Certain notions of single-valued neutrosophic K-algebras

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Abstract. We apply the notion of single-valued neutrosophic sets to K-algebras. We develop the concept of single-valued neutrosophic K-subalgebras, and present some of their properties. Moreover, we study the behavior of single-valued neutrosophic K-subalgebras under homomorphism. Finally, we discuss $(\in, \in \vee q)$-single-valued neutrosophic K-algebras.

Keywords: Single-valued neutrosophic sets, K-algebras, homomorphism, $(\in, \in \vee q)$-single-valued neutrosophic K-algebras.

1. Introduction

A new kind of logical algebra, known as K-algebra, was introduced by Dar and Akram [9]. A K-algebra was built on a group G by adjoining the induced binary operation on G. The group G is particularly of the type in which each non-identity element is not of order 2. This algebraic structure is, in general, non-commutative and non-associative with right identity element [5, 10, 11]. Akram et.al [2, 3, 4] introduced fuzzy K-algebras. They then developed fuzzy K-algebras with other researchers worldwide. The concepts and results of K-algebras have been broadened to the fuzzy setting frames by applying Zadeh's fuzzy set theory and its generalizations, namely, interval- valued fuzzy sets, intuitionistic fuzzy sets, interval-valued intuitionistic fuzzy sets, bipolar fuzzy sets and vague sets.

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In handling information regarding various aspects of uncertainty, non-classical logic (a great extension and development of classical logic) is considered to be a more powerful technique than the classical logic. The non-classical logic has nowadays become a useful tool in computer science. Moreover, non-classical logic deals with fuzzy information and uncertainty. In 1998, Smarandache [15] introduced neutrosophic sets as a generalization of fuzzy sets [19] and intuitionistic fuzzy sets [6]. A neutrosophic set is identified by three functions called truth-membership (T), indeterminacy-membership (I) and falsity-membership (F) whose values are real standard or non-standard subset of unit interval $]^{-}0, 1^{+}[$, where $^{-}0 = 0 - \epsilon$, $1^{+} = 1 + \epsilon$, ϵ is an infinitesimal number. To apply neutrosophic set in real-life problems more conveniently, Smarandache [15] and Wang et al. [16] defined single-valued neutrosophic sets which takes the value from the subset of [0, 1]. Thus, a single-valued neutrosophic set is an instance of neutrosophic set, and can be used expediently to deal with real-world problems, especially in decision support. Algebraic structures have a vital place with vast applications in various disciplines. Neutrosophic set theory has been applied to algebraic structures [1, 8, 13]. In this research article, we introduce the notion of single-valued neutrosophic K-subalgebra and investigate some of their properties. We discuss K-subalgebra in terms of level sets using neutrosophic environment. We study the homomorphisms between the single-valued neutrosophic K-subalgebras. We discuss characteristic K-subalgebras and fully invariant K-subalgebras. Finally, we discuss $(\in, \in \vee q)$-single-valued neutrosophic K-algebras.

2. Single-valued neutrosophic K-algebras

The concept of K-algebra was first developed by Dar and Akram in [14].

Definition 2.1. Let $(G, \cdot, e)$ be a group in which each non-identity element is not of order 2. Then a K- *algebra* is a structure $\mathcal{K} = (G, \cdot, \odot, e)$ on a group G in which induced binary operation $\odot : G \times G \to G$ is defined by $\odot(x, y) = x \odot y = x.y^{-1}$ and satisfies the following axioms:

(i) $(x \odot y) \odot (x \odot z) = (x \odot ((e \odot z) \odot (e \odot y))) \odot x$,

(ii) $x \odot (x \odot y) = (x \odot (e \odot y)) \odot x$,

(iii) $x \odot x = e$,

(iv) $x \odot e = x$,

(v) $e \odot x = x^{-1}$,

for all x, y, $z \in G$.

Definition 2.2. [16] Let Z be a space of objects with a general element $z \in Z$. A single-valued neutrosophic set $\mathcal{A}$ in Z is characterized by three membership functions, $\mathcal{T}_{\mathcal{A}}$-truth membership function, $\mathcal{I}_{\mathcal{A}}$-indeterminacy membership function and $\mathcal{F}_{\mathcal{A}}$-falsity membership function, where $\mathcal{T}_{\mathcal{A}}(z), \mathcal{I}_{\mathcal{A}}(z), \mathcal{F}_{\mathcal{A}}(z) \in [0,1]$, for all $z \in Z$.

$\mathcal{A}$ can also be written as $\mathcal{A} = \{< z, \mathcal{T}_{\mathcal{A}}(z), \mathcal{I}_{\mathcal{A}}(z), \mathcal{F}_{\mathcal{A}}(z) > \ | \ z \in Z\}$.

Definition 2.3. A single-valued neutrosophic set $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ in a K-algebra $\mathcal{K}$ is called a single-valued neutrosophic K-subalgebra of $\mathcal{K}$ if it satisfies the following conditions:

(a) $\mathcal{T}_{\mathcal{A}}(s \odot t) \geq \min\{\mathcal{T}_{\mathcal{A}}(s), \mathcal{T}_{\mathcal{A}}(t)\}$,
(b) $\mathcal{I}_{\mathcal{A}}(s \odot t) \geq \min\{\mathcal{I}_{\mathcal{A}}(s), \mathcal{I}_{\mathcal{A}}(t)\}$,
(c) $\mathcal{F}_{\mathcal{A}}(s \odot t) \leq \max\{\mathcal{F}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(t)\}$, for all $s, t \in G$.

Note that $\mathcal{T}_{\mathcal{A}}(e) \geq \mathcal{T}_{\mathcal{A}}(s)$, $\mathcal{I}_{\mathcal{A}}(e) \geq \mathcal{I}_{\mathcal{A}}(s)$, $\mathcal{F}_{\mathcal{A}}(e) \leq \mathcal{F}_{\mathcal{A}}(s)$, for all $s \in G$.

Example 2.1. Let $\mathcal{K} = (G, \cdot, \odot, e)$ be a K-algebra, where $G = \{e, x, x^2, x^3, x^4, x^5, x^6, x^7, x^8\}$ is the cyclic group of order 9 and Cayley's table for $\odot$ is given as:

$\odot$	e	x	x^2	x^3	x^4	x^5	x^6	x^7	x^8
e	e	x^8	x^7	x^6	x^5	x^4	x^3	x^2	x
x	x	e	x^8	x^7	x^6	x^5	x^4	x^3	x^2
x^2	x^2	x	e	x^8	x^7	x^6	x^5	x^4	x^3
x^3	x^3	x^2	x	e	x^8	x^7	x^6	x^5	x^4
x^4	x^4	x^3	x^2	x	e	x^8	x^7	x^6	x^5
x^5	x^5	x^4	x^3	x^2	x	e	x^8	x^7	x^6
x^6	x^6	x^5	x^4	x^3	x^2	x	e	x^8	x^7
x^7	x^7	x^6	x^5	x^4	x^3	x^2	x	e	x^8
x^8	x^8	x^7	x^6	x^5	x^4	x^3	x^2	x	e

We define a single-valued neutrosophic set $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ in K-algebra as follows:

$\mathcal{T}_{\mathcal{A}}(e) = 0.8, \mathcal{I}_{\mathcal{A}}(e) = 0.7, \mathcal{F}_{\mathcal{A}}(e) = 0.4$,

$\mathcal{T}_{\mathcal{A}}(s) = 0.2, \mathcal{I}_{\mathcal{A}}(s) = 0.3, \mathcal{F}_{\mathcal{A}}(s) = 0.6$, for all $s \neq e \in G$.

Clearly, $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}$.

Example 2.2. Let $\mathcal{K} = (G, \cdot, \odot, e)$ be a K-algebra on dihedral group $D4$ given as $G = \{e, a, b, c, x, y, u, v\}$, where $c = ab, x = a^2, y = a^3$, $u = a^2b, v = a^3b$ and

Cayley's table for $\odot$ is given as:

$\odot$	e	a	b	c	x	y	u	v
e	e	y	b	c	x	a	u	v
a	a	e	c	u	y	x	v	b
b	b	c	e	y	u	v	x	a
c	c	u	a	e	v	b	y	x
x	x	a	u	v	e	y	b	c
y	y	x	v	b	a	e	c	u
u	u	v	x	a	b	c	e	y
v	v	b	y	x	c	u	a	e

We define a single-valued neutrosophic set $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ in K-algebra as follows: $\mathcal{T}_{\mathcal{A}}(e) = 0.9, \mathcal{I}_{\mathcal{A}}(e) = 0.3, \mathcal{F}_{\mathcal{A}}(e) = 0.3$, $\mathcal{T}_{\mathcal{A}}(s) = 0.6, \mathcal{I}_{\mathcal{A}}(s) = 0.2, \mathcal{F}_{\mathcal{A}}(s) = 0.4$, for all $s \neq e \in G$. By routine calculations, it can be verified that $\mathcal{A}$ is a single-valued neutrosophic K-subalgebra ok $\mathcal{K}$.

Proposition 2.1. If $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}$, then

1. $(\forall s, t \in G), (\mathcal{T}_{\mathcal{A}}(s \odot t) = \mathcal{T}_{\mathcal{A}}(t) \Rightarrow \mathcal{T}_{\mathcal{A}}(s) = \mathcal{T}_{\mathcal{A}}(e))$.
   $(\forall s, t \in G)(\mathcal{T}_{\mathcal{A}}(s) = \mathcal{T}_{\mathcal{A}}(e) \Rightarrow \mathcal{T}_{\mathcal{A}}(s \odot t) \geq \mathcal{T}_{\mathcal{A}}(t))$.
2. $(\forall s, t \in G), (\mathcal{I}_{\mathcal{A}}(s \odot t) = \mathcal{I}_{\mathcal{A}}(t) \Rightarrow \mathcal{I}_{\mathcal{A}}(s) = \mathcal{I}_{\mathcal{A}}(e))$.
   $(\forall s, t \in G)(\mathcal{I}_{\mathcal{A}}(s) = \mathcal{I}_{\mathcal{A}}(e) \Rightarrow \mathcal{I}_{\mathcal{A}}(s \odot t) \geq \mathcal{I}_{\mathcal{A}}(t))$.
3. $(\forall s, t \in G), (\mathcal{F}_{\mathcal{A}}(s \odot t) = \mathcal{F}_{\mathcal{A}}(t) \Rightarrow \mathcal{F}_{\mathcal{A}}(s) = \mathcal{F}_{\mathcal{A}}(e))$.
   $(\forall s, t \in G)(\mathcal{F}_{\mathcal{A}}(s) = \mathcal{F}_{\mathcal{A}}(e) \Rightarrow \mathcal{F}_{\mathcal{A}}(s \odot t) \leq \mathcal{F}_{\mathcal{A}}(t))$.

Proof. 1. Assume that $\mathcal{T}_{\mathcal{A}}(s \odot t) = \mathcal{T}_{\mathcal{A}}(t)$, for all $s, t \in G$. Taking $t = e$ and using (iii) of Definition 2.1, we have $\mathcal{T}_{\mathcal{A}}(s) = \mathcal{T}_{\mathcal{A}}(s \odot e) = \mathcal{T}_{\mathcal{A}}(e)$. Let for $s, t \in G$ be such that $\mathcal{T}_{\mathcal{A}}(s) = \mathcal{T}_{\mathcal{A}}(e)$. Then $\mathcal{T}_{\mathcal{A}}(s \odot t) \geq \min\{\mathcal{T}_{\mathcal{A}}(s), \mathcal{T}_{\mathcal{A}}(t)\} = \min\{\mathcal{T}_{\mathcal{A}}(e), \mathcal{T}_{\mathcal{A}}(t)\} = \mathcal{T}_{\mathcal{A}}(t)$.

2. Assume that $\mathcal{I}_{\mathcal{A}}(s \odot t) = \mathcal{I}_{\mathcal{A}}(t)$, for all $s, t \in G$. Taking $t = e$ and by (iii) of Definition 2.1, we have $\mathcal{I}_{\mathcal{A}}(s) = \mathcal{I}_{\mathcal{A}}(s \odot e) = \mathcal{I}_{\mathcal{A}}(e)$. Also let $s, t \in G$ be such that $\mathcal{I}_{\mathcal{A}}(s) = \mathcal{I}_{\mathcal{A}}(e)$. Then $\mathcal{I}_{\mathcal{A}}(s \odot t) \geq \min\{\mathcal{I}_{\mathcal{A}}(s), I_(t)\} = \min\{\mathcal{I}_{\mathcal{A}}(e), \mathcal{I}_{\mathcal{A}}(t)\} = \mathcal{I}_{\mathcal{A}}(t)$.

