

See discussions, stats, and author profiles for this publication at: <https://www.researchgate.net/publication/369522457>

Seperation Axioms of Neutrosophic Beta Omega Closed Sets

Article · March 2023

DOI: 10.9790/5728-1803011922

CITATIONS

0

1 author:



[Anusuya .A](#)

Don Bosco College of Arts and Science, Keela Eral, Thoothukudi, Tamilnadu, India

7 PUBLICATIONS 3 CITATIONS

[SEE PROFILE](#)

Some of the authors of this publication are also working on these related projects:



INTUITIONISTIC FUZZY TOPOLOGY [View project](#)

Seperation Axioms of Neutrosophic Beta Omega Closed Sets

Pious Missier S¹, Anusuya A², Nagarajan A³

¹Head & Associate Professor, Department of Mathematics, Don Bosco College of Arts and Science, Keela Eral, Thoothukudi, Tamilnadu-628 908, India.
spmissier@gmail.com

²Research Scholar (Reg.No-19222232092024), Department of Mathematics, V. O. Chidambaram College, (Affiliated to Manonmaniam Sundaranar University, Tirunelveli), Thoothukudi, Tamilnadu-628 008, India. anuanishnakul@gmail.com

³Head & Associate Professor, V. O. Chidambaram College, Thoothukudi, Tamilnadu-628 008, India.
nagarajan.voc@gmail.com

Abstract:

In this paper, we introduce the concepts of neutrosophic beta omega locally indiscrete space, neutrosophic weakly hausdorff spaces, neutrosophic ultra hausdorff spaces and analyze the properties of these spaces. Furthermore, we have defined neutrosophic beta omega T_0 , neutrosophic beta omega T_1 , neutrosophic beta omega T_2 and neutrosophic beta omega normal spaces. We also have studied the concept of neutrosophic almost beta omega continuous mapping.

Keywords: neutrosophic beta omega T_0 , neutrosophic beta omega T_1 , neutrosophic beta omega T_2 , neutrosophic beta omega normal, neutrosophic almost beta omega continuous mapping.

AMS Mathematics Subject Classification: 18B30, 03E72

Date of Submission: 04-05-2022

Date of Acceptance: 19-05-2022

I. Introduction

Fuzzy set theory introduced by Zadeh[13] has laid the foundation for the new mathematical theories in the research of mathematics. The concept “neutrosophic set” was first given by Smarandache[7]. Neutrosophic operations and Neutrosophic topological spaces have been investigated by Salama[11]. Later, Dhavaseelan[6] introduced neutrosophic almost continuous function, neutrosophic strongly normal and ultra normal spaces. Here, we shall introduce separation axioms of neutrosophic beta omega closed sets and neutrosophic beta omega almost continuous mapping. Also we present characteristics of this mapping.

II. Preliminaries

Definition 2.1. [7] Let X be a non-empty fixed set. A neutrosophic set (NS) G is an object having the form $G = \{ \langle x, \mu_G(x), \sigma_G(x), \nu_G(x) \rangle : x \in X \}$ where $\mu_G(x)$, $\sigma_G(x)$ and $\nu_G(x)$ represent the degree of membership, degree of indeterminacy and the degree of nonmembership respectively of each element $x \in X$ to the set G . A neutrosophic set $G = \{ \langle x, \mu_G(x), \sigma_G(x), \nu_G(x) \rangle : x \in X \}$ can be identified as an ordered triple $\langle \mu_G, \sigma_G, \nu_G \rangle$ in $]0, 1]^+$ on X .

Definition 2.2. [2] For any two sets G and H ,

1. $G \subseteq H \Leftrightarrow \mu_G(x) \leq \mu_H(x), \sigma_G(x) \leq \sigma_H(x)$ and $\nu_G(x) \geq \nu_H(x), x \in X$
2. $G \cap H = \langle x, \mu_G(x) \wedge \mu_H(x), \sigma_G(x) \wedge \sigma_H(x), \nu_G(x) \vee \nu_H(x) \rangle$
3. $G \cup H = \langle x, \mu_G(x) \vee \mu_H(x), \sigma_G(x) \vee \sigma_H(x), \nu_G(x) \wedge \nu_H(x) \rangle$
4. $G^c = \{ \langle x, \nu_G(x), 1 - \sigma_G(x), \mu_G(x) \rangle : x \in X \}$
5. $0_N = \{ \langle x, 0, 0, 1 \rangle : x \in X \}$
6. $1_N = \{ \langle x, 1, 1, 0 \rangle : x \in X \}$.

