VIKOR based MAGDM Strategy Revisited in Bipolar Neutrosophic Set Environment

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Abstract: VIKOR strategy was proposed to solve Multi-Attribute Group Decision Making (MAGDM) in Bipolar Neutrosophic Set (BNS) environment, where compromise solutions were not identified. To overcome the shortcomings, the VIKOR strategy is revisited by incorporating compromise solutions in the BNS environment. Using the revisited VIKOR strategy, an MAGDM problem is solved. Sensitivity analysis is presented to reflect the impact of the decision-making mechanism coefficient on ranking of the alternatives.

Keywords: neutrosophic set; bipolar neutrosophic set; MADM; MAGDM; VIKOR; compromise solution

1. Introduction

Smarandache (1998) introduced the Neutrosophic Set (NS) that extended Fuzzy Set (FS) (Zadeh, 1965), and intuitionistic FS (Atanassov & Stoeva, 1983; Atanassov, 1983). New trends in neutrosophic research have been documented by many researchers (El-Hefenawy, 2016; Peng, & Dai, 2018; Otay,& Kahraman, 2018; Nguyen et al., 2019; Zhang et al., 2020; Muzaffar et al., 2020; Broumi et al., 2022).

Deli et al. (2015) grounded the Bipolar NS (BNS) by hybridizing the Bipolar FS (BFS)(Zhang, 1994, 1998) and the NS (Smarandache, 1998). Pramanik et al. (2017) presented the projection based MADM strategy under the BNS environment. Uluçay et al. (2018) presented the outranking approach for MADM in the BNS setting. Wang et al. (2018) developed the MADM strategy using "Frank Choquet Bonferroni operators" under the BNS setting. Pramanik et al. (2018a) developed the TODIM strategy under the BNS environment. Abdel-Basset et al. (2019) defined cosine SMs and established their properties to develop MADM strategies in BNS and Interval BN(IBNS) environments. Many researchers contributed to the development of BNSs (Akram & Sarwar, 2017; Chakraborty et al., 2019). Akram and Sarwar (2017) developed a new graph theory on the BNS environment. Chakraborty et al. (2019) presented the MADM strategy for the triangular BNS setting. Ali et al. (2017) grounded the bipolar neutrosophic soft set (BNSS) by combining soft sets (Molodtsov, 1999). and BNSs (Deli et al., 2015) and developed the MADM strategy using the aggregation operator under the BNSS setting. Hashim et al. (2020) presented the grey relational analysis-based MADM strategy in the neutrosophic bipolar FS setting.

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Opricovic (1998) proposed the "VlseKriterijuska Optimizacija I Komoromisno Resenje" (VIKOR) strategy with conflicting criteria. Opricovic and Tzeng (2003) presented the VIKOR strategy using FSs in analyzing land-use techniques to deal with natural hazards. Chen and Wang (2009) presented the fuzzy VIKOR strategy to present the optimal compromise solution. Chang (2010) presented the modified VIKOR strategy that has a logical judgment to improve the conventional VIKOR strategy. Vahdani et al. (2010) proposed the VIKOR strategy under interval FS assessment having unequal weights of criteria. Devi (2011) presented the VIKOR strategy in the Intuitionistic Fuzzy (IF) setting where the rating values and the weights of the criteria are presented in the form of triangular IF numbers. Park et al. (2011) presented the VIKOR strategy for MAGDM in interval-valued IF (IVIF) setting where ratings are presented in terms of IVIF numbers. Zhang and Wei (2013) presented the VIKOR and TOPSIS to deal with MADM problems under the hesitant FS environment. Mardani et al. (2016) documented a systematic review of the VIKOR strategy dealing with its methodologies and applications.

Rani et al. (2019) presented the VIKOR strategy in Pythagorean FS setting**.** VIKOR strategy under the trapezoidal BFS environment was presented by Shumaiza et al. (2019). Pouresmaeil et al.(2017) presented the VIKOR and TOPSIS for MAGDM in SVNS setting. VIKOR strategy under the Interval NS (INS) (Wang et al., 2005) environment was studied by Bausys and Zavadskas (2015), and Huang et al. (2017). Hu et al. (2017) discussed the VIKOR strategy based on projection measures under the INS setting. Unver et al. (2022) presented the VIKOR strategy for MAGDM in IF valued neutrosophic setting.

