0.260], 0.665, 0.432, 0.708)$$

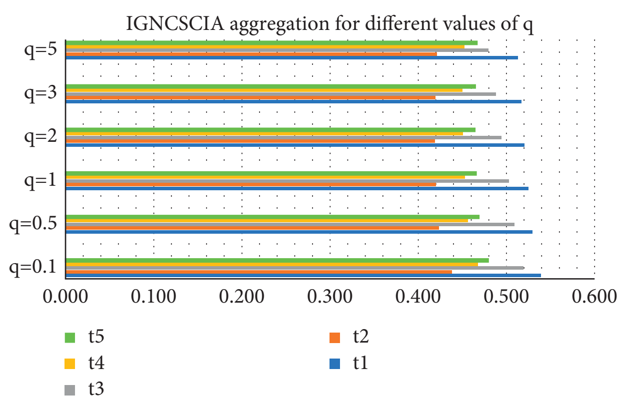

Figure 2: Graphical view of IGNCSCIA.

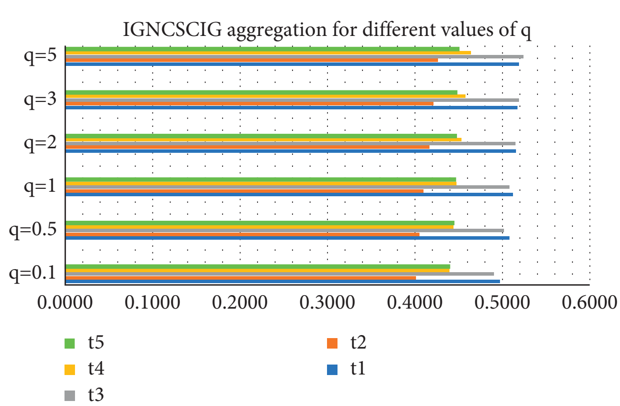

Figure 3: Graphical view of IGNCSCIG.

Table 4: The ranking of alternatives subject to the different values of q using the IGNCSCIA operator.

Tabulated view of IGNCSCIA for different values of q	
q	Ranks
$q = 0.1$	$Ŧ_1 > Ŧ_3 > Ŧ_5 > Ŧ_4 > Ŧ_2$
$q = 0.5$	$Ŧ_1 > Ŧ_3 > Ŧ_5 > Ŧ_4 > Ŧ_2$
$q = 1$	$Ŧ_1 > Ŧ_3 > Ŧ_5 > Ŧ_4 > Ŧ_2$
$q = 2$	$Ŧ_1 > Ŧ_3 > Ŧ_5 > Ŧ_4 > Ŧ_2$
$q = 3$	$Ŧ_1 > Ŧ_3 > Ŧ_5 > Ŧ_4 > Ŧ_2$
$q = 5$	$Ŧ_1 > Ŧ_3 > Ŧ_5 > Ŧ_4 > Ŧ_2$

Table 5: The ranking of alternatives subject to the different values of q using the IGNCSCIG operator.

Tabulated view of IGNCSCIG for different values of q	
q	Ranks
$q = 0.1$	$Ŧ_1 > Ŧ_3 > Ŧ_5 > Ŧ_4 > Ŧ_2$
$q = 0.5$	$Ŧ_1 > Ŧ_3 > Ŧ_5 > Ŧ_4 > Ŧ_2$
$q = 1$	$Ŧ_1 > Ŧ_3 > Ŧ_4 > Ŧ_5 > Ŧ_2$
$q = 2$	$Ŧ_1 > Ŧ_3 > Ŧ_4 > Ŧ_5 > Ŧ_2$
$q = 3$	$Ŧ_3 > Ŧ_1 > Ŧ_5 > Ŧ_4 > Ŧ_2$
$q = 5$	$Ŧ_3 > Ŧ_1 > Ŧ_5 > Ŧ_4 > Ŧ_2$

where $k = \{1, 2, \ldots, n\}$ and t expresses a threshold function in accordance with the parameter θ assigned by the decision makers determined as $t(\theta) = \begin{cases} 1, & \text{if } |\theta| \geq 1, \\ 0, & \text{if } |\theta| < 1. \end{cases}$ Note: for better result, different values of q are used in Minkwoski distance in $(MD_i - MD_k)$ which is used twice. In general, (38) is considered as

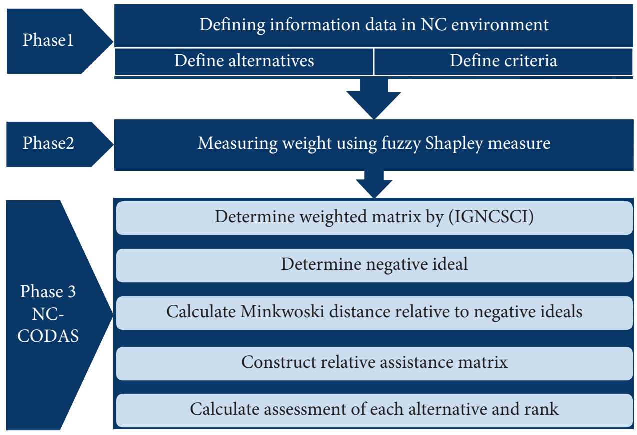

Figure 4: Flowchart.

