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Research Article

Hibrid Δ-Statistical Convergence for Neutrosophic Normed Space

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This study is about neutrosophic structures, which is one of the popular topics of recent days. In this study, different types of convergence concepts were applied to difference sequences. With the help of the properties of double type sequences, the concept of difference sequences is combined with structures that are advantageous to work like Lacunary sequences.

1. Introduction

The idea of difference sequences emerged in the days when researchers focused on the idea of constructing new sequence spaces. The difference sequences were given by Kizmaz in [1]. After that, Basarir [2] applied this concept to statistical analysis. After statistical convergence was introduced in [3], different versions of statistical convergence have been defined in area of functional analysis, e.g., [4–9]. Fridy and Orhan gave this concept for Lacunary sequences in [10]. Esi and Araci applied this concept to operator theory in [11]. The properties of Lacunary A-convergence are given in [12]. Then, Hazarika applied that definition of Basarir to Lacunary sequences in normed spaces in [13]. Altundag and Kamber made an important study by evaluating Lacunary sequence and difference sequence structures together in n-dimensional intuitionistic fuzzy normed space, where $(\Delta x_k) = x_k - x_{k+1}$, in [14]. Colak introduced Lacunary strong convergence using difference sequences for modulus function in [15]. Et et al. gave a generalization of difference sequences in [16]. Statistical convergence is defined for double sequences in [17]. Later, this concept attracted great attention of researchers. Tripathy and Sarma applied the idea of this work to difference sequences in [18]. In [19], Patterson and Savas transferred Lacunary statistical convergence to double sequences. Fuzzy-intuitionistic fuzzy sets are generalizations of classical sets established for compelling reasons in daily life. Due to the inadequacy of fuzzy set and intuitionistic set concepts, a new set concept was needed. Thus, a new concept emerged with the help of neutrosophy, which is called a subdivision of philosophy that studies the structure of neutrals: neutrosophic sets. Neutrosophic sets are a concept introduced to investigate the degrees of correctness, wrongness, and uncertainty of the elements in the set in [20]. While classical statistics uses precise data and inferences, neutrophic statistics uses methods that contain uncertain, contradictory, and partially unknown data. Kirisci and Simsek introduced classical statistical convergence in neutrosophic normed spaces in [21]. After that, Khan et al. carried this work to Lacunary sequences in [22]. Granados and Dhital adapted this worked for double sequences in [23]. In 2022, Kisi and Gurdal introduced triple difference sequences in neutrosophic normed spaces [24]. Sahin and Kargın worked neutrosophic tripled normed spaces in [25]. Apart from this, the concept of neutrosophic set has many applications in many different fields, i.e., [26, 27].

Now, by evaluating all this information together, a study was prepared to fill the relevant gap in the literature. Here, $\Delta-$ statistical convergence will be applied to neutrosophic normed spaces single-double sequences, and also, $\Delta-$Lacunary statistical convergence will be given for these two types of sequences. Many important results were given, especially the relations between these two concepts. The properties of $\Delta-$ statistical Cauchy and $\Delta-$ Lacunary statistical Cauchy sequences will be examined for these sequences.

2. Preliminaries

First of all, some necessary definitions will be given.

Definition 1 (see [24]). Let us consider m crisp-components: $\iota_1, \iota_2, \ldots, \iota_m \in [0, 1]$. If all of them are 100% independent two by two, then their sum is

$$0 \le \iota_1 + \iota_2 + \cdots \in + \iota_m \le m. \tag{1}$$

On the contrary, if all of them are 100% dependent, then

$$0 \le \iota_1 + \iota_2 + \cdots + \iota_m \le 1. \tag{2}$$

As stated by Smarandache in [28], in this study, $0 \le \iota_1 + \iota_2 + \iota_3 \le 2$ will be taken when two components are dependent, while the third one is independent from them.

Definition 2 (see [20]). Let $\mathscr{U} \ne \varnothing$ and $\mu_{(\mathfrak{N},\mathscr{T})}(s), \varrho_{(\mathfrak{N},\mathscr{I})}(s)$, and $\zeta_{(\mathfrak{N},\mathscr{F})}(s)$ are the degrees of correctness, uncertainty, and falsity. A neutrosophic set $\mathfrak{N}$ is in the next form: $\mathfrak{N} = \{(s, \mu_{(\mathfrak{N},\mathscr{C})}(s), \varrho_{(\mathfrak{N},\mathscr{U})}(s), \zeta_{(\mathfrak{N},\mathscr{F})}(s)): s \in \mathscr{U}\}$, where, for all s in $\mathscr{U}$, $\mu_{(\mathfrak{N},\mathscr{C})}(s), \varrho_{(\mathfrak{N},\mathscr{U})}(s)$ and $\zeta_{(\mathfrak{N},\mathscr{F})}(s) \in [0, 1]$, $0 \le \mu_{(\mathfrak{N},\mathscr{C})}(s) + \zeta_{(\mathfrak{N},\mathscr{F})}(s) + \varrho_{(\mathfrak{N},\mathscr{U})}(s) \le 2$.

It should be noted that $\varrho_{(\mathfrak{N},\mathscr{U})}(s)$ is an independent component and $\mu_{(\mathfrak{N},\mathscr{T})}(s)$ and $\zeta_{(\mathfrak{N},\mathscr{F})}(s)$ are dependent components.

Definition 3 (see [21]). Let $\mathscr{U}$ be a linear spaces and $\circledast$ and $\boxdot$ show the continuous $\mathbf{t}-$ norm and continuous $\mathbf{t}-$ conorm on $\mathbb{R}$. The notation of neutrosophic normed is $\{((s, r), \mu_{(\mathfrak{N},\mathscr{C})}(s, r), \varrho_{(\mathfrak{N},\mathscr{U})}(s, r), \zeta_{(\mathfrak{N},\mathscr{F})}(s, r)): (s, r) \in \mathscr{U} \times (0, \infty)\}$, where $\mu_{(\mathfrak{N},\mathscr{C})}$, $\zeta_{(\mathfrak{N},\mathscr{F})}$, and $\varrho_{(\mathfrak{N},\mathscr{I})}$ demonstrate the degree of correctness, uncertainty, and falsity of (s, r) on $\mathscr{U} \times (0, \infty)$ which satisfies the following conditions, for all $s_1, s_2 \in \mathscr{U}$:

(i) For every $r \in \mathbb{R}^+$, $\mu_{(\mathfrak{N},\mathscr{C})}(s, r) + \varrho_{(\mathfrak{N},\mathscr{U})}(s, r) + \zeta_{(\mathfrak{N},\mathscr{F})}(s, r) \le 2$.

(ii) For every $r_1, r_2 \in \mathbb{R}^+$,

$$\begin{aligned}
&\mu_{(\mathfrak{N},\mathscr{C})}(s_1, r_1) \circledast \mu_{(\mathfrak{N},\mathscr{C})}(s_2, r_2) \le \mu_{(\mathfrak{N},\mathscr{C})}(s_1 + s_2, r_1 + r_2),\\
&\varrho_{(\mathfrak{N},\mathscr{U})}(s_1, r_1) \boxdot \varrho_{(\mathfrak{N},\mathscr{U})}(s_2, r_2) \ge \varrho_{(\mathfrak{N},\mathscr{U})}(s_1 + s_2, r_1 + r_2),\\
&\zeta_{(\mathfrak{N},\mathscr{F})}(s_1, r_1) \boxdot \zeta_{(\mathfrak{N},\mathscr{F})}(s_2, r_2) \ge \zeta_{(\mathfrak{N},\mathscr{F})}(s_1 + s_2, r_1 + r_2).
\end{aligned} \tag{3}$$

(iii) For every $r \in \mathbb{R}^+$, $\mu_{(\mathfrak{N},\mathscr{C})}(s, r) = 1 \Leftrightarrow s = 0$, $\varrho_{(\mathfrak{N},\mathscr{U})}(s, r) = 0 \Leftrightarrow s = 0$, and $\zeta_{(\mathfrak{N},\mathscr{F})}(s, r) = 0 \Leftrightarrow s = 0$.

(iv) For each $K \ne 0$, $\mu_{(\mathfrak{N},\mathscr{C})}(Ks, r) = \mu_{(\mathfrak{N},\mathscr{C})}(s, (r/|K|))$, $\varrho_{(\mathfrak{N},\mathscr{U})}(Ks, r) = \varrho_{(\mathfrak{N},\mathscr{U})}(s, (r/|K|))$, and

$$\zeta_{(\mathfrak{N},\mathscr{F})}(Ks, r) = \zeta_{(\mathfrak{N},\mathscr{F})}\left(s, \frac{r}{|K|}\right). \tag{4}$$

(v) $\mu_{(\mathfrak{N},\mathscr{C})}(s, \cdot)$ is a continuous nondecreasing function; $\varrho_{(\mathfrak{N},\mathscr{U})}(s, \cdot)$ and $\nu^F_{P_N}(s, \cdot)$ are continuous nonincreasing function,

(vi) $\lim_{u\longrightarrow\infty}\mu_{(\mathfrak{N},\mathscr{C})}(s, r) = 1$, $\lim_{u\longrightarrow\infty}\varrho_{(\mathfrak{N},\mathscr{U})}(s, r) = 0$, and $\lim_{u\longrightarrow\infty}\zeta_{(\mathfrak{N},\mathscr{F})}(s, r) = 0$.

