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NEUTROSOPHIC GENERALIZED SEMI ALPHA STAR CLOSED SETS IN NEUTROSOPHIC TOPOLOGICAL SPACES

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Abstract
The aim of this paper is to introduce a new concept of Neutrosophic closed sets namely Neutrosophic generalized semi alpha star closed sets (Neutrosophic $gs\alpha^* -$ closed sets) in Neutrosophic topological spaces. Properties and characterizations of Neutrosophic generalized semi alpha star closed sets are derived and compared with already existing sets.
Keywords: $N_{eu}gs\alpha^*$ −closed sets , $N_{eu}gs\alpha^*$ −open sets , $N_{eu}gs\alpha^*$ −interior , $N_{eu}gs\alpha^*$ − closure.
抽象的
本文的目的是在中智拓扑空间中引入一个新的中智闭集概念，即中智广义半阿尔法星闭集（**Neutrosophic gsα^*-**闭集）。 导出了中智广义半阿尔法星封闭集的性质和特征，并与现有的集进行了比较。
关键词：N_eu gsα^*-闭集，**N_eu gsα^*-开集**，**N_eu gsα^*-内部**，**N_eu gsα^*-**闭包。

I. INTRODUCTION

The term "neutrosophic" etymologically comes from "neutrosophy" which means knowledge of neutral thought . F.Smarandache[6] first introduced the concept of Neutrosophic set theory and it is based on intuitionistic fuzzy sets by K.Atanassov's[2] and also based on fuzzy sets by L.A.Zadeh's[15] . It includes three components , truth , indeterminancy and false membership function . The real life application of neutrosophic topology is applied in Information Systems , Applied Mathematics etc . R.Dhavaseelan and S.Jafari[4] has discussed about the concept of generalized neutrosophic closed sets .

In this paper, we introduce some new concepts in neutrosophic topological spaces such as Neutrosophic $gs\alpha^*$ −closed sets and Neutrosophic $gs\alpha^*$ −open sets. We also studied the relationship between Neutrosophic β −closed set , Neutrosophic α −closed set, Neutrosophic pre-closed set, Neutrosophic semi-closed set, Neutrosophic generalized Closed set,etc.

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II. PRELIMINARIES

Definition 2.1:[13] Let $\mathbb{P}$ be a non-empty fixed set . A Neutrosophic set H on the universe $\mathbb{P}$ is defined as $H=\{\langle \wp, (t_{H}(\wp), i_{H}(\wp), f_{H}(\wp))\rangle : \wp \in \mathbb{P}\}$ where $t_{H}(\wp), i_{H}(\wp), f_{H}(\wp)$ represent the degree of membership function $t_{H}(\wp)$, the degree of indeterminacy $i_{H}(\wp)$ and the degree of non-membership function $f_{H}(\wp)$ respectively for each element $\wp \in \mathbb{P}$ to the set H . Also , t_{H} , i_{H} , $f_{H} : \mathbb{P} \to]^{-}0 , 1^{+}[$ and $^{-}0 \leq t_{H}(\wp) + i_{H}(\wp) + f_{H}(\wp) \leq 3^{+}$. Set of all Neutrosophic set over $\mathbb{P}$ is denoted by $N_{eu}(\mathbb{P})$.

Definition 2.2:[13] Let $\mathbb{P}$ be a non-empty set. $\mathbb{A} = \{\langle \wp, (t_{\mathbb{A}}(\wp), i_{\mathbb{A}}(\wp), f_{\mathbb{A}}(\wp)) \rangle : \wp \in \mathbb{P}\}$ and $\mathbb{B} = \{\langle \wp, (t_{\mathbb{B}}(\wp), i_{\mathbb{B}}(\wp), f_{\mathbb{B}}(\wp)) \rangle : \wp \in \mathbb{P}\}$ are neutrosophic sets , then

(i) $\mathbb{A} \subseteq \mathbb{B}$ if $t_{\mathbb{A}}(\wp) \leq t_{\mathbb{B}}(\wp), i_{\mathbb{A}}(\wp) \leq i_{\mathbb{B}}(\wp), f_{\mathbb{A}}(\wp) \geq f_{\mathbb{B}}(\wp)$ for all $\wp \in \mathbb{P}$.

(ii) $\mathbb{A} \cap \mathbb{B} = \{\langle \wp, (\min(t_{\mathbb{A}}(\wp), t_{\mathbb{B}}(\wp)), \min(i_{\mathbb{A}}(\wp), i_{\mathbb{B}}(\wp)), \max(f_{\mathbb{A}}(\wp), f_{\mathbb{B}}(\wp))) \rangle : \wp \in \mathbb{P}\}$.

(iii) $\mathbb{A} \cup \mathbb{B} = \{\langle \wp, (\max(t_{\mathbb{A}}(\wp), t_{\mathbb{B}}(\wp)), \max(i_{\mathbb{A}}(\wp), i_{\mathbb{B}}(\wp)), \min(f_{\mathbb{A}}(\wp), f_{\mathbb{B}}(\wp))) \rangle : \wp \in \mathbb{P}\}$.

(iv) $\mathbb{A}^{c} = \{\langle \wp, (f_{\mathbb{A}}(\wp) , 1 - i_{\mathbb{A}}(\wp) , t_{\mathbb{A}}(\wp)) \rangle : \wp \in \mathbb{P}\}$.

(v) $0_{N_{eu}} = \{\langle \wp, (0,0,1)\rangle : \wp \in \mathbb{P}\}$ and $1_{N_{eu}} = \{\langle \wp, (1,1,0)\rangle : \wp \in \mathbb{P}\}$.

Definition 2.3:[13] A neutrosophic topology (N_{eu}T) on a non-empty set $\mathbb{P}$ is a family $\tau_{N_{eu}}$ of neutrosophic sets in $\mathbb{P}$ satisfying the following axioms ,

(i) $0_{N_{eu}}$, $1_{N_{eu}} \in \tau_{N_{eu}}$.

(ii) $\mathbb{A}_1 \cap \mathbb{A}_2 \in \tau_{N_{eu}}$ for any $\mathbb{A}_1$, $\mathbb{A}_2 \in \tau_{N_{eu}}$.

(iii) $\cup \mathbb{A}_i \in \tau_{N_{eu}}$ for every family $\{ \mathbb{A}_i / i \in \Omega \} \subseteq \tau_{N_{eu}}$.

In this case , the ordered pair $(\mathbb{P}, \tau_{N_{eu}})$ or simply $\mathbb{P}$ is called a neutrosophic topological space (N_{eu}TS) . The elements of $\tau_{N_{eu}}$ is neutrosophic open set ($N_{eu} - OS$) and $\tau_{N_{eu}}{}^{c}$ is neutrosophic closed set ($N_{eu} - CS$) .

Definition 2.4: A neutrosophic set $\mathbb{A}$ of a N_{eu}TS $(\mathbb{P}, \tau_{N_{eu}})$ is said to be

(i) a neutrosophic pre – closed set ($N_{eu}P - CS$) [7] if $N_{eu} - cl(N_{eu} - \mathrm{int}(\mathbb{A})) \subseteq \mathbb{A}$.

(ii) a neutrosophic semi – closed set ($N_{eu}S - CS$) [7] if $N_{eu} - int(N_{eu} - \mathrm{cl}(\mathbb{A})) \subseteq \mathbb{A}$.

(iii) a neutrosophic α – closed set ($N_{eu}\alpha - CS$) [7] if $N_{eu} - cl(N_{eu} - \mathrm{int}(N_{eu} - cl(\mathbb{A}))) \subseteq \mathbb{A}$.

(iv) a neutrosophic β – closed set ($N_{eu}\beta - CS$) [7] if $N_{eu} - int(N_{eu} - \mathrm{cl}(N_{eu} - int(\mathbb{A}))) \subseteq \mathbb{A}$.

(v) a neutrosophic regular – closed set ($N_{eu}R - CS$)[7] if $N_{eu} - cl(N_{eu} - \mathrm{int}(\mathbb{A})) = \mathbb{A}$.

(vi) a neutrosophic b – closed set ($N_{eu}b - CS$) [9] if $(Neu - cl(Neu - \mathrm{int}(\mathbb{A}))) \cap (Neu - int(Neu - \mathrm{cl}(\mathbb{A}))) \subseteq \mathbb{A}$.

(vii) a neutrosophic semi α – closed set ($N_{eu}S\alpha - CS$) [7] if $N_{eu} - int(N_{eu}\alpha - \mathrm{cl}(\mathbb{A})) \subseteq \mathbb{A}$.

(viii) a neutrosophic π – open set ($N_{eu}\pi - OS$) [10] if $\mathbb{A} = \cup\{ \mathcal{G} : \mathcal{G}\ is\ a\ N_{eu}R - OS\ in\ \mathbb{P} \}$.

Definition 2.5: Let $\mathbb{A}$ be a neutrosophic set in N_{eu}TS $(\mathbb{P}, \tau_{N_{eu}})$. Then ,

(1) $N_{eu} - int(\mathbb{A}) = \cup\{ \mathcal{G} : \mathcal{G}\ is\ a\ N_{eu} - OS\ in\ \mathbb{P}\ and\ \mathcal{G} \subseteq \mathbb{A} \}$ [4] .

(2) $N_{eu} - cl(\mathbb{A}) = \cap\{\mathcal{K} : \mathcal{K}\ is\ a\ N_{eu} - CS\ in\ \mathbb{P}\ and\ \mathbb{A} \subseteq \mathcal{K} \}$ [4] .

(3) $N_{eu}-\alpha int(\mathbb{A}) = \cup \{ \mathcal{G} : \mathcal{G}\ is\ a\ N_{eu}\alpha - OS\ in\ \mathbb{P}\ and\ \mathcal{G} \subseteq \mathbb{A} \} = \mathbb{A} \cap N_{eu} - int\left(N_{eu} - \text{cl}\left(N_{eu} - int(\mathbb{A})\right)\right)$ [7] .

(4) $N_{eu}-\alpha cl(\mathbb{A}) = \cap \{ \mathcal{K} : \mathcal{K}\ is\ a\ N_{eu}\alpha - CS\ in\ \mathbb{P}\ and\ \mathbb{A} \subseteq \mathcal{K}\} = \mathbb{A} \cup N_{eu} - cl\left(N_{eu} - \text{int}\left(N_{eu} - cl(\mathbb{A})\right)\right)$ [7] .

(5) $N_{eu}\beta - int(\mathbb{A}) = \mathbb{A} \cap N_{eu} - cl\left(N_{eu} - \text{int}\left(N_{eu} - cl(\mathbb{A})\right)\right)$, $N_{eu}\beta - cl(\mathbb{A}) = \mathbb{A} \cup N_{eu} - int\left(N_{eu} - \text{cl}\left(N_{eu} - int(\mathbb{A})\right)\right)$ [10] .

(6) $N_{eu}P - int(\mathbb{A}) = \mathbb{A} \cap N_{eu} - int\left(N_{eu} - \text{cl}(\mathbb{A})\right)$, $N_{eu}P - cl(\mathbb{A}) = \mathbb{A} \cup N_{eu} - cl\ \left(N_{eu} - \text{int}(\mathbb{A})\right)$ [5] .

(7) $N_{eu}S - int(\mathbb{A}) = \mathbb{A} \cap N_{eu} - cl\left(N_{eu} - \text{int}(\mathbb{A})\right)$, $N_{eu}S - cl(\mathbb{A}) = \mathbb{A} \cup N_{eu} - int\ \left(N_{eu} - \text{cl}(\mathbb{A})\right)$ [5] .

(8) $N_{eu}b - int(\mathbb{A}) = (N_{eu}S - int(\mathbb{A})) \cup (N_{eu}P - int(\mathbb{A}))$, $N_{eu}b - cl(\mathbb{A}) = \left(N_{eu}S - cl(\mathbb{A})\right) \cap \left(N_{eu}P - cl(\mathbb{A})\right)$ [5] .

Definition 2.6: A neutrosophic set $\mathbb{A}$ of a N_{eu}TS $(\mathbb{P}, \tau_{N_{eu}})$ is said to be

(1) a neutrosophic generalized closed set $(N_{eu}g - CS)$ [11] if $N_{eu} - cl(\mathbb{A}) \subseteq \mathcal{G}$, whenever $\mathbb{A} \subseteq \mathcal{G}$ and $\mathcal{G}$ is $N_{eu} - OS$ in $\mathbb{P}$.

(2) a neutrosophic generalized semi - closed set $(N_{eu}gs - CS)$ [11] if $N_{eu}S - cl(\mathbb{A}) \subseteq \mathcal{G}$, whenever $\mathbb{A} \subseteq \mathcal{G}$ and $\mathcal{G}$ is $N_{eu} - OS$ in $\mathbb{P}$.

(3) a neutrosophic generalized b - closed set $(N_{eu}gb - CS)$ [9] if $N_{eu}b - cl(\mathbb{A}) \subseteq \mathcal{G}$, whenever $\mathbb{A} \subseteq \mathcal{G}$ and $\mathcal{G}$ is $N_{eu} - OS$ in $\mathbb{P}$.

(4) a neutrosophic $\alpha -$ generalized closed set $(N_{eu}\alpha g - CS)$ [7] if $N_{eu}\alpha - cl(\mathbb{A}) \subseteq \mathcal{G}$, whenever $\mathbb{A} \subseteq \mathcal{G}$ and $\mathcal{G}$ is $N_{eu} - OS$ in $\mathbb{P}$.

(5) a neutrosophic generalized $\alpha -$closed set $(N_{eu}g\alpha - CS)$ [8] if $N_{eu}\alpha - cl(\mathbb{A}) \subseteq \mathcal{G}$, whenever $\mathbb{A} \subseteq \mathcal{G}$ and $\mathcal{G}$ is $N_{eu}\alpha - OS$ in $\mathbb{P}$.

(6) a neutrosophic generalized $\beta -$closed set $(N_{eu}g\beta - CS)$ [10] if $N_{eu}\beta - cl(\mathbb{A}) \subseteq \mathcal{G}$, whenever $\mathbb{A} \subseteq \mathcal{G}$ and $\mathcal{G}$ is $N_{eu} - OS$ in $\mathbb{P}$.

(7) a neutrosophic $b -$ generalized closed set $(N_{eu}bg - CS)$ [8] if $N_{eu}b - cl(\mathbb{A}) \subseteq \mathcal{G}$, whenever $\mathbb{A} \subseteq \mathcal{G}$ and $\mathcal{G}$ is $N_{eu}b - OS$ in $\mathbb{P}$.

(8) a neutrosophic generalized regular closed set $(N_{eu}gR - CS)$ [3] if $N_{eu}R - cl(\mathbb{A}) \subseteq \mathcal{G}$, whenever $\mathbb{A} \subseteq \mathcal{G}$ and $\mathcal{G}$ is $N_{eu} - OS$ in $\mathbb{P}$.

(9) a neutrosophic $\pi -$generalized beta closed set $(N_{eu}\pi g\beta - CS)$ [10] if $N_{eu}\beta - cl(\mathbb{A}) \subseteq \mathcal{G}$, whenever $\mathbb{A} \subseteq \mathcal{G}$ and $\mathcal{G}$ is $N_{eu}\pi - OS$ in $\mathbb{P}$.

(10) a neutrosophic $\alpha^* -$ open set $(N_{eu}\alpha^* - OS)$ [1] if $\mathbb{A} \subseteq N_{eu}\alpha - int\ \left(N_{eu} - cl\left(N_{eu}\alpha - int\ (\mathbb{A})\right)\right)$

III.NEUTROSOPHIC gsα^* −CLOSED SETS

Definition 3.1: A neutrosophic set $\mathbb{A}$ in a N_{eu}TS $(\mathbb{P}, \tau_{N_{eu}})$ is called a neutrosophic generalized semi alpha star closed set $(N_{eu}gs\alpha^* - CS)$ if $N_{eu}\alpha - int\left(N_{eu}\alpha - cl(\mathbb{A})\right) \subseteq N_{eu} - int(\mathcal{G})$, whenever $\mathbb{A} \subseteq \mathcal{G}$ and $\mathcal{G}$ is $N_{eu}\alpha^* -$ open set .

Theorem 3.2: Every $N_{eu} - CS$ is $N_{eu}gs\alpha^* - CS$, but not conversely .

Proof:

Let $\mathbb{A}\subseteq$W , W is $N_{eu}\alpha^* - OS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Since $\mathbb{A}$ is $N_{eu} - CS$, then $N_{eu} - cl(\mathbb{A}) = \mathbb{A}$ [14] . Now , $N_{eu}\alpha - int\left(N_{eu}\alpha - cl(\mathbb{A})\right) \subseteq N_{eu}\alpha - int\left(N_{eu} - cl(\mathbb{A})\right) \supseteq N_{eu} - int\ \left(N_{eu} - cl(\mathbb{A})\right) = N_{eu} - int(\mathbb{A}) \subseteq N_{eu} - int(\text{W}) \Rightarrow N_{eu}\alpha - int\left(N_{eu}\alpha - cl(\mathbb{A})\right) \subseteq N_{eu} - int(\text{W})$ [14,12] . Hence , $\mathbb{A}$ is $N_{eu}gs\alpha^* - CS$.

