Computational and Applied Mathematics (2021) 40:308
https://doi.org/10.1007/s40314-021-01600-9

Octagonal fuzzy neutrosophic number and its application to reusable container inventory model with container shrinkage

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Received: 22 January 2021 / Revised: 3 July 2021 / Accepted: 28 July 2021

Abstract

In supply chain management, the reusable containers are the main source of import and export trading. Import and export of the cargoes need some reusable transport items like containers for proper transportation process. Generally, the container management organization supplies the empty containers to multiple consigners for the purpose of transportation of cargoes. This paper scrutinizes the effect of inventory shrinkage on the reusable container inventory management system under neutrosophic arena. The octagonal fuzzy neutrosophic number (OFNN) is applied in this research and its de-neutrosophication is framed using 'removal area method'. The container inventory model is proposed and formulated under neutrosophic environment by treating the demand of each consigner and the return rate as octagonal fuzzy neutrosophic numbers and interpreted with numerical computation. The notion of neutrosophic arena leads the model to reach approximate solutions under uncertain parameters. The sensitivity analysis is conferred to show the impact of the proportion of returned units on optimal solution under both crisp and neutrosophic arena.

Keywords Inventory · Supplychain · Neutrosophic · Uncertainty

Mathematics Subject Classification 62A86

Communicated by Anibal Tavares de Azevedo.

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Published online: 16 November 2021

1 Introduction

In practicality, the necessary needs for everyone day-to-day life has been accomplished with the help of import and export trading. This depends upon the consumer's requirement which is uncertain and unpredictable. In accordance with this situation, many researchers utilized the concepts of fuzzy set, intuitionistic fuzzy set, neutrosophic set and so on. The notion of intuitionistic fuzzy set is flourished by Atanassov (1986) as the generalization of fuzzy set which introduced by Zadeh (1965). As the extension of crisp set, fuzzy set and intuitionistic fuzzy sets, Smarandache (1998) established the neutrosophic set. This research scrutinizes the octagonal fuzzy neutrosophic number in which the 'removal area method' is performed to compute the de-neutrosophication of OFNN and enforced this method to the reusable container inventory model under shrinkage of container.

Import and export of the cargoes like garments, electronics, fruits, vegetables, seafood, etc. needs some reusable transport items for safe and proper transportation process. Different types of containers are used for particular products, for example general purpose (GP) containers are used to carry general cargoes garments, rough blocks, electronics, etc. also for food items like fruits, vegetables, and seafood are transported by reefer containers and so on. In accordance with this, many shipping and logistic organizations allow the shipper to stuff the cargo in empty reusable containers and shipped the same to the particular discharge port. The exported laden containers are de-stuffed by the consignee and return the empty containers at particular empty yard and vice-versa. Some container inventory management issue says container inaccuracy or container shrinkage like receiving errors, clerical errors and misplacement of containers may occur. Because of container shrinkage, the return rate of the used container is reduced. As a part of the container shrinkage, the misplacement of containers plays a vital role in the Container Management Organization (CMO). This paper mainly focuses on the container misplacement that occurs due to clerical errors, invisibility of the container number (damage in the place of container number) and so on. Sometimes, the containers stored at some other yard because of the receiving errors at empty yard. Such containers and the delayed returns are restored to the CMO's yard as soon as possible but the misplaced containers will take more time to restore. This research is developed to compute the CMO's total cost under the situation of container misplacement. Since the misplaced units and the container demand are uncertain, the proposed research is designed in the neutrosophic environment to reach the maximum approximate outcome. Due to the misplacement of the container, the CMO unable to satisfy the entire demand and the revenue is reduced because of this situation. This reduced amount is computed as the penalty cost for misplaced units.

To the best of our knowledge, a few studies on the misplacement of inventories have been done. So far, no one has analyzed the misplacement of containers in the supply chain management problems. This study initiated the concept of misplacement of containers and developed the container inventory model for CMO with the knowledge of the misplacement of containers. When comparing with the product inventories, the containers are very costlier, and also without loss of generality, the proportion of misplaced containers is fewer than the proportion of misplaced product inventories. Since the quantity of misplaced containers is unpredictable, the present research utilizing the neutrosophic environment to obtain the most approximate result. The OFNN is considered and the new de-neutrosophication of the OFNN using the 'removal area method' is computed and implemented. In this paper, the single-vendor multi-consigner container inventory model is developed with the effect of container misplacement under crisp and neutrosophic environment. The storage charge is the main costs spent by the container management companies. The empty container storage yard is fixing the storage price according to the size of the container, type of the containers, and the number of storage days, and also they offer some beneficial storage systems like free days, free pool, slab storage system, etc. The slab storage system utilized in this paper helps the CMO to minimize the container storage cost. The main contributions of the proposed study are:

- Computed the container inventory model under the situation of container misplacement.
- Container inventory model in the neutrosophic arena for unpredictable parameters.
- Slab storage system is utilized to reduce the container storage charge.

The major issue faced by any supply chain management system is the misplacement of inventory. In 2010, Camdereli and Swaminathan (2010) examined the effect of misplacement of inventory in supply chain management and rectified the issue of misplacement of inventory by RFID adoption. The benefits of adopting RFID in a coordinated supply chain have discussed. Zhang et al. (2014) studied the centralized and decentralized supply chain model under the misplacement of stock. They examined that while adopting the RFID technology, the lead time of the supply chain has reduced and also the problem of misplacement of stock rectified. In 2016, Wang et al. (2016) discussed about the stock inaccuracy and its effect by considering the demand as stock dependent. They formulated the inventory models under without knowledge of stock inaccuracy, with some rectification of known inaccuracy and partially adopting the RFID to reduce the inaccuracy of stock.

The following studies scrutinize the concept of container inventory management. In 2011, Yun et al. (2011) performed a container inventory model to repositioning of empty container with renting system under (s,S) strategy. A closed loop supply chain model with the implementation of RFID technology to containers is proposed by Kim and Glock (2014). They designed a container inventory model by presuming the proportion of returned unit as stochastic and discussed the benefits of RFID technology. Further, in 2016, Cobb (2016) formulated a closed supply chain container inventory model with the effect of loss ratio and restored the deficit units by purchasing the new containers. The remaining research conferred the usage of buffer containers which returned earlier before the cycle. Recently, in 2019, Hosseini and Sahlin (2019) presented a container management inventory model by discussing about ECR option with respect to time under unpredictable environment. In the same year, Göçen et al. (2019) explored an inventory model by utilizes some strategy to reduce the storage cost to minimize the cost of repositioning the empty containers. Recently, Rajeswari et al. (2021) analyzed (2016) and proposed a model by utilizing the empty container reposition option and the leasing of empty containers instead of purchasing new containers with price sensitive demand under fuzzy environment is presented by considering fuzzy return rate and fuzzy proportion of repaired units.

For extending (Zadeh 1965), many research works presented to determine various fuzzy numbers say triangular fuzzy number, trapezoidal fuzzy numbers, pentagonal fuzzy number and so on. Inspite of this, Malini and Felbin (2013) established the octagonal fuzzy number and its applications to assignment problem. In 2017, Menaga (2017) designed the octagonal intuitionistic fuzzy number and its ranking method and Saqlain (2020) formulated the octagonal neutrosophic number along with its α-cuts under linear and non-linear systems. For simple understanding, Wang et al. (2010) flourished the neutrosophic set into the singled valued neutrosophic set. For effective results, [2, 3 and 22] utilizes various measures on neutrosophic numbers to decision making models under various environments. Selvakumari and Lavanya (2018) framed the octagonal neutrosophic number and determined its ranking method using alpha cuts and also the effective outcome of game problem is attained under neutrosophic arena. In 2019, Chakraborty et al. (2019) introduced the new de-neutrosophication formulation of pentagonal neutrosophic number by removal area method and utilized this method to minimal spanning tree. Further, (Mullai et al. 2020; Pal and Chakraborty 2020) designed the inventory models with various strategies under neutrosophic situation. Singh (2019) applied single-value neutrosophic attributes to multi-decision process on using the metric of multi-granulation and Galois connection. As a special categorization of a neutrosophic set, Freen et al. (2020) initiated the four-valued refined neutrosophic set and applied this notion to multi-objective optimization techniques.

In the below section, some basic definitions are provided and the de-neutrosophication of octagonal fuzzy neutrosophic number is derived by 'removal area method'. Followed by the notations, the container inventory model is developed and the proposed model under the octagonal neutrosophic arena is designed in Sect. 3 also the optimal cycle length is provided along with the convexity of total cost. The numerical computation and sensitivity analysis are provided in Sect. 4. The conclusions of the study are deliberated in the last section.

2 Preliminaries

Fuzzy set (Zadeh (1965)): A fuzzy set $\tilde{F}$ in the universal set $\mathcal{U}$ is defined as a set of ordered pairs and it is expressed as

$$\tilde{F} = \left(g, \mu_{\tilde{F}}(g)\right) : g \in \mathcal{U},$$

in which $\mu_{\tilde{F}}(g)$ is a membership function of g which assumes values in the range from 0 to 1 (ie.,) $\mu_{\tilde{\mathcal{F}}}(g) \in [0, 1]$.

Fuzzy number (Zadeh (1965)): A fuzzy number $\tilde{\mathcal{F}}$ is a subset of real line R, which has the membership function $\mu_{\tilde{\mathcal{F}}}$ fullfilling the given features:

(1) $\mu_{\tilde{\mathcal{F}}}(g)$ is piecewise continuous in its domain.
(2) $\tilde{\mathcal{F}}$ is normal, i.e., there is a $g_0 \in \tilde{\mathcal{F}}$, such that $\mu_{\tilde{\mathcal{F}}}(g_0) = 1$.
(3) $\tilde{\mathcal{F}}$ is convex, i.e., $\mu_{\tilde{\mathcal{F}}}\left(\varepsilon g_1 + (1-\varepsilon) g_2\right) \geq \min\left(\mu_{\tilde{\mathcal{F}}}\left(g_1\right), \mu_{\tilde{\mathcal{F}}}\left(g_2\right)\right) \forall g_1, g_2$ in $\mathcal{U}$.

