Types of Energy in Nover Top Graphs

G.Muthumari, R.Narmada Devi
Department of Mathematics
Vel Tech Rangarajan Dr. Sagunthala R& D Institute of Science and Technology, Chennai
Emails: mathsgmm@gmail.com; narmadadevi23@gmail.com

Abstract

Energy of graphs plays a vital role in the field of application in energy. Neutrosophic over top graph theory is more efficient and accurate results than other existing methods. In this research study, we present concepts of Energy, Laplaican energy and Signless laplacian energy in Nover top graphs. Describe some of their properties, develop relationship among them and their application.

Keywords: Nover top graph; Energy; Laplacian energy; Signless Laplacian energy of Nover top graph.

1 Introduction

In 1983, Atanassove [2] proposed the notion of intuitionistic fuzzy set as the generalization of fuzzy sets by introduced by Zadeh [19] considering the degree of membership and non-membership in order to refer to the details of intuitionistic fuzzy sets. In 1996, Coker [7] introduced the concept of an intuitionistic set (called an intuitionistic crisp set by Salama et al.) as the generalzation of an ordinary set and the specialization of an intuitionistic fuzzy set. The concept of topologized graph was introduced by Antoine Vella in 2005 [1]. Antoine Vella extended topology to the topologized graph by the S_1 space and the boundary of every vertex and edges of a graph G. Chang [6] introduced the concept of the notion of fuzzy topology. Smarandadhe[7] has described the neutrosophic set, by extending the idea of a fuzzy set. It can manage with inderminate, vague, uncertain and inconsistent data of any real-world problem. The neutrsophic set is mainly an extension of the classical set, fuzzy set and intuitionstic fuzzy set. A neutrosophic [16,17,18] has three membership grades are always independent and lie between the interval[0,1]. It has been applied in various field image processing, medical diagnosis, decision making. Narmada devi et.al [11,12,13,14] were discuss the neutrosophic over topologized graph.Guatman[9,10]introduced notion of energy of a graph in chemistry, because of its relevance to the total π-Electron energy of certain molecules and found upper and lower bounds for the energy of graph. Later, Gutman and Zhan defined the Laplacian energy of a graph as the sum of the absolutes values. In this research study, we present concepts of energy, Laplacian energy and Signless Laplacian energy in Neutrosophic over topologized graphs. Some of the properties and relations are developed among them. Also consider practical examples to illustrate the applicability of the our proposed concepts.

2 Preliminaries

[3] A Nover graph is a pair $G = (\text{A,B})$ of a crisp graph $G^* = (V, E)$ where A is Nvertex over set on V and B is a Nedge over set on E such that $T_B(xy) \leq (T_A(x) \wedge T_A(y))$, $I_B(xy) \leq (I_A(x) \wedge I_A(y))$, $F_B(xy) \geq (F_A(x) \vee F_A(y))$.

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[2]
Let G be a Nover top graph. Let $x, y \in V$. Then x dominate y in G if edge xy is effective edge
$T_B(xy) = (T_A(x) \wedge T_A(y)), I_B(xy) = (I_A(x) \wedge I_A(y)), F_B(xy) = (F_A(x) \vee F_A(y))$.
A subset D_N of V is called a Nover top dominating set in G if every vertex $V \notin D_N$ there exists $u \in D_N$ such that u dominates V.

3 Energy of Nover Top Graphs

The adjacency matrix $A(G)$ of a Nover top graph $G(A, B)$ is defined as a square matrix $A(G) = [a_{ij}]$, $a_{ij} = \langle T_B(u_{ij}), I_B(u_{ij}), F_B(u_{ij})\rangle$, where $T_B(u_{ij})$, $I_B(u_{ij})$ and $F_B(u_{ij})$ represent the strength of relationship, strength of undecided relationship and strength of non-relationship between u_i and u_j. The adjacency matrix of Nover top graph can be expressed as three matrices, first matrix contains the entries as truth-membership values, second contains the entries as indeterminacy membership values and the third contains the entries as falsity membership values.
i.e., $A(G) = \langle A(T_B(u_{ij})), A(I_B(u_{ij})), A(F_B(u_{ij}))\rangle$.

The spectrum of adjacency matrix of Nover top graph $A(G)$ is defined as $\langle P, Q, R\rangle$ where P, Q and R are the sets of eigen values of $A(T_B(u_{ij}))$, $A(I_B(u_{ij}))$, $A(F_B(u_{ij}))$ respectively.

A	u_1	u_2	u_3
T_A	0.3	0.2	0.5
I_A	0.5	0.6	1.2
F_A	1.3	1.1	0.8

B	u_1u_2	u_2u_3
T_B	0.2	0.2
I_B	0.5	0.6
F_B	1.3	1.1

The adjacency matrix of Nover top graph given in Fig. 1 is

$$A(G) = \begin{bmatrix} (0,0,0) & (0.2,0.5,1.3) & (0,0,0) \\ (0.2,0.5,1.3) & (0,0,0) & (0.2,0.6,1.1) \\ (0,0,0) & (0.2,0.6,1.1) & (0,0,0) \end{bmatrix}$$

$$T_B(G) = \begin{bmatrix} 0 & 0.2 & 0 \\ 0.2 & 0 & 0.2 \\ 0 & 0.2 & 0 \end{bmatrix}$$

$$I_B(G) = \begin{bmatrix} 0 & 0.5 & 0 \\ 0.5 & 0 & 0.6 \\ 0 & 0.6 & 0 \end{bmatrix}$$

$$\text{and } F_B(G) = \begin{bmatrix} 0 & 1.3 & 0 \\ 1.3 & 0 & 1.1 \\ 0 & 1.1 & 0 \end{bmatrix}$$

The spectrum of a Nover top graph G, given in Figure 1 as follows.

$$\begin{aligned} \text{spec}(T_B)(u_{ij})) &= (-.2828, 0, 0.2828) \\ \text{spec}(I_B)(u_{ij})) &= (-.7818, 0, 0.7810) \\ \text{spec}(F_B)(u_{ij})) &= (-1.7029, 0, 1.7029) \\ \therefore \text{spec}(F) &= \{\langle -0.2828, 0.7818, -1.7029\rangle, \langle 0\rangle, \langle 0.2828, 0.7818, 1.7029\rangle\} \end{aligned}$$

The energy of a Nover top graph $G = (A, B)$ is defined as

$$\begin{aligned} E(G) &= \langle E(T_B(u_{ij})), E(I_B(u_{ij})), E(F_B(u_{ij}))\rangle \\ &= \left\langle \sum_{\substack{i=1 \\ \lambda_i \in P}}^{n} |\lambda_i|, \sum_{\substack{i=1 \\ \delta_i \in Q}}^{n} |\delta_i|, \sum_{\substack{i=1 \\ \rho_i \in R}}^{n} |\rho_i| \right\rangle \end{aligned}$$

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Two Nover top graphs with the same number of vertices and the same energy are called equienergetic.

Let $G = (A, B)$ be a Nover top graph and $A(G)$ be its adjacency matrix. If $\lambda_1 \geq \lambda_2 \geq \cdots \lambda_n$, $\delta_1 \geq \delta_2 \geq \cdots \delta_n$ and $\rho_1 \geq \rho_2 \geq \cdots \rho_n$ are the eigen values of $A(T_B(u_{ij})), A(I_B(u_{ij}))$ and $A(F_B(u_{ij}))$ respectively. Then

(1) $\sum\limits_{\substack{i=1\\ \lambda_i \in P}}^{n} \lambda_i = 0, \quad \sum\limits_{\substack{i=1\\ \delta_i \in Q}}^{n} \delta_i = 0, \quad \sum\limits_{\substack{i=1\\ \rho_i \in R}}^{n} \rho_i = 0$

(2) $$\sum_{\substack{i=1\\ \lambda_i \in P}}^{n} \lambda_i^2 = 2 \sum_{1 \leq i < j \leq n} (T_B(u_{ij}))^2$$

$$\sum_{\substack{i=1\\ \delta_i \in Q}}^{n} \lambda_i^2 = 2 \sum_{1 \leq i < j \leq n} (I_B(u_{ij}))^2$$

$$\sum_{\substack{i=1\\ \rho_i \in R}}^{n} \rho_i^2 = 2 \sum_{1 \leq i < j \leq n} (F_B(u_{ij}))^2$$

Proof:

(1) Since $A(G)$ is a symmetric matrix whose trace is zero, so its eigen values are real with zero sum.