Table 6: The aggregated neutrosophic cubic value data by IGNCSCA and IGNCSCG.

IGNCSCA
$\tilde{u}_{31} = ([0.735, 0.862], [0.412, 0.557], [0.158, 0.204], 0.172, 0.267, 0.191)$
$\tilde{u}_{32} = ([0.594, 0.746], [0.383, 0.468], [0.137, 0.329], 0.531, 0.207, 0.375)$
$\tilde{u}_{33} = ([0.648, 0.750], [0.528, 0.670], [0.165, 0.270], 0.588, 0.516, 0.480)$
$\tilde{u}_{34} = ([0.662, 0.801], [0.406, 0.511], [0.156, 0.267], 0.444, 0.449, 0.358)$
$\tilde{u}_{35} = ([0.761, 0.872], [0.451, 0.528], [0.093, 0.188], 0.573, 0.348, 0.369)$
IGNCSCG
$\tilde{u}_{31} = ([0.644, 0.803], [0.339, 0.468], [0.251, 0.271], 0.888, 0.843, 0.869)$
$\tilde{u}_{32} = ([0.518, 0.663], [0.286, 0.372], [0.268, 0.421], 0.634, 0.343, 0.757)$
$\tilde{u}_{33} = ([0.560, 0.664], [0.436, 0.597], [0.261, 0.370], 0.731, 0.646, 0.598)$
$\tilde{u}_{34} = ([0.540, 0.664], [0.311, 0.414], [0.317, 0.415], 0.560, 0.573, 0.714)$
$\tilde{u}_{35} = ([0.645, 0.786], [0.350, 0.436], [0.158, 0.260], 0.665, 0.432, 0.708)$

$Ra = [h_{ik}]_{mn} by\, h_{ik} = (ED_i - ED_k) + t \quad (ED_i + ED_k)(HD_i - HD_k)$, where HD is hamming and ED is Euclidean distance.

Step 6: calculate the assessment of each alternative by

$$AS_i = \sum_{k=1}^{m} h_{ik}. \tag{39}$$

Step 7: rank the alternative in the ascending order. The alternative with highest rank is desirable alternative.

3.3.2. A Case Study of Foreign Country Investment. A company wants to expand its foreign country investment to choose the best country out of five alternatives (countries) $\widehat{T} = \{\widehat{l}_1, \widehat{l}_2, \widehat{l}_3, \widehat{l}_4, \widehat{l}_5\}$ and to make investment. The four main factors (criteria) E to decide are resources, policies, economy, and infrastructure. $E = \{e_1, e_2, e_3, e_4\}$. Jiao et al. [26] used different techniques to evaluate the results. The proposed method will be applied to the given data, and results will be compared with some existing methods.

3.4. Application of NCCODAS. The proposed method includes IGNCSCIA, IGNCSCIG aggregation operators, and NCCODAS decision-making techniques. The steps defined in NCCODAS are follows:

Step 1: the data are organized in the form of NC values considered.

It shows that a firm is interested to finance in a country out of five countries (attributes). The countries are represented by $\widehat{T} = \{\widehat{l}_1, \widehat{l}_2, \widehat{l}_3, \widehat{l}_4, \widehat{l}_5\}$ subject to the criteria resources, politics and policies, economy, and infrastructure of countries represented by $E = \{e_1, e_2, e_3, e_4\}$ tabulated in Table 1.

Step 2: to obtain the weighted matrix, the Shapley–Choquet measure is used to find the weight with the help of equations (10) and (11). Then, the aggregated values of IGNCSCA and IGNCSCG are tabulated in Tables 3 and 4. The measured values are tabulated in Table 2.

Step 3: the minimum value is calculated using equation (23).

The minimum value in IGNCSCA is $\min_i\{\tilde{u}_{ij}\} = ([0.594, 0.746], [0.383, 0.468], [0.165, 0.329], 0.588, 0.516, 0.191)$.

The minimum value in IGNCSCG is $\min_i\{\tilde{u}_{ij}\} = ([0.518, 0.663], [0.286, 0.372], [0.317, 0.421], 0.888, 0.843, 0.598)$.

TABLE 7: The table represents the numerical data calculated by steps 4–7 for different values of q.

Alternatives	ED_i	HD_i	Ra	AS_i
R_1	0.1955	0.4856	0.18444	1
R_2	0.1591	0.3847	−0.1766	3
R_3	0.1840	0.4661	0.088546	2
R_4	0.1674	0.4143	−0.08639	4
R_5	0.1786	0.4263	−0.00999	3
Ranking		$\widehat{t}_1 > \widehat{t}_3 > \widehat{t}_5 > \widehat{t}_4 > \widehat{t}_2$		

TABLE 8: The graphical interpretation of all alternatives subject to the different values of q.

