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Article · January 2022

DOI: 10.54216/IJNS.190301

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Neutrosophic BCK-algebra and Ω-BCK-algebra

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Abstract

In this paper, we study neutrosophic of one important types of algebra namely BCK-algebra. Some new results of a generalization of BCK-algebra (Ω-BCK-algebra) have been introduced. Several facts about neutrosophic Ω-BCK-algebra are presented such as neutrosophic of homomorphic image and neutrosophic of kernel homomorphism. Finally, some definitions, examples, and other properties of neutrosophic BCK-algebra and neutrosophic Ω-BCK-algebra are given.

Keywords: Neutrosophic set; Neutrosophic algebra; BCK-algebra; BCI-algebra; Fuzzy set

1. Introduction

In 1998 Smarandache [1,2] came up with the idea of a neutrosophic set to cover the weaknesses discovered in fuzzy sets [3] and intuitionistic fuzzy sets [4]. Based on this mathematical idea, many researchers have made a lot of contributions by linking this tool with other branches of mathematics, such as complex analysis, topology, algebra, numerical analysis, and so on. In neutrosophic logic each matter is approach to the percentage of truth inside T, the percentage of indeterminacy inside I, and the percentage of falsity inside F. In fact, the neutrosophic set is the generalization of fuzzy set. We denote (NE) to neutrosophic property. The concept of fuzzy algebra was developed by Zadeh's student [5]; since then, many researchers have made significant contributions to fuzzy sets and their extension with various algebraic structures; for example, Youssef and Dib [6] studied the concept of a fuzzy group and demonstrated numerous results. Fathi and Salleh [7] generalized the idea of the fuzzy group to the intuitionistic fuzzy subgroup. Agboola et al. [8] created a neutrosophic group by adding a third membership to the definition of intuitionistic fuzzy groups [9]. Smarandache and others contributed to the neutrosophic algebraic environment [10-22]. In an environment of BCK/BCI BCK/BCI -algebras, there were several contributions discussed in [23-29]. Several authors inspired of some algebras, for example BCK-algebra, BCI-algebra and KU-algebra. An algebra $(A, *, \circ)$ is said to be BCK-algebra if $[(a*b)*(a*c)]*(c*b)=0]$, $[(a*(a*b))*b=0]$, $[(a*a)=0]$, $[(0*a)=0]$ and $[(a*b)=(b*a)=0$, so $a=b]$ for all a, b, c $\in A$. In this paper we introduced a new idea that is: neutrosophic BCK-algebra and neutrosophic Ω-BCK-algebra as a generalization of neutrosophic BCK-algebra. In section two, we define neutrosophic BCK-algebra, neutrosophic Ω-BCK-algebra, and some of their properties are discussed. In the next section, neutrosophic Ω-BCK-algebra are presented with their generalization. Finally, some their properties have been constructed with some remarks and examples.

2. Preliminaries

This section is about some of the vital definitions, remarks and examples which are used later in this paper.

Doi: https://doi.org/10.54216/IJNS.190301
Received: April 20, 2021 Accepted: October 04, 2022

Definition 2.1. [3]

Let X be a nonempty set. A fuzzy set A = {<x, $\mu_A(x)$> | x ∈ X} is characterized by a membership function μ_A: X ⟶ [0, 1], where $\mu_A(x)$ is interpreted as the degree of membership of the element x in the fuzzy subset A for any x ∈ X.

Definition 2.2. [1,2]

Let A be a universal set. The neutrosophic A, in short NE (A) is defined as

B={(ξ, $tH(\xi)$, $iH(\xi)$, $fH(\xi)$:ξ ∈ A} ∋ tH, iH, fH : A⟶[0, 1].