(2) By matrix trace properties, we have

$$\text{tr}((A(T_B(u_{ij})) = \sum_{\substack{i=1\\ \lambda_i \in P}}^{n} \lambda_i^2$$

where

$$\begin{aligned}\text{tr}((A(T_B(u_{ij})^2) &= \big(0 + {T_B}^2(u_1u_2) + \cdots {T_B}^2(u_1u_n)\big)\\ &+ \big({T_B}^2(u_2u_1) + 0 + \cdots {T_B}^2(u_2u_n)\big)\\ &+ \cdots\\ &+ \big({T_B}^2(u_nu_1) + {T_B}^2(u_nu_2) \cdots 0\big)\\ &= 2 \sum_{1 \leq i < j \leq n} (T_B(u_{ij}))^2\end{aligned}$$

Hence $\sum\limits_{\substack{i=1\\ \lambda_i \in P}}^{n} \lambda_i^2 = 2 \sum\limits_{1 \leq i < j \leq n} (T_B(u_{ij}))^2.$

Similarly we can show that

$$\sum_{\substack{i=1\\ \delta_i \in Q}}^{n} \delta_i^2 = 2 \sum_{1 \leq i < j \leq n} (I_B(u_{ij}))^2$$

and $$\sum_{\substack{i=1\\ \rho_i \in R}}^{n} \rho_i^2 = 2 \sum_{1 \leq i < j \leq n} (F_B(u_{ij}))^2$$

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Consider a Nover top graph $G(A,B)$ as show in Figure 1.

Then $E(T_B(u_{ij})) = 0.5656$, $E(I_B(u_{ij})) = 1.562$, $F(F_B(u_{ij})) = 3.4058$.

$\therefore E(G) = \langle 0.5656, 1.562, 3.4058\rangle$

$$\sum_{\substack{i=1\\ \lambda_i\in P}}^{3} \lambda_i = -.2828 + 0 + 0.2828 = 0$$

$$\sum_{\substack{i=1\\ \delta_i\in Q}}^{3} \delta_i = -.7810 + 0 + 0.7810 = 0$$

$$\sum_{\substack{i=1\\ \rho_i\in R}}^{3} \rho_i = -1.7029 + 0 + 1.7029 = 0$$

$$\sum_{\substack{i=1\\ \lambda_i\in P}}^{3} \lambda_i^2 = 0.15995 = 2(0.07998) = 2(0.08) = 2\sum_{1\le i<j\le 3}(T_B(u_{ij}))^2$$

$$\sum_{\substack{i=1\\ \delta_i\in Q}}^{3} \delta_i^2 = 1.21992 = 2(0.60996) = 2(0.61) = 2\sum_{1\le i<j\le 3}(I_B(u_{ij}))^2$$

$$\sum_{\substack{i=1\\ \rho_i\in R}}^{3} \rho_i^2 = 5.79974 = 2(2.89987) = 2(2.9) = 2\sum_{1\le i<j\le 3}(F_B(u_{ij}))^2$$

We now give upper and lower bounds on energy of a Nover top graph G, in terms of the number of vertices and the sum of squares of truth-membership, indeterminacy-membership and falsity-membership values of edges. Let $G = (A,B)$ be a Nover top graph on n vertices with adjacency matrix
$A(G) = \langle A(T_B(u_{ij})), A(I_B(u_{ij})), A(F_B(u_{ij}))\rangle$. Then

(i) $\sqrt{2\sum_{1\le i<j\le n}(T_B(u_{ij}))^2 + n(n-1)|T|^{2/n}} \le E(T_B(u_{ij})) \le \sqrt{2n\sum_{1\le i<j\le n}(T_B(u_{ij}))^2}$

(ii) $\sqrt{2\sum_{1\le i<j\le n}(I_B(u_{ij}))^2 + n(n-1)|I|^{2/n}} \le E(I_B(u_{ij})) \le \sqrt{2n\sum_{1\le i<j\le n}(I_B(u_{ij}))^2}$

(iii) $\sqrt{2\sum_{1\le i<j\le n}(F_B(u_{ij}))^2 + n(n-1)|F|^{2/n}} \le E(F_B(u_{ij})) \le \sqrt{2n\sum_{1\le i<j\le n}(F_B(u_{ij}))^2}$

where $|T|$, $|I|$ and $|F|$ are the determinant $A(T_B(u_{ij})), A(I_B(u_{ij})), A(F_B(u_{ij}))$ respectively.
Proof:

(i) Upper bound:
Apply Cauchy-Schwarz inequality to the n numbers $1, 1, \cdots, 1$ and $|\lambda_1|, |\lambda_2|, \cdots |\lambda_n|$, then

$$\sum_{i=1}^{n}|\lambda_i| \le \sqrt{n}\sqrt{\sum_{i=1}^{n}|\lambda_i|^2} \tag{1}$$

$$\left(\sum_{i=1}^{n}\lambda_i\right)^2 = \sum_{i=1}^{n}|\lambda_i|^2 + 2\sum_{1\le i<j\le n}\lambda_i\lambda_j \tag{2}$$

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By compaing the coefficients of λ^{n-2} in the characteristic polynomial

$$\prod_{i=1}^{n}(\lambda-\lambda_i)=|A(G)-\lambda I|$$

we have

$$\sum_{1\leq i<j\leq n}\lambda_i\lambda_j=-\sum_{1\leq i<j\leq n}(T_B(U_{ij}))^2 \tag{3}$$

Substituting (3) in (2), we obtain

$$\sum_{i=1}^{n}|\lambda_i|^2=2\sum_{1\leq i<j\leq n}(T_B(u_{ij}))^2 \tag{4}$$

Substituting (4) in (1), we obtain

$$\begin{aligned}\sum_{i=1}^{n}|\lambda_i| &\leq \sqrt{n}\sqrt{2\sum_{1\leq i<j\leq n}(T_B(u_{ij}))^2}\\ &=\sqrt{2n\sum_{1\leq i<j\leq n}(T_B(u_{ij}))^2}\\ \therefore E(T_B(u_{ij})) &\leq \sqrt{2n\sum_{1\leq i<j\leq n}(T_B(u_{ij}))^2}\end{aligned}$$

(ii) Lower bound:

$$\begin{aligned}(E(T_B(u_{ij})))^2 &= \left(\sum_{i=1}^{n}|\lambda_i|\right)^2\\ &=\sum_{i=1}^{n}|\lambda_i|^2+2\sum_{1\leq i<j\leq n}|\lambda_i\lambda_j|\\ &=2\sum_{1\leq i<j\leq n}(T_B(u_{ij}))^2+\frac{2n(n-1)}{2}\text{AM}\{|\lambda_i\lambda_j|\}\end{aligned}$$