Example 3.3: Let $\mathbb{P}$ =$\{\wp\}$ and $\mathbb{A}$= $\{\langle\wp, (0.5,0.3,0.8)\rangle\}$ be $N_{eu}(\mathbb{P})$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ is a N_{eu}TS on $(\mathbb{P}, \tau_{N_{eu}})$. $\mathbb{A}^c =$

$\{\langle \wp, (0.8,0.7,0.5)\rangle\}$. Let $\mathcal{G} = \{\langle \wp, (0.4,0.2,0.9)\rangle\}$ be any $N_{eu}(\mathbb{P})$. $N_{eu}\alpha^* - OS = N_{eu}\alpha - OS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ and $N_{eu}\alpha - CS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}^c\}$. $N_{eu}\alpha - cl(\mathcal{G}) = \mathbb{A}^c \cap 1_{N_{eu}} = \mathbb{A}^c$. Now , $N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = N_{eu}\alpha - int(\mathbb{A}^c) = 0_{N_{eu}} \cup \mathbb{A} = \mathbb{A} \Rightarrow N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = \mathbb{A} \subseteq N_{eu} - int(\mathbb{A}), N_{eu} - int(1_{N_{eu}}) = \mathbb{A}, 1_{N_{eu}}$ whenever $\mathcal{G} \subseteq \mathbb{A}, 1_{N_{eu}}$. Hence , $\mathcal{G}$ is $N_{eu}gs\alpha^* - CS$. But $\mathcal{G}$ is not $N_{eu} - CS$, because $N_{eu} - cl(\mathcal{G}) = \mathbb{A}^c \cap 1_{N_{eu}} = \mathbb{A}^c \neq \mathcal{G}$.

Theorem 3.4: Every $N_{eu}\alpha - CS$ is $N_{eu}gs\alpha^* - CS$, but not conversely .

Proof:

Let $\mathbb{A} \subseteq \mathrm{W}$, W is $N_{eu}\alpha^* - OS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Since $\mathbb{A}$ is $N_{eu}\alpha - CS$, then $N_{eu} - cl\left(N_{eu} - int(N_{eu} - cl(\mathbb{A}))\right) \subseteq \mathbb{A}$. Now , $N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathbb{A})) \subseteq N_{eu}\alpha - int\ (N_{eu} - cl(\mathbb{A})) \supseteq N_{eu} - int(N_{eu} - cl(\mathbb{A})) \subseteq N_{eu} - cl\left(N_{eu} - int(N_{eu} - cl(\mathbb{A}))\right) \supseteq N_{eu} - int\left(N_{eu} - cl\left(N_{eu} - int(N_{eu} - cl(\mathbb{A}))\right)\right) \subseteq N_{eu} - int(\mathbb{A}) \subseteq N_{eu} - int(\mathrm{W}) \Rightarrow N_{eu}\alpha - int\ (N_{eu}\alpha - cl(\mathbb{A})) \subseteq N_{eu} - int(\mathrm{W})$. Hence , $\mathbb{A}$ is $N_{eu}gs\alpha^* - CS$.

Example 3.5: Let $\mathbb{P} = \{\wp\}$ and $\mathbb{A} = \{\langle \wp, (0.5,0.3,0.8)\rangle\}$ be $N_{eu}(\mathbb{P})$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ is a N_{eu}TS on $(\mathbb{P}, \tau_{N_{eu}})$. $\mathbb{A}^c = \{\langle \wp, (0.8,0.7,0.5)\rangle\}$. Let $\mathcal{G} = \{\langle \wp, (0.7,0.8,0.7)\rangle\}$ be any $N_{eu}(\mathbb{P})$. $N_{eu}\alpha^* - OS = N_{eu}\alpha - OS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ and $N_{eu}\alpha - CS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}^c\}$. $N_{eu}\alpha - cl(\mathcal{G}) = 1_{N_{eu}}$. Now , $N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = N_{eu}\alpha - int(1_{N_{eu}}) = 1_{N_{eu}} \Rightarrow N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = 1_{N_{eu}} \subseteq N_{eu} - int(1_{N_{eu}}) = 1_{N_{eu}}$ whenever $\mathcal{G} \subseteq 1_{N_{eu}}$. Hence , $\mathcal{G}$ is $N_{eu}gs\alpha^* - CS$. But $\mathcal{G}$ is not $N_{eu}\alpha - CS$, because $N_{eu} - cl\left(N_{eu} - int(N_{eu} - cl(\mathcal{G}))\right) = 1_{N_{eu}} \nsubseteq \mathcal{G}$.

Theorem 3.6: Every $N_{eu}S - CS$ is $N_{eu}gs\alpha^* - CS$, but not conversely .

Proof:

Let $\mathbb{A} \subseteq \mathrm{W}$, W is $N_{eu}\alpha^* - OS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Since $\mathbb{A}$ is $N_{eu}S - CS$, then $N_{eu} - int\ (N_{eu} - cl(\mathbb{A})) \subseteq \mathbb{A}$. Now , $N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathbb{A})) \subseteq N_{eu}\alpha - int(N_{eu} - cl(\mathbb{A})) \supseteq N_{eu} - int(N_{eu} - cl(\mathbb{A})) \subseteq \mathbb{A} \supseteq N_{eu} - int(\mathbb{A}) \subseteq N_{eu} - int(\mathrm{W}) \Rightarrow N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathbb{A})) \subseteq N_{eu} - int(\mathrm{W})$. Hence , $\mathbb{A}$ is $N_{eu}gs\alpha^* - CS$.

Example 3.7: Let $\mathbb{P} = \{\wp\}$ and $\mathbb{A} = \{\langle \wp, (0.4,0.5,0.7)\rangle\}$ be $N_{eu}(\mathbb{P})$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ is a N_{eu}TS on $(\mathbb{P}, \tau_{N_{eu}})$. $\mathbb{A}^c = \{\langle \wp, (0.7,0.5,0.4)\rangle\}$. Let $\mathcal{G} = \{\langle \wp, (0.2,0.3,0.5)\rangle\}$ be any $N_{eu}(\mathbb{P})$. $N_{eu}\alpha^* - OS = N_{eu}\alpha - OS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ and $N_{eu}\alpha - CS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}^c\}$. $N_{eu}\alpha - cl(\mathcal{G}) = \mathbb{A}^c \cap 1_{N_{eu}} = \mathbb{A}^c$. Now , $N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = N_{eu}\alpha - int(\mathbb{A}^c) = 0_{N_{eu}} \cup \mathbb{A} = \mathbb{A} \Rightarrow N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = \mathbb{A} \subseteq N_{eu} - int(1_{N_{eu}}) = 1_{N_{eu}}$ whenever $\mathcal{G} \subseteq 1_{N_{eu}}$. Hence , $\mathcal{G}$ is $N_{eu}gs\alpha^* - CS$. But $\mathcal{G}$ is not $N_{eu}S - CS$, because $N_{eu} - int(N_{eu} - cl(\mathcal{G})) = N_{eu} - int(\mathbb{A}^c \cap 1_{N_{eu}}) = N_{eu} - int(\mathbb{A}^c) = 0_{N_{eu}} \cup \mathbb{A} = \mathbb{A} \nsubseteq \mathcal{G}$.

Theorem 3.8: Every $N_{eu}\alpha^* - CS$ is $N_{eu}gs\alpha^* - CS$, but not conversely .

Proof:

Let $\mathbb{A} \subseteq \mathrm{W}$, W is $N_{eu}\alpha^* - OS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Since $\mathbb{A}$ is $N_{eu}\alpha^* - CS$, then $N_{eu}\alpha - cl\ \left(N_{eu} - int(N_{eu}\alpha - cl(\mathbb{A}))\right) \subseteq \mathbb{A}$. Now, $N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathbb{A})) \supseteq N_{eu} -$

$int(N_{eu}\alpha - cl(\mathbb{A})) \subseteq N_{eu} - cl\left(N_{eu} - int(N_{eu}\alpha - cl(\mathbb{A}))\right) \supseteq N_{eu}\alpha - cl\left(N_{eu} - int(N_{eu}\alpha - cl(\mathbb{A}))\right) \subseteq \mathbb{A} \supseteq N_{eu} - int(\mathbb{A}) \subseteq N_{eu} - int(W) \Rightarrow N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathbb{A})) \subseteq N_{eu} - int(W)$. Hence , $\mathbb{A}$ is $N_{eu}gs\alpha^* - CS$.

Example 3.9: Let $\mathbb{P}$ $=\{\wp\}$ and $\mathbb{A}=\{\langle\wp,(0.2,0.4,0.6)\rangle\}$ be $N_{eu}(\mathbb{P})$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ is a N_{eu}TS on $(\mathbb{P},\tau_{N_{eu}})$. $\mathbb{A}^c = \{\langle\wp,(0.6,0.6,0.2)\rangle\}$. Let $\mathcal{G} = \{\langle\wp,(0.4,0.8,0.7)\rangle\}$ be any $N_{eu}(\mathbb{P})$. $N_{eu}\alpha^* - OS = N_{eu}\alpha - OS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ and $N_{eu}\alpha - CS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}^c\}$. $N_{eu}\alpha - cl(\mathcal{G}) = 1_{N_{eu}}$. Now , $N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = N_{eu}\alpha - int(1_{N_{eu}}) = 1_{N_{eu}} \Rightarrow N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = 1_{N_{eu}} \subseteq N_{eu} - int(1_{N_{eu}}) = 1_{N_{eu}}$ whenever $\mathcal{G} \subseteq 1_{N_{eu}}$. Hence , $\mathcal{G}$ is $N_{eu}gs\alpha^* - CS$. But $\mathcal{G}$ is not $N_{eu}\alpha^* - CS$, because $N_{eu}\alpha - cl$ $\left(N_{eu} - int(N_{eu}\alpha - cl(\mathcal{G}))\right) = N_{eu}\alpha - cl$ $\left(N_{eu} - int(1_{N_{eu}})\right) = N_{eu}\alpha - cl\ (1_{N_{eu}}) = 1_{N_{eu}} \nsubseteq \mathcal{G}$.

Theorem 3.10: Every $N_{eu}R - CS$ is $N_{eu}gs\alpha^* - CS$, but not conversely .

Proof:

Let $\mathbb{A} \subseteq W$, W is $N_{eu}\alpha^* - OS$ in $(\mathbb{P},\tau_{N_{eu}})$. Since $\mathbb{A}$ is $N_{eu}R - CS$, then $N_{eu} - cl$ $(N_{eu} - int(\mathbb{A})) = \mathbb{A}$. Now , $N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathbb{A})) \subseteq N_{eu}\alpha - int(N_{eu} - cl(\mathbb{A}))$ $\supseteq N_{eu} - int(N_{eu} - cl(\mathbb{A})) = \mathbb{A} \supseteq N_{eu} - int(\mathbb{A}) \subseteq N_{eu} - int(W) \Rightarrow N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathbb{A})) \subseteq N_{eu} - int(W)$. Hence , $\mathbb{A}$ is $N_{eu}gs\alpha^* - CS$.

Example 3.11: Let $\mathbb{P}$ $=\{\wp\}$ and $\mathbb{A}=\{\langle\wp,(0.4,0.6,0.2)\rangle\}$ be $N_{eu}(\mathbb{P})$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ is a N_{eu}TS on $(\mathbb{P},\tau_{N_{eu}})$. Let $\mathcal{G} = \{\langle\wp,(0.1,0.3,0.5)\rangle\}$ be any $N_{eu}(\mathbb{P})$. Here $N_{eu}\alpha^* - OS = N_{eu}\alpha - OS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}, E\}$ and $N_{eu}\alpha - CS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}^c, D\}$, where $E = \{\langle\wp,([0.4,1], [0.6,1], [0,0.2])\rangle\}, D = \{\langle\wp,([0,0.2],[0,0.4],[0.4,1])\rangle\}$. Also , $\mathbb{A}^c = \{\langle\wp,(0.2,0.4,0.4)\rangle\}$ and $F = \{\langle\wp,([0.1,0.2],[0.3,0.4],[0.4,0.5])\rangle\}$. Now , $N_{eu}\alpha - int$ $(N_{eu}\alpha - cl(\mathcal{G})) = N_{eu}\alpha - int(\mathbb{A}^c \cap F \cap 1_{N_{eu}}) = N_{eu}\alpha - int(F) = 0_{N_{eu}} \Rightarrow N_{eu}\alpha - int$ $(N_{eu}\alpha - cl(\mathcal{G})) = 0_{N_{eu}} \subseteq N_{eu} - int(E), N_{eu} - int(\mathbb{A}), N_{eu} - int(1_{N_{eu}}) = \mathbb{A}, 1_{N_{eu}}$, where $\mathcal{G} \subseteq \mathbb{A}$, E , $1_{N_{eu}}$. Hence , $\mathcal{G}$ is $N_{eu}gs\alpha^* - CS$. But $\mathcal{G}$ is not $N_{eu}R - CS$, because $N_{eu} - cl$ $(N_{eu} - int(\mathcal{G})) = N_{eu} - cl\ (0_{N_{eu}}) = 0_{N_{eu}} \neq \mathcal{G}$.

Theorem 3.12: Every $N_{eu}g\alpha - CS$ is $N_{eu}gs\alpha^* - CS$, but not conversely .

Proof:

Let $\mathbb{A} \subseteq W$, W is $N_{eu}\alpha^*$ –OS in $(\mathbb{P},\tau_{N_{eu}})$. Since $\mathbb{A}$ is $N_{eu}g\alpha -$ CS , then $N_{eu}\alpha - cl(\mathbb{A}) \subseteq M$, whenever $\mathbb{A} \subseteq M$, M is $N_{eu}\alpha -$ OS . Now , $N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathbb{A})) \subseteq N_{eu}\alpha - int$ (M) $\supseteq N_{eu} - int(M) \supseteq N_{eu} - int(\mathbb{A}) \subseteq N_{eu} - int(W) \Rightarrow N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathbb{A})) \subseteq N_{eu} - int(W)$. Hence , $\mathbb{A}$ is $N_{eu}gs\alpha^* -$ CS .

Example 3.13: Let $\mathbb{P}$ $=\{\wp\}$ and $\mathbb{A}=\{\langle\wp,(0.3,0.2,0.8)\rangle\}$ be $N_{eu}(\mathbb{P})$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ is a N_{eu}TS on $(\mathbb{P},\tau_{N_{eu}})$. $\mathbb{A}^c = \{\langle\wp,(0.8,0.8,0.3)\rangle\}$. Let $\mathcal{G} = \{\langle\wp,(0.1,0.2,0.9)\rangle\}$ be any $N_{eu}(\mathbb{P})$. $N_{eu}\alpha^* - OS = N_{eu}\alpha - OS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ and $N_{eu}\alpha - CS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}^c\}$. Now , $N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = N_{eu}\alpha - int(\mathbb{A}^c \cap 1_{N_{eu}}) = N_{eu}\alpha - int(\mathbb{A}^c) = \mathbb{A} \cup 0_{N_{eu}} = \mathbb{A} \Rightarrow N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = \mathbb{A} \subseteq N_{eu} - int(\mathbb{A}), N_{eu} - int(1_{N_{eu}}) = \mathbb{A}, 1_{N_{eu}}$ where $\mathcal{G} \subseteq$

$\mathbb{A}$, $1_{N_{eu}}$. Hence , $\mathcal{G}$ is $N_{eu}gs\alpha^* - CS$. But $\mathcal{G}$ is not $N_{eu}g\alpha - CS$, because $N_{eu}\alpha - cl\ (\mathcal{G}) = \mathbb{A}^c \nsubseteq \mathbb{A}$, when $\mathcal{G} \subseteq \mathbb{A}$.

Theorem 3.14: Every $N_{eu}S\alpha - CS$ is $N_{eu}gs\alpha^* - CS$, but not conversely .

Proof:

Let $\mathbb{A} \subseteq W$, W is $N_{eu}\alpha^* - OS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Since $\mathbb{A}$ is $N_{eu}S\alpha - CS$, then $N_{eu} - int\ (N_{eu}\alpha - cl(\mathbb{A})) \subseteq \mathbb{A}$. Now , $N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathbb{A})) \supseteq N_{eu} - int(N_{eu}\alpha - cl(\mathbb{A})) \subseteq \mathbb{A} \supseteq N_{eu} - int(\mathbb{A}) \subseteq N_{eu} - int(W) \Rightarrow N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathbb{A})) \subseteq N_{eu} - int(W)$. Hence , $\mathbb{A}$ is $N_{eu}gs\alpha^* - CS$.

Example 3.15: Let $\mathbb{P} = \{\wp\}$ and $\mathbb{A} = \{\langle \wp, (0.4,0.3,0.6)\rangle\}$ be $N_{eu}(\mathbb{P})$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ is a N_{eu}TS on $(\mathbb{P}, \tau_{N_{eu}})$. $\mathbb{A}^c = \{\langle \wp, (0.6,0.7,0.4)\rangle\}$. Let $\mathcal{G} = \{\langle \wp, (0.6,0.9,0.9)\rangle\}$ be any $N_{eu}(\mathbb{P})$. $N_{eu}\alpha^* - OS = N_{eu}\alpha - OS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ and $N_{eu}\alpha - CS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}^c\}$. Now , $N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = N_{eu}\alpha - int(1_{N_{eu}}) = 1_{N_{eu}}$ $\Rightarrow N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = 1_{N_{eu}} \subseteq N_{eu} - int(1_{N_{eu}}) = 1_{N_{eu}}$ whenever $\mathcal{G} \subseteq 1_{N_{eu}}$. Hence , $\mathcal{G}$ is $N_{eu}gs\alpha^* - CS$. But $\mathcal{G}$ is not $N_{eu}S\alpha - CS$, because $N_{eu} - int\ (N_{eu}\alpha - cl(\mathcal{G})) = N_{eu} - int\ (1_{N_{eu}}) = 1_{N_{eu}} \nsubseteq \mathcal{G}$.