(vii) If $r \le 0$, then $\mu_{(\mathfrak{N},\mathscr{C})}(s, r) = 0$, $\varrho_{(\mathfrak{N},\mathscr{U})}(s, r) = 1$, and $\zeta_{(\mathfrak{N},\mathscr{F})}(s, r) = 1$.

In this case, $(\mathscr{U}, \mu_{(\mathfrak{N},\mathscr{C})}, \varrho_{(\mathfrak{N},\mathscr{U})}, \zeta_{(\mathfrak{N},\mathscr{F})}, \circledast, \boxdot)$ is called neutrosophic normed spaces. Here, $\mu_{(\mathfrak{N},\mathscr{C})}$ and $\varrho_{(\mathfrak{N},\mathscr{U})}$ are interdependent and $\zeta_{(\mathfrak{N},\mathscr{F})}$ is an independent components.

Definition 4 (see [3]). Let $\mathscr{G} \subseteq \mathbb{N}$ and $|\mathscr{G}|$ denote the cardinality of $\mathscr{G}$. The density of the set $\mathscr{G}$ is given by the following equation and is denoted by $\delta(\mathscr{G})$:

$$\delta(\mathscr{G}) = \lim_{p\longrightarrow\infty} \frac{1}{p} |\{k \le p: k \in \mathscr{G}\}|. \tag{5}$$

Definition 5 (see [3]). (s_k) is called to be statistically convergent to s, where

$$\delta(\{k \in \mathbb{N}: |s_k - s| \ge \varepsilon\}) = 0, \tag{6}$$

for all $\varepsilon > 0$. Then, it will be represented by $st - \lim s_k = s$. S is demonstrated, set of statistical convergence sequences.

Definition 6 (see [2]). (s_k) is called to be $\Delta-$ statistically convergent to s, where

$$\delta(\{k \in \mathbb{N}: |\Delta s_k - s| \ge \varepsilon\}) = 0, \tag{7}$$

for all $\varepsilon > 0$ and $\Delta s_k = s_k - s_{k+1}$, i.e.,

$$\lim_{p\longrightarrow\infty} \frac{1}{p} \left|\{k \le p: |\Delta s_k - s| \ge \varepsilon\}\right| = 0. \tag{8}$$

Then, it is demonstrated $st - \lim \Delta s_k = s$. S_Δ is denoted, set of all $\Delta-$ statistical convergence sequences.

Definition 7 (see [10]). Let $\theta = \{k_r\}$ be a sequence of increasing integers, $k_0 = 0$ and also $\lim_{r\longrightarrow\infty} h_r$: $\lim_{r\longrightarrow\infty} k_r - k_{r-1} = \infty$. Then, θ is called to be Lacunary sequences. Let $A \subset \mathbb{N}$, $l_r = (k_r / k_{r-1})$, and $I_r = (k_{r-1}, k_r]$.

$$\delta^\theta(A) = \lim_{r\longrightarrow\infty} \frac{1}{l_r} |k \in I_r: k \in A|, \tag{9}$$

is said to be the $\theta-$ density of A if limit is exhibited. Let $A_\varepsilon = \{k \in I_r: |s_k - s| \ge \varepsilon\}$; for all $\varepsilon > 0$, if

$$\delta^\theta(A_\varepsilon) = \lim_{r\longrightarrow\infty} \frac{1}{l_r} \left|\{k \in I_r: |s_k - s| \ge \varepsilon\}\right| = 0, \tag{10}$$

in this case, (s_k) is called to be Lacunary statistical convergent to s. Then, it is represented as $st^\theta - \lim s_k = s$. S^θ is a denoted set of every Lacunary statistical convergence sequences.

3. Materials and Methods

In this section where important properties for difference sequences will be given, two separate parts will be given where convergence studies will be made for single and double sequences.

3.1. $\Delta-$Statistical Convergence in Neutrosophic Normed Spaces. Now, $\Delta-$convergence, $\Delta-$statistical convergence, and Lacunary $\Delta-$statistical convergence will be defined in neutrosophic normed spaces. The relations between these concepts will be given.

Definition 8. Let $(\mathcal{U}, \mu_{(\mathfrak{N},\mathcal{C})}, \varrho_{(\mathfrak{N},\mathcal{U})}, \zeta_{(\mathfrak{N},\mathcal{F})}, \circledast, *)$ be neutrosophic normed spaces and $\Delta s_k = s_k - s_{k+1}$. (s_k) is called to be $\Delta-$ convergent to s according to neutrosophic normed if, for all $\varepsilon \in (0,1)$ and $r > 0$, there exists a $\widetilde{k} \in \mathbb{N}$ such that, for every $k \geq \widetilde{k}$,

$$\mu_{(\mathfrak{N},\mathcal{C})}(\Delta s_k - s, r) \leq 1 - \varepsilon,\ \varrho_{(\mathfrak{N},\mathcal{U})}(\Delta s_k - s, r) \geq \varepsilon,\ \zeta_{(\mathfrak{N},\mathcal{F})}(\Delta s_k - s, r) \geq \varepsilon. \tag{11}$$

This sequences is shown with ${}^{\mathfrak{N}}_{\Delta}-\lim s_k = s$.

Definition 9. Let $(\mathcal{U}, \mu_{(\mathfrak{N},\mathcal{C})}, \varrho_{(\mathfrak{N},\mathcal{U})}, \zeta_{(\mathfrak{N},\mathcal{F})}, \circledast, \boxdot)$ be neutrosophic normed spaces. If there exist $r > 0$ and $0 < \varepsilon < 1$, for all Δs_k where $\mu_{(\mathfrak{N},\mathcal{C})}(\Delta s_k, r) \leq 1 - \varepsilon$, $\varrho_{(\mathfrak{N},\mathcal{U})}(\Delta s_k, r) \geq \varepsilon$, and $\zeta_{(\mathfrak{N},\mathcal{F})}(\Delta s_k, r) \geq \varepsilon$, then (s_k) is called $\Delta-$bounded sequences in $(\mathcal{U}, \mu_{(\mathfrak{N},\mathcal{C})}, \varrho_{(\mathfrak{N},\mathcal{U})}, \zeta_{(\mathfrak{N},\mathcal{F})}, \circledast, \boxdot)$.

Definition 10. Let $(\mathcal{U}, \mu_{(\mathfrak{N},\mathcal{C})}, \varrho_{(\mathfrak{N},\mathcal{U})}, \zeta_{(\mathfrak{N},\mathcal{F})}, \circledast, \boxdot)$ be neutrosophic normed spaces, (s_k) is called to be $\Delta-$statistical convergence with respect to $(\mu_{(\mathfrak{N},\mathcal{C})}, \varrho_{(\mathfrak{N},\mathcal{U})}, \zeta_{(\mathfrak{N},\mathcal{F})})$ if, for every $\varepsilon \in (0,1)$ and $r > 0$, there exist s such that

$$\left\{k \leq n:\ \mu_{(\mathfrak{N},\mathcal{C})}(\Delta s_k - s, r) \leq 1 - \varepsilon \text{ or } \varrho_{(\mathfrak{N},\mathcal{U})}(\Delta s_k - s, r) \geq \varepsilon, \zeta_{(\mathfrak{N},\mathcal{F})}(\Delta s_k - s, r) \geq \varepsilon\right\}, \tag{12}$$

has natural density zero, i.e.,

$$\lim_{n} \frac{1}{n}\left|\left\{k \leq n:\ \mu_{(\mathfrak{N},\mathcal{C})}(\Delta s_k - s, r) \leq 1 - \varepsilon \text{ or } \varrho_{(\mathfrak{N},\mathcal{U})}(\Delta s_k - s, r) \geq \varepsilon, \zeta_{(\mathfrak{N},\mathcal{F})}(\Delta s_k - s, r) \geq \varepsilon\right\}\right| = 0. \tag{13}$$

Therefore, it will be denoted as $st_{\Delta}^{\mathfrak{N}} - \lim s_k = s$ or $s_k \longrightarrow s(S_{\Delta}^{\mathfrak{N}})$, where $k \longrightarrow \infty$. $S_{\Delta}^{\mathfrak{N}}$ denotes set of all $\Delta-$statistical convergence sequences.