Since AM$\{|\lambda_i\lambda_j|\}\geq$ GM$\{|\lambda_i\lambda_j|\}, 1\leq i<j\leq n$, so

$$E(T_B(u_{ij}))\geq\sqrt{2\sum_{1\leq i<j\leq n}(T_B(u_{ij}))^2+n(n-1)\text{GM}\{|\lambda_i\lambda_j|\}}$$

also since

$$\text{GM}\{|\lambda_i\lambda_j|\}=\left(\prod_{1\leq i<j\leq n}|\lambda_i\lambda_j|\right)^{\frac{2}{n(n-1)}}=\left(\prod_{i=1}^{n}|\lambda_i|\right)^{\frac{2}{n(n-1)}}=\left(\prod_{i=1}^{n}|\lambda_i|\right)^{\frac{2}{n}}=|T|^{\frac{2}{n}}$$

So

$$E(T_B(u_{ij}))\geq\sqrt{2\sum_{1\leq i<j\leq n}(T_B(u_{ij}))^2+n(n-1)|T|^{\frac{2}{3}}}$$

Thus

$$\sqrt{2\sum_{1\leq i<j\leq n}(T_B(u_{ij}))^2+n(n-1)|T|^{\frac{2}{n}}}\leq E(T_B(u_{ij}))\leq\sqrt{2\sum_{1\leq i<j\leq n}(T_B(u_{ij}))^2}$$

Similarly we can show that

$$\sqrt{2\sum_{1\leq i<j\leq n}(I_B(u_{ij}))^2+n(n-1)|I|^{\frac{2}{n}}}\leq E(I_B(u_{ij}))\leq\sqrt{2\sum_{1\leq i<j\leq n}(I_B(u_{ij}))^2}$$

$$\sqrt{2\sum_{1\leq i<j\leq n}(F_B(u_{ij}))^2+n(n-1)|F|^{\frac{2}{n}}}\leq E(F_B(u_{ij}))\leq\sqrt{2\sum_{1\leq i<j\leq n}(F_B(u_{ij}))^2}$$

(Showing to theorem 2) For the Nover top graph G, given in Figure 1 $E(T_B(u_{ij})) = 0.5656$, Lower bound $= 0.399$ and upper bound $= 0.6927$

$\therefore 0.399 \leq 0.5656 \leq 0.6927$

Similarly $E(I_B(u_{ij})) = 1.562$, Lower bound $= 1.1044$ and upper bound $= 1.9130$

$\therefore 1.1044 \leq 1.562 \leq 1.9130$

$E(F_B(u_{ij})) = 3.4058$, Lower bound $= 2.408$ and upper bound $= 4.171$

$\therefore 2.408 \leq 3.4058 \leq 4.171$

4 Laplacian Energy of Nover Top Graphs

Let $G = (A, B)$ be a Nover top graph on n vertices.
The degree matrix $D(G) = \langle D(T_B(u_{ij}), D(I_B(u_{ij})), D(F_B(u_{ij}))\rangle = [d_{ij}]$ of G is a $n \times n$ diagonal matrix defined as

$$d_{ij} = \begin{cases} d_G(u_i), & \text{if } i = j \\ 0, & \text{otherwise} \end{cases}$$

The Laplacian matrix of a Nover top graph $G = (A, B)$ is defined as
$L(G) = \langle L(T_B(u_{ij}), L(I_B(u_{ij})), L(F_B(u_{ij}))\rangle = D(G) - A(G)$, where $A(G)$ is an adjacency matrix and $D(G)$ is a degree matrix of a Nover top graph G.

The spectrum of Laplacian matrix of a Nover top graph $L(G)$ is defined as $\langle P_L, Q_L, R_L\rangle$, where P_L, Q_L and R_L are the sets of Laplacian eigen values of $L(T_B(u_{ij}))$, $L(I_B(u_{ij}))$ and $L(F_B(u_{ij}))$.

Consider a Nover top graph $G = (A, B)$
The adjacency and the Laplacian matrices of the Nover top graph shown in Figure 2 are as follows

$$A(G) = \begin{bmatrix} (0,0,0) & (0.3,0.3,1.2) & (0,0,0) & (0,0,0) & (0.3,0.5,1.2) \\ (0.3,0.3,1.2) & (0,0,0) & (1.1,0.3,0.9) & (0,0,0) & (0,0,0) \\ (0,0,0) & (1.1,0.3,0.9) & (0,0,0) & (0.5,0.4,1.3) & (0,0,0) \\ (0,0,0) & (0,0,0) & (0.5,0.4,1.3) & (0,0,0) & (0.5,0.6,1.3) \\ (0.3,0.5,1.2) & (0,0,0) & (0,0,0) & (0.5,0.6,1.3) & (0,0,0) \end{bmatrix}$$

$$L(G) = \begin{bmatrix} (0.6,0.8,2.4) & (-.3,-.3,-1.2) & (0,0,0) & (0,0,0) & (-.3,-.5,-1.2) \\ (-.3,-.3,-1.2) & (1.4,0.6,2.1) & (-1.1,-.3,-.9) & (0,0,0) & (0,0,0) \\ (0,0,0) & (-1.1,-.3,-.9) & (1.6,0.7,2.2) & (-.5,-.4,-1.3) & (0,0,0) \\ (0,0,0) & (0,0,0) & (-.5,-.4,-1.3) & (1,1,2.6) & (-.5,-.6,-1.3) \\ (-.3,-.5,-1.2) & (0,0,0) & (0,0,0) & (-.5,-.6,-1.3) & (0.8,1.1,2.5) \end{bmatrix}$$

$$L(T_A(G)) = \begin{bmatrix} 0.6 & -.3 & 0 & 0 & -.3 \\ -.3 & 1.4 & -1.1 & 0 & 0 \\ 0 & -1.1 & 1.6 & -.5 & 0 \\ 0 & 0 & -.5 & 1 & -.5 \\ -.3 & 0 & 0 & -.5 & .8 \end{bmatrix}$$

$$L(I_A(G)) = \begin{bmatrix} 0.8 & -.3 & 0 & 0 & -.5 \\ -.3 & 0.6 & -0.3 & 0 & 0 \\ 0 & -0.3 & 0.7 & -0.4 & 0 \\ 0 & 0 & -0.4 & 1 & -0.6 \\ -0.5 & 0 & 0 & -.6 & 1.1 \end{bmatrix}$$

$$L(F_A(G)) = \begin{bmatrix} 2.4 & -1.2 & 0 & 0 & -1.2 \\ -1.2 & 2.1 & -0.9 & 0 & 0 \\ 0 & -0.9 & 2.2 & -1.3 & 0 \\ 0 & 0 & -1.3 & 2.6 & -1.3 \\ -1.2 & 0 & 0 & -1.3 & 2.5 \end{bmatrix}$$

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The Laplacian spectrum of a Nover top graph G, given in Figure 2 is

$$\text{Laplacian spec } (T_A(u_{ij})) = (0, 0.5443, 0.6483, 1.4985, 2.7089)$$
$$\text{Laplacian spec } (I_A(u_{ij})) = (0, 0.5202, 0.5726, 1.2586, 1.8487)$$
$$\text{Laplacian spec } (F_A(u_{ij})) = (0, 1.4923, 1.7275, 4.0266, 4.5536)$$

$$\text{Laplacian spec } (G) = \{\langle 0,0,0\rangle, \langle 0.5443, 0.5202, 1.4923\rangle, \langle 0.6483, 0.5726, 1.7275\rangle, \langle 1.4985, 1.2586, 4.0266\rangle, \langle 2.7089, 1.8487, 4.5536\rangle\}$$

Let $G = (A, B)$ be a Nover top graph and let $L(G) = \langle L(T_B(u_{ij})), L(I_B(u_{ij})), L(F_B(u_{ij}))\rangle$ be the Laplacian matrix of G. If $\mu_1 \geq \mu_2 \geq \cdots \geq \mu_n$, $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$ and $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n$ are the eigen values $L(T_B(u_{ij})), L(I_B(u_{ij})), L(F_B(u_{ij}))$. Then

