On the Structure of Number of Neutrosophic Clopen Topological Space

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Abstract

Let $\mathcal{X}$ be a finite set having n elements. The formula for giving the number of topologies $T(n)$ is still not obtained. If the number of elements n of a finite set is small, we can compute it by hand. However, the difficulty of finding the number of the topology increases when n becomes large. A topology describes how elements of a set are spatially related to each other, and the same set can have different topologies. Studying this particular area is also a highly valued part of the topology, and this is one of the fascinating and challenging research areas. Note that the explicit formula for finding the number of topologies is undetermined till now, and many researchers are researching this particular area. This paper is towards the formulae for finding the number of neutrosophic clopen topological spaces having two, three, four, and five open sets. In addition, some properties related to formulae are determined.

Keywords: Combinatorics; Neutrosophic Set; Neutrosophic Clopen Topological Space; Number of Neutrosophic Clopen Topological Space

1 Introduction

The notion of the NS (neutrosophic set) was originally introduced by Smarandache[1] by the generalization of fuzzy set[22] and intuitionistic fuzzy sets (IFSs).[2] Salama and Alblowi[3] introduced the concept of NTS (neutrosophic topological space) after Coker[4] introduced IFTS (intuitionistic fuzzy topological space). Nowadays, several researchers have contributed to NS and NTS (,[5],[67]). Neutrosophic semi-open sets were introduced by Ishwarya and Bageerathi[8] in NTS. Dhavaseelan and Jafari[9] introduced generalized Neutrosophic closed sets. Later, Shanthi, Chandrasekar, and Safina[10] generalized semi-closed sets in NTS. Many authors (,[1112]) also studied the notion of NSs and discussed various properties.

Recall thar a topology in which every open set is closed will be called a clopen topology (CLT). Many results for finding the number of topologies (NoTs) have been developed by several researchers (,[13],[14],[15],[1617]). Authors in (,[18],[1921]) also studied FTS (fuzzy topological spaces) and computed some formulas for finding NoTs and the number of FTS on a finite set. Recently, Francis, and Adenji[20] studied on clopen sets in topological spaces (TS). In this paper we focus on number of neutrosophic clopen topologies (NCLTs) and related results.

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2 Preliminaries

In the following, $\mathcal{X}$ denotes a non-empty finite set of cardinality n, M is a totally ordered set of cardinality $m \geq 2$, and let $\mathcal{N}_{\mathcal{X}}$ be the collection of all neutrosophic subsets of $\mathcal{X}$ whose membership values lies in M. Also, $0^{NCL} = \langle \frac{x}{(0,1,1)} : x \in \mathcal{X} \rangle$ and $1^{NCL} = \langle \frac{x}{(1,0,0)} : x \in \mathcal{X} \rangle$ are taken to denote neutrosophic empty set and neutrosophic universal set respectively. The following definitions will be useful in the results. Moreover, open set is used to mean neutrosophic open set (NOS) in this paper.

Definition 2.1. [1] A NS A^{NCL} on a universe of discourse $\mathcal{X}$ is defined as $A^{NCL} = \langle \frac{x}{(T(x),I(x),F(x))} : x \in \mathcal{X} \rangle$ where $T, I, F : \mathcal{X} \rightarrow]^{-}0, 1^{+}[$. Note that $^{-}0 \leq T(x) + I(x) + F(x) \leq 3^{+}$; $T(x), I(x)$ and $F(x)$ represents degree of membership function, degree of indeterminacy and degree of non-membership function respectively.

Definition 2.2. [3] Let $\tau^{NCL} \subseteq N_{\mathcal{X}}$ then τ^{NCL} is called a NT (neutrosophic topology) on $\mathcal{X}$ if
(i) $0^{NCL}, 1^{NCL} \in \tau^{NCL}$
(ii) $A_1^{NCL} \cap A_2^{NCL} \in \tau^{NCL}$, for any $A_1^{NCL}, A_2^{NCL} \in \tau^{NCL}$.
(iii) $\bigcup A_i^{NCL} \in \tau^{NCL}$, for arbitrary family $\{A_i^{NCL} : i \in I\} \in \tau^{NCL}$.
The pair $(\mathcal{X}, \tau^{NCL})$ is called NTS and any NS in τ^{NCL} is called NOS in $\mathcal{X}$.

Definition 2.3. [16] The Stirling number of the second kind is the number of partitions of a finite set with n elements into k blocks. It is denoted by $S(n,k)$ or $S_{n,k}$ and its explicit formula is
$S(n,k) = S_{n,k} = \frac{1}{k!}\sum_{j=0}^{k}(-1)^j\binom{k}{j}(k-j)^n$.

3 Results

Definition 3.1. A NT τ^{NCL} on a non-empty set $\mathcal{X}$ is said a NCLT if it consists of neutrosophic clopen sets, i.e., if every its NOSs is closed too. The pair $(\mathcal{X}, \tau^{NCL})$ is called neutrosophic clopen topological space (NCLTS) and if τ^{NCL} contains k-open sets then $(\mathcal{X}, \tau^{NCL})$ is called NCLTS having k-open sets.

Example 3.2. Let $\mathcal{X} = \{u_1, v_1, w_1\}$ and consider the family $\tau^{NCL} = \{0^{NCL}, 1^{NCL}, A_1^{NCL}, A_2^{NCL}\}$, Where $0^{NCL} = \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0,1,1)}, \frac{w_1}{(0,1,1)} \rangle$, $1^{NCL} = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(1,0,0)}, \frac{w_1}{(1,0,0)} \rangle$,
$A_1^{NCL} = \langle \frac{u_1}{(0.2,0.5,0.3)}, \frac{v_1}{(0.3,0.6,0.5)}, \frac{w_1}{(0,0.7,0.4)} \rangle$, $A_2^{NCL} = \langle \frac{u_1}{(0.3,0.5,0.2)}, \frac{v_1}{(0.5,0.4,0.3)}, \frac{w_1}{(0.4,0.3,0)} \rangle$.
Then τ^{NCL} is a NCLT on $\mathcal{X}$ and so $(\mathcal{X}, \tau^{NCL})$ is a NCLTS on $\mathcal{X}$.