Theorem 1. *Let $(\mathcal{U}, \mu_{(\mathfrak{N},\mathcal{C})}, \varrho_{(\mathfrak{N},\mathcal{U})}, \zeta_{(\mathfrak{N},\mathcal{F})}, \circledast, \boxdot)$ be neutrosophic normed spaces. If (s_k) is $\Delta-$statistically convergent in this case, $st_{\Delta}^{\mathfrak{N}} - \lim s_k$ is unique.*

Lemma 1. *Let $(\mathcal{U}, \mu_{(\mathfrak{N},\mathcal{C})}, \varrho_{(\mathfrak{N},\mathcal{U})}, \zeta_{(\mathfrak{N},\mathcal{F})}, \circledast, \boxdot)$ be neutrosophic normed spaces and (s_k) be a $\Delta-$statistical convergence sequences. Then, for each $\varepsilon > 0, r > 0$, the next properties are equivalent:*

(i) $st_{\Delta}^{\mathfrak{N}} - \lim s_k = s$

(ii) $\lim_n (1/n)\left|\left\{k \leq n:\ \mu_{(\mathfrak{N},\mathcal{C})}(\Delta s_k - s, r) > 1 - \varepsilon, \varrho_{(\mathfrak{N},\mathcal{U})}(\Delta s_k - s, r) < \varepsilon, \zeta_{(\mathfrak{N},\mathcal{F})}(\Delta s_k - s, r) < \varepsilon\right\}\right| = 1$

(iii) $\lim_{n\longrightarrow\infty} (1/n)\left|\left\{k \leq n:\ \mu_{(\mathfrak{N},\mathcal{C})}(\Delta s_k - s, r) \leq 1 - \varepsilon\right\}\right| = 0$, $\lim_{n\longrightarrow\infty} (1/n)\left|\left\{k \leq n:\ \varrho_{(\mathfrak{N},\mathcal{U})}(\Delta s_k - s, r) \geq \varepsilon\right\}\right| = 0$, *and* $\lim_{n\longrightarrow\infty} (1/n)\left|\left\{k \leq n:\ \zeta_{(\mathfrak{N},\mathcal{F})}(\Delta s_k - s, r) \geq \varepsilon\right\}\right| = 0$

(iv) $\lim_{n\longrightarrow\infty} (1/n)\left|\left\{k \leq n:\ \mu_{(\mathfrak{N},\mathcal{C})}(\Delta s_k - s, r) > 1 - \varepsilon\right\}\right| = 1$, $\lim_{n\longrightarrow\infty} (1/n)\left|\left\{k \leq n:\ \varrho_{(\mathfrak{N},\mathcal{U})}(\Delta s_k - s, r) < \varepsilon\right\}\right| = 1$, *and* $\lim_{n\longrightarrow\infty} (1/n)\left|\left\{k \leq n:\ \zeta_{(\mathfrak{N},\mathcal{F})}(\Delta s_k - s, r) < \varepsilon\right\}\right| = 1$

(v) $st - \lim_{n\longrightarrow\infty} \mu_{(\mathfrak{N},\mathcal{C})}(\Delta s_k - s, r) = 1$, $st - \lim_{n\longrightarrow\infty} \varrho_{(\mathfrak{N},\mathcal{U})}(\Delta s_k - s, r) = 0$, *and* $st - \lim_{n\longrightarrow\infty} \zeta_{(\mathfrak{N},\mathcal{F})}(\Delta s_k - s, r) = 0$.

Using Definition 10, the equivalence of statements is easily demonstrated.

Definition 11. Let $(\mathcal{U}, \mu_{(\mathfrak{N},\mathcal{C})}, \varrho_{(\mathfrak{N},\mathcal{U})}, \zeta_{(\mathfrak{N},\mathcal{F})}, \circledast, \boxdot)$ be neutrosophic normed spaces, (s_k) is called to be $\Delta-$ Cauchy sequences if, for every $\varepsilon \in (0,1)$ and $r > 0$, there exist a $k_0 \in \mathbb{N}$ such that, for every $k, p \geq k_0$, $\mu_{(\mathfrak{N},\mathcal{C})}(\Delta s_k - \Delta s_p, r) \leq 1 - \varepsilon$ or $\varrho_{(\mathfrak{N},\mathcal{U})}(\Delta s_k - \Delta s_p, r) \geq \varepsilon, \zeta_{(\mathfrak{N},\mathcal{F})}(\Delta s_k - \Delta s_p, r) \geq \varepsilon$.

Now, a new type of convergence will be defined by including the Lacunary sequence structure in the investigations for difference sequences in neutrosophic normed spaces.

Definition 12. Let $(\mathcal{U}, \mu_{(\mathfrak{N},\mathcal{C})}, \varrho_{(\mathfrak{N},\mathcal{U})}, \zeta_{(\mathfrak{N},\mathcal{F})}, \circledast, \boxdot)$ be neutrosophic normed spaces and (θ) be Lacunary sequence in this spaces. (s_k) is named to be $\Delta-$ Lacunary statistical convergence with respect to $(\mu_{(\mathfrak{N},\mathcal{C})}, \varrho_{(\mathfrak{N},\mathcal{U})}, \zeta_{(\mathfrak{N},\mathcal{F})})$ if, for every $\varepsilon \in (0,1)$ and $r > 0$, there exist s such that

$$\left\{k \leq n:\ \mu_{(\mathfrak{N},\mathcal{C})}(\Delta s_k - s, r) \leq 1 - \varepsilon \text{ or } \varrho_{(\mathfrak{N},\mathcal{U})}(\Delta s_k - s, r) \geq \varepsilon, \zeta_{(\mathfrak{N},\mathcal{F})}(\Delta s_k - s, r) \geq \varepsilon\right\}, \tag{14}$$

has natural $\theta-$density zero, i.e.,

$$\delta_{\Delta}^{\theta}\left\{k \leq n:\ \mu_{(\mathfrak{N},\mathcal{C})}(\Delta s_k -, r) \leq 1 - \varepsilon \text{ or } \varrho_{(\mathfrak{N},\mathcal{U})}(\Delta s_k - s, r) \geq \varepsilon, \zeta_{(\mathfrak{N},\mathcal{F})}(\Delta s_k - s, r) \geq \varepsilon\right\} = 0, \tag{15}$$

or

$$\lim_{r\longrightarrow\infty}\frac{1}{l_r}\left|\left\{k\in I_r:\ \mu_{(\mathfrak{N},\mathscr{C})}(\Delta s_k-s,r)\leq 1-\varepsilon \text{ or } \varrho_{(\mathfrak{N},\mathscr{U})}(\Delta s_k-s,r)\geq\varepsilon,\zeta_{(\mathfrak{N},\mathscr{F})}(\Delta s_k-s,r)\geq\varepsilon\right\}\right|=0. \tag{16}$$

Therefore, it will denote $st_{\Delta}^{\mathfrak{N}}(\theta)-\lim s_k=s$ or $s_k\longrightarrow s(S_{\Delta}^{\mathfrak{N}})$ as $k\longrightarrow\infty$. $S_{\Delta}^{\mathfrak{N}}(\theta)$ denotes set of all $\Delta-$Lacunary statistical convergence sequences.

Lemma 2. *Let $(\mathscr{U},\mu_{(\mathfrak{N},\mathscr{C})},\varrho_{(\mathfrak{N},\mathscr{U})},\zeta_{(\mathfrak{N},\mathscr{F})},\circledast,\boxdot)$ be neutrosophic normed spaces, (θ) be a Lacunary sequence, and (s_k) be a $\Delta-$Lacunary statistical convergence sequences. Then, for every $\varepsilon\in(0,1)$ and $r>0$, the following properties are equivalent:*

(i) $st_{\Delta}^{\mathfrak{N}}(\theta)-\lim s_k=s$

(ii) $\lim_n (1/l_r)|\{k\in I_r:\ \mu_{(\mathfrak{N},\mathscr{C})}(\Delta s_k-s,r)>1-\varepsilon,\ \varrho_{(\mathfrak{N},\mathscr{U})}(\Delta s_k-s,r)<\varepsilon,\zeta_{(\mathfrak{N},\mathscr{F})}(\Delta s_k-s,r)<\varepsilon\}|=1$

(iii) $\lim_{n\longrightarrow\infty}(1/l_r)|\{k\in I_r:\mu_{(\mathfrak{N},\mathscr{C})}(\Delta s_k-s,r)\leq 1-\varepsilon\}|=0$, $\lim_{n\longrightarrow\infty}(1/l_r)|\{k\in I_r:\ \varrho_{(\mathfrak{N},\mathscr{U})}(\Delta s_k-s,r)\geq\varepsilon\}|=0$, *and* $\lim_{n\longrightarrow\infty}(1/l_r)|\{k\in I_r:\ \zeta_{(\mathfrak{N},\mathscr{F})}(\Delta s_k-s,r)\geq\varepsilon\}|=0$

(iv) $\lim_{n\longrightarrow\infty}(1/l_r)|\{k\in I_r:\mu_{(\mathfrak{N},\mathscr{C})}(\Delta s_k-,r)>1-\varepsilon\}|=1$, $\lim_{n\longrightarrow\infty}(1/l_r)|\{k\in I_r:\ \varrho_{(\mathfrak{N},\mathscr{U})}(\Delta s_k-s,r)<\varepsilon\}|=1$, *and* $\lim_{n\longrightarrow\infty}(1/l_r)|\{k\in I_r:\ \zeta_{(\mathfrak{N},\mathscr{F})}(\Delta s_k-s,r)<\varepsilon\}|=1$

(v) $st_{\Delta}^{\mathfrak{N}}(\theta)-\lim_{n\longrightarrow\infty}\mu_{(\mathfrak{N},\mathscr{C})}(\Delta s_k-s,r)=1$, $st_{\Delta}^{\mathfrak{N}}(\theta)-\lim_{n\longrightarrow\infty}\varrho_{(\mathfrak{N},\mathscr{U})}(\Delta s_k-s,r)=0$, *and* $st_{\Delta}^{\mathfrak{N}}(\theta)-\lim_{n\longrightarrow\infty}\zeta_{(\mathfrak{N},\mathscr{F})}(\Delta s_k-s,r)=0$.

They are easily demonstrated using Definition 12.

