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INTERVAL- VALUED FERMATEAN NEUTROSOPHIC GRAPHS

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Abstract: *In this work, we define Interval-valued Fermatean neutrosophic graphs (IVFNS) and present some operations on Interval-valued Fermatean neutrosophic graphs. Further, we introduce the concepts of Regular interval-valued Fermatean neutrosophic graphs, Strong interval-valued Fermatean neutrosophic graphs, Cartesian, Composition, Lexicographic product of interval-valued Fermatean neutrosophic graphs. Finally, we give the applications of Interval-valued Fermatean neutrosophic graphs.*

Key words: *Interval-valued Fermatean Fuzzy sets, Interval-valued Fermatean Neutrosophic sets, Interval-valued Fermatean Neutrosophic graphs*

1. Introduction

The concept of neutrosophic set theory was proposed by Jun (2017). The idea of neutrosophic set which is a generalization of the fuzzy set (Zadeh, 1965), intuitionistic

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fuzzy set (Atanassov, 1986). The neutrosophic sets are characterized by a truth function (T), an indeterminate function (I) and a false function (F) independently. Smarandache (2019) introduced the concept of spherical neutrosophic oversets as generalization of spherical fuzzy sets. By bending the concept of single valued neutrosophic set and graph theory, different classes of neutrosophic graphs is discussed by Broumi (2016) and many works available in the literature (Broumi et al., 2016a, 2016b, 2016c, 2016d, 2022). Nagarajan et al. (2019) investigated the interval-valued neutrosophic graphs and its applications. Recently, Ajay et al. (2020, 2021) extended the concept of Pythagorean neutrosophic sets to graphs and called it Pythagorean neutrosophic graph (PNG) and investigated some of their properties. The same authors presented the idea of labelling of Pythagorean neutrosophic fuzzy graphs and investigate their properties. Ajay et al. (2022) studied the concept of regularity in PNG and introduced the ideas of regular, full edge regular, edge regular, and partially edge regular Pythagorean Neutrosophic graphs. In addition, a new MCDM method has been introduced using the Pythagorean neutrosophic graphs with an illustrative example. By integrating the concepts pythagorean neutrosophic fuzzy graph and Dombi operator. Furthermore, Ajay et al. (2021) proposed a new extension of neutrosophic graph called Pythagorean Neutrosophic Dombi fuzzy graphs (PNDFG) and suggested some basic operations of PNDFG and computed the degree and total degree of a vertex of PNDFG. Akalyadevi et al. (2022) introduced the concept of spherical neutrosophic graph coloring and discussed some of their important properties also they suggested the chromatic number of spherical neutrosophic graph as a crisp number. Duleba et al. (2021) applied the concept of Interval-valued spherical fuzzy AHP method to public transportation problem. Aydın et al. (2021) proposed a novel fuzzy MULTIMOORA method based on interval-valued spherical fuzzy sets to evaluate companies that are using Industry 4.0 technologies. Lathamaheswari et al. (2021) proposed the concept of Interval Valued Spherical Fuzzy Aggregation Operators and applied it for solving a Decision-Making Problem. Kutlu Gündoğdu et al. (2021) extended spherical fuzzy analytic hierarchy process to interval-valued spherical fuzzy AHP (IVSF-AHP) method and applied it to compare the service performances of several hospitals. Kutlu Gündoğdu et al. (2019) presented the idea of Spherical fuzzy sets (SFS) as an integration of Pythagorean fuzzy sets and neutrosophic sets. Smarandache (2017) proposed the concept of Spherical Neutrosophic Numbers. Senapati et al. (2019) defined basic operators over the FFSs. On the other hand, division, and subtraction operations on FFSs were proposed. Donghai Liu et al. (2019) focused on a distance measure for Fermatean fuzzy linguistic term sets. Ganie et al. (2022) proposed some novel distance measures for Fermatean fuzzy sets using t-conorms. On the other hand, Jeevaraj et al. (2021) proposed the concept of interval-valued Fermatean fuzzy sets (IVFFSs) and establishes some Mathematical operations on the class of IVFFSs. A new total ordering principle on the class of IVFFNs is introduced. They implemented the interval-valued Fermatean fuzzy TOPSIS (IVFFTOPSIS) method for solving multi-criteria decision-making problems. Based on neutrosophic Pythagorean sets, Stephen et al. (2021) introduced the concept of interval-valued neutrosophic Pythagorean sets with dependent interval valued Pythagorean components and discussed some of its properties. Recently, Lakhwani et al. (2022) introduced a novel concept of Dombi neutrosophic graph and presented some kinds of Dombi neutrosophic graph such as a regular Dombi neutrosophic graph, strong Dombi neutrosophic graph, complete Dombi neutrosophic graph, and complement Dombi neutrosophic graph and described some of their properties, also, and discussed some operations on Dombi neutrosophic graphs are defined.

In this paper, we present the concept of Interval-valued Fermatean neutrosophic graphs (IVFNG) and the concepts of Regular interval-valued Fermatean neutrosophic graphs, Strong interval-valued Fermatean neutrosophic graphs, Cartesian, Composition, Lexicographic product of interval-valued Fermatean neutrosophic graphs. We also introduce some theorems and examples on IVFNG's Finally, we give the applications of Interval-valued Fermatean neutrosophic graphs.

2. Preliminaries

The extension of crisp set theory with membership degree is known as Fuzzy set theory in which each element of the set gets a real number between 0 and 1. But in many real time situations, it is not always possible to give an exact degree of membership because of lack of knowledge, vague information, and so forth. To overcome this problem, we can use interval-valued fuzzy sets, which assign to each element a closed interval which approximates the "real," but unknown, membership degree. The length of this interval is a measure for the uncertainty about the membership degree. An interval number I is an interval $[c^-, c^+]$ with $0 \leq c^- \leq c^+ \leq 1$. The interval $[c, c]$ is identified with the number $c \in [0,1]$. Let $I[0,1]$ be the set of all closed subintervals of $[0,1]$. An extension of fuzzy sets by Zadeh (1965), Interval-valued fuzzy sets which stated that the values of the membership degrees are intervals of numbers instead of the numbers. It provides a more sufficient information about uncertainty than traditional fuzzy sets. In this section, we provide all the basic definitions of interval valued sets and its corresponding graphs. Table 1 depicts the types of sets and graphs for interval-valued fuzzy and neutrosophic environments.

Table 1. Different types of Interval-valued sets and its graphs

Type of Sets	Definition	Type of Graphs	Definition
Interval-valued Fuzzy set (IVFS) -Zadeh, 1975	$A = \{(x, [\mu_A^-(x), \mu_A^+(x)]): x \in V\}$	Interval-valued Fuzzy graph (IVFG) - Muhammad Akram, Wieslaw A. Dudek, 2011.	G = (A, B), where A = $[\mu_A^-(x), \mu_A^+(x)]$ is an interval-valued fuzzy set on V and B = $[\mu_B^-(x), \mu_B^+(x)]$is an interval-valued fuzzy relation on E.
Interval-valued Intuitionistic Fuzzy set (IVIFS) - Atanassov, K., Gargov, G., 1989	$A = \{(x, [T_A^-(x), T_A^+(x)]): \in V\}$; $B = \{(x, [F_A^-(x), F_A^+(x)]): x \in V\}$ such that $0 \leq T_A^+(x) + F_A^+(x) \leq 1 \; for \; all \; x \epsilon X$	Interval-valued Intuitionistic Fuzzy graph (IVIFG) - S. N. Mishra and A. Pal, 2013	• $\mu_A: V \to D[0,1]; \eta_A: V \to D[0,1]$ such that $0 \leq \mu_A(x) + \eta_A(x) \leq 1, \; \forall x \in V$ • $\mu_B: E \subseteq V \times V \to D[0,1]$; $\eta_B: E \subseteq V \times V \to D[0,1]$ $\mu_B^-((x,y)) \leq \min(\mu_A^-(x), \mu_A^-(y))$; $\eta_B^-((x,y)) \geq \min(\eta_A^-(x), \eta_A^-(y))$ $\mu_B^+((x,y)) \leq \min(\mu_A^+(x), \mu_A^+(y))$; $\eta_B^+((x,y)) \geq \min(\eta_A^+(x), \eta_A^+(y))$ such that $0 \leq \mu_B^+((x,y)) + \eta_B^+((x,y)) \leq 1, \forall (x,y) \in E$
Interval-valued Neutrosophic set (IVNS) - Said Broumi , Mohamed Talea , Assia Bakali , Florentin Smarandache, (2016)	For each point $x \in X$, we have that $T_A(x) = [T_A^-(x), T_A^+(x)]$, $I_A(x) = [I_A^-(x), I_A^+(x)]$, $F_A(x) = [F_A^-(x), F_A^+(x)] \subseteq [0,1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.	Interval-valued Neutrosophic graph (IVNG) - Said Broumi, Mohamed Talea, Assia Bakali, Florentin Smarandache, 2016	$G = (A, B)$, where $A =< [T_A^-, T_A^+], [I_A^-, I_A^+], [F_A^-, F_A^+] >$ is an interval-valued neutrosophic set on V; and $B =< [T_B^-, T_B^+], [I_B^-, I_B^+], [F_B^-, F_B^+] > T_B^-: V \times V \to [0,1], T_B^+: V \times V \to [0,1], I_B^-: V \times V \to [0,1], I_B^+: V \times V \to [0,1]$ and $F_B^-: V \times V \to [0,1], F_B^+: V \times V \to [0,1]$ are such that $T_B^-(\{v_i, v_j\}) \leq \min[T_A^-(v_i), T_A^-(v_j)]$, $T_B^+(\{v_i, v_j\}) \leq \min[T_A^+(v_i), T_A^+(v_j)]$, $I_B^-(\{v_i, v_j\}) \geq \max[I_B^-(v_i), I_B^-(v_j)]$, $I_B^+(\{v_i, v_j\}) \geq \max[I_B^+(v_i), I_B^+(v_j)]$, $F_B^-(\{v_i, v_j\}) \geq \max[F_B^-(v_i), F_B^-(v_j)]$,