Theorem 3.16: Every $N_{eu}gs\alpha^* - CS$ is $N_{eu}\beta - CS$, but not conversely .

Proof:

Let $\mathbb{A} \subseteq W$, W is $N_{eu}\alpha^* - OS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Since $\mathbb{A}$ is $N_{eu}gs\alpha^* - CS$, then $N_{eu}\alpha - int\ (N_{eu}\alpha - cl(\mathbb{A})) \subseteq N_{eu} - int(W)$. Now , $N_{eu} - int\left(N_{eu} - cl(N_{eu} - int(\mathbb{A}))\right) \subseteq N_{eu} - int(N_{eu} - cl(\mathbb{A})) \supseteq N_{eu} - int(N_{eu}\alpha - cl(\mathbb{A})) \subseteq N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathbb{A})) \subseteq N_{eu} - int(W) \supseteq N_{eu} - int(\mathbb{A}) \subseteq \mathbb{A} \Rightarrow N_{eu} - int\left(N_{eu} - cl(N_{eu} - int(\mathbb{A}))\right) \subseteq \mathbb{A}$. Hence , $\mathbb{A}$ is $N_{eu}\beta - CS$.

Example 3.17: Let $\mathbb{P} = \{\wp\}$ and $\mathbb{A} = \{\langle \wp, (0.7,0.4,0.6)\rangle\}$ be $N_{eu}(\mathbb{P})$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ is a N_{eu}TS on $(\mathbb{P}, \tau_{N_{eu}})$. $\mathbb{A}^c = \{\langle \wp, (0.6,0.6,0.7)\rangle\}$. Let $\mathcal{G} = \{\langle \wp, (0.4,0.2,0.6)\rangle\}$ be any $N_{eu}(\mathbb{P})$. Since , $N_{eu} - int\left(N_{eu} - cl(N_{eu} - int(\mathcal{G}))\right) = 0_{N_{eu}} \subseteq \mathcal{G}$. Hence , $\mathcal{G}$ is $N_{eu}\beta - CS$. But $\mathcal{G}$ is not $N_{eu}gs\alpha^* - CS$. Also, $N_{eu}\alpha^* - OS = N_{eu}\alpha - OS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}, D, E\}$, $N_{eu}\alpha - CS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}^c, F, H\}$, where $D = \{\langle \wp, ([0.7, 1], [0.6, 1], [0,0.6])\rangle\}$, $E = \{\langle \wp, ([0.7, 1], [0.4, 0.5], [0, 0.6])\rangle\}$, $F = \{\langle \wp, ([0, 0.6], [0, 0.4], [0.7, 1])\rangle\}$, $H = \{\langle \wp, ([0, 0.6], [0.5, 0.6], [0.7, 1])\rangle\}$. Now , $N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = N_{eu}\alpha - int(1_{N_{eu}}) = 1_{N_{eu}} \Rightarrow N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = 1_{N_{eu}} \nsubseteq N_{eu} - int(\mathbb{A}), N_{eu} - int(D)$, $N_{eu} - int(E) = \mathbb{A}$, whenever $\mathcal{G} \subseteq \mathbb{A}$, D , E . Hence , $\mathcal{G}$ is not $N_{eu}gs\alpha^* - CS$.

Theorem 3.18: Every $N_{eu}gs\alpha^* - CS$ is $N_{eu}gs - CS$, but not conversely .

Proof:

Let $\mathbb{A} \subseteq M$, M is $N_{eu} - OS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Since $\mathbb{A}$ is $N_{eu}gs\alpha^* - CS$, then $N_{eu}\alpha - int\ (N_{eu}\alpha - cl(\mathbb{A})) \subseteq N_{eu} - int(W)$, whenever $\mathbb{A} \subseteq W$, W is $N_{eu}\alpha^* - OS$. Since every $N_{eu} - OS$ is $N_{eu}\alpha^* - OS$, then $W = M$. Now , $N_{eu}S - cl(\mathbb{A}) = \mathbb{A} \cup \left(N_{eu} - int(N_{eu} - cl(\mathbb{A}))\right) \subseteq \mathbb{A} \cup \left(N_{eu}\alpha - int(N_{eu} - cl(\mathbb{A}))\right) \supseteq \mathbb{A} \cup \left(N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathbb{A}))\right) \subseteq \mathbb{A} \cup (N_{eu} - int(W)) \subseteq W = M \Rightarrow N_{eu}s - cl\ (\mathbb{A}) \subseteq M$,whenever $\mathbb{A} \subseteq M$, M is

$N_{eu}-OS$ in $(\mathbb{P},\tau_{N_{eu}})$. Hence , $\mathbb{A}$ is $N_{eu}gs-CS$.

Example 3.19: Let $\mathbb{P}=\{\wp\}$ and $\mathbb{A}=\{\langle\wp,(0.4,0.6,0.8)\rangle\}$ be $N_{eu}(\mathbb{P})$. $\tau_{N_{eu}}=\{0_{N_{eu}},1_{N_{eu}},\mathbb{A}\}$ is a N_{eu}TS on $(\mathbb{P},\tau_{N_{eu}})$. $\mathbb{A}^c=\{\langle\wp,(0.8,0.4,0.4)\rangle\}$. Let $\mathcal{G}=\{\langle\wp,(0.9,0.4,0.2)\rangle\}$ be any $N_{eu}(\mathbb{P})$. Since , $N_{eu}S-cl(\mathcal{G})=\mathcal{G}\cup\left(N_{eu}-int(N_{eu}-cl(\mathcal{G}))\right)=\mathcal{G}\cup N_{eu}-int(1_{N_{eu}})=\mathcal{G}\cup 1_{N_{eu}}=1_{N_{eu}}\subseteq 1_{N_{eu}}$, when $\mathcal{G}\subseteq 1_{N_{eu}}$. Hence , $\mathcal{G}$ is $N_{eu}gs-CS$. But $\mathcal{G}$ is not $N_{eu}gs\alpha^*-CS$. Also, $N_{eu}\alpha^*-OS=N_{eu}\alpha-OS=\{0_{N_{eu}},1_{N_{eu}},\mathbb{A},D,E,F\}$, $N_{eu}\alpha-CS=\{0_{N_{eu}},1_{N_{eu}},\mathbb{A}^c,H,L,M\}$, where $D=\{\langle\wp,([0.8,1],[0.6,1],[0,0.4])\rangle\}$, $E=\{\langle\wp,([0.8,1],[0.6,1],[0.5,0.8])\rangle\}$, $F=\{\langle\wp,([0.4,0.7],[0.6,1],[0,0.8])\rangle\}$, $H=\{\langle\wp,([0,0.4],[0,0.4],[0.8,1])\rangle\}$, $L=\{\langle\wp,([0,0.8],[0,0.4],[0.4,0.7])\rangle\}$, $M=\{\langle\wp,([0.5,0.8],[0,0.4],[0.8,1])\rangle\}$. Now , $N_{eu}\alpha-int(N_{eu}\alpha-cl(\mathcal{G}))=N_{eu}\alpha-int(1_{N_{eu}})=1_{N_{eu}}\Rightarrow N_{eu}\alpha-int(N_{eu}\alpha-cl(\mathcal{G}))=1_{N_{eu}}\nsubseteq N_{eu}-int(N)=\mathbb{A}$ whenever $\mathcal{G}\subseteq$ N & $N=\{\langle\wp,([0.9,1],[0.6,1],[0,0.2])\rangle\}$. Hence , $\mathcal{G}$ is not $N_{eu}gs\alpha^*-CS$.

Theorem 3.20: Every $N_{eu}gs\alpha^*-CS$ is $N_{eu}gb-CS$, but not conversely .

Proof:

Let $\mathbb{A}\subseteq Ɱ$, Ɱ is $N_{eu}-OS$ in $(\mathbb{P},\tau_{N_{eu}})$. Since $\mathbb{A}$ is $N_{eu}gs\alpha^*-CS$, then $N_{eu}\alpha-int(N_{eu}\alpha-cl(\mathbb{A}))\subseteq N_{eu}-int(W)$, whenever $\mathbb{A}\subseteq W$, W is $N_{eu}\alpha^*-OS$. Since every $N_{eu}-OS$ is $N_{eu}\alpha^*-OS$, then W = Ɱ . Now , $N_{eu}b-cl(\mathbb{A})=N_{eu}S-cl(\mathbb{A})\cap N_{eu}P-cl(\mathbb{A})=\mathbb{A}\cup\left(\left(N_{eu}-int(N_{eu}-cl(\mathbb{A}))\right)\cap\left(N_{eu}-cl(N_{eu}-int(\mathbb{A}))\right)\right)\supseteq\mathbb{A}\cup\left(\left(N_{eu}-int(N_{eu}-cl(\mathbb{A}))\right)\cap\left(N_{eu}-int(\mathbb{A})\right)\right)\subseteq\mathbb{A}\cup\left(N_{eu}-int(N_{eu}-cl(\mathbb{A}))\right)\subseteq\mathbb{A}\cup\left(N_{eu}\alpha-int(N_{eu}-cl(\mathbb{A}))\right)\supseteq\mathbb{A}\cup\left(N_{eu}\alpha-int(N_{eu}\alpha-cl(\mathbb{A}))\right)\subseteq\mathbb{A}\cup(N_{eu}-int(W))\subseteq W=Ɱ$ $\Rightarrow N_{eu}b-cl(\mathbb{A})\subseteq Ɱ$, whenever $\mathbb{A}\subseteq Ɱ$, Ɱ is $N_{eu}-OS$ in $(\mathbb{P},\tau_{N_{eu}})$. Hence , $\mathbb{A}$ is $N_{eu}gb-CS$.

Example 3.21: Let $\mathbb{P}=\{\wp\}$ and $\mathbb{A}=\{\langle\wp,(0.6,0.8,0.4)\rangle\}$ be $N_{eu}(\mathbb{P})$. $\tau_{N_{eu}}=\{0_{N_{eu}},1_{N_{eu}},\mathbb{A}\}$ is a N_{eu}TS on $(\mathbb{P},\tau_{N_{eu}})$. $\mathbb{A}^c=\{\langle\wp,(0.4,0.2,0.6)\rangle\}$. Let $\mathcal{G}=\{\langle\wp,(0.2,0.7,0.4)\rangle\}$ be any $N_{eu}(\mathbb{P})$. Now , $N_{eu}b-cl(\mathcal{G})=\mathcal{G}\cup\left(\left(N_{eu}-int(N_{eu}-cl(\mathcal{G}))\right)\cap\left(N_{eu}-cl(N_{eu}-int(\mathcal{G}))\right)\right)=\mathcal{G}\cup\left(\left(N_{eu}-int(1_{N_{eu}})\right)\cap\left(N_{eu}-cl(0_{N_{eu}})\right)\right)=\mathcal{G}\cup(0_{N_{eu}}\cap 1_{N_{eu}})=\mathcal{G}\cup 0_{N_{eu}}=\mathcal{G}\subseteq\mathbb{A},1_{N_{eu}}$, whenever $\mathcal{G}\subseteq\mathbb{A},1_{N_{eu}}$. Hence , $\mathcal{G}$ is $N_{eu}gb-CS$. But $\mathcal{G}$ is not $N_{eu}gs\alpha^*-CS$. Also, $N_{eu}\alpha^*-OS=N_{eu}\alpha-OS=\{0_{N_{eu}},1_{N_{eu}},\mathbb{A},D\}$, $N_{eu}\alpha-CS=\{0_{N_{eu}},1_{N_{eu}},\mathbb{A}^c,E\}$, where $D=\{\langle\wp,([0.6,1],[0.8,1],[0,0.4])\rangle\}$, $E=\{\langle\wp,([0,0.4],[0,0.2],[0.6,1])\rangle\}$. Now , $N_{eu}\alpha-int(N_{eu}\alpha-cl(\mathcal{G}))=N_{eu}\alpha-int(1_{N_{eu}})=1_{N_{eu}}\Rightarrow N_{eu}\alpha-int(N_{eu}\alpha-cl(\mathcal{G}))=1_{N_{eu}}\nsubseteq N_{eu}-int(\mathbb{A}),N_{eu}-int(D)=\mathbb{A}$ whenever $\mathcal{G}\subseteq\mathbb{A}$, D . Hence , $\mathcal{G}$ is not $N_{eu}gs\alpha^*-CS$.

Theorem 3.22: Every $N_{eu}gs\alpha^*-CS$ is $N_{eu}g\beta-CS$, but not conversely .

Proof:

Let $A \subseteq M$, M is $N_{eu}-OS$ in $(\mathbb{P},\tau_{N_{eu}})$. Since A is $N_{eu}gs\alpha^*-CS$, then $N_{eu}\alpha-int\,(N_{eu}\alpha-cl(A)) \subseteq N_{eu}-int(W)$, whenever $A \subseteq W$, W is $N_{eu}\alpha^*-OS$. Since every $N_{eu}-OS$ is $N_{eu}\alpha^*-OS$, then $W = M$. Now , $N_{eu}\beta-cl(A) = A \cup \left(N_{eu}-int\left(N_{eu}-cl\left(N_{eu}-int\,(A)\right)\right)\right) \subseteq A \cup \left(N_{eu}-int\left(N_{eu}-cl\,(A)\right)\right) \subseteq A \cup \left(N_{eu}\alpha-int\left(N_{eu}-cl(A)\right)\right) \supseteq A \cup \left(N_{eu}\alpha-int\left(N_{eu}\alpha-cl(A)\right)\right) \subseteq A \cup \left(N_{eu}-int(W)\right) \subseteq W = M \Rightarrow N_{eu}\beta-cl\,(A) \subseteq M$, whenever $A\subseteq M$, M is $N_{eu}-OS$ in $(\mathbb{P},\tau_{N_{eu}})$. Hence , A is $N_{eu}g\beta-CS$.

Example 3.23: Let $\mathbb{P}=\{\wp\}$ and $A=\{\langle\wp,(0.3,0.8,0.6)\rangle\}$ be $N_{eu}(\mathbb{P})$. $\tau_{N_{eu}}=\{0_{N_{eu}},1_{N_{eu}},A\}$ is a N_{eu}TS on $(\mathbb{P},\tau_{N_{eu}})$. $A^c=\{\langle\wp,(0.6,0.2,0.3)\rangle\}$. Let $\mathcal{G}=\{\langle\wp,(0.8,0.1,0.5)\rangle\}$ be any $N_{eu}(\mathbb{P})$. Since , $N_{eu}\beta-cl(\mathcal{G}) = \mathcal{G} \cup \left(N_{eu}-int\left(N_{eu}-cl\left(N_{eu}-int\,(\mathcal{G})\right)\right)\right) = \mathcal{G} \cup \left(N_{eu}-int\left(N_{eu}-cl(0_{N_{eu}})\right)\right) = \mathcal{G} \cup \left(N_{eu}-int(0_{N_{eu}})\right) = \mathcal{G} \cup 0_{N_{eu}} = \mathcal{G} \subseteq 1_{N_{eu}}$, when $\mathcal{G} \subseteq 1_{N_{eu}}$. Hence , $\mathcal{G}$ is $N_{eu}g\beta-CS$. But is not $N_{eu}gs\alpha^*-CS$. Also, $N_{eu}\alpha^*-OS = N_{eu}\alpha-OS = \{0_{N_{eu}},1_{N_{eu}},A,D,E,F\}$, $N_{eu}\alpha-CS=\{0_{N_{eu}},1_{N_{eu}},A^c,L,M,N\}$, where $D=\{\langle\wp,([0.6,1],[0.8,1],[0,0.3])\rangle\}$, $E=\{\langle\wp,([0.6,1],[0.8,1],[0.4,0.6])\rangle\}$, $F=\{\langle\wp,([0.3,0.5],[0.8,1],[0,0.6])\rangle\}$, $L=\{\langle\wp,([0,0.3],[0,0.2],[0.6,1])\rangle\}$, $M=\{\langle\wp,([0,0.6],[0,0.2],[0.3,0.5])\rangle\}$, $N=\{\langle\wp,([0.4,0.6],[0,0.2],[0.6,1])\rangle\}$. Now , $N_{eu}\alpha-int\left(N_{eu}\alpha-cl(\mathcal{G})\right) = N_{eu}\alpha-int(1_{N_{eu}}) = 1_{N_{eu}} \Rightarrow N_{eu}\alpha-int\left(N_{eu}\alpha-cl(\mathcal{G})\right) = 1_{N_{eu}} \nsubseteq N_{eu}-int(O), N_{eu}-int(R) = A$ whenever $\mathcal{G} \subseteq$ O, R , $O=\{\langle\wp,([0.8,1],[0.8,1],[0,0.3])\rangle\}$ and $R=\{\langle\wp,([0.8,1],[0.8,1],[0.4,0.5])\rangle\}$. Hence , $\mathcal{G}$ is not $N_{eu}gs\alpha^*-CS$.