Definition 2.3. [11] A neutrosophic topology (NT) on a non-empty set X is a family τ of neutrosophic subsets in X satisfies the following axioms:

1. $0_N, 1_N \subseteq \tau$
2. $G_1 \cap G_2 \subseteq \tau$ for any $G_1, G_2 \subseteq \tau$
3. $\bigcup G_i \subseteq \tau$ where $\{G_i : i \in J\} \subseteq \tau$

Here the pair (X, τ) is a neutrosophic topological space (NTS) and any neutrosophic set in τ is known as a neutrosophic open set (N-open set) in X . A neutrosophic set G is a neutrosophic closed set (N-closed set) if and only if its complement G^c is a neutrosophic open set in X .

Definition 2.4. [6] A space (X, τ) is called as N-strongly normal if for each pair of disjoint non-empty N-closed sets G and H , there exists disjoint N-open sets U and V such that $G \subset U$, $H \subset V$ and $cl_N(U) \cap cl_N(V) = 0_N$.

Definition 2.5. [6] A space (X, τ) is called a N-ultra normal if each pair of non-empty disjoint N-closed sets can be separated by disjoint N-clopen sets.

Definition 2.6. [6] A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is termed as N-almost continuous if $f^{-1}(V)$ is N-open in (X, τ) for each NR-open set V in (Y, σ) .

Definition 2.7. [10] A neutrosophic set G of a neutrosophic topological space (X, τ) is called neutrosophic beta omega closed ($N\beta\omega$ -closed) if $\beta cl_N(G) \subseteq U$ whenever $G \subseteq U$ and U is $N\omega$ -open in (X, τ) .

Definition 2.8. Let (X, τ) and (Y, σ) be any two neutrosophic topological spaces. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called as neutrosophic almost contra beta omega continuous (almost contra- $N\beta\omega$ -continuous) if inverse image of each NR-open set in (Y, σ) is $N\beta\omega$ -closed set in (X, τ) .

III. Seperation Axioms of neutrosophic Beta Omega Closed Sets

Definition 3.1. A space (X, τ) is called a $N\beta\omega$ -locally-indiscrete if every $N\beta\omega$ -open set is N-closed in (X, τ) .

Definition 3.2. A space (X, τ) is called as N-weakly hausdorff if for each pair of distinct N-points $x_{r,s,t}$ and $y_{r,s,t}$ in (X, τ) , there exist NR-closed sets G and H containing $x_{r,s,t}$ and $y_{r,s,t}$ respectively, so as $y_{r,s,t} \notin G$ and $x_{r,s,t} \notin H$.

Definition 3.3. A space (X, τ) is called a N-ultra hausdorff if for each pair of distinct N-points $x_{r,s,t}$ and $y_{r,s,t}$ in (X, τ) , there exists N-clopen set G containing $x_{r,s,t}$ and not containing $y_{r,s,t}$ respectively.

Definition 3.4. A space (X, τ) is called as

1. $N\beta\omega$ - T_0 if for each pair of distinct N-points $x_{r,s,t}$ and $y_{r,s,t}$ in (X, τ) , there exists $N\beta\omega$ -open set G such that $x_{r,s,t} \in G, y_{r,s,t} \notin G$ or $x_{r,s,t} \notin G, y_{r,s,t} \in G$.
2. $N\beta\omega$ - T_1 if for each pair of distinct N-points $x_{r,s,t}$ and $y_{r,s,t}$ in (X, τ) , there exist $N\beta\omega$ -open sets G and H containing $x_{r,s,t}$ and $y_{r,s,t}$ respectively, so as $y_{r,s,t} \notin G$ and $x_{r,s,t} \notin H$.
3. $N\beta\omega$ - T_2 if for each pair of distinct N-points $x_{r,s,t}$ and $y_{r,s,t}$ in (X, τ) , there exist $N\beta\omega$ -open set G containing $x_{r,s,t}$ and $N\beta\omega$ -open set H containing $y_{r,s,t}$ so as $G \cap H = 0_N$.
4. A space (X, τ) is termed as $N\beta\omega$ -normal if each pair of non-empty disjoint N-closed sets can be separated by disjoint $N\beta\omega$ -open sets.

Theorem 3.1. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is an almost contra- $N\beta\omega$ -continuous injection and (Y, σ) is N-weakly hausdorff space, then (X, τ) is $N\beta\omega$ - T_1 .

Proof. Let (Y, σ) be a N-weakly hausdorff space. For any distinct N-points $x_{r,s,t}$ and $y_{r,s,t}$ in (X, τ) , there exist G and H, NR-closed sets in (Y, σ) such that $f(x_{r,s,t}) \in G$, $f(y_{r,s,t}) \notin G$, $f(y_{r,s,t}) \in H$ and $f(x_{r,s,t}) \notin H$. Since f is almost contra-N $\beta\omega$ -continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are N $\beta\omega$ -open subsets of (X, τ) such that $x_{r,s,t} \in f^{-1}(G)$, $y_{r,s,t} \notin f^{-1}(G)$, $y_{r,s,t} \in f^{-1}(H)$ and $x_{r,s,t} \notin f^{-1}(H)$. Hence (X, τ) is N $\beta\omega$ - T_1 .