Note that there is an equivalent definition to Definition 2.2 and by the following:

Let X be a nonempty set. A neutrosophic set (NS, for short) A on X is an object of the form A = {<x, $\mu_A(x)$, $\sigma_A(x)$, $\nu_A(x)$> | x ∈ X} characterized by a membership function μ_A : X ⟶] −0, 1+[and an indeterminacy function σ_A : X ⟶] −0, 1+[and a non-membership function ν_A : X ⟶] −0, 1+[which satisfy the condition: $-0 \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3+$, for any x ∈ X.

Definition 2.3. [1,2]

Let S = {⟨ x, $\mu_S(x)$, $\sigma_S(x)$, $\gamma_S(x)$⟩ : x ∈ X} and T = {⟨ x, $\mu_T(x)$, $\sigma_T(x)$, $\gamma_T(x)$⟩: x ∈ X} be the two neutrosophic sets in X. Then the following statements are true in X:

(i) S ⊆ T ⇔ {⟨x, $\mu_S(x)$, ≤ $\mu_T(x)$, $\sigma_A(x) \leq \sigma_T(x)$, $\gamma_S(x) \geq \gamma_T(x)$⟩ : x ∈ X}
(ii) S ∩ T = {⟨x, $\mu_S(x)$, ∧ $\mu_T(x)$, $\sigma_S(x) \wedge \sigma_T(x)$, $\gamma_S(x) \vee \gamma_T(x)$⟩: x ∈ X}
(iii) S ∪ T = {⟨x, $\mu_S(x)$, ∨ $\mu_T(x)$, $\sigma_S(x) \vee \sigma_T(x)$, $\gamma_S(x) \wedge \gamma_T(x)$⟩: x ∈ X}
(iv) 0N = {⟨x, 0, 0, 1⟩ : x ∈ X}
(v) 1N = {⟨x, 1, 1, 0⟩ : x ∈ X}.

Definition 2.4. [8]

Let (G, *) be a group with (G∪I)={a+bI; a and b in G} We say NE(G)=(<G∪I>,*) is neutrosophic group which it is generated by G, I.

Remarks 2.5.

1) I denote to neutrosophic element such that I^2=I. Also, 0. I=0.I^{-1} is the inverse of I which is not exists.

2) NE(G) is commutative if ab=ba such that a and b in NE(G).

3) N(G)< is not group in general, but it contains a group.

Definition 2.6. [8]

Let NE(G) be a neutrosophic group. We say NE(A) is a neutrosophic subgroup if it satisfies all conditions of neutrosophic group; NE(A) contains a subset and it is group.

Definition 2.7. [9]

Let A be a semigroup, the semigroup generated by A and I (<A∪I>) denoted by <A∪I> is defined to be a neutrosophic semigroup where I is indeterminacy element and termed as neutrosophic element.

Remark 2.8.

All neutrosophic semigroups contain a proper subset which is a semigroup.

Example 2.9.

Z is a semigroup with (.) and let NE(A) = {<Z∪I>} be a neutrosophic semigroup with (.). Then Z subset of NE(A) is a semigroup.

Definition 2.10. [9]

Let NE(A) be a neutrosophic monoid under the binary operation ∗. Suppose e is the identity in N(A), that is s ∗ e = e ∗ s = s for all s ∈ NE(A).

Doi: https://doi.org/10.54216/IJNS.190301
Received: April 20, 2021 Accepted: October 04, 2022

Remark 2.11.

A proper subset B of NE(A) is a neutrosophic submonoid if B is a neutrosophic semigroup with ∗ and e ∈ B, i.e., B is a monoid with ∗.

3. Neutrosophic BCK-algebra

In this section, we present neutrosophic BCK-algebra with some their properties and some examples about it.

Definition 3.1.

An algebra (A, *, ∘) is called neutrosophic BCK-algebra if:

1){[T((a*b) *(a*c))] *T(c*b)=0, [I(a*b)*(a*c)]*I(c*b)=0, [F(a*b)*(a*c)]*F(c*b)=0}.

