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Developing a Novel Approach for Determining the Reliability of Bipolar Neutrosophic Sets and its Application in Multi-Criteria Decision-Making

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This article aims to propose an approach for determining reliability of information collected using questionnaires and bipolar neutrosophic sets. Bipolar neutrosophic sets use six membership functions that express the truth membership, the falsity membership, as well as the indeterminacy membership to the set and complementary set. Therefore, bipolar neutrosophic numbers may be suitable for applying in multi-criteria evaluation when a smaller number of complex evaluation criteria are used. Unfortunately, a significant number of membership functions make them somewhat complex for collecting data by

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surveying respondents. Using reliability of data decision-makers can identify respondents who did not want to participate in the survey, or did not understand the application of BNNs, and take appropriate action. The usability of the proposed approach is presented through two illustrative examples and conclusions were drawn based on obtained results.

Keywords: Bipolar fuzzy set, neutrosophistic, single-valued bipolar fuzzy number, data reliability, MCDM, decision making

1 INTRODUCTION

Decision-making is a process that is continuously taking place everywhere and by everyone. Therefore, the challenge that arises is to make the optimal decision for a given problem or situation. Decision-making is an essential part of operational research. Due to its popularity and rapid growth, so far it has been used for solving numerous complex decision-making problems [1-5].

MCDM enables the choice of the suitable alternative from the finite set of alternatives, respecting the values of the criterion attributes, i.e. enables the decision-making process in the presence of multiple, generally conflicting criteria [6-7]. Multi-criteria decision-making (MCDM) allows tolerating ambiguities that arise when solving many problems in the decision-making system [8]. Depending on the domain of alternatives, MCDM is divided into Multi-Objective Decision Making (MODM) and Multi-Attribute Decision Making (MADM). The fundamental problem of multi-criteria decision-making is how to reconcile criteria, different preferences, and conflicting interests. Therefore, the primary task of multi-criteria decision-making is to find the best solution in terms of evaluated criteria [9-10].

In due course of time, due to extremely rapid development, many prominent and widely-used MCDM methods are developed, such as: Simple additive weighting – SAW [11]; Analytic hierarchy process – AHP [12]; Elimination et choix traduisant la realit – ELECTRE [13]; Preference ranking organization method for enrichment evaluation – PROMETHEE [14]; Technique for order performance by similarity to ideal solution – TOPSIS [15]; Višekriterijumska optimizacija i kompromisno rešenje – VIKOR [16]; and so forth. Besides, it is worth mentioning a new generation of the MCDM methods and MCDM approaches, such as: A new additive ratio assessment method – ARAS [17]; Multiobjective optimization by ratio analysis plus full multiplicative form – MULTIMOORA [18]; Pivot pair-wise relative criteria importance assessment method – PIPRECIA [19]; Full consistency method – FUCOM [20]; Measurement of alternatives and ranking according to COmpromise solution – MARCOS [21]; Multi-attributive ideal-real comparative analysis method – MAIRCA [22]; A new combinative distance-based assessment – CODAS [23]; and so forth.

To provide a reliable methodology for solving complex decision-making problems, Zadeh [24] introduced a fuzzy set theory. Fuzzy set theory has been used successfully for solving a number of decision-making problems, which is why Bellman and Zadeh proposed fuzzy MCDM [25]. A concise overview of the application of fuzzy logic can be found in [26-29]. However, the membership function to the set, presented in this theory, has not been sufficient for solving some classes of complex decision-making problems, or its determination was difficult. Therefore, some extensions of the fuzzy set theory have been proposed. For example, Attanasov [30] proposed intuitionistic fuzzy sets by introducing non-membership function to the set, while Zhang [31] introduced the concept of bipolar fuzzy sets and suggested the usage of the two membership functions that represent membership to a set and membership to a complementary set.

Smarandache [32] introduced the neutrosophic sets (NS) theory, as the generalization of fuzzy sets and intuitionistic fuzzy sets, introducing truth-membership function, indeterminacy membership function and falsity-membership function, while Wang, Smarandache, Zhang, and Sunderraman [33] further introduced the single-valued neutrosophic sets (SVNS) that are more suitable for solving many real-world decision-making problems. Deli, Ali and Smarandache [34] introduced Bipolar Neutrosophic Sets (BNS) by generalizing the concept of bipolar fuzzy sets.