3. Consider that $\mathcal{F}_{\mathcal{A}}(s \odot t) = \mathcal{F}_{\mathcal{A}}(t)$, for all $s, t \in G$. Taking $t = e$ and again by (iii) of Definition 2.1, we have $\mathcal{F}_{\mathcal{A}}(s) = \mathcal{F}_{\mathcal{A}}(s \odot e) = \mathcal{F}_{\mathcal{A}}(e)$. Let $s, t \in G$ be such that $\mathcal{F}_{\mathcal{A}}(s) = F_(e)$. Then $\mathcal{F}_{\mathcal{A}}(s \odot t) \leq \max\{\mathcal{F}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(t)\} = \max\{\mathcal{F}_{\mathcal{A}}(e), \mathcal{F}_{\mathcal{A}}(t)\} = \mathcal{F}_{\mathcal{A}}(t)$.

   This completes the proof.

□

Definition 2.4. Let $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic set in a K-algebra $\mathcal{K}$ and let $(\alpha, \beta, \gamma) \in [0,1] \times [0,1] \times [0,1]$ with $\alpha + \beta + \gamma \leq 3$. Then level subsets of $\mathcal{A}$ are defined as:

$$\begin{aligned}
&\mathcal{A}_{(\alpha,\beta,\gamma)} = \{s \in G \mid \mathcal{T}_{\mathcal{A}}(s) \geq \alpha, \mathcal{I}_{\mathcal{A}}(s) \geq \beta, \mathcal{F}_{\mathcal{A}}(s) \leq \gamma\},\\
&\mathcal{A}_{(\alpha,\beta,\gamma)} = \{s \in G \mid \mathcal{T}_{\mathcal{A}}(s) \geq \alpha\} \cap \{s \in G \mid \mathcal{I}_{\mathcal{A}}(s) \geq \beta\} \cap \{s \in G \mid \mathcal{F}_{\mathcal{A}}(s) \leq \gamma\},\\
&\mathcal{A}_{(\alpha,\beta,\gamma)} = \cup(\mathcal{T}_{\mathcal{A}}, \alpha) \cap \cup^{'}(\mathcal{I}_{\mathcal{A}}, \beta) \cap L(\mathcal{F}_{\mathcal{A}}, \gamma)
\end{aligned}$$

are called (α, β, γ) -level subsets of single-valued neutrosophic set $\mathcal{A}$.

The set of all $(\alpha, \beta, \gamma) \in \mathrm{Im}(\mathcal{T}_{\mathcal{A}}) \times \mathrm{Im}(\mathcal{I}_{\mathcal{A}}) \times \mathrm{Im}(\mathcal{I}_{\mathcal{A}})$ is known as image of $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$. The set $\mathcal{A}_{(\alpha,\beta,\gamma)} = \{s \in G \mid \mathcal{T}_{\mathcal{A}}(s) > \alpha, \mathcal{I}_{\mathcal{A}}(s) > \beta, \mathcal{F}_{\mathcal{A}}(s) < \gamma\}$ is known as strong (α, β, γ)- level subset of $\mathcal{A}$.

Proposition 2.2. If $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}$, then the level subsets $\cup(\mathcal{T}_{\mathcal{A}}, \alpha) = \{s \in G \mid \mathcal{T}_{\mathcal{A}}(s) \geq \alpha\}$, $\cup^{'}(\mathcal{I}_{\mathcal{A}}, \beta) = \{s \in G \mid \mathcal{I}_{\mathcal{A}}(s) \geq \beta\}$ and $L(\mathcal{F}_{\mathcal{A}}, \gamma) = \{s \in G \mid \mathcal{F}_{\mathcal{A}}(s) \leq \gamma\}$ are k-subalgebras of $\mathcal{K}$, for every $(\alpha, \beta, \gamma) \in \mathrm{Im}(\mathcal{T}_{\mathcal{A}}) \times \mathrm{Im}(\mathcal{I}_{\mathcal{A}}) \times \mathrm{Im}(\mathcal{F}_{\mathcal{A}}) \subseteq [0,1]$, where $\mathrm{Im}(\mathcal{T}_{\mathcal{A}})$, $\mathrm{Im}(\mathcal{I}_{\mathcal{A}})$ and $\mathrm{Im}(\mathcal{F}_{\mathcal{A}})$ are sets of values of $T_(\mathcal{A})$, $\mathcal{I}_(\mathcal{A})$ and $F_(\mathcal{A})$, respectively.

Proof. Assume that $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}$ and let $(\alpha, \beta, \gamma) \in \mathrm{Im}(\mathcal{T}_{\mathcal{A}}) \times \mathrm{Im}(\mathcal{I}_{\mathcal{A}}) \times \mathrm{Im}(\mathcal{F}_{\mathcal{A}})$ be such that $\cup(\mathcal{T}_{\mathcal{A}}, \alpha) \neq \emptyset, \cup^{'}(\mathcal{I}_{\mathcal{A}}, \beta) \neq \emptyset$ and $L(\mathcal{F}_{\mathcal{A}}, \gamma) \neq \emptyset$. Now to prove that $\cup, \cup^{'}$ and L are level K-subalgebras. Let for $s, t \in \cup(\mathcal{T}_{\mathcal{A}}, \alpha)$, $\mathcal{T}_{\mathcal{A}}(s) \geq \alpha$ and $\mathcal{T}_{\mathcal{A}}(t) \geq \alpha$. It follows from Definition 2.3 that $\mathcal{T}_{\mathcal{A}}(s \odot t) \geq \min\{\mathcal{T}_{\mathcal{A}}(s), \mathcal{T}_{\mathcal{A}}(t)\} \geq \alpha$. It implies that $s \odot t \in \cup(\mathcal{T}_{\mathcal{A}}, \alpha)$. Hence $\cup(\mathcal{T}_{\mathcal{A}}, \alpha)$ is a level K-subalgebra of $\mathcal{K}$. Similar result can be proved for $\cup^{'}(\mathcal{I}_{\mathcal{A}}, \beta)$ and $L(\mathcal{F}_{\mathcal{A}}, \gamma)$. □

Theorem 2.1. Let $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic set in K-algebra $\mathcal{K}$. Then $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}$ if and only if $\mathcal{A}_{(\alpha,\beta,\gamma)}$ is a K-sublagebra of $\mathcal{K}$, for every $(\alpha, \beta, \gamma) \in \mathrm{Im}(\mathcal{T}_{\mathcal{A}}) \times \mathrm{Im}(\mathcal{I}_{\mathcal{A}}) \times \mathrm{Im}(\mathcal{F}_{\mathcal{A}})$ with $\alpha + \beta + \gamma \leq 3$.

Proof. Let $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic set in a K-algebra $\mathcal{K}$. Assume that $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic K-subalgebra of $\mathcal{K}$.

Let for $(\alpha, \beta, \gamma) \in \mathrm{Im}(\mathcal{T}_{\mathcal{A}}) \times \mathrm{Im}(\mathcal{I}_{\mathcal{A}}) \times \mathrm{Im}(\mathcal{F}_{\mathcal{A}})$ with $\alpha + \beta + \gamma \leq 3$ be such that $\mathcal{A}_{(\alpha,\beta,\gamma)} \neq \emptyset$. Let $s, t \in \mathcal{A}_{(\alpha,\beta,\gamma)}$ be such that

$$\begin{aligned}
&\mathcal{T}_{\mathcal{A}}(s) \geq \alpha, \mathcal{T}_{\mathcal{A}}(t) \geq \alpha^{'},\\
&\mathcal{I}_{\mathcal{A}}(s) \geq \beta, \mathcal{I}_{\mathcal{A}}(t) \geq \beta^{'},\\
&\mathcal{F}_{\mathcal{A}}(s) \leq \gamma, \mathcal{F}_{\mathcal{A}}(t) \leq \gamma^{'}.
\end{aligned}$$

Without loss of generality we can assume that $\alpha \leq \alpha'$, $\beta \leq \beta'$ and $\gamma \geq \gamma'$. It follows from Definition 2.3 that

$$\begin{aligned}\mathcal{T}_{\mathcal{A}}(s \odot t) &\geq \alpha = \min\{\mathcal{T}_{\mathcal{A}}(s), \mathcal{T}_{\mathcal{A}}(t)\},\\ \mathcal{I}_{\mathcal{A}}(s \odot t) &\geq \beta = \min\{\mathcal{I}_{\mathcal{A}}(s), \mathcal{I}_{\mathcal{A}}(t)\},\\ \mathcal{F}_{\mathcal{A}}(s \odot t) &\leq \gamma = \max\{\mathcal{F}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(t)\}.\end{aligned}$$

It implies that $s \odot t \in \mathcal{A}_{(\alpha,\beta,\gamma)}$. So, $\mathcal{A}_{(\alpha,\beta,\gamma)}$ is a K-subalgebra of $\mathcal{K}$.

Conversely, we suppose that $\mathcal{A}_{(\alpha,\beta,\gamma)}$ is a K-subalgebra of $\mathcal{K}$. If the condition of the Definition 2.3 is not true, then there exist $u, v \in G$ such that

$$\begin{aligned}\mathcal{T}_{\mathcal{A}}(u \odot v) &< \min\ \{\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v)\},\\ \mathcal{I}_{\mathcal{A}}(u \odot v) &< \min\ \{\mathcal{I}_{\mathcal{A}}(u), \mathcal{I}_{\mathcal{A}}(v)\},\\ \mathcal{F}_{\mathcal{A}}(u \odot v) &> \max\{\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v)\}.\end{aligned}$$

Taking $\alpha_1 = \frac{1}{2}(\mathcal{T}_{\mathcal{A}}(u \odot v) + \min\{\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v)\})$, $\beta_1 = \frac{1}{2}(\mathcal{I}_{\mathcal{A}}(u \odot v) + \min\{\mathcal{I}_{\mathcal{A}}(u), \mathcal{I}_{\mathcal{A}}(v)\})$, $\gamma_1 = \frac{1}{2}(\mathcal{F}_{\mathcal{A}}(u \odot v) + \min\{\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v)\})$.

We have $\mathcal{T}_{\mathcal{A}}(u \odot v) < \alpha_1 < \min\{\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v)\}$, $\mathcal{I}_{\mathcal{A}}(u \odot v) < \beta_1 < \min\{\mathcal{I}_{\mathcal{A}}(u), \mathcal{I}_{\mathcal{A}}(v)\}$ and $\mathcal{F}_{\mathcal{A}}(u \odot v) > \gamma_1 > \max\{\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v)\}$. It implies that $u, v \in \mathcal{A}_{(\alpha,\beta,\gamma)}$ and $u \odot v \notin \mathcal{A}_{(\alpha,\beta,\gamma)}$, a contradiction. Therefore, the condition of Definition 2.3 is true. Hence $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic k-subalgebra of $\mathcal{K}$. □

Theorem 2.2. Let $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic k-subalgebra and $(\alpha_1, \beta_1, \gamma_1), (\alpha_2, \beta_2, \gamma_2) \in \mathrm{Im}(\mathcal{T}_{\mathcal{A}}) \times \mathrm{Im}(\mathcal{I}_{\mathcal{A}}) \times \mathrm{Im}(\mathcal{F}_{\mathcal{A}})$ with $\alpha_j + \beta_j + \gamma_j \leq 3$ for $j = 1, 2$. Then $\mathcal{A}_{(\alpha_1,\beta_1,\gamma_1)} = \mathcal{A}_{(\alpha_2,\beta_2,\gamma_2)}$ if $(\alpha_1, \beta_1, \gamma_1) = (\alpha_2, \beta_2, \gamma_2)$.

Proof. If $(\alpha_1, \beta_1, \gamma_1) = (\alpha_2, \beta_2, \gamma_2)$, then clearly $\mathcal{A}_{(\alpha_1,\beta_1,\gamma_1)} = \mathcal{A}_{(\alpha_2,\beta_2,\gamma_2)}$.