Theorem 3.24: Every $N_{eu}gs\alpha^*-CS$ is $N_{eu}\pi g\beta-CS$, but not conversely .

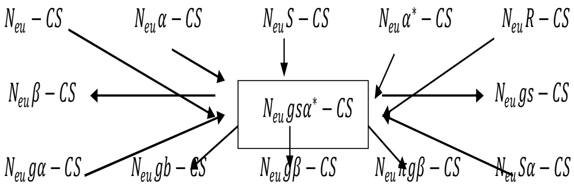
Proof:

Let $A\subseteq M$, M is $N_{eu}\pi-OS$ in $(\mathbb{P},\tau_{N_{eu}})$. Since A is $N_{eu}gs\alpha^*-CS$, then $N_{eu}\alpha-int\,(N_{eu}\alpha-cl(A)) \subseteq N_{eu}-int(W)$, whenever $A\subseteq W$, W is $N_{eu}\alpha^*-OS$. Since every $N_{eu}\pi-OS$ is $N_{eu}-OS$, then M is $N_{eu}-$OS . Also , since every $N_{eu}-OS$ is $N_{eu}\alpha^*-OS$, then $W = M$. Now by theorem 3.22 , $N_{eu}\beta-cl\,(A) \subseteq M$, whenever $A\subseteq M$, M is $N_{eu}\pi-OS$ in $(\mathbb{P},\tau_{N_{eu}})$. Hence , A is $N_{eu}\pi g\beta-CS$.

Example 3.25: Let $\mathbb{P}=\{\wp\}$ and $A=\{\langle\wp,(0.7,0.6,0.5)\rangle\}$ be $N_{eu}(\mathbb{P})$. $\tau_{N_{eu}}=\{0_{N_{eu}},1_{N_{eu}},A\}$ is a N_{eu}TS on $(\mathbb{P},\tau_{N_{eu}})$. $A^c=\{\langle\wp,(0.5,0.4,0.7)\rangle\}$. Let $\mathcal{G}=\{\langle\wp,(0.9,0.2,0.4)\rangle\}$ be any $N_{eu}(\mathbb{P})$. Since , $N_{eu}\beta-cl(\mathcal{G}) = \mathcal{G} \cup \left(N_{eu}-int\left(N_{eu}-cl\left(N_{eu}-int(\mathcal{G})\right)\right)\right) = \mathcal{G} \cup \left(N_{eu}-int\left(N_{eu}-cl(0_{N_{eu}})\right)\right) = \mathcal{G} \cup \left(N_{eu}-int(0_{N_{eu}})\right) = \mathcal{G} \cup 0_{N_{eu}} = \mathcal{G} \subseteq 1_{N_{eu}}$, when $\mathcal{G} \subseteq 1_{N_{eu}}$& $N_{eu}\pi-OS=\{0_{N_{eu}}\}$. Hence , $\mathcal{G}$ is $N_{eu}\pi g\beta-CS$. But is not $N_{eu}gs\alpha^*-CS$. Also, $N_{eu}\alpha^*-OS = N_{eu}\alpha-OS = \{0_{N_{eu}},1_{N_{eu}},A,D\}$, $N_{eu}\alpha-CS=\{0_{N_{eu}},1_{N_{eu}},A^c,E\}$, where $D=\{\langle\wp,([0.7,1],[0.6,1],[0,0.5])\rangle\}$, $E=\{\langle\wp,([0,0.5],[0,0.4],[0.7,1])\rangle\}$. Now , $N_{eu}\alpha-int\left(N_{eu}\alpha-cl(\mathcal{G})\right) = N_{eu}\alpha-int(1_{N_{eu}}) = 1_{N_{eu}} \Rightarrow N_{eu}\alpha-int\left(N_{eu}\alpha-cl(\mathcal{G})\right) = 1_{N_{eu}} \nsubseteq N_{eu}-int(F) = A$ whenever $\mathcal{G} \subseteq$ F , $F=$

$\{\langle \wp, ([0.9,1],[0.6,1],[0,0.4])\rangle\}$. Hence , $\mathcal{G}$ is not $N_{eu}gs\alpha^* - CS$.

Inter-relationship 3.26:



Theorem 3.27: Let $(\mathbb{P}, \tau_{N_{eu}})$ be a N_{eu}TS . Then intersection of two $N_{eu}gs\alpha^* - CS$ is a $N_{eu}gs\alpha^* - CS$ in N_{eu}TS $(\mathbb{P}, \tau_{N_{eu}})$.

Proof:

Let Ⱥ and Ƀ are $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Then $N_{eu}\alpha - int\ (N_{eu}\alpha - cl(Ⱥ)) \subseteq N_{eu} - int(Ⱳ)$, whenever Ⱥ⊆Ⱳ , Ⱳ is $N_{eu}\alpha^* - OS$ and $N_{eu}\alpha - int\ (N_{eu}\alpha - cl(Ƀ)) \subseteq N_{eu} - int(Ɱ)$, whenever Ƀ ⊆Ɱ , Ɱ is $N_{eu}\alpha^* - OS$. Since Ⱳ is $N_{eu}\alpha^* - OS$, then $Ⱳ \subseteq N_{eu}\alpha - int\ \left(N_{eu} - cl\left(N_{eu}\alpha - int(Ⱳ)\right)\right)$ and Ɱ is $N_{eu}\alpha^* - OS$, then $Ɱ \subseteq N_{eu}\alpha - int\ \left(N_{eu} - cl\left(N_{eu}\alpha - int(Ɱ)\right)\right)$. Now , $Ⱳ \cap Ɱ \subseteq \left(N_{eu}\alpha - int\ \left(N_{eu} - cl\left(N_{eu}\alpha - int(Ⱳ)\right)\right)\right) \cap \left(N_{eu}\alpha - int\ \left(N_{eu} - cl\left(N_{eu}\alpha - int(Ɱ)\right)\right)\right) \supseteq \left(N_{eu} - int\ \left(N_{eu} - cl\left(N_{eu} - int(Ⱳ)\right)\right)\right) \cap \left(N_{eu} - int\left(N_{eu} - cl\left(N_{eu} - int(Ɱ)\right)\right)\right) = N_{eu} - int\left(\left(N_{eu} - cl\left(N_{eu} - int(Ⱳ)\right)\right) \cap \left(N_{eu} - cl\left(N_{eu} - int(Ɱ)\right)\right)\right) \supseteq N_{eu} - int\left(N_{eu} - cl\left(\left(N_{eu} - int(Ⱳ)\right) \cap \left(N_{eu} - int(Ɱ)\right)\right)\right) = N_{eu} - int\ \left(N_{eu} - cl\left(N_{eu} - int\ (Ⱳ \cap Ɱ)\right)\right) \Rightarrow Ⱳ \cap Ɱ \subseteq N_{eu}\alpha - int\ \left(N_{eu} - cl\left(N_{eu}\alpha - int(Ⱳ \cap Ɱ)\right)\right) \Rightarrow Ⱳ \cap Ɱ$ is $N_{eu}\alpha^* - OS$.

Now , $\left(N_{eu}\alpha - int\ \left(N_{eu}\alpha - cl(Ⱥ)\right)\right) \cap \left(N_{eu}\alpha - int\ \left(N_{eu}\alpha - cl(Ƀ)\right)\right) \subseteq \left(N_{eu} - int(Ⱳ)\right) \cap \left(N_{eu} - int(Ɱ)\right) = N_{eu} - int(Ⱳ \cap Ɱ) \Rightarrow N_{eu} - int(Ⱳ \cap Ɱ) \supseteq \left(N_{eu}\alpha - int\ \left(N_{eu}\alpha - cl(Ⱥ)\right)\right) \cap \left(N_{eu}\alpha - int\ \left(N_{eu}\alpha - cl(Ƀ)\right)\right) \subseteq \left(N_{eu}\alpha - int\ \left(N_{eu} - cl(Ⱥ)\right)\right) \cap \left(N_{eu}\alpha - int\left(N_{eu} - cl(Ƀ)\right)\right) \supseteq \left(N_{eu} - int\ \left(N_{eu} - cl(Ⱥ)\right)\right) \cap \left(N_{eu} - int\ \left(N_{eu} - cl(Ƀ)\right)\right) = N_{eu} - int\left(\left(N_{eu} - cl(Ⱥ)\right) \cap \left(N_{eu} - cl(Ƀ)\right)\right) \supseteq N_{eu} - int\left(N_{eu} - cl(Ⱥ \cap Ƀ)\right) \subseteq N_{eu}\alpha - int\ \left(N_{eu} - cl(Ⱥ \cap Ƀ)\right) \supseteq N_{eu}\alpha - int\ \left(N_{eu}\alpha - cl(Ⱥ \cap Ƀ)\right) \Rightarrow N_{eu}\alpha - int\ \left(N_{eu}\alpha - cl(Ⱥ \cap Ƀ)\right) \subseteq N_{eu} - int(Ⱳ \cap Ɱ)$, whenever $Ⱥ \cap Ƀ \subseteq Ⱳ \cap Ɱ$ and $Ⱳ \cap Ɱ$ is $N_{eu}\alpha^* - OS$. Hence , $Ⱥ \cap Ƀ$ is $N_{eu}gs\alpha^* - CS$.

Theorem 3.28: Let $\{Ⱥ_\gamma\}_{\gamma\epsilon\Delta}$ be a collection of $N_{eu}gs\alpha^* - CS$ in a N_{eu}TS $(\mathbb{P}, \tau_{N_{eu}})$. Then $\cap_{\gamma\epsilon\Delta}\{Ⱥ_\gamma\}$ is $N_{eu}gs\alpha^* - CS$ in N_{eu}TS $(\mathbb{P}, \tau_{N_{eu}})$. (ie) Arbitrary intersection of $N_{eu}gs\alpha^* - CS$ is $N_{eu}gs\alpha^* - CS$ in N_{eu}TS $(\mathbb{P}, \tau_{N_{eu}})$.

Proof:

Since $\{Ⱥ_\gamma\}_{\gamma\epsilon\Delta}$ is $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Then $N_{eu}\alpha - int\ \left(N_{eu}\alpha - cl(Ⱥ_\gamma)\right) \subseteq N_{eu} - int(Ⱳ_\gamma)$, whenever $Ⱥ_\gamma \subseteq Ⱳ_\gamma$, $Ⱳ_\gamma$ is $N_{eu}\alpha^* - OS$, for all $\gamma\epsilon\Delta$. Since $Ⱳ_\gamma$ is $N_{eu}\alpha^* - OS$, then $Ⱳ_\gamma \subseteq N_{eu}\alpha - int\ \left(N_{eu} - cl\left(N_{eu}\alpha - int(Ⱳ_\gamma)\right)\right)$ for all Δ . Now , $\cap_{\gamma\epsilon\Delta}\{Ⱳ_\gamma\} \subseteq$

$\cap_{\gamma\epsilon\Delta}\Big\{ N_{eu}\alpha - int\Big(N_{eu} - cl\Big(N_{eu}\alpha - int(W_\gamma)\Big)\Big)\Big\} \supseteq \cap_{\gamma\epsilon\Delta}\Big\{ N_{eu} - int\Big(N_{eu} - cl\Big(N_{eu} - int(W_\gamma)\Big)\Big)\Big\} = N_{eu} - int\Big(\cap_{\gamma\epsilon\Delta}\Big\{ N_{eu} - cl\Big(N_{eu} - int(W_\gamma)\Big)\Big\}\Big) \supseteq N_{eu} - int \Big(N_{eu} - cl\big(\cap_{\gamma\epsilon\Delta}\{ N_{eu} - int(W_\gamma)\}\big)\Big) = N_{eu} - int\Big(N_{eu} - cl\Big(N_{eu} - int\big(\cap_{\gamma\epsilon\Delta}\{W_\gamma\}\big)\Big)\Big) \subseteq N_{eu}\alpha - int\Big(N_{eu} - cl\Big(N_{eu}\alpha - int\big(\cap_{\gamma\epsilon\Delta}\{W_\gamma\}\big)\Big)\Big) \Rightarrow \cap_{\gamma\epsilon\Delta}\{W_\gamma\}$ is $N_{eu}\alpha^* - OS$.

Now , $\cap_{\gamma\epsilon\Delta}\Big\{N_{eu}\alpha - int\Big(N_{eu}\alpha - cl(A_\gamma)\Big)\Big\} \subseteq \cap_{\gamma\epsilon\Delta}\{ N_{eu} - int(W_\gamma)\} = N_{eu} - int\big(\cap_{\gamma\epsilon\Delta}\{W_\gamma\}\big) \Rightarrow N_{eu} - int\big(\cap_{\gamma\epsilon\Delta}\{W_\gamma\}\big) \supseteq \cap_{\gamma\epsilon\Delta}\Big\{N_{eu}\alpha - int\Big(N_{eu}\alpha - cl(A_\gamma)\Big)\Big\} \subseteq \cap_{\gamma\epsilon\Delta}\Big\{N_{eu}\alpha - int\Big(N_{eu} - cl(A_\gamma)\Big)\Big\} \supseteq \cap_{\gamma\epsilon\Delta}\Big\{N_{eu} - int\Big(N_{eu} - cl(A_\gamma)\Big)\Big\} = N_{eu} - int\Big(\cap_{\gamma\epsilon\Delta}\Big(N_{eu} - cl(A_\gamma)\Big)\Big) \supseteq N_{eu} - int\Big(N_{eu} - cl\big(\cap_{\gamma\epsilon\Delta}\{A_\gamma\}\big)\Big) \subseteq N_{eu}\alpha - int\Big(N_{eu} - cl\big(\cap_{\gamma\epsilon\Delta}\{A_\gamma\}\big)\Big) \supseteq N_{eu}\alpha - int\Big(N_{eu}\alpha - cl\big(\cap_{\gamma\epsilon\Delta}\{A_\gamma\}\big)\Big) \Rightarrow N_{eu}\alpha - int\Big(N_{eu}\alpha - cl\big(\cap_{\gamma\epsilon\Delta}\{A_\gamma\}\big)\Big) \subseteq N_{eu} - int\big(\cap_{\gamma\epsilon\Delta}\{W_\gamma\}\big)$, whenever $\cap_{\gamma\epsilon\Delta}\{A_\gamma\} \subseteq \cap_{\gamma\epsilon\Delta}\{W_\gamma\}$ and $\cap_{\gamma\epsilon\Delta}\{W_\gamma\}$is $N_{eu}\alpha^* - OS$.

Hence , $\cap_{\gamma\epsilon\Delta}\{A_\gamma\}$ is $N_{eu}gs\alpha^* - CS$.

Theorem 3.29: Let $(\mathbb{P}, \tau_{N_{eu}})$ be a N_{eu}TS . Then union of any two $N_{eu}gs\alpha^* - CS$ is a $N_{eu}gs\alpha^* - CS$ in N_{eu}TS $(\mathbb{P}, \tau_{N_{eu}})$.

Proof:

Let A and B are $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Then $N_{eu}\alpha - int\big(N_{eu}\alpha - cl(A)\big) \subseteq N_{eu} - int(W)$, whenever $A \subseteq W$, W is $N_{eu}\alpha^* - OS$ and $N_{eu}\alpha - int\big(N_{eu}\alpha - cl(B)\big) \subseteq N_{eu} - int(M)$, whenever $B \subseteq M$, M is $N_{eu}\alpha^* - OS$. Since W is $N_{eu}\alpha^* - OS$, then $W \subseteq N_{eu}\alpha - int\Big(N_{eu} - cl\big(N_{eu}\alpha - int(W)\big)\Big)$ and M is $N_{eu}\alpha^* - OS$, then $M \subseteq N_{eu}\alpha - int\Big(N_{eu} - cl\big(N_{eu}\alpha - int(M)\big)\Big)$. Now , $W \cup M \subseteq \Big(N_{eu}\alpha - int\Big(N_{eu} - cl\big(N_{eu}\alpha - int(W)\big)\Big)\Big) \cup \Big(N_{eu}\alpha - int\Big(N_{eu} - cl\big(N_{eu}\alpha - int(M)\big)\Big)\Big) \supseteq \Big(N_{eu} - int\Big(N_{eu} - cl\big(N_{eu} - int(W)\big)\Big)\Big) \cup \Big(N_{eu} - int\Big(N_{eu} - cl\big(N_{eu} - int(M)\big)\Big)\Big) \subseteq N_{eu} - int\Big(\Big(N_{eu} - cl\big(N_{eu} - int(W)\big)\Big) \cup \Big(N_{eu} - cl\big(N_{eu} - int(M)\big)\Big)\Big) = N_{eu} - int\Big(N_{eu} - cl\Big(\big(N_{eu} - int(W)\big) \cup \big(N_{eu} - int(M)\big)\Big)\Big) \subseteq N_{eu} - int\Big(N_{eu} - cl\big(N_{eu} - int(W \cap M)\big)\Big) \Rightarrow W \cap M \subseteq N_{eu}\alpha - int\Big(N_{eu} - cl\big(N_{eu}\alpha - int(W \cap M)\big)\Big) \Rightarrow W \cap M$ is $N_{eu}\alpha^* - OS$.