Neutrosophic set theory has enabled forming extensions of a number of MCDM methods, such as the AHP [35] and EDAS [36] methods, and has also been used for solving a number of decision-making problems, such as the diagnosis of bipolar disorder diseases [37]. A comprehensive overview of the application of neutrosophic sets to solve decision-making problems can be found in [40-41].

One of the areas where NS and BNS can be effectively applied is multiple decision-making criteria based on the use of complex evaluation criteria. However, the use of several membership functions can make evaluation somewhat complex, especially when the evaluation is based on data collected by the survey. Therefore, Smarandache, Stanujkic and Karabasevic [42] proposed an approach to determine the reliability of information collected using questionnaires and single-valued neutrosophic numbers. This research was continued in [43].

In this article, the mentioned approach is extended to the use of bipolar neutrosophic numbers. Therefore, the rest of the manuscript is organized as follows: in Section 2, the basic concepts of the bipolar neutrosophic sets are presented and in Section 3, a procedure for determining the reliability of the information contained in bipolar neutrosophic numbers is proposed. In

Section 4, the usability of the proposed approach is shown in two numerical illustrations. Finally, the conclusions are given.

2 THE BASIC CONCEPTS OF A BIPOLAR NEUTROSOPHIC SET

As is previously mentioned, Zadeh [24] proposed a fuzzy set theory and introduced the membership function.

Definition 1. ***A fuzzy set****. Let X be a nonempty set. Then, a fuzzy set A in X is a set of ordered pairs [24]:*

$$A = \{ \langle x, \mu_A(x)\rangle |\, x \in X\} \tag{1}$$

where the membership function $\mu_{A(x)}$ denotes the degree of the membership of an element x to the set A, and $\mu_A(x) \in [0, 1]$.

Definition 2. ***A bipolar fuzzy set****. Let X be a nonempty set. Then, a bipolar fuzzy set is defined as follows [44]:*

$$A = \left\{ \left\langle x, \mu_A^+(x), v_A^+(x)\right\rangle \middle|\, x \in X\right\} \tag{2}$$

where: the positive membership function $\mu_A^+(x)$denotes the satisfaction degree of the element x to the property corresponding to a bipolar-valued fuzzy set, and the negative membership function $v_A^+(x)$, denotes the degree of the satisfaction degree of the element x to a corresponding complementary bipolar-valued fuzzy set, respectively; $\mu_A^+ : X \to [0, 1]$ and $v_A^+ : X \to [0, 1]$.

Definition 3. ***A neutrosophic set****. Let X be a nonempty set. Then, NS A in X is defined as [32]:*

$$A = \{ \langle x, T_A(x), I_A(x), F_A(x)\rangle |\, x \in X\} \tag{3}$$

where: $T_A(x)$, $I_A(x)$ and $F_A(x)$, denote the truth-membership $T_A(x)$, the indeterminacy-membership $I_A(x)$ and the falsity-membership functions $F_A(x)$, and T_A, I_A, $F_A : X \to \left]0^-, 1^+\right[$.

In contrast to intuitionistic sets, the restriction regarding the sum of the membership functions is eliminated, so that $0^- \leq T_A(x)+ I_A(x)+U_A(x) \leq 3^+$.

Definition 4. ***A single-valued neutrosophic set****. Let X be a nonempty set. Then, an SVNS A over X is an object having the form [32, 33]:*

$$A = \{ \langle x, T_A(x), I_A(x), F_A(x)\rangle |\, x \in X\} \tag{4}$$

where $T_A(x)$, $I_A(x)$ and $F_A(x)$ are the truth-membership function, the indeterminacy-membership function and the falsity-membership function, respectively, T_A, I_A, $F_A : X \to [0,1]$and $0 \leq T_A(x) + I_A(x) + U_A(x) \leq 3$.

Smarandache [32] also introduced the Single Valued Neutrosophic Number (SVNN), which can be designated as follows $x = \langle t_x, i_x, f_x \rangle$ for convenience.

