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Neutrosophic Data Envelopment Analysis: a Comprehensive Review and Current Trends

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Abstract

The concept of a neutrosophic set is a comprehensive extension of both fuzzy sets and Intuitionistic Fuzzy Sets (IFSs). It allows decision makers to assign three distinct membership degrees, enabling a more precise representation of uncertainty. Neutrosophic Data Envelopment Analysis (Neu-DEA) is an extended version of the Data Envelopment Analysis (DEA) and Fuzzy DEA (FDEA) concepts. It aims to assess the performance of Decision Making Units (DMUs) within a neutrosophic environment. Neu-DEA specifically tackles the challenges associated with evaluating performance when the input and output data are incomplete, ambiguous, or unsure. As a result, the Neu-DEAs have attracted substantial attention from scholars and academics. This article aims to provide an academic overview of the present state, development patterns, and future research directions of the Neu-DEA research. To do this, the study examines relevant publications using two analytical approaches: description analysis and literature review.

Keywords: Neutrosophic data envelopment analysis, Performance analysis, Efficiency analysis, Neutrosophic number.

1 | Introduction

The measuring of efficiency has significant importance in the field of management as it enables a deeper comprehension of a unit's previous achievements and facilitates strategic planning for its future growth. The evaluation of performance is a critical factor in decision-making across a range of domains and real-world situations. The technique of Data Envelopment Analysis (DEA) was first introduced over 45 years ago by Charnes et al. [1]. This method, also referred to as the CCR model, is widely recognized as a practical mathematical approach for evaluating the efficiency of Decision Making Units (DMUs). Following that,

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Banker et al. [2] modified the CCR model by including the convexity criterion and developed the BCC model for assessing efficiency. The primary benefit of the DEA model lies in its non-parametric nature, use of linear programming methods, capacity to handle multiple inputs and outputs, and its ability to measure efficiency and identify potential improvements. Therefore, the DEA models have garnered significant scholarly interest and has been extensively used in several practical domains. In addition to the commonly used CCR and BCC models, numerous other DEA models have been created from different points of view. These include the Range Adjusted Measure (RAM) [3], additive model [4], Slacks-Based Measure (SBM) model [5], super-efficiency model [6], cross-efficiency DEA model [7], undesirable DEA model [8] and many others. These models provide different approaches to measuring efficiency in DEA and have been widely used in various fields such as finance, economics, healthcare, energy, agriculture, sustainability and operations research [9]-[13]. However, a notable challenge associated with using these conventional approaches is to the acquisition of precise input and output data. Data ambiguity may arise due to a variety of factors, including a lack of understanding or awareness of the data, the unavailability of some data, reliance on expert opinions, the use of language concepts that are difficult to quantify, and the inclusion of data that cannot be measured or quantified. The use of traditional DEA models for performance assessment and ranking of DMUs is not feasible when dealing with imprecise and unclear data. Consequently, it is important to construct DEA models in order to properly address these concerns; otherwise, the reliability and validity of the efficiency scores and rankings of the DMUs may be compromised [14], [15].

Real-world data often exhibits several sources of uncertainty that need the proper management of such uncertainty. Fuzzy set theory, introduced by Zadeh [16], is a mathematical framework that provides an approach to define and express uncertainty. Fuzzy set theory enables the management of imprecise or ambiguous data by assigning degrees of membership to elements in a set. Fuzzy DEA (FDEA) is a variant of the conventional DEA technique, which integrates fuzzy logic and fuzzy sets to effectively manage imprecise and uncertain data. After that, many authors developed different techniques to solve FDEA models [17]. The solution techniques for the FDEA model may be categorized into several approaches, such as the tolerance technique, α -cut approach, fuzzy ranking approach, the possibility approach, fuzzy arithmetic, multi-objective approach and fuzzy random/type-2 fuzzy set approach. Zhou and Xu [18] provided an overview of FDEA models, hybrid FDEA models and some real-life applications. However, it is important to note that only FS has a single membership value for the uncertain and imprecise data, which may become inadequate in several real-life situations when attempting to depict both supportive and opposing evidence simultaneously. In order to tackle this issue, Atanassov [19] introduced the concept of Intuitionistic Fuzzy Set (IFS), which considers both the degree of truth and the degree of falsehood associated with every piece of information. Several methods have been established in the scientific literature for addressing the DEA model by using IFSs. These approaches may be categorized as the expected value approach [20], the α , β -cut approach [21], parametric approach [22], possibility mean approach [23] and the ranking approach [24]. Recently, other extension of FS like fermatean fuzzy set [25] and spherical fuzzy set [26], [27] are used to construct the DEA models and its solution techniques are developed. The concept of neutrosophic sets was first proposed in the academic literature by Smarandache [28]. This was done as a response to the limitations of IFSs, which were only capable of handling partial information and not indeterminate or contradictory information. neutrosophic set is a generalization of crisp set, fuzzy set, IFS, inconsistent IFS (picture fuzzy set, ternary fuzzy set), Pythagorean fuzzy set, fermatean set, q-rung orthopair fuzzy set, spherical fuzzy set, and n-hyperspherical fuzzy set *Fig. 1*. neutrosophic sets have been used in several disciplines, including medicine, economics, image processing, decision-making, and risk assessment. This application has facilitated a more sophisticated and comprehensive methodology for representing and analyzing ambiguity and vagueness within real-life scenarios.