			$F_B^+(\{v_i, v_j\}) \geq max[F_B^+(v_i), F_B^+(v_j)]$
Interval-valued Pythagorean Fuzzy set (IVPFS) - F. Teng, Z. Liu, and P. Liu, (2018).	For each point $x \in X$, we have that $T_A(x) = [T_A^-(x), T_A^+(x)]$, $F_A(x) = [F_A^-(x), F_A^+(x)] \subseteq [0,1]$ and $0 \leq T_A^+(x)^2 + F_A^+(x)^2 \leq 1$.	Interval-valued Pythagorean Fuzzy graph (IVPFG) - Mohamed S.Y., Ali A.M.,2018	$G = (A, B)$, where $A =< [T_A^-, T_A^+], [F_A^-, F_A^+] >$ is an interval-valued neutrosophic set on V; and $B = < [T_B^-, T_B^+], [F_B^-, F_B^+] > T_B^-: V \times V \to [0,1], T_B^+: V \times V \to [0,1]$, and $F_B^-: V \times V \to [0,1], F_B^+: V \times V \to [0,1]$ are such that $T_B^-(\{v_i, v_j\}) \leq \min[T_A^-(v_i), T_A^-(v_j)]$, $T_B^+(\{v_i, v_j\}) \leq \min[T_A^+(v_i), T_A^+(v_j)]$, $F_B^-(\{v_i, v_j\}) \geq \max[F_B^-(v_i), F_B^-(v_j)]$, $F_B^+(\{v_i, v_j\}) \geq \max[F_B^+(v_i), F_B^+(v_j)]$ such that $0 \leq T_A^+(x)^2 + F_A^+(x)^2 \leq 1$.
Interval-valued Fermatean Fuzzy set (IVFFS) - Jeevaraj S, (2021)	For each point $x \in X$, we have that $T_A(x) = [T_A^-(x), T_A^+(x)], F_A(x) = [F_A^-(x), F_A^+(x)] \subseteq [0,1]$ and $0 \leq T_A^+(x)^3 + F_A^+(x)^3 \leq 1$.	Interval-valued Fermatean Fuzzy graph (IVFFG)	$G = (A, B)$, where $A =< [T_A^-, T_A^+], [F_A^-, F_A^+] >$ is an interval-valued neutrosophic set on V; and $B = < [T_B^-, T_B^+], [F_B^-, F_B^+] > T_B^-: V \times V \to [0,1], T_B^+: V \times V \to [0,1]$, and $F_B^-: V \times V \to [0,1], F_B^+: V \times V \to [0,1]$ are such that $T_B^-(\{v_i, v_j\}) \leq \min[T_A^-(v_i), T_A^-(v_j)]$, $T_B^+(\{v_i, v_j\}) \leq \min[T_A^+(v_i), T_A^+(v_j)]$, $F_B^-(\{v_i, v_j\}) \geq \max[F_B^-(v_i), F_B^-(v_j)]$, $F_B^+(\{v_i, v_j\}) \geq max[F_B^+(v_i), F_B^+(v_j)]$ such that $0 \leq T_A^+(x)^3 + F_A^+(x)^3 \leq 1$.
Interval-valued Fermatean Neutrosophic set (IVFNS) – Said Broumi, Raman Sundareswaran, Marayanagaraj Shanmugapriya, Giorgio Nordo Mohamed Talea, Assia Bakali, and Florentin Smarandache, (2022)	A=$\{\langle x, T_A(x), I_A(x), F_A(x) \rangle \mid x \in X\}$ where $T_A(x) = [T_A^-(x), T_A^+(x)]$, $I_A(x)=[I_A^-(x), I_A^+(x)]$ and $F_A(x) = [F_A^-(x), F_A^+(x)]$, $T_A(x): X \to D[0,1]$ $I_A(x): X \to D[0,1]$, $F_A(x): X \to D[0,1]$ and $0 \leq (T_A(x))^3 + (F_A(x))^3 \leq 1$ and $0 \leq (I_A(x))^3 \leq 1$ $0 \leq (T_A(x))^3 + (F_A(x))^3 + (I_A(x))^3 \leq 2$ means $0 \leq (T_A^+(x))^3 + (I_A^+(x))^3 + (F_A^+(x))^3 \leq 2$ $\forall x \in X$	Interval-valued Fermatean Neutrosophic graph (IVFNG) - Said Broumi, et al., 2022	$G = (A, B)$, where $A =< [T_A^-, T_A^+], [I_A^-, I_A^+], [F_A^-, F_A^+] >$ is an interval-valued Fermatean neutrosophic set on V; and $B = < [T_B^-, T_B^+], [I_B^-, I_B^+], [F_B^-, F_B^+] >$ E satisfying the following condition: $T_A^-: V \to [0,1]$, $T_A^+: V \to [0,1]$, $I_A^-: V \to [0,1]$, $I_A^+: V \to [0,1]$ and $F_A^-: V \to [0,1]$, $F_A^+: V \to [0,1]$, and $0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3$, $for\ all\ v_i \in V\ (i = 1, 2, \dots, n$ The functions $T_B^-: V \times V \to [0,1], T_B^+: V \times V \to [0,1], I_B^-: V \times V \to [0,1], I_B^+: V \times V \to [0,1]$ and $F_B^-: V \times V \to [0,1], F_B^+: V \times V \to [0,1]$ are such that $T_B^-(\{v_i, v_j\}) \leq \min[T_A^-(v_i), T_A^-(v_j)]$, $T_B^+(\{v_i, v_j\}) \leq \min[T_A^+(v_i), T_A^+(v_j)]$, $I_B^-(\{v_i, v_j\}) \geq \max[I_B^-(v_i), I_B^-(v_j)]$, $I_B^+(\{v_i, v_j\}) \geq \max[I_B^+(v_i), I_B^+(v_j)]$, $F_B^-(\{v_i, v_j\}) \geq \max[F_B^-(v_i), F_B^-(v_j)]$, $F_B^+(\{v_i, v_j\}) \geq \max[F_B^+(v_i), F_B^+(v_j)]$, denoting the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $(v_i, v_j) \in E$ respectively, where $0 \leq T_B(\{v_i, v_j\})^3 + I_B(\{v_i, v_j\})^3 + F_B(\{v_i, v_j\})^3 \leq 2$ for all $\{v_i, v_j\} \in E\ (i, j = 1, 2, \dots, n)$ means $0 \leq (T_B^+(v_i, v_j))^3 + (I_B^+(v_i, v_j))^3 + (F_B^+(v_i, v_j))^3 \leq 2$ $\forall x \in X$.

Definition 2.1 (Akram et al., 2013)

The Interval-valued Fuzzy Set (IVFS) A in V is defined by $= \{(x, \{\mu_A^-(x), \mu_A^+(x)\}) : x \in V\}$, where $\mu_A^-(x)$ and $\mu_A^+(x)$ are fuzzy subsets of V such that $\mu_A^-(x) \leq \mu_A^+(x)$ for all $x \in V$. For any two interval-valued sets $A = [\mu_A^-(x), \mu_A^+(x)]$ and $B = [\mu_B^-(x), \mu_B^+(x)]$ in V.

Define: • $A \cup B = \{(x, max(\mu_A^-(x), \mu_B^-(x)), max(\mu_A^+(x), \mu_B^+(x))) : x \in V\}$,

• $A \cap B = \{(x, min(\mu_A^-(x), \mu_B^-(x)), min(\mu_A^+(x), \mu_B^+(x))) : x \in V\}$.

Definition 2.2 (Akram et al., 2013)

If $G^* = (V, E)$ is a graph, then by an **Interval-valued Fuzzy Relation (IVFR)** B on a set E we mean an interval-valued fuzzy set such that $\mu_B^-(xy) \leq min(\mu_A^-(x), \mu_A^-(y)), \mu_B^+(xy) \leq min(\mu_A^+(x), \mu_A^+(y))$ for all $xy \in E$.

Definition 2.3 (Akram et al., 2013)

By an Interval-valued Fuzzy Graph (IVFG) of a graph $G^* = (V, E)$ we mean a pair $G = (A, B)$, where $A = [\mu_A^-, \mu_A^+]$ is an interval-valued fuzzy set on V and $B = [\mu_B^-, \mu_B^+]$ is an interval-valued fuzzy relation on E.

Example 2.4 (Akram et al., 2013)

Consider a graph $G^* = (V, E)$ such that $V = \{x, y, z\}, E = \{xy, yz, zx\}$. Let A be an interval-valued fuzzy set of V and B be an interval-valued fuzzy set of $E \subseteq V \times V$ defined by

$$A = \langle\left(\frac{x}{0.2}, \frac{y}{0.3}, \frac{z}{0.4}\right), \left(\frac{x}{0.4}, \frac{y}{0.5}, \frac{z}{0.5}\right)\rangle, B = \langle\left(\frac{x}{0.2}, \frac{y}{0.3}, \frac{z}{0.4}\right), \left(\frac{xy}{0.3}, \frac{yz}{0.4}, \frac{zx}{0.4}\right)\rangle$$

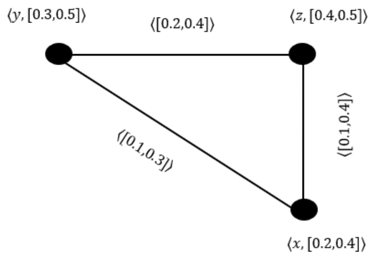

Figure 1. Interval-Valued Fuzzy Graph G

Akram et al. (2013) introduced certain types of interval-valued fuzzy graphs including balanced interval-valued fuzzy graphs, neighbourly irregular interval-valued fuzzy graphs, neighbourly total irregular interval-valued fuzzy graphs, highly irregular interval-valued fuzzy graphs, and highly total irregular interval-valued fuzzy graph. Hossein et al. (2013) define three new operations on interval-valued fuzzy graphs; namely direct product, semi strong product and strong product.

Definition 2.5 (Mishra et al., 2013; Ismayil et al., 2014)

An Interval-valued Intuitionistic Fuzzy Set (IVIFS) A in X, is given by $A = \{\langle x, \mu_A(x), \eta_A(x)\rangle / x \epsilon X\}$ where $\mu_A: X \rightarrow [0,1]$, $\eta_A: X \rightarrow D[0,1]$. The intervals $\mu_A(x)$ and $\eta_A(x)$ denote the degree of membership and the degree of non-membership of the element x to the set, where $\mu_A(x) = [\mu_A^-(x), \mu_A^+(x)]$ and $\eta_A(x) = [\eta_A^-(x), \eta_A^+(x)]$ with the condition $0 \leq \mu_A^+(x) + \eta_A^+(x) \leq 1$ for all $x \epsilon X$.

Definition 2.6 (Mishra et al., 2013; Ismayil et al., 2014)

An Interval-valued Intuitionistic Fuzzy Graph (IVIFG) with underlying set V is defined to be a pair $G = (A, B)$ where

- the functions $\mu_A: V \rightarrow D[0,1]; \eta_A: V \rightarrow D[0,1]$ denote the degree of membership and non-membership of the element $x \in V$ respectively, such that $0 \leq \mu_A(x) + \eta_A(x) \leq 1$, $\forall x \in V$
- the functions $\mu_B: E \subseteq V \times V \rightarrow D[0,1]$; $\eta_B: E \subseteq V \times V \rightarrow D[0,1]$ are defined by

$\mu_B^-((x,y)) \leq \min(\ \mu_A^-(x), \mu_A^-(y))\ ;\ \eta_B^-((x,y)) \geq \min(\ \eta_A^-(x), \eta_A^-(y))$
$\mu_B^+((x,y)) \leq \min(\ \mu_A^+(x), \mu_A^+(y))\ ;\ \eta_B^+((x,y)) \geq \min(\ \eta_A^+(x), \eta_A^+(y))$
such that $0 \leq \mu_B^+((x,y)) + \eta_B^+((x,y)) \leq 1, \forall (x,y) \in E$

Example 2.7

$G = (A, B)$ defined on a graph $G^* = (V, E)$ such that $V = \{x, y, z\}, E = \{xy, yz, zx\}$, A is an interval valued intuitionistic fuzzy set of $V$and let B is an interval-valued intuitionistic fuzzy set of $E \subseteq V\ X\ V$.

here $A = \{\langle x, [0.5,0.7], [0.1,0.3]\rangle, \langle y, [0.6,0.7], [0.1,0.3]\rangle, \langle z, [0.4,0.6], [0.2,0.4]\rangle\}$
$B = \{\langle xy, [0.3,0.6], [0.2,0.4]\rangle, \langle yz, [0.3,0.5], [0.3,0.4]\rangle, \langle xz, [0.3,0.5], [0.2,0.4]\rangle\}$

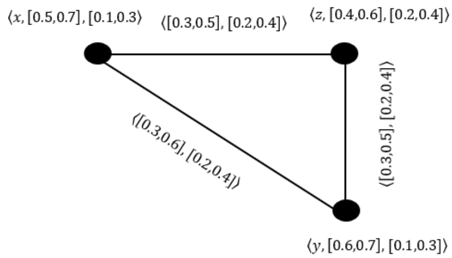

Figure 2. Interval-Valued Intuitionistic Fuzzy Graph G

Mishra et al. (2013) introduced product of IVIFG and Ismayil et al. (2014) defined On Strong Interval-Valued Intuitionistic Fuzzy Graph. Akram et al. (2013) studied the certain types of interval-valued fuzzy graphs. Peng Xu et al. (2022) studied the concept of certain interval-valued intuitionistic fuzzy graphs and its applications. Xindong et al. (2016) introduced the concept of interval-valued Pythagorean fuzzy set. Mohamed et. al. (2018) introduced and studied interval-valued Pythagorean fuzzy graphs.