Theorem 2. *Let $(\mathscr{U},\mu_{(\mathfrak{N},\mathscr{C})},\varrho_{(\mathfrak{N},\mathscr{U})},\zeta_{(\mathfrak{N},\mathscr{F})},\circledast,\boxdot)$ be neutrosophic normed spaces, $\varepsilon\in(0,1)$ and $r>0$. If (s_k) is $\Delta-$Lacunary statistically convergent in this case, $st_{\Delta}^{\mathfrak{N}}(\theta)-\lim s_k$ is unique.*

Theorem 3. *Let $(\mathscr{U},\mu_{(\mathfrak{N},\mathscr{C})},\varrho_{(\mathfrak{N},\mathscr{U})},\zeta_{(\mathfrak{N},\mathscr{F})},\circledast,\boxdot)$ be neutrosophic normed spaces. If (s_k) is $\Delta-$ statistically convergent, then this sequences is $\Delta-$Lacunary statistically convergent.*

Proof. Let (s_k) be $\Delta-$statistically convergent to s and $\delta_{\Delta}^{\theta}(A)$ be $\theta-$density of A obtained with the help of the difference sequence. Then, for every $\varepsilon\in(0,1)$ and $r>0$, there exists a $\tilde{k}\in\mathbb{N}$ such that, for every $k\geq\tilde{k}$, $\mu_{(\mathfrak{N},\mathscr{C})}(\Delta s_k-s,r)\leq 1-\varepsilon$ or $\varrho_{(\mathfrak{N},\mathscr{U})}(\Delta s_k-s,r)\geq\varepsilon$, and $\zeta_{(\mathfrak{N},\mathscr{F})}(\Delta s_k-s,r)<\varepsilon$. So,

$$\left\{k\in\mathbb{N}:\ \mu_{(\mathfrak{N},\mathscr{C})}(\Delta s_k-s,r)\leq 1-\varepsilon \text{ or } \varrho_{(\mathfrak{N},\mathscr{U})}(\Delta s_k-s,r)\geq\varepsilon,\zeta_{(\mathfrak{N},\mathscr{F})}(\Delta s_k-s,r)\geq\varepsilon\right\}, \tag{17}$$

has finite number of terms. Hence, $\theta-$density of this set is zero, i.e.,

$$\delta_{\Delta}^{\theta}\left(\left\{k\in I_r:\ \mu_{(\mathfrak{N},\mathscr{C})}(\Delta s_k-s,r)\leq 1-\varepsilon \text{ or } \varrho_{(\mathfrak{N},\mathscr{U})}(\Delta s_k-s,r)\geq\varepsilon,\zeta_{(\mathfrak{N},\mathscr{F})}(\Delta s_k-s,r)\right\}\right)=0. \tag{18}$$

Thus, (s_k) is $\Delta-$Lacunary statistically convergent to s. □

Definition 13. Let $(\mathscr{U},\mu_{(\mathfrak{N},\mathscr{C})},\varrho_{(\mathfrak{N},\mathscr{U})},\zeta_{(\mathfrak{N},\mathscr{F})},\circledast,\boxdot)$ be neutrosophic normed spaces and (θ) be Lacunary sequence. (s_k) is called to be $\Delta-$Lacunary statistical Cauchy sequences when, for all $\varepsilon\in(0,1)$ and $r>0$, there exist a $k_0\in\mathbb{N}$, for every $k,p\geq k_0$:

$$\delta_{\Delta}^{\theta}\left(\left\{k\in\mathbb{N}:\ \mu_{(\mathfrak{N},\mathscr{C})}\left(\Delta s_k-\Delta s_p,r\right)\leq 1-\varepsilon \text{ or } \varrho_{(\mathfrak{N},\mathscr{U})}\left(\Delta s_k-\Delta s_p,r\right)\geq\varepsilon,\zeta_{(\mathfrak{N},\mathscr{F})}\left(\Delta s_k-\Delta s_p,r\right)\right\}\right)=0. \tag{19}$$

Theorem 4. *Let $(\mathscr{U},\mu_{(\mathfrak{N},\mathscr{C})},\varrho_{(\mathfrak{N},\mathscr{U})},\zeta_{(\mathfrak{N},\mathscr{F})},\circledast,\boxdot)$ be neutrosophic normed spaces and (θ) be a Lacunary sequence. (s_k) is $\Delta-$Lacunary statistical convergent $\Leftrightarrow (s_k)$ is $\Delta-$Lacunary statistical Cauchy in neutrosophic normed spaces.*

Proof. Let (s_k) be a $\Delta-$Lacunary statistical convergent sequence and $st_{\Delta}^{\mathfrak{N}}(\theta)-\lim s_k=s$. For a given $\varepsilon\in(0,1)$, choose $\vartheta>0$ such that $(1-\varepsilon)\circledast(1-\varepsilon)>1-\vartheta$ and $\varepsilon\boxdot\varepsilon<\vartheta$. For any $r>0$,

$$\delta_{\Delta}^{\theta}(G) := \delta_{\Delta}^{\theta}\left\{k \in \mathbb{N}: \mu_{(\mathfrak{N},\mathscr{C})}\left(\Delta s_k - s, \frac{r}{2}\right) \leq 1 - \varepsilon, \varrho_{(\mathfrak{N},\mathscr{U})}\left(\Delta s_k - s, \frac{r}{2}\right) \geq \varepsilon, \zeta_{(\mathfrak{N},\mathscr{F})}\left(\Delta s_k - s, \frac{r}{2}\right) \geq \varepsilon\right\} = 0, \tag{20}$$

can be written so $\delta_{\Delta}^{\theta}(G^c) = 1$. For $m \in G^c$, $\mu_{(\mathfrak{N},\mathscr{C})}(\Delta s_m - s, (r/2)) > 1 - \varepsilon$ and $\varrho_{(\mathfrak{N},\mathscr{I})}(\Delta s_k - s, (r/2)) < \varepsilon$, $\zeta_{(\mathfrak{N},\mathscr{F})}(\Delta s_k - s, (r/2)) < \varepsilon$ Let

$$H = \left\{k \in \mathbb{N}: \mu_{(\mathfrak{N},\mathscr{C})}(\Delta s_k - \Delta s_m, r) \leq 1 - \varepsilon \text{ or } \varrho_{(\mathfrak{N},\mathscr{U})}(\Delta s_k - \Delta s_m, r) \geq \varepsilon, \zeta_{(\mathfrak{N},\mathscr{F})}(\Delta s_k - \Delta s_m, r) \geq \varepsilon\right\}. \tag{21}$$

It is necessary to show that $H \subset G$. So, to show this, let $k \in (H \cap G^c)$. In this case, $\mu_{(\mathfrak{N},\mathscr{C})}(\Delta s_k - \Delta s_m, r) \leq 1 - \vartheta$ and $\mu_{(\mathfrak{N},\mathscr{C})}(\Delta s_k - s, (r/2)) > 1 - \vartheta$, especially $\mu_{(\mathfrak{N},\mathscr{C})}(\Delta s_m - s, (r/2)) > 1 - \vartheta$. So,

$$1 - \vartheta \geq \mu_{(\mathfrak{N},\mathscr{C})}(\Delta s_k - \Delta s_m, r) \geq \mu_{(\mathfrak{N},\mathscr{C})}\left(\Delta s_k - s, \frac{r}{2}\right) \circledast \mu_{(\mathfrak{N},\mathscr{C})}\left(\Delta s_m - s, \frac{r}{2}\right) > (1 - \varepsilon) \circledast (1 - \varepsilon) > 1 - \vartheta. \tag{22}$$

However, this is not possible. Moreover, $\varrho_{(\mathfrak{N},\mathscr{U})}(\Delta s_k - \Delta s_m, r) \geq \vartheta$ and $\varrho_{(\mathfrak{N},\mathscr{U})}(\Delta s_k - s, (r/2)) < \varepsilon$. Especially, $\varrho_{(\mathfrak{N},\mathscr{U})}(\Delta s_m - s, (r/2)) < \varepsilon$. Hence,

$$\vartheta \leq \varrho_{(\mathfrak{N},\mathscr{U})}(\Delta s_k - \Delta s_m, r) \leq \varrho_{(\mathfrak{N},\mathscr{U})}\left(\Delta s_k - s, \frac{r}{2}\right) \boxdot \varrho_{(\mathfrak{N},\mathscr{U})}\left(\Delta s_m - s, \frac{r}{2}\right) < \varepsilon \boxdot \varepsilon < \vartheta, \tag{23}$$

which is impossible. With a similar technique can be applied for $\zeta_{(\mathfrak{N},\mathscr{F})}(\Delta s_k - \Delta s_m, r)$. So, $H \subset G$ and $\delta_{\Delta}^{\theta}(G) = 0$. Then, (s_k) is $\Delta-$Lacunary statistical Cauchy sequences in neutrosophic normed spaces.