Assume that $\mathcal{A}_{(\alpha_1,\beta_1,\gamma_1)} = \mathcal{A}_{(\alpha_2,\beta_2,\gamma_2)}$. Since $(\alpha_1, \beta_1, \gamma_1) \in \mathrm{Im}(\mathcal{T}_{\mathcal{A}}) \times \mathrm{Im}(\mathcal{I}_{\mathcal{A}}) \times \mathrm{Im}(\mathcal{F}_{\mathcal{A}})$, there exists $s \in G$ such that $\mathcal{T}_{\mathcal{A}}(s) = \alpha_1, \mathcal{I}_{\mathcal{A}}(s) = \beta_1$ and $\mathcal{F}_{\mathcal{A}}(s) = \gamma_1$. It follows that $s \in \mathcal{A}_{(\alpha_1,\beta_1,\gamma_1)} = \mathcal{A}_{(\alpha_2,\beta_2,\gamma_2)}$. So that $\alpha_1 = \mathcal{T}_{\mathcal{A}}(s) \geq \alpha_2, \beta_1 = \mathcal{I}_{\mathcal{A}}(s) \geq \beta_2$ and $\gamma_1 = \mathcal{F}_{\mathcal{A}}(s) \leq \gamma_2$. Also $(\alpha_2, \beta_2, \gamma_2) \in \mathrm{Im}(\mathcal{T}_{\mathcal{A}}) \times \mathrm{Im}(\mathcal{I}_{\mathcal{A}}) \times \mathrm{Im}(\mathcal{F}_{\mathcal{A}})$, there exists $t \in G$ such that $\mathcal{T}_{\mathcal{A}}(t) = \alpha_2, \mathcal{I}_{\mathcal{A}}(t) = \beta_2$ and $\mathcal{F}_{\mathcal{A}}(t) = \gamma_2$. It follows that $t \in \mathcal{A}_{(\alpha_2,\beta_2,\gamma_2)} = \mathcal{A}_{(\alpha_1,\beta_1,\gamma_1)}$. So that $\alpha_2 = \mathcal{T}_{\mathcal{A}}(t) \geq \alpha_1$, $\beta_2 = \mathcal{I}_{\mathcal{A}}(t) \geq \beta_1$ and $\gamma_2 = \mathcal{F}_{\mathcal{A}}(t) \leq \gamma_1$. Hence $(\alpha_1, \beta_1, \gamma_1) = (\alpha_2, \beta_2, \gamma_2)$. □

Theorem 2.3. Let H be a K-subalgebra of K-algebra $\mathcal{K}$. Then there exists a single-valued neutrosophic K-subalgebra $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ of K-algebra $\mathcal{K}$ such that $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}}) = H$, for some $\alpha, \beta \in (0, 1], \gamma \in [0, 1)$.

Proof. Let $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic set in K-algebra $\mathcal{K}$ given by

$$\mathcal{T}_{\mathcal{A}}(s) = \begin{cases} \alpha \in (0, 1], & \text{if } s \in H,\\ 0, & \text{otherwise.}\end{cases}$$

$$\mathcal{I}_{\mathcal{A}}(s)=\begin{cases}\beta\in(0,1], & \text{if } s\in H,\\ 0, & \text{otherwise,}\end{cases}, \quad \mathcal{F}_{\mathcal{A}}(s)=\begin{cases}\gamma\in[0,1), & \text{if } s\in H,\\ 0, & \text{otherwise.}\end{cases}$$

Let $s,t\in G$. If $s,t\in H$, then $s\odot t\in H$ and so

$\mathcal{T}_{\mathcal{A}}(s\odot t)\geq\min\{\mathcal{T}_{\mathcal{A}}(s),\mathcal{T}_{\mathcal{A}}(t)\}$, $\mathcal{I}_{\mathcal{A}}(s\odot t)\geq\min\{\mathcal{I}_{\mathcal{A}}(s),\mathcal{I}_{\mathcal{A}}(t)\}$, $\mathcal{F}_{\mathcal{A}}(s\odot t)\leq\max\{\mathcal{F}_{\mathcal{A}}(s),\mathcal{F}_{\mathcal{A}}(t)\}$. But if $s\notin H$ or $t\notin H$, then $\mathcal{T}_{\mathcal{A}}(s)=0$ or $\mathcal{T}_{\mathcal{A}}(t),\mathcal{I}_{\mathcal{A}}(s)=0$ or $\mathcal{I}_{\mathcal{A}}(t)$ and $\mathcal{F}_{\mathcal{A}}(s)=0$ or $\mathcal{F}_{\mathcal{A}}(t)$. It follows that $\mathcal{T}_{\mathcal{A}}(s\odot t)\geq\min\{\mathcal{T}_{\mathcal{A}}(s),\mathcal{T}_{\mathcal{A}}(t)\}$, $\mathcal{I}_{\mathcal{A}}(s\odot t)\geq\min\{\mathcal{I}_{\mathcal{A}}(s),\mathcal{I}_{\mathcal{A}}(t)\}$, $\mathcal{F}_{\mathcal{A}}(s\odot t)\leq\max\{\mathcal{F}_{\mathcal{A}}(s),\mathcal{F}_{\mathcal{A}}(t)\}$. Hence $\mathcal{A}=(\mathcal{T}_{\mathcal{A}},\mathcal{I}_{\mathcal{A}},\mathcal{F}_{\mathcal{A}})$ is a SVN K-subalgebra of $\mathcal{K}$. Consequently $\mathcal{A}_{(\alpha,\beta,\gamma)}=H$. □

The following Theorem shows that any K-subalgebra of $\mathcal{K}$ can be perceived as a level K-subalgebra of some single-valued neutrosophic K-subalgebras of $\mathcal{K}$.

Theorem 2.4. Let $\mathcal{K}$ be a K-algebra. Given a chain of K-subalgebras: $\mathcal{A}_0\subset\mathcal{A}_1\subset\mathcal{A}_2\subset\ldots\subset A_n=G$. Then there exists a single-valued neutrosophic K-subalgebra whose level K-subalgebras are exactly the K-subalgebras in this chain.

Proof. Let $\{\alpha_k\mid k=0,1,...,n\},\{\beta_k\mid k=0,1,...,n\}$ be finite decreasing sequences and $\{\gamma_k\mid k=0,1,...,n\}$ be finite increasing sequence in $[0,1]$ such that $\alpha_i+\beta_i+\gamma_i\leq 3$, for $i=0,1,2,...,n$. Let $\mathcal{A}=(\mathcal{T}_{\mathcal{A}},\mathcal{I}_{\mathcal{A}},\mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic set in $\mathcal{K}$ defined by $\mathcal{T}_{\mathcal{A}}(\mathcal{A}_0)=\alpha_0,\mathcal{I}_{\mathcal{A}}(\mathcal{A}_0)=\beta_0,\mathcal{F}_{\mathcal{A}}(\mathcal{A}_0)=\gamma_0$, $\mathcal{T}_{\mathcal{A}}(\mathcal{A}_k\setminus\mathcal{A}_{k-1})=\alpha_k,\mathcal{I}_{\mathcal{A}}(\mathcal{A}_k\setminus\mathcal{A}_{k-1})=\beta_k$ and $\mathcal{F}_{\mathcal{A}}(\mathcal{A}_k\setminus\mathcal{A}_{k-1})=\gamma_k$, for $0<k\leq n$. We claim that $\mathcal{A}=(\mathcal{T}_{\mathcal{A}},\mathcal{I}_{\mathcal{A}},\mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}$. Let $s,t\in G$. If $s,t\in\mathcal{A}_k\setminus\mathcal{A}_{k-1}$, then it implies that $\mathcal{T}_{\mathcal{A}}(s)=\alpha_k=\mathcal{T}_{\mathcal{A}}(t),\mathcal{I}_{\mathcal{A}}(s)=\beta_k=\mathcal{I}_{\mathcal{A}}(t)$ and $\mathcal{F}_{\mathcal{A}}(s)=\gamma_k=\mathcal{F}_{\mathcal{A}}(t)$. Since each $\mathcal{A}_k$ is a K-subalgebra, it follows that $s\odot t\in\mathcal{A}_k$. So that either $s\odot t\in\mathcal{A}_k\setminus\mathcal{A}_{k-1}$ or $s\odot t\in\mathcal{A}_{k-1}$. In any case, we conclude that

$$\begin{aligned}\mathcal{T}_{\mathcal{A}}(s\odot t)&\geq\alpha_k=\min\{\mathcal{T}_{\mathcal{A}}(s),\mathcal{T}_{\mathcal{A}}(t)\},\\ \mathcal{I}_{\mathcal{A}}(s\odot t)&\geq\beta_k=\min\{\mathcal{I}_{\mathcal{A}}(s),\mathcal{I}_{\mathcal{A}}(t)\},\\ \mathcal{F}_{\mathcal{A}}(s\odot t)&\leq\gamma_k=\max\{\mathcal{F}_{\mathcal{A}}(s),\mathcal{F}_{\mathcal{A}}(t)\}.\end{aligned}$$

For $i>j$, if $s\in\mathcal{A}_i\setminus\mathcal{A}_{i-1}$ and $t\in\mathcal{A}_j\setminus\mathcal{A}_{j-1}$, then $\mathcal{T}_{\mathcal{A}}(s)=\alpha_i$, $\mathcal{T}_{\mathcal{A}}(t)=\alpha_j,\mathcal{I}_{\mathcal{A}}(s)=\beta_i,\mathcal{I}_{\mathcal{A}}(t)=\beta_j$ and $\mathcal{F}_{\mathcal{A}}(s)=\gamma_i,\mathcal{F}_{\mathcal{A}}(t)=\gamma_j$ and $s\odot t\in\mathcal{A}_i$ because $\mathcal{A}_i$ is a K-subalgebra and $\mathcal{A}_j\subset\mathcal{A}_i$. It follows that

$$\begin{aligned}\mathcal{T}_{\mathcal{A}}(s\odot t)&\geq\alpha_i=\min\{\mathcal{T}_{\mathcal{A}}(s),\mathcal{T}_{\mathcal{A}}(t)\},\\ \mathcal{I}_{\mathcal{A}}(s\odot t)&\geq\beta_i=\min\{\mathcal{I}_{\mathcal{A}}(s),\mathcal{I}_{\mathcal{A}}(t)\},\\ \mathcal{F}_{\mathcal{A}}(s\odot t)&\leq\gamma_i=\max\{\mathcal{F}_{\mathcal{A}}(s),\mathcal{F}_{\mathcal{A}}(t)\}.\end{aligned}$$

Thus, $\mathcal{A}=(\mathcal{T}_{\mathcal{A}},\mathcal{I}_{\mathcal{A}},\mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}$ and all its non empty level subsets are level K-subalgebras of $\mathcal{K}$. Since $\mathrm{Im}(\mathcal{T}_{\mathcal{A}})=\{\alpha_0,\alpha_1,...,\alpha_{\mathrm{n}}\},\mathrm{Im}(\mathcal{I}_{\mathcal{A}})=\{\beta_0,\beta_1,...,\beta_{\mathrm{n}}\}$, $\mathrm{Im}(\mathcal{F}_{\mathcal{A}})=\{\gamma_0,\gamma_1,...,\gamma_{\mathrm{n}}\}$. Therefore, the level K-subalgebras of $\mathcal{A} = (\mathcal{T}_\mathcal{A}, \mathcal{I}_\mathcal{A}, \mathcal{F}_\mathcal{A})$ are given by the chain of K-subalgebras:

$$\begin{aligned}
\cup(\mathcal{T}_\mathcal{A}, \alpha_0) \subset \cup(\mathcal{T}_\mathcal{A}, \alpha_1) \subset ... \subset \cup(\mathcal{T}_\mathcal{A}, \alpha_n) = G,\\
\cup^{'}(\mathcal{I}_\mathcal{A}, \beta_0) \subset \cup^{'}(\mathcal{I}_\mathcal{A}, \beta_1) \subset ... \subset \cup^{'}(\mathcal{I}_\mathcal{A}, \beta_n) = G,\\
\mathrm{L}(\mathcal{F}_\mathcal{A}, \gamma_0) \subset \mathrm{L}(\mathcal{F}_\mathcal{A}, \gamma_1) \subset ... \subset \mathrm{L}(\mathcal{F}_\mathcal{A}, \gamma_n) = G,
\end{aligned}$$

respectively. Indeed,

$$\begin{aligned}
\cup(\mathcal{T}_\mathcal{A}, \alpha_0) = \{s \in G \mid \mathcal{T}_\mathcal{A}(s) \geq \alpha_0\} = \mathcal{A}_0,\\
\cup^{'}(\mathcal{I}_\mathcal{A}, \beta_0) = \{s \in G \mid \mathcal{I}_\mathcal{A}(s) \geq \beta_0\} = \mathcal{A}_0,\\
\mathrm{L}(\mathcal{F}_\mathcal{A}, \gamma_0) = \{s \in G \mid \mathcal{F}_\mathcal{A}(s) \leq \gamma_0\} = \mathcal{A}_0.
\end{aligned}$$

Now we prove that $\cup(\mathcal{T}_\mathcal{A}, \alpha_k) = \mathcal{A}_k, \cup^{'}(\mathcal{I}_\mathcal{A}, \beta_k) = \mathcal{A}_k$ and $L(\mathcal{F}_\mathcal{A}, \gamma_k) = \mathcal{A}_k$, for $0 < k \leq n$. Clearly, $\mathcal{A}_k \subseteq \cup(\mathcal{T}_\mathcal{A}, \alpha_k), \mathcal{A}_k \subseteq \cup^{'}(\mathcal{I}_\mathcal{A}, \beta_k)$ and $\mathcal{A}_k \subseteq L(\mathcal{F}_\mathcal{A}, \gamma_k)$. If $s \in \cup(\mathcal{T}_\mathcal{A}, \alpha_k)$, then $\mathcal{T}_\mathcal{A}(s) \geq \alpha_k$ and so $s \notin \mathcal{A}_i$, for $i > k$.