Now , $\Big(N_{eu}\alpha - int\big(N_{eu}\alpha - cl(A)\big)\Big) \cup \Big(N_{eu}\alpha - int\big(N_{eu}\alpha - cl(B)\big)\Big) \subseteq \big(N_{eu} - int(W)\big) \cup \big(N_{eu} - int(M)\big) \subseteq N_{eu} - int(W \cup M) \Rightarrow N_{eu} - int(W \cup M) \supseteq \Big(N_{eu}\alpha - int\big(N_{eu}\alpha - cl(A)\big)\Big) \cup \Big(N_{eu}\alpha - int\big(N_{eu}\alpha - cl(B)\big)\Big) \subseteq \Big(N_{eu}\alpha - int\big(N_{eu} - cl(A)\big)\Big) \cup \Big(N_{eu}\alpha - int\big(N_{eu} - cl(B)\big)\Big) \supseteq \Big(N_{eu} -$

$int\ (N_{eu}-cl(Ⱥ))\big) \cup \big(N_{eu}-int\ (N_{eu}-cl(Ƀ))\big) \subseteq N_{eu}-int\big((N_{eu}-cl(Ⱥ)) \cup (N_{eu}-cl(Ƀ))\big) = N_{eu} - int\,(N_{eu} - cl(Ⱥ \cup Ƀ)) \subseteq N_{eu}\alpha - int\ (N_{eu}-cl(Ⱥ \cup Ƀ)) \supseteq N_{eu}\alpha - int\ (N_{eu}\alpha - cl(Ⱥ \cup Ƀ)) \Rightarrow N_{eu}\alpha - int\ (N_{eu}\alpha - cl(Ⱥ \cup Ƀ)) \subseteq N_{eu} - int(W \cup M)$, whenever $Ⱥ \cup Ƀ \subseteq W \cup M$ and $W \cup M$ is $N_{eu}\alpha^* - OS$. Hence, $Ⱥ \cup Ƀ$ is $N_{eu}gs\alpha^* - CS$.

Theorem 3.30: In a N_{eu}TS $(\mathbb{P}, \tau_{N_{eu}})$, we have the following conditions

(i) $0_{N_{eu}}$ and $1_{N_{eu}}$ are $N_{eu}gs\alpha^* - CS$.

(ii) The intersection of any number of $N_{eu}gs\alpha^* - CS$ subsets is a $N_{eu}gs\alpha^* - CS$.

(iii) The union of any two $N_{eu}gs\alpha^* - CS$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$.

Proof:

(i) Since $0_{N_{eu}}$ and $1_{N_{eu}}$ are $N_{eu} - CS$, then by theorem 3.2, $0_{N_{eu}}$ and $1_{N_{eu}}$ are $N_{eu}gs\alpha^* - CS$.

(ii) Proof follows from theorem 3.28.

(iii) Proof follows from theorem 3.29.

Remark 3.31: The collection of $N_{eu}gs\alpha^* - CS$ form a topology. (by theorem 3.30)

Remark 3.32: The concept of $N_{eu}G^* - CS$ and $N_{eu}gs\alpha^* - CS$ are independent.

Example 3.33: Let $\mathbb{P} = \{\wp\}$ and $Ⱥ = \{\langle \wp, (0.4, 0.5, 0.7)\rangle\}$ be $N_{eu}(\mathbb{P})$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, Ⱥ\}$ is a N_{eu}TS on $(\mathbb{P}, \tau_{N_{eu}})$. $Ⱥ^c = \{\langle \wp, (0.7, 0.5, 0.4)\rangle\}$. Let $\mathcal{G} = \{\langle \wp, (0.6, 0.5, 0.5)\rangle\}$ be any $N_{eu}(\mathbb{P})$. $N_{eu}\alpha^* - OS = N_{eu}\alpha - OS = \{0_{N_{eu}}, 1_{N_{eu}}, Ⱥ\}$ and $N_{eu}\alpha - CS = \{0_{N_{eu}}, 1_{N_{eu}}, Ⱥ^c\}$. Now, $N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = N_{eu}\alpha - int(Ⱥ^c \cap 1_{N_{eu}}) = N_{eu}\alpha - int(Ⱥ^c) = Ⱥ \cup 0_{N_{eu}} = Ⱥ \Rightarrow N_{eu}\alpha - int\,(N_{eu}\alpha - cl(\mathcal{G})) = Ⱥ \subseteq N_{eu} - int\,(1_{N_{eu}}) = 1_{N_{eu}}$, whenever $\mathcal{G} \subseteq 1_{N_{eu}}$. Hence, $\mathcal{G}$ is $N_{eu}gs\alpha^* - CS$. But $\mathcal{G}$ is not $N_{eu}G^* - CS$, because $N_{eu} - cl\ (\mathcal{G}) = Ⱥ^c \not\subseteq$ F, J, K. where, $F = \{\langle \wp, ([0.8, 1], 0.5, 0.5)\rangle\}$, J= $\{\langle \wp, (0.6, [0.6, 1], [0, 0.4])\rangle\}$, K= $\{\langle \wp, ([0.6,1], [0.6,1], 0.5)\rangle\}$.

Example 3.34: Let $\mathbb{P} = \{\wp\}$ and $Ⱥ = \{\langle \wp, (0.2, 0.7, 0.8)\rangle\}$ be $N_{eu}(\mathbb{P})$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, Ⱥ\}$ is a N_{eu}TS on $(\mathbb{P}, \tau_{N_{eu}})$. $Ⱥ^c = \{\langle \wp, (0.8, 0.3, 0.2)\rangle\}$. Let $\mathcal{G} = \{\langle \wp, (0.9, 0.8, 0.1)\rangle\}$ be any $N_{eu}(\mathbb{P})$. Since, $N_{eu} - cl\ (\mathcal{G}) = 1_{N_{eu}} \subseteq 1_{N_{eu}}$, when $\mathcal{G} \subseteq 1_{N_{eu}}$. Hence, $\mathcal{G}$ is $N_{eu}G^* - CS$. But $\mathcal{G}$ is not $N_{eu}gs\alpha^* - CS$. Also, $N_{eu}\alpha^* - OS = N_{eu}\alpha - OS = \{0_{N_{eu}}, 1_{N_{eu}}, Ⱥ, D, E, F\}$, $N_{eu}\alpha - CS = \{0_{N_{eu}}, 1_{N_{eu}}, Ⱥ^c, L, M, N\}$, where $D = \{\langle \wp, ([0.8, 1], [0.7, 1], [0, 0.2])\rangle\}$, $E = \{\langle \wp, ([0.2, 0.7], [0.7, 1], [0, 0.8])\rangle\}$, $F = \{\langle \wp, ([0.8, 1], [0.7, 1], [0.3, 0.8])\rangle\}$, $L = \{\langle \wp, ([0, 0.2], [0, 0.3], [0.8, 1])\rangle\}$, $M = \{\langle \wp, ([0, 0.8], [0, 0.3], [0.2, 0.7])\rangle\}$, $N = \{\langle \wp, ([0.3, 0.8], [0, 0.3], [0.8, 1])\rangle\}$. Now, $N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = N_{eu}\alpha - int(1_{N_{eu}}) = 1_{N_{eu}} \Rightarrow N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = 1_{N_{eu}} \not\subseteq N_{eu} - int\ (O) = Ⱥ$, whenever $\mathcal{G} \subseteq$ O, $O = \{\langle \wp, ([0.9, 1], [0.8, 1], [0, 0.1])\rangle\}$. Hence, $\mathcal{G}$ is not $N_{eu}gs\alpha^* - CS$.

Remark 3.35: The concept of $N_{eu}g - CS$ and $N_{eu}gs\alpha^* - CS$ are independent.

Example 3.36: Let $\mathbb{P} = \{\wp\}$ and $Ⱥ = \{\langle \wp, (0.3, 0.6, 0.7)\rangle\}$ be $N_{eu}(\mathbb{P})$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, Ⱥ\}$ is a N_{eu}TS on $(\mathbb{P}, \tau_{N_{eu}})$. $Ⱥ^c = \{\langle \wp, (0.7, 0.4, 0.3)\rangle\}$. Let $\mathcal{G} = \{\langle \wp, (0.2, 0.3, 0.9)\rangle\}$ be any $N_{eu}(\mathbb{P})$. $N_{eu}\alpha^* - OS = N_{eu}\alpha - OS = \{0_{N_{eu}}, 1_{N_{eu}}, Ⱥ, D, E, F\}$ and $N_{eu}\alpha - CS = \{0_{N_{eu}}, 1_{N_{eu}}, Ⱥ^c, L, M, N\}$, where $D = \{\langle \wp, ([0.7, 1], [0.6, 1], [0, 0.3])\rangle\}$, $E = \{\langle \wp, ([0.3, 0.6], [0.6, 1], [0, 0.7])\rangle\}$, $F = \{\langle \wp, ([0.7, 1],$

[0.6 , 1] , [0.4 , 0.7])⟩} , $L = \{\langle \wp, ([0,0.3], [0,0.4], [0.7,1])\rangle\}$, $M = \{\langle \wp, ([0,0.7], [0,0.4], [0.3,0.6])\rangle\}$, $N = \{\langle \wp, ([0.4,0.7], [0,0.4], [0.7,1])\rangle\}$. Now , $N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = N_{eu}\alpha - int(J) = 0_{N_{eu}}$, where $J = \{\langle \wp, ([0.2,0.3], [0.3,0.4], [0.7,0.9])\rangle\}$. $N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = 0_{N_{eu}} \subseteq N_{eu} - int(A)$, $N_{eu} - int(D)$, $N_{eu} - int(E)$, $N_{eu} - int(F)$, $N_{eu} - int(1_{N_{eu}}) = \mathbb{A}$, $1_{N_{eu}}$, whenever $\mathcal{G} \subseteq \mathbb{A}, D, E, F, 1_{N_{eu}}$. Hence , $\mathcal{G}$ is $N_{eu}gs\alpha^* - CS$. But $\mathcal{G}$ is not $N_{eu}g - CS$, because $N_{eu} - cl(\mathcal{G}) = \mathbb{A}^c \nsubseteq \mathbb{A}$, where $\mathcal{G} \subseteq \mathbb{A}$.

Example 3.37: Let $\mathbb{P} = \{\wp\}$ and $\mathbb{A} = \{\langle \wp, (0.8,0.5,0.2)\rangle\}$ be $N_{eu}(\mathbb{P})$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ is a N_{eu}TS on $(\mathbb{P}, \tau_{N_{eu}})$. $\mathbb{A}^c = \{\langle \wp, (0.2,0.5,0.8)\rangle\}$. Let $\mathcal{G} = \{\langle \wp, (0.9,0.7,0.2)\rangle\}$ be any $N_{eu}(\mathbb{P})$. Since , $N_{eu} - cl(\mathcal{G}) = 1_{N_{eu}} \subseteq 1_{N_{eu}}$, when $\mathcal{G} \subseteq 1_{N_{eu}}$. Hence , $\mathcal{G}$ is $N_{eu}g - CS$. But $\mathcal{G}$ is not $N_{eu}gs\alpha^* - CS$. Also, $N_{eu}\alpha^* - OS = N_{eu}\alpha - OS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}, D\}$, $N_{eu}\alpha - CS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}^c, E\}$, where $D = \{\langle \wp, ([0.8,1], [0.5,1], [0,0.2])\rangle\}$, $E = \{\langle \wp, ([0,0.2], [0,0.5], [0.8,1])\rangle\}$. Now , $N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = N_{eu}\alpha - int(1_{N_{eu}}) = 1_{N_{eu}}$ $\Rightarrow N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = 1_{N_{eu}} \nsubseteq N_{eu} - int(F) = \mathbb{A}$ whenever $\mathcal{G} \subseteq F$, $F = \{\langle \wp, ([0.9,1], [0.7,1], [0,0.2])\rangle\}$. Hence , $\mathcal{G}$ is not $N_{eu}gs\alpha^* - CS$.

Remark 3.38: The concept of $N_{eu}P - CS$ and $N_{eu}gs\alpha^* - CS$ are independent .

Example 3.39: Let $\mathbb{P} = \{\wp\}$ and $\mathbb{A} = \{\langle \wp, (0.2,0.4,0.6)\rangle\}$ be $N_{eu}(\mathbb{P})$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ is a N_{eu}TS on $(\mathbb{P}, \tau_{N_{eu}})$. $\mathbb{A}^c = \{\langle \wp, (0.6,0.6,0.2)\rangle\}$. Let $\mathcal{G} = \{\langle \wp, (0.4,0.8,0.6)\rangle\}$ be any $N_{eu}(\mathbb{P})$. $N_{eu}\alpha^* - OS = N_{eu}\alpha - OS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ and $N_{eu}\alpha - CS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}^c\}$, Now , $N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = N_{eu}\alpha - int(1_{N_{eu}}) = 1_{N_{eu}}$. $N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = 1_{N_{eu}} \subseteq N_{eu} - int(1_{N_{eu}}) = 1_{N_{eu}}$, whenever $\mathcal{G} \subseteq 1_{N_{eu}}$. Hence , $\mathcal{G}$ is $N_{eu}gs\alpha^* - CS$. But $\mathcal{G}$ is not $N_{eu}P - CS$, because $N_{eu} - cl(N_{eu} - int(\mathcal{G})) = N_{eu} - cl(\mathbb{A}) = \mathbb{A}^c \nsubseteq \mathcal{G}$.

Example 3.40: Let $\mathbb{P} = \{\wp\}$ and $\mathbb{A} = \{\langle \wp, (0.7,0.8,0.3)\rangle\}$ be $N_{eu}(\mathbb{P})$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ is a N_{eu}TS on $(\mathbb{P}, \tau_{N_{eu}})$. $\mathbb{A}^c = \{\langle \wp, (0.3,0.2,0.7)\rangle\}$. Let $\mathcal{G} = \{\langle \wp, (0.6,0.5,0.9)\rangle\}$ be any $N_{eu}(\mathbb{P})$. Since , $N_{eu} - cl(N_{eu} - int(\mathcal{G})) = N_{eu} - cl(0_{N_{eu}}) = 0_{N_{eu}} \subseteq \mathcal{G}$. Hence , $\mathcal{G}$ is $N_{eu}P - CS$. But $\mathcal{G}$ is not $N_{eu}gs\alpha^* - CS$. $N_{eu}\alpha^* - OS = N_{eu}\alpha - OS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}, D\}$, $N_{eu}\alpha - CS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}^c, E\}$, where $D = \{\langle \wp, ([0.7,1], [0.8,1], [0,0.3])\rangle\}$, $E = \{\langle \wp, ([0,0.3], [0,0.2], [0.7,1])\rangle\}$. Now , $N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = N_{eu}\alpha - int(1_{N_{eu}}) = 1_{N_{eu}}$ $\Rightarrow N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = 1_{N_{eu}} \nsubseteq N_{eu} - int(\mathbb{A})$, $N_{eu} - int(D) = \mathbb{A}$ whenever $\mathcal{G} \subseteq \mathbb{A}$, D . Hence , $\mathcal{G}$ is not $N_{eu}gs\alpha^* - CS$.

Remark 3.41: The concept of $N_{eu}b - CS$ and $N_{eu}gs\alpha^* - CS$ are independent .

Example 3.42: Let $\mathbb{P} = \{\wp\}$ and $\mathbb{A} = \{\langle \wp, (0.3,0.2,0.8)\rangle\}$ be $N_{eu}(\mathbb{P})$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ is a N_{eu}TS on $(\mathbb{P}, \tau_{N_{eu}})$. $\mathbb{A}^c = \{\langle \wp, (0.8,0.8,0.3)\rangle\}$. Let $\mathcal{G} = \{\langle \wp, (0.7,0.9,0.8)\rangle\}$ be any $N_{eu}(\mathbb{P})$. $N_{eu}\alpha^* - OS = N_{eu}\alpha - OS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ and $N_{eu}\alpha - CS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}^c\}$, Now , $N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = N_{eu}\alpha - int(1_{N_{eu}}) = 1_{N_{eu}}$. $N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = 1_{N_{eu}} \subseteq$

$N_{eu}-int(1_{N_{eu}})=1_{N_{eu}}$, whenever $\mathcal{G}\subseteq 1_{N_{eu}}$. Hence , $\mathcal{G}$ is $N_{eu}gs\alpha^*-CS$. But $\mathcal{G}$ is not $N_{eu}b-CS$, because $N_{eu}-cl\ (N_{eu}-int(\mathcal{G}))\cap N_{eu}-int\ (N_{eu}-cl(\mathcal{G}))=N_{eu}-cl(\mathbb{A})\cap N_{eu}-int(1_{N_{eu}})=\mathbb{A}^c\cap 1_{N_{eu}}=\mathbb{A}^c\nsubseteq \mathcal{G}$.