(1) $\sum_{i=1}^{n} \mu_i = 2 \sum_{1\leq i<j\leq n} T_B(u_{ij}), \sum_{i=1}^{n} \gamma_i = 2 \sum_{1\leq i<j\leq n} I_B(u_{ij}), \sum_{i=1}^{n} \sigma_i = 2 \sum_{1\leq i<j\leq n} F_B(u_{ij})$

(2) $\sum_{i=1}^{n} \mu_i^2 = 2 \sum_{1\leq i<j\leq n} (T_B(u_{ij}))^2 + \sum_{i=1}^{n} d^2_{T_B(u_{ij})}(u_i)$

$\sum_{i=1}^{n} \gamma_i^2 = 2 \sum_{1\leq i<j\leq n} (I_B(u_{ij}))^2 + \sum_{i=1}^{n} d^2_{I_B(u_{ij})}(u_i)$

$\sum_{i=1}^{n} \sigma_i^2 = 2 \sum_{1\leq i<j\leq n} (F_B(u_{ij}))^2 + \sum_{i=1}^{n} d^2_{F_B(u_{ij})}(u_i)$

Proof: Since $L(G)$ is a symmetric matrix with non-negative eigen values, such that

$$\sum_{\substack{i=1 \\ \lambda_i \in P_L}}^{n} \mu_i = \text{tr}(L(G)) = \sum_{i=1}^{n} d_{T_B(u_{ij})}(u_i) = 2 \sum_{1\leq i<j\leq n} T_B(u_{ij})$$

Similarly we can show that

$$\sum_{\substack{i=1 \\ \gamma_i \in Q_L}}^{n} \gamma_i = 2 \sum_{1\leq i<j\leq n} I_B(u_{ij})$$
$$\sum_{\substack{i=1 \\ \sigma_i \in R_L}}^{n} \sigma_i = 2 \sum_{1\leq i<j\leq n} F_B(u_{ij})$$

By definition of Laplacian matrix, we have

$$L(T_B(u_{ij})) = \begin{bmatrix} d_{T_B(u_{ij})}(u_1) & -T_B(u_1u_2) & \cdots & -T_B(u_1u_n) \\ -T_B(u_2u_1) & d_{T_B(u_{ij})}(u_2) & \cdots & -T_B(u_2u_n) \\ \vdots & \vdots & \vdots & \vdots \\ -T_B(u_nu_1) & -T_B(u_nu_2) & \cdots & d_{T_B(u_{ij})}(u_n) \end{bmatrix}$$

By trace properties of a matrix,

$$\text{tr}((L(T_B(u_{ij})))^2) = \sum_{\substack{i=1 \\ \lambda_i \in P_L}}^{n} \mu_i^2$$

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where

$$\begin{aligned}
\text{tr}((L(T_B(u_{ij})))^2) = &\left(d^2_{T_B(u_{ij})}(u_1) + {T_B}^2(u_1u_2) \cdots + {T_B}^2(u_1u_n)\right) \\
&+ \left({T_B}^2(u_2u_1) + d^2_{T_B(u_{ij})}(u_2) + \cdots + {T_B}^2(u_2u_n)\right) \\
&\cdots \\
&+ \left({T_B}^2(u_nu_1) + {T_B}^2(u_nu_2) + \cdots + d^2_{T_B(u_{ij})}(u_n)\right)
\end{aligned}$$

$$\therefore \sum_{\substack{i=1 \\ \lambda_i \in P_L}}^{n} \mu_i^2 = 2\sum_{1\le i<j\le n}(T_B(u_{ij}))^2 + \sum_{i=1}^{n} d^2_{T_B(u_{ij})}(u_i)$$

Similarly

$$\sum_{\substack{i=1 \\ \gamma_i \in Q_L}}^{n} \gamma_i^2 = 2\sum_{1\le i<j\le n}(I_B(u_{ij}))^2 + \sum_{i=1}^{n} d^2_{I_B(u_{ij})}(u_i)$$

$$\text{and} \quad \sum_{\substack{i=1 \\ \sigma_i \in R_L}}^{n} \sigma_i^2 = 2\sum_{1\le i<j\le n}(F_B(u_{ij}))^2 + \sum_{i=1}^{n} d^2_{F_B(u_{ij})}(u_i)$$

The Laplacian energy of Nover top graph $G = (A, B)$ is defined as

$$\begin{aligned}
\text{LE}(G) &= \langle \text{LE}(T_B(u_{ij})), \text{LE}(I_B(u_{ij})), \text{LE}(F_B(u_{ij}))\rangle \\
&= \langle \sum_{i=1}^{n} |\mu_i|, \sum_{i=1}^{n} |\gamma_i|, \sum_{i=1}^{n} |\sigma_i|\rangle
\end{aligned}$$

where

$$\begin{aligned}
e_i &= \mu_i - \frac{2\sum\limits_{1\le i<j\le n} T_B(u_{ij})}{n} \\
\xi_i &= \gamma_i - \frac{2\sum\limits_{1\le i<j\le n} I_B(u_{ij})}{n} \\
\tau_i &= \sigma_i - \frac{2\sum\limits_{1\le i<j\le n} F_B(u_{ij})}{n}
\end{aligned}$$

Let $G = (A, B)$ be a Nover top graph and let $L(G)$ be the Laplacian matrix of G. If $\mu_1 \ge \mu_2 \ge \cdots \ge \mu_n$, $\lambda_1 \ge \lambda_2 \ge \cdots \ge \lambda_n$ and $\sigma_1 \ge \sigma_2 \ge \cdots \ge \sigma_n$ are the eigen values of $L(T_B(u_{ij})), L(I_B(u_{ij})), L(F_B(u_{ij}))$ respectively and $e_i = \mu_i - \frac{2\sum\limits_{1\le i<j\le n} T_B(u_{ij})}{n}$, $\xi_i = \gamma_i - \frac{2\sum\limits_{1\le i<j\le n} I_B(u_{ij})}{n}$, $\tau_i = \sigma_i - \frac{2\sum\limits_{1\le i<j\le n} F_B(u_{ij})}{n}$. Then

$$\begin{aligned}
&\sum_{i=1}^{n} e_i = 0, \sum_{i=1}^{n} \xi_i = 0, \sum_{i=1}^{n} \tau_i = 0 \\
&\sum_{i=1}^{n} e_i^2 = 2M_T, \sum_{i=1}^{n} \xi_i^2 = 2M_I, \sum_{i=1}^{n} \tau_i = 2M_F
\end{aligned}$$

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where

$$M_T = \sum_{1\le i<j\le n} (T_B(u_{ij}))^2 + \frac{1}{2}\sum_{i=1}^{n}\left(d_{T_B(u_{ij})}(u_i) - \frac{2\sum\limits_{1\le i<j\le n} T_B(u_{ij})}{n}\right)^2$$

$$M_I = \sum_{1\le i<j\le n} (I_B(u_{ij}))^2 + \frac{1}{2}\sum_{i=1}^{n}\left(d_{I_B(u_{ij})}(u_i) - \frac{2\sum\limits_{1\le i<j\le n} I_B(u_{ij})}{n}\right)^2$$

$$M_F = \sum_{1\le i<j\le n} (F_B(u_{ij}))^2 + \frac{1}{2}\sum_{i=1}^{n}\left(d_{F_B(u_{ij})}(u_i) - \frac{2\sum\limits_{1\le i<j\le n} F_B(u_{ij})}{n}\right)^2$$

Consider a Nover top graph $G = (A, B)$ as shown in Figure 2. Then $I(G) = \langle 5.4, 4.2, 11.8\rangle$. Also we have

$$\sum_{i=1}^{5} e_i = 0, \sum_{i=1}^{5} \xi_i = 0, \sum_{i=1}^{5} \tau_i = 0$$

$$\sum_{i=1}^{5} e_i^2 = 10.3002 = 2M_T, \sum_{i=1}^{5} \xi_i^2 = 5.6002 = 2M_I, \sum_{i=1}^{5} \tau_i = 42.16 = 2M_F$$

