An Introduction To The Symbolic 3-Plithogenic Modules

Article · January 2023

DOI: 10.54216/GJMSA.060102

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Abstract

The objective of this paper is to define and study for the first time the concept of symbolic 3-plithogenic module based on symbolic 3-plithogenic sets and classical modules.Also, many related substructures will be defined and handled such as AH-functions, AH-submodules, and symbolic 3-plithogenic homomorphisms.

Keywords: 3-plithogenic symbolic set; 3-plithogenic module; 3-plithogenic homomorphism

1. Introduction

The concept of symbolic plithogenic sets was defined by Smarandache in [13-17 ,30], and he suggested an algebraic approach of these sets. Laterally, the concept of symbolic 2-plithogenic rings [31], where the concepts such as symbolic AH-ideals, and AH-homomorphisms were presented and discussed. In [35-39] many algebraic structures about symbolic 2-plithogenic structures were studied such as number theory, algebraic equations, and symbolic 3-plithogenic rings.

In general, we can say that symbolic plithogenic structures are very close to neutrosophic algebraic structures with many differences in the definition of multiplication operation [1-10].

Let R be a ring, the symbolic 3-plithogenic ring is defined as follows:
$3-SP_R=\{a_0+a_1P_1+a_2P_2+a_3P_3;\ a_i\in R, {P_j}^2=P_j, P_i\times P_j=P_{max(i,j)}\}$.
Smarandache has defined algebraic operations on $3-SP_R$ as follows:
Addition:
$[a_0+a_1P_1+a_2P_2+a_3P_3]+[b_0+b_1P_1+b_2P_2+b_3P_3]=(a_0+b_0)+(a_1+b_1)P_1+(a_2+b_2)P_2+(a_3+b_3)P_3$.
Multiplication:
$[a_0+a_1P_1+a_2P_2+a_3P_3].[b_0+b_1P_1+b_2P_2+b_3P_3]=a_0b_0+P_1[a_0b_1+a_1b_0+a_1b_1]+P_2[a_0b_2+a_1b_2+a_2b_2+a_2b_0+a_2b_1]+P_3[a_0b_3+a_1b_3+a_2b_3+a_3b_3+a_3b_0+a_3b_1+a_3b_2]$.
It is clear that $(3-SP_R)$ is a ring.

Main Discussion

Definition.
Let M be a module over the ring R, let $3-SP_R$ be the corresponding symbolic 3-plithogenic ring.
$3-SP_R=\{x+yP_1+zP_2+tP_3;\ x,y,z,t\in R, {P_i}^2=P_i, P_1P_2=P_2P_1=P_2\}$.
We define the symbolic 3-plithogenic module as follows:
$3-SP_M=M+MP_1+MP_2+MP_3=\{a+bP_1+cP_2+dP_3;\ a,b,c,d\in M\}$.
Operations on $3-SP_M$ can be defined as follows:
Addition: $(+):3-SP_M\rightarrow 3-SP_M$, such that:

Doi: https://doi.org/10.54216/GJMSA.060102
Received: December 15, 2022 Revised: April 01, 2023 Accepted: May 01, 2023

$[x_0 + x_1P_1 + x_2P_2 + x_3P_3] + [y_0 + y_1P_1 + y_2P_2 + y_3P_3] = (x_0 + y_0) + (x_1 + y_1)P_1 + (x_2 + y_2)P_2 + (x_3 + y_3)P_3$.

Multiplication: $(.): 3 - SP_R \times 3 - SP_M \to 3 - SP_M$, such that:

$[a + bP_1 + cP_2 + dP_3].[x_0 + x_1P_1 + x_2P_2 + x_3P_3] = ax_0 + (ax_1 + bx_0 + bx_1)P_1 + (ax_2 + bx_2 + cx_0 + cx_1 + cx_2)P_2 + (ax_3 + bx_3 + cx_3 + dx_0 + dx_1 + dx_2 + dx_3)P_3$.

where $x_i, y_i \in V, a, b, c, d \in R$

Theorem.

Let $(3 - SP_M, +, .)$ Is a module over the ring $3 - SP_R$.

Proof.

Let $X = x_0 + x_1P_1 + x_2P_2 + x_3P_3, Y = y_0 + y_1P_1 + y_2P_2 + y_3P_3 \in 3 - SP_V$, $A = a_0 + a_1P_1 + a_2P_2 + a_3P_3, B = b_0 + b_1P_1 + b_2P_2 + b_3P_3 \in 3 - SP_R$ we have:

1. $X = X, (X + Y) + Z = X + (Y + Z), X + (-X) = -X + X = 0, X + 0 = 0 + X = X$

Also

$A(X + Y) = (a_0 + a_1P_1 + a_2P_2 + a_3P_3)[(x_0 + y_0) + (x_1 + y_1)P_1 + (x_2 + y_2)P_2 + (x_3 + y_3)P_3] = A.X + A.Y$

$(A + B)X = A.X + B.X$

$(A.B).X = A(B.X)$

Example.

Let $V = Z^3$ be a module over the ring of integers $Z = R$.

The corresponding symbolic 3-plithogenic module over $3 - SP_Z$ is:

$3 - SP_{Z^3} = \{(x_0, y_0, z_0) + (x_1, y_1, z_1)P_1 + (x_2, y_2, z_2)P_2 + (x_3, y_3, z_3)P_3; x_i, y_i, z_i \in Z\}$

Definition.

Let $3 - SP_M$ be a symbolic 3-plithogenic module over $3 - SP_M$, let V_0, V_1, V_2, V_3 be the three submodules of M, we define the AH-submodule as follows:

$W = V_0 + V_1P_1 + V_2P_2 + V_3P_3 = \{x + yP_1 + zP_2 + tP_3;\ x \in V_0, y \in V_1, z \in V_2, t \in V_3\}$.