Proposition 3.3. *Arbitrary intersection of NCLTs on $\mathcal{X}$ is clopen.*

Proof. Let
$$\mathcal{N}_{\tau}^{NCL} = \bigcap_{i \in \lambda} \tau_i^{NCL},$$
where λ is an index set and $\tau_i^{NCL} \in \mathcal{N}_{\mathcal{X}}^{NCL}$, collection of all NCLTs on $\mathcal{X}$. Clearly $\mathcal{N}_{\tau}^{NCL} \neq \emptyset$ as $0^{NCL}, 1^{NCL} \in \mathcal{N}_{\tau}^{NCL}$.
Let A^{NCL}, B^{NCL} be any two members of $\mathcal{N}_{\tau}^{NCL}$. Then
$A^{NCL}, B^{NCL} \in \mathcal{N}_{\tau}^{NCL}$
$\implies A^{NCL}, B^{NCL} \in \bigcap_{i \in \lambda} \tau_i^{NCL}$
$\implies A^{NCL}, B^{NCL} \in \tau_i^{NCL} \quad ; \forall i$
$\implies A^{NCL} \cap B^{NCL} \in \tau_i^{NCL}$ and $A^{NCL} \cup B^{NCL} \in \tau_i^{NCL} \quad ; \forall i$
$\implies A^{NCL} \cap B^{NCL} \in \bigcap_{i \in \lambda} \tau_i^{NCL}$ and $A^{NCL} \cup B^{NCL} \in \bigcap_{i \in \lambda} \tau_i^{NCL} \quad ; \forall i$
$\implies A^{NCL} \cap B^{NCL} \in \mathcal{N}_{\tau}^{NCL}$ and $A^{NCL} \cup B^{NCL} \in \mathcal{N}_{\tau}^{NCL}$
Therefore $\mathcal{N}_{\tau}^{NCL}$ is a NT on $\mathcal{X}$.
Let A^{NCL} be any element of $\mathcal{N}_{\tau}^{NCL}$, then
$A^{NCL} \in \mathcal{N}_{\tau}^{NCL} \implies A^{NCL} \in \bigcap_{i \in \lambda} \tau_i^{NCL}$
$\implies A^{NCL} \in \tau_i^{NCL} \quad ; \forall i$
$\implies C(A^{NCL}) \in \tau_i^{NCL} \quad ; \forall i$
$\implies C(A^{NCL}) \in \bigcap_{i \in \lambda} \tau_i^{NCL};$
$\implies C(A^{NCL}) \in \mathcal{N}_{\tau}^{NCL}$
Since A^{NCL} is arbitrary element of $\mathcal{N}_{\tau}^{NCL}$ and $C(A^{NCL}) \in \mathcal{N}_{\tau}^{NCL}$ implies that every element of $\mathcal{N}_{\tau}^{NCL}$ is NCLS. Hence arbitrary intersection of NCLTs is clopen. □

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Remark 3.4. Union of NCLTs on $\mathcal{X}$ is not clopen.
Illustration: Let $\mathcal{X} = \{u_1, v_1\}$ and $M = \{(0,1,1), (0.5, 0.5, 0.5), (1,0,0)\}$. Then number of elements in $\mathcal{N}_{\mathcal{X}}$, i.e.,$|\mathcal{N}_{\mathcal{X}}| = 3^2 = 9$. These are
$0^{NCL} = \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0,1,1)} \rangle$, $1^{NCL} = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(1,0,0)} \rangle$, $A_1^{NCL} = \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0.5,0.5,0.5)} \rangle$,
$A_2^{NCL} = \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(1,0,0)} \rangle$, $A_3^{NCL} = \langle \frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(0,1,1)} \rangle$,
$A_4^{NCL} = \langle \frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(0.5,0.5,0.5)} \rangle$, $A_5^{NCL} = \langle \frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(1,0,0)} \rangle$,
$A_6^{NCL} = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(0,1,1)} \rangle$, $A_7^{NCL} = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(0.5,0.5,0.5)} \rangle$.
Here $n = 2$, $m = 3$, so $\mathcal{J}_{\mathcal{N}_{\mathcal{X}}}(2,3,3) = 3^2 - 2 = 9 - 2 = 7$.
In this case
$\tau_1^{NCL} = \{0^{NCL}, 1^{NCL}, A_4^{NCL}\}$ and
$\tau_2^{NCL} = \{0^{NCL}, 1^{NCL}, A_1^{NCL}, A_2^{NCL}, A_6^{NCL}, A_7^{NCL}\}$ are NCLTs on $\mathcal{X}$.
But $\tau_1^{NCL} \cup \tau_2^{NCL} = \{0^{NCL}, 1^{NCL}, A_1^{NCL}, A_2^{NCL}, A_4^{NCL}, A_6^{NCL}, A_7^{NCL}\}$ is not a NCLT on $\mathcal{X}$ as $A_4^{NCL} \cap A_6^{NCL} = A_3^{NCL} \notin \tau_1^{NCL} \cup \tau_2^{NCL}$.

Proposition 3.5. *Union of two NCLTs is again a NCLT if one is contained in the other.*

Proof. Let A^{NCL} and B^{NCL} be two NCLTs on $\mathcal{X}$. Let $A^{NCL} \subseteq B^{NCL}$, then $A^{NCL} \cup B^{NCL} = B^{NCL}$, which is a NCLT on $\mathcal{X}$. Similarly, if $B^{NCL} \subseteq A^{NCL}$, then $A^{NCL} \cup B^{NCL} = A^{NCL}$, which is a NCLT on $\mathcal{X}$. This shows that union of two NCLTs is again a NCLT if one is contained in the other. □

Number of Neutrosophic clopen topology having 2-open Sets:
1. If $\mathcal{X} = \{u_1\}$ or $|\mathcal{X}| = 1$ whose neutrosophic values lies in M.
Case I: If $M = \{(0,1,1), (1,0,0)\}$.
Then, $|\mathcal{N}_{\mathcal{X}}| = 2^1 = 2$. These are
$0^{NCL} = \langle \frac{u_1}{(0,1,1)} \rangle$, $1^{NCL} = \langle \frac{u_1}{(1,0,0)} \rangle$.
In this case we will get only one NT which is $\tau_1^{NCL} = \{0^{NCL}, 1^{NCL}\}$. This NT is also NCLT.
Case II: If $M = \{(0,1,1), (0.5, 0.5, 0.5), (1,0,0)\}$
Then, $|\mathcal{N}_{\mathcal{X}}| = 3^1 = 3$. These are
$0^{NCL}, 1^{NCL}, A_1^{NCL} = \langle \frac{u_1}{(0.5,0.5,0.5)}$.
In this case also we will get only one NT having 2-open set which is $\tau_1^{NCL} = \{0^{NCL}, 1^{NCL}\}$ and hence a NCLT.
Case III: If $M = \{(0,1,1), (T, I, F), (F, 1-I, T), (1,0,0)\}; T, I, F \in [0,1]$
In this case, $|\mathcal{N}_{\mathcal{X}}| = 4^1 = 4$. These are
$0^{NCL}, 1^{NCL}, A_1^{NCL} = \langle \frac{u_1}{(T,I,F)} \rangle$, $A_2^{NCL} = \langle \frac{u_1}{(F,1-I,T)} \rangle$.
In this case also we will get only one NT having 2-open set which is $\tau_1^{NCL} = \{0^{NCL}, 1^{NCL}\}$ and therefore a NCLT.