Let $G = (A, B)$ be a Nover top graph on n vertices and
let $L(G) = \langle L(T_B(u_{ij})), L(I_B(u_{ij})), L(F_B(u_{ij}))\rangle$ be the Laplacian matrix of G. Then

$$\text{(i)}\;\; \text{LE}(T_B(u_{ij})) \le \sqrt{2n\sum_{1\le i<j\le n} (T_B(u_{ij}))^2 + n\sum_{i=1}^{n}\left(d_{T_B(u_{ij})}(u_i) - \frac{2\sum\limits_{1\le i<j\le n} T_B(u_{ij})}{n}\right)^2}$$

$$\text{(ii)}\;\; \text{LE}(I_B(u_{ij})) \le \sqrt{2n\sum_{1\le i<j\le n} (I_B(u_{ij}))^2 + n\sum_{i=1}^{n}\left(d_{I_B(u_{ij})}(u_i) - \frac{2\sum\limits_{1\le i<j\le n} I_B(u_{ij})}{n}\right)^2}$$

$$\text{(iii)}\;\; \text{LE}(F_B(u_{ij})) \le \sqrt{2n\sum_{1\le i<j\le n} (F_B(u_{ij}))^2 + n\sum_{i=1}^{n}\left(d_{F_B(u_{ij})}(u_i) - \frac{2\sum\limits_{1\le i<j\le n} F_B(u_{ij})}{n}\right)^2}$$

Proof: Apply Cauchy-Schwarz inequality to the n numbers $1, 1, \cdots 1$ and $|e_1|, |e_2|, \cdots, |e_n|$, we have

$$\sum_{i=1}^{n} |e_i| \le \sqrt{n}\sqrt{\sum_{i=1}^{n} |e_i|^2}$$

$$\text{LE}(T_B(u_{ij})) \le \sqrt{n}\sqrt{2M_T} = \sqrt{2nM_T}$$

Since

$$M_T = \sum_{1\le i<j\le n} (T_B(u_{ij}))^2 + \frac{1}{2}\sum_{i=1}^{n}\left(d_{T_B(u_{ij})}(u_i) - \frac{2\sum\limits_{1\le i<j\le n} T_B(u_{ij})}{n}\right)^2$$

$$\therefore \text{LE}(T_B(u_{ij})) \le \sqrt{2n\sum_{1\le i<j\le n} (T_B(u_{ij}))^2 + n\sum_{i=1}^{n}\left(d_{T_B(u_{ij})}(u_i) - 2\sum_{1\le i<j\le n} T_B(u_{ij})\right)^2}$$

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Similarly

$$\text{LE}(I_B(u_{ij})) \leq \sqrt{2n \sum_{1\leq i<j\leq n} (I_B(u_{ij}))^2 + n \sum_{i=1}^{n} \left(d_{I_B(u_{ij})}(u_i) - 2 \sum_{1\leq i<j\leq n} I_B(u_{ij}) \right)^2}$$

$$\text{LE}(F_B(u_{ij})) \leq \sqrt{2n \sum_{1\leq i<j\leq n} (F_B(u_{ij}))^2 + n \sum_{i=1}^{n} \left(d_{F_B(u_{ij})}(u_i) - 2 \sum_{1\leq i<j\leq n} F_B(u_{ij}) \right)^2}$$

Let $G = (A, B)$ be a Nover top graph on n vertices and
let $L(G) = \langle L(T_B(u_{ij})), L(I_B(u_{ij})), L(F_B(u_{ij}))\rangle$ be the Laplacian matrix of G. Then

(i) $\text{LE}(T_B(u_{ij})) \geq 2\sqrt{\sum_{1\leq i<j\leq n} (T_B(u_{ij}))^2 + \frac{1}{2}\sum_{i=1}^{n} \left(d_{T_B(u_{ij})}(u_i) - \frac{2\sum_{1\leq i<j\leq n} T_B(u_{ij})}{n} \right)^2}$

(ii) $\text{LE}(I_B(u_{ij})) \geq 2\sqrt{\sum_{1\leq i<j\leq n} (I_B(u_{ij}))^2 + \frac{1}{2}\sum_{i=1}^{n} \left(d_{I_B(u_{ij})}(u_i) - \frac{2\sum_{1\leq i<j\leq n} I_B(u_{ij})}{n} \right)^2}$

(iii) $\text{LE}(F_B(u_{ij})) \geq 2\sqrt{\sum_{1\leq i<j\leq n} (F_B(u_{ij}))^2 + \frac{1}{2}\sum_{i=1}^{n} \left(d_{F_B(u_{ij})}(u_i) - \frac{2\sum_{1\leq i<j\leq n} F_B(u_{ij})}{n} \right)^2}$

Proof:

$$\begin{aligned} \left(\sum_{i=1}^{n} |e_i|\right)^2 &= \sum_{i=1}^{n} |e_i|^2 + 2\sum_{1\leq i<j\leq n} |e_i e_j| \\ &\geq 4M_T \\ \text{LE}(T_B(u_{ij})) &\geq 2\sqrt{M_T} \end{aligned}$$

Since

$$M_T = \sum_{1\leq i<j\leq n} (T_B(u_{ij}))^2 + \frac{1}{2}\sum_{i=1}^{n} \left(d_{T_B(u_{ij})}(u_i) - \frac{2\sum_{1\leq i<j\leq n} T_B(u_{ij})}{n} \right)^2$$

$$\therefore \text{LE}(T_B(u_{ij})) \geq 2\sqrt{\sum_{1\leq i<j\leq n} (T_B(u_{ij}))^2 + \frac{1}{2}\sum_{i=1}^{n} \left(d_{T_B(u_{ij})}(u_i) - \frac{2\sum_{1\leq i<j\leq n} T_B(u_{ij})}{n} \right)^2}$$

Similarly

$$\text{LE}(I_B(u_{ij})) \geq 2\sqrt{\sum_{1\leq i<j\leq n} (I_B(u_{ij}))^2 + \frac{1}{2}\sum_{i=1}^{n} \left(d_{I_B(u_{ij})}(u_i) - \frac{2\sum_{1\leq i<j\leq n} I_B(u_{ij})}{n} \right)^2}$$

$$\text{LE}(F_B(u_{ij})) \geq 2\sqrt{\sum_{1\leq i<j\leq n} (F_B(u_{ij}))^2 + \frac{1}{2}\sum_{i=1}^{n} \left(d_{F_B(u_{ij})}(u_i) - \frac{2\sum_{1\leq i<j\leq n} F_B(u_{ij})}{n} \right)^2}$$

Let$G = (A, B)$ be a Nover top graph on n vertices and
let $L(G) = \langle L(T_B(u_{ij})), L(I_B(u_{ij})), L(F_B(u_{ij}))\rangle$ be the Laplacian matrix of G. Then

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(i) $\mathrm{LE}(T_B(u_{ij})) \leq |e_i| + \sqrt{(n-1)\left(2\sum_{1\leq i<j\leq n}(T_B(u_{ij}))^2 + \sum_{i=1}^{n}\left(d_{T_B(u_{ij})}(u_i) - \frac{2\sum_{1\leq i<j\leq n} T_B(u_{ij})}{n}\right)^2 - e_1^2\right)}$

(ii) $\mathrm{LE}(I_B(u_{ij})) \leq |e_i| + \sqrt{(n-1)\left(2\sum_{1\leq i<j\leq n}(I_B(u_{ij}))^2 + \sum_{i=1}^{n}\left(d_{I_B(u_{ij})}(u_i) - \frac{2\sum_{1\leq i<j\leq n} I_B(u_{ij})}{n}\right)^2 - \xi_1^2\right)}$