*Definition 2.8 (*Mohamed et. al., 2018*)*

An Interval- valued Pythagorean Fuzzy set (IVPFS) A defined in a finite universe of discourse X is given by $A = \{\langle x, \mu_A(x) = [\ \mu_A^-(x), \mu_A^+(x)], \eta_A(x) = [\eta_A^-(x), \eta_A^+(x)]\rangle / x \epsilon\ X\}$ where $\mu_A^-(x), \mu_A^+(x) : X \to [0,1]$ and $\eta_A^-(x), \eta_A^+(x): X \to [0,1]$ and $0 \leq (\mu_A^+(x))^2\ + (\eta_A^+(x))^2 \leq 1$. Here $\mu_A(x)$ and $\eta_A(x)$ denote the degree of membership and degree of non-membership of $x \in X$ in A.

*Definition 2.9 (*Mohamed et. al., 2018*)*

An Pythagorean Fuzzy Graph (PFG) with underlying set V defined to be a pair $G = (A, B)$where

- the functions $\mu_A: V \to D[0,1]; \eta_A: V \to D[0,1]$ denote the degree of membership and non-membership of the element $x \in V$ respectively, such that $0 \leq \mu_A(x) + \eta_A(x) \leq 1,\ \forall\ x \in V$
- the functions $\mu_B: E \subseteq V\ \times\ V \to D[0,1]\ ;\ \eta_B: E \subseteq V\ \times\ V \to D[0,1]$ are defined by
  $\mu_B^-((x,y)) \leq \min(\ \mu_A^-(x), \mu_A^-(y))\ ;\ \eta_B^-((x,y)) \geq \min(\ \eta_A^-(x), \eta_A^-(y))$
  $\mu_B^+((x,y)) \leq \min(\ \mu_A^+(x), \mu_A^+(y))\ ;\ \eta_B^+((x,y)) \geq \min(\ \eta_A^+(x), \eta_A^+(y))$
  such that $0 \leq \mu_B^+((x,y))^2 + \eta_B^+((x,y))^2 \leq 1, \forall (x,y) \in E$

Yahya et al. (2018) defined the strong interval-valued Pythagorean fuzzy graph and Cartesian product, composition and join of two strong interval-valued Pythagorean fuzzy graph are studied.

Definition 2.10 (Broumi et al., 2016d)

An Interval-valued Neutrosophic Set (IVNS) A in X is characterized by truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$ and falsity-membership function $F_A(x)$. For each point $x \in X$, we have that $T_A(x) = [T_A^-(x), T_A^+(x)], I_A(x) = [I_A^-(x), I_A^+(x)], F_A(x) = [F_A^-(x), F_A^+(x)] \subseteq [0,1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.11 (Broumi et al., 2016d)

An **Interval- valued Neutrosophic Graph (IVNG)** of a graph $G^* = (V, E)$ we mean a pair $G = (A, B)$, where $A =< [T_A^-, T_A^+], [I_A^-, I_A^+], [F_A^-, F_A^+] >$ is an interval-valued neutrosophic set on V; and $B = \langle [T_B^-, T_B^+], [I_B^-, I_B^+], [F_B^-, F_B^+] \rangle$ is an interval valued neutrosophic relation on E satisfying the following condition:

i. $V = \{ v_1, v_2, \dots, v_n \}$, such that $T_A^- : V \to [0,1]$, $T_A^+ : V \to [0,1], I_A^- : V \to [0,1]$, $I_A^+: V \to [0,1]$ and $F_A^-: V \to [0,1], F_A^+: V \to [0,1]$ denote the degree of truth-membership, the degree of indeterminacy-membership and falsity-membership of the element $y \in V$, respectively, and $0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3, for\ all\ v_i \in V\ (i = 1, 2, \dots, n)$

ii. The functions $T_B^-: V \times V \to [0,1], T_B^+: V \times V \to [0,1], I_B^-: V \times V \to [0,1], I_B^+: V \times V \to [0,1]$ and $F_B^-: V \times V \to [0,1], F_B^+: V \times V \to [0,1]$ are such that

$$T_B^-(\{v_i, v_j\}) \leq \min[T_A^-(v_i), T_A^-(v_j)],$$
$$T_B^+(\{v_i, v_j\}) \leq \min[T_A^+(v_i), T_A^+(v_j)],$$
$$I_B^-(\{v_i, v_j\}) \geq \max[I_B^-(v_i), I_B^-(v_j)],$$
$$I_B^+(\{v_i, v_j\}) \geq \max[I_B^+(v_i), I_B^+(v_j)],$$
$$F_B^-(\{v_i, v_j\}) \geq \max[F_B^-(v_i), F_B^-(v_j)],$$
$$F_B^+(\{v_i, v_j\}) \geq max[F_B^+(v_i), F_B^+(v_j)],$$

denoting the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $(v_i, v_j) \in E$ respectively, where $0 \leq T_B(\{v_i, v_j\}) + I_B(\{v_i, v_j\}) + F_B(\{v_i, v_j\}) \leq 3$ for all $\{v_i, v_j\} \in E\ (i, j = 1, 2, \dots, n)$.

3. Interval-valued Fermatean neutrosophic graphs

Fuzzy sets, Intuitionistic fuzzy sets, Neutrosophic sets are the generalization of the classical set and which are also the most popular mathematical tools in the study uncertainty. Later, researchers combined these sets with graph structures and studied its properties in literature. These combinations, Fuzzy graphs, Intuitionistic fuzzy graphs and Neutrosophic graphs are useful in decision making problems. In an administrative setup, electing a leader among a group of people through the voting process, a judgement may give based on a candidate satisfies his expectations with a possibility of 0.80 and this candidate dissatisfies the expectations with a possibility of 0.95 and neutrally give 0.85 But their sum is 2.80 (>2) and their square sum is 2.265 (>2) and the sum of the cubes is equal to 1.9835 (<2). It is impossible to give an exact degree of membership in every instant, because the lack of knowledge, vague information, and so forth may produce higher values to the membership values. To overcome this problem, we can use interval-valued fuzzy sets, which assign to each element a closed interval which approximates the "real," but unknown, membership degree. In this series, we are adding one more class of graphs namely, interval-valued Fermatean neutrosophic graphs and certain types of interval-valued Fermatean neutrosophic graphs are introduced and discussed in this section.

Definition 3.1

An interval-valued Fermatean neutrosophic set (IVFNS) A on the universe of discourse X is of the structure:

$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}$, where $T_A(x) = [T_A^-(x), T_A^+(x)], I_A(x) = [I_A^-(x), I_A^+(x)]$ and $F_A(x) = [F_A^-(x), F_A^+(x)]$ represents the truth-membership degree, indeterminacy-membership degree and falsity-membership degree, respectively. Consider the mapping $T_A(x): X \to D[0,1]$, $I_A(x): X \to D[0,1]$, $F_A(x): X \to D[0,1]$ and

$0 \le (T_A(x))^3 + (F_A(x))^3 \le 1$ and $0 \le (I_A(x))^3 \le 1$

$0 \le (T_A(x))^3 + (F_A(x))^3 + (I_A(x))^3 \le 2$ means

$0 \le (T_A^+(x))^3 + (I_A^+(x))^3 + (F_A^+(x))^3 \le 2 \quad \forall x \in X$

Definition 3.2

An Interval-Valued Fermatean Neutrosophic Graph (IVFNG) of a graph $G^* = (V, E)$ we mean a pair $G = (A, B)$, where $A = \langle [T_A^-, T_A^+], [I_A^-, I_A^+], [F_A^-, F_A^+]\rangle$ is an interval-valued Fermatean neutrosophic set on V; and $B = \langle [T_B^-, T_B^+], [I_B^-, I_B^+], [F_B^-, F_B^+\rangle]$ is an interval valued Fermatean neutrosophic relation on E satisfying the following condition:

i. $V = \{v_1, v_2, \ldots, v_n\}$, such that $T_A^- : V \to [0,1]$, $T_A^+ : V \to [0,1]$, $I_A^- : V \to [0,1]$, $I_A^+ : V \to [0,1]$ and $F_A^- : V \to [0,1]$, $F_A^+ : V \to [0,1]$ denote the degree of truth -membership, the degree of indeterminacy-membership and falsity-membership of the element $y \in V$, respectively, and $0 \le T_A(v_i) + I_A(v_i) + F_A(v_i) \le 3$, $for\ all\ v_i \in V\ (i = 1, 2, \ldots, n)$.

ii. The functions $T_B^-: V \times V \to [0,1]$, $T_B^+: V \times V \to [0,1]$, $I_B^-: V \times V \to [0,1]$, $I_B^+: V \times V \to [0,1]$ and $F_B^-: V \times V \to [0,1]$, $F_B^+: V \times V \to [0,1]$ are such that

$$T_B^-(\{v_i, v_j\}) \le \min[T_A^-(v_i), T_A^-(v_j)],\ T_B^+(\{v_i, v_j\}) \le \min[T_A^+(v_i), T_A^+(v_j)],$$
$$I_B^-(\{v_i, v_j\}) \ge \max[I_B^-(v_i), I_B^-(v_j)],\ I_B^+(\{v_i, v_j\}) \ge \max[I_B^+(v_i), I_B^+(v_j)],$$
$$F_B^-(\{v_i, v_j\}) \ge \max[F_B^-(v_i), F_B^-(v_j)],\ F_B^+(\{v_i, v_j\}) \ge \max[F_B^+(v_i), F_B^+(v_j)]$$

denoting the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $(v_i, v_j) \in E$ respectively, where $0 \le T_B(\{v_i, v_j\})^3 + I_B(\{v_i, v_j\})^3 + F_B(\{v_i, v_j\})^3 \le 2$ for all $\{v_i, v_j\} \in E\ (i, j = 1, 2, \ldots, n)$ means $0 \le (T_B^+(v_i, v_j))^3 + (I_B^+(v_i, v_j))^3 + (F_B^+(v_i, v_j))^3 \le 2 \quad \forall x \in X$.