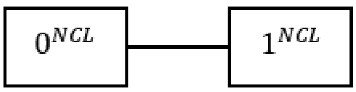

Figure 1: NCLT having 2-open sets

2. If $\mathcal{X} = \{u_1, v_1\}$ or $|\mathcal{X}| = 2$ whose neutrosophic values lies in M.
Case I: If $M = \{(0,1,1), (1,0,0)\}$
Then, $|\mathcal{N}_{\mathcal{X}}| = 2^2 = 4$. These are
$0^{NCL} = \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0,1,1)} \rangle$, $1^{NCL} = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(1,0,0)} \rangle$,
$A_1^{NCL} = \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(1,0,0)}$, $A_2^{NCL} = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(0,1,1)} \rangle$.
In this case, NCLT having 2-open set is one i.e., $\tau_1^{NCL} = \{0^{NCL}, 1^{NCL}\}$.
Case II: If $\mathcal{M} = \{(0,1,1), (0.5, 0.5, 0.5), (1,0,0)\}$
Then, $|\mathcal{N}_{\mathcal{X}}| = 3^2 = 9$. These are
$0^{NCL}, 1^{NCL}, A_1^{NCL} = \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0.5,0.5,0.5)} \rangle$, $A_2^{NCL} = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(0,1,1)} \rangle$,
$A_3^{NCL} = \langle \frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(0,1,1)} \rangle$, $A_4^{NCL} = \langle \frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(0.5,0.5,0.5)} \rangle$,
$A_5^{NCL} = \langle \frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(1,0,0)} \rangle$, $A_6^{NCL} = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(0,1,1)} \rangle$,
$A_7^{NCL} = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(0.5,0.5,0.5)} \rangle$.

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In this case also NCLT having 2-open set is one i.e., $\tau_1^{NCL} = \{0^{NCL}, 1^{NCL}\}$.
Case III: If $M = \{(0,1,1), (T,I,F), (F,1-I,T), (1,0,0)\}; T, I, F \in [0,1]$
In this case, $|\mathcal{N}_X| = 4^2 = 16$. These are

$0^{NCL}, 1^{NCL}, A_1^{NCL} = \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(T,I,F)} \rangle, A_2^{NCL} = \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(1,0,0)} \rangle,$
$A_3^{NCL} = \langle \frac{u_1}{(T,I,F)}, \frac{v_1}{(0,1,1)} \rangle, \quad A_4^{NCL} = \langle \frac{u_1}{(T,I,F)}, \frac{v_1}{(T,I,F)} \rangle,$
$A_5^{NCL} = \langle \frac{u_1}{(T,I,F)}, \frac{v_1}{(1,0,0)} \rangle, \quad A_6^{NCL} = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(0,1,1)} \rangle,$
$A_7^{NCL} = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(T,I,F)} \rangle, \quad A_8^{NCL} = \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(F,1-I,T)} \rangle,$
$A_9^{NCL} = \langle \frac{u_1}{(T,I,F)}, \frac{v_1}{(F,1-I,T)} \rangle, \quad A_{10}^{NCL} = \langle \frac{u_1}{(F,1-I,T)}, \frac{v_1}{(0,1,1)} \rangle,$
$A_{11}^{NCL} = \langle \frac{u_1}{(F,1-I,T)}, \frac{v_1}{(F,1-I,T)} \rangle, \quad A_{12}^{NCL} = \langle \frac{u_1}{(F,1-I,T)}, \frac{v_1}{(1,0,0)} \rangle,$
$A_{13}^{NCL} = \langle \frac{u_1}{(F,1-I,T)}, \frac{v_1}{(T,I,F)} \rangle, \quad A_{14}^{NCL} = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(F,1-I,T)} \rangle.$

In this case also NCLT having 2-open set is one i.e., $\tau_1^{NCL} = \{0^{NCL}, 1^{NCL}\}$.

Proposition 3.6. *For $|\mathcal{X}| = n, |M| = m$, M is the any set of neutrosophic values containing $(1,0,0)$ and $(0,1,1)$, then number of NCLT having 2-open sets is one.*

Proof. The NT having 2-open sets is indiscrete NT only i.e., $\tau^{NCL} = \{0^{NCL}, 1^{NCL}\}$. This NT is NCLT as 0^{NCL} and 1^{NCL} are complements of each other. Therefore, the number of NCLTS having 2-open sets is one. □

Number of Neutrosophic clopen topology having 3-open Sets:
1. If $\mathcal{X} = \{p\}$ or $|\mathcal{X}| = 1$ whose neutrosophic values lies in M.
Case I: If $\mathcal{M} = \{(0,1,1), (1,0,0)\}$
In this case, $|\mathcal{N}_\mathcal{X}| = 2^1 = 2$. We will get only one NCLT which is
$\tau_1^{NCL} = \{0^{NCL}, 1^{NCL}\}$ and so no NCLT having 3-open sets. Therefore, the number of NCLT having 3-open sets is zero.
Case II: If $M = \{(0,1,1), (0.5,0.5,0.5), (1,0,0)\}$
In this case, $|\mathcal{N}_\mathcal{X}| = 3^1 = 3$, we will get only one NCLT having 3-open sets, which is
$\tau_1^{NCL} = \{0^{NCL}, A_1^{NCL}, 1^{NCL}\}$ as complement of A_1^{NCL} i.e., $C(A_1^{NCL}) = A_1^{NCL}$. Therefore, the number of NCLT having 3-open sets is one.
Case III: If $M = \{(0,1,1), (T,I,F), (F,1-I,T), (1,0,0)\}; T, I, F \in [0,1]$
In this case, $|\mathcal{N}_\mathcal{X}| = 4^1 = 4$. We will get no NT having 3-open sets. Therefore, the number of NCLT having 3-open sets is zero.

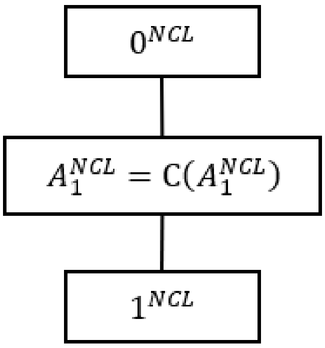

Figure 2: NCLT having 3-open sets

2. If $\mathcal{X} = \{p, q\}$ or $|\mathcal{X}| = 2$ whose neutrosophic values lies in M.
Case I: If $M = \{(0,1,1), (1,0,0)\}$
In this case, $|\mathcal{N}_\mathcal{X}| = 2^2 = 4$. We will get no NCLT having 3-open sets. Therefore, the number of NCLT having 3-open sets is zero.
Case II: If $M = \{(0,1,1), (0.5,0.5,0.5), (1,0,0)\}$
Then, $|\mathcal{N}_\mathcal{X}| = 3^2 = 9$. We will get only one NT having 3-open sets, which is $\tau_1^{NCL} = \{0^{NCL}, A_1^{NCL}, 1^{NCL}\}$. Therefore, the number of NCLT having 3-open sets is one.
Case III: If $M = \{(0,1,1), (T,I,F), (F,1-I,T), (1,0,0)\}$, where $T, I, F \in [0,1]$
In this case, $|\mathcal{N}_\mathcal{X}| = 4^2 = 16$. We will get no NT having 3-open sets. Therefore, the number of NCLT having 3-open sets is zero.