Pramanik et al. (2018b) presented the VIKOR strategy under the BNS environment where Compromise Solutions (CSs) are not identified. So there exists a research gap.

Motivation: To deal with the research gap, the VIKOR strategy is modified by defining CSs under the BNS environment

The structure of the remaining part of the paper is presented in Table 1.

Table 1. Outline of the paper

Section	**Content**
2	Some basic concepts and operations related to BNSs.
3	The modified VIKOR strategy is developed by incorporating CS.
4	Illustration of the developed VIKOR strategy.
5	Sensitivity analysis
6	The conclusion and direction of further research

2. Preliminaries

The basics of the BNS are recalled in this section.

Definition 2.1 BNS (Deli et al., 2015)
Let W be a space of objects, and $\omega \in W$ be a generic element. A BNS θ is defined as:

$$\theta = \{< \omega, \tau_\theta^+(\omega), \iota_\theta^+(\omega), \pi_\theta^+(\omega), \tau_\theta^-(\omega), \iota_\theta^-(\omega), \pi_\theta^-(\omega) >: \omega \in \Omega\},$$

where, $\tau_\theta^+(\omega), \iota_\theta^+(\omega), \pi_\theta^+(\omega): \Omega \to [0,1]$ and

$\tau_\theta^-(\omega), \iota_\theta^-(\omega), \pi_\theta^-(\omega): \Omega \to [-1,0]$.

Here, $\theta = < \tau_\theta^+, \iota_\theta^+, \pi_\theta^+, \tau_\theta^-, \iota_\theta^-, \pi_\theta^- >$ represents a Bipolar Neutrosophic Number (BNN).
Definition 2.2 Containment **(**Deli et al, 2015)

Assume that and
$$\theta_1 = \{< \omega, \tau_{\theta_1}^+(\omega), \iota_{\theta_1}^+(\omega), \pi_{\theta_1}^+(\omega), \tau_{\theta_1}^-(\omega), \iota_{\theta_1}^-(\omega), \pi_{\theta_1}^-(\omega) >: \omega \in \Omega\}$$

and

$$\theta_2 = \{< \omega, \tau_{\theta_2}^+(\omega), \iota_{\theta_2}^+(\omega), \pi_{\theta_2}^+(\omega), \tau_{\theta_2}^-(\omega), \iota_{\theta_2}^-(\omega), \pi_{\theta_2}^-(\omega) >: \omega \in \Omega\}$$

are any two BNSs in Ω. Then $\theta_1 \subseteq \theta_2$, iff

$\tau_{\theta_1}^+(\omega) \le \tau_{\theta_2}^+(\omega)$, $\iota_{\theta_1}^+(\omega) \ge \iota_{\theta_2}^+(\omega)$, $\pi_{\theta_1}^+(\omega) \ge \pi_{\theta_2}^+(\omega)$ and $\tau_{\theta_1}^-(\omega) \ge \tau_{\theta_2}^-(\omega)$, $\iota_{\theta_1}^-(\omega) \le \iota_{\theta_2}^-(\omega)$, $\pi_{\theta_1}^-(\omega) \le \pi_{\theta_2}^-(\omega)$, $\forall \omega \in \Omega$.

Definition 2.3 Equality (Deli et al., 2015)
Assume that
$$\theta_1 = \{< \omega, \tau_{\theta_1}^+(\omega), \iota_{\theta_1}^+(\omega), \pi_{\theta_1}^+(\omega), \tau_{\theta_1}^-(\omega), \iota_{\theta_1}^-(\omega), \pi_{\theta_1}^-(\omega) >: \omega \in \Omega\}$$
and
$$\theta_2 = \{< \omega, \tau_{\theta_2}^+(\omega), \iota_{\theta_2}^+(\omega), \pi_{\theta_2}^+(\omega), \tau_{\theta_2}^-(\omega), \iota_{\theta_2}^-(\omega), \pi_{\theta_2}^-(\omega) >: \omega \in \Omega\}$$
are any two BNSs in Ω. Then, $\theta_1 = \theta_2$, iff

$\tau_{\theta_1}^+(\omega) = \tau_{\theta_2}^+(\omega)$, $\iota_{\theta_1}^+(\omega) = \iota_{\theta_2}^+(\omega)$, $\pi_{\theta_1}^+(\omega) = \pi_{\theta_2}^+(\omega)$ and

$\tau^{-}_{\theta_1}(\omega)= \tau^{-}_{\theta_2}(\omega)$, $\iota^{-}_{\theta_1}(\omega) =\iota^{-}_{\theta_2}(\omega)$,
$\pi^{-}_{\theta_1}(\omega)= \iota^{-}_{\theta_2}(\omega), \forall \omega \in \Omega$.