Example 3.3

Consider a graph G^*, such that $V = \{x_1, x_2, x_3\}$, $E = \{x_1x_2, x_2x_3, x_3x_4, x_4x_1\}$. Let A be an interval valued Fermatean neutrosophic subset of V and B be an interval valued Fermatean neutrosophic subset of E, denoted by

$$A = \left\{ \begin{matrix} \langle x_1 [0.85,0.95], [0.90,0.95], [0.85,0.85]\rangle, \langle x_2, [0.85,0.90], [0.90,0.95], [0.85,0.90]\rangle, \\ \langle x_3, [0.85,0.95], [0.95,0.95], [0.85,0.95]\rangle \end{matrix} \right\}$$

$$B = \left\{ \begin{matrix} \langle x_1x_2, [0.80,0.90], [0.90,0.95], [0.80,0.85]\rangle, \langle x_2x_3, [0.85,0.90], [0.90,0.95], [0.85,0.85]\rangle, \\ \langle x_3x_1, [0.85,0.95], [0.90,0.95], [0.85,0.85]\rangle \end{matrix} \right\}$$

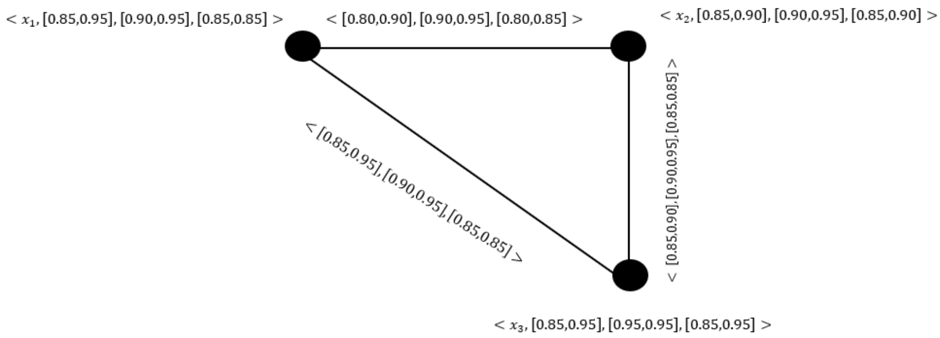

Figure 3. Interval-valued Fermatean Neutrosophic Graph G

Definition 3.4.

Let $G = (A, B)$ be an IVFNG. G is an interval valued regular Fermatean neutrosophic graph if it satisfies the following conditions:

$$\sum_{v_1 \neq v_2} T_B^-(v_1, v_2) = constant \quad ; \sum_{v_1 \neq v_2} T_B^+(v_1, v_2) = constant$$
$$\sum_{v_1 \neq v_2} I_B^-(v_1, v_2) = constant \quad ; \sum_{v_1 \neq v_2} I_B^+(v_1, v_2) = constant$$
$$\sum_{v_1 \neq v_2} F_B^-(v_1, v_2) = constant \quad ; \sum_{v_1 \neq v_2} F_B^+(v_1, v_2) = constant$$

Definition 3.5.

Let G = (A, B) be an IVFNG. G is an interval valued regular strong neutrosophic graph if it satisfies the following conditions

$$T_B^-(v_1, v_2) = \min\left(T_A^-(v_1), T_A^-(v_2)\right); \sum_{v_1 \neq v_2} T_B^-(v_1, v_2) = constant$$
$$T_B^+(v_1, v_2) = \min\left(T_A^+(v_1), T_A^+(v_2)\right); \sum_{v_1 \neq v_2} T_B^+(v_1, v_2) = constant$$
$$I_B^-(v_1, v_2) = \max\left(I_A^-(v_1), I_A^-(v_2)\right); \sum_{v_1 \neq v_2} I_B^-(v_1, v_2) = constant$$
$$I_B^+(v_1, v_2) = \max\left(I_A^+(v_1), I_A^+(v_2)\right); \sum_{v_1 \neq v_2} I_B^+(v_1, v_2) = constant$$
$$F_B^-(v_1, v_2) = \max\left(F_A^-(v_1), F_A^-(v_2)\right); \sum_{v_1 \neq v_2} F_B^-(v_1, v_2) = constant$$
$$F_B^+(v_1, v_2) = \max\left(F_A^+(v_1), F_A^+(v_2)\right); \sum_{v_1 \neq v_2} F_B^+(v_1, v_2) = constant$$

Definition 3.6.

Let G = (A, B) be an IVFNG. G is a strong interval valued regular strong neutrosophic graph if it satisfies the following conditions:

$$T_B^-(v_1, v_2) = \min\left(T_A^-(v_1), T_A^-(v_2)\right);$$
$$I_B^-(v_1, v_2) = \max\left(I_A^-(v_1), I_A^-(v_2)\right);$$
$$F_B^-(v_1, v_2) = \max\left(F_A^-(v_1), F_A^-(v_2)\right);$$
$$T_B^+(v_1, v_2) = \min\left(T_A^+(v_1), T_A^+(v_2)\right);$$
$$I_B^+(v_1, v_2) = \max\left(I_A^+(v_1), I_A^+(v_2)\right);$$
$$F_B^+(v_1, v_2) = \max\left(F_A^+(v_1), F_A^+(v_2)\right);$$

such that $0 \leq T_B^+(v_1, v_2)) + I_B^+(v_1, v_2)) + F_B^+(v_1, v_2)) \leq 3$, for all $v_1, v_2 \in E$ and $0 \leq \left(T_B^+(v_i, v_j)\right)^3 + \left(I_B^+(v_i, v_j)\right)^3 + \left(F_B^+(v_i, v_j)\right)^3 \leq 2 \quad \forall x \in X$

Example 3.7.

Let $G = (A, B)$be an Interval-valued Fermatean Neutrosophic graph with$V = \{x_1, x_2, x_3\}$.

$$A = \left\{ \begin{matrix} \langle x_1, [0.85,0.95], [0.90,0.95], [0.85,0.85] \rangle, \langle x_2, [0.85,0.90], [0.90,0.95], [0.85,0.90] \rangle, \\ \langle x_3, [0.85,0.95], [0.95,0.95], [0.85,0.95] \rangle, \end{matrix} \right\},$$

$$B = \left\{ \begin{matrix} \langle x_1x_2, [0.85,0.90], [0.90,0.95], [0.80,0.90] \rangle, \langle x_2x_3, [0.85,0.90], [0.95,0.95], [0.85,0.95] \rangle, \\ \langle x_1x_3, [0.85,0.95], [0.95,0.95], [0.85,0.95] \rangle, \end{matrix} \right\},$$

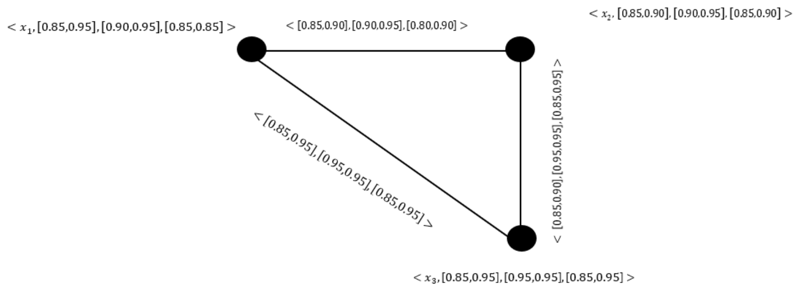

Figure 4. Strong Interval-valued Fermatean Neutrosophic Graph G

Definition 3.8.

Let A_1 and A_2 be interval-valued neutrosophic subsets of V_1 and V_2 respectively. Let B_1 and B_2 interval-valued neutrosophic subsets of E_1 and E_2 respectively. The Cartesian product of two IVFNGs G_1 and G_2 is denoted by $G_1 \times G_2 = (A_1 \times A_2, B_1 \times B_2)$ and is defined as follows:

i. $(T_{A_1}^- \times T_{A_2}^-)(x_1, x_2) = \min\left(T_{A_1}^-(x_1), T_{A_2}^-(x_2)\right)$

$(T_{A_1}^+ \times T_{A_2}^+)(x_1, x_2) = \min\left(T_{A_1}^+(x_1), T_{A_2}^+(x_2)\right)$

$(I_{A_1}^- \times I_{A_2}^-)(x_1, x_2) = \max\left(I_{A_1}^-(x_1), I_{A_2}^-(x_2)\right)$

$(I_{A_1}^+ \times I_{A_2}^+)(x_1, x_2) = \max\left(I_{A_1}^+(x_1), I_{A_2}^+(x_2)\right)$

$(F_{A_1}^- \times F_{A_2}^-)(x_1, x_2) = \max\left(F_{A_1}^-(x_1), F_{A_2}^-(x_2)\right)$

$(F_{A_1}^+ \times F_{A_2}^+)(x_1, x_2) = \max\left(F_{A_1}^+(x_1), F_{A_2}^+(x_2)\right) \textit{ for all } (x_1, x_2) \in V$

ii. $(T_{B_1}^- \times T_{B_2}^-)\left((x, x_2)(x, y_2)\right) = \min\left(T_{A_1}^-(x), T_{B_1}^-(x_2y_2)\right)$

$(T_{B_1}^+ \times T_{B_2}^+)\left((x, x_2)(x, y_2)\right) = \min\left(T_{A_1}^+(x), T_{B_1}^+(x_2y_2)\right)$

$(I_{B_1}^- \times I_{B_2}^-)\left((x, x_2)(x, y_2)\right) = \max\left(I_{A_1}^-(x), I_{B_2}^-(x_2y_2)\right)$

$(I_{B_1}^+ \times I_{B_2}^+)\left((x, x_2)(x, y_2)\right) = \max\left(I_{A_1}^+(x), I_{B_2}^+(x_2y_2)\right)$

$(F_{B_1}^- \times F_{B_2}^-)\left((x, x_2)(x, y_2)\right) = \max\left(F_{A_1}^-(x), F_{B_2}^-(x_2y_2)\right)$

$(F_{B_1}^+ \times F_{B_2}^+)\left((x, x_2)(x, y_2)\right) = \max\left(F_{A_1}^+(x), F_{B_2}^+(x_2y_2)\right)$

$\forall x \in V_1 \textit{ and } \forall x_2y_2 \in E_2$

iii. $(T_{B_1}^- \times T_{B_2}^-)\left((x_1, z)(y_1, z)\right) = \min(T_{B_1}^-(x_1y_1), T_{A_2}^-(z))$

$$(T_{B_1}^{+} \times T_{B_2}^{+})((x_1, z)(y_1, z)) = \min\left(T_{B_1}^{+}(x_1 y_1), T_{A_2}^{+}(z)\right)$$
$$\left(I_{B_1}^{-} \times I_{B_2}^{-}\right)((x_1, z)(y_1, z)) = \max\left(I_{B_1}^{-}(x_1 y_1), I_{A_2}^{-}(z)\right)$$
$$\left(I_{B_1}^{+} \times I_{B_2}^{+}\right)((x_1, z)(y_1, z)) = \max\left(I_{B_1}^{+}(x_1 y_1), I_{A_2}^{+}(z)\right)$$
$$\left(F_{B_1}^{-} \times F_{B_2}^{-}\right)((x_1, z)(y_1, z)) = \max\left(F_{B_1}^{-}(x_1 y_1), F_{A_2}^{-}(z)\right)$$
$$\left(F_{B_1}^{+} \times F_{B_2}^{+}\right)\left((x_1, z)(y_1, z)\right) = \max\left(F_{B_1}^{+}(x_1 y_1), F_{A_2}^{+}(z)\right)$$
$$\forall z \in V_2 \text{ and } \forall x_1 y_1 \in E_1$$

Example 3.9.

Let $G_1^* = (A_1, B_1)$and $G_2^* = (A_2, B_2)$ be two graphs where $V_1 = \{u_1, u_2\}, V_2 = \{v_1, v_2\}$. Consider two interval valued Fermatean neutrosophic graphs:
$A_1 = \{\langle u_1, [0.85,0.95], [0.95,0.95], [0.95,0.95]\rangle, \langle u_2, [0.90,0.90], [0.95,0.95], [0.85,0.85]\rangle, \}$,
$B_1 = \{\langle u_1 u_2, [0.85,0.90], [0.95,0.95], [0.95,0.95]\rangle\}$;
$A_2 = \{\langle v_1, [0.80,0.90], [0.85,0.95], [0.95,0.85]\rangle, \langle v_2, [0.95,0.90], [0.95,0.95], [0.80,0.85]\rangle, \}$,
$B_2 = \{\langle v_1 v_2, [0.80,0.90], [0.95,0.95], [0.95,0.85]\rangle\}$.