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Proposition 3.7. *For $|\mathcal{X}| = n$, the number of NCLT having 3-open sets is always one for M containing $(0,1,1),(1,0,0)$ and $(T,0.5,F),T = F;T,F \in [0,1]$, and zero for M that does not include neutrosophic values which are complement to each other i.e., $(T,0.5,F),T = F;T,F \in [0,1]$.*

Proof. Let, M be the set containing $(0,1,1),(1,0,0),(T,0.5,F),T = F;T,F \in [0,1]$, and other neutrosophic values.
Case 1: Let $|\mathcal{X}| = 1$, say $\mathcal{X} = \{a\}$
In this case the NCLT having 3-open sets is $\{0^{NCL}, A_1^{NCL}, 1^{NCL}\}$, where $A_1^{NCL} = \langle \frac{a}{(T,0.5,F)} \rangle$.
Case II: Let $|\mathcal{X}| = 2$, say $\mathcal{X} = \{a,b\}$
In this case the NCLT having 3-open sets is $\{0^{NCL}, 1^{NCL}, A_2^{NCL}\}$, where $A_2^{NCL} = \langle \frac{a}{(T,0.5,F)}, \frac{b}{(T,0.5,F)} \rangle$.
Case III: Let $|\mathcal{X}| = 3$, say $\mathcal{X} = \{a,b,c\}$
In this case the NCLT having 3-open sets is $\{0^{NCL}, 1^{NCL}, A_3^{NCL}\}$, where $A_3^{NCL} = \langle \frac{a}{(T,0.5,F)}, \frac{b}{(T,0.5,F)}, \frac{c}{(T,0.5,F)} \rangle$.
Case IV: Let $|\mathcal{X}| = n$(finite), say $\mathcal{X} = \{a_1, a_2, a_3, \dots, a_n\}$
In this case the NCLT having 3-open sets is $\{0^{NCL}, 1^{NCL}, A_n^{NCL}\}$, where
$A_n^{NCL} = \langle \frac{a_1}{(T,0.5,F)}, \frac{a_2}{(T,0.5,F)}, \frac{a_3}{(T,0.5,F)}, \dots, \frac{a_n}{(T,0.5,F)} \rangle$.
It is seen that there exists only one NCLT having 3-open sets. This NCLT contains $0^{NCL}, 1^{NCL}$ and A_n^{NCL}.
Note that, in A_n^{NCL} every member of $\mathcal{X}$ has neutrosophic membership value as $(T,0.5,F),T = F;T,F \in [0,1]$, whose complement is itself i.e., $A_n^{NCL} = C(A_n^{NCL})$.
On the other hand, if M does not contain $(T,0.5,F),T = F;T,F \in [0,1]$, then there exist no neutrosophic subset of $\mathcal{X}$ of the form A_n^{NCL}, such that $A_n^{NCL} \neq C(A_n^{NCL})$. Hence there exists no NCLT having 3-open sets. □

Number of Neutrosophic clopen topology having 4-open Sets:
1. If $\mathcal{X} = \{p\}$ or $|\mathcal{X}| = 1$ whose neutrosophic values lies in M.
Case I: If $M = \{(0,1,1),(1,0,0)\}$
In this case, there exists no NCLT having 4-open sets. Therefore, the number of NCLT having 4-open sets is zero.
Case II: If $M = \{(0,1,1),(0.5,0.5,0.5),(1,0,0)\}$
In this case, $|\mathcal{N}_{\mathcal{X}}| = 3^1 = 3$, we will get no NT having 4-open set. Therefore, the number of NCLT having 4-open sets is zero.
Case III: If $M = \{(0,1,1),(T,I,F),(F,1-I,T),(1,0,0)\};T,I,F \in [0,1]$.
In this case, $|\mathcal{N}_{\mathcal{X}}| = 4^1 = 4$. We will get one NCLT having 4-open set. Therefore, the number of NCLT having 4-open sets is one.
2. If $\mathcal{X} = \{p,q\}$ or $|\mathcal{X}| = 2$ whose neutrosophic values lies in M.
Case I: If $M = \{(0,1,1),(1,0,0)\}$
In this case, $|\mathcal{N}_X| = 2^2 = 4$, we will get one NCLT having 4-open sets. Therefore, the number of NCLT having 4-open sets is one i.e., $\tau_1^{NCL} = \{0^{NCL}, A_1^{NCL}, A_2^{NCL}, 1^{NCL}\}$.
Case II: If $M = \{(0,1,1),(0.5,0.5,0.5),(1,0,0)\}$
In this case, $|\mathcal{N}_{\mathcal{X}}| = 3^2 = 9$. We will get three NCLTs having 4-open set. These are

$$\tau_1^{NCL} = \{0^{NCL}, A_1^{NCL}, A_7^{NCL}, 1^{NCL}\}, \quad \tau_2^{NCL} = \{0^{NCL}, A_2^{NCL}, A_6^{NCL}, 1^{NCL}\},$$
$$\tau_3^{NCL} = \{0^{NCL}, A_3^{NCL}, A_5^{NCL}, 1^{NCL}\}.$$

Therefore, the number of NCLTs having 4-open sets is three.
Case III: If $M = \{(0,1,1),(T,I,F),(F,1-I,T),(1,0,0)\};T,I,F \in [0,1]$.
In this case, $|\mathcal{N}_{\mathcal{X}}| = 4^2 = 16$. We will get four NCLT having 4-open sets. These are

$$\tau_1^{NCL} = \{0^{NCL}, A_1^{NCL}, A_{14}^{NCL}, 1^{NCL}\}, \quad \tau_2^{NCL} = \{0^{NCL}, A_4^{NCL}, A_{11}^{NCL}, 1^{NCL}\},$$
$$\tau_3^{NCL} = \{0^{NCL}, A_2^{NCL}, A_6^{NCL}, 1^{NCL}\}, \quad \tau_4^{NCL} = \{0^{NCL}, A_3^{NCL}, A_{12}^{NCL}, 1^{NCL}\}.$$

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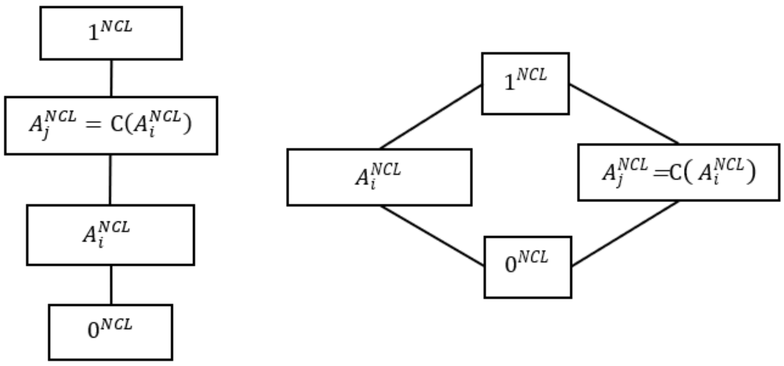