Definition 2.4 Union (Deli et al., 2015)

Assume that

$\theta_1 = \{<\omega, \tau^{+}_{\theta_1}(\omega), \iota^{+}_{\theta_1}(\omega), \pi^{+}_{\theta_1}(\omega), \tau^{-}_{\theta_1}(\omega), \iota^{-}_{\theta_1}(\omega), \pi^{-}_{\theta_1}(\omega) >: \omega \in \Omega\}$

and

$\theta_2 = \{<\omega, \tau^{+}_{\theta_2}(\omega), \iota^{+}_{\theta_2}(\omega), \pi^{+}_{\theta_2}(\omega), \tau^{-}_{\theta_2}(\omega), \iota^{-}_{\theta_2}(\omega), \pi^{-}_{\theta_2}(\omega) >: \omega \in \Omega\}$

are any two BNSs in Ω. Then,

$$\theta_1(\omega) \cup \theta_2(\omega) = \{<\omega, \max(\tau^{+}_{\theta_1}(\omega), \tau^{+}_{\theta_2}(\omega)), \min(\iota^{+}_{\theta_1}(\omega), \iota^{+}_{\theta_2}(\omega)), \min(\pi^{+}_{\theta_1}(\omega), \pi^{+}_{\theta_2}(\omega)), \min(\tau^{-}_{\theta_1}(\omega), \tau^{-}_{\theta_2}(\omega)), \max(\iota^{-}_{\theta_1}(\omega), \iota^{-}_{\theta_2}(\omega)), \max(\pi^{-}_{\theta_1}(\omega), \pi^{-}_{\theta_2}(\omega)) >: \omega \in \Omega\}, \forall \omega \in \Omega\}.$$

Definition 2.5 Intersection (Deli et al., 2015)

Assume that

$\theta_1 = \{<\omega, \tau^{+}_{\theta_1}(\omega), \iota^{+}_{\theta_1}(\omega), \pi^{+}_{\theta_1}(\omega), \tau^{-}_{\theta_1}(\omega), \iota^{-}_{\theta_1}(\omega), \pi^{-}_{\theta_1}(\omega) >: \omega \in \Omega\}$

and

$\theta_2 = \{<\omega, \tau^{+}_{\theta_2}(\omega), \iota^{+}_{\theta_2}(\omega), \pi^{+}_{\theta_2}(\omega), \tau^{-}_{\theta_2}(\omega), \iota^{-}_{\theta_2}(\omega), \pi^{-}_{\theta_2}(\omega) >: \omega \in \Omega\}$

are any two BNSs in Ω. Then,

$$\theta_1(\omega) \cap \theta_2(\omega) = \{<\omega, \min(\tau^{+}_{\theta_1}(\omega), \tau^{+}_{\theta_2}(\omega)), \max(\iota^{+}_{\theta_1}(\omega), \iota^{+}_{\theta_2}(\omega)), \max(\pi^{+}_{\theta_1}(\omega), \pi^{+}_{\theta_2}(\omega)), \max(\tau^{-}_{\theta_1}(\omega), \tau^{-}_{\theta_2}(\omega)), \min(\iota^{-}_{\theta_1}(\omega), \iota^{-}_{\theta_2}(\omega)), \min(\pi^{-}_{\theta_1}(\omega), \pi^{-}_{\theta_2}(\omega)) >: \omega \in \Omega\}.$$

Definition 2.6 Compliment (Deli et al. 2015)

Assume that

$\theta_1 = \{<\omega, \tau^{+}_{\theta_1}(\omega), \iota^{+}_{\theta_1}(\omega), \pi^{+}_{\theta_1}(\omega), \tau^{-}_{\theta_1}(\omega), \iota^{-}_{\theta_1}(\omega), \pi^{-}_{\theta_1}(\omega) >: \omega \in \Omega\}$

is a BNS. Then the compliment of θ_1 is denoted as:

$$\theta_1' = \{<\omega, 1-\tau^{+}_{\theta_1}(\omega), 1-\iota^{+}_{\theta_1}(\omega), 1-\pi^{+}_{\theta_1}(\omega), \{-1\}-\tau^{-}_{\theta_1}(\omega), \{-1\}-\iota^{-}_{\theta_1}(\omega), \{-1\}-\pi^{-}_{\pi_1}(\omega) >: \omega \in \Omega\}$$