Octagonal fuzzy number (Malini and Kennedy Felbin (2013)): An octagonal fuzzy number denoted by $\tilde{\mathcal{F}}$ is defined to be the ordered quadruple $\tilde{\mathcal{F}} = \left(L_1(x), S_1\left(x'\right), S_2\left(x'\right), L_2(x)\right)$ for $x \in [0, \tau]$ and $x' \in \left[\tau, p\right]$ where

(i) $L_1(x)$ is a bounded left continuous non decreasing function over $\left[0, p_1\right], 0 \leq p_1 \leq \tau$.
(ii) $S_1\left(x'\right)$ is a bounded left continuous non decreasing function over $\left[\tau, p_2\right], \tau \leq p_2 \leq p$.
(iii) $S_2\left(x'\right)$ is a bounded left continuous non increasing function over $\left[\tau, p_2\right], \tau \leq p_2 \leq p$.
(iv) $L_2(x)$ is a bounded left continuous non increasing function over $\left[0, p_1\right], 0 \leq p_1 \leq \tau$.

If $p = 1$ then the above defined number is called a normal octagonal fuzzy number. That is, a fuzzy number $\tilde{\mathcal{F}} = (u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8)$ is a normal octagonal fuzzy number, where $u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8$ are real and its membership function $\mu_{\tilde{\mathcal{F}}}(g)$

is given by

$$\mu_{\tilde{\mathcal{F}}}(g) = \begin{cases} \tau\left(\frac{g-u_1}{u_2-u_1}\right), & u_1 \le g \le u_2 \\ \tau, & u_2 \le g \le u_3 \\ \tau + (1-\tau)\left(\frac{g-u_3}{u_4-u_3}\right), & u_3 \le g \le u_4 \\ 1, & u_4 \le g \le u_5 \\ \tau + (1-\tau)\left(\frac{u_6-g}{u_6-u_5}\right), & u_5 \le g \le u_6 \\ \tau, & u_6 \le g \le u_7 \\ \tau\left(\frac{u_8-g}{u_8-u_7}\right), & u_7 \le g \le u_8 \\ 0, & \text{otherwise} \end{cases}$$

where $\tau \in [0, 1]$.

Remark: When $\tau = 0$, the above octagonal fuzzy number reduces to trapezoidal fuzzy number (u_3, u_4, u_5, u_6) and if the value of τ is 1, it reduces to trapezoidal fuzzy number (u_1, u_4, u_5, u_8).

Intuitionistic fuzzy set (Atanassov (1986)): Let $\mathcal{U}$ be a finite universe of discourse then an intuitionistic fuzzy set $\tilde{\mathcal{A}}$ is defined as.

$\tilde{\mathcal{A}} = \left\{\langle g, '_{\tilde{\mathcal{A}}}(g), \psi_{\tilde{\mathcal{A}}}(g)\rangle | g \in \mathcal{U}\right\}$,

where $'_{\tilde{\mathcal{A}}} : \mathcal{U} \to [0, 1]$ and $\psi_{\tilde{\mathcal{A}}} : \mathcal{U} \to [0, 1]$ satisfying the condition $0 \le '_{\tilde{\mathcal{A}}}(g) + \psi_{\tilde{\mathcal{A}}}(g) \le 1$. The number $'_{\tilde{\mathcal{A}}}(g)$ represents the degree of membership of $g \in \mathcal{U}$ in $\tilde{\mathcal{A}}$ and the number $\psi_{\tilde{\mathcal{A}}}(g)$ represents the degree of non-membership of $g \in \mathcal{U}$ in $\tilde{\mathcal{A}}$. In addition with this, the hesitance degree of $g \in \mathcal{U}$ in $\tilde{\mathcal{A}}$ is denoted as $\varepsilon_{\tilde{\mathcal{A}}}(g)$ and is given as $\varepsilon_{\tilde{\mathcal{A}}}(g) = 1 - '_{\tilde{\mathcal{A}}}(g) - \psi_{\tilde{\mathcal{A}}}(g)$. For convenience, the intuitionistic fuzzy number is considered as $\tilde{\mathcal{A}} = \left('_{\tilde{\mathcal{A}}}(g), \psi_{\tilde{\mathcal{A}}}(g)\right)$.

Octagonal intuitionistic fuzzy number (Menaga (2017)): A fuzzy number $\tilde{\mathcal{F}} = (u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8), \left(u_1^1, u_2^1, u_3^1, u_4^1, u_5^1, u_6^1, u_7^1, u_8^1\right)$ is an octagonal intuitionistic fuzzy number where $u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_1^1, u_2^1, u_3^1, u_4^1, u_5^1, u_6^1, u_7^1, u_8^1$ are real and its membership function $\mu_{\tilde{\mathcal{F}}}(g)$ and non-membership function $\vartheta_{\tilde{\mathcal{F}}}(g)$ are given as follows:

$$\mu_{\tilde{\mathcal{F}}}(g) = \begin{cases} \tau\left(\frac{g-u_1}{u_2-u_1}\right), & u_1 \le g \le u_2 \\ \tau, & u_2 \le g \le u_3 \\ \tau + (1-\tau)\left(\frac{g-u_3}{u_4-u_3}\right), & u_3 \le g \le u_4 \\ 1, & u_4 \le g \le u_5 \\ \tau + (1-\tau)\left(\frac{u_6-g}{u_6-u_5}\right), & u_5 \le g \le u_6 \\ \tau, & u_6 \le g \le u_7 \\ \tau\left(\frac{u_8-g}{u_8-u_7}\right), & u_7 \le g \le u_8 \\ 0, & \text{otherwise} \end{cases}$$

and

$$\vartheta_{\tilde{\mathcal{F}}}(g) = \begin{cases} \tau + (1-\tau)\left(\frac{u_2^{'}-g}{u_2^{'}-u_1^{'}}\right), & u_1^{'} \le g \le u_2^{'} \\ \tau, & u_2^{'} \le g \le u_3^{'} \\ \tau\left(\frac{u_4^{'}-g}{u_4^{'}-u_3^{'}}\right), & u_3^{'} \le g \le u_4^{'} \\ 0, & u_4^{'} \le g \le u_5^{'} \\ \tau\left(\frac{g-u_6^{'}}{u_6^{'}-u_5^{'}}\right), & u_5^{'} \le g \le u_6^{'} \\ \tau, & u_6^{'} \le g \le u_7^{'} \\ \tau + (1-\tau)\left(\frac{g-u_8^{'}}{u_8^{'}-u_7^{'}}\right), & u_7^{'} \le g \le u_8^{'} \\ 1, & \text{otherwise} \end{cases}$$

2.1 Some basic definitions of neutrosophic set

Neutrosophic set (Smarandache (1998)): Let $\mathcal{U}$ be a space of points (objects) with generic element $g \in \mathcal{U}$. Then a neutrosophic set $\tilde{\mathcal{A}}$ in $\mathcal{U}$ is characterized by a truth membership function $\mathrm{T}_{\tilde{\mathcal{A}}}$, an indeterminacy membership function $\mathrm{I}_{\tilde{\mathcal{A}}}$ and a falsity membership function $\mathrm{F}_{\tilde{\mathcal{A}}}$. The functions $\mathrm{T}_{\tilde{\mathcal{A}}}$, $\mathrm{I}_{\tilde{\mathcal{A}}}$ and $\mathrm{F}_{\tilde{\mathcal{A}}}$ are real standard or nonstandard subsets of $]^{-}0, 1^{+}[$ that is $\mathrm{T}_{\tilde{\mathcal{A}}} : \mathcal{U} \to]^{-}0, 1^{+}[; \mathrm{I}_{\tilde{\mathcal{A}}} : \mathcal{U} \to]^{-}0, 1^{+}[; \mathrm{F}_{\tilde{\mathcal{A}}} : \mathcal{U} \to]^{-}0, 1^{+}[$. $\mathrm{T}_{\tilde{\mathcal{A}}}(g)$, $\mathrm{I}_{\tilde{\mathcal{A}}}(g)$, and $\mathrm{F}_{\tilde{\mathcal{A}}}(g)$ satisfy the relation i.e., $0 \le \sup \mathrm{T}_{\tilde{\mathcal{A}}}(g) \le \sup \mathrm{I}_{\tilde{\mathcal{A}}}(g) \le \sup \mathrm{F}_{\tilde{\mathcal{A}}}(g) \le 3^{+}$

Fuzzy neutrosophic number (Biswas et al. (2016)): A neutrosophic set $\tilde{\mathcal{A}}_N$ in a universe of discourse $\mathcal{U}$ is defined as $\tilde{\mathcal{A}}_N = \left\{\langle \mathrm{T}_{\tilde{\mathcal{A}}_N}(g), \mathrm{I}_{\tilde{\mathcal{A}}_N}(g), \mathrm{F}_{\tilde{\mathcal{A}}_N}(g)\rangle | g \in \mathcal{U}\right\}$ where the truth membership function $\mathrm{T}_{\tilde{\mathcal{A}}}(g)$, an indeterminacy membership function $\mathrm{I}_{\tilde{\mathcal{A}}}(g)$ and a falsity membership function $\mathrm{F}_{\tilde{\mathcal{A}}}(g)$ are the real subsets of [0, 1]. Fuzzy neutrosophic number can be defined by extending a discrete set to a continuous set.