2) {T[a*(a*b)*T(b)]=0, I[(a*(a*b))*I(b)]=0, F[(a*(a*b))*F(b)]=0}.

3) {T[(a*a) =0, I(a*a) *=0, F(a*a) =0}.

4) {T[(0*a) =0, I(0*a) =0, F(0*a)=0}.

5) {T[((a*b)=(b*a))=0, so a=b), I[((a*b)=(b*a))=0, so a=b), F[((a*b)=(b*a))=0, so a=b} for all a, b, c in A.

Remarks 3.2.

Let (A, *, ∘) be a neutrosophic BCK-algebra. Then the following statements are holds:

1) {[T((a*b)*(b*c))≤ T(c*b), (I(a*b)*(b*c)] ≤I(c*b), (F(a*b)*(b*c)] ≤ F(c*b)} for all a, b, c in A.

2) {T(a*(a*(a*b))) ≤ T(a*b), (I(a*(a*(a*b))) ≤ I(a*b), (F(a*(a*(a*b))) ≤ F(a*b)}.

3) {(0) ≤ T(a), (0≤I(a), (0≤F(a)}.

4) {(T(a*b)=0 iff a≤ b), (I(a*b)=0 iff a≤ b), (F(a*b)=0 iff a≤ b)}.

5) {If T((a≤ b), then T(a*c) ≤ T(b*c) and T(c*b) ≤ T(c*a)), (If I((a≤ b), then I(a*c) ≤ I(b*c) and I(c*b) ≤ I(c*a), (If F((a≤ b), then F(a*c) ≤ F(b*c) and F(c*b) ≤ F(c*a)).

6) {T((a*b)*c)=T((a*c)*b), {I((a*b)*c)=I((a*c)*b), {F((a*b)*c)=F((a*c)*b).

7) {T(a*b)≤c iff T(a*c)≤b, (I((a*b)≤c iff I(a*c)≤b, F(a*b)≤c iff F(a*c)≤b}.

8) {0*T(a*b)=T((0*a)*(0*b)), {0*I(a*b)=I((0*a)*(0*b)), {0*F(a*b)=F((0*a)*(0*b))}.

9) {T((a*c)*(b*c)≤T(a*b)), I((a*c)*(b*c)≤I(a*b)), F((a*c)*(b*c)≤F(a*b))}.

10) {T((a*c)*(b*c)≤T(a*b)), I((a*c)*(b*c)≤I(a*b)), F((a*c)*(b*c)≤F(a*b))} for all a, b and c in A.

Example 3.3.

Let A={0, x, y}. Then there is a neutrosophic binary operation * which define by:

0*0=0, 0*x=0, 0*y=0,

x*0=x, x*x=0, x*y=0,

y*0=y, y*x=x, y*y=0.

Hence the neutrosophic BCK-algebra is commutative.

Definition 3.4.

Let J be a noon zero set of the neutrosophic BCK-algebra A. We say J is a neutrosophic ideal (NE(J)) of A if :

Doi: https://doi.org/10.54216/IJNS.190301
Received: April 20, 2021 Accepted: October 04, 2022

1) NE(0) ∈J. i.e. [0 ∈ T(0) ∈ J, I(0) ∈ J, F(0) ∈ J].

2) NE(a*b) ∈ J. i.e. [(T(a*b) ∈ J), (I(a*b) ∈ J), (F(a*b) ∈ J)]. OR (NE(a*b) ∈ NE(J)).

3) [(T(b) ∈ J), (I(b) ∈ J), (F(b) ∈ J)]. OR {NE(b) ∈ NE(J)}.

So [(T(a) ∈ J), (I(a) ∈ J), (F(a) ∈ J)]. OR {NE(a) ∈ NE(J)}.