Definition 2.7 Hamming Distance (HD) (Pramanik et al., 2018a)

Assume that $\theta_1 = <\tau^{+}_{\theta_1}, \iota^{+}_{\theta_1}, \pi^{+}_{\theta_1}, \tau^{-}_{\theta_1}, \iota^{-}_{\theta_1}, \pi^{-}_{\theta_1}>$ **and** $\theta_2 = <\tau^{+}_{\theta_2}, \iota^{+}_{\theta_2}, \pi^{+}_{\theta_2}, \tau^{-}_{\theta_2}, \iota^{-}_{\theta_2}, \pi^{-}_{\theta_2}>$ are any two BNNs. HD between θ_1 and θ_2 is defined as:

$$\aleph(\theta_1, \theta_2) = \frac{1}{6}[|\tau^{+}_{\theta_1} - \tau^{+}_{\theta_2}| + |\iota^{+}_{\theta_1} - \iota^{+}_{\theta_2}| + |\pi^{+}_{\theta_1} - \pi^{+}_{\theta_2}| + |\tau^{-}_{\theta_1} - \tau^{-}_{\theta_2}| + |\iota^{-}_{\theta_1} - \iota^{-}_{\theta_2}| + |\pi^{-}_{\theta_1} - \pi^{-}_{\theta_2}| \quad (1)$$

3. Modified VIKOR Strategy for MAGDM under BNS environment

Assume that

i. ξ'_r, (r= 1, 2, ..., R) is the r-th alternative of R alternatives,
ii. λ'_s (s= 1,2, …,S) is the s-th attribute of S attributes,
iii. $\rho_m (\geq 0)$(m= 1, 2, …, S) is the weight of the k-th attribute and $\sum_{m=1}^{S} \rho_m = 1$,
iv. $\omega_t (\geq 0)$ is the Weight of t-th Decision Maker (DM) and ω_t (t = 1, 2, …T) and $\sum_{t=1}^{T} \omega_t = 1$,

Modified VIKOR strategy is developed as follows:

Step: 1. **Formulate the decision matrices**

Assume that $X^t = (\theta'^t_{rs})_{R\times S}$ (t = 1, 2, 3, ..., T) is the decision matrix of t-th DM, where rating of the alternative ξ_r is provided by the t-th DM over the attribute λ'_s (s = 1, 2, 3, ..., S). Assume that BNN $\theta'_{rs} = <\tau'^{+}_{rs}, \iota'^{+}_{rs}, \pi'^{+}_{rs}, \tau'^{-}_{rs}, \iota'^{-}_{rs}, \pi'^{-}_{rs}>$ reflects the rating value of thr r-th alternative over the s-th attribute. Then X^t is constructed as follows:

$$X^t = [\theta'^t_{rs}]_{R\times S} = \begin{pmatrix} & \lambda'_1 & \lambda'_2 & \dots & \lambda'_S \\ \xi'_1 & \theta'^t_{11} & \theta'^t_{12} & \dots & \theta'^t_{1S} \\ \xi'_2 & \theta'^t_{21} & \theta'^t_{22} & \dots & \theta'^t_{2S} \\ . & . & . & \dots & . \\ \xi'_R & \theta'^t_{R1} & \theta'^t_{R2} & \dots & \theta'^t_{RS} \end{pmatrix} \quad (2)$$

Here, $\theta'^t_{rs} = <\tau'^{t+}_{rs}, \iota'^{t+}_{rs}, \pi'^{t+}_{rs}, \tau'^{t-}_{rs}, \iota'^{t-}_{rs}, \pi'^{t-}_{rs}>$

Step: 2. Normalize the decision matrices

For cost type attribute, the normalization technique (Pramanik et al., 2018b) is employed as follows:

$$\tilde{\theta}'^{t*}_{rs} = < \{1\} - \tau'^{t+}_{rs}, \{1\} - \iota'^{t+}_{rs}, \{1\} - \pi'^{t+}_{rs}, \{-1\} - \tau'^{t-}_{rs}, \{-1\} - \iota'^{t-}_{rs}, \{-1\} - \pi'^{t-}_{rs} > \quad (3)$$