Definition 5. ***Bipolar neutrosophic sets***. *Let X be a nonempty set. Then, a BNS A in X is as follows [34]:*

$$A = \left\{ \left\langle x, T_A^+(x), I_A^+(x), F_A^+(x), T_A^-(x), I_A^-(x), F_A^-(x) \right\rangle \middle| \, x \in X \right\} \quad (5)$$

where: T_A^+, I_A^+, F_A^+ denote the truth-membership, the indeterminacy-membership and the falsity-membership of x to the BNS A, and T_A^-, I_A^-, F_A^- denote the truth membership, the indeterminacy-membership, and the falsity-membership of x to a complementary BNS; $T_A^+, I_A^+, F_A^+ : X \to [0,1]$ and $T_A^-, I_A^-, F_A^- : X \to [-1, 0]$.

Deli, Ali and Smarandache [34] also introduced the Bipolar Neutrosophic Number (BNN), which can be denoted as follows $x = \left\langle t_x^+, i_x^+, f_x^+, t_x^-, i_x^-, f_x^- \right\rangle$ for convenience.

Definition 6. ***Score function of BNN***. *Let be a $x = \left\langle t_x^+, i_x^+, f_x^+, t_x^-, i_x^-, f_x^- \right\rangle$ BNN. The score function $s_{(x)}$ of a BNN is as follows [34]:*

$$s_{(x)} = \left(t_x^+ + 1 - i_x^+ + 1 - f_x^+ + 1 + t_x^- - i_x^- - f_x^- \right)/6 \quad (6)$$

Definition 7. ***A bipolar neutrosophic weighted average operator of BNNs***. *Let $a_j = \left\langle t_j^+, i_j^+, f_j^+, t_j^-, i_j^-, f_j^- \right\rangle$be a collection of BNNs. The bipolar neutrosophic weighted average operator (A_w) of the n dimensions is mapping $A_w : Q_n \to Q$ as follows [34]:*

$$\begin{aligned} &A_w(a_1, a_2, \ldots, a_n) \\ &= \sum_{j=1}^{n} w_j a_j = \left(1 - \prod_{j=1}^{n} \left(1 - t_j^+\right)^{w_j}, \prod_{j=1}^{n} \left(i_j^+\right)^{w_j}, \prod_{j=1}^{n} \left(f_j^+\right)^{w_j}, \right. \\ &\left. - \prod_{j=1}^{n} \left(-t_j^-\right)^{w_j}, - \left(1 - \prod_{j=1}^{n} \left(1 - \left(-i_j^-\right)\right)^{w_j}\right), - \left(1 - \prod_{j=1}^{n} \left(1 - \left(-f_j^-\right)\right)^{w_j}\right) \right) \end{aligned} \quad (7)$$

where: w_j is the element j of the weighting vector, $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$.

3 RELIABILITY OF THE INFORMATION CONTAINED IN BNNS

NS theory has numerous applications. One of them is in decision making where different types of neutrosophic numbers are used for gathering attitudes from DMs, experts, and/or respondents. Therefore, Smarandache, Stanujkic and Karabasevic [42] proposed an approach to assess the reliability of the information $r_{(x)}$ contained in an SVNN, as follows:

$$r_{(x)} = \frac{t - f}{1 + i^{1/n}} \tag{8}$$

where: t, i, f denote the truth, the indeterminacy and the falsity of information contained in SVNN $x = \langle t,\ \mathrm{i},\ \mathrm{f}\ \rangle$, n denotes a parameter and $n \in [0, 1]$.

For simplicity, the value of the variable n can be set to 0, in which case the previous Eq. (9) has the following form:

$$r_{(x)} = \frac{t - f}{1 + i} \tag{9}$$

Due to their complexity, the reliability of the information contained in the BNN should also include the reliability of the information contained in the complementary NS, as follows:

$$r_{(x)} = r^{+}_{(x)} + r^{-}_{(x)} = 0.5\left(\frac{\left|t^{+} - f^{+}\right|}{1 + i^{-}} + \frac{\left|t^{-} - f^{-}\right|}{1 - i^{-}}\right) \tag{10}$$

where: $r_{(x)}$ denote reliability of the information contained in a BNN, $r_{(x)} \in [0, 1]$ and higher value of $r_{(x)}$ indicates the higher reliability, $r^{+}_{(x)}$ and $r^{-}_{(x)}$ denote reliability contained in the complementary NS and complementary NS.