If $V_0 = V_1 = V_2 = V_3$, then W is called an AHS-submodule.

Example.

Consider $3 - SP_{R^3}$, we have $V_0 = \{(a, 0,0);\ a \in R\}, V_1 = \{(0, b, 0);\ b \in R\}, V_2 = \{(0,0, c);\ c \in R\}$ are three submodules of $M = R^3$.

$W = V_0 + V_1P_1 + V_2P_2 + V_2P_3 = \{(a, 0,0) + (0, b, 0)P_1 + (0,0, c)P_2 + (0,0, d)P_2;\ a, b, c, d \in R\}$ is an AH-submodule of $3 - SP_{R^3}$.

$T = V_1 + V_1P_1 + V_1P_2 + V_1P_3 = \{(0, a, 0) + (0, b, 0)P_1 + (0, c, 0)P_2 + (0, d, 0)P_2;\ a, b, c, d \in R\}$ is an AHS-submodule.

Theorem.

Let $3 - SP_M$ be a symbolic 3-plithogenic module over $3 - SP_R$, let W be an AHS-submodule of $3 - SP_M$, then W is a submodule of $3 - SP_M$.

Proof.

The proof is similar to the case of 2-plithogenic modules.

Definition.

Let V, W be two modules over the ring R. Let $3 - SP_V$, $3 - SP_W$ be the corresponding symbolic 3-plithogenic modules over $3 - SP_R$.

Let $L_0, L_1, L_2, L_3: V \to W$ be three homomorphisms, we define the AH-homomorphism as follows:

$L: 3 - SP_V \to 3 - SP_W, L = L_0 + L_1P_1 + L_2P_2 + L_3P_3; L(x + yP_1 + zP_2 + dP_3) = L_0(x) + L_1(y)P_1 + L_2(z)P_2 + L_3(d)P_3$.

If $L_0 = L_1 = L_2 = L_3$, then L is called AHS-homomorphism.

Definition.

Let $L = L_0 + L_1P_1 + L_2P_2 + L_3P_3: 3 - SP_V \to 3 - SP_W$ be an AH-homomorphism, we define:

1. $AH - ker(L) = ker(L_0) + ker(L_1)P_1 + ker(L_2)P_2 + ker(L_3)P_3 = \{x + yP_1 + zP_2 + dP_3\}; x \in ker(L_0), y \in ker(L_1), z \in ker(L_2), d \in ker(L_3)$.
2. $AH - Im(L) = Im(L_0) + Im(L_1)P_1 + Im(L_2)P_2 + Im(L_3)P_3 = \{a + bP_1 + cP_2 + dP_3\}; a \in Im(L_0), b \in Im(L_1), c \in Im(L_2), d \in Im(L_3)$

If L is AHS-homomorphism, then we get $AHS - kernel$, $AHS - Image$.

Theorem.

Let $L = L_0 + L_1P_1 + L_2P_2 + L_3P_3: 3 - SP_V \to 3 - SP_W$ be an AH-homomorphism, then:

$AH - ker(L)$ is AH-submodule of $3 - SP_V$.

$AH - Im(L)$ is AH-submodule of $3 - SP_W$.

Example.

Take $V = Z^3$, $W = Z^3$, $L_0, L_1, L_2: V \to W$ such that:

$L_0(x, y, z) = (x, y), L_1(x, y, z) = (2x, z), L_2(x, y, z) = (x - y, y - z)$

Doi: https://doi.org/10.54216/GJMSA.060102
Received: December 15, 2022 Revised: April 01, 2023 Accepted: May 01, 2023

The corresponding AH-homomorphism is:

$L = L_0 + L_1P_1 + L_2P_2 + L_2P_3 : 3 - SP_{Z^3} \to 3 - SP_{Z^2}$:

$$L[(x_0, y_0, z_0) + (x_1, y_1, z_1)P_1 + (x_2, y_2, z_2)P_2 + +(x_3, y_3, z_3)P_3]$$
$$= L_0(x_0, y_0, z_0) + L_1(x_1, y_1, z_1)P_1 + L_2(x_2, y_2, z_2)P_2 + L_2(x_3, y_3, z_3)P_3$$
$$= (x_0, y_0) + (2x_1, z_1)P_1 + (x_2 - y_2, y_2 - z_2)P_2 + (x_3 - y_3, y_3 - z_3)P_3$$

$$\begin{cases} ker(L_0) = \{(0,0,z_0);\ z_0 \in R\} \\ ker(L_1) = \{(0, y_1, 0);\ y_1 \in R\} \\ ker(L_2) = \{(x_2, x_2, x_2);\ x_2 \in R\} \\ AH - ker(L) = \{(0,0,z_0) + (0, y_1, 0)P_1 + (x_2, x_2, x_2)P_2 + (x_3, x_3, x_3)P_3; z_0, y_1, x_2, x_3 \in Z\} \end{cases}$$

Also,

$$\begin{cases} Im(L_0) = R^2 \\ Im(L_1) = R^2 \\ Im(L_2) = R^2 \\ AH - Im(L) = R^2 + R^2P_1 + R^2P_2 + R^2P_3 = 3 - SP_W \end{cases}$$

3. Conclusion

In this paper we have defined the concept of symbolic 3-plithogenic modules over a symbolic 3-plithogenic ring, where we have presented some of their elementary properties such as basis, linear transformations, and AH-submodules. On the other hand, we have suggested many examples to clarify the validity of our work.

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Received: December 15, 2022 Revised: April 01, 2023 Accepted: May 01, 2023

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Received: December 15, 2022 Revised: April 01, 2023 Accepted: May 01, 2023