Hence $\mathcal{T}_\mathcal{A}(s) \in \{\alpha_0, \alpha_1, ..., \alpha_k\}$ which implies that $s \in \mathcal{A}_i$, for some $i \leq k$ since $\mathcal{A}_i \subseteq \mathcal{A}_k$. It follows that $s \in \mathcal{A}_k$. Consequently, $\cup(\mathcal{T}_\mathcal{A}, \alpha_k) = \mathcal{A}_k$ for some $0 < k \leq n$. Similar case can be proved for $\cup^{'}(\mathcal{I}_\mathcal{A}, \beta_k) = \mathcal{A}_k$. Now if $t \in L(\mathcal{F}_\mathcal{A}, \gamma_k)$, then $\mathcal{F}_\mathcal{A}(s) \leq \gamma_k$ and so $t \notin \mathcal{A}_i$, for some $j \leq k$.Thus, $\mathcal{F}_\mathcal{A}(s) \in \{\gamma_0, \gamma_1, ..., \gamma_k\}$ which implies that $s \in \mathcal{A}_j$, for some $j \leq k$. Since $\mathcal{A}_j \subseteq \mathcal{A}_k$. It follows that $t \in \mathcal{A}_k$. Consequently, $L(\mathcal{F}_\mathcal{A}, \gamma_k) = \mathcal{A}_k$, for some $0 < k \leq n$. Hence the proof. □

2.1 Homomorphism of single-valued neutrosophic K-algebras

Definition 2.5. Let $\mathcal{K}_1 = (G_1, \cdot, \odot, e_1)$ and $\mathcal{K}_2 = (G_2, \cdot, \odot, e_2)$ be two K-algebras and let ϕ be a function from $\mathcal{K}_1$ into $\mathcal{K}_2$. If $\mathcal{B} = (\mathcal{T}_\mathcal{B}, \mathcal{I}_\mathcal{B}, \mathcal{F}_\mathcal{B})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_2$, then the *preimage* of $\mathcal{B} = (\mathcal{T}_\mathcal{B}, \mathcal{I}_\mathcal{B}, \mathcal{F}_\mathcal{B})$ under ϕ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_1$ defined by $\phi^{-1}(\mathcal{T}_\mathcal{B})(s) = \mathcal{T}_\mathcal{B}(\phi(s))$, $\phi^{-1}(\mathcal{I}_\mathcal{B})(s) = \mathcal{I}_\mathcal{B}(\phi(s))$ and $\phi^{-1}(\mathcal{F}_\mathcal{B})(s) = \mathcal{F}_\mathcal{B}(\phi(s))$, for all $s \in G_1$.

Theorem 2.5. Let $\phi : \mathcal{K}_1 \to \mathcal{K}_2$ be an epimorphism of K-algebras. If $\mathcal{B} = (\mathcal{T}_\mathcal{B}, \mathcal{I}_\mathcal{B}, \mathcal{F}_\mathcal{B})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_2$, then $\phi^{-1}(\mathcal{B})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_1$.

Proof. It is easy to see that $\phi^{-1}(\mathcal{T}_\mathcal{B})(e) \geq \phi^{-1}(\mathcal{T}_\mathcal{B})(s)$, $\phi^{-1}(\mathcal{I}_\mathcal{B})(e) \geq \phi^{-1}(\mathcal{I}_\mathcal{B})(s)$ and $\phi^{-1}(\mathcal{F}_\mathcal{B})(e) \leq \phi^{-1}(\mathcal{F}_\mathcal{B})(s)$ for all $s \in G_1$. Let $s, t \in G_1$, then

$$\begin{aligned}
&\phi^{-1}(\mathcal{T}_\mathcal{B})(s \odot t) = \mathcal{T}_\mathcal{B}(\phi(s \odot t))\\
&\phi^{-1}(\mathcal{T}_\mathcal{B})(s \odot t) = \mathcal{T}_\mathcal{B}(\phi(s) \odot \phi(t))\\
&\phi^{-1}(\mathcal{T}_\mathcal{B})(s \odot t) \geq \min\{\mathcal{T}_\mathcal{B}(\phi(s)), \mathcal{T}_\mathcal{B}(\phi(t))\}\\
&\phi^{-1}(\mathcal{T}_\mathcal{B})(s \odot t) \geq \min\{\phi^{-1}(\mathcal{T}_\mathcal{B})(s), \phi^{-1}(\mathcal{T}_\mathcal{B})(t)\},
\end{aligned}$$

$$\begin{aligned}
\phi^{-1}(\mathcal{I}_{\mathcal{B}})(s \odot t) &= \mathcal{I}_{\mathcal{B}}(\phi(s \odot t))\\
\phi^{-1}(\mathcal{I}_{\mathcal{B}})(s \odot t) &= \mathcal{I}_{\mathcal{B}}(\phi(s) \odot \phi(t))\\
\phi^{-1}(\mathcal{I}_{\mathcal{B}})(s \odot t) &\geq \min\{\mathcal{I}_{\mathcal{B}}(\phi(s)), \mathcal{I}_{\mathcal{B}}(\phi(t))\}\\
\phi^{-1}(\mathcal{I}_{\mathcal{B}})(s \odot t) &\geq \min\{\phi^{-1}(\mathcal{I}_{\mathcal{B}})(s), \phi^{-1}(\mathcal{I}_{\mathcal{B}})(t)\},
\end{aligned}$$

$$\begin{aligned}
\phi^{-1}(\mathcal{F}_{\mathcal{B}})(s \odot t) &= \mathcal{F}_{\mathcal{B}}(\phi(s \odot t))\\
\phi^{-1}(\mathcal{F}_{\mathcal{B}})(s \odot t) &= \mathcal{F}_{\mathcal{B}}(\phi(s) \odot \phi(t))\\
\phi^{-1}(\mathcal{F}_{\mathcal{B}})(s \odot t) &\leq \max\{\mathcal{F}_{\mathcal{B}}(\phi(s)), \mathcal{F}_{\mathcal{B}}(\phi(t))\}\\
\phi^{-1}(\mathcal{F}_{\mathcal{B}})(s \odot t) &\leq \max\{\phi^{-1}(\mathcal{F}_{\mathcal{B}})(s), \phi^{-1}(\mathcal{F}_{\mathcal{B}})(t)\}.
\end{aligned}$$

Hence $\phi^{-1}(\mathcal{B})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_1$. □

Theorem 2.6. Let $\phi : \mathcal{K}_1 \to \mathcal{K}_2$ be an epimorphism of K-algebras. If $\mathcal{B} = (\mathcal{T}_{\mathcal{B}}, \mathcal{I}_{\mathcal{B}}, \mathcal{F}_{\mathcal{B}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_2$ and $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is the *preimage* of $\mathcal{B}$ under ϕ. Then $\mathcal{A}$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_1$.

Proof. It is easy to see that $\mathcal{T}_{\mathcal{A}}(e) \geq \mathcal{T}_{\mathcal{A}}(s)$, $\mathcal{I}_{\mathcal{A}}(e) \geq \mathcal{I}_{\mathcal{A}}(s)$ and $\mathcal{F}_{\mathcal{A}}(e) \leq \mathcal{F}_{\mathcal{A}}(s)$, for all $s \in G_1$. Now for any $s, t \in G_1$,

$$\begin{aligned}
\mathcal{T}_{\mathcal{A}}(s \odot t) &= \mathcal{T}_{\mathcal{B}}(\phi(s \odot t))\\
\mathcal{T}_{\mathcal{A}}(s \odot t) &= \mathcal{T}_{\mathcal{B}}(\phi(s) \odot \phi(t))\\
\mathcal{T}_{\mathcal{A}}(s \odot t) &\geq \min\{\mathcal{T}_{\mathcal{B}}(\phi(s)), \mathcal{T}_{\mathcal{B}}(\phi(t))\}\\
\mathcal{T}_{\mathcal{A}}(s \odot t) &\geq \min\{\mathcal{T}_{\mathcal{A}}(s), \mathcal{T}_{\mathcal{A}}(t)\},
\end{aligned}$$

$$\begin{aligned}
\mathcal{I}_{\mathcal{A}}(s \odot t) &= \mathcal{I}_{\mathcal{B}}(\phi(s \odot t))\\
\mathcal{I}_{\mathcal{A}}(s \odot t) &= \mathcal{I}_{\mathcal{B}}(\phi(s) \odot \phi(t))\\
\mathcal{I}_{\mathcal{A}}(s \odot t) &\geq \min\{\mathcal{I}_{\mathcal{B}}(\phi(s)), \mathcal{I}_{\mathcal{B}}(\phi(t))\}\\
\mathcal{I}_{\mathcal{A}}(s \odot t) &\geq \min\{\mathcal{I}_{\mathcal{A}}(s), \mathcal{I}_{\mathcal{A}}(t)\},
\end{aligned}$$

$$\begin{aligned}
\mathcal{F}_{\mathcal{A}}(s \odot t) &= \mathcal{F}_{\mathcal{B}}(\phi(s \odot t))\\
\mathcal{F}_{\mathcal{A}}(s \odot t) &= \mathcal{F}_{\mathcal{B}}(\phi(s) \odot \phi(t))\\
\mathcal{F}_{\mathcal{A}}(s \odot t) &\leq \max\{\mathcal{F}_{\mathcal{B}}(\phi(s)), \mathcal{F}_{\mathcal{B}}(\phi(t))\}\\
\mathcal{F}_{\mathcal{A}}(s \odot t) &\leq \max\{\mathcal{F}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(t)\}.
\end{aligned}$$

Hence $\mathcal{A}$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_1$. □

Definition 2.6. Let a mapping $\phi : \mathcal{K}_1 \to \mathcal{K}_2$ from $\mathcal{K}_1$ into $\mathcal{K}_2$ of K-algebras and let $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic set of $\mathcal{K}_2$. The map $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is called the *preimage* of $\mathcal{A}$ under ϕ, if $\mathcal{T}^{\phi}_{\mathcal{A}}(s) = \mathcal{T}_{\mathcal{A}}(\phi(s))$, $\mathcal{I}^{\phi}_{\mathcal{A}}(s) = \mathcal{I}_{\mathcal{A}}(\phi(s))$ and $\mathcal{F}^{\phi}_{\mathcal{A}}(s) = \mathcal{F}_{\mathcal{A}}(\phi(s))$ for all $s \in G_1$.

Proposition 2.3. Let $\phi : \mathcal{K}_1 \to \mathcal{K}_2$ be an epimorphism of K-algebras. If $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_2$, then $\mathcal{A}^{\phi} = (\mathcal{T}^{\phi}_{\mathcal{A}}, \mathcal{I}^{\phi}_{\mathcal{A}}, \mathcal{F}^{\phi}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_1$.