And finally, each decision matrix in the MCDM contains more rows and more columns, which is why the following equations can be used to determine the average reliability of the information contained in their rows, columns or them, respectively:

$$r_r = \frac{1}{m}\sum_{i=1}^{m} r_{(x_{\mathrm{ij}})} \tag{11}$$

$$r_c = \frac{1}{n}\sum_{j=1}^{n} r_{(x_{\mathrm{ij}})} \tag{12}$$

$$r_m = \frac{1}{m \times n}\sum_{i=1}^{m}\sum_{j=1}^{n} r_{(x_{\mathrm{ij}})} \tag{13}$$

where: m denotes the number of alternatives, n denotes the number of criteria, and x_{ij} denotes rating of alternative i in relation to the criterion j in the form of BNN.

4 NUMERICAL ILLUSTRATIONS

In order to demonstrate the usability of the proposed approach, two numerical illustrations are presented and discussed in this section. The first example is borrowed from Ulucay, Deli and Sahin [45] and slightly modified to accurately demonstrate the usability of the proposed approach. The second example shows the application of the proposed approach in the case of a cloud service provider selection.

4.1 The First Numerical Illustration

Ulucay, Deli and Sahin [45] evaluated four green suppliers based on three criteria, namely: product quality, technology capability, and pollution control.

The decision matrix, whose elements are BNN, is shown in Table 1. The reliability of the information contained in the elements of the decision matrix, determined using Eq. (10), and the average reliability of data contained in their columns and rows, determined by Eqs. (11) and (12) are shown in Table 2. Table 2 also shows the overall reliability, which is 0.39.

It has already been stated that the higher value of the reliability of the information contained in the BNN is more preferable. The average reliability of all information contained in the decision matrix is 0.39. Such a decision

	C_1	C_2	C_3
A_1	$< 0.8, 0.0, 0.2, -0.6, -0.1, -0.1 >$	$< 0.6, 0.1, 0.2, -0.4, 0.0, -0.2 >$	$< 0.8, 0.1, 0.5, -0.3, 0.0, -0.1 >$
A_2	$< 0.7, 0.0, 0.2, -0.4, 0.0, -0.2 >$	$< 0.6, 0.2, 0.3, -0.6, -0.1, -0.3 >$	$< 0.7, 0.2, 0.5, -0.1, 0.0, -0.8 >$
A_3	$< 0.7, 0.1, 0.2, -0.2, -0.1, -0.7 >$	$< 0.9, 0.1, 0.6, 0.1, 0.0, -0.5 >$	$< 0.6, 0.1, 0.5, -0.2, -0.1, -0.6 >$
A_4	$< 0.8, 0.0, 0.1, -0.2, 0.0, -0.8 >$	$< 0.6, 0.0, 0.3, -0.1, -1.0, -0.4 >$	$< 0.9, 0.1, 0.4, 0.0, 0.0, -0.8 >$

TABLE 1
The initial decision matrix.

Source: Ulucay et al. (2018)

	C_1	C_2	C_3	***Average***
A_1	0.53	0.28	0.24	0.35
A_2	0.35	0.26	0.43	0.35
A_3	0.45	0.44	0.23	0.37
A_4	0.65	0.23	0.63	0.50
Average	0.50	0.30	0.38	0.39

TABLE 2
The reliability of the information.

Source: Authors' calculation

matrix can be accepted for further evaluation, but it is noticeable that there is a possibility to increase the reliability of the information.

Using values calculated by Eqs. (10), (11) and (12) decision-makers could analyze the reliability of collected information, make some modifications, and accept the decision matrix for further evaluation or rejected it.

4.2 The Second Numerical Illustration

The second illustration shows the partial results of an evaluation of four cloud service providers based on four criteria. For the sake of simplicity and more precise presentation, only the characteristic results obtained from three DMs and four evaluation criteria are presented below. The criteria used for evaluation are: Security – (C_1), Service levels - (C_2), Support - (C_3), and Costs - (C_4).

During the completion of the questionnaire, the respondents expressed their attitudes using values from the interval [0, 1], for positive membership functions, and values from the interval [0, 1], for negative membership functions. The use of linguistic variables in this research would significantly facilitate the collection of respondents' attitudes. However, the use of linguistic variables, i.e. predefined values of membership functions, also can limit expressing real respondents' attitudes. Therefore, linguistic variables are omitted in this case in order to perform the most realistic collection of respondents' attitudes and realistically consider the use of bipolar neutrosophic numbers for that purpose. The results of the evaluation obtained from the first decision-maker are shown in Table 3.