Using the formula (3), the normalized decision matrix (4) is formulated as:

$$X^t = \begin{pmatrix} & \lambda'_1 & \lambda'_2 & \ldots & \lambda'_S \\ \xi'_1 & \theta^t_{11} & \theta^t_{12} & \ldots & \theta^t_{1S} \\ \xi'_2 & \theta^t_{21} & \theta^t_{22} & \ldots & \theta^t_{2S} \\ . & . & . & \ldots & . \\ \xi'_R & \theta^t_{R1} & \theta^t_{R2} & \ldots & \theta^t_{RS} \end{pmatrix} \quad (4)$$

Here, $$\theta^t_{rs} = \begin{cases} \theta'^t_{rs}, & \text{when } \lambda_s \text{ is a benefit type attribute.} \\ \tilde{\theta}'^{t*}_{rs}, & \text{when } \lambda_s \text{ is a cost type attribute.} \end{cases}$$

Step: 3. Aggregate the decision matrices

BNN Weighted Averaging (BNWA) operator (Pramanik, et al. 2018b) is employed to aggregate the decision matrices (X^t) to formulate the Aggregated Decision Matrix (AGM) X as :

$$\theta'_{rs} = BNWA(\theta^1_{rs}, \theta^2_{rs}, \ldots, \theta^T_{rs}) = (\omega_1 \theta^1_{rs} \oplus \omega'_2 \theta^2_{rs} \oplus \omega'_3 \theta^3_{rs} \oplus \ldots \oplus \omega'_t \theta^T_{rs}) =$$
$$< \frac{1}{T}\left(\sum_{t=1}^{T} \omega_t \tilde{\tau}'^{+(t)}_{rs}, \sum_{t=1}^{T} \omega_t \tilde{\iota}'^{+(t)}_{rs}, \sum_{t=1}^{T} \omega_t \tilde{\pi}'^{+(t)}_{ij}, \sum_{t=1}^{T} \omega_t \tilde{\tau}'^{-(t)}_{rs}, \sum_{t=1}^{T} \omega_t \tilde{\iota}'^{-(t)}_{rs}, \sum_{t=1}^{T} \omega_t \tilde{\pi}'^{-(t)}_{rs} \right) > \quad (5)$$

r = 1, 2, 3, ..., R; s = 1, 2, 3, ..., S.

The ADM is obtained as follows:

$$X = \begin{pmatrix} & \lambda'_1 & \lambda'_2 & \ldots & \lambda'_S \\ \xi'_1 & \theta'_{11} & \theta'_{12} & \ldots & \theta'_{1S} \\ \xi'_2 & \theta'_{21} & \theta'_{22} & \ldots & \theta'_{2S} \\ . & . & . & \ldots & . \\ \xi'_R & \theta'_{R1} & \theta'_{R2} & \ldots & \theta'_{RS} \end{pmatrix} \quad (6)$$

Step: 4. Calculate **the "Positive Ideal Solution (PIS)" and "Negative Ideal Solution(NIS)"**

The "PIS" and "NIS" are calculated respectively as:

$$\theta'^+_{rs} = < \max_r \tau'^+_{rs}, \min_r \iota'^+_{rs}, \min_r \pi'^+_{rs}, \min_r \tau'^-_{rs}, \max_r \iota'^-_{rs}, \max_r \pi'^-_{rs} > \quad (7)$$

$$\theta'^-_{rs} = < \min_r \tau'^+_{rs}, \max_r \iota'^+_{rs}, \max_r \pi'^+_{rs}, \max_r \tau'^-_{rs}, \min_r \iota'^-_{rs}, \min_r \pi'^-_{rs} > \quad (8)$$

Step: 5. Compute the values of χ'_i and ζ'_i

"Maximum group utility" χ'_i and the "minimum individual regret of the opponent" ζ'_i are calculated as follows:

$$\chi'_r = \sum_{s=1}^{S} \frac{\rho_s \times \aleph(\theta'^+_{rs}, \theta_{rs})}{\aleph(\theta'^+_{rs}, \theta'^-_{rs})} \quad (9)$$

$$\zeta'_r = \max_s \left\{ \frac{\rho_s \times \aleph(\theta'^+_{rs}, \theta_{rs})}{\aleph(\theta'^+_{rs}, \theta'^-_{rs})} \right\} \quad (10)$$

Here, ρ_s presents the weight for λ'_s.