Definition 3.5. [10]

Let A be a single valued neutrosophic set on X and B be a single valued neutrosophic set on Y, let f: Supp A → Supp B be an ordinary mapping and R be a single valued neutrosophic relation on X × Y. Then f_R is called a single valued neutrosophic mapping if for all (x, y) ∈ Supp A × Supp B the following condition is satisfied:

$\mu_R(x, y) = \{\min(\mu_A(x), \mu_B(f(x))$, if y = f(x) or 0, otherwise} and

$\sigma_R(x, y) = \{\min(\sigma_A(x), \sigma_B(f(x))$, if y = f(x) or 0 , otherwise} and

$\nu_R(x, y) = \{ \max(\nu_A(x), \nu_B(f(x))$, if y = f(x) , or 1, otherwise ,

Definition 3.6.

Let f: $A_1 \rightarrow A_2$ be neutrosophic mapping where A_1 and A_2 are two neutrosophic BCK-algebras. Then f is called neutrosophic homomorphism if:

$\{T_f(a*b) = T_f(a)* T_f(b), I_f(a*b) = I_f(a)* I_f(b), F_f(a*b) = F_f(a)* F_f(b)\}$ for all a, b in A_1.

Now we present another notion namely neutrosophic subalgebra of neutrosophic BCK-algebra A

Definition 3.7.

Let φ ≠J be a neutrosophic subset of a neutrosophic BCK-algebra A. Then NE(J) is called neutrosophic subalgebra of A if:

{T(a*b) ∈NE(J), (I(a*b)∈NE(J), (F(a*b)∈ NE(J)}. OR NE(a*b)∈NE(J) for all a, b∈J.

4. Neutrosophic Ω-BCK-algebra

In this section, we present a neutrosophic of a generalization of BCK-algebra namely Ω-BCK-algebra with some properties about the topic, but before that we need to define some concepts which will used later like Ω-group and Ω-semigroup.

Definition 4.1.

Let A be a set has an element 0 and let φ≠Ω be a set. If g: A× Ω ×A → A ∀ a, b in A and α in Ω such that the images denoted by a α b and the following statement are hold:

1)[(a α b) β (a α c)] β (c α b) =0,

2) (a α b) = (b α a) =0, so a=b,

3) a α a=0,

4) 0 α a=0 ∀ a, b, c in A and α, β in Ω.

Then A is called Ω-BCK-algebra.

Definition 4.2.

Let A be a neutrosophic set has an element (0) and let φ≠Ω be a neutrosophic set. Suppose that g: A× Ω ×A → A be a neutrosophic mapping ∀ a, b in A and α in Ω such that the images denoted by a α b and the following statement are hold:

Doi: https://doi.org/10.54216/IJNS.190301
Received: April 20, 2021 Accepted: October 04, 2022

1) {[T((a α b) β (a α c)) β (c α b) =0], [I((a α b) β (a α c)) β (c α b) =0], [F((a α b) β (a α c)) β (c α b) =0]},

2) {[T ((a α b) = (b α a) =0], so a=b), [I ((a α b) = (b α a) =0], so a=b), [F ((a α b) = (b α a) =0], so a=b)},

3) {T(a α a)=0, I(a α a)=0, F(a α a)=0},

4) {[T((0 α a)=0)], [I((0 α a)=0)], [F((0 α a)=0)]}, ∀ a, b, c in A and α, β in **Ω**.

Then A is called neutrosophic **Ω**-BCK-algebra.

Remark 4.3.

Let A be a neutrosophic **Ω**-BCK-algebra with α in **Ω**. If *: A× A→A such that NE(a*b) = NE(a α b), then (A, *, ∘) is a neutrosophic BCK-algebra (Aα) for all a, b in A.

Example 4.4.