(iii) $\mathrm{LE}(F_B(u_{ij})) \leq |e_i| + \sqrt{(n-1)\left(2\sum_{1\leq i<j\leq n}(F_B(u_{ij}))^2 + \sum_{i=1}^{n}\left(d_{F_B(u_{ij})}(u_i) - \frac{2\sum_{1\leq i<j\leq n} F_B(u_{ij})}{n}\right)^2 - \tau_1^2\right)}$

Proof: Using Cauchy-Schwarz inequality

$$\sum_{i=1}^{n}|e_i| \leq \sqrt{n\sum_{i=1}^{n}|e_i|^2}$$

$$\sum_{i=2}^{n}|e_i| \leq \sqrt{(n-1)\sum_{i=2}^{n}|e_i|^2}$$

$$\mathrm{LE}(T_B(u_{ij})) - |e_1| \leq \sqrt{(n-1)\left(2M_T - e_1^2\right)}$$

$$\mathrm{LE}(T_B(u_{ij})) \leq |e_1| + \sqrt{(n-1)\left(2M_T - e_1^2\right)}$$

$$\therefore \mathrm{LE}(T_B(u_{ij})) \leq |e_1| + \sqrt{(n-1)\left(2\sum_{1\leq i<j\leq n}(T_B(u_{ij}))^2 + \sum_{i=1}^{n}\left(d_{T_B(u_{ij})}(u_i) - \frac{2\sum_{1\leq i<j\leq n} T_B(u_{ij})}{n}\right)^2 - e_1^2\right)} \quad \text{(A)}$$

Similarly

$$\mathrm{LE}(I_B(u_{ij})) \leq |\xi_1| + \sqrt{(n-1)\left(2\sum_{1\leq i<j\leq n}(I_B(u_{ij}))^2 + \sum_{i=1}^{n}\left(d_{I_B(u_{ij})}(u_i) - \frac{2\sum_{1\leq i<j\leq n} I_B(u_{ij})}{n}\right)^2 - \xi_1^2\right)}$$

$$\mathrm{LE}(F_B(u_{ij})) \leq |\tau_1| + \sqrt{(n-1)\left(2\sum_{1\leq i<j\leq n}(F_B(u_{ij}))^2 + \sum_{i=1}^{n}\left(d_{F_B(u_{ij})}(u_i) - \frac{2\sum_{1\leq i<j\leq n} F_B(u_{ij})}{n}\right)^2 - \tau_1^2\right)}$$

If the Novertop graph $G = (A, B)$ is regular, then

(i) $\mathrm{LE}(T_B(u_{ij})) \leq |e_1| + \sqrt{(n-1)\left(2\sum_{1\leq i<j\leq n}(T_B(u_{ij}))^2 - e_1^2\right)}$

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(ii) $\text{LE}(I_B(u_{ij})) \leq |\xi_1| + \sqrt{(n-1)\left(2\sum_{1\leq i<j\leq n}(I_B(u_{ij}))^2 - \xi_1^2\right)}$

(iii) $\text{LE}(F_B(u_{ij})) \leq |\tau_1| + \sqrt{(n-1)\left(2\sum_{1\leq i<j\leq n}(F_B(u_{ij}))^2 - \tau_1^2\right)}$

Proof: Let $G = (A, B)$ be a regular Nover top graph , then

$$d_{T_B(u_{ij})}(u_i) = \frac{2\sum_{1\leq i<j\leq n} T_B(u_{ij})}{n} \tag{B}$$

Substituting (B) in (A), we get

$$\text{LE}(T_B(u_{ij})) \leq |e_1| + \sqrt{(n-1)\left(2\sum_{1\leq i<j\leq n}(T_B(u_{ij}))^2 - e_1^2\right)}$$

Similarly

$$\text{LE}(I_B(u_{ij})) \leq |\xi_1| + \sqrt{(n-1)\left(2\sum_{1\leq i<j\leq n}(I_B(u_{ij}))^2 - \xi_1^2\right)}$$

$$\text{LE}(F_B(u_{ij})) \leq |\tau_1| + \sqrt{(n-1)\left(2\sum_{1\leq i<j\leq n}(F_B(u_{ij}))^2 - \tau_1^2\right)}$$

5 Signless Laplacian Energy of Nover Top Graphs

The signless Laplacian matrix of Nover top graph $G = (A, B)$ is defined as $L^+(G) = \langle L^+(T_B(u_{ij})), L^+(I_B(u_{ij})), L^+(F_B(u_{ij})$ $D(G) + A(G)$ where $D(G)$ and $A(G)$ are the degree matrix and the adjacency matrix of a Nover top graph G.

The spectrum of signless Laplacian matrix of Nover top graph L^+G is defined as $\langle P_{L^+}, Q_{L^+}, R_{L^+}\rangle$ where $P_{\text{SLE}}, Q_{\text{SLE}}$, and R_{SLE} are the sets of signless Laplacian eigen values of $L^+(T_B(u_{ij}))$, $L^+(I_B(u_{ij}))$ and

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$L^{+}(F_B(u_{ij}))$. Consider a Nover top graph $G=(A,B)$ as shown in Figure 2

$$L^{+}(G)=\begin{bmatrix}(.6,.8,2.4) & (.3,.3,1.2) & (0,0,0) & (0,0,0) & (.3,.5,1.2)\\ (.3,.3,1.2) & (1.4,.6,2.1) & (1.1,.3,.9) & (0,0,0) & (0,0,0)\\ (0,0,0) & (1.1,.3,.9) & (1.6,.7,2.2) & (.5,.4,1.3) & (0,0,0)\\ (0,0,0) & (0,0,0) & (.5,.4,1.3) & (1,1,2.6) & (.5,.6,1.3)\\ (.3,.5,1.2) & (0,0,0) & (0,0,0) & (.5,.6,1.3) & (.8,1.1,2.5)\end{bmatrix}$$

$$L^{+}(T_A(G))=\begin{bmatrix}.6 & .3 & 0 & 0 & .3\\ .3 & 1.4 & 1.1 & 0 & 0\\ 0 & 1.1 & 1.6 & .5 & 0\\ 0 & 0 & .5 & 1 & .5\\ .3 & 0 & 0 & .5 & .8\end{bmatrix}$$

$$L^{+}(I_A(G))=\begin{bmatrix}.8 & .3 & 0 & 0 & .5\\ .3 & .6 & .3 & 0 & 0\\ 0 & .3 & .7 & .4 & 0\\ 0 & 0 & .4 & 1 & .6\\ .5 & 0 & 0 & .6 & 1.1\end{bmatrix}$$

$$L^{+}(F_A(G))=\begin{bmatrix}2.4 & 1.2 & 0 & 0 & 1.2\\ 1.2 & 2.1 & .9 & 0 & 0\\ 0 & 0.9 & 2.2 & 1.3 & 0\\ 0 & 0 & 1.3 & 2.6 & 1.3\\ 1.2 & 0 & 0 & 1.3 & 2.5\end{bmatrix}$$

$$\text{Signless Laplacian spec }(T_B(u_{ij}))=(0.1322,0.2105,0.9237,1.4181,2.7155)$$
$$\text{Signless Laplacian spec }(I_B(u_{ij}))=(0.1250,0.1810,0.9569,1.0669,1.8702)$$
$$\text{Signless Laplacian spec }(F_B(u_{ij}))=(0.4099,0.4783,2.8619,3.2594,4.7905)$$
$$E(G)=\langle 5.4,4.2,11.8\rangle$$