Figure 3: NCLT having 4-open sets

Number of Neutrosophic clopen topology having 5-open Sets:
1. If $\mathcal{X} = \{p\}$ or $|\mathcal{X}| = 1$ whose neutrosophic values lies in M.
Case I: If $M = \{(0, 1, 1), (1, 0, 0)\}$
In this case, $|\mathcal{N}_{\mathcal{X}}| = 2^1 = 2$, we will get only one NT which is $\tau_1^{NCL} = \{0^{NCL}, 1^{NCL}\}$ and so, there exists no NCLT having 5-open sets. Therefore, the number of NCLT having 5-open sets is zero.
Case II: If $M = \{(0, 1, 1), (0.5, 0.5, 0.5), (1, 0, 0)\}$
In this case, $|\mathcal{N}_{\mathcal{X}}| = 3^1 = 3$, we will get only two NCLTs having 2 and 3-open sets. Therefore, the number of NCLTs having 5-open sets is zero.
Case III: If $M = \{(0, 1, 1), (T, I, F), (F, 1 - I, T), (1, 0, 0)\}; T, I, F \in [0, 1]$
In this case, $|\mathcal{N}_{\mathcal{X}}| = 4^1 = 4$. We will get no NT having 5-open sets. Therefore, the number of NCLT having 5-open sets is zero.

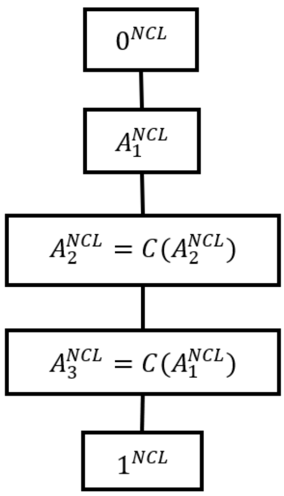

Figure 4: NCLT having 5-open sets

2. If $\mathcal{X} = \{p, q\}$ or $|\mathcal{X}| = 2$ whose neutrosophic values lies in M.
Case I: If $M = \{(0, 1, 1), (1, 0, 0)\}$
In this case, $|\mathcal{N}_{\mathcal{X}}| = 2^2 = 4$, we will get no NCLT having 5-open sets. Therefore, the number of NCLT having 5-open sets is zero.
Case II: If $M = \{(0, 1, 1), (0.5, 0.5, 0.5), (1, 0, 0)\}$
Then $|\mathcal{N}_{\mathcal{X}}| = 3^2 = 9$. We will get two NCLTs having 5-open sets. These are

$$\tau_1^{NCL} = \{0^{NCL}, A_1^{NCL}, A_4^{NCL}, A_7^{NCL}, 0^{NCL}\}, \quad \tau_2^{NCL} = \{0^{NCL}, A_3^{NCL}, A_4^{NCL}, A_5^{NCL}, 0^{NCL}\}.$$

Therefore, the number of NCLTs having 5-open sets is two.
Case III: If $M = \{(0, 1, 1), (T, I, F), (F, 1 - I, T), (1, 0, 0)\}; T, I, F \in [0, 1]$
In this case, $|\mathcal{N}_{\mathcal{X}}| = 4^2 = 16$. We will get no NCLT having 5-open sets. Therefore, the number of NCLT having 5-open sets is zero.

Proposition 3.8. *For* $|X| = n, |M| = m$, *M is the any finite set of neutrosophic values containing* $(1, 0, 0), (0, 1, 1)$

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and other neutrosophic values which are complement to each other, then number of NCLTs having m^n-open sets is one and for any other M it is zero.

Proof. In this case, the NT having m^n-open sets is discrete NT. This NCLT is also clopen as it contains all neutrosophic subsets and complement of each subset is also in that NT. Therefore, the number of NCLTS having m^n-open sets is one.
Further, it is found that any other M, with $|M| = m > 2$, is equivalent to $M = \{(1, 0, 0), (0, 1, 1)\}$ for NCLT. But for $M = \{(1, 0, 0), (0, 1, 1)\}$, the maximum number of open sets in NCLT is 2^n. Since $m^n \geq 2^n$, the number of NCLTS having m^n-open sets is zero. □

Corollary 3.9. *The minimum number of NOSs in a NCLT is 2 and the maximum number of NOSs in a NCLT is m^n, where n is the number of elements in $\mathcal{X}$ and m is the number of elements in M.*

Proof. This result is obtained by using Proposition 3.8. □

Proposition 3.10. *For $|\mathcal{X}| = n$, $M = \{(0, 1, 1), (T, I, F), (F, 1 - I, T), (1, 0, 0)\}$, where $T \neq F; T, I, F \in [0, 1]$, then the number of NCLTS having odd number of open sets is always zero.*

Proposition 3.11. *For $|\mathcal{X}| = n$, $M = \{(1, 0, 0), (0, 1, 1)\}$ and $\mathcal{N}_{\mathcal{X}}^{NCL}$ be the set of all NCLTSs on $\mathcal{X}$ whose membership values lies in M then*
(i) $\mathcal{N}_{\mathcal{X}}^{NCL}$ contains only NCLTs having 2^k-open sets where $k = 1, 2, 3, \ldots, n$.
(ii) Number of NCLTs having 2^k-open sets is $S(n, k), k = 1, 2, \ldots, n$.