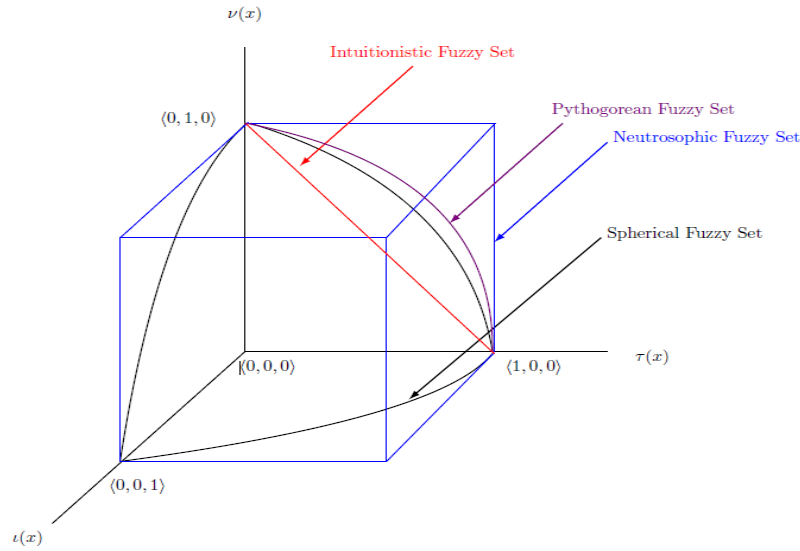


Fig. 1. Geometric representation of fuzzy set and its extension sets.

In contrast to the conventional literature review approach, this work undertakes a comprehensive examination of existing research on Neu-DEA via the use of bibliometric analysis. Therefore, we are able to provide a thorough overview of the current state of Neu-DEA research, including several dimensions such as authors, publications, keywords, co-citations, publication types, publications by year, and models. These findings may provide further support for the conclusions drawn from the comprehensive literature review. This study examines the existing literature relevant to the Neu-DEA model, which was obtained from the Web of Science (WoS), Scopus and Google Scholar, database.

This review article is organized in the following manner: Section 2 provides a comprehensive assessment of the existing literature on the latest advancements in the Neu-DEA model. Additionally, it conducts a comparative analysis of this model with other approaches in the field. Section 3 provides a bibliometric study of the literature on Neu-DEA, focusing on the publishing trends by year, author, and citation. Section 4 concludes with performing an in-depth review of the future pathways for further research, as well as highlighting a notable limitation that exists in the existing approaches employed in the field of Neu-DEA.

2 | Literature Review: Neutrosophic DEA

Neutrosophic Data Envelopment Analysis (Neu-DEA) is a novel approach that has emerged in the realm of performance assessment, specifically designed to handle situations when the observed input and output variables exhibit a high degree of uncertainty. This area has been established by the integration of neutrosophic set and DEA, enabling policymakers to make more precise decisions in the study of performance problems within highly unpredictable environments. In the year 2018, a theoretical development in the form of the Neu-DEA model was proposed by Edalatpanah [29]. This groundbreaking study included several researchers and authors in the development of a solution approach for the Neu-DEA model, aiming to properly quantify the performance of DMUs within a neutrosophic environment. Abdelfattah [30] used triangular neutrosophic number as input and output, and developed a unique solution strategy for the Neu-DEA model, referred known as the parametric approach. This approach involves converting the Neu-DEA model into a crisp interval value DEA model, allowing for the determination of efficiency scores in interval form. Kahraman et al. [31] employed interval value neutrosophic numbers as both input and output in their study. They introduced a new approach that combines Analytic Hierarchy Process (AHP) and DEA. The weights of the inputs and outputs were determined using pairwise comparison matrices derived from neutrosophic AHP. Additionally, deneutrosophication was applied in the DEA application to calculate the relative efficiency of the DMUs. Edalatpanah and Smarandache [32] employed simplified neutrosophic