Example 3.43: Let $\mathbb{P}=\{\wp\}$ and $\mathbb{A}=\{\langle\wp,(0.7,0.4,0.6)\rangle\}$ be $N_{eu}(\mathbb{P})$. $\tau_{N_{eu}}=\{0_{N_{eu}},1_{N_{eu}},\mathbb{A}\}$ is a N_{eu}TS on $(\mathbb{P},\tau_{N_{eu}})$. $\mathbb{A}^c=\{\langle\wp,(0.6,0.6,0.7)\rangle\}$. Let $\mathcal{G}=\{\langle\wp,(0.4,0.3,0.6)\rangle\}$ be any $N_{eu}(\mathbb{P})$. Since , $N_{eu}-cl\ (N_{eu}-int(\mathcal{G}))\cap N_{eu}-int\ (N_{eu}-cl(\mathcal{G}))=N_{eu}-cl(0_{N_{eu}})\cap N_{eu}$ - $int\ (1_{N_{eu}})=0_{N_{eu}}\cap 1_{N_{eu}}=0_{N_{eu}}\subseteq\mathcal{G}$. Hence , $\mathcal{G}$ is $N_{eu}b-CS$. But $\mathcal{G}$ is not $N_{eu}gs\alpha^*-CS$. $N_{eu}\alpha^*-OS=N_{eu}\alpha-OS=\{0_{N_{eu}},1_{N_{eu}},\mathbb{A},D,E\}$, $N_{eu}\alpha-CS=\{0_{N_{eu}},1_{N_{eu}},\mathbb{A}^c,F,L\}$, where $D=\{\langle\wp,([0.7,1],[0.6,1],[0,0.6])\rangle\}$, $E=\{\langle\wp,([0.7,1],[0.4,0.5],[0,0.6])\rangle\}$, $F=\{\langle\wp,([0,0.6],[0,0.4],[0.7,1])\rangle\}$, $L=\{\langle\wp,([0,0.6],[0.5,0.6],[0.7,1])\rangle\}$. Now , $N_{eu}\alpha-int(N_{eu}\alpha-cl(\mathcal{G}))=N_{eu}\alpha-int(1_{N_{eu}})=1_{N_{eu}}\Rightarrow N_{eu}\alpha$ - $int\ (N_{eu}\alpha-cl(\mathcal{G}))=1_{N_{eu}}\nsubseteq N_{eu}-int(\mathbb{A})$, $N_{eu}-int(D),N_{eu}-int(E)=\mathbb{A}$, whenever $\mathcal{G}\subseteq\mathbb{A}$, D , E . Hence , $\mathcal{G}$ is not $N_{eu}gs\alpha^*-CS$.

Remark 3.44: The concept of $N_{eu}bg-CS$ and $N_{eu}gs\alpha^*-CS$ are independent .

Example 3.45: Let $\mathbb{P}=\{\wp\}$ and $\mathbb{A}=\{\langle\wp,(0.5,0.3,0.8)\rangle\}$ be $N_{eu}(\mathbb{P})$. $\tau_{N_{eu}}=\{0_{N_{eu}},1_{N_{eu}},\mathbb{A}\}$ is a N_{eu}TS on $(\mathbb{P},\tau_{N_{eu}})$. $\mathbb{A}^c=\{\langle\wp,(0.8,0.7,0.5)\rangle\}$. Let $\mathcal{G}=\{\langle\wp,(0.7,0.8,0.7)\rangle\}$ be any $N_{eu}(\mathbb{P})$. $N_{eu}\alpha^*-OS=N_{eu}\alpha-OS=\{0_{N_{eu}},1_{N_{eu}},\mathbb{A}\}$ and $N_{eu}\alpha-CS=\{0_{N_{eu}},1_{N_{eu}},\mathbb{A}^c\}$, Now , $N_{eu}\alpha$ - $int\ (N_{eu}\alpha-cl(\mathcal{G}))=N_{eu}\alpha$ - $int\ (1_{N_{eu}})=1_{N_{eu}}\Rightarrow N_{eu}\alpha$ - $int\ (N_{eu}\alpha-cl(\mathcal{G}))=1_{N_{eu}}\subseteq N_{eu}-int(1_{N_{eu}})=1_{N_{eu}}$, whenever $\mathcal{G}\subseteq 1_{N_{eu}}$. Hence , $\mathcal{G}$ is $N_{eu}gs\alpha^*-CS$. But $\mathcal{G}$ is not $N_{eu}bg-CS$, because $N_{eu}b-cl\ (\mathcal{G})=\mathcal{G}\cup\Big(\big(N_{eu}-int\ (N_{eu}-cl(\mathcal{G}))\big)\cap\big(N_{eu}-cl(N_{eu}-int(\mathcal{G}))\big)\Big)=\mathcal{G}\cup\Big(\big(N_{eu}-int\ (1_{N_{eu}})\big)\cap\big(N_{eu}-cl(\mathbb{A})\big)\Big)=\mathcal{G}\cup(1_{N_{eu}}\cap\mathbb{A}^c)=\mathcal{G}\cup\mathbb{A}^c=S$, where $S=\{\langle\wp,(0.8,0.8,0.5)\rangle\}\Rightarrow N_{eu}b-cl\ (\mathcal{G})=S\nsubseteq D,E$, whenever $\mathcal{G}\subseteq D,E$, where $D=\{\langle\wp,([0.7],[0.8,1],[0,0.7])\rangle\}$, $E=\{\langle\wp,([0.8,1],[0.8,1],[0.6,0.7])\rangle\}$.

Example 3.46: Let $\mathbb{P}=\{\wp\}$ and $\mathbb{A}=\{\langle\wp,(0.4,0.6,0.8)\rangle\}$ be $N_{eu}(\mathbb{P})$. $\tau_{N_{eu}}=\{0_{N_{eu}},1_{N_{eu}},\mathbb{A}\}$ is a N_{eu}TS on $(\mathbb{P},\tau_{N_{eu}})$. $\mathbb{A}^c=\{\langle\wp,(0.8,0.4,0.4)\rangle\}$. Let $\mathcal{G}=\{\langle\wp,(0.3,0.5,0.9)\rangle\}$ be any $N_{eu}(\mathbb{P})$. Since , $N_{eu}b-cl(\mathcal{G})=\mathcal{G}\cup\Big(\big(N_{eu}-int\ (N_{eu}-cl(\mathcal{G}))\big)\cap\big(N_{eu}-cl(N_{eu}-int(\mathcal{G}))\big)\Big)=\mathcal{G}\cup\Big(\big(N_{eu}-int\ (1_{N_{eu}})\big)\cap\big(N_{eu}-cl(0_{N_{eu}})\big)\Big)=\mathcal{G}\cup(0_{N_{eu}}\cap 1_{N_{eu}})=\mathcal{G}\cup 0_{N_{eu}}=\mathcal{G}\subseteq W$, whenever $\mathcal{G}\subseteq W$ and W is $N_{eu}b-OS$. Hence , $\mathcal{G}$ is $N_{eu}bg-CS$. But $\mathcal{G}$ is not $N_{eu}gs\alpha^*-CS$. $N_{eu}\alpha^*-OS=N_{eu}\alpha-OS=\{0_{N_{eu}},1_{N_{eu}},\mathbb{A},D,E,F\}$, $N_{eu}\alpha-CS=\{0_{N_{eu}},1_{N_{eu}},\mathbb{A}^c,L,M,N\}$, where $D=\{\langle\wp,([0.8,1],[0.6,1],[0,0.4])\rangle\}$, $E=\{\langle\wp,([0.8,1],[0.6,1],[0.5,0.8])\rangle\}$, $F=\{\langle\wp,([0.4,0.7],[0.6,1],[0,0.8])\rangle\}$, $L=\{\langle\wp,([0,0.4],[0,0.4],[0.8,1])\rangle\}$, $M=\{\langle\wp,([0,0.8],[0,0.4],[0.4,0.7])\rangle\}$, $N=\{\langle\wp,([0.5,0.8],[0,0.4],[0.8,1])\rangle\}$. Now , $N_{eu}\alpha$ - $int\ (N_{eu}\alpha-cl(\mathcal{G}))=N_{eu}\alpha-$

$int(1_{N_{eu}}) = 1_{N_{eu}} \Rightarrow N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = 1_{N_{eu}} \nsubseteq N_{eu} - int(\mathbb{A})$, $N_{eu} - int(D)$, $N_{eu} - int(E)$, $N_{eu} - int(F) = \mathbb{A}$, whenever $\mathcal{G} \subseteq \mathbb{A}$, D , E , F . Hence , $\mathcal{G}$ is not $N_{eu}gs\alpha^* - CS$.

Remark 3.47: The concept of $N_{eu}\alpha g - CS$ and $N_{eu}gs\alpha^* - CS$ are independent .

Example 3.48: Let $\mathbb{P} =\{\wp\}$ and $\mathbb{A}= \{\langle \wp, (0.4,0.3,0.6)\rangle\}$ be $N_{eu}(\mathbb{P})$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ is a N_{eu}TS on $(\mathbb{P}, \tau_{N_{eu}})$. $\mathbb{A}^c = \{\langle \wp, (0.6,0.7,0.4)\rangle\}$. Let $\mathcal{G} = \{\langle \wp, (0.2,0.3,0.8)\rangle\}$ be any $N_{eu}(\mathbb{P})$. $N_{eu}\alpha^* - OS = N_{eu}\alpha - OS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ and $N_{eu}\alpha - CS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}^c\}$, Now , $N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = N_{eu}\alpha - int(\mathbb{A}^c \cap 1_{N_{eu}}) = N_{eu}\alpha - int(\mathbb{A}^c) = \mathbb{A} \cup 0_{N_{eu}} = \mathbb{A} \Rightarrow N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = \mathbb{A} \subseteq N_{eu} - int(\mathbb{A})$, $N_{eu} - int(1_{N_{eu}}) = \mathbb{A}$, $1_{N_{eu}}$, whenever $\mathcal{G} \subseteq \mathbb{A}$, $1_{N_{eu}}$. Hence , $\mathcal{G}$ is $N_{eu}gs\alpha^* - CS$. But $\mathcal{G}$ is not $N_{eu}\alpha g - CS$, because $N_{eu}\alpha - cl(\mathcal{G}) = \mathbb{A}^c \nsubseteq \mathbb{A}$, whenever $\mathcal{G} \subseteq \mathbb{A}$.

Example 3.49: Let $\mathbb{P} =\{\wp\}$ and $\mathbb{A}= \{\langle \wp, (0.6,0.8,0.4)\rangle\}$ be $N_{eu}(\mathbb{P})$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ is a N_{eu}TS on $(\mathbb{P}, \tau_{N_{eu}})$. $\mathbb{A}^c = \{\langle \wp, (0.4,0.2,0.6)\rangle\}$. Let $\mathcal{G} = \{\langle \wp, (0.2,0.7,0.3)\rangle\}$ be any $N_{eu}(\mathbb{P})$. $N_{eu}\alpha^* - OS = N_{eu}\alpha - OS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}, D\}$, $N_{eu}\alpha - CS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}^c, E\}$, where $D = \{\langle \wp, ([0.6,1], [0.8,1], [0,0.4])\rangle\}$, $E = \{\langle \wp, ([0,0.4], [0,0.2], [0.6,1])\rangle\}$ .Since , $N_{eu}\alpha - cl(\mathcal{G}) = 1_{N_{eu}} \subseteq 1_{N_{eu}}$, whenever $\mathcal{G} \subseteq 1_{N_{eu}}$. Hence , $\mathcal{G}$ is $N_{eu}\alpha g - CS$. But $\mathcal{G}$ is not $N_{eu}gs\alpha^* - CS$. Now , $N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = N_{eu}\alpha - int(1_{N_{eu}}) = 1_{N_{eu}} \Rightarrow N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = 1_{N_{eu}} \nsubseteq N_{eu} - int(F) = \mathbb{A}$, whenever $\mathcal{G} \subseteq$ F , where $F = \{\langle \wp, ([0.6,1], [0.8,1], [0,0.3])\rangle\}$. Hence , $\mathcal{G}$ is not $N_{eu}gs\alpha^* - CS$.

Remark 3.50: The concept of $N_{eu}gR - CS$ and $N_{eu}gs\alpha^* - CS$ are independent .

Example 3.51: Let $\mathbb{P} =\{\wp\}$ and $\mathbb{A}= \{\langle \wp, (0.7, 0.6, 0.5)\rangle\}$ be $N_{eu}(\mathbb{P})$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ is a N_{eu}TS on $(\mathbb{P}, \tau_{N_{eu}})$. $\mathbb{A}^c = \{\langle \wp, (0.5,0.4,0.7)\rangle\}$. Let $\mathcal{G} = \{\langle \wp, (0.4,0.2,0.8)\rangle\}$ be any $N_{eu}(\mathbb{P})$. $N_{eu}\alpha^* - OS = N_{eu}\alpha - OS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}, D\}$ and $N_{eu}\alpha - CS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}^c, E\}$, where $D = \{\langle \wp, ([0.7, 1], [0.6, 1], [0, 0.5])\rangle\}$, $E = \{\langle \wp, ([0, 0.5], [0, 0.4], [0.7, 1])\rangle\}$. Now , $N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = N_{eu}\alpha - int(F) = 0_{N_{eu}}$, where $F = \{\langle \wp, ([0.4, 0.5], [0.2, 0.4], [0.7, 0.8])\rangle\} \Rightarrow N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = 0_{N_{eu}} \subseteq N_{eu} - int(\mathbb{A})$, $N_{eu} - int(D)$, $N_{eu} - int(1_{N_{eu}}) = \mathbb{A}$, $1_{N_{eu}}$, whenever $\mathcal{G} \subseteq \mathbb{A}$, D , $1_{N_{eu}}$. Hence , $\mathcal{G}$ is $N_{eu}gs\alpha^* - CS$. But $\mathcal{G}$ is not $N_{eu}gR - CS$, because $N_{eu}R - cl(\mathcal{G}) = 1_{N_{eu}} \nsubseteq \mathbb{A}$, whenever $\mathcal{G} \subseteq \mathbb{A}$.

Example 3.52: Let $\mathbb{P} =\{\wp\}$ and $\mathbb{A}= \{\langle \wp, (0.7,0.4,0.6)\rangle\}$ be $N_{eu}(\mathbb{P})$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ is a N_{eu}TS on $(\mathbb{P}, \tau_{N_{eu}})$. $\mathbb{A}^c = \{\langle \wp, (0.6,0.6,0.7)\rangle\}$. Let $\mathcal{G} = \{\langle \wp, (0.8,0.7,0.2)\rangle\}$ be any $N_{eu}(\mathbb{P})$. $N_{eu}R - CS = \{0_{N_{eu}}, 1_{N_{eu}}\}$. Since , $N_{eu}R - cl(\mathcal{G}) = 1_{N_{eu}} \subseteq 1_{N_{eu}}$, whenever $\mathcal{G} \subseteq 1_{N_{eu}}$. Hence , $\mathcal{G}$ is $N_{eu}gR - CS$. But $\mathcal{G}$ is not $N_{eu}gs\alpha^* - CS$. $N_{eu}\alpha^* - OS = N_{eu}\alpha - OS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}, D, E\}$, $N_{eu}\alpha - CS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}^c, L, M\}$, where $D = \{\langle \wp, ([0.7, 1], [0.6, 1], [0, 0.6])\rangle\}$, $E = \{\langle \wp, ([0.7, 1], [0.4, 0.5], [0, 0.6])\rangle\}$, $L = \{\langle \wp, ([0, 0.6], [0, 0.4], [0.7,1])\rangle\}$, $M = \{\langle \wp, ([0,0.6], [0.5,0.6], [0.7,1])\rangle\}$. Now ,

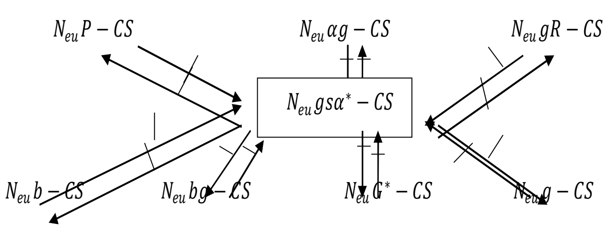
$N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = N_{eu}\alpha - int(1_{N_{eu}}) = 1_{N_{eu}} \Rightarrow N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{G})) = 1_{N_{eu}} \nsubseteq N_{eu} - int(J)$, whenever $\mathcal{G} \subseteq$ J , where $J = \{\langle \wp, ([0.8,1], [0.7,1], [0,0.2]) \rangle\}$. Hence , $\mathcal{G}$ is not $N_{eu}gs\alpha^* - CS$.

Theorem 3.53: Let $(\mathbb{P}, \tau_{N_{eu}})$ be a N_{eu}TS . If Ƀ is a $N_{eu}gs\alpha^* - CS$ and Ƀ ⊆ Ⱥ , then Ⱥ is $N_{eu}gs\alpha^* - CS$.