Step: 6. Calculate the values of $\mho'_r$

$$\mho'_r = \wp' \frac{(\chi'_r - \chi'^-)}{(\chi'^+ - \chi'^-)} + (1-\wp') \frac{(\zeta'_r - \zeta'^-)}{(\zeta'^+ - \zeta'^-)} \quad (11)$$

Here, $\chi'^- = \min_r \chi'_r$, $\chi'^+ = \max_r \chi'_r$,
$\zeta'^- = \min_r \zeta'_r$, $\zeta'^+ = \max_r \zeta'_r$, (r= 1, 2, 3, ..., R) (12)

and $\wp'$ is the "Decision-Making Mechanism Coefficient" (DMMC).

Step: 7. Prepare the ranking

Ranking of the alternatives is done using χ', ζ', and $\mho'$ in decreasing order.

Step 8. Fix the CS

The alternative ξ'^1 is a CS if it attains the best rank based on the measure $\mho'$ (minimum) subject to the two conditions:

C_1."Acceptable advantage"

$$\mho'(\xi'^2) - \mho'(\xi'^2) \geq D\mho', \quad (13)$$

where $D\mho' = \frac{1}{(R-1)}$, ξ'^{1}, ξ'^{2} are in the 1st and the 2nd rank by $\mho'$.

C2: "Acceptable stability in decision making":

Alternative ξ'^{1} must also be the best ranked by χ' or/and ς'.

The CS is stable within whole decision-making process that could be:

1. "voting by majority rule" (when $\wp' > 0.5$).

2.. "by consensus" (when $\wp' = 0.5$),

iii. "with veto" (If $\wp' < 0.5$)

If one of the conditions is not met, then CSs are identified as follows:

- Here, ξ'^{1} and ξ'^{2} are CSs if only C2 is not satisfied, or
- ξ'^{1}, ξ'^{2}, ξ'^{3},..., ξ'^{P} are CSs if C1 is not satisfied; and
- ξ'^{P} is determined by $\mho'(\xi'^{P}) - \mho'(\xi'^{1}) \le D\mho'$ for maximum P.

4. Illustrative Example

The MAGDM problem (Pramanik et al., 2018b) which is adapted from (Ye, 2014) is considered here. The MAGDM is described as follows:

An investment company forms an expert committee consisting of three DMs in order to invest a sum of money in the best option. The four alternatives are namely, Car company (ξ_1), Food company (ξ_2), Computer company (ξ_3) and Arms company (ξ_4). Three criteria are namely, risk factor (λ_1), growth factor (λ_2), environment impact (λ_3).

Weight vector of the attributes $\rho = (0.37, 0.33, 0.3)^T$ and weight vector of DMs $\omega = (0.38, 0.32, 0.3)^T$.

Following Zhang et al. (2016)., the criteria are considered as benefit type.

The problem is to find the best option for investment.

Step: 1.
Using the ratings provided by the DMs, the decision matrices are constructed in terms of BNNs as:

Decision matrix for 1st DM

$$M^1 = \begin{pmatrix} & \lambda_1 & \lambda_2 & \lambda_3 \\ \xi_1 & (.5, .6, .7, -.3, -.6, -.3) & (.8, .5, .6, -.4, -.6, -.3) & (.9, .4, .6, -.1, -.6, -.5) \\ \xi_2 & (.6, .2, .2, -.4, -.5, -.3) & (.6, .3, .7, -.4, -.3, -.5) & (.7, .5, .3, -.4, -.3, -.3) \\ \xi_3 & (.8, .3, .5, -.6, -.4, -.5) & (.5, .2, .4, -.1, -.5, -.3) & (.4, .2, .8, -.5, -.3, -.2) \\ \xi_4 & (.7, .5, .3, -.6, -.3, -.3) & (.8, .7, .2, -.8, -.6, -.1) & (.6, .3, .4, -.3, -.4, -.7) \end{pmatrix}$$