numbers as input and output in BCC model, and then used weighted arithmetic average operator and logarithm technique to convert the neutrosophic BCC model into a corresponding crisp LP model in order to find the relative efficiency of the DMUs. The method suggested by Edalatpanah and Smarandache [32] to solve the neutrosophic BCC model is also used to solve the neutrosophic undesirable BCC model when both desirable and undesirable outputs are present [33]. Again, Edalatpanah [34] introduced a new ranking function for TNN and presented a unique approach to solve the neutrosophic CCR model when TNN inputs and outputs are present. This approach transforms the model into a crisp LP model, making it possible to the determination of the relative efficiency of the DMUs. In their study, Yang et al. [35] used single value neutrosophic numbers as both input and output variables. They utilized a ranking function proposed by Chakraborty et al. [36] to address the neutrosophic CCR model. This approach was applied to evaluate the efficiency of 13 hospitals affiliated with Tehran university of medical sciences in Iran. This study is the first attempt to address the issue of real-world performance assessment by using the Neu-DEA model. Nabeeh [37] developed a novel approach that combines neutrosophic DEMATEL with AR-DEA to enhance the process of technology selection. The DEMATEL first identifies positive and negative areas in the form of cause-and-effect relationships before introducing a system of evaluations for input-output relationships in the context of the technology selection process. Second, the application of AR-DEA to assess the efficiency of DMUs based on AR weight constraints involves multiple decision makers. Öztaş et al. [38] used a Plitogenic set as both the input and output variables. The performance evaluation of 16 hotels located in Cesme, Turkey is conducted using DEA, which depends on the use of subjective assessments that have been aggregated by Plithogenic operators. Tapia [39] developed a novel Neu-DEA model that is specifically designed for the evaluation of Negative Emissions Technologies (NETs). This model incorporates the use of interval value neutrosophic numbers and employs sensitivity analysis to assess the impact of various levels of risk attitudes among experts. Abdelfattah [40] used both a ranking technique and a parametric strategy in order solve the Neu-CCR model, which was utilized to evaluate the relative efficiency of 32 regional hospitals in Tunisia. Monazzam et al. [41] employed the interval value neutrosophic number as an input and output in the Neutrosophic cross efficiency DEA model in order to determine the relative efficiency of the automotive industrial plant in Iran. Jaber Hafshjani et al. [42] suggested a hybrid BSC-DEA model to calculate DMU efficiency and rank DMUs in a neutrosophic environment. In the hybrid BSC-DEA model, the BSC model is used to measure performance indices, while the neutrosophic DEA model is used to evaluate the efficiency of the DMUs. Again, Tapia et al. [43] utilized the robust tolerance technique to solve the modified neutrosophic CCR model and evaluate the performance of eight energy storage technologies. Mohanta and Toragay [44] introduced a new ranking function specifically designed for pentagonal neutrosophic numbers. This ranking function was used to address the pentagonal neutrosophic DEA model. A unique raking function for the trapezoidal neutrosophic number was developed by Mohanta and Sharanappa [45] and is used to solve the neutrosophic DEA and neutrosophic super efficiency model, which evaluate seaport efficiency and rank them completely. Mohanta et al. [46] created a novel method for solving neutrosophic DEA model termed the possibility mean approach, which converts it into corresponding crisp LP model that assess the efficiency of the DMUs. Mohanta et al. [47] used the possibilistic mean technique to determine the relative efficiency of the 8 AIIMS by considering the inputs and outputs of the SVTrNN. In order to solve the Neu-DEA model and determine the relative efficiency of hospitals, Mohanta and Sharanappa [48] developed a unique ranking technique based on the value and ambiguity index of SVTNN. Again, the possibilistic mean approach is utilized to solve the independent and relational neutrosophic two stage network DEA model to determine the relative efficiency of the Indian insurance sector [48]. Li et al. [49] used the neutrosophic DEA model, which uses neutrosophic numbers as input and output, to measure and assess the safety performance of 35 construction projects in China. Rasinojehdehi and Valami [50] suggested a parametric technique for solving the neutrosophic network DEA model in order to determine the relative efficiency of 13 Iranian airlines. However, by an extensive review of the existing literature, it becomes apparent that several solution techniques have been developed to address various forms of uncertainty in the Neu-DEA model. However, there is a limited number of authors that have used these models to tackle real-

world problems. *Table 1* presents a comprehensive comparison of the published works on Neu-DEA that use neutrosophic data type, approaches, models, and their applications in real-world problems.