Suppose that A={0, 1, 2, 3} and let **Ω**={ α, β}. The following results satisfies neutrosophic **Ω**-BCK-algebra for A:

{$\mathbf{T_A}$ (0α0=0, 0α1=0, 0α2=0, 0 α3=0, 1α 0=1, 1α 1=0, 1α2=2, 1α3=2, 2α0=2, 2α1=3, 2α2=0, 2α3=3, 3α0=3, 3α1=1, 3α2=1, 3α3=0), $\mathbf{I_A}$ (0α0=0, 0α1=0, 0 α2=0, 0 α3=0, 1α0=1, 1α1=0, 1α2=2, 1α3=2, 2α0=2, 2α1=3, 2α2=0, 2α3=3, 3α0=3, 3α1=1, 3α2=1, 3α3=0), $\mathbf{F_A}$ (0α0=0, 0α1=0, 0α2=0, 0 α3=0, 1α 0=1, 1α1=0, 1α2=2, 1α3=2, 2α0=2, 2α1=3, 2α2=0, 2α3=3, 3α0=3, 3α1=1, 3α2=1, 3α3=0)}.

Example 4.5.

Suppose that A={0, a, b} and let **Ω**={α, β}. The following results satisfies neutrosophic **Ω**-BCK-algebra for A:

{$\mathbf{T_A}$ (0 α 0=0, 0 α a=0, 0 α b=0, a α 0=a, a α a=0, aαb=0, b α 0=b, b α a=a, b α b=0), $\mathbf{I_A}$ (0 α 0=0, 0 α a=0, 0 α b=0, a α 0=a, a α a=0, aαb=0, b α 0=b, b α a=a, b α b=0), $\mathbf{F_A}$ (0 α 0=0, 0 α a=0, 0 α b=0, a α 0=a, a α a=0, aαb=0, b α 0=b, b α a=a, b α b=0)}.

Example 4.6. Suppose that A={0, y_1, y_2, y_3} and let **Ω**={0}. Then A is a neutrosophic **Ω**-BCK-algebra if:

{$\mathbf{T_A}$ (0 ∘0=0, 0 ∘y_1=0, 0 ∘y_2=0, 0∘y_3, y_1∘0=y_1, y_1∘y_1=0, y_1∘y_2=0, y_1 ∘y_3=0,y_2∘0=y_2, y_2∘y_1=y_1, y_2∘y_2=0, y_2∘y_3=0, y_3 ∘0=y_3, y_3∘y_1=y_3, y_3∘y_2=y_3, y_3∘y_3=0), $\mathbf{I_A}$ (0 ∘0=0, 0 ∘y_1=0, 0 ∘y_2=0, 0∘y_3, y_1∘0=y_1, y_1∘y_1=0, y_1∘y_2=0, y_1 ∘y_3=0,y_2∘0=y_2, y_2∘y_1=y_1, y_2∘y_2=0, y_2∘y_3=0, y_3 ∘0=y_3, y_3∘y_1=y_3, y_3∘y_2=y_3, y_3∘y_3=0), $\mathbf{F_A}$ (0 ∘0=0, 0 ∘y_1=0, 0 ∘y_2=0, 0∘y_3, y_1∘0=y_1, y_1∘y_1=0, y_1∘y_2=0, y_1 ∘y_3=0,y_2∘0=y_2, y_2∘y_1=y_1, y_2∘y_2=0, y_2∘y_3=0, y_3 ∘0=y_3, y_3∘y_1=y_3, y_3∘y_2=y_3, y_3∘y_3=0)}.

Definition 4.7.

Let ϕ≠A_1 be a neutrosophic subset of a neutrosophic **Ω**-BCK-algebra A. If NE(aαb) belong to NE(A), then A_1 is called neutrosophic subalgebra.

Example 4.8.

Let A={0, 1, 2, 3} and let **Ω**={α}. Then J={0, 1} is a normal neutrosophic subalgebra where a neutrosophic normal subalgebra means:

Let ϕ≠A_1 be a neutrosophic subset of neutrosophic **Ω**-BCK-algebra A. So A_1 is called neutrosophic normal subalgebra if NE(a αa) α (b αb) belong to NE(A_1).