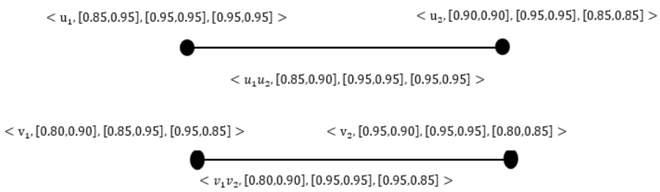

Figure 5. Interval − valued Fermatean Neutrosophic Graphs G_1 , G_2

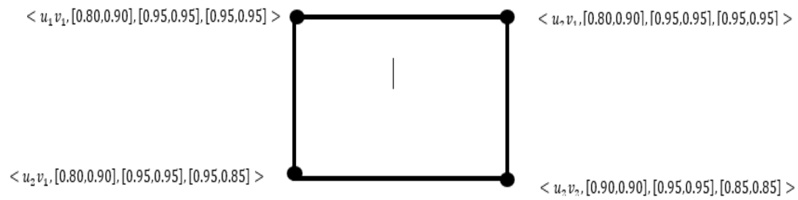

Figure 6. Cartesian product of two IVFNGs $G_1 \times G_2$

Definition 3.10.

Let $G^{\$} = G_1^{\$} \times G_2^{\$} = (V_1 \times V_2, E)$ be the composition of two graphs where $E = \{(x, x_2)(x, y_2) / x \in V_1, x_2 y_2 \in E_2\} \cup \{(x_1, z)(y_1, z) / z \in V_2, x_1 y_1 \in E_1\} \cup \{(x_1, x_2)(y_1, y_2) | x_1 y_1 \in E_1, x_2 \neq y_2\}$, then the composition of interval valued Fermatean neutrosophic graphs $G_1[G_2] = (A_1 \circ A_2, B_1 \circ B_2)$ is an interval valued Fermatean neutrosophic graphs defined by:

i. $(T_{A_1}^{-} \circ T_{A_2}^{-})(x_1, x_2) = \min(T_{A_1}^{-}(x_1), T_{A_2}^{-}(x_2))$

$(T_{A_1}^{+} \circ T_{A_1}^{+})(x_1, x_2) = \min(T_{A_1}^{+}(x_1), T_{A_1}^{+}(x_2))$

$\left(I_{A_1}^{-} \circ I_{A_2}^{-}\right)(x_1, x_2) = \max\left(I_{A_1}^{-}(x_1), I_{A_2}^{-}(x_2)\right)$

$\left(I_{A_1}^{+} \circ I_{A_2}^{+}\right)(x_1, x_2) = \max\left(I_{A_1}^{+}(x_1), I_{A_2}^{+}(x_2)\right)$

$\left(F_{A_1}^{-} \circ F_{A_2}^{-}\right)(x_1, x_2) = \max\left(F_{A_1}^{-}(x_1), F_{A_2}^{-}(x_2)\right)$

$(F_{A_1}^{+} \circ F_{A_2}^{+})(x_1, x_2) = \max(F_{A_1}^{+}(x_1), F_{A_2}^{+}(x_2)) \ \forall x_1 \in V_1, x_2 \in V_2$

ii. $\left(T_{A_1}^{-} \circ T_{A_2}^{-}\right)\left((x, x_2)(x, y_2)\right) = \min\left(T_{A_1}^{-}(x), T_{B_2}^{-}(x_2 y_2)\right)$

$\left(T_{A_1}^{+} \circ T_{A_1}^{+}\right)\left((x, x_2)(x, y_2)\right) = \min\left(T_{A_1}^{+}(x), T_{B_2}^{+}(x_2y_2)\right)$

$(I_{A_1}^{-} \circ I_{A_2}^{-})\ ((x, x_2)(x, y_2)) = \max(I_{A_1}^{-}(x), I_{B_2}^{-}(x_2y_2))$

$(I_{A_1}^{+} \circ I_{A_2}^{+})\ ((x, x_2)(x, y_2)) = \max(I_{A_1}^{+}(x), I_{B_2}^{+}(x_2y_2))$

$\left(F_{A_1}^{-} \circ F_{A_2}^{-}\right)\left((x, x_2)(x, y_2)\right) = \max\left(F_{A_1}^{-}(x), F_{B_2}^{-}(x_2y_2)\right)$

$(F_{A_1}^{+} \circ F_{A_2}^{+})\ ((x, x_2)(x, y_2)) = \max(F_{A_1}^{+}(x), F_{B_2}^{+}(x_2y_2))\ \forall\, x \in V_1, \forall x_2y_2 \in E_2$

iii. $(T_{B_1}^{-} \circ T_{B_2}^{-})\ ((x_1, z)\ (y_1, z)) = \min(T_{B_1}^{-}(x_1y_1), T_{A_2}^{-}(z))$

$(T_{B_1}^{+} \circ T_{B_2}^{+})\ ((x_1, z)\ (y_1, z)) = \min(T_{B_1}^{+}(x_1y_1), T_{A_2}^{+}(z))$

$(I_{B_1}^{-} \circ I_{B_2}^{-})\ ((x1, z)\ (y1, z)) = \max(I_{B_1}^{-}(x_1y_1), I_{A_2}^{-}(z))$

$(I_{B_1}^{+} \circ I_{B_2}^{+})\ ((x_1, z)\ (y_1, z)) = \max(I_{B_1}^{+}(x_1y_1), I_{A_2}^{+}(z))$

$(F_{B_1}^{-} \circ F_{B_2}^{-})\ ((x_1, z)\ (y_1, z)) = \max(F_{B_1}^{-}(x_1y_1), F_{A_2}^{-}(z))$

$(F_{B_1}^{+} \circ F_{B_2}^{+})\ ((x_1, z)\ (y_1, z)) = \max(F_{B_1}^{+}(x_1y_1), F_{A_2}^{+}(z))\ \forall\, z \in V_2, \forall x_1y_1 \in E_1$;

iv. $(T_{B_1}^{-} \circ T_{B_2}^{-})\ ((x_1, x_2)\ (y_1, y_2)) = \min(T_{A_2}^{-}(x_2), T_{A_2}^{-}(y_2), T_{B_1}^{-}(x_1y_1))$

$\left(T_{B_1}^{+} \circ T_{B_2}^{+}\right)\left((x_1, x_2)(y_1, y_2)\right) = \min\left(T_{A_2}^{+}(x_2), T_{A_2}^{+}(y_2), T_{B_1}^{+}(x_1y_1)\right)$

$\left(I_{B_1}^{-} \circ I_{B_2}^{-}\right)\left((x_1, x_2)(y_1, y_2)\right) = \max\left(I_{A_2}^{-}(x_2), I_{A_2}^{-}(y_2), I_{B_1}^{-}(x_1y_1)\right)$

$\left(I_{B_1}^{+} \circ I_{B_2}^{+}\right)\left((x_1, x_2)(y_1, y_2)\right) = \max\left(I_{A_2}^{+}(x_2), I_{A_2}^{+}(y_2), I_{B_1}^{+}(x_1y_1)\right)$

$\left(F_{B_1}^{-} \circ F_{B_2}^{-}\right)\left((x_1, x_2)(y_1, y_2)\right) = \max\left(F_{A_2}^{-}(x_2), F_{A_2}^{-}(y_2), F_{B_1}^{-}(x_1y_1)\right)$

$\left(F_{B_1}^{+} \circ F_{B_2}^{+}\right)\left((x_1, x_2)(y_1, y_2)\right) = \max\left(F_{A_2}^{+}(x_2), F_{A_2}^{+}(y_2), F_{B_1}^{+}(x_1y_1)\right)$,

$\forall\, (x_1, x_2)(y_1, y_2) \in E^0 - E$, *where* $E^0 = E \cup \{(x_1, x_2)\ (y_1, y_2)\ | x_1y_1 \in E_1, x_2 \neq y_2\}$.

Example 3.11.

Let $G_1^* = (A_1, B_1)$ and $G_2^* = (A_2, B_2)$ be two graphs where $V_1 = \{u_1, u_2\}, V_2 = \{v_1, v_2\}$. Consider two interval valued Fermatean neutrosophic graphs:

$A_1 = \{\langle u_1, [0.85,0.95], [0.95,0.95], [0.95,0.95]\rangle, \langle u_2, [0.90,0.90], [0.95,0.95], [0.85,0.85]\rangle\}$,
$B_1 = \{\langle u_1u_2, [0.85,0.90], [0.95,0.95], [0.95,0.95]\rangle\}$;
$A_2 = \{\langle v_1, [0.80,0.90], [0.85,0.95], [0.95,0.85]\rangle, \langle v_2, [0.95,0.90], [0.95,0.95], [0.80,0.85]\rangle\}$,
$B_2 = \{\langle v_1v_2, [0.80,0.90], [0.95,0.95], [0.95,0.85]\rangle\}$.

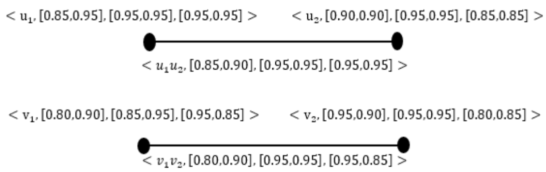

Figure 7. Interval − valued Fermatean Neutrosophic Graphs G_1 , G_2

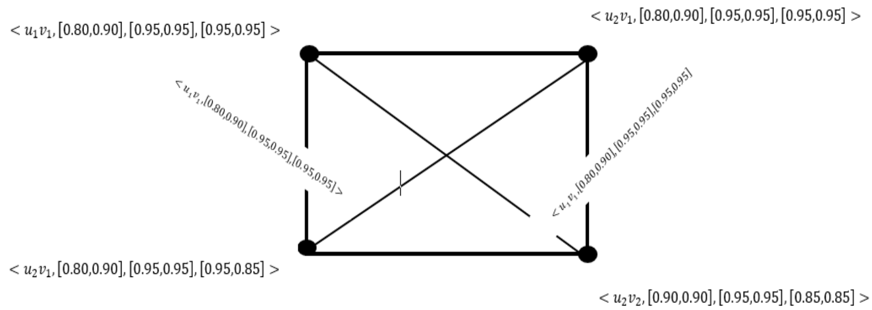

Figure 8. Composition of interval valued Fermatean neutrosophic graphs $G_1[\ G_2]$

Definition 3.12.

The union $G_1 \cup G_2 = (A_1 \cup A_2, B_1 \cup B_2)$ of two interval valued Fermatean neutrosophic graphs of the graphs G_1^* and G_2^* is an interval-valued Fermatean neutrosophic graph of $G_1^* \cup G_2^*$.