Table 1. Comparative analysis of the neutrosophic DEA literature.

Source	Data Type	Concept	Model	Application
[29]	Single valued triangular Neutrosophic number	Score and Accuracy function	CCR	Theoretical
[31]	Simplified neutrosophic sets	Deneutrosophication	Integrated AHP-DEA	NA
[30]	Triangular neutrosophic numbers	Parametric approach	CCR	NA
[32]	Simplified neutrosophic sets	Logarithm approach	BCC	NA
[34]	Triangular neutrosophic numbers	Ranking approach	CCR	NA
[33]	Simplified neutrosophic sets	Logarithm approach	Undesirable BCC	NA
[35]	Single-valued neutrosophic number	Ranking approach	CCR	Hospital
[37]	Triangular neutrosophic number	MCDM approach	Hybrid DEMATEL-DEA-AR	Technology selection
[38]	Triangular neutrosophic numbers	MCDM approach	Hybrid MCDM-CCR	Hotels' performances
[39]	Interval value numbers	Robust tolerance approach	Modified CCR	NETs
[40]	Triangular neutrosophic numbers	Ranking and Parametric approach	CCR	Hospitals
[41]	Interval value neutrosophic number		Cross- efficiency DEA	Automotive industry
[42]	neutrosophic number	Direct approach	Hybrid BCA-DEA	Bank
[43]	Interval value numbers	Robust tolerance approach	Modified CCR	ESTs
[44]	Pentagonal neutrosophic number	Ranking approach	CCR	NA
[47]	Trapezoidal neutrosophic number	Possibilistic mean approach	CCR	Hospital
[46]	Triangular neutrosophic number	Possibility mean approach	CCR	NA
[45]	Trapezoidal neutrosophic number	Ranking approach	CCR and supper efficiency	Seaport
[48]	Triangular neutrosophic number	Ranking approach	CCR	Hospital
[49]	neutrosophic number		DEA	Construction projects
[50]	Triangular neutrosophic number	Parametric approach	Network SBM DEA	Arline
[48]	Triangular and trapezoidal neutrosophic number	Possibilistic mean approach	Two stage network DEA	Insurance industry

3 | Bibliometric Analysis and Current Research Trends

This section presents a detailed and systematic literature overview of investigations on Neu-DEA. Consequently, a methodical and thorough investigation is conducted across many databases, such as WoS, Scopus, and Google Scholar. The search methodology employed in this study is depicted in *Fig. 2*. The keywords stated here were queried in the “title, abstract, keywords” sections of publications from the WoS, Scopus, and Google Scholar databases, as indicated in *Fig. 2*. Subsequently, the removal of duplicate papers is undertaken, while the exclusion of grey literature is implemented. A total of 21 pertinent studies were identified within the Neu-DEA literature spanning the years 2018 to 2023.