Theorem 3.2. If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a almost contra-N $\beta\omega$ -continuous injective mapping from a space (X, τ) into a N-ultra hausdorff space (Y, σ) , then (X, τ) is N $\beta\omega$ - T_0 .

Proof. Let $x_{r,s,t}$ and $y_{r,s,t}$ be any two distinct N-points in (X, τ) . Since f is an injective, $f(x_{r,s,t}) \neq f(y_{r,s,t})$ and (Y, σ) is a N-ultra hausdorff space, there exist disjoint N-clopen set G of (Y, σ) containing $f(x_{r,s,t})$ and not containing $f(y_{r,s,t})$ respectively. Subsequently, $x_{r,s,t} \in f^{-1}(G)$ and $y_{r,s,t} \notin f^{-1}(G)$, wherein $f^{-1}(G)$ is a N $\beta\omega$ -open set in (X, τ) . Then (X, τ) is N $\beta\omega$ - T_0 .

Proposition 3.1. If (Y, σ) is N-strongly normal and $f: (X, \tau) \rightarrow (Y, \sigma)$ is an almost contra-N $\beta\omega$ -continuous, closed and injection, then (X, τ) is N $\beta\omega$ -normal.

Proof. Suppose G and H are disjoint N-closed sets of (X, τ) . Let f be a N-closed and injective map. Then $f(G)$ and $f(H)$ be disjoint N-closed sets in (Y, σ) . Since (Y, σ) is N-strongly normal, there exists disjoint non empty N-open sets U and V in (Y, σ) , so that $f(G) \subset U$ and $f(H) \subset V$ and $cl_N(U) \cap cl_N(V) = \emptyset$. Since $cl_N(U)$ and $cl_N(V)$ are NR-closed and f is an almost contra-N $\beta\omega$ -continuous, $f^{-1}(cl_N(U))$ and $f^{-1}(cl_N(V))$ are N $\beta\omega$ -open sets in (X, τ) . This implies $G \subseteq f^{-1}(cl_N(U))$ and $H \subseteq f^{-1}(cl_N(V))$. Also $f^{-1}(cl_N(U))$ and $f^{-1}(cl_N(V))$ are disjoint, so that (X, τ) is N $\beta\omega$ -normal.

Theorem 3.3. If $f: (X, \tau) \rightarrow (Y, \sigma)$ is an almost contra-N $\beta\omega$ -continuous, N-closed injection along with (Y, σ) is N-ultra normal, then (X, τ) is N $\beta\omega$ -normal.

Proof. Let G and H be disjoint N-closed sets of (X, τ) . Since f is a N-closed and injective map, $f(G)$ along with $f(H)$ are disjoint N-closed sets in (Y, σ) . Since (Y, σ) is N-ultra normal, there exist disjoint N-clopen sets U and V in (Y, σ) such that $f(G) \subseteq U$ and $f(H) \subseteq V$. This implies $G \subseteq f^{-1}(U)$ with $H \subseteq f^{-1}(V)$. As f is an almost contra-N $\beta\omega$ -continuous and injection, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint N $\beta\omega$ -open sets in (X, τ) . Therefore, (X, τ) is N $\beta\omega$ -normal.

IV. Neutrosophic Almost Beta Omega Continuous Mapping

Definition 4.1. A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called almost-N $\beta\omega$ -continuous if $f^{-1}(V)$ is N $\beta\omega$ -open in (X, τ) for each NR-open set V in (Y, σ) .

Theorem 4.1. If a function $f: (X, \tau) \rightarrow (Y, \sigma)$ is almost contra-N $\beta\omega$ -continuous and (X, τ) is N $\beta\omega$ -locally-indiscrete space, then f is N-almost-continuous function.

Proof. Let G be a NR-closed set in (Y, σ) . Since f is almost contra-N $\beta\omega$ -continuous function, $f^{-1}(G)$ is N $\beta\omega$ -open set in (X, τ) . Also (X, τ) is locally-N $\beta\omega$ -indiscrete space, which implies $f^{-1}(G)$ is a N-closed set in (X, τ) . Hence f is almost-N-continuous function.

Proposition 4.1. If $f: (X, \tau) \rightarrow (Y, \sigma)$ is almost-N $\beta\omega$ -continuous and $g: (Y, \sigma) \rightarrow (Z, \phi)$ is perfectly-N-continuous, then their composition $g \circ f: (X, \tau) \rightarrow (Z, \phi)$ is also N $\beta\omega$ -continuous.