Let $\tilde{\mathcal{A}}_N$ be a fuzzy neutrosophic number in the set of real numbers R. Then its truth membership function, indeterminacy membership function and falsity membership function are, respectively, written as follows:

$$\mathrm{T}_{\tilde{\mathcal{A}}_N}(g) = \left\langle \begin{matrix} \mathrm{T}^{L}_{\tilde{\mathcal{A}}_N}(g) & l_1 \le g \le l_2 \\ 1 & l_2 \le g \le l_3 \\ \mathrm{T}^{U}_{\tilde{\mathcal{A}}_N}(g) & l_3 \le g \le l_4 \\ 0 & \text{otherwise} \end{matrix} \right\rangle,$$

$$\mathrm{I}_{\tilde{\mathcal{A}}_N}(\mathrm{y}) = \left\langle \begin{matrix} \mathrm{I}^{L}_{\tilde{\mathcal{A}}_N}(g) & m_1 \le g \le m_2 \\ 0 & m_2 \le g \le m_3 \\ \mathrm{I}^{U}_{\tilde{\mathcal{A}}_N}(g) & m_3 \le g \le m_4 \\ 1 & \text{otherwise} \end{matrix} \right\rangle,$$

$$\mathrm{F}_{\tilde{\mathcal{A}}_N}(\mathrm{y}) = \left\langle \begin{matrix} \mathrm{F}^{L}_{\tilde{\mathcal{A}}_N}(g) & n_1 \le g \le n_2 \\ 0 & n_2 \le g \le n_3 \\ \mathrm{F}^{U}_{\tilde{\mathcal{A}}_N}(g) & n_3 \le g \le n_4 \\ 1 & \text{otherwise} \end{matrix} \right\rangle,$$

where $0 \leq \sup T_{\tilde{\mathcal{A}}}(g) \leq \sup I_{\tilde{\mathcal{A}}}(g) \leq \sup F_{\tilde{\mathcal{A}}}(g) \leq 3$, $\forall g \in \mathcal{U}$ and for all $i = 1, 2, 3, 4; l_i, m_i, n_i \in R$, such that $l_1 \leq l_2 \leq l_3 \leq l_4; m_1 \leq m_2 \leq m_3 \leq m_4; n_1 \leq n_2 \leq n_3 \leq n_4$. Here $T^L_{\tilde{\mathcal{A}}_N}(g), I^U_{\tilde{\mathcal{A}}_N}(g), F^U_{\tilde{\mathcal{A}}_N}(g) \in [0, 1]$ are continuous monotonic increasing functions and $T^U_{\tilde{\mathcal{A}}_N}(g), I^L_{\tilde{\mathcal{A}}_N}(g), F^L_{\tilde{\mathcal{A}}_N}(g) \in [0, 1]$ are continuous monotonic increasing functions.

Single-valued neutrosophic set (Wang et al. (2010)): Let $\mathcal{U}$ be a space of points (objects) with generic element $g \in \mathcal{U}$. A single-valued neutrosophic set $\tilde{\mathcal{A}}$ in $\mathcal{U}$ is characterized by a truth membership function $T_{\tilde{\mathcal{A}}}(g)$, an indeterminacy membership function $I_{\tilde{\mathcal{A}}}(g)$ and a falsity membership function $F_{\tilde{\mathcal{A}}}(g)$ and is given by.

$$\tilde{\mathcal{A}} = \left\{ g, \langle T_{\tilde{\mathcal{A}}}(g), I_{\tilde{\mathcal{A}}}(g), F_{\tilde{\mathcal{A}}}(g) \rangle | g \in \mathcal{U} \right\}.$$

Here $T_{\tilde{\mathcal{A}}}(g), I_{\tilde{\mathcal{A}}}(g), F_{\tilde{\mathcal{A}}}(g) \in [0, 1]$ and the relation $0 \leq \sup T_{\tilde{\mathcal{A}}}(g) \leq \sup I_{\tilde{\mathcal{A}}}(g) \leq \sup F_{\tilde{\mathcal{A}}}(g) \leq 3$ holds for all $g \in \mathcal{U}$.

Octagonal neutrosophic number (Saqlain et al. (2020)) An octagonal neutrosophic number $\tilde{\mathcal{O}}_N$ in a universe of discourse Y is defined as follows:

$$\tilde{\mathcal{O}}_N = \left\langle \begin{array}{c} [(u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8); \delta_1], \\ [(v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8); \delta_2], \\ [(w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_8); \delta_3] \end{array} \right\rangle,$$

where $\delta_1, \delta_2, \delta_3 \in [0, 1]$. Then its truth membership function $\Omega_{\tilde{\mathcal{O}}_N} : \mathbb{R} \to [0, \delta_1]$, indeterminacy membership function $\Psi_{\tilde{\mathcal{O}}_N} : \mathbb{R} \to [0, \delta_2]$ and falsity membership function $\Phi_{\tilde{\mathcal{O}}_N} : \mathbb{R} \to [0, \delta_3]$ are written as follows:

$$\Omega_{\tilde{\mathcal{O}}_N}(g) = \begin{cases} \Omega^{l1}_{\tilde{\mathcal{O}}_N}(g), & u_1 \leq g \leq u_2 \\ \Omega^{l2}_{\tilde{\mathcal{O}}_N}(g), & u_2 \leq g \leq u_3 \\ \Omega^{l3}_{\tilde{\mathcal{O}}_N}(g), & u_3 \leq g \leq u_4 \\ \delta_1, & u_4 \leq g \leq u_5 \\ \Omega^{r1}_{\tilde{\mathcal{O}}_N}(g), & u_5 \leq g \leq u_6 \\ \Omega^{r2}_{\tilde{\mathcal{O}}_N}(g), & u_6 \leq g \leq u_7 \\ \Omega^{r3}_{\tilde{\mathcal{O}}_N}(g), & u_7 \leq g \leq u_8 \\ 0, & \text{otherwise} \end{cases};$$

$$\Psi_{\tilde{\mathcal{O}}_N}(g) = \begin{cases} \Psi^{l1}_{\tilde{\mathcal{O}}_N}(g), & v_1 \leq g \leq v_2 \\ \Psi^{l2}_{\tilde{\mathcal{O}}_N}(g), & v_2 \leq g \leq v_3 \\ \Psi^{l3}_{\tilde{\mathcal{O}}_N}(g), & v_3 \leq g \leq v_4 \\ \delta_2, & v_4 \leq g \leq v_5 \\ \Psi^{r1}_{\tilde{\mathcal{O}}_N}(g), & v_5 \leq g \leq v_6 \\ \Psi^{r2}_{\tilde{\mathcal{O}}_N}(g), & v_6 \leq g \leq v_7 \\ \Psi^{r3}_{\tilde{\mathcal{O}}_N}(g), & v_7 \leq g \leq v_8 \\ 1, & \text{otherwise} \end{cases};$$

$$\Phi_{\tilde{O}_N}(g) = \begin{cases} \Phi^{l1}_{\tilde{O}_N}(g), & w_1 \le g \le w_2 \\ \Phi^{l2}_{\tilde{O}_N}(g), & w_2 \le g \le w_3 \\ \Phi^{l3}_{\tilde{O}_N}(g), & w_3 \le g \le w_4 \\ \delta_3, & w_4 \le g \le w_5 \\ \Phi^{r1}_{\tilde{O}_N}(g), & w_5 \le g \le w_6 \\ \Phi^{r2}_{\tilde{O}_N}(g), & w_6 \le g \le w_7 \\ \Phi^{r3}_{\tilde{O}_N}(g), & w_7 \le g \le w_8 \\ 1, & \text{otherwise} \end{cases}.$$

3 De-neutrosophication of an octagonal fuzzy neutrosophic number

The crisp value of any neutrosophic number can be obtained by developing the de-neutrosophication strategy. In this paper, the de-neutrosophication of an octagonal fuzzy neutrosophic number is derived by ‘removal area method’, where the same method for a pentagonal neutrosophic number was developed by Chakraborty et al. (2019) (Fig. 1).

A linear octagonal fuzzy neutrosophic number is given as follows:

$$\tilde{O}_{\mathcal{N}} = \left\langle \begin{array}{c} (u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8), \\ (v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8), \\ (w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_8) \end{array} \right\rangle.$$

The above figure represents the octagonal fuzzy neutrosophic number with $\tau = 0.5$ and $p = 1$. The blue colored octagonal diagram shows the truth membership function, red colored octagonal diagram represents the indeterminacy membership function and the green lined

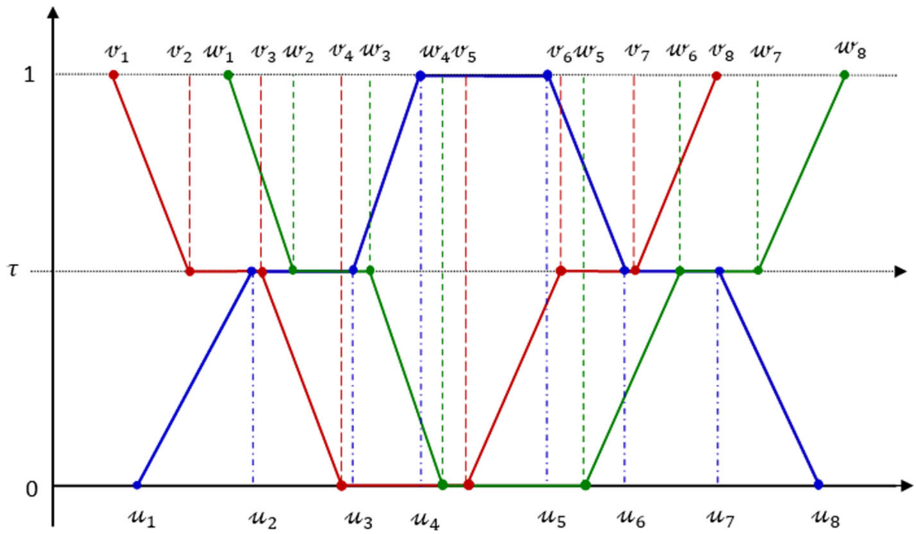


Fig. 1 Graphical representation of linear octagonal fuzzy neutrosophic number

octagonal diagram display the falsity membership function. It is noted that the value of τ lies between 0 and 1. When $\tau = 0 or 1$, this OFNN reduces to trapezoidal fuzzy neutrosophic number.