Otherwise, let (s_k) be $\Delta-$ Lacunary statistical Cauchy but not $\Delta-$Lacunary statistical convergent on $(\mathscr{U}, \mu_{(\mathfrak{N},\mathscr{C})}, \varrho_{(\mathfrak{N},\mathscr{U})}, \zeta_{(\mathfrak{N},\mathscr{F})}, \circledast, \boxdot)$. For a given $\varepsilon \in (0,1)$, choose $\vartheta > 0$ such that $(1 - \varepsilon) \circledast (1 - \varepsilon) > 1 - \vartheta$ and $\varepsilon \boxdot \varepsilon < \vartheta$. Because (s_k) is not $\Delta-$Lacunary statistical convergent,

$$\begin{aligned}
&\mu_{(\mathfrak{N},\mathscr{C})}(\Delta s_k - \Delta s_m, r) \geq \mu_{(\mathfrak{N},\mathscr{C})}\left(\Delta s_k - s, \frac{r}{2}\right) \circledast \mu_{(\mathfrak{N},\mathscr{C})}\left(\Delta s_m - s, \frac{r}{2}\right) > (1 - \varepsilon) \circledast (1 - \varepsilon) > 1 - \vartheta,\\
&\varrho_{(\mathfrak{N},\mathscr{U})}(\Delta s_k - \Delta s_m, r) \leq \varrho_{(\mathfrak{N},\mathscr{U})}\left(\Delta s_k - s, \frac{r}{2}\right) \boxdot \varrho_{(\mathfrak{N},\mathscr{U})}\left(\Delta s_m - s, \frac{r}{2}\right) < \varepsilon \boxdot \varepsilon < \vartheta,\\
&\zeta_{(\mathfrak{N},\mathscr{F})}(\Delta s_k - \Delta s_m, r) \leq \zeta_{(\mathfrak{N},\mathscr{F})}\left(\Delta s_k - s, \frac{r}{2}\right) \boxdot \zeta_{(\mathfrak{N},\mathscr{F})}\left(\Delta s_m - s, \frac{r}{2}\right) < \varepsilon \boxdot \varepsilon < \vartheta.
\end{aligned} \tag{24}$$

So, for

$$I = \left\{k \in \mathbb{N}: \mu_{(\mathfrak{N},\mathscr{C})}(\Delta s_k - \Delta s_m, r) \leq 1 - \vartheta, \varrho_{(\mathfrak{N},\mathscr{U})}(\Delta s_k - \Delta s_m, r) \geq \vartheta, \zeta_{(\mathfrak{N},\mathscr{F})}(\Delta s_k - \Delta s_m, r) \geq \vartheta\right\}, \tag{25}$$

$\delta_{\Delta}^{\theta}(I^c) = 0$ and also $\delta_{\Delta}^{\theta}(I) = 1$. Since (s_k) is $\Delta-$Lacunary statistical Cauchy, this is impossible. Thus, (s_k) is $\Delta-$Lacunary statistical convergent in $(\mathscr{U}, \mu_{(\mathfrak{N},\mathscr{C})}, \varrho_{(\mathfrak{N},\mathscr{U})}, \zeta_{(\mathfrak{N},\mathscr{F})}, \circledast, \boxdot)$. □

3.2. $\Delta-$Lacunary Statistical Convergence with Double Sequences. Now, $\Delta-$lacunary statistical convergence will be applied to neutrosophic normed spaces using double sequences. First of all, let us remind a few definitions necessary for the section.

Let $\delta_{(n,\widetilde{n})}$ be numbers of $(k,l) \in D$, where $k \leq n$ and $l \leq \widetilde{n}$. $D \subset \mathbb{N} \times \mathbb{N}$ with double density is defined [17] as

$$\lim_{n,\widetilde{n} \longrightarrow \infty} \frac{\delta_{(n,\widetilde{n})}}{n\widetilde{n}} = \delta_{(n,\widetilde{n})}(D). \tag{26}$$

In [17], statistical convergence is defined using double sequences: (s_{kl}) is called statistical convergence if, for all $\varepsilon > 0$, $\delta_{(n,\widetilde{n})}(\{(k,l),\ k \leq n \text{ and } l \leq \widetilde{n}\colon |s_{kl} - s| \geq \varepsilon\}) = 0$.

Then, in [19], double Lacunary sequences are defined as follows. Let there exist two increasing integers sequences, where $k_0, \widetilde{k}_0 = 0$, $\lim_{r \longrightarrow \infty} h_r := \lim_{r \longrightarrow \infty} k_r - k_{r-1} = \infty$, and $\lim_{\widetilde{r} \longrightarrow \infty} h_{\widetilde{r}} := \lim_{\widetilde{r} \longrightarrow \infty} \widetilde{k}_{\widetilde{r}} - \widetilde{k}_{\widetilde{r}-1} = \infty$; then, $\theta = \{k_r, \widetilde{k}_{\widetilde{r}}\}$ is called double Lacunary sequences. Here, $l_r = (k_r / k_{r-1})$, $\widetilde{l}_{\widetilde{r}} = (\widetilde{k}_{\widetilde{r}} / \widetilde{k}_{\widetilde{r}-1})$, and $l_{r,\widetilde{r}} = l_r \widetilde{l}_{\widetilde{r}}$; also, $I_r = (k_{r-1}, k_r]$, $I_{\widetilde{r}} = (\widetilde{k}_{\widetilde{r}-1}, \widetilde{k}_{\widetilde{r}}]$, and $I_{r,\widetilde{r}} = \{(k,\widetilde{k})\colon k \in I_r \text{ and } \widetilde{k} \in I_{\widetilde{r}}\}$. Also, statistical convergence for double Lacunary sequences is given in [19]. Then, for neutrosophic normed spaces, statistical convergence is defined using double sequences in [23].

Double difference sequences is given in [29] as $(\Delta s_{k,l}) = s_{k+1,\,l+1} - s_{k,l+1} + s_{k,l} - s_{k+1,l}$.

Definition 14. Let $(\mathscr{U}, \mu_{(\mathfrak{N},\mathscr{C})}, \varrho_{(\mathfrak{N},\mathscr{U})}, \zeta_{(\mathfrak{N},\mathscr{F})}, \circledast, \boxdot)$ be neutrosophic normed spaces and $(\Delta s_{k,l}) = s_{k+1,\,l+1} - s_{k,l+1} + s_{k,l} - s_{k+1,l}$. Double sequences (s_{kl}) is called to be $\Delta-$ convergent to s according to neutrosophic normed if, for all $\varepsilon \in (0,1)$ and $r > 0$, there exists $\widetilde{k}, \widetilde{l} \in \mathbb{N}$ such that, for every $\widetilde{k} \leq k$ and $\widetilde{l} \leq l$,

$$\mu_{(\mathfrak{N},\mathscr{C})}\left(\Delta s_{k,l} - s, r\right) \leq 1 - \varepsilon,\ \varrho_{(\mathfrak{N},\mathscr{U})}\left(\Delta s_{k,l} - s, r\right) \geq \varepsilon,\ \zeta_{(\mathfrak{N},\mathscr{F})}\left(\Delta s_{k,l} - s, r\right) \geq \varepsilon. \tag{27}$$

This sequences is shown with ${}^{\mathfrak{N}}_{\Delta}{-}\lim_{k,l} s_{kl} = s$.

Definition 15. Let $(\mathscr{U}, \mu_{(\mathfrak{N},\mathscr{C})}, \varrho_{(\mathfrak{N},\mathscr{U})}, \zeta_{(\mathfrak{N},\mathscr{F})}, \circledast, \boxdot)$ be neutrosophic normed spaces. If there exist $r > 0$ and $0 < \varepsilon < 1$, for each $\Delta s_{k,l}$, where $\mu_{(\mathfrak{N},\mathscr{C})}(\Delta s_{k,l}, r) \leq 1 - \varepsilon$, $\varrho_{(\mathfrak{N},\mathscr{U})}(\Delta s_{k,l}, r) \geq \varepsilon$, and $\zeta_{(\mathfrak{N},\mathscr{F})}(\Delta s_{k,l}, r) \geq \varepsilon$, then (s_{kl}) is called bounded sequences in $(\mathscr{U}, \mu_{(\mathfrak{N},\mathscr{C})}, \varrho_{(\mathfrak{N},\mathscr{U})}, \zeta_{(\mathfrak{N},\mathscr{F})}, \circledast, \boxdot)$.

Lemma 3. *Let $(\mathscr{U}, \mu_{(\mathfrak{N},\mathscr{C})}, \varrho_{(\mathfrak{N},\mathscr{U})}, \zeta_{(\mathfrak{N},\mathscr{F})}, \circledast, \boxdot)$ be neutrosophic normed spaces.*

(1) If a sequence (s_{kl}) is $\Delta-$ convergent to s, then ${}^{\mathfrak{N}}_{\Delta}{-}\lim_{k,l} s_{kl}$ is unique

(2) If (s_{kl}) is $\Delta-$ convergent to s and (t_{kl}) is $\Delta-$ convergent to t, then $(s_{kl} + t_{kl})$ is $\Delta-$ convergent to $s + t$

(3) If (s_{kl}) is $\Delta-$ convergent to s and $c \neq 0$, then (cs_{kl}) is $\Delta-$ convergent to cs

Definition 16. Let $(\mathscr{U}, \mu_{(\mathfrak{N},\mathscr{C})}, \varrho_{(\mathfrak{N},\mathscr{U})}, \zeta_{(\mathfrak{N},\mathscr{F})}, \circledast, \boxdot)$ be neutrosophic normed spaces; (s_{kl}) is called to be $\Delta_{kl}-$ Cauchy sequences if, for every $\varepsilon \in (0,1)$ and $r > 0$, there exist a $k_0 \in \mathbb{N}$ such that, for every $k, l, j, m \geq k_0$,

$$\mu_{(\mathfrak{N},\mathscr{C})}\left(\Delta s_{k,l} - \Delta s_{j,m}, r\right) \leq 1 - \varepsilon \text{ or } \varrho_{(\mathfrak{N},\mathscr{U})}\left(\Delta s_{k,l} - \Delta s_{j,m}, r\right) \geq \varepsilon,\ \zeta_{(\mathfrak{N},\mathscr{F})}\left(\Delta s_{k,l} - \Delta s_{j,m}, r\right) \geq \varepsilon. \tag{28}$$