- $(T_{A_1}^- \cup T_{A_2}^-)(x) = \begin{cases} T_{A_1}^-(x), if\ x \in V_1\ and\ x \notin V_2 \\ T_{A_2}^-(x) if\ x \notin V_1\ and\ x \in V_2 \\ \min(T_{A_1}^-(x), T_{A_2}^-(x)) \text{ if } x \in V_1 \cap V_2, \end{cases},$
- $(T_{A_1}^+ \cup T_{A_2}^+)(x) = \begin{cases} T_{A_1}^+(x), if\ x \in V_1\ and\ x \notin V_2 \\ T_{A_2}^+(x) if\ x \notin V_1\ and\ x \in V_2 \\ \min(T_{A_1}^+(x), T_{A_2}^+(x)) \text{ if } x \in V_1 \cap V_2, \end{cases}$
- $(I_{A_1}^- \cup I_{A_2}^-)(x) = \begin{cases} I_{A_1}^-(x), if\ x \in V_1\ and\ x \notin V_2 \\ I_{A_2}^-(x) if\ x \notin V_1\ and\ x \in V_2 \\ \max(I_{A_1}^-(x), I_{A_2}^-(x)) \text{ if } x \in V_1 \cap V_2, \end{cases}$
- $(I_{A_1}^+ \cup I_{A_2}^+)(x) = \begin{cases} I_{A_1}^+(x), if\ x \in V_1\ and\ x \notin V_2 \\ I_{A_2}^+(x) if\ x \notin V_1\ and\ x \in V_2 \\ \max(I_{A_1}^+(x), I_{A_2}^+(x)) \text{ if } x \in V_1 \cap V_2, \end{cases}$
- $(F_{A_1}^- \cup F_{A_2}^-)(x) = \begin{cases} F_{A_1}^-(x), if\ x \in V_1\ and\ x \notin V_2 \\ F_{A_2}^-(x) if\ x \notin V_1\ and\ x \in V_2 \\ \max(F_{A_1}^-(x), F_{A_2}^-(x)) \text{ if } x \in V_1 \cap V_2, \end{cases}$
- $(F_{A_1}^+ \cup F_{A_2}^+)(x) = \begin{cases} F_{A_1}^+(x), if\ x \in V_1\ and\ x \notin V_2 \\ F_{A_2}^+(x) if\ x \notin V_1\ and\ x \in V_2 \\ \max(F_{A_1}^+(x), F_{A_2}^+(x)) \text{ if } x \in V_1 \cap V_2, \end{cases}$
- $(T_{B_1}^- \cup T_{B_2}^-)(xy) = \begin{cases} T_{B_1}^-(xy), if\ xy \in E_1\ and\ xy \notin E_2 \\ T_{B_2}^-(xy)\ if\ xy \notin E_1\ and\ xy \in E_2 \\ \min(T_{B_1}^-(xy), T_{B_2}^-(xy)) \text{ if } xy \in E_1 \cap E_2, \end{cases}$
- $(T_{B_1}^+ \cup T_{B_2}^+)(xy) = \begin{cases} T_{B_1}^+(xy), if\ xy \in E_1\ and\ xy \notin E_2 \\ T_{B_2}^+(xy)\ if\ xy \notin E_1\ and\ xy \in E_2 \\ \min(T_{B_1}^+(xy), T_{B_2}^+(xy)) \text{ if } xy \in E_1 \cap E_2, \end{cases}$

- $(I^-_{B_1} \cup I^-_{B_2})(xy) = \begin{cases} I^-_{B_1}(xy), if\ xy \in E_1\ and\ xy \notin E_2 \\ I^-_{B_2}(xy)\ if\ xy \notin E_1\ and\ xy \in E_2 \\ \min(I^-_{B_1}(xy), I^-_{B_2}(xy)) \text{ if } xy \in \mathrm{E}_1 \cap E_2, \end{cases}$
- $(I^+_{B_1} \cup I^+_{B_2})(xy) = \begin{cases} I^+_{B_1}(xy), if\ xy \in E_1\ and\ xy \notin E_2 \\ I^+_{B_2}(xy)\ if\ xy \notin E_1\ and\ xy \in E_2 \\ \max(I^+_{B_1}(xy), I^+_{B_2}(xy)) \text{ if } xy \in \mathrm{E}_1 \cap E_2, \end{cases}$
- $F^-_{B_1} \cup F^-_{B_2})(xy) = \begin{cases} F^-_{B_1}(xy), if\ xy \in E_1\ and\ xy \notin E_2 \\ F^-_{B_2}(xy)\ if\ xy \notin E_1\ and\ xy \in E_2 \\ \max(F^-_{B_1}(xy), F^-_{B_2}(xy)) \text{ if } xy \in \mathrm{E}_1 \cap E_2, \end{cases}$
- $(F^+_{B_1} \cup F^+_{B_2})(xy) = \begin{cases} F^+_{B_1}(xy), if\ xy \in E_1\ and\ xy \notin E_2 \\ F^+_{B_2}(xy)\ if\ xy \notin E_1\ and\ xy \in E_2 \\ \max(F^+_{B_1}(xy), F^+_{B_2}(xy)) \text{ if } xy \in \mathrm{E}_1 \cap E_2, \end{cases}$

Definition 3.13.

The join of $G_1 + G_2 = (A_1 + A_2, B_1 + B_2)$ interval valued neutrosophic graphs G_1 and G_2 of the graphs G_1^* and G_2^* is defined as follows:

- $(T^-_{A_1} + T^-_{A_2})(x) = \begin{cases} T^-_{A_1}(x)\ if\ x \in V_1 \\ T^-_{A_2}(x)\ if\ x \in V_2 \\ \min(T^-_{A_1}, T^-_{A_2})(x) \text{ if } x \in \mathrm{V}_1 \cup \mathrm{V}_2, \end{cases}$,
- $(T^+_{A_1} + T^+_{A_2})(x) = \begin{cases} T^+_{A_1}(x)\ if\ x \in V_1 \\ T^+_{A_2}(x)\ if\ x \in V_2 \\ \min(T^+_{A_1}, T^+_{A_2})(x) \text{ if } x \in \mathrm{V}_1 \cup \mathrm{V}_2, \end{cases}$
- $(I^-_{A_1} + I^-_{A_2})(x) = \begin{cases} I^-_{A_1}(x)\ if\ x \in V_1 \\ I^-_{A_2}(x)\ if\ x \in V_2 \\ \max(I^-_{A_1}, I^-_{A_2})(x) \text{ if } x \in \mathrm{V}_1 \cup \mathrm{V}_2, \end{cases}$
- $(I^+_{A_1} + I^+_{A_2})(x) = \begin{cases} I^+_{A_1}(x)\ if\ x \in V_1 \\ I^+_{A_2}(x)\ if\ x \in V_2 \\ \max(I^+_{A_1}, I^+_{A_2})(x) \text{ if } x \in \mathrm{V}_1 \cup \mathrm{V}_2, \end{cases}$
- $(F^-_{A_1} + F^-_{A_2})(x) = \begin{cases} F^-_{A_1}(x)\ if\ x \in V_1 \\ F^-_{A_2}(x)\ if\ x \in V_2 \\ \max(F^-_{A_1}, F^-_{A_2})(x) \text{ if } x \in \mathrm{V}_1 \cup \mathrm{V}_2, \end{cases}$
- $(F^+_{A_1} + F^+_{A_2})(x) = \begin{cases} F^+_{A_1}(x)\ if\ x \in V_1 \\ F^+_{A_2}(x)\ if\ x \in V_2 \\ \max(F^+_{A_1}, F^+_{A_2})(x) \text{ if } x \in \mathrm{V}_1 \cup \mathrm{V}_2, \end{cases}$
- $(T^-_{B_1} + T^-_{B_2})(xy) = \begin{cases} T^-_{B_1}(xy), if\ xy \in E_1 \\ T^-_{B_2}(xy)\ if\ xy \in E_2 \\ \min(T^-_{B_1}(xy), T^-_{B_2}(xy)) \text{ if } xy \in \mathrm{E}_1 \cup E_2, \end{cases}$

- $(T_{B_1}^{+} + T_{B_2}^{+})(xy) = \begin{cases} T_{B_1}^{+}(xy), if\ xy \in E_1 \\ T_{B_2}^{+}(xy)\ if\ xy \in E_2 \\ \min(T_{B_1}^{+}(xy), T_{B_2}^{+}(xy)) \text{ if } xy \in \mathrm{E}_1 \cup E_2, \end{cases}$
- $I_{B_1}^{-} + I_{B_2}^{-})(xy) = \begin{cases} I_{B_1}^{-}(xy), if\ xy \in E_1 \\ I_{B_2}^{-}(xy)\ if\ xy \in E_2 \\ \max(I_{B_1}^{-}(xy), I_{B_2}^{-}(xy)) \text{ if } xy \in \mathrm{E}_1 \cup E_2, \end{cases}$
- $(I_{B_1}^{+} + I_{B_2}^{+})(xy) = \begin{cases} I_{B_1}^{+}(xy), if\ xy \in E_1 \\ I_{B_2}^{+}(xy)\ if\ xy \in E_2 \\ \max(I_{B_1}^{+}(xy), I_{B_2}^{+}(xy)) \text{ if xy } \in \mathrm{E}_1 \cup E_2, \end{cases}$
- $F_{B_1}^{-} + F_{B_2}^{-})(xy) = \begin{cases} F_{B_1}^{-}(xy), if\ xy \in E_1 \\ F_{B_2}^{-}(xy)\ if\ xy \in E_2 \\ \max(F_{B_1}^{-}(xy), F_{B_2}^{-}(xy)) \text{ if } xy \in \mathrm{E}_1 \cup E_2, \end{cases}$
- $(F_{B_1}^{+} + F_{B_2}^{+})(xy) = \begin{cases} F_{B_1}^{+}(xy), if\ xy \in E_1 \\ F_{B_2}^{+}(xy)\ if\ xy \in E_2 \\ \max(F_{B_1}^{+}(xy), F_{B_2}^{+}(xy)) \text{ if } xy \in \mathrm{E}_1 \cup E_2, \end{cases}$
- $(T_{B_1}^{-} + T_{B_2}^{-})(x\,y) = \min(T_{B_1}^{-}(x), T_{B_2}^{-}(x))$
- $(T_{B_1}^{+} + T_{B_2}^{+})(xy) = \min(T_{B_1}^{+}(x), T_{B_2}^{+}(x))$
- $(I_{B_1}^{-} + I_{B_2}^{-})(xy) = \max(I_{B_1}^{-}(x), I_{B_2}^{-}(x))$
- $(I_{B_1}^{+} + I_{B_2}^{+})(x\,y) = \max(I_{B_1}^{+}(x), I_{B_2}^{+}(x)$
- $(F_{B_1}^{-} + F_{B_2}^{-})(xy) = \max(F_{B_1}^{-}(x), F_{B_2}^{-}(x))$
- $(F_{B_1}^{+} + F_{B_2}^{+})(x\,y) = \max(F_{B_1}^{+}(x), F_{B_1}^{+}(x)) if xy \in E'$,

where E' is the set of all edges joining the nodes of V_1 and V_2, assuming $V_1 \cap V_2 = \emptyset$.

Example 3.14.