According to Chakraborty et al. (2019), the de-neutrosophication of an OFNN is given as,

$$\mathcal{O}_{\mathcal{N}}\left(\widetilde{\mathcal{D}e_{Oct}}, \ddagger\right) = \frac{\mathcal{O}_{\mathcal{N}}\left(\tilde{\Delta}_1, \ddagger\right) + \mathcal{O}_{\mathcal{N}}\left(\tilde{\Delta}_2, \ddagger\right) + \mathcal{O}_{\mathcal{N}}\left(\tilde{\Delta}_3, \ddagger\right)}{3}$$

where$\mathcal{O}_{\mathcal{N}}\left(\tilde{\Delta}_1, \ddagger\right) = \frac{\mathcal{O}^l_{\mathcal{N}}\left(\tilde{\Delta}_1, \ddagger\right) + \mathcal{O}^r_{\mathcal{N}}\left(\tilde{\Delta}_1, \ddagger\right)}{2}$, $\mathcal{O}_{\mathcal{N}}\left(\tilde{\Delta}_2, \ddagger\right) = \frac{\mathcal{O}^l_{\mathcal{N}}\left(\tilde{\Delta}_2, \ddagger\right) + \mathcal{O}^r_{\mathcal{N}}\left(\tilde{\Delta}_2, \ddagger\right)}{2}$ and $\mathcal{O}_{\mathcal{N}}\left(\tilde{\Delta}_3, \ddagger\right) = \frac{\mathcal{O}^l_{\mathcal{N}}\left(\tilde{\Delta}_3, \ddagger\right) + \mathcal{O}^r_{\mathcal{N}}\left(\tilde{\Delta}_3, \ddagger\right)}{2}$.

Here, $\mathcal{O}^l_{\mathcal{N}}\left(\tilde{\Delta}_1, \ddagger\right)$ is the area of the left side of blue colored octagon and $\mathcal{O}^r_{\mathcal{N}}\left(\tilde{\Delta}_1, \ddagger\right)$ is the area of the right side of the same octagon. Similarly, $\mathcal{O}^l_{\mathcal{N}}\left(\tilde{\Delta}_2, \ddagger\right)$ and $\mathcal{O}^r_{\mathcal{N}}\left(\tilde{\Delta}_2, \ddagger\right)$ represent the left side area and the right side area of the red lined octagon. Also, $\mathcal{O}^l_{\mathcal{N}}\left(\tilde{\Delta}_3, \ddagger\right)$ and $\mathcal{O}^r_{\mathcal{N}}\left(\tilde{\Delta}_3, \ddagger\right)$ denotes, respectively, the area of left side and the area of right side of the green lined octagon.

By setting $z = 0, \mathcal{O}_{\mathcal{N}}\left(\tilde{\Delta}_1, 0\right) = \frac{\mathcal{O}^l_{\mathcal{N}}\left(\tilde{\Delta}_1, 0\right) + \mathcal{O}^r_{\mathcal{N}}\left(\tilde{\Delta}_1, 0\right)}{2}$, $\mathcal{O}_{\mathcal{N}}\left(\tilde{\Delta}_2, 0\right) = \frac{\mathcal{O}^l_{\mathcal{N}}\left(\tilde{\Delta}_2, 0\right) + \mathcal{O}^r_{\mathcal{N}}\left(\tilde{\Delta}_2, 0\right)}{2}$ and $\mathcal{O}_{\mathcal{N}}\left(\tilde{\Delta}_3, 0\right) = \frac{\mathcal{O}^l_{\mathcal{N}}\left(\tilde{\Delta}_3, 0\right) + \mathcal{O}^r_{\mathcal{N}}\left(\tilde{\Delta}_3, 0\right)}{2}$.

Thus, $\mathcal{O}_{\mathcal{N}}(\tilde{\mathcal{D}e}_{Oct}, 0) = \frac{\mathcal{O}_{\mathcal{N}}\left(\tilde{\Delta}_1, 0\right) + \mathcal{O}_{\mathcal{N}}\left(\tilde{\Delta}_2, 0\right) + \mathcal{O}_{\mathcal{N}}\left(\tilde{\Delta}_3, 0\right)}{3}$

Hence,

$\mathcal{O}^l_{\mathcal{N}}\left(\tilde{\Delta}_1, 0\right)$ = Shaded area of Fig. 2a = $\frac{(u_1+u_2)\tau}{2} + \frac{(u_3+u_4)(1-\tau)}{2}$,

$\mathcal{O}^r_{\mathcal{N}}\left(\tilde{\Delta}_1, 0\right)$ = Shaded area of Fig. 2b = $\frac{(u_7+u_8)\tau}{2} + \frac{(u_5+u_6)(1-\tau)}{2}$,

$\mathcal{O}^l_{\mathcal{N}}\left(\tilde{\Delta}_2, 0\right)$ = Shaded area of Fig. 3a = $\frac{(v_1+v_2)(1-\tau)}{2} + \frac{(v_3+v_4)\tau}{2}$,

$\mathcal{O}^r_{\mathcal{N}}\left(\tilde{\Delta}_2, 0\right)$ = Shaded area of Fig. 3b = $\frac{(v_7+v_8)(1-\tau)}{2} + \frac{(v_5+v_6)\tau}{2}$,

$\mathcal{O}^l_{\mathcal{N}}\left(\tilde{\Delta}_3, 0\right)$ = Shaded area of Fig. 4a = $\frac{(w_1+w_2)(1-\tau)}{2} + \frac{(w_3+w_4)\tau,}{2}$

$\mathcal{O}^r_{\mathcal{N}}\left(\tilde{\Delta}_3, 0\right)$ = Shaded area of Fig. 4b = $\frac{(w_7+w_8)(1-\tau)}{2} + \frac{(w_5+w_6)\tau}{2}$.

Therefore, $\mathcal{O}_{\mathcal{N}}\left(\tilde{\Delta}_1, 0\right) = \frac{\frac{(u_1+u_2)\tau}{2} + \frac{(u_3+u_4)(1-\tau)}{2} + \frac{(u_7+u_8)\tau}{2} + \frac{(u_5+u_6)(1-\tau)}{2}}{2}$, $\mathcal{O}_{\mathcal{N}}\left(\tilde{\Delta}_2, 0\right) = \frac{\frac{(v_1+v_2)(1-\tau)}{2} + \frac{(v_3+v_4)\tau}{2} + \frac{(v_7+v_8)(1-\tau)}{2} + \frac{(v_5+v_6)\tau}{2}}{2}$ and$\mathcal{O}_{\mathcal{N}}\left(\tilde{\Delta}_3, 0\right) = \frac{\frac{(w_1+w_2)(1-\tau)}{2} + \frac{(w_3+w_4)\tau}{2} + \frac{(w_7+w_8)(1-\tau)}{2} + \frac{(w_5+w_6)\tau}{2}}{2}$.

Thus, the de-neutrosophication of an octagonal fuzzy neutrosophic number is obtained as.

$$\mathcal{O}_{\mathcal{N}}\left(\widetilde{\mathcal{D}e_{Oct}}, 0\right) = \frac{(u_1+u_2+u_7+u_8+v_3+v_4+v_5+v_6+w_3+w_4+w_5+w_6)\tau + (u_3+u_4+u_5+u_6+v_1+v_2+v_7+v_8+w_1+w_2+w_7+w_8)(1-\tau)}{12}. \quad (1)$$