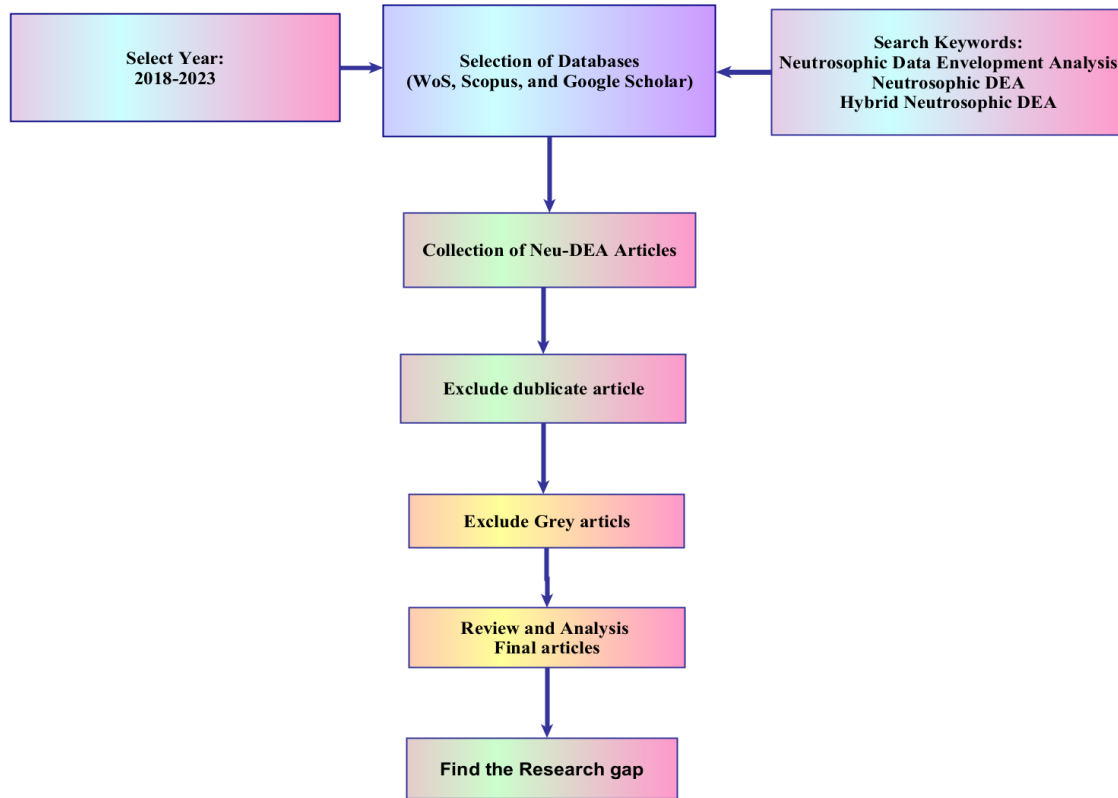


Fig. 2. Research methodology and search strategy.

Table 2 presents comprehensive data on the total number of articles published within the domain of neutrosophic DEA, categorized by year, title of article, journal, and citation.

Table 2. The neutrosophic DEA studies.

Source	Year	Title of Article	Journal	Citation
[29]	2018	Neutrosophic perspective on DEA	Journal of applied research on industrial engineering	42
[31]	2019	An integrated AHP and DEA methodology with neutrosophic sets	Springer book chapter	26
[30]	2019	DEA with neutrosophic inputs and outputs	Expert systems	26
[32]	2019	DEA for simplified neutrosophic sets	neutrosophic sets and systems	55
[34]	2020	DEA based on triangular neutrosophic numbers	CAAI transactions on intelligence technology	60
[33]	2020	A neutrosophic-based approach in DEA with undesirable outputs	Mathematical problems in engineering	35
[35]	2020	Triangular single valued Neu-DEA : application to hospital performance measurement	Symmetry	49
[37]	2020	A hybrid neutrosophic approach of DEMATEL with AR-DEA in technology selection	Neutrosophic sets and systems	15
[38]	2020	An alternative approach for performance evaluation: Plithogenic sets and DEA	International conference on intelligent and fuzzy systems	8
[39]	2021	Evaluating NETs using Neu-DEA	Journal of cleaner production	20
[40]	2021	Neu-DEA: an application to regional hospitals in Tunisia	neutrosophic sets and systems	7
[41]	2021	Online simulation optimization using neutrosophic cross-efficiency DEA and box-behnken experimental design (a case study on the automotive paint shop)	International journal of information technology and decision making	1

Table 2. Continued.

Source	Year	Title of Article	Journal	Citation
[42]	2021	A hybrid BCA-DEA model with indeterminate information	Journal of mathematics	10
[43]	2022	Selection of energy storage technologies under neutrosophic decision environment	Cleaner engineering and technology	2
[44]	2023	Enhanced performance evaluation through Neu-DEA leveraging pentagonal neutrosophic numbers	Journal of operational and strategic analytics	1
[47]	2023	Neu-DEA based on the possibilistic mean approach	Operations research and decisions	3
[46]	2023	A novel modified approach for solving Neu-DEA	Croatian operational research review	1
[45]	2023	A novel method for solving Neu-DEA models based on single-valued trapezoidal neutrosophic numbers	Soft computing	1
[51]	2023	Value and ambiguity index-based ranking approach for solving Neu-DEA	neutrosophic sets and systems	0
[49]	2023	Evaluation of safety-based performance in construction projects with Neu-DEA	Management decision	8
[50]	2023	A comprehensive neutrosophic model for evaluating the efficiency of airlines based on SBM model of network DEA	Decision making: applications in management and engineering	0
[48]	2023	Development of the neutrosophic two stage network DEA to measure the performance of the insurance industry	Soft computing	0

Fig. 3 displays the distribution of 22 articles, with one published as a book chapter, one published in conference proceedings, and the other 20 published in academic journals. The majority of publications, including 90.9%, are journal articles, while book chapters and conference papers each account for 4.5% of the total publications.