Lemma 4. *Let $(\mathscr{U}, \mu_{(\mathfrak{N},\mathscr{C})}, \varrho_{(\mathfrak{N},\mathscr{U})}, \zeta_{(\mathfrak{N},\mathscr{F})}, \circledast, \boxdot)$ be neutrosophic normed spaces and (s_{kl}) be a $\Delta_{kl}-$ Cauchy sequences. Then,*

(1) $(\mathscr{U}, \mu_{(\mathfrak{N},\mathscr{C})}, \varrho_{(\mathfrak{N},\mathscr{U})}, \zeta_{(\mathfrak{N},\mathscr{F})}, \circledast, \boxdot)$ is a complete if all $\Delta_{kl}-$ Cauchy sequences have a $\Delta-$ convergent subsequence in this spaces

(2) For the $\Delta_{kl}-$ Cauchy (s_{kl}) and (t_{kl}) sequences, $(s_{kl} + t_{kl})$ is $\Delta_{kl}-$ Cauchy sequences in $(\mathscr{U}, \mu_{(\mathfrak{N},\mathscr{C})}, \varrho_{(\mathfrak{N},\mathscr{U})}, \zeta_{(\mathfrak{N},\mathscr{F})}, \circledast, \boxdot)$

(3) Let (c_{kl}) be scalar and (s_{kl}) be a $\Delta_{kl}-$ Cauchy sequences in $(\mathscr{U}, \mu_{(\mathfrak{N},\mathscr{C})}, \varrho_{(\mathfrak{N},\mathscr{U})}, \zeta_{(\mathfrak{N},\mathscr{F})}, \circledast, \boxdot)$. Then, $(c_{kl} s_{kl})$ is a $\Delta_{kl}-$ Cauchy sequences in this spaces.

Definition 17. Let $(\mathscr{U}, \mu_{(\mathfrak{N},\mathscr{C})}, \varrho_{(\mathfrak{N},\mathscr{U})}, \zeta_{(\mathfrak{N},\mathscr{F})}, \circledast, \boxdot)$ be neutrosophic normed spaces; (c_{kl}) is called to be $\Delta_{kl}-$ statistical convergence with respect to $(\mu_{(\mathfrak{N},\mathscr{C})}, \varrho_{(\mathfrak{N},\mathscr{U})}, \zeta_{(\mathfrak{N},\mathscr{F})})$, if, for every $\varepsilon \in (0,1)$ and $r > 0$, there exist s such that

$$\left\{k \leq n \text{ and } l \leq \widetilde{n}\colon \mu_{(\mathfrak{N},\mathscr{C})}\left(\Delta s_{k,l} - s, r\right) \leq 1 - \varepsilon \text{ or } \varrho_{(\mathfrak{N},\mathscr{U})}\left(\Delta s_{k,l} - s, r\right) \geq \varepsilon, \zeta_{(\mathfrak{N},\mathscr{F})}\left(\Delta s_{k,l} - s, r\right) \geq \varepsilon\right\}, \tag{29}$$

has natural density zero, i.e.,

$$\lim_{n,\widetilde{n}} \frac{1}{n\widetilde{n}} \left|\left\{k \leq n \text{ and } l \leq \widetilde{n}\colon \mu_{(\mathfrak{N},\mathscr{C})}\left(\Delta s_{k,l} - s, r\right) \leq 1 - \varepsilon \text{ or } \varrho_{(\mathfrak{N},\mathscr{U})}\left(\Delta s_{k,l} - s, r\right) \geq \varepsilon, \zeta_{(\mathfrak{N},\mathscr{F})}\left(\Delta s_{k,l} - s, r\right) \geq \varepsilon\right\}\right| = 0. \tag{30}$$

Therefore, it will be denote $st^{\mathfrak{N}}_{\Delta_{kl}} - \lim s_{kl} = s$ or $s_{kl} \longrightarrow s(S^{\mathfrak{N}}_{\Delta_{kl}})$ as $k \longrightarrow \infty$. $S^{\mathfrak{N}}_{\Delta_{kl}}$ denote set of each $\Delta_{kl}-$ statistical convergence sequences.

Lemma 5. *Let $(\mathscr{U}, \mu_{(\mathfrak{N},\mathscr{C})}, \varrho_{(\mathfrak{N},\mathscr{U})}, \zeta_{(\mathfrak{N},\mathscr{F})}, \circledast, \boxdot)$ be neutrosophic normed spaces, and (s_{kl}) is $\Delta_{kl}-$ statistical convergence; then, $st^{\mathfrak{N}}_{\Delta_{kl}} - \lim s_{kl} = s$ is unique.*

Definition 18. Let $(\mathscr{U}, \mu_{(\mathfrak{N},\mathscr{C})}, \varrho_{(\mathfrak{N},\mathscr{U})}, \zeta_{(\mathfrak{N},\mathscr{F})}, \circledast, \boxdot)$ be neutrosophic normed spaces; (s_{kl}) is called to be $\Delta_{kl}-$ statistical Cauchy sequences if, for every $\varepsilon \in (0,1)$ and $r > 0$, there exist $j, m \in \mathbb{N}$ such that

$$\delta_{(n,\widetilde{n})}\left(\left\{k \le n \,\text{and}\, l \le \widetilde{n} \le l\colon\ \mu_{(\mathfrak{N},\mathscr{C})}\left(\Delta s_{kl} - \Delta s_{jm}, r\right) \le 1 - \varepsilon \,\text{or}\, \varrho_{(\mathfrak{N},\mathscr{U})}\left(\Delta s_{kl} - \Delta s_{jm}, r\right) \ge \varepsilon, \zeta_{(\mathfrak{N},\mathscr{F})}\left(\Delta s_{kl} - \Delta s_{jm}, r\right) \ge \varepsilon\right\}\right) = 0. \tag{31}$$

Lemma 6. *Let $(\mathscr{U}, \mu_{(\mathfrak{N},\mathscr{C})}, \varrho_{(\mathfrak{N},\mathscr{U})}, \zeta_{(\mathfrak{N},\mathscr{F})}, \circledast, \boxdot)$ be neutrosophic normed spaces. Every $\Delta_{kl}-$ statistical convergence sequences is $\Delta_{kl}-$ statistical Cauchy sequences.*

Proof. Let (s_{kl}) be a $\Delta_{kl}-$ statistical convergence sequences in $(\mathscr{U}, \mu_{(\mathfrak{N},\mathscr{C})}, \varrho_{(\mathfrak{N},\mathscr{U})}, \zeta_{(\mathfrak{N},\mathscr{F})}, \circledast, \boxdot)$ and $st^{\mathfrak{N}}_{\Delta_{kl}} - \lim s_{kl} = s$. For a given $\varepsilon \in (0,1)$, choose $\vartheta > 0$ such that $(1-\varepsilon) \circledast (1-\varepsilon) > 1 - \vartheta$ and $\varepsilon \boxdot \varepsilon < \vartheta$. For any $r > 0$,

$$\delta_{(n,\widetilde{n})}\left(G_{n,\widetilde{n}}\right) \coloneqq \delta_{(n,\widetilde{n})}\left\{k \le n \,\text{and}\, l \le \widetilde{n}\colon\ \mu_{(\mathfrak{N},\mathscr{C})}\left(\Delta s_{kl} - s, \frac{r}{2}\right) \le 1 - \varepsilon, \varrho_{(\mathfrak{N},\mathscr{U})}\left(\Delta s_{kl} - s, \frac{r}{2}\right) \ge \varepsilon, \zeta_{(\mathfrak{N},\mathscr{F})}\left(\Delta s_{kl} - s, \frac{r}{2}\right) \ge \varepsilon\right\} = 0, \tag{32}$$

can be written, so $\delta_{(n,\widetilde{n})}(G^c_{n,\widetilde{n}}) = 1$. For $j, m \in G^c_{n,\widetilde{n}}$, $\mu_{(\mathfrak{N},\mathscr{C})}(\Delta s_{jm} - s, r) > 1 - \varepsilon$ and $\varrho_{(\mathfrak{N},\mathscr{U})}(\Delta s_{jm} - s, r) < \varepsilon$, $\zeta_{(\mathfrak{N},\mathscr{F})}(\Delta s_{jm} - s, r) < \varepsilon$. Let

$$H_{n,\widetilde{n}} = \left\{k \le n \,\text{and}\, l \le \widetilde{n}\colon\ \mu_{(\mathfrak{N},\mathscr{C})}\left(\Delta s_{kl} - \Delta s_{jm}, r\right) \le 1 - \vartheta \,\text{or}\, \varrho_{(\mathfrak{N},\mathscr{U})}\left(\Delta s_{kl} - \Delta s_{jm}, r\right) \ge \vartheta, \zeta_{(\mathfrak{N},\mathscr{F})}\left(\Delta s_{kl} - \Delta s_{jm}, r\right) \ge \vartheta\right\}. \tag{33}$$