As can be seen from Table 4, the reliability of all responses obtained from the first respondent is 0.76, where the highest value of reliability is achieved based on criterion C_1 and the lowest value of reliability is achieved based on criterion C_2. In relation to the considered alternatives, the highest reliability of the collected responses is achieved in relation to alternative A_2, i.e. 0.84, and the lowest in relation to alternative A_4, and it is 0.62. The difference in the reliability of these alternatives is 0.22, which indicates that the evaluation

	C_1	C_2	C_3	C_4
A_1	$< 1.0, 0.0, 0.0, 0.0, 0.0, -1.0 >$	$< 0.9, 1.0, 0.1, 0.0, -0.1, -0.7 >$	$< 0.9, 0.0, 0.1, -0.1, 0.0, -0.9 >$	$< 0.9, 0.0, 0.0, 0.0, 0.0, -0.8 >$
A_2	$< 1.0, 0.0, 0.0, 0.0, 0.0, -1.0 >$	$< 0.8, 0.0, 0.0, -0.2, 0.0, -0.8 >$	$< 0.9, 0.0, 0.0, 0.0, -0.1, -0.9 >$	$< 0.8, 0.0, 0.0, 0.0, 0.0, -0.8 >$
A_3	$< 1.0, 0.1, 0.1, 0.0, 0.0, -1.0 >$	$< 1.0, 0.1, 0.1, -0.4, 0.0, 0.0 >$	$< 0.9, 0.0, 0.0, 0.0, 0.0, -0.8 >$	$< 0.7, 0.0, 0.0, 0.0, 0.0, -0.9 >$
A_4	$< 0.9, 0.1, 0.1, -0.1, -0.1, -0.9 >$	$< 0.9, 0.1, 0.2, 0.0, 0.0, -0.8 >$	$< 0.7, 2.0, 0.2, -0.1, -0.1, -0.5 >$	$< 0.8, 0.0, 0.0, 0.0, 0.0, -0.7 >$

TABLE 3
The responses obtained from the first respondent.

Source: Authors' calculation

	C_1	C_2	C_3	C_4	***Average***
A_1	1.00	0.52	0.80	0.85	0.79
A_2	1.00	0.70	0.86	0.80	0.84
A_3	0.91	0.61	0.85	0.80	0.79
A_4	0.73	0.72	0.27	0.75	0.62
Average	0.91	0.64	0.69	0.80	0.76

TABLE 4
The reliability of the information obtained from the first DM.

Source: Authors' calculation

	C_1	C_2	C_3	C_4
A_1	$< 1.0, 0.0, 0.0, 0.0, 0.0, -1.0 >$	$< 0.4, 0.1, 0.5, -0.5, -0.1, -0.4 >$	$< 0.7, 0.0, 0.5, -0.8, 0.0, -0.6 >$	$< 0.1, 0.0, 0.7, -0.1, 0.0, -0.8 >$
A_2	$< 0.9, 0.0, 0.0, 0.0, 0.0, -1.0 >$	$< 0.7, 0.0, 0.8, -0.2, 0.0, -0.4 >$	$< 0.9, 0.0, 0.6, -0.1, 0.0, -0.5 >$	$< 0.5, 0.1, 0.7, -0.3, 0.0, -0.9 >$
A_3	$< 0.9, 0.2, 0.1, 0.0, 0.0, -1.0 >$	$< 0.3, 0.0, 0.2, -0.4, -1.0, -0.8 >$	$< 0.9, 0.0, 0.5, -0.6, 0.0, -0.2 >$	$< 0.7, 0.0, 0.3, -0.2, 0.0, -0.2 >$
A_4	$< 0.9, 0.1, 0.1, -0.1, -0.1, -0.9 >$	$< 0.6, 0.0, 0.2, -0.5, 0.0, 0.1 >$	$< 0.7, 0.0, 0.1, -0.1, 0.0, -0.9 >$	$< 0.4, 0.0, 0.2, -0.6, 0.0, -0.3 >$

TABLE 5
The responses obtained from the second respondent.