Proof. For any $s \in G_1$, we have

$$\begin{aligned}
\mathcal{T}^{\phi}_{\mathcal{A}}(e_1) &= \mathcal{T}_{\mathcal{A}}(\phi(e_1)) = \mathcal{T}_{\mathcal{A}}(e_2) \geq \mathcal{T}_{\mathcal{A}}(\phi(s)) = \mathcal{T}^{\phi}_{\mathcal{A}}(s),\\
\mathcal{I}^{\phi}_{\mathcal{A}}(e_1) &= \mathcal{I}_{\mathcal{A}}(\phi(e_1)) = \mathcal{I}_{\mathcal{A}}(e_2) \geq \mathcal{I}_{\mathcal{A}}(\phi(s)) = \mathcal{I}^{\phi}_{\mathcal{A}}(s),\\
\mathcal{F}^{\phi}_{\mathcal{A}}(e_1) &= \mathcal{F}_{\mathcal{A}}(\phi(e_1)) = \mathcal{F}_{\mathcal{A}}(e_2) \leq \mathcal{F}_{\mathcal{A}}(\phi(s)) = \mathcal{F}^{\phi}_{\mathcal{A}}(s).
\end{aligned}$$

For any $s, t \in G_1$, since $\mathcal{A}$ ia a single-valued neutrosophic K-subalgebra of $\mathcal{K}_2$

$$\begin{aligned}
\mathcal{T}^{\phi}_{\mathcal{A}}(s \odot t) &= \mathcal{T}_{\mathcal{A}}(\phi(s \odot t))\\
\mathcal{T}^{\phi}_{\mathcal{A}}(s \odot t) &= \mathcal{T}_{\mathcal{A}}(\phi(s) \odot \phi(t))\\
\mathcal{T}^{\phi}_{\mathcal{A}}(s \odot t) &\geq \min\{\mathcal{T}_{\mathcal{A}}(\phi(s)), \mathcal{T}_{\mathcal{A}}(\phi(t))\}\\
\mathcal{T}^{\phi}_{\mathcal{A}}(s \odot t) &\geq \min\{\mathcal{T}^{\phi}_{\mathcal{A}}(s), \mathcal{T}^{\phi}_{\mathcal{A}}(s)\},
\end{aligned}$$

$$\begin{aligned}
\mathcal{I}^{\phi}_{\mathcal{A}}(s \odot t) &= \mathcal{I}_{\mathcal{A}}(\phi(s \odot t))\\
\mathcal{I}^{\phi}_{\mathcal{A}}(s \odot t) &= \mathcal{I}_{\mathcal{A}}(\phi(s) \odot \phi(t))\\
\mathcal{I}^{\phi}_{\mathcal{A}}(s \odot t) &\geq \min\{\mathcal{I}_{\mathcal{A}}(\phi(s)), \mathcal{I}_{\mathcal{A}}(\phi(t))\}\\
\mathcal{I}^{\phi}_{\mathcal{A}}(s \odot t) &\geq \min\{\mathcal{I}^{\phi}_{\mathcal{A}}(s), \mathcal{I}^{\phi}_{\mathcal{A}}(s)\},
\end{aligned}$$

$$\begin{aligned}
\mathcal{F}^{\phi}_{\mathcal{A}}(s \odot t) &= \mathcal{F}_{\mathcal{A}}(\phi(s \odot t))\\
\mathcal{F}^{\phi}_{\mathcal{A}}(s \odot t) &= \mathcal{F}_{\mathcal{A}}(\phi(s) \odot \phi(t))\\
\mathcal{F}^{\phi}_{\mathcal{A}}(s \odot t) &\leq \max\{\mathcal{F}_{\mathcal{A}}(\phi(s)), \mathcal{F}_{\mathcal{A}}(\phi(t))\}\\
\mathcal{F}^{\phi}_{\mathcal{A}}(s \odot t) &\leq \max\{\mathcal{F}^{\phi}_{\mathcal{A}}(s), \mathcal{F}^{\phi}_{\mathcal{A}}(s)\}.
\end{aligned}$$

Hence $\mathcal{A}^{\phi} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_1$. □

Proposition 2.4. Let $\phi : \mathcal{K}_1 \to \mathcal{K}_2$ be an epimorphism of K-algebras. If $\mathcal{A}^{\phi} = (\mathcal{T}^{\phi}_{\mathcal{A}}, \mathcal{I}^{\phi}_{\mathcal{A}}, \mathcal{F}^{\phi}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_2$, then $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_1$.

Proof. Since there exists $s \in G_1$ such that $t = \phi(s)$, for any $t \in G_2$

$$\begin{aligned}
\mathcal{T}_{\mathcal{A}}(t) &= \mathcal{T}_{\mathcal{A}}(\phi(s)) = \mathcal{T}^{\phi(s)}_{\mathcal{A}} \leq \mathcal{T}^{\phi(e_1)}_{\mathcal{A}} = \mathcal{T}_{\mathcal{A}}(\phi(e_1)) = \mathcal{T}_{\mathcal{A}}(e_2),\\
\mathcal{I}_{\mathcal{A}}(t) &= \mathcal{I}_{\mathcal{A}}(\phi(s)) = \mathcal{I}^{\phi(s)}_{\mathcal{A}} \leq \mathcal{I}^{\phi(e_1)}_{\mathcal{A}} = \mathcal{I}_{\mathcal{A}}(\phi(e_1)) = \mathcal{I}_{\mathcal{A}}(e_2),\\
\mathcal{F}_{\mathcal{A}}(t) &= \mathcal{F}_{\mathcal{A}}(\phi(s)) = \mathcal{F}^{\phi(s)}_{\mathcal{A}} \geq \mathcal{F}^{\phi(e_1)}_{\mathcal{A}} = \mathcal{F}_{\mathcal{A}}(\phi(e_1)) = \mathcal{F}_{\mathcal{A}}(e_2).
\end{aligned}$$

for any $s, t \in G_2$, $u, v \in G_1$ such that $s = \phi(u)$ and $t = \phi(v)$. It follows that

$$\begin{aligned}
\mathcal{T}_{\mathcal{A}}(s \odot t) &= \mathcal{T}_{\mathcal{A}}(\phi(u \odot v)) \\
\mathcal{T}_{\mathcal{A}}(s \odot t) &= \mathcal{T}^{\phi}_{\mathcal{A}}(u \odot v) \\
\mathcal{T}_{\mathcal{A}}(s \odot t) &\geq \min\{\mathcal{T}^{\phi}_{\mathcal{A}}(u), \mathcal{T}^{\phi}_{\mathcal{A}}(v)\} \\
\mathcal{T}_{\mathcal{A}}(s \odot t) &\geq \min\{\mathcal{T}_{\mathcal{A}}(\phi(u)), \mathcal{T}_{\mathcal{A}}(\phi(v))\} \\
\mathcal{T}_{\mathcal{A}}(s \odot t) &\geq \min\{\mathcal{T}_{\mathcal{A}}(s), \mathcal{T}_{\mathcal{A}}(t)\},
\end{aligned}$$

$$\begin{aligned}
\mathcal{I}_{\mathcal{A}}(s \odot t) &= \mathcal{I}_{\mathcal{A}}(\phi(u \odot v)) \\
\mathcal{I}_{\mathcal{A}}(s \odot t) &= \mathcal{I}^{\phi}_{\mathcal{A}}(u \odot v) \\
\mathcal{I}_{\mathcal{A}}(s \odot t) &\geq \min\{\mathcal{I}^{\phi}_{\mathcal{A}}(u), \mathcal{I}^{\phi}_{\mathcal{A}}(v)\} \\
\mathcal{I}_{\mathcal{A}}(s \odot t) &\geq \min\{\mathcal{I}_{\mathcal{A}}(\phi(u)), \mathcal{I}_{\mathcal{A}}(\phi(v))\} \\
\mathcal{I}_{\mathcal{A}}(s \odot t) &\geq \min\{\mathcal{I}_{\mathcal{A}}(s), \mathcal{I}_{\mathcal{A}}(t)\},
\end{aligned}$$

$$\begin{aligned}
\mathcal{F}_{\mathcal{A}}(s \odot t) &= \mathcal{F}_{\mathcal{A}}(\phi(u \odot v)) \\
\mathcal{F}_{\mathcal{A}}(s \odot t) &= \mathcal{F}^{\phi}_{\mathcal{A}}(u \odot v) \\
\mathcal{F}_{\mathcal{A}}(s \odot t) &\leq \max\{\mathcal{F}^{\phi}_{\mathcal{A}}(u), \mathcal{F}^{\phi}_{\mathcal{A}}(v)\} \\
\mathcal{F}_{\mathcal{A}}(s \odot t) &\leq \max\{\mathcal{F}_{\mathcal{A}}(\phi(u)), \mathcal{F}_{\mathcal{A}}(\phi(v))\} \\
\mathcal{F}_{\mathcal{A}}(s \odot t) &\leq \max\{\mathcal{F}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(t)\}.
\end{aligned}$$

Hence $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_2$. □

From above two propositions, we obtain the following theorem.

Theorem 2.7. Let $\phi : \mathcal{K}_1 \to \mathcal{K}_2$ be an epimorphism of K-algebras. Then $\mathcal{A}^{\phi} = (\mathcal{T}^{\phi}_{\mathcal{A}}, \mathcal{I}^{\phi}_{\mathcal{A}}, \mathcal{F}^{\phi}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_1$ if and only if $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_2$.

Definition 2.7. A single-valued neutrosophic K-subalgebra $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ of a K-algebra $\mathcal{K}$ is called *characteristic* if $\mathcal{T}_{\mathcal{A}}(\phi(s)) = \mathcal{T}_{\mathcal{A}}(s)$, $\mathcal{I}_{\mathcal{A}}(\phi(s)) = \mathcal{I}_{\mathcal{A}}(s)$ and $\mathcal{F}_{\mathcal{A}}(\phi(s)) = \mathcal{F}_{\mathcal{A}}(s)$, for all $s \in G$ and $\phi \in Aut(\mathcal{K})$.

Definition 2.8. A K-subalgebra S of a K-algebra $\mathcal{K}$ is said to be *fully invariant* if $\phi(S) \subseteq S$, for all $\phi \in End(\mathcal{K})$, where $End(\mathcal{K})$ is the set of all endomorphisms of a K-algebra $\mathcal{K}$. A single-valued neutrosophic K-subalgebra $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ of a K-algebra $\mathcal{K}$ is called *fully invariant* if $\mathcal{T}_{\mathcal{A}}(\phi(s)) \leq \mathcal{T}_{\mathcal{A}}(s)$, $\mathcal{I}_{\mathcal{A}}(\phi(s)) \leq \mathcal{I}_{\mathcal{A}}(s)$ and $\mathcal{F}_{\mathcal{A}}(\phi(s)) \leq \mathcal{F}_{\mathcal{A}}(s)$, for all $s \in G$ and $\phi \in End(\mathcal{K})$.

Definition 2.9. Let $\mathcal{A}_1 = (\mathcal{T}_{\mathcal{A}_1}, \mathcal{I}_{\mathcal{A}_1}, \mathcal{F}_{\mathcal{A}_1})$ and $\mathcal{A}_2 = (\mathcal{T}_{\mathcal{A}_2}, \mathcal{I}_{\mathcal{A}_2}, \mathcal{F}_{\mathcal{A}_2})$ be single-valued neutrosophic K-subalgebras of $\mathcal{K}$. Then $\mathcal{A}_1 = (\mathcal{T}_{\mathcal{A}_1}, \mathcal{I}_{\mathcal{A}_1}, {}_{\mathcal{A}_1})$ is said to be the same type of $\mathcal{A}_2 = (\mathcal{T}_{\mathcal{A}_2}, \mathcal{I}_{\mathcal{A}_2}, \mathcal{F}_{\mathcal{A}_2})$ if there exists $\phi \in Aut(\mathcal{K})$ such that $\mathcal{A}_1 = \mathcal{A}_2 \circ \phi$, i.e., $\mathcal{T}_{\mathcal{A}_1}(s) = \mathcal{T}_{\mathcal{A}_2}(\phi(s))$, $\mathcal{I}_{\mathcal{A}_1}(s) = \mathcal{I}_{\mathcal{A}_2}(\phi(s))$ and $\mathcal{F}_{\mathcal{A}_1}(s) = \mathcal{F}_{\mathcal{A}_2}(\phi(s))$, for all $s \in G$.