Proof. Let, $|\mathcal{X}| = n$, $M = \{(1, 0, 0), (0, 1, 1)\}$ and η_k be the number of NCLTs having k-open sets.
Case I: If $|\mathcal{X}| = 1$ say, $\mathcal{X} = \{u_1\}$ whose neutrosophic values lies in M. Then $|\mathcal{N}_{\mathcal{X}}| = 2^1 = 2$. These are
$$0^{NCL} = \langle \tfrac{u_1}{(0,1,1)} \rangle, 1^{NCL} = \langle \tfrac{u_1}{(1,0,0)} \rangle.$$
In this case we will get only one NCLT which is the indiscrete NT i.e., $\tau_1^{NCL} = \{0^{NCL}, 1^{NCL}\}$. This shows that for $|\mathcal{X}| = 1$,
(a) there exists only NCLT having $2^n, n = 1$ open sets.
(b) there exists 1 NCLT having 2^1-open sets i.e., $\eta_{2^1} = S(1, 1)$.
Case II: If $|\mathcal{X}| = 2$ say, $\mathcal{X} = \{u_1, v_1\}$ whose neutrosophic values lies in M. Then $|\mathcal{N}_{\mathcal{X}}| = 2^2 = 4$. These are
$$0^{NCL} = \langle \tfrac{u_1}{(0,1,1)}, \tfrac{v_1}{(0,1,1)} \rangle, 1^{NCL} = \langle \tfrac{u_1}{(1,0,0)}, \tfrac{v_1}{(1,0,0)} \rangle,$$
$$A_1^{NCL} = \langle \tfrac{u_1}{(0,1,1)}, \tfrac{v_1}{(1,0,0)} \rangle, A_2^{NCL} = \langle \tfrac{u_1}{(1,0,0)}, \tfrac{v_1}{(0,1,1)} \rangle.$$
These NCLTs are
$$\tau_1^{NCL} = \{0^{NCL}, 1^{NCL}\}, \tau_2^{NCL} = \{0^{NCL}, A_1^{NCL}, A_2^{NCL}, 1^{NCL}\}.$$
This gives that the number of open sets in τ_1^{NCL} is 2 and in τ_2^{NCL} is $4 = 2^2$. Therefore, for $|X| = 2$,
(a) there exist only NCLTs having $2^n, n = 1, 2$ open sets i.e., τ_1^{NCL} and τ_2^{NCL}.
(b) there exists 1 NCLT having 2^1-open set and 1 NCLT having 2^2-open sets i.e., $\eta_{2^1} = 1 = S(2, 1)$ and $\eta_{2^2} = 1 = S(2, 2)$ respectively.
Case III: If $|\mathcal{X}| = 3$ say, $\mathcal{X} = \{u_1, v_1, w_1\}$ whose neutrosophic values lies in M. Then $|\mathcal{N}_{\mathcal{X}}| = 2^3 = 8$. These are
$$0^{NCL} = \langle \tfrac{u_1}{(0,1,1)}, \tfrac{v_1}{(0,1,1)}, \tfrac{w_1}{(0,1,1)} \rangle, 1^{NCL} = \langle \tfrac{u_1}{(1,0,0)}, \tfrac{v_1}{(1,0,0)}, \tfrac{w_1}{(1,0,0)} \rangle,$$
$$A_1^{NCL} = \langle \tfrac{u_1}{(1,0,0)}, \tfrac{v_1}{(0,1,1)}, \tfrac{w_1}{(0,1,1)} \rangle, A_2^{NCL} = \langle \tfrac{u_1}{(0,1,1)}, \tfrac{v_1}{(1,0,0)}, \tfrac{w_1}{(0,1,1)} \rangle,$$
$$A_3^{NCL} = \langle \tfrac{u_1}{(0,1,1)}, \tfrac{v_1}{(0,1,1)}, \tfrac{w_1}{(1,0,0)} \rangle, A_4^{NCL} = \langle \tfrac{u_1}{(1,0,0)}, \tfrac{v_1}{(1,0,0)}, \tfrac{w_1}{(0,1,1)} \rangle,$$
$$A_5^{NCL} = \langle \tfrac{u_1}{(1,0,0)}, \tfrac{v_1}{(0,1,1)}, \tfrac{w_1}{(1,0,0)} \rangle \; A_6^{NCL} = \langle \tfrac{u_1}{(0,1,1)}, \tfrac{v_1}{(1,0,0)}, \tfrac{w_1}{(1,0,0)} \rangle.$$
In this case the NCLTs are
$$\tau_1^{NCL} = \{0^{NCL}, 1^{NCL}\}, \tau_2^{NCL} = \{0^{NCL}, A_1^{NCL}, A_6^{NCL}, 1^{NCL}\},$$
$$\tau_3^{NCL} = \{0^{NCL}, A_2^{NCL}, A_5^{NCL}, 1^{NCL}\},$$
$$\tau_4^{NCL} = \{0^{NCL}, A_3^{NCL}, A_4^{NCL}, 1^{NCL}\},$$
$$\tau_5^{NCL} = \{0^{NCL}, A_1^{NCL}, A_2^{NCL}, A_3^{NCL}, A_4^{NCL}, A_5^{NCL}, A_6^{NCL}, 1^{NCL}\}.$$
This gives that the number of open sets in τ_1^{NCL} is 2, in $\tau_2^{NCL}, \tau_3^{NCL}, \tau_4^{NCL}$ is $4 = 2^2$, and in τ_6^{NCL} is $8 = 2^3$. Therefore, for $|\mathcal{X}| = 2$,
(a) there exist only NCLTs having $2^n, n = 1, 2, 3$ open sets.
(b) there exists 1 NCLT having 2^1-open set, 3 NCLT having 2^2-open sets and 1 NCLT having 2^3-open sets i.e., $\eta_{2^1} = 1 = S(3, 1), \eta_{2^2} = 3 = S(3, 2)$ and $\eta_{2^3} = 1 = S(3, 3)$ respectively.
Continuing in this way, it is seen that for $|\mathcal{X}| = n$ (finite), and

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$\mathcal{M} = \{(0,1,1),(1,0,0)\}$,
(i) there exist only NCLTs having $2^n, n = 1,2,3,\ldots,n$ open sets.
(ii) $\eta_{2^1} = 1 = S(n,1), \eta_{2^2} = S(n,2), \eta_{2^3} = S(n,3), \cdots, \eta_{2^n} = S(n,n)$.
Hence, $\mathcal{N}_{\mathcal{X}}^{NCL}$ contains only NCLTs having 2^k-open sets where $k = 1,2,3,\ldots,n.$ and number of NCLTs having 2^k-open sets is $S(n,k), k = 1,2,\ldots,n$. □

Table 1: Number of NCLTSs on $\mathcal{X}$

$M = \{(0,1,1),(1,0,0)\}$	Number of NCLTSs having k-open sets					
$\lvert\mathcal{X}\rvert$	$k = 2^1$	$k = 2^2$	$k = 2^3$	$k = 2^4$	$\ldots$	$k = 2^n$
1	$S(1,1)$	-	-	-	-	-
2	$S(2,1)$	$S(2,2)$	-	-	-	
3	$S(3,1)$	$S(3,2)$	$S(3,3)$	-	-	-
4	$S(4,1)$	$S(4,2)$	$S(4,3)$	$S(4,4)$	-	-
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	$S(n,1)$	$S(n,2)$	$S(n,3)$	$S(n,4)$	$\ldots$	$S(n,n)$

Proposition 3.12. *Let $\mathcal{X}$ be a finite set with $|\mathcal{X}| = n$ and $M = \{(0,1,1),(1,0,0),(0.5,0.5,0.5)\}$. Then the number of NCLTSs having 4-open sets is obtained by*
$t_n = 0 + 3.2^0 + 3.2^1 + 3.2^2 + 3.2^3 + \ldots + 3.2^{n-2} = 6.2^{n-2} - 3$, *where t_n is the sum of first n^{th} term and $t_1 = 0$.*