Proof:

Let Ⱥ ⊆ W and W is a $N_{eu}\alpha^* - OS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Since Ƀ ⊆ Ⱥ , then Ƀ ⊆ W . Now , Ƀ is $N_{eu}gs\alpha^* - CS$, then $N_{eu}\alpha - int(N_{eu}\alpha - cl(Ƀ)) \subseteq N_{eu} - int$ (W) . But $N_{eu} - cl(Ⱥ) \subseteq N_{eu} - cl(Ƀ) \Rightarrow N_{eu}\alpha - cl(Ⱥ) \subseteq N_{eu} - cl(Ⱥ) \subseteq N_{eu}\alpha - cl(Ƀ) \subseteq N_{eu} - cl(Ƀ) \Rightarrow N_{eu} - int(N_{eu}\alpha - cl(Ⱥ)) \subseteq N_{eu} - int(N_{eu}\alpha - cl(Ƀ)) \Rightarrow N_{eu} - int(N_{eu}\alpha - cl(Ⱥ)) \subseteq N_{eu}\alpha - int(N_{eu}\alpha - cl(Ⱥ)) \subseteq N_{eu} - int(N_{eu}\alpha - cl(Ƀ)) \subseteq N_{eu}\alpha - int(N_{eu}\alpha - cl(Ƀ)) \Rightarrow N_{eu}\alpha - int(N_{eu}\alpha - cl(Ⱥ)) \subseteq N_{eu}\alpha - int(N_{eu}\alpha - cl(Ƀ)) \subseteq N_{eu} - int$ (W) $\Rightarrow N_{eu}\alpha - int (N_{eu}\alpha - cl(Ⱥ)) \subseteq N_{eu} - int$(W) , whenever Ⱥ ⊆ W and W is a $N_{eu}\alpha^* - OS$. Hence , Ⱥ is $N_{eu}gs\alpha^* - CS$.

Inter-relationship 3.54:



IV.NEUTROSOPHIC gsα^* –OPEN SETS

Definition 4.1: A neutrosophic set Ⱥ in a N_{eu}TS $(\mathbb{P}, \tau_{N_{eu}})$ is called a neutrosophic generalized semi alpha star open set (N$_{eu}gs\alpha^*$ – OS) if $N_{eu}\alpha - cl(N_{eu}\alpha - int(Ⱥ)) \supseteq N_{eu} - cl(\mathcal{G})$, whenever Ⱥ ⊇ $\mathcal{G}$ and $\mathcal{G}$ is $N_{eu}\alpha^*$ – closed set .

Example 4.2: Let $\mathbb{P}$ ={$\wp$} and Ⱥ= $\{\langle \wp, (0.4,0.5,0.7) \rangle\}$ be $N_{eu}(\mathbb{P})$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, Ⱥ\}$ is a N_{eu}TS on $(\mathbb{P}, \tau_{N_{eu}})$. Ⱥ^c = $\{\langle \wp, (0.7,0.5,0.4) \rangle\}$. Let $\mathcal{G} = \{\langle \wp, (0.2,0.5,0.6) \rangle\}$ be any $N_{eu}(\mathbb{P})$. $N_{eu}\alpha^* - OS = N_{eu}\alpha - OS = \{0_{N_{eu}}, 1_{N_{eu}}, Ⱥ\}$ and $N_{eu}\alpha - CS = \{0_{N_{eu}}, 1_{N_{eu}}, Ⱥ^c\}$. Now , $N_{eu}\alpha - cl(N_{eu}\alpha - int(\mathcal{G})) = N_{eu}\alpha - cl(0_{N_{eu}}) = 0_{N_{eu}} \Rightarrow N_{eu}\alpha - cl(N_{eu}\alpha - int(\mathcal{G})) = 0_{N_{eu}} \supseteq N_{eu} - cl(0_{N_{eu}}) = 0_{N_{eu}}$, whenever $\mathcal{G} \supseteq 0_{N_{eu}}$. Hence , $\mathcal{G}$ is $N_{eu}gs\alpha^* - OS$.

Theorem 4.3: Let $(\mathbb{P}, \tau_{N_{eu}})$ be a N_{eu}TS . Then

(1) Every $N_{eu} - OS$ is $N_{eu}gs\alpha^* - OS$, but not conversely .

(2) Every $N_{eu}\alpha - OS$ is $N_{eu}gs\alpha^* - OS$, but not conversely .

(3) Every $N_{eu}S - OS$ is $N_{eu}gs\alpha^* - OS$, but not conversely .

(4) Every $N_{eu}\alpha^* - OS$ is $N_{eu}gs\alpha^* - OS$, but not conversely .

(5) Every $N_{eu}R - Os$ is $N_{eu}gs\alpha^* - OS$, but not conversely .

(6) Every $N_{eu}g\alpha - OS$ is $N_{eu}gs\alpha^* - OS$, but not conversely .

(7) Every $N_{eu}S\alpha - OS$ is $N_{eu}gs\alpha^* - OS$, but not conversely .

(8) Every $N_{eu}gs\alpha^* - OS$ is $N_{eu}\beta - OS$, but not conversely .

(9) Every $N_{eu}gs\alpha^* - OS$ is $N_{eu}gs - OS$, but not conversely .

(10) Every $N_{eu}gs\alpha^* - OS$ is $N_{eu}gb - OS$, but not conversely .

(11) Every $N_{eu}gs\alpha^* - OS$ is $N_{eu}g\beta - OS$, but not conversely .

(12) Every $N_{eu}gs\alpha^* - OS$ is $N_{eu}\pi g\beta - OS$, but not conversely .

(13) Let $(\mathbb{P}, \tau_{N_{eu}})$ be a N_{eu}TS . Then union of two $N_{eu}gs\alpha^* - OS$ is a $N_{eu}gs\alpha^* - OS$ in N_{eu}TS $(\mathbb{P}, \tau_{N_{eu}})$.

(14) Let $\{\mathbb{A}_\gamma\}_{\gamma\epsilon\Delta}$ be a collection of $N_{eu}gs\alpha^* - OS$ in a N_{eu}TS $(\mathbb{P}, \tau_{N_{eu}})$. Then $\cup_{\gamma\epsilon\Delta}\{\mathbb{A}_\gamma\}$ is $N_{eu}gs\alpha^* - OS$ in N_{eu}TS $(\mathbb{P}, \tau_{N_{eu}})$. (ie) Arbitrary union of $N_{eu}gs\alpha^* - OS$ is $N_{eu}gs\alpha^* - OS$ in N_{eu}TS $(\mathbb{P}, \tau_{N_{eu}})$.

(15) Let $(\mathbb{P}, \tau_{N_{eu}})$ be a N_{eu}TS . Then intersection of any two $N_{eu}gs\alpha^* - OS$ is a $N_{eu}gs\alpha^* - OS$ in N_{eu}TS $(\mathbb{P}, \tau_{N_{eu}})$.

(16) In a N_{eu}TS $(\mathbb{P}, \tau_{N_{eu}})$, we have the following conditions

(i) $0_{N_{eu}}$ and $1_{N_{eu}}$ are $N_{eu}gs\alpha^* - OS$.

(ii) The intersection of any number of $N_{eu}gs\alpha^* - OS$ subsets is a $N_{eu}gs\alpha^* - OS$.

(iii) The union of any two $N_{eu}gs\alpha^* - OS$ is a $N_{eu}gs\alpha^* - OS$ in $(\mathbb{P}, \tau_{N_{eu}})$.

(17) The collection of $N_{eu}gs\alpha^* - OS$ form a topology .

(18) The concept of $N_{eu}G^* - OS$ and $N_{eu}gs\alpha^* - OS$ are independent .

(19) The concept of $N_{eu}g - OS$ and $N_{eu}gs\alpha^* - OS$ are independent .

(20) The concept of $N_{eu}P - OS$ and $N_{eu}gs\alpha^* - OS$ are independent .

(21) The concept of $N_{eu}b - OS$ and $N_{eu}gs\alpha^* - OS$ are independent .

(22) The concept of $N_{eu}bg - OS$ and $N_{eu}gs\alpha^* - OS$ are independent .

(23) The concept of $N_{eu}\alpha g - OS$ and $N_{eu}gs\alpha^* - OS$ are independent .

(24) The concept of $N_{eu}gR - OS$ and $N_{eu}gs\alpha^* - OS$ are independent .

(25) Let $(\mathbb{P}, \tau_{N_{eu}})$ be a N_{eu}TS . If $\mathbb{B}$ is a $N_{eu}gs\alpha^* - OS$ and $\mathbb{B} \subseteq \mathbb{A}$, then $\mathbb{A}$ is $N_{eu}gs\alpha^* - OS$.

Proof :

The proof follows from theorem 3.2 to 3.54 .

V. $N_{eu}gs\alpha^*$ –INTERIOR AND $N_{eu}gs\alpha^*$ –CLOSURE

Definition 5.1: A neutrosophic set $\mathbb{A}$ in a N_{eu}TS $(\mathbb{P}, \tau_{N_{eu}})$ is called a neutrosophic generalized semi alpha star interior ($N_{eu}gs\alpha^* - int$) and neutrosophic generalized semi alpha star closure ($N_{eu}gs\alpha^* - cl$) of $\mathbb{A}$ are defined by ,

(i) $N_{eu}gs\alpha^* - int(\mathbb{A}) = \cup\{\mathcal{G} : \mathcal{G}\ is\ a\ N_{eu}gs\alpha^* - OS\ in\ \mathbb{P}\ and\ \mathcal{G} \subseteq \mathbb{A}\}$

(ii) $N_{eu}gs\alpha^* - cl(\mathbb{A}) = \cap\{\mathcal{K} : \mathcal{K}\ is\ a\ N_{eu}gs\alpha^* - CS\ in\ \mathbb{P}\ and\ \mathbb{A} \subseteq \mathcal{K}\}$.

Theorem 5.2: Let $(\mathbb{P}, \tau_{N_{eu}})$ be a N_{eu}TS . Then for any neutrosophic subsets $\mathbb{A}$ and $\mathbb{B}$ of a N_{eu}TS $\mathbb{P}$, we have

(1) $N_{eu}gs\alpha^* - int(\mathbb{A}) \subseteq \mathbb{A}$

(2) $\mathbb{A} \subseteq N_{eu}gs\alpha^* - cl(\mathbb{A})$

(3) $\mathbb{A}$ is $N_{eu}gs\alpha^* - OS$ in $\mathbb{P}$ iff $N_{eu}gs\alpha^* - int(\mathbb{A}) = \mathbb{A}$

(4) $\mathbb{A}$ is $N_{eu}gs\alpha^* - CS$ in $\mathbb{P}$ iff $N_{eu}gs\alpha^* - cl(\mathbb{A}) = \mathbb{A}$

(5) $N_{eu}gs\alpha^* - int(N_{eu}gs\alpha^* - int(\mathbb{A})) = N_{eu}gs\alpha^* - int(\mathbb{A})$

(6) $N_{eu}gs\alpha^* - cl(N_{eu}gs\alpha^* - cl(\mathbb{A})) = N_{eu}gs\alpha^* - cl(\mathbb{A})$

(7) If $\mathbb{A} \subseteq \mathbb{B}$, then $N_{eu}gs\alpha^* - int(\mathbb{A}) \subseteq N_{eu}gs\alpha^* - int(\mathbb{B})$

(8) If $\mathbb{A} \subseteq \mathbb{B}$, then $N_{eu}gs\alpha^* - cl(\mathbb{A}) \subseteq N_{eu}gs\alpha^* - cl(\mathbb{B})$

Proof:

(1) $N_{eu}gs\alpha^* - int(\mathbb{A}) = \cup\{\mathcal{G} : \mathcal{G}\ is\ a\ N_{eu}gs\alpha^* - OS\ in\ \mathbb{P}\ and\ \mathcal{G} \subseteq \mathbb{A}\} \subseteq \mathbb{A}$. Clearly , $N_{eu}gs\alpha^* - int(\mathbb{A}) \subseteq \mathbb{A}$.

(2) $N_{eu}gs\alpha^* - cl(\mathbb{A}) = \cap\{\mathcal{K} : \mathcal{K}\ is\ a\ N_{eu}gs\alpha^* - CS\ in\ \mathbb{P}\ and\ \mathbb{A} \subseteq \mathcal{K}\} \supseteq \mathbb{A}$. Clearly , $\mathbb{A} \subseteq N_{eu}gs\alpha^* - cl(\mathbb{A})$.

(3) Let $\mathbb{A}$ be $N_{eu}gs\alpha^* - OS$ in $\mathbb{P}$. Since $\mathbb{A} \subseteq \mathbb{A}$ and $\mathbb{A}$ is $N_{eu}gs\alpha^* - OS$ in $\mathbb{P}$, then $\mathbb{A} \epsilon\{\mathcal{G} :$

$\mathcal{G}\ is\ a$ $N_{eu}gs\alpha^* - OS\ in\ \mathbb{P}\ and\ \mathcal{G} \subseteq Ⱥ\} \Rightarrow$ $Ⱥ = \cup\{\mathcal{G} : \mathcal{G}\ is\ a\ N_{eu}gs\alpha^* - OS\ in\ \mathbb{P}\ and\ \mathcal{G} \subseteq Ⱥ\}$. Hence , $N_{eu}gs\alpha^* - int(Ⱥ) = Ⱥ$. Conversely , Let $N_{eu}gs\alpha^* - int(Ⱥ) = Ⱥ$. Then , $Ⱥ = \cup\{\mathcal{G} : \mathcal{G}\ is\ a\ N_{eu}gs\alpha^* - OS\ in\ \mathbb{P}\ and\ \mathcal{G} \subseteq Ⱥ\} \Rightarrow Ⱥ \epsilon \{\mathcal{G} : \mathcal{G}\ is\ a\ N_{eu}gs\alpha^* - OS\ in\ \mathbb{P}\ and\ \mathcal{G} \subseteq Ⱥ\} \Rightarrow Ⱥ$ is $N_{eu}gs\alpha^* - OS$ in $\mathbb{P}$.

(4) Let $Ⱥ$ be $N_{eu}gs\alpha^* - CS$ in $\mathbb{P}$. Since $Ⱥ \subseteq Ⱥ$ and $Ⱥ$ is $N_{eu}gs\alpha^* - CS$ in $\mathbb{P}$, then $Ⱥ \epsilon \{\mathcal{K} : \mathcal{K}\ is\ a\ N_{eu}gs\alpha^* - CS\ in\ \mathbb{P}\ and\ Ⱥ \subseteq \mathcal{K}\} \Rightarrow$ $Ⱥ = \cap\{\mathcal{K} : \mathcal{K}\ is\ a\ N_{eu}gs\alpha^* - CS\ in\ \mathbb{P}\ and\ Ⱥ \subseteq \mathcal{K}\}$. Hence , $N_{eu}gs\alpha^* - cl(Ⱥ) = Ⱥ$. Conversely , Let $N_{eu}gs\alpha^* - cl(Ⱥ) = Ⱥ$. Then , $Ⱥ = \cap\{\mathcal{K} : \mathcal{K}\ is\ a\ N_{eu}gs\alpha^* - CS\ in\ \mathbb{P}\ and\ Ⱥ \subseteq \mathcal{K}\} \Rightarrow Ⱥ\ \epsilon\ \{\mathcal{K} : \mathcal{K}\ is\ a\ N_{eu}gs\alpha^* - CS\ in\ \mathbb{P}\ and\ Ⱥ \subseteq \mathcal{K}\} \Rightarrow Ⱥ$ is $N_{eu}gs\alpha^* - CS$ in $\mathbb{P}$.

(5) $N_{eu}gs\alpha^* - int(Ⱥ) = \cup\{\mathcal{G} : \mathcal{G}\ is\ a\ N_{eu}gs\alpha^* - OS\ in\ \mathbb{P}\ and\ \mathcal{G} \subseteq Ⱥ\} \Rightarrow N_{eu}gs\alpha^* - int\left(N_{eu}gs\alpha^* - int(Ⱥ)\right) = \cup\{N_{eu}gs\alpha^* - int(\mathcal{G}) : N_{eu}gs\alpha^* - int(\mathcal{G})\ is\ a\ N_{eu}gs\alpha^* - OS\ in\ \mathbb{P}\ and\ N_{eu}gs\alpha^* - int(\mathcal{G}) \subseteq N_{eu}gs\alpha^* - int(Ⱥ)\} \Rightarrow N_{eu}gs\alpha^* - int(Ⱥ)$ is $N_{eu}gs\alpha^* - OS$ in $\mathbb{P}$. Hence , $N_{eu}gs\alpha^* - int\left(N_{eu}gs\alpha^* - int(Ⱥ)\right) = N_{eu}gs\alpha^* - int(Ⱥ)$.

(6) $N_{eu}gs\alpha^* - cl(Ⱥ) = \cap\{\mathcal{K} : \mathcal{K}\ is\ a\ N_{eu}gs\alpha^* - CS\ in\ \mathbb{P}\ and\ Ⱥ \subseteq \mathcal{K}\} \Rightarrow N_{eu}gs\alpha^* - cl\left(N_{eu}gs\alpha^* - cl(Ⱥ)\right) = \cap\{N_{eu}gs\alpha^* - cl(\mathcal{K}) : N_{eu}gs\alpha^* - cl(\mathcal{K})\ is\ a\ N_{eu}gs\alpha^* - CS\ in\ \mathbb{P}\ and\ N_{eu}gs\alpha^* - cl(Ⱥ) \subseteq N_{eu}gs\alpha^* - cl(\mathcal{K})\} \Rightarrow N_{eu}gs\alpha^* - cl(Ⱥ)$ is $N_{eu}gs\alpha^* - CS$ in $\mathbb{P}$. Hence , $N_{eu}gs\alpha^* - cl\left(N_{eu}gs\alpha^* - cl(Ⱥ)\right) = N_{eu}gs\alpha^* - cl(Ⱥ)$.