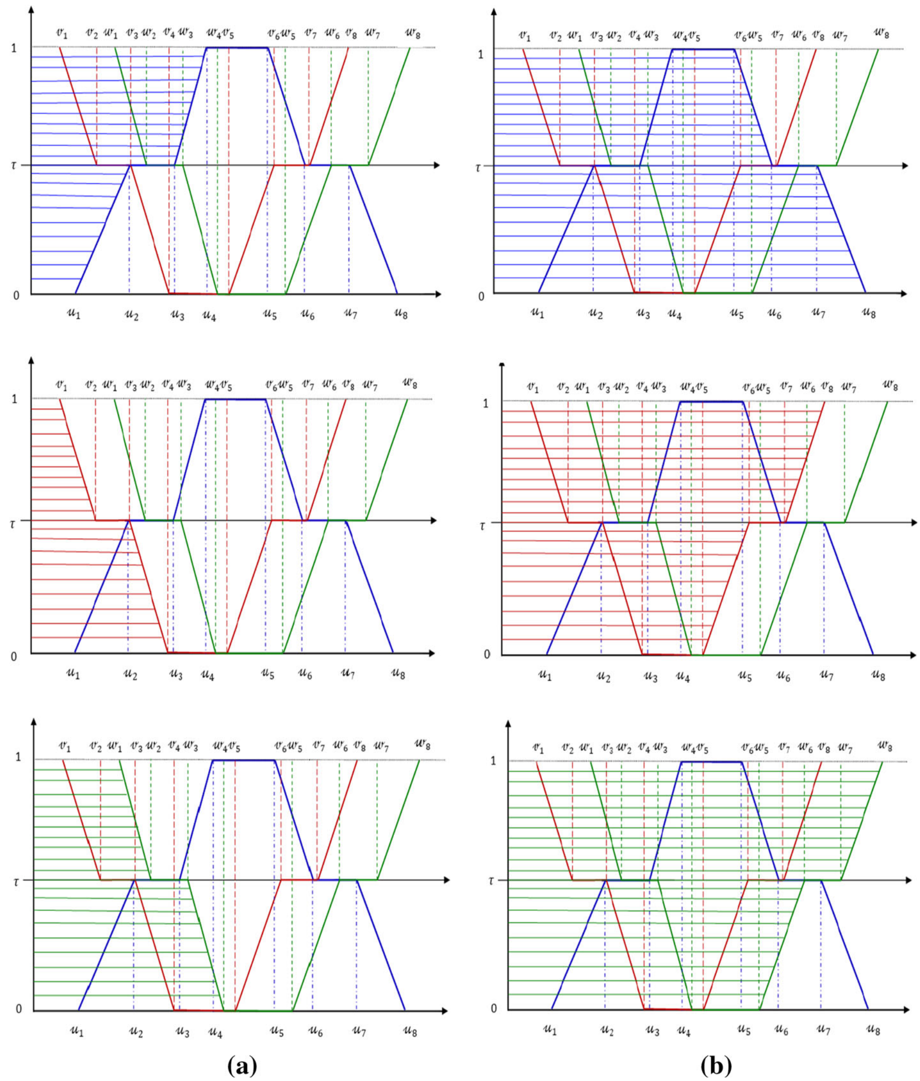


Fig. 2 Diagrammatic representation of removal area method of OFNN

4 Model development

Notation:

$\mathcal{D}$	Demand rate, where $\mathcal{D} = \sum_{k\in\mathcal{R}} \mathcal{D}_k$
$\mathcal{R}$	Number of consigner
$\mathcal{V}$	Set of containers, where $\mathcal{V} = \pi \sum_{k\in\mathcal{R}} \mathcal{D}_k$
π	Proportion of containers returned without misplacement, $0 \leq \pi \leq 1$
$\tilde{\pi}$	Neutrosophic proportion of containers returned without misplacement
$\mathcal{D}_k$	Demand rate for kth consigner
$\tilde{\mathcal{D}_k}$	Neutrosophic demand rate for kth consigner

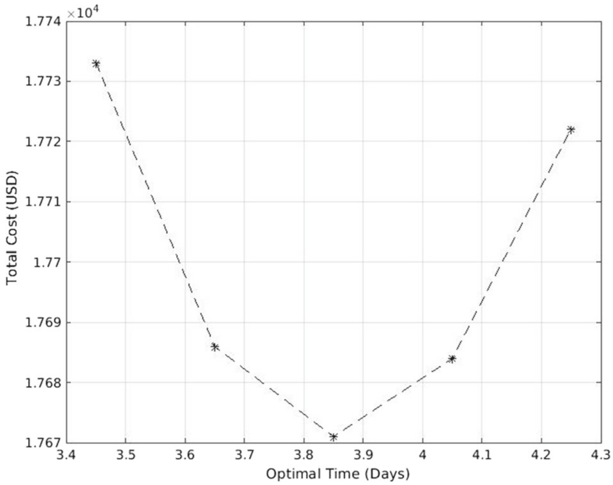
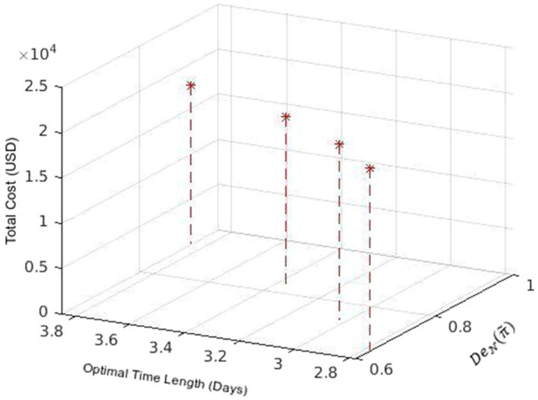


Fig. 3 Convexity of the inventory model

Fig. 4 Effect of neutrosophic proportion of returned units on T^* and TC^*



$\mathcal{M}$ Fixed maintenance and repair (M&R) cost

m Variable M&R cost

d' Depreciation cost per container/day

l Lift-On Lift-Off (LOLO) charge per container

c' Penalty charge for misplaced container

n_j Number of storage days for container j, $j \in \mathcal{V}$

T Cycle time

$\hbar_r$ Unit storage charge (where r varies from 1 to 4 based on the total number of storage days)

$\mathcal{S}_j, \mathcal{S}_j^{'}$ Decision variables depend upon the size of the container (where $\mathcal{S}_j = 1 \text{or} s_1 \text{and} \mathcal{S}_j^{'} = 1 \text{or} s_2$; for all $s_1, s_2 \in \mathbb{R}^+$)

$C_{j,k}$ Decision variable for the allocation of j th container to k th consignor

$\mathcal{S}_{j,\hbar_r}$ Storage charge for container j per day

$$\mathcal{S}_{j,\hbar_r} = \hbar_r = \begin{cases} \hbar_1, & n_j \leq 7 \\ \hbar_2, & 7 < n_j \leq 14 \\ \hbar_3, & 14 < n_j \leq 21 \\ \hbar_4, & n_j < 21 \end{cases} \tag{2}$$

Decision variables:

$$C_{j,k} = \begin{cases} 1, & \text{if container } j \text{ is allocated to consigner } k \\ 0, & \text{otherwise} \end{cases}$$

The type of containers discussed in this study is both 20 ft as well as 40 ft containers. Since, for particular cost such as handling cost, depreciation cost, storage cost etc. the charges assigned for 20 ft and 40 ft containers varies, the decision variables $\mathcal{S}_j$ and $\mathcal{S}_j^{'}$ are considered. The decision variables $\mathcal{S}_j$ and $\mathcal{S}_j^{'}$ represents the proportion assigned for the containers and its value depends upon the size of the containers, say, TEU (Twenty-foot Equivalent Unit).

$$\mathcal{S}_j = \begin{cases} 1, & \text{if TEU of container } j \text{ is 20} \\ s_1, & \text{if TEU of container } j \text{ is 40} \end{cases}$$

$$\mathcal{S}_j^{'} = \begin{cases} 1, & \text{if TEU of container } j \text{ is 20} \\ s_2, & \text{if TEU of container } j \text{ is 40} \end{cases}$$

4.1 Formulation of container inventory model

For import and export trading purpose, the CMO used to supply the empty container to multiple consigners ($\mathcal{R}$) at some price during the time (T) at the rate of demand $\mathcal{D} = \sum_{k\in\mathcal{R}} \mathcal{D}_k$, where $\mathcal{D}_k$ is the constant demand rate for kth consigner. In this study the shrinkage of container is considered, that is a proportion of container say, $1-\pi$ is misplaced because of some clerical and receiving errors. According to the hypothesis, $1-\pi$ fraction of container is misplaced and the CMO supplied only $\mathcal{V}$, ($\mathcal{V} = \pi\mathcal{D}$) units to their consigners per cycle. While managing the container inventory, the CMO should consume some expenses for management of empty containers. The costs involving in the management of container inventory in this model per cycle are maintenance and repair cost, depreciation cost, lift-on lift-off charge, penalty cost for misplaced units and storage cost which are discussed as follows.

Maintenance and repair (M & R) cost is the cost spent for the purpose of surveying, cleaning and repairing process of the containers and is given as

$$\text{MRC} = \frac{\mathcal{M}}{\text{T}} + \sum_{k\in\mathcal{R}} \pi\mathcal{D}_k m. \tag{3}$$

The value of an asset or the fixed product value is reduced over time due to the usage can be stated as depreciation cost. Each container is depreciated with respect to the time period and the cost of the depreciated value of reusable containers in this model per cycle is written as

$$\text{DC} = \sum_{k\in\mathcal{R}} \sum_{j\in\mathcal{V}} \pi\mathcal{S}_j\mathcal{D}_k d^{'}\text{T}. \tag{4}$$

Lift-on lift-off charge is the charge for loading and unloading the empty container of the trailer which is given as

$$\text{LOLO} = \sum_{k\in\mathcal{R}}\sum_{j\in\mathcal{V}} \pi \mathcal{S}_j^{'} \mathcal{D}_k l. \tag{5}$$

In this model, the revenue reduced due to the misplacement of containers is considered as the penalty cost. Thus, the penalty cost for misplaced containers during the cycle time T is derived as

$$\text{PC} = \sum_{k\in\mathcal{R}}\sum_{j\in\mathcal{V}} (1-\pi) \mathcal{S}_j \mathcal{D}_k c^{'} \text{T}. \tag{6}$$

The storage charge is the major expense for every CMO and the holding charge is based on the type and the size of the container. The storage cost is charged by the empty container storage yard according to the size of the container, type of the container and the number of storage days. To minimize the storage cost, the CMO should utilizes some beneficial storage systems like free days, free pool, slab storage system that are offered by the container storage yard. In this research, slab storage system is considered and the storage cost under slab system is derived as per the equation for $\mathcal{S}_{j,\hbar_r}$ given in (2). The derivation part of storage cost is provided in Appendix.