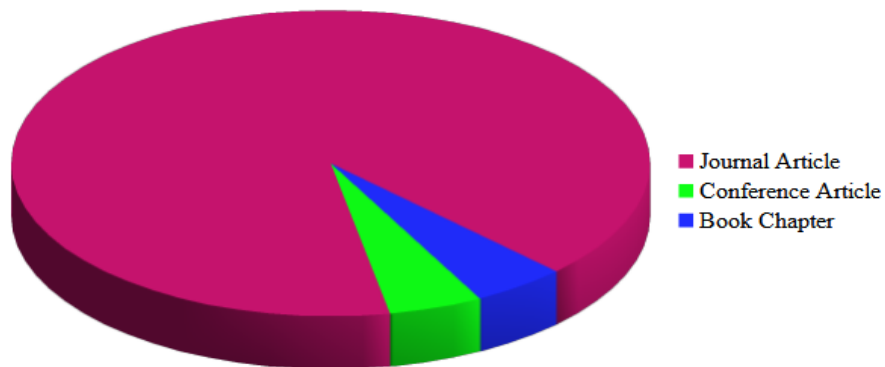


Fig. 3. Distribution of Neu-DEA articles across various publication types.

Fig. 4 depicts the quantitative representation of scholarly works related to the neutrosophic DEA approach till the year 2023. In the year 2018, there was a single publication, however, in the following year of 2019, the number of publications increased to three. The year 2020 had a notable rise in the quantity of publications reaching a count of 5. In the following year, 2021, this figure decreased to 4. However, in the year 2022, the number of publications experienced a comparatively lower count of 1. However, there was a decline in the quantity of articles in the year 2022. Currently, a total of eight articles have been published in the year 2023. However, it is anticipated that this figure will see growth before the conclusion of the year.

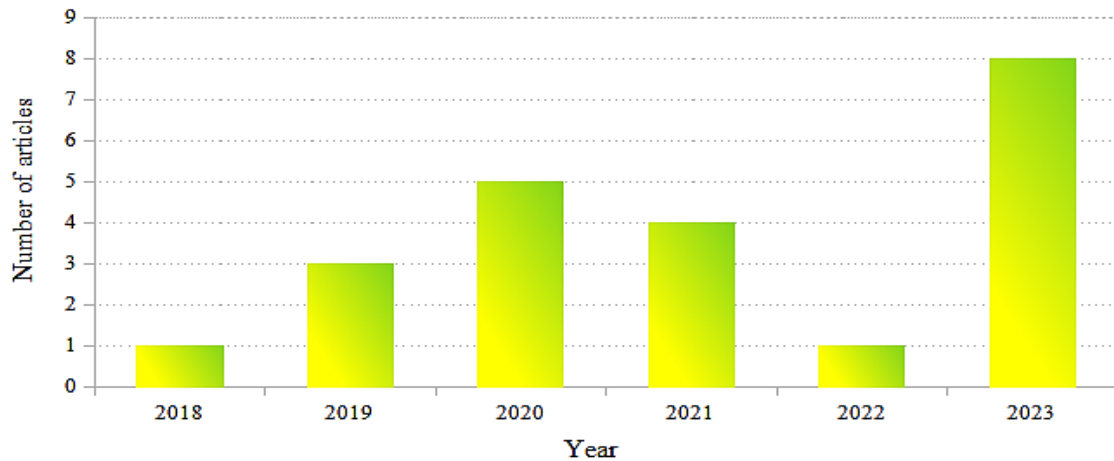


Fig. 4. Year wise article published in Neu-DEA area.

Fig. 5 depicts the distribution of publications among the prominent writers in the field of Neu-DEA literature. K. K. Mohanta is the most prominent researcher in the subject of neutrosophic DEA, having authored six publications, which is around 27% of the overall scholarly contributions in this domain. S. A. Edalatpanah and D. S. Sharanappa are the second most prolific researchers, having authored five publications apiece, which accounts for nearly 23% of the total reports. Following closely behind are F. Smarandache, A. Aggarwal, W. Abdelfattah, and J. F. D. Tapia, each having contributed two articles. Several other writers have made significant contributions to this area via their own works. These authors include W. Yang, L. Cai, M. Jaber Hafshjani, S.E. Najafi, F. Hosseinzadeh Lotfi, S.M. Hajimolana, X. Mao, Z. Guoxi, M. Fallah, and so on.