Theorem 2.8. Let $\mathcal{A}_1 = (\mathcal{T}_{\mathcal{A}_1}, \mathcal{I}_{\mathcal{A}_1}, \mathcal{F}_{\mathcal{A}_1})$ and $\mathcal{A}_2 = (\mathcal{T}_{\mathcal{A}_2}, \mathcal{I}_{\mathcal{A}_2}, \mathcal{F}_{\mathcal{A}_2})$ be single-valued neutrosophic K-subalgebras of $\mathcal{K}$. Then $\mathcal{A}_1 = (\mathcal{T}_{\mathcal{A}_1}, \mathcal{I}_{\mathcal{A}_1}, \mathcal{F}_{\mathcal{A}_1})$ is a single-valued neutrosophic K-subalgebra is of the same type of $\mathcal{A}_2 = (\mathcal{T}_{\mathcal{A}_2}, \mathcal{I}_{\mathcal{A}_2}, \mathcal{F}_{\mathcal{A}_2})$ if and only if $\mathcal{A}_1$ is isomorphic to $\mathcal{A}_2$.

Proof. Sufficient condition holds trivially so we only need to prove the necessary condition. Let $\mathcal{A}_1 = (\mathcal{T}_{\mathcal{A}_1}, \mathcal{I}_{\mathcal{A}_1}, \mathcal{F}_{\mathcal{A}_1})$ be a single-valued neutrosophic K-subalgebra having same type of $\mathcal{A}_2 = (\mathcal{T}_{\mathcal{A}_2}, \mathcal{I}_{\mathcal{A}_2}, \mathcal{F}_{\mathcal{A}_2})$. Then there exists $\phi \in Aut(\mathcal{K})$ such that $\mathcal{T}_{\mathcal{A}_1}(s) = \mathcal{T}_{\mathcal{A}_2}(\phi(s))$, $\mathcal{I}_{\mathcal{A}_1}(s) = \mathcal{I}_{\mathcal{A}_2}(\phi(s))$ and $\mathcal{F}_{\mathcal{A}_1} = \mathcal{F}_{\mathcal{A}_2}(\phi(s))$, for all $s \in G$.

Let $f : \mathcal{A}_1(K) \to \mathcal{A}_2(K)$ be a mapping defined by $f(\mathcal{A}_1(s)) = \mathcal{A}_2(\phi(s))$, for all $s \in G$, that is, $f(\mathcal{T}_{\mathcal{A}_1}(s)) = \mathcal{T}_{\mathcal{A}_2}(\phi(s))$, $f(\mathcal{I}_{\mathcal{A}_1}(s)) = \mathcal{I}_{\mathcal{A}_2}(\phi(s))$ and $f(\mathcal{F}_{\mathcal{A}_1}(s)) = \mathcal{F}_{\mathcal{A}_2}(\phi(s))$, for all $s \in G$. Clearly, f is surjective. Also, f is injective because if $f(\mathcal{T}_{\mathcal{A}_1}(s)) = f(\mathcal{T}_{\mathcal{A}_1}(t))$, for all $s, t \in G$, then $\mathcal{T}_{\mathcal{A}_2}(\phi(s)) = \mathcal{T}_{\mathcal{A}_2}(\phi(t))$ and we have $\mathcal{T}_{\mathcal{A}_1}(s) = \mathcal{T}_{\mathcal{A}_1}(t)$. Similarly, $\mathcal{I}_{\mathcal{A}_1}(s) = \mathcal{I}_{\mathcal{A}_1}(t)$, $\mathcal{F}_{\mathcal{A}_1}(s) = \mathcal{F}_{\mathcal{A}_1}(t)$.

Therefore, f is a homomorphism, such that for $s, t \in G$, we have

$$\begin{aligned} f(\mathcal{T}_{\mathcal{A}_1}(s \odot t)) &= \mathcal{T}_{\mathcal{A}_2}(\phi(s \odot t)) = \mathcal{T}_{\mathcal{A}_2}(\phi(s) \odot \phi(t)), \\ f(\mathcal{I}_{\mathcal{A}_1}(s \odot t)) &= \mathcal{I}_{\mathcal{A}_2}(\phi(s \odot t)) = \mathcal{I}_{\mathcal{A}_2}(\phi(s) \odot \phi(t)), \\ f(\mathcal{F}_{\mathcal{A}_1}(s \odot t)) &= \mathcal{F}_{\mathcal{A}_2}(\phi(s \odot t)) = \mathcal{F}_{\mathcal{A}_2}(\phi(s) \odot \phi(t)). \end{aligned}$$

Hence $\mathcal{A}_1 = (\mathcal{T}_{\mathcal{A}_1}, \mathcal{I}_{\mathcal{A}_1}, \mathcal{F}_{\mathcal{A}_1})$ is isomorphic to $\mathcal{A}_2 = \mathcal{T}_{\mathcal{A}_2}, \mathcal{I}_{\mathcal{A}_2}, \mathcal{F}_{\mathcal{A}_2})$. Hence the proof. □

3. $(\tilde{a}, \tilde{b})$-single-valued neutrosophic K-algebras

Definition 3.1. A single-valued neutrosophic set $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ in a set G is called an $(\tilde{a}, \tilde{b})-$single-valued neutrosophic K-subalgebra of $\mathcal{K}$ if it satisfies the following conditions:

- $u_{(\alpha_1,\beta_1,\gamma_1)}\ \tilde{a}\ \mathcal{A},\ v_{(\alpha_2,\beta_2,\gamma_2)}\ \tilde{a}\ \mathcal{A} \Rightarrow (u \odot v)_{(\min(\alpha_1,\alpha_2),\min(\beta_1,\beta_2),\max(\gamma_1,\gamma_2))}\ \tilde{b}\ \mathcal{A}$, for all $u, v \in G, \alpha_1, \alpha_2 \in (0, 1], \beta_1, \beta_2 \in (0, 1], \gamma_1, \gamma_2 \in [0, 1)$.

Twelve different types of single-valued neutrosophic K-subalgebras can be obtained by replacing the values of $\tilde{a}(\neq\in \wedge q)$ and $\tilde{b}$ by any two values in the set $\{\in, q, \in \vee q, \in \wedge q\}$ in Definition 3.1.

Remark 3.1. Every $(\in, \in)$-single-valued neutrosophic K-subalgebra is in fact, a single-valued neutrosophic K-subalgebra.

Proposition 3.1. Every $(\in, \in)$-single-valued neutrosophic K-subalgebra is an $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebra.

Proof. Let $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic K-subalgebra of $\mathcal{K}$. Let $u, v \in G$ and $\alpha_1, \alpha_2 \in (0, 1], \beta_1, \beta_2 \in (0, 1], \gamma_1, \gamma_2 \in [0, 1)$ be such that $u_{(\alpha_1,\beta_1,\gamma_1)} \in \mathcal{A}$, $v_{(\alpha_2,\beta_2,\gamma_2)} \in \mathcal{A}$. Then $u_{(\alpha_1,\beta_1,\gamma_1)} \in \mathcal{A}$, $v_{(\alpha_2,\beta_2,\gamma_2)} \in \mathcal{A} \Rightarrow (u \odot v)_{(\min(\alpha_1,\alpha_2),\min(\beta_1,\beta_2),\max(\gamma_1,\gamma_2))} \in \vee q\ \mathcal{A}$. Hence $\mathcal{A}$ is an $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$. □

Proposition 3.2. Every $(\in\vee q, \in\vee q)$-single-valued neutrosophic K-subalgebra is an $(\in, \in\vee q)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$.

Definition 3.2. Let $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic set in G. The set $\underline{\mathcal{A}}$=$\{u \in G \mid \mathcal{T}_{\mathcal{A}}(u) \neq 0,\ \mathcal{I}_{\mathcal{A}}(u) \neq 0,\ \mathcal{F}_{\mathcal{A}}(u) \neq 0\}$ is called the *support* of $\mathcal{A}$.

Lemma 3.1. If $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a non-zero $(\in, \in)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$, then $\underline{\mathcal{A}}$ is a K-subalgebra of $\mathcal{K}$.

Proof. Let $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ be a non-zero $(\in, \in)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$ and let $u, v \in \underline{\mathcal{A}}$. Then $\mathcal{T}_{\mathcal{A}}(u) \neq 0$ and $\mathcal{T}_{\mathcal{A}}(v) \neq 0$, $\mathcal{I}_{\mathcal{A}}(u) \neq 0$ and $\mathcal{I}_{\mathcal{A}}(v) \neq 0$ and $\mathcal{F}_{\mathcal{A}}(u) \neq 0$, $\mathcal{F}_{\mathcal{A}}(v) \neq 0$. Let $\mathcal{T}_{\mathcal{A}}(u \odot v) = 0, \mathcal{I}_{\mathcal{A}}(u \odot v) = 0$ and $\mathcal{F}_{\mathcal{A}}(u \odot v) = 0$. Since $u_{\mathcal{T}_{\mathcal{A}}}(u) \in \mathcal{A}$ and $v_{\mathcal{T}_{\mathcal{A}}}(v) \in \mathcal{A}$, $u_{\mathcal{I}_{\mathcal{A}}}(u) \in \mathcal{A}$ and $v_{\mathcal{I}_{\mathcal{A}}}(v) \in \mathcal{A}$, $u_{\mathcal{F}_{\mathcal{A}}}(u) \in \mathcal{A}$ and $v_{\mathcal{F}_{\mathcal{A}}}(v) \in \mathcal{A}$ but
$(u \odot v)_{(\min(\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v)), \min(\mathcal{I}_{\mathcal{A}}(u), \mathcal{I}_{\mathcal{A}}(v)), \max(\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v)))} \notin \mathcal{A}$.

Since $\mathcal{T}_{\mathcal{A}}(u \odot v) = 0, \mathcal{I}_{\mathcal{A}}(u \odot v) = 0$ and $\mathcal{F}_{\mathcal{A}}(u \odot v) = 0$. A contradiction. Hence $\mathcal{T}_{\mathcal{A}}(u \odot v) \neq 0, \mathcal{I}_{\mathcal{A}}(u \odot v) \neq 0$ and $\mathcal{F}_{\mathcal{A}}(u \odot v) \neq 0$ which shows that $(u \odot v) \in \underline{\mathcal{A}}$, consequently $\underline{\mathcal{A}}$ is a K-subalgebra of $\mathcal{A}$. □

Lemma 3.2. If $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a non-zero $(\in, q)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$, then $\underline{\mathcal{A}}$ is a K-subalgebra of $\mathcal{K}$.

Lemma 3.3. If $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a non-zero $(q, \in)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$, then $\underline{\mathcal{A}}$ is a K-subalgebra of $\mathcal{K}$.

Lemma 3.4. If $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a non-zero (q, q)-single-valued neutrosophic K-subalgebra of $\mathcal{K}$, then $\underline{\mathcal{A}}$ is a K-subalgebra of $\mathcal{K}$.

Theorem 3.1. If $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a non-zero $(\tilde{a}, \tilde{b})$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$, then $\underline{\mathcal{A}}$ is a K-subalgebra of $\mathcal{K}$.

Definition 3.3. A neutrosophic set $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ in a K-algebra $\mathcal{K}$ is called an $(\in, \in\vee q)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$ if it satisfies the following conditions:

(a) $e_{(\alpha,\beta,\gamma)} \in \mathcal{A} \Rightarrow (u)_{(\alpha,\beta,\gamma)} \in \vee q\ \mathcal{A}$,

(b) $u_{(\alpha_1,\beta_1,\gamma_1)} \in \mathcal{A}, v_{(\alpha_2,\beta_2,\gamma_2)} \in \mathcal{A} \Rightarrow (u \odot v)_{(\min(\alpha_1,\alpha_2),\min(\beta_1,\beta_2),\max(\gamma_1,\gamma_2))} \in \vee q\ \mathcal{A}$,

For all $u, v \in G, \alpha, \alpha_1, \alpha_2 \in (0,1], \beta, \beta_1, \beta_2 \in (0,1], \gamma, \gamma_1, \gamma_2 \in [0,1)$.