Proof. For $n = 1$, we have $t_1 = 0 = 6.2^{1-2} - 3$.
Let $\mathcal{X} = \{a\}$ then $|\mathcal{N}_{\mathcal{X}}| = 3^1 = 3$. So there is no NCLT having 4-open set.
Therefore, for $n = 1$ the result is true.
For $n = 2$, we have $t_2 = 0 + 3.2^0 = 3 = 6.2^{2-2} - 3$.
Let $\mathcal{X} = \{u_1, v_1\}$ then $|\mathcal{N}_{\mathcal{X}}| = 3^2 = 9$. In this case NCLTs having 4-open sets are

$$\tau_1^{NCL} = \{\langle \tfrac{u_1}{(0,1,1)}, \tfrac{v_1}{(0,1,1)} \rangle, \langle \tfrac{u_1}{(1,0,0)}, \tfrac{v_1}{(1,0,0)} \rangle, \langle \tfrac{u_1}{(0,1,1)}, \tfrac{v_1}{(0.5,0.5,0.5)} \rangle, \langle \tfrac{u_1}{(1,0,0)}, \tfrac{v_1}{(0.5,0.5,0.5)} \rangle\},$$
$$\tau_2^{NCL} = \{\langle \tfrac{u_1}{(0,1,1)}, \tfrac{v_1}{(0,1,1)} \rangle, \langle \tfrac{u_1}{(1,0,0)}, \tfrac{v_1}{(1,0,0)} \rangle, \langle \tfrac{u_1}{(0.5,0.5,0.5)}, \tfrac{v_1}{(0,1,1)} \rangle, \langle \tfrac{u_1}{(0.5,0.5,0.5)}, \tfrac{v_1}{(1,0,0)} \rangle\},$$
$$\tau_3^{NCL} = \{\langle \tfrac{u_1}{(0,1,1)}, \tfrac{v_1}{(0,1,1)} \rangle, \langle \tfrac{u_1}{(1,0,0)}, \tfrac{v_1}{(1,0,0)} \rangle, \langle \tfrac{u_1}{(0,1,1)}, \tfrac{v_1}{(1,0,0)} \rangle, \langle \tfrac{u_1}{(1,0,0)}, \tfrac{v_1}{(0.1,1)} \rangle\}.$$

Let us consider, the result is true for $n = k$ i.e., $t_k = 6.2^{k-2} - 3$.
We now try to prove the result for $n = k + 1$.
Therefore, $t_n = t_{k+1} = t_k + 3.2^{(k+1)-2}$
$= 6.2^{k-2} - 3 + 3.2^{k-1}$
$= 3.2^{k-2}(2 + 2) - 3$
$= 3.4.2^{k-2} - 3$
$= 6.2^{k-1} - 3$
$= 6.2^{(k+1)-2} - 3.$
Thus, for $n = k + 1$ the result is true. Hence, for all the natural number the result is true .

Table 2: Number of NCLTSs having 4-open sets on $\mathcal{X}$

	$M = \{(0,1,1),(0.5,0.5,0.5),(1,0,0)\}$						
$\lvert\mathcal{X}\rvert$	1	2	3	4	5	$\ldots$	n
Number of NCLT having 4-open sets	0	3	9	21	45	$\ldots$	$6.2^{n-2} - 3$

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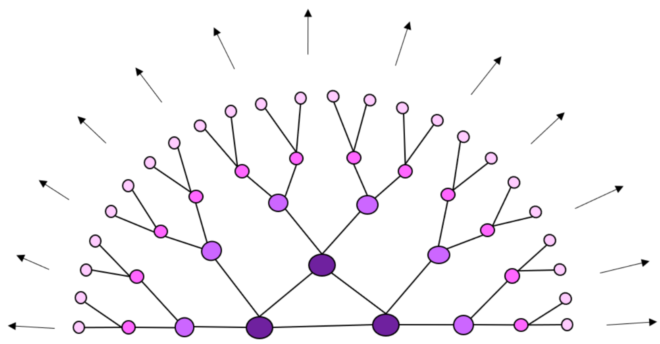

Figure 5: Representation of NCLTSs having 4-open sets on $\mathcal{X}$

□

Proposition 3.13. *Let $\mathcal{X}$ be a finite set, $|\mathcal{X}| = n$ and $M = \{(0,1,1),(1,0,0),(0.5,0.5,0.5)\}$. Then the number of NCLTs having 5-open sets is obtained by*

$t_n = 0 + 2^1 + 2^2 + 2^3 + \ldots + 2^{n-1} = 2^n - 2$, *where t_n is the sum of first n^{th} term and $t_1 = 0$.*

Proof. For $n = 1$, we have $t_1 = 0 = 2^1 - 2$.
Let $\mathcal{X} = \{a\}$ then $|\mathcal{N}_{\mathcal{X}}| = 3^1 = 3$. So there is no NCLT having 4-open set.
Therefore, for $n = 1$ the result is true.
For $n = 2$, we have $t_2 = 2^1 = 2 = 2^2 - 2$.
Let $X = \{u_1, v_1\}$ then $|\mathcal{N}_{\mathcal{X}}| = 3^2 = 9$. In this case NCLTs having 5-open sets are

$$\tau_1^{NCL} = \left\{ \begin{array}{c} \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0,1,1)} \rangle, \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(1,0,0)} \rangle, \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0.5,0.5,0.5)} \rangle, \\ \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(0.5,0.5,0.5)} \rangle, \langle \frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(0.5,0.5,0.5)} \rangle \end{array} \right\},$$

$$\tau_2^{NCL} = \left\{ \begin{array}{c} \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0,1,1)} \rangle, \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(1,0,0)} \rangle, \langle \frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(0,1,1)} \rangle, \\ \langle \frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(1,0,0)} \rangle, \langle \frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(0.5,0.5,0.5)} \rangle \end{array} \right\}.$$

Let us consider, the result is true for $n = k$ i.e., $t_k = 2^k - 2$.
We now try to prove the result for $n = k + 1$.
Therefore, $t_n = t_{k+1} = t_k + 2^{(k+1)-1} = 2^k - 2 + 2^k = 2.2^k - 2 = 2^{k+1} - 2$.
Hence, for $n = k + 1$ the result is true. So, for all the natural number the result is true. □

Table 3: NCLTSs having 5-open sets on $\mathcal{X}$

	$M = \{(0,1,1),(1,0,0),(0.5,0.5,0.5)\}$						
$\lvert\mathcal{X}\rvert$	1	2	3	4	5	...	n
Number of NCLT having 5-open sets	0	2	6	14	30	...	$2^n - 2$

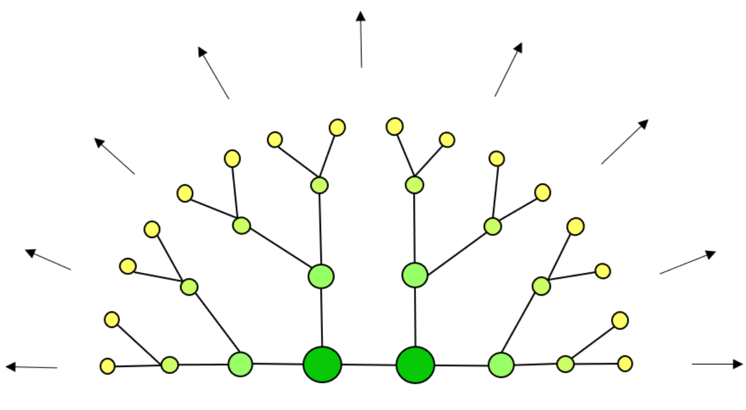

Figure 6: Representation of NCLTSs having 5-open sets on $\mathcal{X}$

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Proposition 3.14. *Let $\mathcal{X}$ be a finite set, $|\mathcal{X}| = n$ and $M = \{(0,1,1),(T,I,F),(1,0,0)\}, T = F; I = 0.5$. Then the number of NCLTs having*
(i) 4*-open sets is obtained by*
$t_n = 0 + 3.2^0 + 3.2^1 + 3.2^2 + 3.2^3 + \ldots + 3.2^{n-2} = 6.2^{n-2} - 3$ *and*
(ii) 5*-open sets is obtained by*
$t_n = 0 + 2^1 + 2^2 + 2^3 + \ldots + 2^{n-1} = 2^n - 2.$