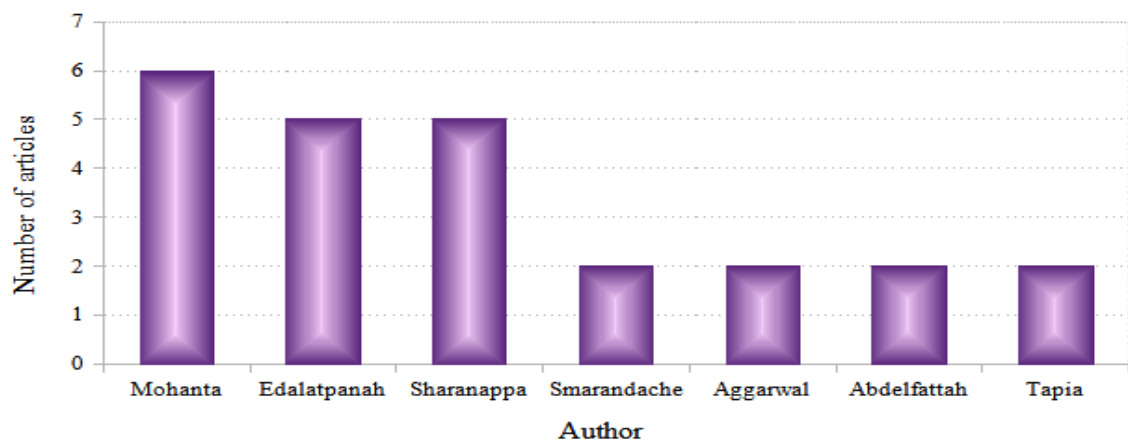


Fig. 5. Top contributor author in Neu-DEA area.

Fig. 6 is an illustration of the distribution of publications among the top articles in the field of Neu-DEA literature, with a particular emphasis on the top 10 works that have received the most citations. The article with the highest number of citations in the field of Neu-DEA is referenced as [34] and was published in 2020 by Edalatpanah. Following this, the second most cited article is referenced as [32] and was published in 2019 by Edalatpanah, and Smarandache. The third most cited article, referenced as [35], was published in 2020 by Yang, Cai, Edalatpanah, and Smarandache. Moving on, the fourth most cited article, referenced as [29], was published in 2018 by Edalatpanah. The fifth most cited article, referenced as [33], was published in 2020 by Mao, Guoxi, Fallah, and Edalatpanah. Similarly, the sixth most cited article, referenced as [30], was published in 2019 by Abdelfattah. The seventh most cited article, referenced as [31], was published in 2019 by Kaharaman et al. Continuing, the eighth most cited article, referenced as [39], was published in 2021 by Tapia.

The ninth most cited article, referenced as [37], was published in 2020 by Nabeeh. Lastly, the tenth most cited article, referenced as [42], was published in 2021 by Jaber Hafshjani, Najafi, Hosseinzadeh Lotfi, and Hajimolana.

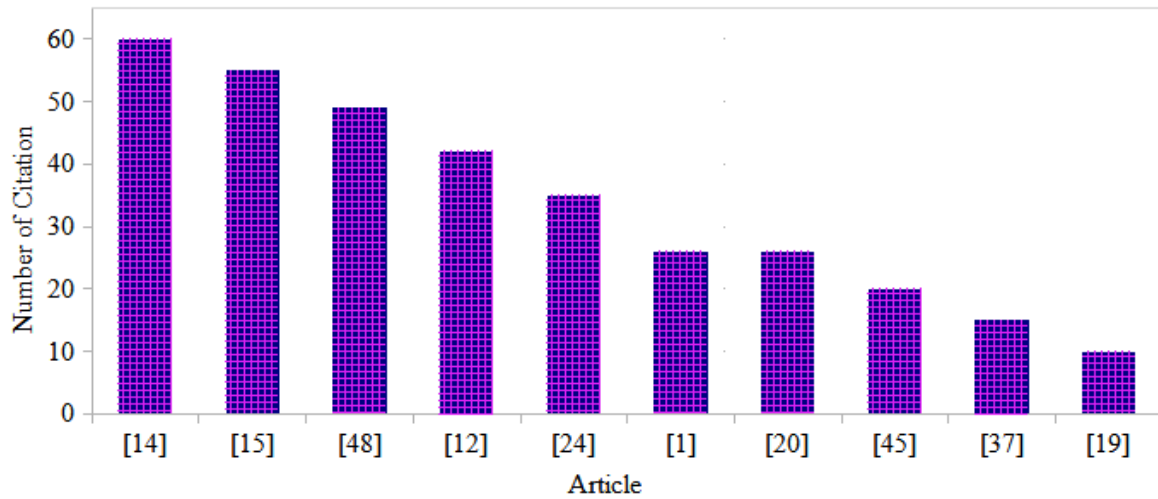


Fig. 6. Top 10 most cited article.