Remarks 4.9.

i) Any neutrosophic neutrosophic **Ω**-BCK-algebra A is commutative if NE(bα(bα β a))=NE(a α (a β b)) with b belong to A and α, β belong to **Ω**.

ii) A neutrosophic **Ω**-BCK-algebra A is a neutrosophic partially ordered by a ≤ b if and only if NE(aαb) =0 where α belong to Ω and namely neutrosophic Ω-BCK-ordering.

iii) If A= {0, a, b, c} and **Ω**= { α, β} such that:

Doi: https://doi.org/10.54216/IJNS.190301
Received: April 20, 2021 Accepted: October 04, 2022

{$\mathbf{T_A}$ (0 α 0= 0, 0 α a= 0, 0 α b= 0, 0 α c =0, a α 0= a, a α a= 0, a α b= a, a α c =a, b α 0= b, b α a= b, b α b= 0, b α c =b, cα0= c, c α a= c, c α b= c, c α c =0, 0 β 0= 0, 0 β a= 0, 0 β b= 0, 0 β c =0, a β 0= a, a β a= 0, a β b= a, a β c =a, b β 0= b, b β a= a, b β b= 0, b β c =a, c β 0= c, c β a= b, c β b= a, c β c =0), $\mathbf{I_A}$(0 α 0= 0, 0 α a= 0, 0 α b= 0, 0 α c =0, a α 0= a, a α a= 0, a α b= a, a α c =a, b α 0= b, b α a= b, b α b= 0, b α c =b, cα0= c, c α a= c, c α b= c, c α c =0, 0 β 0= 0, 0 β a= 0, 0 β b= 0, 0 β c =0, a β 0= a, a β a= 0, a β b= a, a β c =a, b β 0= b, b β a= a, b β b= 0, b β c =a, c β 0= c, c β a= b, c β b= a, c β c =0), $\mathbf{F_A}$(0 α 0= 0, 0 α a= 0, 0 α b= 0, 0 α c =0, a α 0= a, a α a= 0, a α b= a, a α c =a, b α 0= b, b α a= b, b α b= 0, b α c =b, cα0= c, c α a= c, c α b= c, c α c =0, 0 β 0= 0, 0 β a= 0, 0 β b= 0, 0 β c =0, a β 0= a, a β a= 0, a β b= a, a β c =a, b β 0= b, b β a= a, b β b= 0, b β c =a, c β 0= c, c β a= b, c β b= a, c β c =0)},

then the neutrosophic **Ω**-BCK-algebra A is commutative.

Definition 4.10.

Let A_1 and A_2 be two neutrosophic **Ω**-BCK-algebras. So, the neutrosophic mapping g: $A_1 \rightarrow A_2$ is called neutrosophic homomorphism if:

{(T(aαb) =T(a) α T(b)), (I(aαb) =T(a) α I(b)), (F(aαb) =T(a) α F(b))}

where a, b belongs to A and α belong to **Ω**.

Remark 4.11.

A neutrosophic Kernel of the homomorphism g (NE(Ker(g))) can define it by the following:

Let g: $A_1 \rightarrow A_2$ be a neutrosophic homomorphism where A_1 and A_2 are two neutrosophic **Ω**-BCK-algebras. Then the NE(Ker(g)) = {a $\in A_1$: NE(g(a)) =0}.

On the other hand, NE(Img(g)) ={NE(g(a): a $\in$ A_1} and refer to neutrosophic Img(g).

Proposition 4.12.

Let A be a **Ω**-BCK-algebra. Then NE(a α(aβb)) =0 for all a, b belongs to A and α, β belongs to **Ω**.

Proof: From NE(**Ω**-BCK-algebra), for all a, b belongs to A and α, β belongs to **Ω**

NE(aβb) α (aβb) = 0 ---------> (*).

Now we put b=0, c=b in (*), so we obtain NE (a α (aβb) α b) = 0.

Proposition 4.13.