Let $G_1^* = (A_1, B_1)$ and $G_2^* = (A_2, B_2)$ be two graphs where $V_1 = \{u_1, u_2, u_3, u_4\}$, $V_2 = \{v_1, v_2, v_3\}$. Consider two interval valued fermatean neutrosophic graphs:

$$A_1 = \begin{Bmatrix} \langle u_1, [0.85,0.95], [0.95,0.95], [0.95,0.95]\rangle, \langle u_2, [0.90,0.90], [0.95,0.95], [0.85,0.85]\rangle, \\ \langle u_3, [0.90,0.95], [0.85,0.95], [0.85,0.85]\rangle, \langle u_4, [0.90,0.95], [0.95,0.90], [0.80,0.85]\rangle \end{Bmatrix},$$

$$B_1 = \begin{Bmatrix} \langle u_1u_2, [0.85,0.90], [0.95,0.95], [0.95,0.95]\rangle, \langle u_2u_3, [0.90,0.90], [0.95,0.95], [0.85,0.85]\rangle, \\ \langle u_3u_4, [0.90,0.95], [0.95,0.95], [0.85,0.85]\rangle, \langle u_1u_4, [0.85,0.95], [0.95,0.95], [0.95,0.95]\rangle, \\ \langle u_1u_3, [0.85,0.95], [0.95,0.95], [0.95,0.95]\rangle \end{Bmatrix};$$

$$A_2 = \begin{Bmatrix} \langle u_1, [0.80,0.90], [0.85,0.95], [0.95,0.85]\rangle, \langle u_2, [0.95,0.90], [0.95,0.95], [0.80,0.85]\rangle, \\ \langle u_3, [0.90,0.90], [0.95,0.95], [0.80,0.80]\rangle \end{Bmatrix},$$

$$B_2 = \begin{Bmatrix} \langle u_1u_2, [0.80,0.90], [0.95,0.95], [0.95,0.85]\rangle, \langle u_2u_3, [0.90,0.90], [0.95,0.95], [0.80,0.85]\rangle, \\ \langle u_1u_3, [0.80,0.90], [0.95,0.95], [0.95,0.85]\rangle \end{Bmatrix}.$$

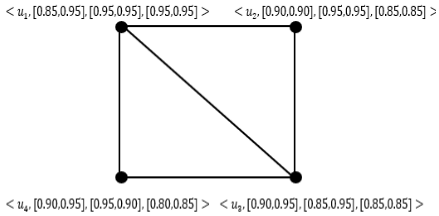

Figure 9. Interval − valued Fermatean Neutrosophic Graph G_1

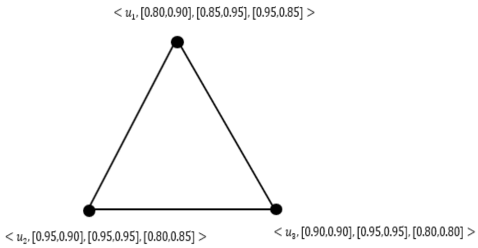

Figure 10. Interval − valued Fermatean Neutrosophic Graph G_2

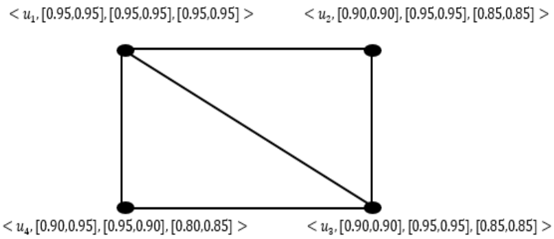

Figure 11. Union two Interval − valued Fermatean Neutrosophic Graphs $G_1 \cup G_2$

$$\left\{\begin{matrix} \langle u_1u_2, [0.80,0.90], [0.95,0.95], [0.95,0.95]\rangle, \langle u_2u_3, [0.90,0.90], [0.95,0.95], [0.85,0.85]\rangle, \\ \langle u_3u_4, [0.90,0.95], [0.95,0.95], [0.85,0.85]\rangle, \langle u_1u_4, [0.85,0.95], [0.95,0.95], [0.95,0.95]\rangle, \\ \langle u_1u_3, [0.80,0.90], [0.95,0.95], [0.95,0.95]\rangle \end{matrix}\right\}$$

Example 3.15

Let $G_1^* = (A_1, B_1)$and $G_2^* = (A_2, B_2)$ be two graphs where $V_1 = \{x_1, x_2, x_3\}, V_2 = \{y_1, y_2, y_3\}$. Consider two interval valued Fermatean neutrosophic graphs :

$$A_1 = \left\{\begin{matrix} \langle x_1, [0.85,0.95], [0.95,0.95], [0.95,0.95]\rangle, \langle x_2, [0.90,0.90], [0.95,0.95], [0.85,0.85]\rangle, \\ \langle x_3, [0.90,0.95], [0.85,0.95], [0.85,0.85]\rangle \end{matrix}\right\},$$

$$B_1 = \{\langle x_1x_2, [0.85,0.90], [0.95,0.95], [0.95,0.95]\rangle, \langle x_2x_3, [0.90,0.90], [0.95,0.95], [0.85,0.85]\rangle \}$$

$$A_2 = \left\{\begin{matrix} \langle y_1, [0.85,0.85], [0.95,0.95], [0.90,0.90]\rangle, \langle y_2, [0.95,0.90], [0.90,0.95], [0.80,0.85]\rangle, \\ \langle y_3, [0.95,0.95], [0.85,0.85], [0.85,0.85]\rangle \end{matrix}\right\},$$

$$B_2 = \{\langle y_1y_2, [0.85,0.85], [0.95,0.95], [0.90,0.90]\rangle, \langle y_2y_3, [0.95,0.90], [0.90,0.95], [0.85,0.85]\rangle\}$$

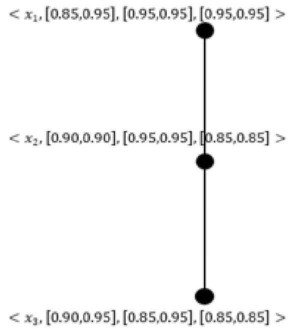

Figure 12. Interval − valued Fermatean Neutrosophic Graph G_1

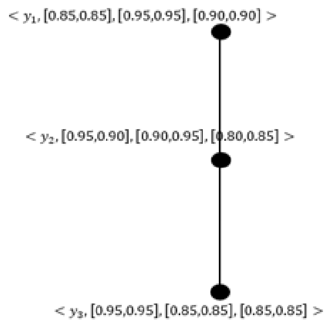

Figure 13. Interval − valued Fermatean Neutrosophic Graph G_2

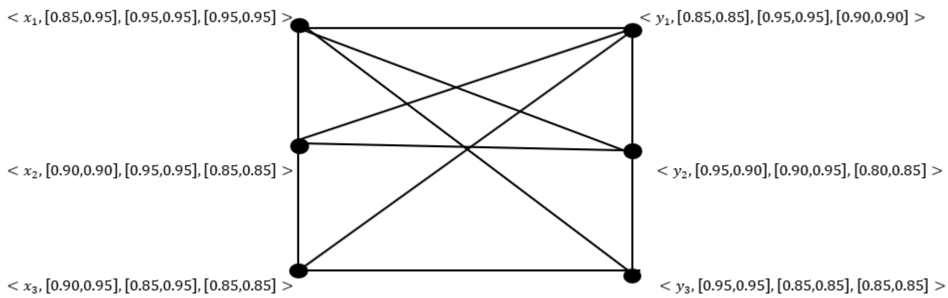

Figure 14. Join of Interval − valued Fermatean Neutrosophic Graphs $G_1 + G_2$

$E(G_1 + G_2)$:
$< x_1x_2$, [0.85,0.90], [0.95,0.95], [0.95,0.95] $>$, $< x_2x_3$, [0.90,0.90], [0.95,0.95], [0.85,0.85] $>$
$< y_1y_2$, [0.85,0.85], [0.95,0.95], [0.90,0.90] $>$, $< y_2y_3$, [0.95,0.90], [0.90,0.95], [0.85,0.85] $>$
$< x_1y_1$, [0.85,0.85], [0.95,0.95], [0.95,0.95] $>$, $< x_1y_2$, [0.85,0.90], [0.95,0.95], [0.95,0.95] $>$,
$< x_1y_3$, [0.85,0.95], [0.95,0.95], [0.95,0.95] $>$
$< x_2y_1$, [0.85,0.90], [0.95,0.95], [0.90,0.90] $>$, $< x_2y_2$, [0.90,0.90], [0.95,0.95], [0.85,0.85] $>$,
$< x_2y_3$, [0.90,0.90], [0.95,0.95], [0.85,0.85] $>$
$< x_3y_1$, [0.85,0.85], [0.95,0.95], [0.90,0.90] $>$, $< x_3y_2$, [0.90,0.90], [0.90,0.95], [0.85,0.85] $>$,
$< x_3y_3$, [0.90,0.95], [0.85,0.95], [0.85,0.85] $>$

Definition 3.16.

An interval valued Fermatean neutrosophic graph G = (A, B) is called complete if

$$T_B^-(\{v_i, vj\}) = \min\left[T_A^-(v_i), T_A^-(v_j)\right], T_B^+(\{v_i, v_j\}) = \min\left[T_A^+(v_i), T_A^+(v_j)\right]$$

$$I_B^-(\{v_i, v_j\}) = \max\left[I_B^-(v_i), I_B^-(v_j)\right], I_B^+(\{v_i, v_j\}) = \max\left[I_B^+(v_i), I_B^+(v_j)\right]$$

$$F_B^-(\{v_i, v_j\}) = \max\left[F_B^-(v_i), F_B^-(v_j)\right],\ F_B^+(\{v_i, v_j\}) = \max[F_B^+(v_i), F_B^+(v_j)]$$

Definition 3.17.

Let G = (A,B) be an interval-valued Fermatean neutrosophic graph where $A = \langle [T_A^-, T_A^+],\ [I_A^-, I_A^+],\ [F_A^-, F_A^+] \rangle$ is an interval-valued Fermatean neutrosophic set on V; and $B = \langle [T_B^-, T_B^+], [I_B^-, I_B^+], [F_B^-, F_B^+] \rangle$ is an interval valued Fermatean neutrosophic relation on E satisfying $V = \{v_1, v_2, \dots, v_n\}$, such that $T_A^- : V \to [0,1]$, $T_A^+ : V \to [0,1]$, $I_A^- : V \to [0,1]$, $I_A^+ : V \to [0,1]$ and $F_A^- : V \to [0,1]$, $F_A^+ : V \to [0,1]$ denote the degree of truth-membership, the degree of indeterminacy-membership and falsity-membership of the element $y \in V$, respectively. The positive degree of a vertex $u \in V(G)$ is $T^+(u) = \sum_{uv \in E(G)} [T_A^+]$; $I^+(u) = \sum_{uv \in E(G)} [I_A^+]$; $F^+(u) = \sum_{uv \in E(G)} [F_A^+]$ and $d^+(u) = (T_A^+, I_A^+, F_A^+)$. $T^-(u) = \sum_{uv \in E(G)} [T_A^-]$; $I^-(u) = \sum_{uv \in E(G)} [I_A^-]$; $F^-(u) = \sum_{uv \in E(G)} [F_A^-]$ and $d^-(u) = (T_A^-, I_A^-, F_A^-)$. The degree of a vertex u is $d(u) = [d^+(u), d^-(u)]$.

If $d^+(u) = k_1, d^-(u) = k_2\ for\ all\ u \in V,\ k_1, k_2$ are two real numbers, then the graph is called $[k_1, k_2]$ -regular interval valued Fermatean neutrosophic graph.

Example 3.18.

We consider an interval-valued Fermatean neutrosophic graph.