Example 3.1. Consider a K-algebra $\mathcal{K} = (G, \cdot, \odot, e)$, where $G = \{e, x, x^2, x^3, x^4, x^5, x^6\}$ is the cyclic group of order 7 and Cayley's table for $\odot$ is given as:

$\odot$	e	x	x^2	x^3	x^4	x^5	x^6
e	e	x^6	x^5	x^4	x^3	x^2	x
x	x	e	x^6	x^5	x^4	x^3	x^2
x^2	x^2	x	e	x^6	x^5	x^4	x^3
x^3	x^3	x^2	x	e	x^6	x^5	x^4
x^4	x^4	x^3	x^2	x	e	x^6	x^5
x^5	x^5	x^4	x^3	x^2	x	e	x^6
x^6	x^6	x^5	x^4	x^3	x^2	x	e

We define a single-valued neutrosophic set $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ in $\mathcal{K}$ as follows:

$$\mathcal{T}_{\mathcal{A}}(u) = \begin{cases} 1, & \text{if } u = e, \\ 0.7, & \text{otherwise} \end{cases}$$

$$\mathcal{I}_{\mathcal{A}}(u) = \begin{cases} 1, & \text{if } u = e, \\ 0.6, & \text{otherwise} \end{cases}$$

$$\mathcal{F}_{\mathcal{A}}(u) = \begin{cases} 0, & \text{if } u = e, \\ 0.5, & \text{otherwise} \end{cases}$$

Now take $\alpha = 0.4, \alpha_1 = 0.5, \alpha_2 = 0.3$, $\beta = 0.5, \beta_1 = 0.6, \beta_2 = 0.3$, $\gamma = 0.6, \gamma_1 = 0.6, \gamma_2 = 0.5$, where $\alpha, \alpha_1, \alpha_2 \in (0, 1], \beta, \beta_1, \beta_2 \in (0, 1], \gamma, \gamma_1, \gamma_2 \in [0, 1)$.

By direct calculations, it is easy to see that $\mathcal{A}$ is an $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$.

Theorem 3.2. Let $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic set in $\mathcal{K}$. Then $\mathcal{A}$ is an $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$ if and only if

$$\begin{aligned} \text{(i) } & \mathcal{T}_{\mathcal{A}}(u) \geq \min(\mathcal{T}_{\mathcal{A}}(e), 0.5), \\ & \mathcal{I}_{\mathcal{A}}(u) \geq \min(\mathcal{I}_{\mathcal{A}}(e), 0.5), \\ & \mathcal{F}_{\mathcal{A}}(u) \leq \max(\mathcal{F}_{\mathcal{A}}(e), 0.5). \end{aligned}$$

$$\begin{aligned} \text{(ii) } & \mathcal{T}_{\mathcal{A}}(u \odot v) \geq \min(\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v), 0.5), \\ & \mathcal{I}_{\mathcal{A}}(u \odot v) \geq \min(\mathcal{I}_{\mathcal{A}}(u), \mathcal{I}_{\mathcal{A}}(v), 0.5), \\ & \mathcal{F}_{\mathcal{A}}(u \odot v) \leq \max(\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v), 0.5), \text{ for all } u, v \in G. \end{aligned}$$

Proof. Assume that $\mathcal{A}$ is an $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebra.

Let for $u, v \in G$. Assume that $\mathcal{T}_{\mathcal{A}}(u \odot v) < \min(\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v), 0.5)$, $\mathcal{I}_{\mathcal{A}}(u \odot v) < \min(\mathcal{I}_{\mathcal{A}}(u),\ \mathcal{I}_{\mathcal{A}}(v), 0.5)$, $\mathcal{F}_{\mathcal{A}}(u \odot v) > \max(\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v), 0.5)$. Then $\mathcal{T}_{\mathcal{A}}(u \odot v) < \min(\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v))$, $\mathcal{I}_{\mathcal{A}}(u \odot v) < \min(\mathcal{I}_{\mathcal{A}}(u), \mathcal{I}_{\mathcal{A}}(v))$ and $\mathcal{F}_{\mathcal{A}}(u \odot v) > \max(\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v))$. Take α, β, γ such that $\mathcal{T}_{\mathcal{A}}(u \odot v) < \alpha < \min(\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v),$

$\mathcal{I}_{\mathcal{A}}(u \odot v) < \beta < \min(\mathcal{I}_{\mathcal{A}}(u), \mathcal{I}_{\mathcal{A}}(v)$, $\mathcal{F}_{\mathcal{A}}(u \odot v) > \gamma > \max(\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v)$. Then $u_\alpha, v_\alpha \in \mathcal{T}_{\mathcal{A}}$, $u_\beta, v_\beta \in \mathcal{I}_{\mathcal{A}}$ and $u_\gamma, v_\gamma \in \mathcal{F}_{\mathcal{A}}$ but $(u \odot v)_{(\min(\alpha_1,\alpha_2),\min(\beta_1,\beta_2),\max(\gamma_1,\gamma_2))}$ $\overline{\in \vee q}\ \mathcal{A}$, a contradiction.

Now if $\mathcal{T}_{\mathcal{A}}(u \odot v) < 0.5$, $\mathcal{I}_{\mathcal{A}}(u \odot v) < 0.5$, $\mathcal{F}_{\mathcal{A}}(u \odot v) > 0.5$. Then $u_{(0.5,0.5,0.5)}, v_{(0.5,0.5,0.5)} \in \mathcal{A}$, but $(u \odot v)_{(0.5,0.5,0.5)} \overline{\in \vee q} \mathcal{A}$ which is also a contradiction. Hence (i) holds.

Let $u_{(\alpha_1,\beta_1,\gamma_1)}, v_{(\alpha_2,\beta_2,\gamma_2)} \in \mathcal{A}$ which means that $\mathcal{T}_{\mathcal{A}}(u) \geq \alpha_1, \mathcal{T}_{\mathcal{A}}(v) \geq \alpha_2$, $\mathcal{I}_{\mathcal{A}}(u) \geq \beta_1, \mathcal{I}_{\mathcal{A}}(v) \geq \beta_2$, $\mathcal{F}_{\mathcal{A}}(u) \leq \gamma_1, \mathcal{F}_{\mathcal{A}}(v) \leq \gamma_2$. We have $\mathcal{T}_{\mathcal{A}}(u \odot v) \geq \min(\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v), 0.5) \geq \min(\alpha_1, \alpha_2, 0.5)$, $\mathcal{I}_{\mathcal{A}}(u \odot v) \geq \min(\mathcal{I}_{\mathcal{A}}(u), \mathcal{I}_{\mathcal{A}}(v), 0.5) \geq \min(\beta_1, \beta_2, 0.5)$, $\mathcal{F}_{\mathcal{A}}(u \odot v) \leq \max(\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v), 0.5) \leq \max(\gamma_1, \gamma_2, 0.5)$. If $\min(\alpha_1, \alpha_2) > 0.5, \min(\beta_1, \beta_2) > 0.5, \max(\gamma_1, \gamma_2) < 0.5$, then $\mathcal{T}_{\mathcal{A}}(u \odot v) \geq 0.5 \Rightarrow \mathcal{T}_{\mathcal{A}}(u \odot v) + \min(\alpha_1, \alpha_2) > 1$, $\mathcal{I}_{\mathcal{A}}(u \odot v) \geq 0.5 \Rightarrow \mathcal{I}_{\mathcal{A}}(u \odot v) + \min(\beta_1, \beta_2) > 1$, $\mathcal{F}_{\mathcal{A}}(u \odot v) \leq 0.5 \Rightarrow \mathcal{F}_{\mathcal{A}}(u \odot v) + \max(\gamma_1, \gamma_2) < 1$.

But if $\min(\alpha_1, \alpha_2) \leq 0.5, \min(\beta_1, \beta_2) \leq 0.5, \max(\gamma_1, \gamma_2) \geq 0.5$, then $\mathcal{T}_{\mathcal{A}}(u \odot v) \geq \min(\alpha_1, \alpha_2), \mathcal{I}_{\mathcal{A}}(u \odot v) \geq \min(\beta_1, \beta_2)$, $\mathcal{F}_{\mathcal{A}}(u \odot v) \leq \max(\gamma_1, \gamma_2)$. Hence $(u \odot v)_{(\min(\alpha_1,\alpha_2),\min(\beta_1,\beta_2),\max(\gamma_1,\gamma_2))} \in \vee q \mathcal{A}$. Which completes the proof. □

Theorem 3.3. Let $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic set in $\mathcal{K}$. Then $\mathcal{A}$ is an $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$ if and only if each non-empty $\mathcal{A}_{(\alpha,\beta,\gamma)}$ is a K-subalgebra of $\mathcal{K}$, for $\alpha, \beta \in (0.5, 1], \gamma \in [0.5, 1)$.

Proof. Assume that $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is an $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$ and let $\alpha, \beta \in (0.5, 1], \gamma \in [0.5, 1)$. To prove that $\mathcal{A}_{(\alpha,\beta,\gamma)} = \{u \in G \mid \mathcal{T}_{\mathcal{A}}(u) \geq \alpha, \mathcal{I}_{\mathcal{A}}(u) \geq \beta, \mathcal{F}_{\mathcal{A}}(u) \leq \gamma\}$ is a K-subalgebra of $\mathcal{K}$. If $u, v \in \mathcal{A}_{(\alpha,\beta,\gamma)}$, then $\mathcal{T}_{\mathcal{A}}(u) \geq \alpha, \mathcal{T}_{\mathcal{A}}(v) \geq \alpha, \mathcal{I}_{\mathcal{A}}(u) \geq \beta, \mathcal{I}_{\mathcal{A}}(v) \geq \beta, \mathcal{F}_{\mathcal{A}}(u) \leq \gamma, \mathcal{F}_{\mathcal{A}}(v) \leq \gamma$. Thus, $\mathcal{T}_{\mathcal{A}}(e) \geq \min(\mathcal{T}_{\mathcal{A}}(u), 0.5) \geq \min(\alpha, 0.5) = \alpha, \mathcal{I}_{\mathcal{A}}(e) \geq \min(\mathcal{I}_{\mathcal{A}}(u), 0.5) \geq \min(\beta, 0.5) = \beta, \mathcal{F}_{\mathcal{A}}(e) \leq \max(\mathcal{F}_{\mathcal{A}}(u), 0.5) \geq \min(\gamma, 0.5) = \gamma$ and $\mathcal{T}_{\mathcal{A}}(u \odot v) \geq \min(\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v), 0.5) \geq \min(\alpha, 0.5) = \alpha$, $\mathcal{I}_{\mathcal{A}}(u \odot v) \geq \min(\mathcal{I}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v), 0.5) \geq \min(\beta, 0.5) = \beta, \mathcal{F}_{\mathcal{A}}(u \odot v) \leq \max(\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v), 0.5) \leq \max(\gamma, 0.5) = \gamma$. Thus, $u \odot v \in \mathcal{A}_{(\alpha,\beta,\gamma)}$. Hence $\mathcal{A}_{(\alpha,\beta,\gamma)}$ is a K-subalgebra of $\mathcal{K}$. Converse part is obvious. □

Theorem 3.4. Let $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic set in $\mathcal{K}$. Then $\mathcal{A}_{(\alpha,\beta,\gamma)}$ is a K-subalgebra of $\mathcal{K}$ if and only if

(a) $\max(\mathcal{T}_{\mathcal{A}}(u \odot v), 0.5) \geq \min(\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v))$,
$\max(\mathcal{I}_{\mathcal{A}}(u \odot v), 0.5) \geq \min(\mathcal{I}_{\mathcal{A}}(u), \mathcal{I}_{\mathcal{A}}(v))$,
$\min(\mathcal{F}_{\mathcal{A}}(u \odot v), 0.5) \leq \max(\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v))$,

(b) $\max(\mathcal{T}_{\mathcal{A}}(e), 0.5) \geq (\mathcal{T}_{\mathcal{A}}(u)$,
$\max(\mathcal{I}_{\mathcal{A}}(e), 0.5) \geq (\mathcal{I}_{\mathcal{A}}(u)$,
$\min(\mathcal{F}_{\mathcal{A}}(e), 0.5) \leq (\mathcal{F}_{\mathcal{A}}(u)$, for all $u, v \in G$.

Proof. Suppose that $\mathcal{A}_{(\alpha,\beta,\gamma)}$ is a K-subalgebra of $\mathcal{K}$ and let $\max(\mathcal{T}_{\mathcal{A}}(u \odot v), 0.5) < \min(\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v)) = \alpha$, $\max(\mathcal{I}_{\mathcal{A}}(u \odot v), 0.5) < \min(\mathcal{I}_{\mathcal{A}}(u), \mathcal{I}_{\mathcal{A}}(v)) = \beta$, $\min(\mathcal{F}_{\mathcal{A}}(u \odot v), 0.5) > \max(\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v)) = \gamma$. Then for $\alpha, \beta \in (0.5, 1]$ and $\gamma \in [0.5, 1)$ and $u, v \in \mathcal{A}_{(\alpha,\beta,\gamma)}, \mathcal{T}_{\mathcal{A}}(u \odot v) < \alpha, \mathcal{I}_{\mathcal{A}}(u \odot v) < \beta, \mathcal{F}_{\mathcal{A}}(u \odot v) > \gamma$. Since $u, v \in \mathcal{A}_{(\alpha,\beta,\gamma)}$ and $\mathcal{A}_{(\alpha,\beta,\gamma)}$ is a K-subalgebra of $\mathcal{K}$, so $u, v \in \mathcal{A}_{(\alpha,\beta,\gamma)}$ or $\mathcal{T}_{\mathcal{A}}(u \odot v) \geq \alpha, \mathcal{I}_{\mathcal{A}}(u \odot v) \geq \beta, \mathcal{F}_{\mathcal{A}}(u \odot v) \leq \gamma$, a contradiction.