Source: Authors' calculation

was performed correctly, carefully, and with an understanding of the use of BNNs.

The responses obtained from the second and the third respondents are shown in Tables 5 and 6, while the calculated reliability values are shown in Tables 7 and 8.

	C_1	C_2	C_3	C_4
A_1	$< 0.8, 0.0, 0.5, 0.0, 0.0, -0.3 >$	$< 0.4, 0.0, 0.5, -0.4, -0.8, -0.4 >$	$< 0.7, 0.0, 0.5, -0.8, 0.0, -0.6 >$	$< 0.8, 0.0, 0.0, 0.0, 0.0, 0.0 >$
A_2	$< 0.8, 0.2, 0.3, 0.0, -0.3, -0.2 >$	$< 0.7, 0.1, 0.8, -0.3, -0.5, -0.1 >$	$< 0.9, 0.0, 0.6, -0.1, 0.0, -0.5 >$	$< 0.9, 0.0, 0.0, 0.0, 0.0, 0.0 >$
A_3	$< 0.7, 0.0, 0.3, 0.0, -0.2, -0.2 >$	$< 0.2, 0.1, 0.2, -0.4, -0.7, -0.4 >$	$< 0.9, 0.0, 0.5, -0.6, 0.0, -0.2 >$	$< 0.9, 0.0, 0.0, 0.0, 0.0, 0.0 >$
A_4	$< 0.8, 0.0, 0.2, -0.1, -0.1, -0.1 >$	$< 0.3, 0.1, 0.2, -0.5, -0.5, -0.1 >$	$< 0.5, 0.0, 0.5, -0.1, 0.0, -0.2 >$	$< 0.7, 0.1, 0.1, -0.1, 0.0, 0.0 >$

TABLE 6
The responses obtained from the third respondent.

Source: Authors' calculation

	C_1	C_2	C_3	C_4	***Average***
A_1	1.00	0.09	0.20	0.65	0.49
A_2	0.95	0.15	0.35	0.39	0.46
A_3	0.83	0.15	0.40	0.20	0.40
A_4	0.73	0.50	0.70	0.25	0.54
Average	0.88	0.22	0.41	0.37	0.47

TABLE 7
The reliability of the information obtained from the second respondent.

Source: Authors' calculation

	C_1	C_2	C_3	C_4	***Average***
A_1	0.30	0.05	0.20	0.40	0.24
A_2	0.29	0.11	0.35	0.45	0.30
A_3	0.28	0.00	0.40	0.45	0.28
A_4	0.30	0.18	0.05	0.32	0.21
Average	0.29	0.09	0.25	0.41	0.26

TABLE 8
The reliability of the information obtained from the third respondent.

Source: Authors' calculation

As can be seen from Tables 7 and 8, the mentioned decision-makers achieved significantly lower values of reliability, where the reliability of responses obtained from the third decision-maker is evidently low, which is why these responses have to be rechecked or omitted from further evaluation.

In this case, responses obtained from the third decision-maker are omitted from further evaluation, and a group decision matrix was constructed using Eq. (7), as shown in Table 9.

	$\boldsymbol{C_1}$	$\boldsymbol{C_2}$	$\boldsymbol{C_3}$	$\boldsymbol{C_4}$
A_1	$< 1.0, 0.0, 0.0, 0.0, 1.0, 0.0 >$	$< 0.8, 0.3, 0.2, 0.0, 0.9, 0.4 >$	$< 0.8, 0.0, 0.2, -0.3, 1.0, 0.2 >$	$< 0.7, 0.0, 0.0, 0.0, 1.0, 0.2 >$
A_2	$< 1.0, 0.0, 0.0, 0.0, 1.0, 0.0 >$	$< 0.8, 0.0, 0.0, -0.2, 1.0, 0.3 >$	$< 0.9, 0.0, 0.0, 0.0, 0.9, 0.2 >$	$< 0.7, 0.0, 0.0, 0.0, 1.0, 0.1 >$
A_3	$< 1.0, 0.1, 0.1, 0.0, 1.0, 0.0 >$	$< 1.0, 0.0, 0.1, -0.4, 0.0, 0.4 >$	$< 0.9, 0.0, 0.0, 0.0, 1.0, 0.4 >$	$< 0.7, 0.0, 0.0, 0.0, 1.0, 0.3 >$
A_4	$< 0.9, 0.1, 0.1, -0.1, 0.9, 0.1 >$	$< 0.8, 0.0, 0.2, 0.0, 1.0, 0.5 >$	$< 0.7, 0.0, 0.1, -0.1, 0.9, 0.2 >$	$< 0.7, 0.0, 0.0, 0.0, 1.0, 0.5 >$

TABLE 9
A group decision matrix.