$$\begin{aligned}
\text{HC} = &\sum_{k\in\mathcal{R}}\sum_{j\in\mathcal{V}/n_j\le 7} n_j \mathcal{S}_j C_{j,k} \mathcal{S}_{j,\hbar_1} + \sum_{k\in\mathcal{R}}\sum_{j\in\mathcal{V}/7<n_j\le 14} 7\mathcal{S}_j C_{j,k} \mathcal{S}_{j,\hbar_1} \\
&+ \left(\left(n_j - 7\right) \mathcal{S}_j C_{j,k} \mathcal{S}_{j,\hbar_2}\right) + \sum_{k\in\mathcal{R}}\sum_{j\in\mathcal{V}/14<n_j\le 21} 7\mathcal{S}_j C_{j,k} \left(\mathcal{S}_{j,\hbar_1} + \mathcal{S}_{j,\hbar_2}\right) \\
&+ \left(\left(n_j - 14\right) \mathcal{S}_j C_{j,k} \mathcal{S}_{j,\hbar_3}\right) + \sum_{k\in\mathcal{R}}\sum_{j\in\mathcal{V}/n_j>21} 7\mathcal{S}_j C_{j,k} \left(\mathcal{S}_{j,\hbar_1} + \mathcal{S}_{j,\hbar_2} + \mathcal{S}_{j,\hbar_3}\right) \\
&+ \left(\left(n_j - 21\right) \mathcal{S}_j C_{j,k} \mathcal{S}_{j,\hbar_4}\right).
\end{aligned} \tag{7}$$

The total cost is the sum of M & R cost, depreciation cost, LOLO cost, penalty cost and storage charge. The minimizing strategy of the total cost subject to the constraints under crisp model is obtained as

$$\begin{aligned}
\min\text{TC} = &\frac{\mathcal{M}}{\text{T}} + \sum_{k\in\mathcal{R}} \pi \mathcal{D}_k m + \sum_{k\in\mathcal{R}}\sum_{j\in\mathcal{V}} \pi \mathcal{S}_j \mathcal{D}_k d^{'} \text{T} + \sum_{k\in\mathcal{R}}\sum_{j\in\mathcal{V}} \pi \mathcal{S}_j^{'} \mathcal{D}_k l \\
&+ \sum_{k\in\mathcal{R}}\sum_{j\in\mathcal{V}} (1-\pi) \mathcal{S}_j \mathcal{D}_k c^{'} \text{T} + \sum_{k\in\mathcal{R}}\sum_{j\in\mathcal{V}/n_j\le 7} n_j \mathcal{S}_j C_{j,k} \mathcal{S}_{j,\hbar_1} \\
&+ \sum_{k\in\mathcal{R}}\sum_{j\in\mathcal{V}/7<n_j\le 14} 7\mathcal{S}_j C_{j,k} \mathcal{S}_{j,\hbar_1} + \left(\left(n_j - 7\right) \mathcal{S}_j C_{j,k} \mathcal{S}_{j,\hbar_2}\right) \\
&+ \sum_{k\in\mathcal{R}}\sum_{j\in\mathcal{V}/14<n_j\le 21} 7\mathcal{S}_j C_{j,k} \left(\mathcal{S}_{j,\hbar_1} + \mathcal{S}_{j,\hbar_2}\right) + \left(\left(n_j - 14\right) \mathcal{S}_j C_{j,k} \mathcal{S}_{j,\hbar_3}\right) \\
&+ \sum_{k\in\mathcal{R}}\sum_{j\in\mathcal{V}/n_j>21} 7\mathcal{S}_j C_{j,k} \left(\mathcal{S}_{j,\hbar_1} + \mathcal{S}_{j,\hbar_2} + \mathcal{S}_{j,\hbar_3}\right) + \left(\left(n_j - 21\right) \mathcal{S}_j C_{j,k} \mathcal{S}_{j,\hbar_4}\right).
\end{aligned} \tag{8}$$

Subject to

$$\mathcal{V} < \sum_{k\in\mathcal{R}} \mathcal{D}_k, \tag{9}$$

$$n_j \le \mathrm{T}, \quad \forall j \in \mathcal{V}, \tag{10}$$

$$\mathcal{S}_j, \mathcal{S}_j' = 1, \quad \text{if TEU of container } j \text{ is } 20 \forall j \in \mathcal{V}, \tag{11}$$

$$\left.\begin{array}{l} \mathcal{S}_j = s_1 = 2 \\ \mathcal{S}_j' = s_2 = 1.5 \end{array}\right\}, \quad \text{if TEU of container } j \text{ is } 40, \forall j \in \mathcal{V}. \tag{12}$$

Note: Generally, most of the container management organization fixed the depreciation cost, storage cost etc. for 40 ft container is twice the cost fixed for 20 ft containers. Whereas, they fix the handling charges for 40 ft container is 1.5 times the cost fixed for 20 ft containers. Thus, the values of s_1 and s_2 in Eq. (12) are considered in such a way.

4.2 Container inventory model in octagonal neutrosophic arena

Since, the proportion of the returned units without misplacement and the container demand of each consigner are uncertain and unpredictable, the proposed model scrutinizes under neutrosophic environment. So that, the proposed container inventory model under neutrosophic arena is framed by presuming the proportion of returned containers without misplaced and the demand of each consigner as the octagonal fuzzy neutrosophic numbers. Thus, the octagonal neutrosophic proportion of containers returned without misplacement is denoted by $\tilde{\pi}$ and is given as $\tilde{\pi} = \left\langle \begin{array}{c} (\pi_{11}, \pi_{12}, \pi_{13}, \pi_{14}, \pi_{15}, \pi_{16}, \pi_{17}, \pi_{18}), \\ (\pi_{21}, \pi_{22}, \pi_{23}, \pi_{24}, \pi_{25}, \pi_{26}, \pi_{27}, \pi_{28}), \\ (\pi_{31}, \pi_{32}, \pi_{33}, \pi_{34}, \pi_{35}, \pi_{36}, \pi_{37}, \pi_{38}) \end{array} \right\rangle$

Also, the octagonal neutrosophic demand of kth consigner $(\tilde{\mathcal{D}_k})$ is written as $\tilde{\mathcal{D}_k} = \left\langle \begin{array}{c} (\mathcal{D}_{11}, \mathcal{D}_{12}, \mathcal{D}_{13}, \mathcal{D}_{14}, \mathcal{D}_{15}, \mathcal{D}_{16}, \mathcal{D}_{17}, \mathcal{D}_{18}), \\ (\mathcal{D}_{21}, \mathcal{D}_{22}, \mathcal{D}_{23}, \mathcal{D}_{24}, \mathcal{D}_{25}, \mathcal{D}_{26}, \mathcal{D}_{27}, \mathcal{D}_{28}), \\ (\mathcal{D}_{31}, \mathcal{D}_{32}, \mathcal{D}_{33}, \mathcal{D}_{34}, \mathcal{D}_{35}, \mathcal{D}_{36}, \mathcal{D}_{37}, \mathcal{D}_{38}) \end{array} \right\rangle$.

The de-neutrosophication of neutrosophic proportion of returned containers and the neutrosophic demand of kth consigner is obtained using Eq. (1).

$$De_{\mathcal{N}}(\tilde{\pi}) = \frac{(\pi_{11}+\pi_{12}+\pi_{17}+\pi_{18}+\pi_{23}+\pi_{24}+\pi_{25}+\pi_{26}+\pi_{33}+\pi_{34}+\pi_{35}+\pi_{36})\tau+(\pi_{13}+\pi_{14}+\pi_{15}+\pi_{16}+\pi_{21}+\pi_{22}+\pi_{27}+\pi_{28}+\pi_{31}+\pi_{32}+\pi_{37}+\pi_{38})(1-\tau)}{12}$$

and

$$De_{\mathcal{N}}\left(\widetilde{\mathcal{D}_k}\right) = \frac{(\mathcal{D}_{11}+\mathcal{D}_{12}+\mathcal{D}_{17}+\mathcal{D}_{18}+\mathcal{D}_{23}+\mathcal{D}_{24}+\mathcal{D}_{25}+\mathcal{D}_{26}+\mathcal{D}_{33}+\mathcal{D}_{34}+\mathcal{D}_{35}+\mathcal{D}_{36})\tau+(\mathcal{D}_{13}+\mathcal{D}_{14}+\mathcal{D}_{15}+\mathcal{D}_{16}+\mathcal{D}_{21}+\mathcal{D}_{22}+\mathcal{D}_{27}+\mathcal{D}_{28}+\mathcal{D}_{31}+\mathcal{D}_{32}+\mathcal{D}_{37}+\mathcal{D}_{38})(1-\tau)}{12}$$

Thus, the minimizing strategy of total cost subject to the constraints under octagonal neutrosophic view is derived as follows:

$$\begin{aligned} \min T\tilde{C}_{\mathcal{N}}(\mathrm{T}) = {} & \frac{\mathcal{M}}{\mathrm{T}} + \sum_{k\in\mathcal{R}} \tilde{\pi}\tilde{\mathcal{D}}_k m + \sum_{k\in\mathcal{R}}\sum_{j\in\mathcal{V}} \tilde{\pi}\mathcal{S}_j\tilde{\mathcal{D}}_k d'\mathrm{T} + \sum_{k\in\mathcal{R}}\sum_{j\in\mathcal{V}} \tilde{\pi}\mathcal{S}_j'\tilde{\mathcal{D}}_k l \\ & + \sum_{k\in\mathcal{R}}\sum_{j\in\mathcal{V}} (1-\tilde{\pi})\mathcal{S}_j\tilde{\mathcal{D}}_k c'\mathrm{T} + \sum_{k\in\mathcal{R}}\sum_{j\in\mathcal{V}/n_j\le 7} n_j\mathcal{S}_jC_{j,k}\mathcal{S}_{j,\hbar_1} \\ & + \sum_{k\in\mathcal{R}}\sum_{j\in\mathcal{V}/7<n_j\le 14} 7\mathcal{S}_jC_{j,k}\mathcal{S}_{j,\hbar_1} + \left((n_j-7)\mathcal{S}_jC_{j,k}\mathcal{S}_{j,\hbar_2}\right) \\ & + \sum_{k\in\mathcal{R}}\sum_{j\in\mathcal{V}/14<n_j\le 21} 7\mathcal{S}_jC_{j,k}\left(\mathcal{S}_{j,\hbar_1}+\mathcal{S}_{j,\hbar_2}\right) + \left((n_j-14)\mathcal{S}_jC_{j,k}\mathcal{S}_{j,\hbar_3}\right) \end{aligned}$$

$$+\sum_{k\in\mathcal{R}}\sum_{j\in\mathcal{V}/n_j>21} 7\mathcal{S}_j C_{j,k}\left(\mathcal{S}_{j,\hbar_1}+\mathcal{S}_{j,\hbar_2}+\mathcal{S}_{j,\hbar_3}\right)+\left((n_j-21)\mathcal{S}_j C_{j,k}\mathcal{S}_{j,\hbar_4}\right). \quad (13)$$