Fig. 7 is a schematic diagram illustrating the various neutrosophic DEA models. Among the 22 publications analysed, 12 were found to be based on the CCR model, while 4 articles used the hybrid DEA model. Additionally, 2 articles were based on the network DEA approach, while 1 article each focused on the BCC model, undesirable model, the cross-efficiency DEA model, and the super-efficiency DEA model. The article published in the field of Neu-DEA aims to measure efficiency by developing traditional DEA models. Specifically, 54.54% of the articles focus on the CCR model, 18.18% on the Hybrid DEA model, 9.09% on the network DEA model, and each of the BCC model, undesirable model, cross-efficiency DEA model, and super-efficiency DEA model represent 4.54% of the articles.

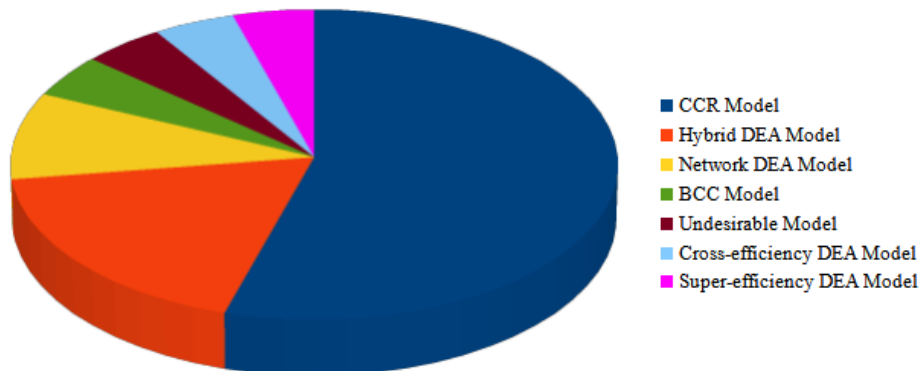


Fig. 7. Type of DEA model extended to Neu-DEA model.

4 | Conclusions and Future Directions

The integration of DEA method and neutrosophic set in context of neutrosophic DEA, is one of the applicable and effective approach that widely employed for performance measurement of DMUs under highly data ambiguity. Accordingly, this paper introduced a comprehensive literature review of neutrosophic DEA. Moreover, all Neu-DEA studies are analysed and classified according to several aspects and factors such as

bibliometric information, the features and objectives of the Neu-DEA model, real-world application and case study, basic DEA model, characteristics of DEA modelling.

The present study has identified that a significant proportion of research pertaining to DEA in a neutrosophic environment has been carried out within the domains of business economics, engineering, and operations research and management science. The bibliometric study revealed K. K. Mohanta, S.A. Edalatpanah, D.S. Sharanappa, F. Smarandache, A. Aggarwal, W. Abdelfattah, and J.F.D. Tapia as the most prolific writers in this particular discipline. The contributors [34], [32] have produced publications that have received significant citations in this particular topic. The following scholarly publications have published works on the topic of neutrosophic DEA: soft computing, journal of cleaner production, neutrosophic sets and systems, expert systems, CAAI transactions on intelligence technology, and operations research and decisions. The current literature in this particular domain has been mostly concerned with the advancement of solution methodologies for Neu-DEA models. Academic researchers are now engaged in efforts to address the difficulties associated with navigating complicated and unpredictable efficiency environments.

Based on the categorization, observations, and comments presented in the preceding parts, this section will outline the suggested research fronts in the area of Neu-DEA. These recommended fronts aim to provide guidance for future research efforts. The following domains are recommended for further investigation:

- I. Neu-DEA models may be formulated in the context of negative data, undesirable factors, non-discretionary factors, and dual-role factors.
- II. The development of Neutrosophic Network Data Envelopment Analysis (Neu-NDEA) enables the evaluation and ranking of homogeneous DMUs with a network topology, including two-stage, series, parallel, and mixed configurations.
- III. The development of Neutrosophic Dynamic Data Envelopment Analysis (Neu-DDEA) enables the assessment of dynamic efficiency of DMUs using neutrosophic panel data. This is achieved via the integration of Dynamic Data Envelopment Analysis (DDEA), Malmquist Productivity Index (MPI), and Window Data Envelopment Analysis (WDEA) techniques.
- IV. The development of hybrid neutrosophic DEA-MCDM techniques by integrating DEA and MCDM techniques using neutrosophic data to measure the relative efficiency and to completely rank the DMUs.
- V. The Neu-DEA models, whether in their original form or with further modifications, may be effectively used for the assessment of performance and productivity in many real-world issues and applications.

Author Contributions

Conceptualization, methodology, writing-creating the initial design: KKM; writing-reviewing and editing: DSS. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest

The authors declare no conflict of interest.

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