Tabulated view of NCCODAS for different values of q	
q	Ranks
$p = 0.1$	$\widehat{t}_1 > \widehat{t}_3 > \widehat{t}_5 > \widehat{t}_4 > \widehat{t}_2$
$p = 0.5$	$\widehat{t}_1 > \widehat{t}_3 > \widehat{t}_5 > \widehat{t}_4 > \widehat{t}_2$
$p = 1$	$\widehat{t}_1 > \widehat{t}_3 > \widehat{t}_5 > \widehat{t}_4 > \widehat{t}_2$
$p = 2$	$\widehat{t}_1 > \widehat{t}_3 > \widehat{t}_5 > \widehat{t}_4 > \widehat{t}_2$
$p = 3$	$\widehat{t}_1 > \widehat{t}_3 > \widehat{t}_5 > \widehat{t}_4 > \widehat{t}_2$
$p = 5$	$\widehat{t}_1 > \widehat{t}_3 > \widehat{t}_5 > \widehat{t}_4 > \widehat{t}_2$

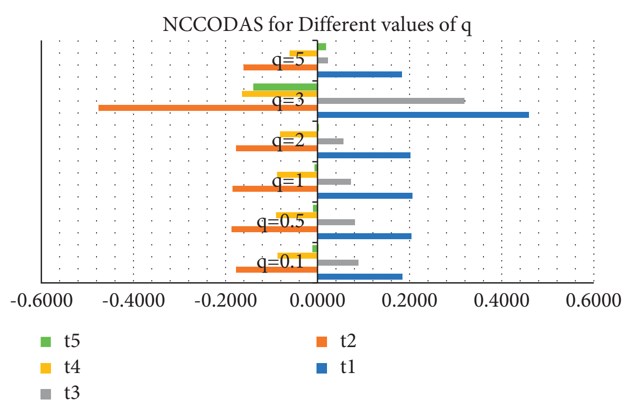

FIGURE 5: NCCODAS graphical view.

TABLE 9: The tabulated view of comparative analysis.

Methodology	Ranking
NC Einstein geometric aggregation operator in MCDM [10]	$\widehat{t}_1 > \widehat{t}_3 > \widehat{t}_2 > \widehat{t}_5 > \widehat{t}_4$
Generalized NC aggregation operators with application MEDM [13]	$\widehat{t}_1 > \widehat{t}_3 > \widehat{t}_5 > \widehat{t}_4 > \widehat{t}_2$
GRA MADM in NCS [14]	$\widehat{t}_1 > \widehat{t}_5 > \widehat{t}_3 > \widehat{t}_2 > \widehat{t}_4$
Cosine measure of NCS for MCDM [15]	$\widehat{t}_1 > \widehat{t}_2 > \widehat{t}_5 > \widehat{t}_3 > \widehat{t}_4$
Proposed NCCODAS	$\widehat{t}_1 > \widehat{t}_3 > \widehat{t}_5 > \widehat{t}_4 > \widehat{t}_2$

Step 4: calculate Minkowski distance from negative ideals by equation (19).

Distance from IGNCSCA $\min_i\{\widetilde{u}_{ij}\}$ is tabulated in Table 4.

Step 5: construction of relative assistance matrix with the help of equation (23) is tabulated in Table 4.

Step 6: calculation of assessment is tabulated in Table 4 by equation (20).

Step 7: ranks of alternative in the ascending order are to get the desired alternative in Table 4.

In Table 2, the data calculated in Steps 4–7 are tabulated for different values of q (Tables 6 and 7):

$$q = 0.1 \quad (40)$$

Different values are assigned to q and alternatives ranked are as shown in Table 8.

The graphical interpretation of tabulated values is shown in Figure 5.

It is observed that changing values of firm do not change the overall ranking of alternatives. To check the validity if method is under NC environment, it is compared with different methods.

3.5. Comparative Analysis. In order to validate the method, comparative analysis is provided with some existing MCDM methods. The analysis is tabulated in Table 9.

From Table 9, it is observed that the NCCODAS method $\widehat{l}_1$ is the best alternative and $\widehat{l}_2$ is the worst alternative evaluated by all the methods considered. Furthermore, it is also observed that the best alternative also obtained by other methods is $\widehat{l}_1$. This validates the validity of the NCCODAS method under neutrosophic cubic environment.

4. Conclusion

This study defined the NCSCI operator based on fuzzy Shapley and Choquet integration operators in NCS. Two aggregation operators IGNCSCIA and IGNCSCIG are defined and furnished upon numerical application of foreign investment of a company. A sensitive analysis is conducted over these investigated cases. A novel CODAS is proposed on IGNCSCIA and IGNCSCIG in NCS environment to handle complex frame of data that occurs in daily life. This method is also furnished upon the same application and observed that it yields the same ranking for different values of q. The comparative analysis is conducted to investigate the validity of the proposed method. It is observed that the proposed method yields the best alternative is the same as existing methods. Comparative analysis is conducted. In future, some extension of this work will be explored in the field of Bonferroni mean operators and unified generalized aggregation operators, and these operators will be furnished upon daily life problems.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

Authors' Contributions

All authors contributed equally in the preparation of this manuscript.

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