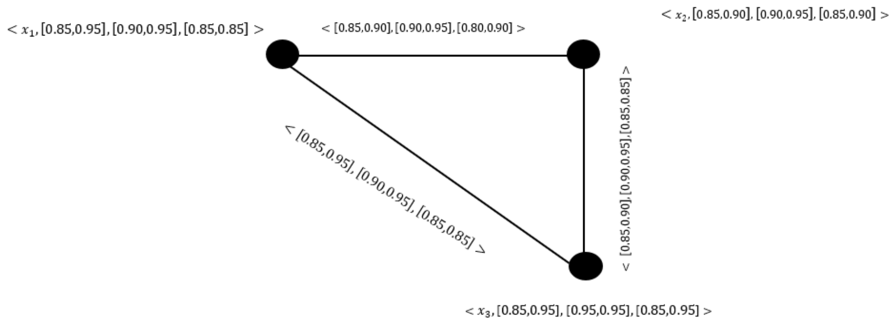

Figure 15. Interval- valued Fermatean Neutrosophic Graph G

$d(x_1) = ([\ 1.65, 1.80, 1.65], [1.85, 1.90, 1.70])$;
$d(x_2) = ([1.65, 1.8, 1.65], [1.8, 1.9, 1.7])$; $d(x_3) = ([1.7, 1.8, 1.7], [1.85, 1.9, 1.7])$.

4. Proposed IVFNG framework for MCDM problem

The most of real life problems deal with uncertain domain. Recently, researchers (Sriganesh et al. 2021; Sundareswaran et al. 2022) have been studied the assessment of structural cracks in buildings using single-valued neutrosophic DEMATEL model and graph theoretical approach. The new concepts of IVFNG are employed to find the best materials that are used for making dental implants in the case of smokers. There are many researchers developed and studied different types uncertainty sets and their application in Multi-Criteria Decision- Making (MCDM) (Duran et al., 2021; Ejegwa et al. 2022; Mohanta et al., 2020; Li et al., 2022; Smarandache, 2020; Smarandache, 2022; Wang et al., 2022; Zhang et al., 2022). Mahesh et al. (2022), made a comparative study of Dental Implant Materials Using Digraph Techniques. Dental implants are the most popular option to replace missing teeth. They create direct contact with the bone which mimics the root of the tooth, upon which dental prosthesis can be fitted. These implants are designed in such a way that they can last for a long time without any failure. They get adhered to the bone without intervening in any connective tissue and this phenomenon is known as osseointegration. Titanium is considered the gold standard as it is the most commonly used dental implant material in use since the 1960s Zirconia is a non-metallic alternative to metal dental implants like $Ti\ alloy\ (Ti - 6Al - 4V)$ and Ti alloys.

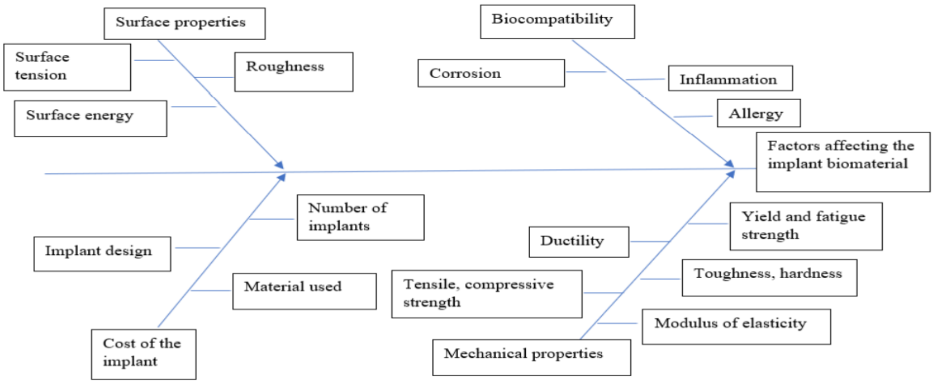

Figure 16. Fishbone diagram with the various factors and subfactors

In this section, the concept of Interval-valued Fermatean neutrosophic graph-theoretic approach has been used to selection of material. The condition of osseointegration in smokers is taken into consideration to compare the different material dental implants namely $Ti\ alloy\ (Ti - 6Al - 4V)$, Ti alloy, and zirconia. The material to be chosen should exhibit certain properties to satisfy the purpose. While designing a dental implant, many factors come into consideration such as materials, dimensions, shape, etc. Material selection is the most important property for a dental implant to serve the required function. The material of the implant must be affordable and available. Following are the factors that are important for the selection of the material.

Biocompatibility (B): A biocompatible material does not invoke an immune response and does not release any toxic substances. The major subfactors of biocompatibility are corrosion, inflammation, and allergy.

Surface Properties (S): Surface properties refer to macroscopic and microscopic features of the implant surface and it plays a major role in determining the level of osseointegration between the implant and the bone. The major subfactors of surface properties are Surface Tension and Surface Energy, Surface Roughness, Porosity.

Mechanical Properties(M): The implant biomaterial should possess a high degree of modulus of elasticity, to withstand the forces applied to the implant, thus preventing its deformation. It also ensures uniform stress distribution, thus reducing the implant movement concerning the bone.

Cost (C): Dental implants in India range from 30,000-50,000 rupees. The price depends on many factors like the type of tooth implant, material, and design of the implant, etc. Titanium is more expensive than stainless steel. The cost of titanium is slightly lower than zirconia.

Titanium (M_1) and Titanium Alloys (M_2): Titanium is an excellent corrosion–resistant material due to the formation of $Ti\ alloy\ (Ti-6Al-4V)$ when Ti atoms react with water molecules and oxygen. They show excellent biocompatibility properties and support osseointegration. Titanium-based dental implants are strong and resist fracture. The cost of titanium is slightly lower than the zirconia. However, titanium implants are less aesthetically pleasing than zirconia and hence they are not preferable to use in the case of front teeth implant placement. Zirconia could be preferred in this case due to its ivory color.

Zirconia (M_3): Zirconia is a non-metallic alternative to metal dental implants like Ti. An advantage of zirconia over titanium is its ivory color. Its low modulus of elasticity and thermal conductivity, low affinity to plaque, and high biocompatibility, in addition to its white color, have made zirconia ceramics a very attractive alternative to titanium. It is highly corrosion resistant and does not involve any release of ions hence no cytotoxicity.

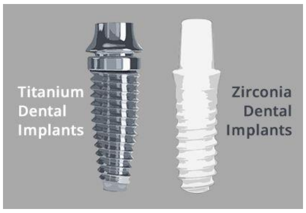

Figure 17. Types of Dental Implants

In the process of applying IVFNG in identifying the best material. IVFNG can be represented as a matrix whose rows and columns are the sub-factors. $V = \{M_1, M_2, M_3\}$ be the three different material under the selection on the basis of wishing param eters or attributes set $A = \{B\ , S\}$.

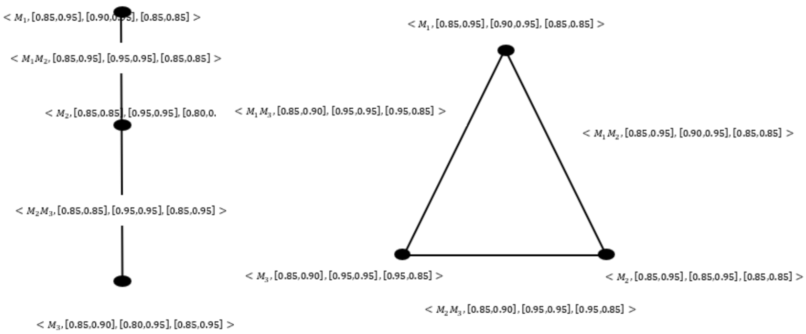

Figure 18. IVFNG based on Biocompatibility & Surface Properties

We construct the adjacency matrix for $M(B)$, $M(S)$ listed below:

$$\boldsymbol{M(B)} = \begin{pmatrix} < [0,0], [0,0], [0,0] > & < [0.85,0.95], [0.95,0.95], [0.85,0.85] > & < [0,0], [0,0], [0,0] > \\ < [0.85,0.95], [0.95,0.95], [0.85,0.85] > & < [0,0], [0,0], [0,0] > & < [0.85,0.85], [0.95,0.95], [0.85,0.95] > \\ < [0,0], [0,0], [0,0] > & < [0.85,0.85], [0.95,0.95], [0.85,0.95] > & < [0,0], [0,0], [0,0] > \end{pmatrix}$$

$$\boldsymbol{M(S)} = \begin{pmatrix} < [0,0], [0,0], [0,0] > & < [0.85,0.95], [0.90,0.95], [0.85,0.85] > & < [0.85,0.90], [0.95,0.95], [0.95,0.85] > \\ < [0.85,0.95], [0.90,0.95], [0.85,0.85] > & < [0,0], [0,0], [0,0] > & < [0.85,0.90], [0.95,0.95], [0.95,0.85] > \\ < [0.85,0.90], [0.95,0.95], [0.95,0.85] > & < [0.85,0.90], [0.95,0.95], [0.95,0.85] > & < [0,0], [0,0], [0,0] > \end{pmatrix}$$

We obtain the resultant interval valued Fermatean neutrosophic graph G by performing some operation (AND or OR). The incidence matrix of resultant interval Fermatean neutrosophic graph is

$$\boldsymbol{M}(\boldsymbol{B}) = \begin{pmatrix} <[0,0],[0,0],[0,0]> & <[0.85,0.95],[0.95,0.95],[0.85,0.85]> & <[0,0],[0,0],[0,0]> \\ <[0.85,0.95],[0.95,0.95],[0.85,0.85]> & <[0,0],[0,0],[0,0]> & <[0.85,0.85],[0.95,0.95],[0.95,0.95]> \\ <[0,0],[0,0],[0,0]> & <[0.85,0.85],[0.95,0.95],[0.95,0.95]> & <[0,0],[0,0],[0,0]> \end{pmatrix}$$

Sahin (2015) defined the average possible membership degree of element x to interval valued neutrosophic set
$A = \langle [T_A^-(x), T_A^+(x)], [I_A^-(x), I_A^+(x)], [F_A^-(x), F_A^+(x)] \rangle$ as follows:

$$S_k(x) = \frac{T_A^-(x) + T_A^+(x) + 4 - I_A^-(x) - I_A^+(x) - F_A^-(x) - F_A^+(x)}{6}$$

Based on $S_k(x)$, Table 2 depicted the score value of adjacency matrix of resultant interval valued Fermatean neutrosophic graph G with S_k and choice value for both materials.

Table 2. Score value of adjacency matrix

Materials	M_1	M_2	M_3	Overall
M_1	0	0.383	0	0.383
M_2	0.383	0	0.317	0.7
M_3	0	0.317	0	0.317

Further, it is noticed from Table 2, $Ti\ alloy\ (Ti - 6Al - 4V)$ has higher level of osseointegration in smokers followed by Ti and zirconia. Therefore, we may claim that IVFNG is a new way to tackle the uncertainty in Fermatean Neutrosophic environment.

5. Conclusion

The concept of uncertainty plays a vital role in all science and engineering problems. Especially, Fuzzy theory, Intuitionistic fuzzy theory and then Neutrosophic theory are the most valuable tools to find the optimum solution in mutli-criteria decision making problems. In this work, we include one more concept called interval-valued Fermatean neutrosophic graphs in the list which has Pythagorean Neutrosophic, Single Valued Neutrosophic, Bipolar Neutrosophic graphs. We have discussed various types of Interval-valued Fermatean Neutrosophic graphs and the other types of these graphs in this paper. We also apply this new type of graph in a decision making problem. We are extending our research on this new concept to introduce Interval-valued Fermatean Neutrosophic number and Interval-valued Fermatean triangle and trapezoidal Neutrosophic number and its applications in our future work.

Interval-valued Fermatean Neutrosophic graph has many advantages in MCDM problems such as mobile networking, supply chain management system, bio-medical applications, e-waste management and networking, etc. In future, one may determine the optimum alternatives in MCDM problems using IVFNG based score and accuracy functions.

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S.B and G.N. All authors have read and agreed to the published version of the manuscript.

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