Conversely, suppose that conditions (a) and (b) holds. Let for $u, v \in \mathcal{A}_{(\alpha,\beta,\gamma)}$, we have $0.5 < \alpha \leq \min(\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v)) \leq \max(\mathcal{T}_{\mathcal{A}}(u \odot v), 0.5) \Rightarrow \mathcal{T}_{\mathcal{A}}(u \odot v) \geq \alpha$, $0.5 < \beta \leq \min(\mathcal{I}_{\mathcal{A}}(u), \mathcal{I}_{\mathcal{A}}(v)) \leq \max(\mathcal{I}_{\mathcal{A}}(u \odot v), 0.5) \Rightarrow \mathcal{I}_{\mathcal{A}}(u \odot v) \geq \beta$, $0.5 > \gamma \geq \max(\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v)) \geq \min(\mathcal{F}_{\mathcal{A}}(u \odot v), 0.5) \Rightarrow \mathcal{F}_{\mathcal{A}}(u \odot v) \leq \gamma$. $0.5 < \alpha \leq \mathcal{T}_{\mathcal{A}}(u) \leq \max(\mathcal{T}_{\mathcal{A}}(e), 0.5) \Rightarrow \mathcal{T}_{\mathcal{A}}(mu) \geq \alpha$, $0.5 < \beta \leq \mathcal{I}_{\mathcal{A}}(u) \leq \max(\mathcal{I}_{\mathcal{A}}(e), 0.5) \Rightarrow \mathcal{I}_{\mathcal{A}}(mu) \geq \beta$, $0.5 > \gamma \geq \mathcal{F}_{\mathcal{A}}(u) \geq \min(\mathcal{F}_{\mathcal{A}}(e), 0.5) \Rightarrow \mathcal{F}_{\mathcal{A}}(mu) \leq \gamma$, for some $m \in G$ $u \odot v \in \mathcal{A}_{(\alpha,\beta,\gamma)}$. Hence $\mathcal{A}_{(\alpha,\beta,\gamma)}$ is a K-subalgebra of $\mathcal{K}$. □

Theorem 3.5. The intersection of any family of $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$ is an $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$.

Proof. Let $\{\mathcal{A}_j : j \in I\}$ be a family of $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebras of $\mathcal{K}$.

Let $\mathcal{A} = \bigcap_{j \in I} \mathcal{A}_j = (\sup_{j \in I} \mathcal{T}_{\mathcal{A}_i}, \sup_{j \in I} \mathcal{I}_{\mathcal{A}_i}, \inf_{j \in I} \mathcal{F}_{\mathcal{A}_i})$, for $u, v \in G$, we have

$$\begin{aligned}
&\mathcal{T}_{\mathcal{A}}(u \odot v) \geq \min(\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v), 0.5),\\
&\mathcal{I}_{\mathcal{A}}(u \odot v) \geq \min(\mathcal{I}_{\mathcal{A}}(u), \mathcal{I}_{\mathcal{A}}(v), 0.5),\\
&\mathcal{F}_{\mathcal{A}}(u \odot v) \leq \max(\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v), 0.5).\\
&\mathcal{T}_{\mathcal{A}}(u \odot v) = \sup_{j \in I} \mathcal{T}_{\mathcal{A}_i}(u \odot v),\\
&\mathcal{T}_{\mathcal{A}}(u \odot v) \geq \sup_{j \in I} \min(\mathcal{T}_{\mathcal{A}_i}(u), \mathcal{T}_{\mathcal{A}_i}(v), 0.5),\\
&\mathcal{T}_{\mathcal{A}}(u \odot v) = \min(\sup_{j \in I} \mathcal{T}_{\mathcal{A}_i}(u), \sup_{j \in I} \mathcal{T}_{\mathcal{A}_i}(v), 0.5),\\
&\mathcal{T}_{\mathcal{A}}(u \odot v) = \min(\bigcap_{j \in I} \mathcal{T}_{\mathcal{A}_i}(u), \bigcap_{j \in I} \mathcal{T}_{\mathcal{A}_i}(v), 0.5),\\
&\mathcal{T}_{\mathcal{A}}(u \odot v) = \min(\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v), 0.5),\\
&\mathcal{I}_{\mathcal{A}}(u \odot v) = \sup_{j \in I} \mathcal{I}_{\mathcal{A}_i}(u \odot v),\\
&\mathcal{I}_{\mathcal{A}}(u \odot v) \geq \sup_{j \in I} \min(\mathcal{I}_{\mathcal{A}_i}(u), \mathcal{I}_{\mathcal{A}_i}(v), 0.5),\\
&\mathcal{I}_{\mathcal{A}}(u \odot v) = \min(\sup_{j \in I} \mathcal{I}_{\mathcal{A}_i}(u), \sup_{j \in I} \mathcal{I}_{\mathcal{A}_i}(v), 0.5),\\
&\mathcal{I}_{\mathcal{A}}(u \odot v) = \min(\bigcap_{j \in I} \mathcal{I}_{\mathcal{A}_i}(u), \bigcap_{j \in I} \mathcal{I}_{\mathcal{A}_i}(v), 0.5),
\end{aligned}$$

$$
\begin{aligned}
&\mathcal{I}_{\mathcal{A}}(u \odot v) = \min(\mathcal{I}_{\mathcal{A}}(u), \mathcal{I}_{\mathcal{A}}(v), 0.5),\\
&\mathcal{F}_{\mathcal{A}}(u \odot v) = \inf_{j\in I} \mathcal{F}_{\mathcal{A}_i}(u \odot v),\\
&\mathcal{F}_{\mathcal{A}}(u \odot v) \leq \inf_{j\in I} \max(\mathcal{F}_{\mathcal{A}_i}(u), \mathcal{F}_{\mathcal{A}_i}(v), 0.5),\\
&\mathcal{F}_{\mathcal{A}}(u \odot v) = \max(\inf_{j\in I} \mathcal{F}_{\mathcal{A}_i}(u), \inf_{j\in I} \mathcal{F}_{\mathcal{A}_i}(v), 0.5),\\
&\mathcal{F}_{\mathcal{A}}(u \odot v) = \max(\bigcap_{j\in I} \mathcal{F}_{\mathcal{A}_i}(u), \bigcap_{j\in I} \mathcal{F}_{\mathcal{A}_i}(v), 0.5),\\
&\mathcal{F}_{\mathcal{A}}(u \odot v) = \max(\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v), 0.5).
\end{aligned}
$$

It follows that $\mathcal{A}$ is an $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$. □

Definition 3.4. Let $\epsilon_1, \epsilon_2 \in [0, 1]$ and $\epsilon_1 < \epsilon_2$. Let $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic K-subalgebra of $\mathcal{K}$. Then $\mathcal{A}$ is called a single-valued neutrosophic K-subalgebra with thresholds (ϵ_1, ϵ_2) of $\mathcal{K}$ if

$$
\begin{aligned}
&\max(\mathcal{T}_{\mathcal{A}}(u \odot v), \epsilon_1) \geq \min(\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v), \epsilon_2),\\
&\max(\mathcal{I}_{\mathcal{A}}(u \odot v), \epsilon_1) \geq \min(\mathcal{I}_{\mathcal{A}}(u), \mathcal{I}_{\mathcal{A}}(v), \epsilon_2),\\
&\min(\mathcal{F}_{\mathcal{A}}(u \odot v), \epsilon_1) \leq \max(\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v), \epsilon_2), \text{ for all } u, v \in G.
\end{aligned}
$$

Example 3.2. Consider a K-algebra on a cyclic group of order 9 and ... table for $\odot$ is given in example 2.1. It is easy to see that $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra with thresholds $(\epsilon_1 = 0.3, \epsilon_2 = 0.56)$ and for $(\epsilon_1 = 0.55, \epsilon_2 = 0.78)$.

Remark 3.2. Let for $\epsilon_1, \epsilon_2 \in [0, 1]$ and $\epsilon_1 < \epsilon_2$ unless otherwise specified.

(i) When $\epsilon_1 = 0$ and $\epsilon_2 = 1$ in single-valued neutrosophic K-subalgebra with thresholds (ϵ_1, ϵ_2), $\mathcal{A}$ is an ordinary single-valued neutrosophic K-subalgebra.

(ii) When $\epsilon_1 = 0$ and $\epsilon_2 = 0.5$ in single-valued neutrosophic K-subalgebra with thresholds (ϵ_1, ϵ_2), $\mathcal{A}$ is an $(\in, \in \vee q)$single-valued neutrosophic K-subalgebra.

Theorem 3.6. A single-valued neutrosophic set $\mathcal{A}$ in $\mathcal{K}$ is a single-valued neutrosophic K-subalgebra with thresholds (ϵ_1, ϵ_2) if and only if $\cup(\mathcal{T}_{\mathcal{A}}, \alpha)$, $\cup^{'}(\mathcal{I}_{\mathcal{A}}, \beta)$, $L(\mathcal{F}_{\mathcal{A}}, \gamma)(\neq \phi), \alpha, \beta, \gamma \in (\epsilon_1, \epsilon_2]$ is a K-subalgebra of $\mathcal{K}$.

Proof. Assume that $\mathcal{A}$ is a single-valued neutrosophic K-subalgebra with thresholds (ϵ_1, ϵ_2). Let for $u, v \in \cup(\mathcal{T}_{\mathcal{A}}, \alpha)$ and $\alpha \in (\epsilon_1, \epsilon_2)$, $\mathcal{T}_{\mathcal{A}}(u) \geq \alpha$ and $\mathcal{T}_{\mathcal{A}}(v) \geq \alpha$. Since $\mathcal{A}$ is a single-valued neutrosophic K-subalgebra, it follows that $\max(\mathcal{T}_{\mathcal{A}}(u\odot v), \epsilon_1) \geq \min(\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v), \epsilon_2) = \alpha$, so that $u \odot v \in \cup(\mathcal{T}_{\mathcal{A}}, \alpha)$. Hence $\cup(\mathcal{T}_{\mathcal{A}}, \alpha)$ is a K-subalgebra of $\mathcal{K}$. Similarly, we can proof for $\cup^{'}(\mathcal{I}_{\mathcal{A}}, \beta)$ and $L(\mathcal{F}_{\mathcal{A}}, \gamma)$. Hence $\mathcal{A}_{(\alpha,\beta,\gamma)}$ is a K-subalgebra of $\mathcal{K}$.

Conversely, suppose that for $(\epsilon_1, \epsilon_2) \in [0, 1]$ and $\epsilon_1 < \epsilon_2$, $\mathcal{A}$ be a ...... K-subalgebra of K such that $\max(\mathcal{T}_{\mathcal{A}}(u \odot v), \epsilon_1) < \min(\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v), \epsilon_2) = \alpha$, then $\mathcal{T}_{\mathcal{A}}(u \odot v) < \alpha$, where $u \in \cup(\mathcal{T}_{\mathcal{A}}, \alpha), v \in \cup(\mathcal{T}_{\mathcal{A}}, \alpha), \alpha \in (\epsilon_1, \epsilon_2]$. Since $u, v \in \cup(\mathcal{T}_{\mathcal{A}}, \alpha)$ and $\cup(\mathcal{T}_{\mathcal{A}}, \alpha)$ is a K-subalgebra, $u \odot v \in \cup(\mathcal{T}_{\mathcal{A}}, \alpha)$, i.e., $\mathcal{T}_{\mathcal{A}}(u \odot v) \geq \alpha$, a contradiction. Similar results can be obtained for $\cup^{'}(\mathcal{I}_{\mathcal{A}}, \beta)$ and $L(\mathcal{F}_{\mathcal{A}}, \gamma)$. □

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