It is necessary to show that $H_{n,\widetilde{n}} \subset G_{n,\widetilde{n}}$. So, to show this, let $u, w \in (H \cap G^c)$. In this case, $\mu_{(\mathfrak{N},\mathscr{C})}(\Delta s_{uw} - \Delta s_{jm}, r) \le 1 - \vartheta$ and $\mu_{(\mathfrak{N},\mathscr{C})}(\Delta s_{uw} - s, (r/2)) > 1 - \vartheta$, especially $\mu_{(\mathfrak{N},\mathscr{C})}(\Delta s_{jm} - s, r) > 1 - \vartheta$. So,

$$1 - \vartheta \ge \mu_{(\mathfrak{N},\mathscr{C})}\left(\Delta s_{uw} - \Delta s_{jm}, r\right) \ge \mu_{(\mathfrak{N},\mathscr{C})}\left(\Delta s_{uw} - s, \frac{r}{2}\right) \circledast \mu_{(\mathfrak{N},\mathscr{C})}\left(\Delta s_{jm} - s, \frac{r}{2}\right) > (1-\varepsilon) \circledast (1-\varepsilon) > 1 - \vartheta. \tag{34}$$

However, this is not possible. Moreover, $\varrho_{(\mathfrak{N},\mathscr{U})}(\Delta s_{uw} - \Delta s_{jm}, r) \ge \vartheta$ and $\varrho_{(\mathfrak{N},\mathscr{U})}(\Delta s_{uw} - s, (r/2)) < \vartheta$. Especially, $\varrho_{(\mathfrak{N},\mathscr{U})}(\Delta s_{jm} - s, (r/2)) < \varepsilon$. Hence,

$$\begin{aligned}\vartheta &\le \varrho_{(\mathfrak{N},\mathscr{U})}\left(\Delta s_{uw} - \Delta s_{jm}, r\right)\\ &\le \varrho_{(\mathfrak{N},\mathscr{U})}\left(\Delta s_{uw} - s, \frac{r}{2}\right) \boxdot \varrho_{(\mathfrak{N},\mathscr{U})}\left(\Delta s_{jm} - s, \frac{r}{2}\right) < \varepsilon \boxdot \varepsilon < \vartheta,\end{aligned} \tag{35}$$

which is impossible. With a similar technique, we can apply for $\zeta_{(\mathfrak{N},\mathscr{F})}(\Delta s_{uw} - \Delta s_{jm}, r)$. So, $H_{n,\widetilde{n}} \subset G_{n,\widetilde{n}}$ and $\delta_{(n,\widetilde{n})}(G_{n,\widetilde{n}}) = 0$. Then, (s_k) is $\Delta-$ statistical Cauchy sequences in $(\mathscr{U}, \mu_{(\mathfrak{N},\mathscr{C})}, \varrho_{(\mathfrak{N},\mathscr{U})}, \zeta_{(\mathfrak{N},\mathscr{F})}, \circledast, \boxdot)$. □

Theorem 5. *Let $(\mathscr{U}, \mu_{(\mathfrak{N},\mathscr{C})}, \varrho_{(\mathfrak{N},\mathscr{U})}, \zeta_{(\mathfrak{N},\mathscr{F})}, \circledast, \boxdot)$ be neutrosophic normed spaces. Then, every $\Delta_{kl}-$ statistical Cauchy sequences is $\Delta_{kl}-$ statistical convergence in this spaces.*

Proof. Let (s_{kl}) be $\Delta-$ statistical Cauchy but not $\Delta-$ statistical convergent on $(\mathscr{U}, \mu_{(\mathfrak{N},\mathscr{C})}, \varrho_{(\mathfrak{N},\mathscr{U})}, \zeta_{(\mathfrak{N},\mathscr{F})}, \circledast, \boxdot)$. For a given $\varepsilon \in (0,1)$, choose $\vartheta > 0$ such that $(1-\varepsilon) \circledast (1-\varepsilon) > 1 - \vartheta$ and $\varepsilon \boxdot \varepsilon < \vartheta$. Then,

$$\begin{aligned}&\mu_{(\mathfrak{N},\mathscr{C})}\left(\Delta s_{k,l} - \Delta s_{j,m}, r\right) \ge \mu_{(\mathfrak{N},\mathscr{C})}\left(\Delta s_{k,l} - s, \frac{r}{2}\right) \circledast \mu_{(\mathfrak{N},\mathscr{C})}\left(\Delta s_{j,m} - s, \frac{r}{2}\right) > (1-\varepsilon) \circledast (1-\varepsilon) > 1 - \vartheta,\\ &\varrho_{(\mathfrak{N},\mathscr{U})}\left(\Delta s_{k,l} - \Delta s_{j,m}, r\right) \le \varrho_{(\mathfrak{N},\mathscr{U})}\left(\Delta s_{k,l} - s, \frac{r}{2}\right) \boxdot \varrho_{(\mathfrak{N},\mathscr{U})}\left(\Delta s_{j,m} - s, \frac{r}{2}\right) < \varepsilon \boxdot \varepsilon < \vartheta,\\ &\zeta_{(\mathfrak{N},\mathscr{F})}\left(\Delta s_{k,l} - \Delta s_{j,m}, r\right) \le \zeta_{(\mathfrak{N},\mathscr{F})}\left(\Delta s_{k,l} - s, \frac{r}{2}\right) \boxdot \zeta_{(\mathfrak{N},\mathscr{F})}\left(\Delta s_{j,m} - s, \frac{r}{2}\right) < \varepsilon \boxdot \varepsilon < \vartheta.\end{aligned} \tag{36}$$

So, for

$$I_{n,\widetilde{n}} = \left\{k \le n \,\text{and}\, l \le \widetilde{n}\colon \mu_{(\mathfrak{N},\mathscr{C})}\left(\Delta s_k - \Delta s_m, r\right) \le 1 - \vartheta, \varrho_{(\mathfrak{N},\mathscr{U})}\left(\Delta s_k - \Delta s_m, r\right) \ge \vartheta, \zeta_{(\mathfrak{N},\mathscr{F})}\left(\Delta s_k - \Delta s_m, r\right) \ge \vartheta\right\}, \tag{37}$$

and $\delta_{(n,\widetilde{n})}(I^c_{n,\widetilde{n}}) = 0$; also, $\delta_{(n,\widetilde{n})}(I_{n,\widetilde{n}}) = 1$. Since (s_{kl}) is $\Delta-$ statistical Cauchy, this is impossible. Thus, (s_k) is $\Delta-$ statistical convergent in $(\mathscr{U}, \mu_{(\mathfrak{N},\mathscr{C})}, \varrho_{(\mathfrak{N},\mathscr{U})}, \zeta_{(\mathfrak{N},\mathscr{F})}, \circledast, \boxdot)$. □

Definition 19. Let $(\mathscr{U}, \mu_{(\mathfrak{N},\mathscr{C})}, \varrho_{(\mathfrak{N},\mathscr{U})}, \zeta_{(\mathfrak{N},\mathscr{F})}, \circledast, \boxdot)$ be neutrosophic normed spaces. If every $\Delta_{kl}-$ statistical Cauchy sequences is $\Delta_{kl}-$ statistical convergence in $(\mathscr{U}, \mu_{(\mathfrak{N},\mathscr{C})}, \varrho_{(\mathfrak{N},\mathscr{U})}, \zeta_{(\mathfrak{N},\mathscr{F})}, \circledast, \boxdot)$, then this spaces is called complete.

Result 1. If $(\mathscr{U}, \mu_{(\mathfrak{N},\mathscr{C})}, \varrho_{(\mathfrak{N},\mathscr{U})}, \zeta_{(\mathfrak{N},\mathscr{F})}, \circledast, \boxdot)$ is neutrosophic normed spaces, then this spaces is complete.

Result 2. Let $(\mathscr{U}, \mu_{(\mathfrak{N},\mathscr{C})}, \varrho_{(\mathfrak{N},\mathscr{U})}, \zeta_{(\mathfrak{N},\mathscr{F})}, \circledast, \boxdot)$ be neutrosophic normed spaces and (s_{kl}) be a double sequences in this spaces. Then, (s_{kl}) is a $\Delta_{kl}-$ statistical convergence sequences, $s(s_{kl})$ is a $\Delta_{kl}-$ statistical Cauchy sequences, and $\Leftrightarrow (\mathscr{U}, \mu_{(\mathfrak{N},\mathscr{C})}, \varrho_{(\mathfrak{N},\mathscr{U})}, \zeta_{(\mathfrak{N},\mathscr{F})}, \circledast, \boxdot)$ is complete neutrosophic normed spaces.