Decision matrix for 2nd DM

$$M^2 = \begin{pmatrix} & \lambda_1 & \lambda_2 & \lambda_3 \\ \xi_1 & (.6, .3, .4, -.5, -.3, -.7) & (.5, .3, .4, -.3, -.3, -.4) & (.1, .5, .7, -.5, -.2, -.6) \\ \xi_2 & (.7, .4, .5, -.3, -.2, -.1) & (.8, .4, .5, -.7, -.3, -.2) & (.6, .2, .7, -.5, -.2, -.9) \\ \xi_3 & (.8, .3, .2, -.5, -.2, -.6) & (.3, .2, .1, -.6, -.3, -.4) & (.7, .5, .4, -.4, -.3, -.2) \\ \xi_4 & (.3, .5, .2, -.5, -.5, -.2) & (.5, .6, .4, -.3, -.6, -.7) & (.4, .3, .8, -.5, -.6, -.5) \end{pmatrix}$$

Decision matrix for 3rd DM

$$M^3 = \begin{pmatrix} & \lambda_1 & \lambda_2 & \lambda_3 \\ \xi_1 & (.9, .6, .4, -.7, -.3, -.2) & (.7, .5, .3, -.6, -.2, -.5) & (.4, .2, .3, -.2, -.5, -.7) \\ \xi_2 & (.5, .3, .2, -.6, -.4, -.1) & (.5, .2, .7, -.3, -.2, -.5) & (.6, .3, .2, -.7, -.6, -.3) \\ \xi_3 & (.2, .5, .6, -.4, -.5, -.7) & (.3, .2, .7, -.2, -.3, -.5) & (.8, .2, .4, -.2, -.3, -.6) \\ \xi_4 & (.8, .5, .5, -.4, -.6, -.3) & (.9, .3, .4, -.5, -.6, -.7) & (.7, .4, .3, -.2, -.5, -.7) \end{pmatrix}$$

Step: 2.
In this problem, step 2 is not required as the criteria are benefit type.

Step: 3.

Utilizing BNWA (See the formula (5), the AGM is constructed as follows:

$$M = \begin{pmatrix} & \lambda_1 & \lambda_2 & \lambda_3 \\ \xi_1 & (.22, .17, .17, -.16, -.14, -.13) & (.22, .14, .15, -.14, -.13, -.13) & (.16, .12, .18, -.10, -.10, -.20) \\ \xi_2 & (.20, .10, .10, -.14, -.12, -.10) & (.21, .10, .21, -.15, -.10, -.13) & (.21, .11, .13, -.17, -.12, -.16) \\ \xi_3 & (.21, .12, .16, -.17, -.12, -.20) & (.13, .10, .13, -.10, -.12, -.13) & (.21, .10, .18, -.13, -.10, -.11) \\ \xi_4 & (.20, .17, .11, -.17, -.15, -.10) & (.24, .18, .11, -.19, -.20, -.16) & (.19, .11, .17, -.11, -.16, -.21) \end{pmatrix}$$

Step: 4.

The PIS =

$$\begin{pmatrix} \lambda_1 & \lambda_2 & \lambda_3 \\ (.22, .10, .10, -.14, -.12, -.10) & (.24, .10, .11, -.19, -.10, -.13) & (.21, .10, .13, -.17, -.10, -.11) \end{pmatrix}$$

and the NIS =

$\left(\overset{\lambda_1}{(.20, .17, .17, -.14, -.15, -.20)} \overset{\lambda_2}{(.13, .18, .21, -.10, -.20, -.16)} \overset{\lambda_3}{(.16, .12, .18, -.10, -.16, -.11)} \right)$

Step: 5.

Utilizing eq. (9), and eq. (10), we obtain

$\chi_1 = 0.75$, $\chi_2 = 0.38$, $\chi_3 = 0.60$, $\chi_4 = 0.75$ and $\zeta_1 =$ 0.34, $\zeta_2 = 0.16$, $\zeta_3 = 0.33$, $\zeta_4 = 0.34$.

Step: 6.

For, $\wp' = 0.5$, using eq. (11), the obtained results are :

$\mho'_1 = 1, \mho'_2 = 0, \mho'_3 = .77, \mho'_4 = 1.$

Step: 7.

The ranking order (see table 2) is obtained as:

$\xi_2 \succ \xi_3 \succ \xi_4 = \xi_1$.