Proof. Prove is straightforward. □

Proposition 3.15. *For $|\mathcal{X}| = n$(finite) and $M = \{(0,1,1),(1,0,0),(T,I,F),(F,1-I,T)\}$. Then the number of NCLTSs having* 4*-open sets is obtained by*
$t_n = 1 + 3.2^0 + 3.2^1 + 3.2^2 + \ldots + 3.2^{n-2} = 3.2^{n-1} - 2,$
where t_n is the sum of first n^{th} term and $t_1 = 1$.

Proof. For $n = 1$, we have $t_1 = 1 = 3.2^{1-1} - 2$.
Let $\mathcal{X} = \{u_1\}$ then $|\mathcal{N}_\mathcal{X}| = 4^1 = 4$. So there is one NCLT having 4-open set which is
$\{\langle\frac{u_1}{(0,1,1)}\rangle, \langle\frac{u_1}{(1,0,0)}\rangle, \langle\frac{u_1}{(T,I,F)}\rangle, \langle\frac{u_1}{(F,1-I,T)}\rangle\}$.
Therefore, for $n = 1$ the result is true .
For $n = 2$, we have $t_2 = 1 + 3.2^0 = 4 = 3.2^1 - 2$.
Let $\mathcal{X} = \{u_1, v_1\}$ and $M = \{(0,1,1),(1,0,0),(T,I,F),(F,1-I,T)\}$.
Then $|\mathcal{N}_\mathcal{X}| = 4^2 = 16$. In this case NCLTs having 4-open sets are

$$\tau_1^{NCL} = \{\langle\tfrac{u_1}{(0,1,1)}, \tfrac{v_1}{(0,1,1)}\rangle, \langle\tfrac{u_1}{(1,0,0)}, \tfrac{v_1}{(1,0,0)}\rangle, \langle\tfrac{u_1}{(0,1,1)}, \tfrac{v_1}{(T,I,F)}\rangle, \langle\tfrac{u_1}{(1,0,0)}, \tfrac{v_1}{(F,1-I,T)}\rangle\},$$
$$\tau_2^{NCL} = \{\langle\tfrac{u_1}{(0,1,1)}, \tfrac{v_1}{(0,1,1)}\rangle, \langle\tfrac{u_1}{(1,0,0)}, \tfrac{v_1}{(1,0,0)}\rangle, \langle\tfrac{u_1}{(T,I,F)}, \tfrac{v_1}{(T,I,F)}\rangle, \langle\tfrac{u_1}{(F,1-I,T)}, \tfrac{v_1}{(F,1-I,T)}\rangle\},$$
$$\tau_3^{NCL} = \{\langle\tfrac{u_1}{(0,1,1)}, \tfrac{v_1}{(0,1,1)}\rangle, \langle\tfrac{u_1}{(1,0,0)}, \tfrac{v_1}{(1,0,0)}\rangle, \langle\tfrac{v_1}{(0,1,1)}, \tfrac{v_1}{(1,0,0)}\rangle, \langle\tfrac{u_1}{(1,0,0)}, \tfrac{v_1}{(0,1,1)}\rangle\},$$
$$\tau_4^{NCL} = \{\langle\tfrac{u_1}{(0,1,1)}, \tfrac{v_1}{(0,1,1)}\rangle, \langle\tfrac{u_1}{(1,0,0)}, \tfrac{v_1}{(1,0,0)}\rangle, \langle\tfrac{u_1}{(T,I,F)}, \tfrac{v_1}{(0,1,1)}\rangle, \langle\tfrac{u_1}{(F,1-I,T)}, \tfrac{v_1}{(1,0,0)}\rangle\}.$$

Let us consider, for $n = k$ the result is true i.e., $t_k = 3.2^{k-1} - 2$.
We now try to prove the result for $n = k + 1$.
Therefore,
$t_n = t_{k+1} = t_k + 3.2^{(k+1)-2} = 3.2^{k-1} - 2 + 3.2^{k-1} = 2.3.2^{k-1} - 2 = 3.2^k - 2 = 3.2^{(k+1)-1} - 2$
Hence, for $n = k + 1$ the result is true. So, for all the natural number the result is true. □

Table 4: Number of NCLTSs having 4-open sets on $\mathcal{X}$

	$M = \{(0,1,1),(1,0,0),(T,I,F),(F,1-I,T)\}, I = 0.5$						
$\lvert\mathcal{X}\rvert$	1	2	3	4	5	...	n
Number of NCLT having 4-open sets	1	4	10	22	46	...	$3.2^{n-1} - 2$

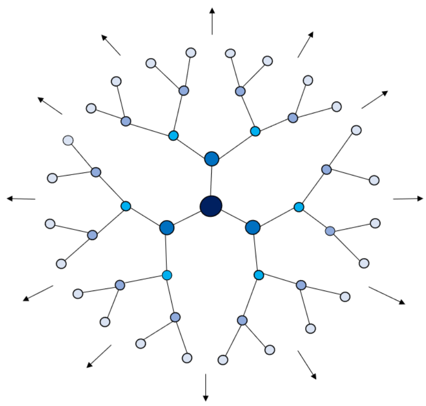

Figure 7: Representation of NCLTSs having 4-open sets on $\mathcal{X}$

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Remark 3.16. Results obtained for $k = 4$ and $k = 5$ are true for all M provided $I = 0.5, T = F$, and $T < F$, $I < 0.5$. Moreover, in this case, the results obtained for $I = 0.5$, $T = F$ coincides with results obtained for $M = \{(0,1,1),(1,0,0),(0.5,0.5,0.5)\}$ and $M = \{(0,1,1),(1,0,0),(T,0.5,F)\}, T = F$ and the results obtained for $T < F$ and $I < 0.5$ coincides with results obtained for $M = \{(0,1,1),(1,0,0)\}$.

Proposition 3.17. *For $n \geq m \geq 2$, the number of NCLTs having k-open set where $m^n - m^{n-2} < k < m^n$ is* 0.

4 Conclusion

In this paper, a number of formulae for finding the number of the NCLTSs are determined. Moreover, relevant propositions are observed, where we can draw logical pictures.

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