Let $G=(A,B)$ be a Nover top graph and let $L^{+}(G)$ be the signless Laplacian matrix of G. If $e_1^{+}\geq e_2^{+}\geq\cdots\geq e_n^{+}, \xi_1^{+}\geq\xi_2^{+}\geq\cdots\geq\xi_n^{+}$ and $\tau_1^{+}\tau e_2^{+}\geq\cdots\geq\tau_n^{+}$ are the eigen values of $L^{+}(T_B(u_{ij})), L^{+}(I_B(u_{ij}))$ and $L^{+}(F_B(u_{ij}))$. Then

(1) $$\sum_{\substack{i=1\\ e_i^{+}\in P_L^{+}}}^{n} e_i^{+}=2\sum_{1\leq i<j\leq n}T_B(u_{ij})$$

$$\sum_{\substack{i=1\\ \xi_i^{+}\in Q_L^{+}}}^{n} \xi_i^{+}=2\sum_{1\leq i<j\leq n}I_B(u_{ij})$$

$$\sum_{\substack{i=1\\ \tau_i^{+}\in R_L^{+}}}^{n} \tau_i^{+}=2\sum_{1\leq i<j\leq n}F_B(u_{ij})$$

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(2) $$\sum_{\substack{i=1\\ e_i^+\in P_L^+}}^{n}(e_i^+)^2 = 2\sum_{1\le i<j\le n}(T_B(u_{ij}))^2+\sum_{i=1}^{n}d^2_{T_B(u_{ij})}(u_i)$$
$$\sum_{\substack{i=1\\ \xi_i^+\in Q_L^+}}^{n}(\xi_i^+)^2 = 2\sum_{1\le i<j\le n}(I_B(u_{ij}))^2+\sum_{i=1}^{n}d^2_{I_B(u_{ij})}(u_i)$$
$$\sum_{\substack{i=1\\ \tau_i^+\in R_L^+}}^{n}(\tau_i^+)^2 = 2\sum_{1\le i<j\le n}(F_B(u_{ij}))^2+\sum_{i=1}^{n}d^2_{F_B(u_{ij})}(u_i)$$

Proof: Let $G=(A,B)$ be a Nover top graph and let $L^+(G)$ be the signless Laplacian matrix of G. If $e_1^+\ge e_2^+\ge\cdots\ge e_n^+, \xi_1^+\ge\xi_2^+\ge\cdots\ge\xi_n^+$ and $\tau_1^+\ge\tau_2^+\ge\cdots\ge\tau_n^+$ are the eigen values of $L^+(T_B(u_{ij})), L^+(I_B(u_{ij}))$ and $L^+(F_B(u_{ij}))$. Then

(1) $$\sum_{\substack{i=1\\ e_i^+\in P_L^+}}^{n} e_i^+ = 2\sum_{1\le i<j\le n}T_B(u_{ij})$$
$$\sum_{\substack{i=1\\ \xi_i^+\in Q_L^+}}^{n} \xi_i^+ = 2\sum_{1\le i<j\le n}I_B(u_{ij})$$
$$\sum_{\substack{i=1\\ \tau_i^+\in R_L^+}}^{n} \tau_i^+ = 2\sum_{1\le i<j\le n}F_B(u_{ij})$$

(2) $$\sum_{\substack{i=1\\ e_i^+\in P_L^+}}^{n}(e_i^+)^2 = 2\sum_{1\le i<j\le n}(T_B(u_{ij}))^2+\sum_{i=1}^{n}d^2_{T_B(u_{ij})}(u_i)$$
$$\sum_{\substack{i=1\\ \xi_i^+\in Q_L^+}}^{n}(\xi_i^+)^2 = 2\sum_{1\le i<j\le n}(I_B(u_{ij}))^2+\sum_{i=1}^{n}d^2_{I_B(u_{ij})}(u_i)$$
$$\sum_{\substack{i=1\\ \tau_i^+\in R_L^+}}^{n}(\tau_i^+)^2 = 2\sum_{1\le i<j\le n}(F_B(u_{ij}))^2+\sum_{i=1}^{n}d^2_{F_B(u_{ij})}(u_i)$$

Proof: Proof follows at once from proof of theorem 4.1 The signless Laplacian energy of a Nover top graph $G=(A,B)$ is defined as
$LE^+(G)=\langle LE^+(T_B(u_{ij})), LE^+(I_B(u_{ij}))LE^+(F_B(u_{ij}))\rangle = \left\langle \sum_{i=1}^{n}|e_i^+|, \sum_{i=1}^{n}|\xi_i^+| \sum_{i=1}^{n}|\tau_i^+|\right\rangle$
where $e_i^+=\mu_i^+-\dfrac{2\sum_{1\le i<j\le n}T_B(u_{ij})}{n}$, $\xi_i^+=\gamma_i^+-\dfrac{2\sum_{1\le i<j\le n}I_B(u_{ij})}{n}$, $\tau_i^+=\sigma_i^+-\dfrac{2\sum_{1\le i<j\le n}F_B(u_{ij})}{n}$. Let $G=(A,B)$ be a Nover top graph and let $L^+(G)$ be the signless Laplacian matrix of G.
If $e_1^+\ge e_2^+\ge\cdots\ge e_n^+$, $\xi_1^+\ge\xi_2^+\ge\cdots\ge\xi_n^+$ and $\tau_1^+\ge\tau_2^+\ge\cdots\ge\tau_n^+$ are the eigen values of $L^+(T_B(u_{ij})), L^+(I_B(u_{ij}))$ and $L^+(F_B(u_{ij}))$ respectively and $e_i^+=\mu_i^+-\dfrac{2\sum_{1\le i<j\le n}T_B(u_{ij})}{n}$,

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$\xi_i^+ = \gamma_i^+ - \frac{2\sum\limits_{1\le i<j\le n} I_B(u_{ij})}{n}, \tau_i^+ = \sigma_i^+ - \frac{2\sum\limits_{1\le i<j\le n} F_B(u_{ij})}{n}.$

Then $\sum\limits_{i=1}^{n} e_i^+ = 0, \sum\limits_{i=1}^{n} \xi_i^+ = 0, \sum\limits_{i=1}^{n} \tau_i^+ = 0, \sum\limits_{i=1}^{n} (e_i^+)^2 = 2M_T^+, \sum\limits_{i=1}^{n} (\xi_i^+)^2 = 2M_I^+, \sum\limits_{i=1}^{n} (\tau_i^+) = 2M_F^+$

where $M_T^+ = \sum\limits_{1\le i<j\le n} (T_B(u_{ij}))^2 + \frac{1}{2}\sum\limits_{i=1}^{n}\left(d_{T_B(u_{ij})}(u_i) - \frac{2\sum\limits_{1\le i<j\le n} T_B(u_{ij})}{n}\right)^2,$

$M_I^+ = \sum\limits_{1\le i<j\le n} (I_B(u_{ij}))^2 + \frac{1}{2}\sum\limits_{i=1}^{n}\left(d_{I_B(u_{ij})}(u_i) - \frac{2\sum\limits_{1\le i<j\le n} I_B(u_{ij})}{n}\right)^2$

$M_F^+ = \sum\limits_{1\le i<j\le n} (F_B(u_{ij}))^2 + \frac{1}{2}\sum\limits_{i=1}^{n}\left(d_{F_B(u_{ij})}(u_i) - \frac{2\sum\limits_{1\le i<j\le n} F_B(u_{ij})}{n}\right)^2$. Consider a Nover graph $G = (A, B)$ as shown in Figure 2. Then $LE^+(T_B(u_{ij})) = 5.4$, $LE^+(I_B(u_{ij})) = 4.2$, $LE^+(F_B(u_{ij})) = 11.8$.
$\therefore LE^(G) = \langle 5.4, 4.2, 11.8\rangle$.