Proof. Let G be a N-open set in (Z, ϕ) . Since g is perfectly-N-continuous, $g^{-1}(G)$ is both N-open and N-closed in (Y, σ) . Also f is almost-N $\beta\omega$ -continuous $f^{-1}(g^{-1}(G)) = (g \circ f)^{-1}(G)$ is N $\beta\omega$ -open in (X, τ) . Thus $g \circ f$ is N $\beta\omega$ -continuous.

Proposition 4.2. If $f: (X, \tau) \rightarrow (Y, \sigma)$ is almost-N $\beta\omega$ -continuous and $g: (Y, \sigma) \rightarrow (Z, \phi)$ is strongly-N-continuous, then their composition $g \circ f: (X, \tau) \rightarrow (Z, \phi)$ is also N $\beta\omega$ -continuous.

Proof. Let G be a N-open set in (Z, ϕ) . Since g is strongly-N-continuous, $g^{-1}(G)$ is both N-open and N-closed in (Y, σ) . Also f is almost-N $\beta\omega$ -continuous, therefore we have $f^{-1}(g^{-1}(G)) = (g \circ f)^{-1}(G)$ is N $\beta\omega$ -open in (X, τ) . Thus $g \circ f$ is N $\beta\omega$ -continuous.

Proposition 4.3. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is almost- $N\beta\omega$ -continuous and $g : (Y, \sigma) \rightarrow (Z, \phi)$ is NR-continuous, then their composition $g \circ f : (X, \tau) \rightarrow (Z, \phi)$ is also $N\beta\omega$ -continuous.

Proof. Let G be a N -open set in (Z, ϕ) . Since g is NR-continuous, $g^{-1}(G)$ is NR-open in (Y, σ) . Also f is almost- $N\beta\omega$ -continuous, therefore $f^{-1}(g^{-1}(G)) = (g \circ f)^{-1}(G)$ is $N\beta\omega$ -open in (X, τ) . Thus $g \circ f$ is $N\beta\omega$ -continuous.

References

- [1]. Anitha S, Mohana K, Florentin Smarandache, "On NGSR Closed Sets in Neutrosophic Topological Spaces", Neutrosophic Sets and Systems, Vol.28, pp.171-178(2019).
- [2]. Arokiarani R, Dhavaseelan S, Jafari, Parimala M, "On Some New Notions and Functions in Neutrosophic Topological Spaces", Neutrosophic Sets and Systems, Vol.16, pp.16-19(2017).
- [3]. Atkinswestley A, Chandrasekar S, "Neutrosophic Weakly G^* -Closed Sets". Advances in Mathematics: Scientific Journal, Vol.9, pp.2853-2864(2020).
- [4]. Banu Priya V, Chandrasekar S, Suresh M, "Neutrosophic α generalized semi homeomorphisms", Malaya Journal of Matematik, Vol.8, pp.1824-1829(2020).
- [5]. Blessie Rebecca, S, Francina Shalini A, "Neutrosophic Generalized Regular Sets in Neutrosophic Topological Spaces", IJRAR Feb 2019, Vol.6, Issue (2019).
- [6]. Dhavaseelan R, Roja E, Uma M K, "Intuitionistic Fuzzy Resolvable and Intuitionistic Fuzzy Irresolvable Spaces, Scientia, MaGa, pp.59-67(2011).
- [7]. Florentin Smarandache, "Single Valued Neutrosophic Sets", Technical Sciences and Applied Mathematics, pp.10-14(2009).
- [8]. MD. Hanaif and Qays Hatem Imran, "Neutrosophic Generalized Homeomorphism", Neutrosophic Sets and Systems, Vol. 35, pp.340-346(2020).
- [9]. Parimala M, Jeevitha R, Smarandache F, Jafari S, Udhaya Kumar R, "Neutrosophic $\alpha\psi$ Homeomorphism in Neutrosophic Topological Spaces", Information, pp.1-10(2018).
- [10]. Pious Missier S, Anusuya A, "Neutrosophic Beta Omega Closed Sets in Neutrosophic Fuzzy Topological Spaces", Proceedings of 24th FAI-ICDBSMD 2021 Vol. 6(i), pp.42(2021).
- [11]. Salama A, Alblowi S. A, "Neutrosophic set and Neutrosophic topological spaces", IOSR Jour. of Mathematics, pp. 31-35(2013).
- [12]. Shaymaa F, Matar, Anas A. Hijab, "Some Properties of Fuzzy Neutrosophic Generalized Semi Continuous Mapping and Alpha Generalized Continuous Mapping", Baghdad Science Journal, pp. 536-541(2021)
- [13]. Zadeh L, "Fuzzy sets", Information and Control, Vol.8, pp.338-353(1965).

Pious Missier S, et. al. "Seperation Axioms of Neutrosophic Beta Omega Closed Sets." *IOSR Journal of Mathematics (IOSR-JM)*, 18(3), (2022): pp. 19-22.