Table 2. Ranking and CS

	ξ_1	ξ_2	ξ_3	ξ_4	Ranking order	CS
χ	0.75	0.38	0.60	0.75	$\xi_2 \succ \xi_3 \succ \xi_4 = \xi_1$	ξ_2
ζ	0.34	0.16	0.33	0.34	$\xi_2 \succ \xi_3 \succ \xi_4 = \xi_1$	ξ_2
$\mho'$ ($\wp' = 0.5$)	1	0	0.77	1	$\xi_2 \succ \xi_3 \succ \xi_4 = \xi_1$	ξ_2

Step 8. Determine the CS

We have $\mho'(\xi_2) = 0$, and $\mho'(\xi_3) = 0.77$.

Therefore, $\mho'(\xi_3) - \mho'(\xi_2) = 0.77 > 0.333$ that satisfies the condition 1

$$\mho'(\xi_2) - \mho'(\xi_1) \geq \frac{1}{(r-1)}.$$

According to χ, ζ, we see that ξ_2 is the best alternative that satisfies the condition 2.

So ξ_2 is the CS. Since ξ_2 satisfies the both conditions, no need to calculate the CS.

5. Sensitivity Analysis

Table 3 reflects impact of ranking orders for different DMMC $(\wp')$

Table 3 Values of, $\wp'$ and ranking of alternatives

$\wp'$	$\mho'_r$	Ranking
0.1	$\mho'_1 = 1$, $\mho'_2 = 0$, $\mho'_3 = 0.915$, $\mho'_4 = 1$	$\xi_2 \succ \xi_3 \succ \xi_4 = \xi_1$

0.2	$\mho'_1 = 1,\ \mho'_2 = 0,\ \mho'_3 = 0.88,\ \mho'_4 = 1$	$\xi_2 \succ \xi_3 \succ \xi_4 = \xi_1$
0.3	$\mho'_1 = 1,\ \mho'_2 = 0,\ \mho'_3 = 0.845,\ \mho'_4 = 1$	$\xi_2 \succ \xi_3 \succ \xi_4 = \xi_1$
0.4	$\mho'_1 = 1,\ \mho'_2 = 0,\ \mho'_3 = 0.81,\ \mho'_4 = 1$	$\xi_2 \succ \xi_3 \succ \xi_4 = \xi_1$
0.5	$\mho'_1 = 1,\ \mho'_2 = 0,\ \mho'_3 = 0.77,\ \mho'_4 = 1$	$\xi_2 \succ \xi_3 \succ \xi_4 = \xi_1$
0.6	$\mho'_1 = 1,\ \mho'_2 = 0,\ \mho'_3 = 0.74,\ \mho'_4 = 1$	$\xi_2 \succ \xi_3 \succ \xi_4 = \xi_1$
0.7	$\mho'_1 = 1,\ \mho'_2 = 0,\ \mho'_3 = 0.7,\ \mho'_4 = 1$	$\xi_2 \succ \xi_3 \succ \xi_4 = \xi_1$.
0.8	$\mho'_1 = 1,\ \mho'_2 = 0,\ \mho'_3 = 0.670,\ \mho'_4 = 1$	$\xi_2 \succ \xi_3 \succ \xi_4 = \xi_1$
0.9	$\mho'_1 = 1,\ \mho'_2 = 0, \mho'_3 =0.64,\ \mho'_4 = 1$	$\xi_2 \succ \xi_3 \succ \xi_4 = \xi_1$

Note 1: The ranking order remains the same for different values of DMMC.

6. Conclusion

In this paper, VIKOR strategy is revisited under the BNS environment to overcome the shortcoming in obtaining CS in the paper (Pramanik et al., 2018b). CS Strategy is incorporated in VIKOR strategy (Pramanik et al., 2018b). An illustrative MAGDM problem is solved to reflect the applicability of the modified VIKOR strategy. The impact of the DMMC on ranking of alternatives is shown by performing sensitivity analysis. The modified VIKOR strategy can be easily extended under the interval BNS setting. The modified VIKOR strategy is applicable in solving MADM and MAGDM such as fault diagnosis (Zhang et al., 2016), supplier selection(Chai et al., 2013, supply chain management (Fan &Stevenson, 2018), fault diagnosis (Zhang, Zhai, li, & MU, 2016)) project selection (Yazdi et al., 2020,), air surveillance (Fan et al., 2018) watershed hydrological system (Garg & Kaur, 2022), etc.

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