(7) $N_{eu}gs\alpha^* - int(Ƀ) = \cup\{\mathcal{G} : \mathcal{G}\ is\ a\ N_{eu}gs\alpha^* - OS\ in\ \mathbb{P}\ and\ Ƀ \supseteq \mathcal{G}\} \supseteq \cup\{\mathcal{G} : \mathcal{G}\ is\ a\ N_{eu}gs\alpha^* - OS\ in\ \mathbb{P}\ and\ Ⱥ \supseteq \mathcal{G}\} \supseteq N_{eu}gs\alpha^* - int(Ⱥ)$. Hence , $N_{eu}gs\alpha^* - int(Ⱥ) \subseteq N_{eu}gs\alpha^* - int(Ƀ)$.

(8) $N_{eu}gs\alpha^* - cl(Ƀ) = \cap\{\mathcal{K} : \mathcal{K}\ is\ a\ N_{eu}gs\alpha^* - CS\ in\ \mathbb{P}\ and\ Ƀ \subseteq \mathcal{K}\} \supseteq \cap\{\mathcal{K} : \mathcal{K}\ is\ a\ N_{eu}gs\alpha^* - CS\ in\ \mathbb{P}\ and\ Ⱥ \subseteq \mathcal{K}\} \supseteq N_{eu}gs\alpha^* - cl(Ⱥ)$. Hence , $N_{eu}gs\alpha^* - cl(Ⱥ) \subseteq N_{eu}gs\alpha^* - cl(Ƀ)$.

Theorem 5.3: Let $Ⱥ$ be a neutrosophic set in a N_{eu}TS $(\mathbb{P}, \tau_{N_{eu}})$. Then ,

(1) $\left(N_{eu}gs\alpha^* - cl(Ⱥ)\right)^c = N_{eu}gs\alpha^* - int(Ⱥ^c)$

(2) $\left(N_{eu}gs\alpha^* - int(Ⱥ)\right)^c = N_{eu}gs\alpha^* - cl(Ⱥ^c)$

(3) $N_{eu}gs\alpha^* - cl(0_{N_{eu}}) = 0_{N_{eu}}$, $N_{eu}gs\alpha^* - cl(1_{N_{eu}}) = 1_{N_{eu}}$

(4) $N_{eu}gs\alpha^* - int(0_{N_{eu}}) = 0_{N_{eu}}$, $N_{eu}gs\alpha^* - int(1_{N_{eu}}) = 1_{N_{eu}}$

Proof:

(1) $N_{eu}gs\alpha^* - cl(Ⱥ) = \cap\{\mathcal{K} : \mathcal{K}\ is\ a\ N_{eu}gs\alpha^* - CS\ in\ \mathbb{P}\ and\ Ⱥ \subseteq \mathcal{K}\} \Rightarrow \left(N_{eu}gs\alpha^* - cl(Ⱥ)\right)^c = \cup\{\mathcal{K}^c : \mathcal{K}^c\ is\ a\ N_{eu}gs\alpha^* - OS\ in\ \mathbb{P}\ and\ Ⱥ^c \supseteq \mathcal{K}^c\} = N_{eu}gs\alpha^* - int\ (Ⱥ^c)$. Hence , $\left(N_{eu}gs\alpha^* - cl(Ⱥ)\right)^c = N_{eu}gs\alpha^* - int(Ⱥ^c)$.

(2) $N_{eu}gs\alpha^* - int(Ⱥ) = \cup\{\mathcal{G} : \mathcal{G}\ is\ a\ N_{eu}gs\alpha^* - OS\ in\ \mathbb{P}\ and\ Ⱥ \supseteq \mathcal{G}\} \Rightarrow \left(N_{eu}gs\alpha^* - int(Ⱥ)\right)^c = \cap\{\mathcal{G}^c : \mathcal{G}^c\ is\ a\ N_{eu}gs\alpha^* - CS\ in\ \mathbb{P}\ and\ Ⱥ^c \subseteq \mathcal{G}^c\} = N_{eu}gs\alpha^* - cl(Ⱥ^c)$. Hence, $\left(N_{eu}gs\alpha^* - int(Ⱥ)\right)^c = N_{eu}gs\alpha^* - cl(Ⱥ^c)$.

(3) Since $0_{N_{eu}}$ and $1_{N_{eu}}$ are $\mathrm{N_{eu}} - CS$, then by theorem 3.2 , $0_{N_{eu}}$ and $1_{N_{eu}}$ are $\mathrm{N_{eu}}gs\alpha^* - CS$. Hence , $\mathrm{N_{eu}}gs\alpha^* - cl(0_{N_{eu}}) = 0_{N_{eu}}$, $\mathrm{N_{eu}}gs\alpha^* - cl(1_{N_{eu}}) = 1_{N_{eu}}$.

(4) Since $0_{N_{eu}}$ and $1_{N_{eu}}$ are $\mathrm{N_{eu}} - OS$, then by theorem 4.3 (1) , $0_{N_{eu}}$ and $1_{N_{eu}}$ are $\mathrm{N_{eu}}gs\alpha^* - OS$. Hence , $\mathrm{N_{eu}}gs\alpha^* - int(0_{N_{eu}}) = 0_{N_{eu}}$, $\mathrm{N_{eu}}gs\alpha^* - int(1_{N_{eu}}) = 1_{N_{eu}}$.

Theorem 5.4: Let $(\mathbb{P}, \tau_{N_{eu}})$ be a N_{eu}TS . Then for any neutrosophic subsets Ⱥ and Ƀ of a N_{eu}TS $\mathbb{P}$, we have

(1) $\mathrm{N_{eu}}gs\alpha^* - int(Ⱥ \cap Ƀ) = \mathrm{N_{eu}}gs\alpha^* - int(Ⱥ) \cap \mathrm{N_{eu}}gs\alpha^* - int(Ƀ)$

(2) $\mathrm{N_{eu}}gs\alpha^* - cl(Ⱥ \cup Ƀ) = \mathrm{N_{eu}}gs\alpha^* - cl(Ⱥ) \cup \mathrm{N_{eu}}gs\alpha^* - cl(Ƀ)$

Proof:

(1) $\mathrm{N_{eu}}gs\alpha^* - int(Ⱥ \cap Ƀ) = \cup\{\mathcal{G} : \mathcal{G}\ is\ a\ \mathrm{N_{eu}}gs\alpha^* - OS\ in\ \mathbb{P}\ and\ \mathcal{G} \subseteq Ⱥ \cap Ƀ\}$. Since $Ⱥ \cap Ƀ \subseteq Ⱥ$ and $Ⱥ \cap Ƀ \subseteq Ƀ$, then by theorem 5.2 (7) , $\mathrm{N_{eu}}gs\alpha^* - int(Ⱥ \cap Ƀ) \subseteq \mathrm{N_{eu}}gs\alpha^* - int(Ⱥ)$ and $\mathrm{N_{eu}}gs\alpha^* - int\,(Ⱥ \cap Ƀ) \subseteq \mathrm{N_{eu}}gs\alpha^* - int(Ƀ) \Rightarrow \mathrm{N_{eu}}gs\alpha^* - int(Ⱥ \cap Ƀ) \subseteq \mathrm{N_{eu}}gs\alpha^* - int(Ⱥ) \cap \mathrm{N_{eu}}gs\alpha^* - int(Ƀ) \rightarrow ①$. Now by theorem 5.2 (1) , $\mathrm{N_{eu}}gs\alpha^* - int(Ⱥ) \subseteq Ⱥ$ and $\mathrm{N_{eu}}gs\alpha^* - int(Ƀ) \subseteq Ƀ \Rightarrow \mathrm{N_{eu}}gs\alpha^* - int(Ⱥ) \cap \mathrm{N_{eu}}gs\alpha^* - int(Ƀ) \subseteq Ⱥ \cap Ƀ \Rightarrow \mathrm{N_{eu}}gs\alpha^* - int(\mathrm{N_{eu}}gs\alpha^* - int(Ⱥ) \cap \mathrm{N_{eu}}gs\alpha^* - int(Ƀ)) \subseteq \mathrm{N_{eu}}gs\alpha^* - int(Ⱥ \cap Ƀ)$. By ① , $\mathrm{N_{eu}}gs\alpha^* - int\,(\mathrm{N_{eu}}gs\alpha^* - int\,(Ⱥ)) \cap \mathrm{N_{eu}}gs\alpha^* - int\,(\mathrm{N_{eu}}gs\alpha^* - int\,(Ƀ)) \subseteq \mathrm{N_{eu}}gs\alpha^* - int(Ⱥ \cap Ƀ)$ .By theorem 5.2 (5) , $\mathrm{N_{eu}}gs\alpha^* - int(Ⱥ) \cap \mathrm{N_{eu}}gs\alpha^* - int(Ƀ) \subseteq \mathrm{N_{eu}}gs\alpha^* - int\,(Ⱥ \cap Ƀ) \rightarrow ②$. By ① and ② , $\mathrm{N_{eu}}gs\alpha^* - int(Ⱥ \cap Ƀ) = \mathrm{N_{eu}}gs\alpha^* - int(Ⱥ) \cap \mathrm{N_{eu}}gs\alpha^* - int(Ƀ)$.

(2) Since $\mathrm{N_{eu}}gs\alpha^* - cl(Ⱥ \cup Ƀ) = \left(\left(\mathrm{N_{eu}}gs\alpha^* - cl(Ⱥ \cup Ƀ)\right)^c\right)^c$, then by theorem 5.3 (1) , $\mathrm{N_{eu}}gs\alpha^* - cl(Ⱥ \cup Ƀ) = \left(\mathrm{N_{eu}}gs\alpha^* - int((Ⱥ \cup Ƀ)^c)\right)^c = \left(\mathrm{N_{eu}}gs\alpha^* - int(Ⱥ^c \cap Ƀ^c)\right)^c = \left(\mathrm{N_{eu}}gs\alpha^* - int(Ⱥ^c) \cap \mathrm{N_{eu}}gs\alpha^* - int(Ƀ^c)\right)^c$ (by (1)) . Now , $\mathrm{N_{eu}}gs\alpha^* - cl(Ⱥ \cup Ƀ) = (\mathrm{N_{eu}}gs\alpha^* - int(Ⱥ)^c)^c \cup (\mathrm{N_{eu}}gs\alpha^* - int(Ƀ)^c)^c = \left(\left(\mathrm{N_{eu}}gs\alpha^* - cl(Ⱥ)\right)^c\right)^c \cup \left(\left(\mathrm{N_{eu}}gs\alpha^* - cl(Ƀ)\right)^c\right)^c$ (by theorem 5.3 (1)) . Hence , $\mathrm{N_{eu}}gs\alpha^* - cl(Ⱥ \cup Ƀ) = \mathrm{N_{eu}}gs\alpha^* - cl(Ⱥ) \cup \mathrm{N_{eu}}gs\alpha^* - cl(Ƀ)$.

Theorem 5.5: Let $(\mathbb{P}, \tau_{N_{eu}})$ be a N_{eu}TS . Then for any neutrosophic subsets Ⱥ and Ƀ of a N_{eu}TS $\mathbb{P}$, we have

(1) $\mathrm{N_{eu}}gs\alpha^* - int(Ⱥ \cup Ƀ) \supseteq \mathrm{N_{eu}}gs\alpha^* - int(Ⱥ) \cup \mathrm{N_{eu}}gs\alpha^* - int(Ƀ)$

(2) $\mathrm{N_{eu}}gs\alpha^* - cl(Ⱥ \cap Ƀ) \subseteq \mathrm{N_{eu}}gs\alpha^* - cl(Ⱥ) \cap \mathrm{N_{eu}}gs\alpha^* - cl(Ƀ)$.

Proof:

(1) Since $Ⱥ \subseteq Ⱥ \cup Ƀ$ and $Ƀ \subseteq Ⱥ \cup Ƀ$, then by theorem 5.2 (7) , $\mathrm{N_{eu}}gs\alpha^* - int(Ⱥ) \subseteq \mathrm{N_{eu}}gs\alpha^* - int(Ⱥ \cup Ƀ)$ and $\mathrm{N_{eu}}gs\alpha^* - int(Ƀ) \subseteq \mathrm{N_{eu}}gs\alpha^* - int(Ⱥ \cup Ƀ) \Rightarrow \mathrm{N_{eu}}gs\alpha^* - int(Ⱥ \cup Ƀ) \supseteq \mathrm{N_{eu}}gs\alpha^* - int(Ⱥ) \cup \mathrm{N_{eu}}gs\alpha^* - int(Ƀ)$.

(2) Since $Ⱥ \cap Ƀ \subseteq Ⱥ$ and $Ⱥ \cap Ƀ \subseteq Ƀ$, then by theorem 5.2 (8) , $\mathrm{N_{eu}}gs\alpha^* - cl(Ⱥ \cap Ƀ) \subseteq \mathrm{N_{eu}}gs\alpha^* - cl(Ⱥ)$ and $\mathrm{N_{eu}}gs\alpha^* - cl(Ⱥ \cap Ƀ) \subseteq \mathrm{N_{eu}}gs\alpha^* - cl(Ƀ) \Rightarrow \mathrm{N_{eu}}gs\alpha^* - cl(Ⱥ \cap Ƀ) \subseteq \mathrm{N_{eu}}gs\alpha^* - cl(Ⱥ) \cap \mathrm{N_{eu}}gs\alpha^* - cl(Ƀ)$.

Remark 5.6: The following example shows that the equality need not be hold in theorem 5.5.

Example 5.7: (1) Let $\mathbb{P} = \{\wp\}$ and $Ⱥ = \{\langle \wp , (0.7, 0.4, 0.6)\rangle\}$ be $N_{eu}(\mathbb{P})$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, Ⱥ\}$ is a N_{eu}TS on $(\mathbb{P}, \tau_{N_{eu}})$. $Ⱥ^c = \{\langle \wp , (0.6, 0.6, 0.7)\rangle\}$. Let $\mathcal{G} = \{\langle \wp , (0.8, 0.5, 0.7)\rangle\}$ and $Ħ = \{\langle \wp , (0.5, 0.3, 0.6)\rangle\}$ are two neutrosophic sets over $\mathbb{P}$.

(1) $N_{eu}gs\alpha^* - OS = \{0_{N_{eu}}, 1_{N_{eu}}, A, D, E\}$, where $D = \{\langle \wp, ([0.7,1], [0.6,1], [0,0.6])\rangle\}$, $E = \{\langle \wp, ([0.7, 1], [0.4, 0.5], [0, 0.6])\rangle\}$. Now, $N_{eu}gs\alpha^* - int(\mathcal{G}) = 0_{N_{eu}}$, $N_{eu}gs\alpha^* - int(H) = 0_{N_{eu}}$. Then, $N_{eu}gs\alpha^* - int(\mathcal{G}) \cup N_{eu}gs\alpha^* - int(H) = 0_{N_{eu}}$. Since, $\mathcal{G} \cup H = \{\langle \wp, (0.8, 0.5, 0.6)\rangle\}$, then $N_{eu}gs\alpha^* - int(\mathcal{G} \cup H) = F$, where $F = \{\langle \wp, ([0.7, 0.8], [0.4, 0.5], 0.6)\rangle\}$. $N_{eu}gs\alpha^* - int(\mathcal{G} \cup H) \neq N_{eu}gs\alpha^* - int(\mathcal{G}) \cup N_{eu}gs\alpha^* - int(H)$, but $N_{eu}gs\alpha^* - int(\mathcal{G} \cup H) \supseteq N_{eu}gs\alpha^* - int(\mathcal{G}) \cup N_{eu}gs\alpha^* - int(H)$. Hence, the equality need not be hold.

(2) $N_{eu}gs\alpha^* - CS = \{0_{N_{eu}}, 1_{N_{eu}}, A^c, D, E\}$, where $D = \{\langle \wp, ([0,0.6], [0,0.4], [0.7,1])\rangle\}$, $E = \{\langle \wp, ([0, 0.6], [0.5, 0.6], [0.7, 1])\rangle\}$. Now, $N_{eu}gs\alpha^* - cl(\mathcal{G}) = 1_{N_{eu}}$, $N_{eu}gs\alpha^* - cl(H) = 1_{N_{eu}}$. Then, $N_{eu}gs\alpha^* - cl(\mathcal{G}) \cap N_{eu}gs\alpha^* - cl(H) = 1_{N_{eu}}$. Since, $\mathcal{G} \cap H = \{\langle \wp, (0.5, 0.3, 0.7)\rangle\}$, then $N_{eu}gs\alpha^* - cl(\mathcal{G} \cap H) = F$, where $F = \{\langle \wp, ([0.5,0.6], [0.3,0.4], 0.7)\rangle\}$. $N_{eu}gs\alpha^* - cl(\mathcal{G} \cap H) \neq N_{eu}gs\alpha^* - cl(\mathcal{G}) \cap N_{eu}gs\alpha^* - cl(H)$, but $N_{eu}gs\alpha^* - cl(\mathcal{G} \cap H) \subseteq N_{eu}gs\alpha^* - cl(\mathcal{G}) \cap N_{eu}gs\alpha^* - cl(H)$. Hence, the equality need not be hold.

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