Source: Authors' calculation

	$\boldsymbol{C_1}$	**Score**	**Rank**
A_1	$< 1.00, 0.00, 0.00, 0.00, 0.98, 0.18 >$	0.47	3
A_2	$< 1.00, 0.00, 0.00, 0.00, 0.99, 0.16 >$	0.48	2
A_3	$< 1.00, 0.00, 0.00, 0.00, 0.70, 0.25 >$	0.51	1
A_4	$< 0.79, 0.00, 0.00, 0.00, 0.96, 0.30 >$	0.42	4

TABLE 10
Overall utilities, values of score function and ranking orders of alternatives.

Source: Authors' calculation

	$\boldsymbol{C_1}$	**Score**	**Rank**
A_1	$< 1.00, 0.00, 0.00, 0.00, 0.88, 0.16 >$	0.493	2
A_2	$< 1.00, 0.00, 0.00, 0.00, 0.91, 0.15 >$	0.491	3
A_3	$< 1.00, 0.00, 0.00, 0.00, 0.68, 0.23 >$	0.515	1
A_4	$< 0.75, 0.00, 0.00, 0.00, 0.91, 0.29 >$	0.425	4

TABLE 11
Overall utilities, values of score function and ranking orders of alternatives based on responses of three DMs

Source: Authors' calculation

The overall utility of each alternative, calculated by using Eq. (6), are shown in Table 10, where the following vector weight $w_j = (0.29, 0.24, 0.21, 0.26)$ was used. The values of the score function and ranking orders for the considered alternatives are also shown in Table 10.

As can be seen from Table 11, the alternative A_3 is the most acceptable. In order to verify the proposed approach, an additional calculation is performed with ratings of all three DMs, as it is shown in Table 11.

As can be seen from Table 11, the scores obtained from the third respondent did not affect the best-ranked alternative but reflected on the rank of the second and third-ranked alternatives. However, in some other cases, the ratings of respondents with low data reliability may affect the ranking order of alternatives. Using reliability of data decision-makers can identify respondents who did not want to participate in the survey, or did not understand the application of BNNs, and take appropriate action.

5 AN ANALYSIS OF THE PROPOSED APPROACH

In order to verify the proposed approach, some characteristic cases, i.e. characteristic BNNs, and their reliability of information are analyzed below. Table 12 shows two "ideal" cases of BNNs, as well as their reliability.

As can be seen from Table 12, both BNNs have the reliability of the information equal to one. Table 13 shows three characteristic, but less desirable, cases in which the respondent fails to consistently express his preferences.

It can also be seen from Table 13 that inconsistency in assessment is reflected in reliability.

The influence of the value of the indeterminacy-membership function on the reliability of the information contained in the BNNs is shown in Table 14.

From Table 14 it can be concluded that the increase in the value of the indeterminacy-membership function affects the decrease in the reliability of the information contained in the BNN.

Based on the cases discussed above, it can be concluded that the proposed approach can be used with high reliability for accessing the reliability of the information contained in BNNs.

BNN	$r^{+}_{(x)}$	$r^{-}_{(x)}$	$r_{(x)}$
$< 1.0, 0.0, 0.0, 0.0, 0.0, -1.0 >$	1.00	1.00	1.00
$< 0.0, 0.0, 1.0, -1.0, 0.0, 0.0 >$	1.00	1.00	1.00

TABLE 12
"Ideal" BNNs and their reliability.

Source: Authors' calculation

	BNN	$r^{+}_{(x)}$	$r^{-}_{(x)}$	$r_{(x)}$
I	$< 1.0, 0.0, 1.0, 0.0, 0.0, -1.0 >$	0.00	1.00	0.50
II	$< 0.0, 0.0, 1.0, -1.0, 0.0, -1.0 >$	1.00	0.00	0.50
II	$< 1.0, 0.0, 1.0, -1.0, 0.0, -1.0 >$	0.00	0.00	0.00

TABLE 13
Some characteristic BNNs and their reliability.