Subject to

$$\mathcal{V} < \sum_{k\in\mathcal{R}} \tilde{\mathcal{D}_k}, \quad (14)$$

$$n_j \leq \mathrm{T}, \forall j \in \mathcal{V}, \quad (15)$$

$$\mathcal{S}_j, \mathcal{S}_j^{'} = 1, if\,TEU of container j is 20, \forall j \in \mathcal{V}, \quad (16)$$

$$\left.\begin{array}{l}\mathcal{S}_j = s_1 = 2\\ \mathcal{S}_j^{'} = s_2 = 1.5\end{array}\right\}, if\,TEU of container j is 40, \forall j \in \mathcal{V}. \quad (17)$$

4.3 Convexity

On observing the above figure, it is clear that the total cost of the proposed model is strictly convex and the optimal cycle length is obtained by equating the first derivative of Eq. (8) with respect to T to zero. That is,

$$\frac{\partial TC}{\partial \mathrm{T}} = \frac{-\mathcal{M}}{\mathrm{T}^2} + \sum_{k\in\mathcal{R}}\sum_{j\in\mathcal{V}} \pi\mathcal{S}_j\mathcal{D}_k d' + \sum_{k\in\mathcal{R}}\sum_{j\in\mathcal{V}}(1-\pi)\mathcal{S}_j\mathcal{D}_k c' = 0. \quad (18)$$

$$T^* = \sqrt{\frac{\mathcal{M}}{\sum_{k\in\mathcal{R}}\sum_{j\in\mathcal{V}} \pi\mathcal{S}_j\mathcal{D}_k d' + \sum_{k\in\mathcal{R}}\sum_{j\in\mathcal{V}}(1-\pi)\mathcal{S}_j\mathcal{D}_k c'}} \quad (19)$$

Similarly, by repeating the same procedure to Eq. (13), the optimal cycle length of neutrosophic inventory model is obtained as:

$$T^* = \sqrt{\frac{\mathcal{M}}{\sum_{k\in\mathcal{R}}\sum_{j\in\mathcal{V}} \tilde{\pi}\mathcal{S}_j\tilde{\mathcal{D}}_k d' + \sum_{k\in\mathcal{R}}\sum_{j\in\mathcal{V}}(1-\tilde{\pi})\mathcal{S}_j\tilde{\mathcal{D}}_k c'}} \quad (20)$$

4.4 Numerical computation

In reality, the storage charge is one of the major costs in the field of container inventory management and also the cost of holding a container is usually based on its type as well the size. The various types of the containers are general purpose container (GP), high cube, open top, flat rack, platform, reefer, and ISO tank with different size say 20 feet or 40 feet. The empty container storage yard is fixing the storage price according to the size of the container, type of the containers and the number of storage days. To minimize the storage cost, the container management companies should utilize some beneficial storage systems like free days, free pool, slab storage system, etc. offered by the container storage yard. In accordance with this, the storage option in the present model is treated as slab storage system which is adopted from Gocen et al. (2019) and is given as follows (Table 1).

For simple understanding, there are three consigners are considered in the proposed model (ie. the number of consigners $k = 1, 2, 3$). The proportion of returned unit is adopted from (Cobb 2016) and some values of the parameters like LOLO, depreciation cost are analyzed and collected from the leading shipping companies like OOCL, COSCO, WAN HAI, etc. in Chennai branch.

Table 1 Variation of storage cost under slab system

Storage cost/day ($)	Storage period (days)
2	0–7
2.5	8–14
3	15–21
4	Above 21

The values of parameter for the proposed crisp inventory model are $\pi = 0.9287$, $\sum_{k\in\mathcal{R}} \mathcal{D}_k = 752; k = 1, 2, 3, d^{'} = \$1/\text{unit/day}, c^{'} = \$5/\text{unit/day}, l = \$2/\text{unit}, \hbar_1 = \$2/\text{day}$, $\hbar_2 = \$2.5/\text{day}, \hbar_3 = \$3/\text{day}, \hbar_4 = \$4/\text{day}$. The optimal solutions are $TC^* = \$17,685$ and $T^* = 3.8425$.

The values of parameter for OFNN container inventory model are same as in the crisp model except the proportion of returned units and the demand rate. The octagonal neutrosophic proportion of retuned container and the octagonal neutrosophic demand rate are given as $\tilde{\pi} = \left\langle \begin{array}{c} (0.90, 0.925, 0.93, 0.945, 0.95, 0.965, 0.98, 0.99), \\ (0.86, 0.88, 0.90, 0.92, 0.94, 0.96, 0.98, 1.0), \\ (0.825, 0.85, 0.875, 0.90, 0.925, 0.95, 0.97, 1.0) \end{array} \right\rangle$,

$\sum_{k\in\mathcal{R}} \tilde{\mathcal{D}_k} = \tilde{\mathcal{D}_1} + \tilde{\mathcal{D}_2} + \tilde{\mathcal{D}_3}$, where $\tilde{\mathcal{D}_1} = \left\langle \begin{array}{c} (164, 172, 180, 190, 200, 212, 220, 230), \\ (125, 150, 175, 200, 225, 250, 275, 300), \\ (176, 180, 185, 190, 195, 200, 210, 220) \end{array} \right\rangle$;

$\tilde{\mathcal{D}_2} = \left\langle \begin{array}{c} (275, 280, 285, 290, 295, 300, 305, 310), \\ (290, 294, 297, 302, 305, 315, 321, 328), \\ (248, 260, 280, 295, 310, 325, 340, 350) \end{array} \right\rangle$ and

$\tilde{\mathcal{D}_3} = \left\langle \begin{array}{c} (230, 238, 242, 246, 255, 260, 270, 280), \\ (184, 200, 230, 245, 260, 275, 312, 322), \\ (214, 223, 231, 243, 250, 262, 271, 281) \end{array} \right\rangle$.

Thus, the optimal solutions of neutrosophic container inventory model are $TC^* = \$17,671$ and $T^* = 3.8503$. It is clearly observed that the total cost under neutrosophic arena is less than that under crisp inventory model.

4.5 Sensitivity analysis

The Table 2 shows that, when OFNN of return proportion reduces then the optimal total increases whereas the optimal time length decreases which is displayed in Fig. 4.

It is noted that from the Fig. 5a and b, the penalty charge for misplaced containers increases the optimal time duration reduces but the optimal total cost raises. Figure 5c indicates that the increase of LOLO charge leads to increase in total cost.

The variation of the number of 20 feet and 40 feet containers in total demand is discussed as follows. The consigners demand includes both 20 ft as well as the 40 ft containers, increase of the number 20 ft container leads to decrease of 40 ft container and vise-versa. Due to more demand of 20 ft container in local market, the return of 20 ft containers by the consignee takes more time when compare to the 40 ft container. And since, the cost for maintaining 20 ft container is less than that of the 40 ft container. Hence, without loss of generality, when the number of 20 ft containers increases the number of 40 ft containers decreases then the

Table 2 Impact of neutrosophic return rate on optimal solution

$\tilde{\pi}$	$De_{\mathcal{N}}(\tilde{\pi})$	T^*	TC^* (USD)
$\left\langle \begin{array}{c}(0.90, 0.925, 0.93, 0.945, 0.95, 0.965, 0.98, 0.99),\\(0.86, 0.88, 0.90, 0.92, 0.94, 0.96, 0.98, 1.0),\\(0.825, 0.85, 0.875, 0.90, 0.925, 0.95, 0.97, 1.0)\end{array}\right\rangle$	0.93	3.8503	17,671
$\left\langle \begin{array}{c}(0.8, 0.825, 0.8, 0.845, 0.85, 0.865, 0.88, 0.89),\\(0.76, 0.78, 0.80, 0.82, 0.84, 0.86, 0.88, 0.90),\\(0.725, 0.75, 0.775, 0.80, 0.825, 0.85, 0.87, 0.90)\end{array}\right\rangle$	0.83	3.3608	18,672
$\left\langle \begin{array}{c}(0.7, 0.725, 0.73, 0.745, 0.75, 0.765, 0.78, 0.79),\\(0.66, 0.68, 0.70, 0.72, 0.74, 0.76, 0.78, 0.80),\\(0.625, 0.65, 0.675, 0.70, 0.725, 0.75, 0.77, 0.80)\end{array}\right\rangle$	0.73	3.0204	19,502
$\left\langle \begin{array}{c}(0.6, 0.625, 0.63, 0.645, .065, 0.665, 0.68, 0.69),\\(0.56, 0.58, 0.60, .062, 0.64, 0.66, 0.68, 0.70),\\(0.525, 0.55, 0.575, 0.60, 0.625, 0.65, 0.67, 0.70)\end{array}\right\rangle$	0.6342	2.7754	20,180