Let A be a neutrosophic **Ω**-BCK-algebra. If A is a neutrosophic commutative **Ω**-BCK-algebra and a≤b, then NE(a)=NE(b α (b β a) such that a, b belongs to A and α, β belongs to **Ω**.

Proof: Suppose that A is a neutrosophic commutative **Ω**-BCK-algebra. Hence

NE (a α (a β b) =NE (b α (b β a)).

Therefore

NE(a α0) =NE (b α(b βa)).

NE(a)=NE (b α(b β a))

Thus, ends the proof.

Corollary 4.14.

Let A be a neutrosophic **Ω**-BCK-algebra. If a≤b, such that NE(a)=NE(b α (b β a), then A is a neutrosophic commutative **Ω**-BCK-algebra.

Proof: Assume that b≤a. So NE(a)=NE (b α(b β a))**.** But a≤b. Hence NE(a αb)=0.

Then

Doi: https://doi.org/10.54216/IJNS.190301
Received: April 20, 2021 Accepted: October 04, 2022

NE(a)=NE(a α 0)

=NE(a α (a α b)

Thus

NE (a α(a β b))=NE(b α (b βa)

Then A is a neutrosophic commutative Ω-BCK-algebra**.**

Corollary 4.15.

Every NE(a)≥0 is a neutrosophic Ω-BCK-algebra A for all a, b belongs to A.

Proof: We know that NE(0αa) =0, α belongs to Ω. Therefore NE(a)≥0. Then 0 is the least an element in a neutrosophic Ω-BCK- algebra.

Proposition 4.16.

Let g: A_1→A_2 be a neutrosophic homomorphism of a neutrosophic Ω-BCK- algebra. Then neutrosophic image of g is a neutrosophic subalgebra of a neutrosophic Ω-BCK- algebra A_2.

Proof: Assume that g: A_1→A_2 be a neutrosophic homomorphism of a neutrosophic Ω-BCK-algebra A_1 and A_2. We have a, b is belonging to NE(Img(g)). So,

∃ S_1, S_2 ∈NE(A_1) ∋ NE(g(S_1)) =NE(a) and NE(g(S_2)) =NE(b).

This implies that NE(g(S_1) α NE(g(S_2) =NE (a α b) ∀ α ∈ Ω. Hence

NE (g (S_1 α S_2)) =NE (S_1 α S_2).

Then

NE(a α b) ∈ NE(Img(g)).

Thus, the neutrosophic of image g is a neutrosophic subalgebra of A_2.

Corollary 4.17.

The neutrosophic of Kernel of g is a neutrosophic subalgebra where g: A_1→A_2 is a neutrosophic homomorphism between two neutrosophic Ω-BCK-algebras A_1 and A_2.

Proof: Suppose that g: A_1→A_2 is a neutrosophic homomorphism of neutrosophic Ω-BCK- algebras A_1 and A_2. For all a, b belongs to NE(Ker(g)), so NE(g(a))=NE(g(b))=0 and hence NE(g(aαb)=NE(g(a)) α NE(g(b))=0α0=0. So, NE(aαb)∈NE(Ker(g)). Thus NE(Ker(g)) is a neutrosophic subalgebra of A_1.

5. Conclusion

In this work, we have studied a new generalization about BCK-algebra named neutrosophic Ω-BCK-algebra. some definitions, examples and other properties of neutrosophic BCK-algebra and neutrosophic Ω-BCK-algebra are proved.

We look forward in the future to study other topics of linear programming and its applications in practical life using neutrosophic logic, accompanying programs, sensitivity analysis...etc.

Funding: "This research received no external funding"

Conflicts of Interest: "The authors declare no conflict of interest."

Doi: https://doi.org/10.54216/IJNS.190301
Received: April 20, 2021 Accepted: October 04, 2022

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Doi: https://doi.org/10.54216/IJNS.190301
Received: April 20, 2021 Accepted: October 04, 2022