Source: Authors' calculation

BNN	$r^{+}_{(x)}$	$r^{-}_{(x)}$	$r_{(x)}$
$< 1.0, 0.5, 1.0, 0.0, -0.5, -1.0 >$	0.00	0.67	0.33
$< 0.0, 0.5, 1.0, -1.0, -0.5, -1.0 >$	0.67	0.00	0.33
$< 1.0, 0.5, 1.0, -1.0, -0.5, -1.0 >$	0.00	0.00	0.00
$< 1.0, 1.0, 1.0, 0.0, -1.0, -1.0 >$	0.00	0.50	0.25
$< 0.0, 1.0, 1.0, -1.0, -1.0, -1.0 >$	0.50	0.00	0.25
$< 1.0, 1.0, 1.0, -1.0, -1.0, -1.0 >$	0.00	0.00	0.00

TABLE 14
Influence of indeterminacy-membership function on the reliability of the information contained in BNNs.

Source: Authors' calculation

BNN	$r^{+}_{(x)}$	$r^{-}_{(x)}$	$r_{(x)}$
$< 1.0, 0.0, 0.0, -1.0, 0.0, 0.0 >$	1.00	1.00	1.00
$< 0.0, 0.0, 1.0, 0.0, 0.0, -1.0 >$	1.00	1.00	1.00

TABLE 15
Some characteristic BNNs and their reliability.

Source: Authors' calculation

Finally, Table 15 also shows two possible cases of BNNs, as well as their reliability.

From Table 15 it can be seen that the proposed approach, i.e. Eq. (10), does not include the interrelationship between positive and negative truth-membership, the indeterminacy-membership, and the falsity-membership functions. The proposed approach is an extension of the approach proposed in [42] and [43] to BNNs, proposed with the aim of simply assessing the reliability of the information contained in BNNs and identifying inconsistently in completed questionnaires. However, the proposed approach can be further extended to include the aforementioned relationships between membership functions.

6 CONCLUSIONS

Bipolar neutrosophic sets use six membership functions that express the truth membership, the falsity membership, as well as the indeterminacy membership to the set and complementary set. Therefore, bipolar neutrosophic numbers may be suitable for applying in multiple criteria evaluation when a smaller number of complex evaluation criteria are used. Unfortunately, a significant number of membership functions make them more complex for collecting data by surveying respondents.

Numerous studies are dedicated to the investigation of the use of fuzzy and neutrosophic numbers for solving complex decision-making problems. However, an evident lack of researches deals with the reliability of data collected by using surveys based on the use of fuzzy or neutrosophic numbers, as well as bipolar neutrosophic numbers. Using reliability of data decision-makers can identify respondents who did not want to participate in the survey, or did not understand the application of bipolar neutrosophic numbers, and take appropriate action. With the main aim of increasing the reliability of the decision-making process based on the bipolar neutrosophic sets, the novel approach for assessing the consistency of the respondents' responses is proposed in this article.

The research conducted in this study indicates that the proposed approach can be used to assess the reliability of the data collected using bipolar fuzzy numbers. As a shortcoming of the proposed approach can be mentioned the fact that the use of bipolar neutrosophic numbers can be complicated and even confusing for some respondents. However, the proposed approach actually allows the identification of such respondents, i.e. respondents who failed to complete the questionnaire correctly. As a weakness of the proposed approach, it can be stated that some characteristic values of reliability of the information contained in a bipolar neutrosophic number on which basis surveys could be classified into certain groups are not been defined in this article. This can be mentioned as a possible direction of further research if the proposed approach is accepted by other researchers.

The proposed approach does not include the interrelationship between positive and negative truth-membership, the indeterminacy-membership, and the falsity-membership functions. However, the proposed approach can be further extended to include the aforementioned relationships, and this can also be mentioned as one of the potential directions of future research.

Finally, the extension of the proposed approach to other types of fuzzy and neutrosophic sets, such as spherical neutrosophic sets, can be mentioned as directions for the future development of the proposed approach.

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