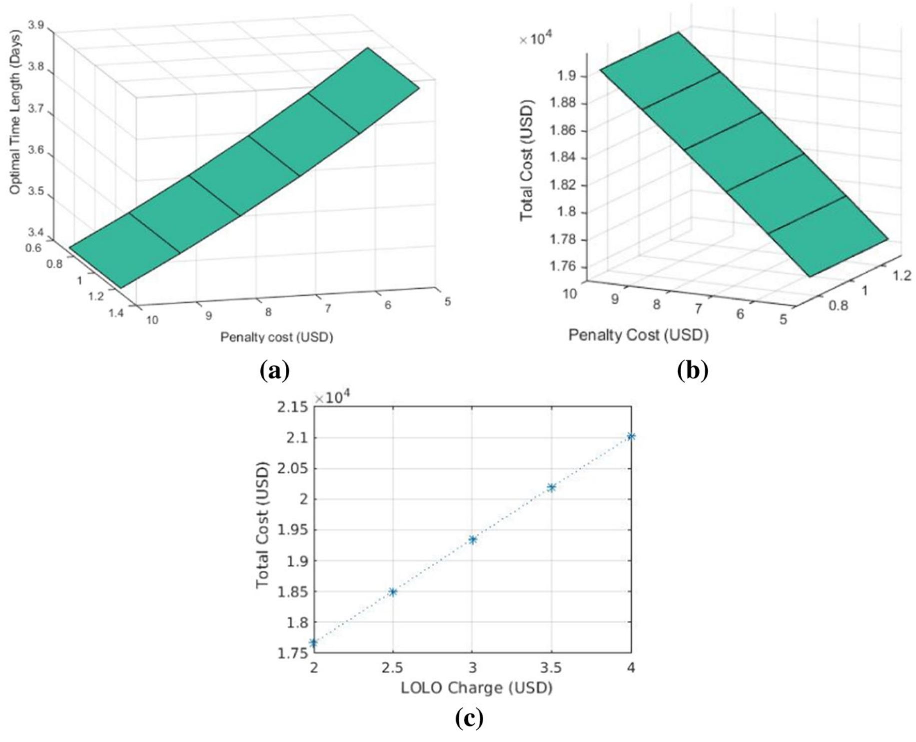


Fig. 5 Effect of variation of c', l, on optimal solution

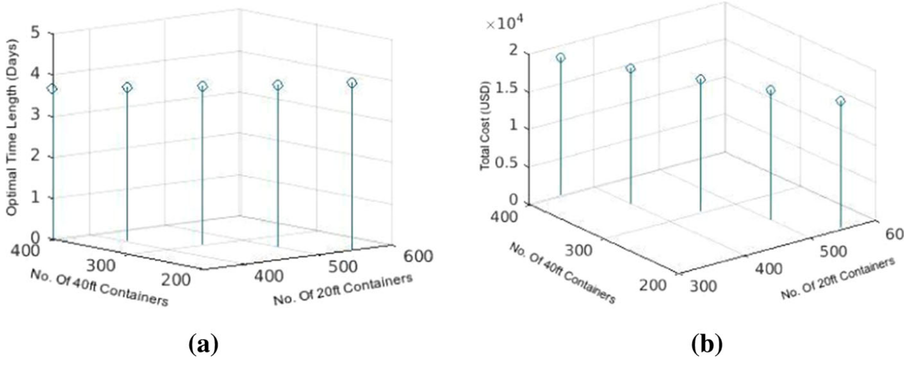


Fig. 6 Analysis of the number of 20 ft & 40 ft containers on TC^*

optimal time period increases slightly but the total cost reduces which is shown in Fig. 6a and b.

5 Conclusion

This research examined and enforced the notion of octagonal fuzzy neutrosophic number to the container inventory model. The current paper studied the container misplacement issue confronted by the container management company with multiple consigners under neutrosophic arena. The de-neutrosophication of OFNN is designed using the removal area method. Initially, the container inventory model under crisp view is developed then the proposed model is framed under neutrosophic situation by presuming the proportion of returned units and the demand of each consigner as OFNN. By knowing the situation of container misplacement, CMO is not necessarily waiting for the returns of the containers, which leads the CMO to decide to rectify the deficit issues as early as possible. The result obtained under the neutrosophic arena is more approximate for the uncertain return proportion and the unpredictable demand. Since the empty yard acquires a low charge for initial slab days under the slab storage option, the slab storage system utilized in this study helps the CMO to abate the container storage cost. This study also suggests the container management organization utilize the slab storage system offered by the empty yards under First In, First Out (FIFO) strategy to reduce the storage charge. The numerical computation demonstrates that the model under neutrosophic arena is comparatively better than the crisp model. The variation of the neutrosophic proportion of returned units, penalty cost for misplaced containers, LOLO charge and its effect are discussed in sensitivity analysis. Finally, the variation of number of 20ft containers and 40ft containers on consigners demand is analyzed and its impact to optimal solution is discussed.

Appendix

The storage cost under slab system is derived as per the equation for $\mathcal{S}_{j,\hbar_r}$ is given in (2). Storage charges for container j per day under four categories are written as follows.

When the number of storage days $n_j \leq 7$, the storage cost is given as

$$\mathrm{HC}_1 = \sum_{k\in\mathcal{R}} \sum_{j\in\mathcal{V}/n_j\leq 7} n_j \mathcal{S}_j C_{j,k} \mathcal{S}_{j,\hbar_1}$$

When the number of storage days $7 < n_j \leq 14$, the storage cost is written as

$$\mathrm{HC}_2 = \sum_{k\in\mathcal{R}} \sum_{j\in\mathcal{V}/7<n_j\leq 14} 7\mathcal{S}_j C_{j,k} \mathcal{S}_{j,\hbar_1} + \left(\left(n_j - 7\right)\mathcal{S}_j C_{j,k} \mathcal{S}_{j,\hbar_2}\right)$$

When the number of storage days $14 < n_j \leq 21$, the storage cost is derived as

$$\begin{aligned}\mathrm{HC}_3 &= \sum_{k\in\mathcal{R}} \sum_{j\in\frac{\mathcal{V}}{14}<n_j\leq 21} \left(7\mathcal{S}_j C_{j,k} \mathcal{S}_{j,\hbar_1}\right) + \left(7\mathcal{S}_j C_{j,k} \mathcal{S}_{j,\hbar_2}\right) + \left(\left(n_j - 14\right)\mathcal{S}_j C_{j,k} \mathcal{S}_{j,\hbar_3}\right) \\ &= \sum_{k\in\mathcal{R}} \sum_{j\in\mathcal{V}/14<n_j\leq 21} 7\mathcal{S}_j C_{j,k}\left(\mathcal{S}_{j,\hbar_1} + \mathcal{S}_{j,\hbar_2}\right) + \left(\left(n_j - 14\right)\mathcal{S}_j C_{j,k} \mathcal{S}_{j,\hbar_3}\right).\end{aligned}$$

When the number of storage days $n_j > 21$, the storage cost is given as

$$\begin{aligned}\mathrm{HC}_4 &= \sum_{k\in\mathcal{R}} \sum_{j\in\frac{\mathcal{V}}{14}<n_j\leq 21} \left(7\mathcal{S}_j C_{j,k} \mathcal{S}_{j,\hbar_1}\right) + \left(7\mathcal{S}_j C_{j,k} \mathcal{S}_{j,\hbar_2}\right) \\ &\quad + \left(7\mathcal{S}_j C_{j,k} \mathcal{S}_{j,\hbar_3}\right) + \left(\left(n_j - 21\right)\mathcal{S}_j C_{j,k} \mathcal{S}_{j,\hbar_4}\right) \\ &= \sum_{k\in\mathcal{R}} \sum_{j\in\mathcal{V}/n_j>21} 7\mathcal{S}_j C_{j,k}\left(\mathcal{S}_{j,\hbar_1} + \mathcal{S}_{j,\hbar_2} + \mathcal{S}_{j,\hbar_3}\right) + \left(\left(n_j - 21\right)\mathcal{S}_j C_{j,k} \mathcal{S}_{j,\hbar_4}\right).\end{aligned}$$

Thus, the storage cost for number of days $n_j, \forall j \in \mathcal{V}$ is obtained as

$$\begin{aligned}\mathrm{HC} = \mathrm{HC}_1 + \mathrm{HC}_2 + \mathrm{HC}_3 + \mathrm{HC}_4 &= \sum_{k\in\mathcal{R}} \sum_{j\in\mathcal{V}/n_j\leq 7} n_j \mathcal{S}_j C_{j,k} \mathcal{S}_{j,\hbar_1} \\ &+ \sum_{k\in\mathcal{R}} \sum_{j\in\mathcal{V}/7<n_j\leq 14} 7\mathcal{S}_j C_{j,k} \mathcal{S}_{j,\hbar_1} + \left(\left(n_j - 7\right)\mathcal{S}_j C_{j,k} \mathcal{S}_{j,\hbar_2}\right) \\ &+ \sum_{k\in\mathcal{R}} \sum_{j\in\mathcal{V}/14<n_j\leq 21} 7\mathcal{S}_j C_{j,k}\left(\mathcal{S}_{j,\hbar_1} + \mathcal{S}_{j,\hbar_2}\right) + \left(\left(n_j - 14\right)\mathcal{S}_j C_{j,k} \mathcal{S}_{j,\hbar_3}\right) \\ &+ \sum_{k\in\mathcal{R}} \sum_{j\in\mathcal{V}/n_j>21} 7\mathcal{S}_j C_{j,k}\left(\mathcal{S}_{j,\hbar_1} + \mathcal{S}_{j,\hbar_2} + \mathcal{S}_{j,\hbar_3}\right) + \left(\left(n_j - 21\right)\mathcal{S}_j C_{j,k} \mathcal{S}_{j,\hbar_4}\right).\end{aligned}$$

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