Lemma 7. *Let $(\mathscr{U}, \mu_{(\mathfrak{N},\mathscr{C})}, \varrho_{(\mathfrak{N},\mathscr{U})}, \zeta_{(\mathfrak{N},\mathscr{F})}, \circledast, \boxdot)$ be neutrosophic normed spaces and (s_{kl}) be a double sequences in this spaces. For all $\varepsilon \in (0,1)$ and $r > 0$,*

$$
\begin{aligned}
& st^{\mathfrak{N}}_{\Delta_{kl}} - \lim s_{kl} = s, \\
& \lim_{n,\widetilde{n}} \frac{1}{n\widetilde{n}} \left|\left\{k \le n \,\text{and}\, l \le \widetilde{n}\colon \mu_{(\mathfrak{N},\mathscr{C})}\left(\Delta s_{k,l} - s, r\right) > 1 - \varepsilon, \varrho_{(\mathfrak{N},\mathscr{U})}\left(\Delta s_{k,l} - s, r\right) < \varepsilon, \zeta_{(\mathfrak{N},\mathscr{F})}\left(\Delta s_{k,l} - s, r\right) < \varepsilon\right\}\right| = 1, \\
& \lim_{n,\widetilde{n}} \frac{1}{n\widetilde{n}} \left|\left\{k \le n \,\text{and}\, l \le \widetilde{n}\colon \mu_{(\mathfrak{N},\mathscr{C})}\left(\Delta s_{k,l} - s, r\right) \le 1 - \varepsilon\right\}\right| = 0, \\
& \lim_{n,\widetilde{n}} \frac{1}{n\widetilde{n}} \left|\left\{k \le n \,\text{and}\, l \le \widetilde{n}\colon \varrho_{(\mathfrak{N},\mathscr{U})}\left(\Delta s_{k,l} - s, r\right) \ge \varepsilon\right\}\right| = 0, \\
& \lim_{n,\widetilde{n}} \frac{1}{n\widetilde{n}} \left|\left\{k \le n \,\text{and}\, l \le \widetilde{n}\colon \zeta_{(\mathfrak{N},\mathscr{F})}\left(\Delta s_{k,l} - s, r\right) \ge \varepsilon\right\}\right| = 0, \\
& \lim_{n,\widetilde{n}} \frac{1}{n\widetilde{n}} \left|\left\{k \le n \,\text{and}\, l \le \widetilde{n}\colon \mu_{(\mathfrak{N},\mathscr{C})}\left(\Delta s_{k,l} - s, r\right) > 1 - \varepsilon\right\}\right| = 1, \\
& \lim_{n,\widetilde{n}} \frac{1}{n\widetilde{n}} \left|\left\{k \le n \,\text{and}\, l \le \widetilde{n}\colon \varrho_{(\mathfrak{N},\mathscr{U})}\left(\Delta s_{k,l} - s, r\right) < \varepsilon\right\}\right| = 1, \\
& \lim_{n,\widetilde{n}} \frac{1}{n\widetilde{n}} \left|\left\{k \le n \,\text{and}\, l \le \widetilde{n}\colon \zeta_{(\mathfrak{N},\mathscr{F})}\left(\Delta s_{k,l} - s, r\right) < \varepsilon\right\}\right| = 1, \\
& st^{\mathfrak{N}}_{\Delta} - \lim_{n\longrightarrow\infty} \mu_{(\mathfrak{N},\mathscr{C})}\left(\Delta s_{k,l} - s, r\right) = 1, \\
& st^{\mathfrak{N}}_{\Delta} - \lim_{n\longrightarrow\infty} \varrho_{(\mathfrak{N},\mathscr{U})}\left(\Delta s_k - s, r\right) = 0, \\
& st^{\mathfrak{N}}_{\Delta} - \lim_{n\longrightarrow\infty} \zeta_{(\mathfrak{N},\mathscr{F})}\left(\Delta s_k - s, r\right) = 0.
\end{aligned} \tag{38}
$$

Now, with the help of double Lacunary sequences, the definition of convergence and important properties for difference sequences will be given.

Definition 20. Let $(\mathscr{U}, \mu_{(\mathfrak{N},\mathscr{C})}, \varrho_{(\mathfrak{N},\mathscr{U})}, \zeta_{(\mathfrak{N},\mathscr{F})}, \circledast, \boxdot)$ be neutrosophic normed spaces and (θ) be a double Lacunary sequences. (s_{kl}) is called to be $\Delta_{kl}-$Lacunary statistical convergence in $(\mathscr{U},\mu_{(\mathfrak{N},\mathscr{C})},\varrho_{(\mathfrak{N},\mathscr{U})},\zeta_{(\mathfrak{N},\mathscr{F})},\circledast,\boxdot)$ if, for every $\varepsilon\in(0,1)$ and $r>0$, there exist a s such that

$$\left\{k\leq n\,\text{and}\,l\leq\tilde{n}\colon\ \mu_{(\mathfrak{N},\mathscr{C})}\left(\Delta s_{k,l}-s,r\right)\leq 1-\varepsilon\,\text{or}\,\varrho_{(\mathfrak{N},\mathscr{U})}\left(\Delta s_{k,l}-s,r\right)\geq\varepsilon,\zeta_{(\mathfrak{N},\mathscr{F})}\left(\Delta s_{k,l}-s,r\right)\geq\varepsilon\right\},\tag{39}$$

has natural $\theta-$ density zero, i.e.,

$$\lim_{r,\tilde{r}\longrightarrow\infty}\frac{1}{l_{r,\tilde{r}}}\left|\left\{(k,\tilde{k})\in I_{r,\tilde{r}}\colon\ \mu_{(\mathfrak{N},\mathscr{C})}\left(\Delta s_{k,l}-s,r\right)\leq 1-\varepsilon\,\text{or}\,_{(\mathfrak{N},\mathscr{U})}\left(\Delta s_{k,l}-s,r\right)\geq\varepsilon,\zeta_{(\mathfrak{N},\mathscr{F})}\left(\Delta s_{k,l}-s,r\right)\geq\varepsilon\right\}\right|=0.\tag{40}$$

Therefore, it will be denote $st^{\mathfrak{N}}_{\Delta_{kl}}(\theta)-\lim s_{kl}=s$ or $s_{kl}\longrightarrow s(S^{\mathfrak{N}}_{\Delta_{kl}})$ as $k\longrightarrow\infty$. $S^{\mathfrak{N}}_{\Delta_{kl}}(\theta)$ denote set of all $\Delta_{kl}-$Lacunary statistical convergence sequences.

Definition 21. Let $(\mathscr{U},\mu_{(\mathfrak{N},\mathscr{C})},\varrho_{(\mathfrak{N},\mathscr{U})},\zeta_{(\mathfrak{N},\mathscr{F})},\circledast,\boxdot)$ be neutrosophic normed spaces and (θ) be a double Lacunary sequence. In this case, (s_{kl}) is a called to be $\Delta_{kl}-$ Lacunary statistical Cauchy sequences if, for all $\varepsilon\in(0,1)$ and $r>0$, there exist a $k_0\in\mathbb{N}$ such that, for every $k,l,j,m\geq k_0$,

$$\delta^{\theta,\Delta}_{(n,\tilde{n})}\left(\left\{k\leq n\,\text{and}\,l\leq\tilde{n}\leq l\colon\ \mu_{(\mathfrak{N},\mathscr{C})}\left(\Delta s_{k,l}-\Delta s_{j,m},r\right)\leq 1-\varepsilon\,\text{or}\,\varrho_{(\mathfrak{N},\mathscr{U})}\left(\Delta s_{k,l}-\Delta s_{j,m},r\right)\geq\varepsilon,\zeta_{(\mathfrak{N},\mathscr{F})}\left(\Delta s_{k,l}-\Delta s_{j,m},r\right)\geq\varepsilon\right\}\right)=0.\tag{41}$$

Theorem 6. *Let $(\mathscr{U},\mu_{(\mathfrak{N},\mathscr{C})},\varrho_{(\mathfrak{N},\mathscr{U})},\zeta_{(\mathfrak{N},\mathscr{F})},\circledast,\boxdot)$ be neutrosophic normed spaces, (θ) be Lacunary sequence. (s_{kl}) is $\Delta-$Lacunary statistical convergent $\Leftrightarrow(s_{kl})$ is $\Delta-$ Lacunary statistical Cauchy in neutrosophic normed spaces.*

4. Results and Discussion

Since the concept of set with membership degrees can explain the situations in daily life more realistically, spaces with neutrosophic norms have been chosen as the space to be studied as it also includes uncertainty situations. By using difference sequences in neutrosophic normed spaces, the characteristic structure of some convergence types for single-double sequences is established.

It is an important study for researchers working in engineering, where convergence situations are important and for approximation theory researchers who want to approach with the help of useful sequences. In this study, the work of Granados and Dhital was transferred to difference sequences, and a more comprehensive study was created since it was studied for single-double sequences. When this study is evaluated together with the study examining the statistical convergence properties for neutrosophic normed spaces using triple difference sequences, in [24], this study can be considered as a study containing more recent information, since it also includes statistical convergence and Lacunary convergence studies using single and double difference sequences.

The concept of Turiyam feature, first defined by Singh, was examined in [30] with the structure of Turiyam R-Modules and Turiyam modulo integers. Since it is considered that the 4th component, which is described as the Turiyam feature, is included in the 3rd component where the uncertainty situations are given, the study was built with 3 components.

5. Conclusions

In this study, which is based on Δ-statistical convergence in neutrosophic normed spaces, the concepts such as Δ-statistical bounded, Lacunary Δ-statistical convergence, related to this concept and important theorems including their transitions to each other are given. Researching these concepts in double sequences is one of the advantages of this study. By making use of the relationship between the concepts of Δ_{kl} −Lacunary statistical Cauchy sequences with Δ_{kl} −lacunary statistical convergence, results related to the completeness of the space were obtained. This study was created due to the need in the literature. It is a guiding study for researchers who will study convergence types in different senses.

Data Availability

The data used to support the findings of the study can be obtained from the author upon request.

Conflicts of Interest

The author declares no conflicts of interest.

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