Also, we have $\sum\limits_{i=1}^{5} e_i^+ = 0, \sum\limits_{i=1}^{5} \xi_i^+ = 0, \sum\limits_{i=1}^{5} \tau_i^+ = 0,$

$\sum\limits_{i=1}^{5} (e_i^+)^2 = 10.3 = 2(5.15) = 2M_T^+, \sum\limits_{i=1}^{n} (\xi_i^+)^2 = 5.6 = 2(2.8) = 2M_I^+,$

$\sum\limits_{i=1}^{n} (\tau_i^+) = 42.16 = 2(21.08) = 2M_F^+$

6 Relation Among Energy, Laplacian Energy and Signless Laplacian Energy of Nover Top Graphs

Let G be a Nover top graph on n vertices and let $A(G)$, $L(G)$ and $L^+(G)$ be the adjacency, the Laplacian and the signless Laplacian matrices of G. Then $|LE^+(G) - LE(E) \le 2E(G)|$.

Proof:

$$L^+(T_B(u_{ij})) - \frac{2\sum\limits_{1\le i<j\le n} T_B(u_{ij})}{n} = D(T_B(u_{ij})) + A(T_B(u_{ij})) - \frac{2\sum\limits_{1\le i<j\le n} T_B(u_{ij})}{n} \quad \text{(a)}$$

$$L(T_B(u_{ij})) - \frac{2\sum\limits_{1\le i<j\le n} T_B(u_{ij})}{n} = D(T_B(u_{ij})) - A(T_B(u_{ij})) - \frac{2\sum\limits_{1\le i<j\le n} T_B(u_{ij})}{n} \quad \text{(b)}$$

From equation (a) & (b), we get

$$\left(L^+(T_B(u_{ij})) - \frac{2\sum\limits_{1\le i<j\le n} T_B(u_{ij})}{n}\right) - \left(L(T_B(u_{ij})) - \frac{2\sum\limits_{1\le i<j\le n} T_B(u_{ij})}{n}\right) = 2A(T_B(u_{ij}))$$

Then

$$\left(L(T_B(u_{ij})) - \frac{2\sum\limits_{1\le i<j\le n} T_B(u_{ij})}{n}\right) = \left(L^+(T_B(u_{ij})) - \frac{2\sum\limits_{1\le i<j\le n} T_B(u_{ij})}{n}\right) - 2A(T_B(u_{ij}))$$

Also

$$\left(L^+(T_B(u_{ij})) - \frac{2\sum\limits_{1\le i<j\le n} T_B(u_{ij})}{n}\right) = 2A(T_B(u_{ij})) + \left(L(T_B(u_{ij})) - \frac{2\sum\limits_{1\le i<j\le n} T_B(u_{ij})}{n}\right)$$

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By well known property of energy of a graph,

$$\begin{aligned}
LE(T_B(u_{ij})) &= E\left(L(T_B(u_{ij})) - \frac{2\sum_{1\le i<j\le n} T_B(u_{ij})}{n}\right)\\
&\le E\left(L^+(T_B(u_{ij})) - \frac{2\sum_{1\le i<j\le n} T_B(u_{ij})}{n}\right) + E(2A(T_B(u_{ij})))\\
&= LE^+(T_B(u_{ij})) + 2E(T_B(u_{ij})) \qquad \text{(c)}\\
LE^+(T_B(u_{ij})) &= E\left(L^+(T_B(u_{ij})) - \frac{2\sum_{1\le i<j\le n} T_B(u_{ij})}{n}\right)\\
&\le E\left(L(T_B(u_{ij})) - \frac{2\sum_{1\le i<j\le n} T_B(u_{ij})}{n}\right) + E(2A(T_B(u_{ij})))\\
&= LE(T_B(u_{ij})) + 2E(T_B(u_{ij})) \qquad \text{(d)}
\end{aligned}$$

Combining (c) and (d) we get

$$|LE^+(T_B(u_{ij})) - LE(T_B(u_{ij}))| \le 2E(T_B(u_{ij}))$$

Similarly

$$\begin{aligned}
|LE^+(I_B(u_{ij})) - LE(I_B(u_{ij}))| &\le 2E(I_B(u_{ij}))\\
|LE^+(F_B(u_{ij})) - LE(F_B(u_{ij}))| &\le 2E(F_B(u_{ij}))
\end{aligned}$$

If the Nover top graph G is regular. Then $E(G) \le LE(G) = LE^+(G)$.

Let $G = (A, B)$ be a Nover top graph on n vertices and let $L(G)$ and $L^+(G)$ be the Laplacian and the signless Laplacian matrices of G. Then

$$\begin{aligned}
LE^+(T_B(u_{ij})) + LE(T_B(u_{ij})) &\ge 4E(T_B(u_{ij})) - \frac{4r\sum_{1\le i<j\le n} T_B(u_{ij})}{n}\\
LE^+(I_B(u_{ij})) + LE(I_B(u_{ij})) &\ge 4E(I_B(u_{ij})) - \frac{4r\sum_{1\le i<j\le n} I_B(u_{ij})}{n}\\
LE^+(F_B(u_{ij})) + LE(F_B(u_{ij})) &\ge 4E(F_B(u_{ij})) - \frac{4r\sum_{1\le i<j\le n} F_B(u_{ij})}{n}
\end{aligned}$$

Let $G = (A, B)$ be a Nover top graph on n vertices and
let $L(G) = \langle L(T_B(u_{ij})), L(I_B(u_{ij})), L(F_B(u_{ij}))\rangle$ be the Laplacian matrix of G. Then

$$\begin{aligned}
LE(T_B(u_{ij})) &\le E(T_B(u_{ij})) + \sum_{i=1}^{n}\left| d_{T_B(u_{ij})}(u_{ij}) - \frac{2\sum_{1\le i<j\le n} T_B(u_{ij})}{n}\right|\\
LE(I_B(u_{ij})) &\le E(I_B(u_{ij})) + \sum_{i=1}^{n}\left| d_{I_B(u_{ij})}(u_{ij}) - \frac{2\sum_{1\le i<j\le n} I_B(u_{ij})}{n}\right|\\
LE(F_B(u_{ij})) &\le E(F_B(u_{ij})) + \sum_{i=1}^{n}\left| d_{F_B(u_{ij})}(u_{ij}) - \frac{2\sum_{1\le i<j\le n} F_B(u_{ij})}{n}\right|
\end{aligned}$$

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Let $G = (A, B)$ be a Nover top graph on n vertices and
let $L^+(G) = \langle L^+(T_B(u_{ij})), L^+(I_B(u_{ij})), L^+(F_B(u_{ij}))\rangle$ be the signless Laplacian matrix of G. Then

$$LE^+(T_B(u_{ij})) \leq E(T_B(u_{ij})) + \sum_{i=1}^{n} \left| d_{T_B(u_{ij})}(u_{ij}) - \frac{2\sum\limits_{1\leq i<j\leq n} T_B(u_{ij})}{n} \right|$$

$$LE^+(I_B(u_{ij})) \leq E(I_B(u_{ij})) + \sum_{i=1}^{n} \left| d_{I_B(u_{ij})}(u_{ij}) - \frac{2\sum\limits_{1\leq i<j\leq n} I_B(u_{ij})}{n} \right|$$

$$LE^+(F_B(u_{ij})) \leq E(F_B(u_{ij})) + \sum_{i=1}^{n} \left| d_{F_B(u_{ij})}(u_{ij}) - \frac{2\sum\limits_{1\leq i<j\leq n} F_B(u_{ij})}{n} \right|$$

7 Real Time Example

In this section, the proposed concepts of Energy, Laplacian energy and Signless laplacian energy of a Nover top graphs are explained through a real-time example. Considering a real-life situation in a hospital where 6 nurses are working in 84 hours working schedule per week with the following constrains. If a nurse is greater than 1(>1) where as if a nurse do her duty for <84 hours than the membership value of the nurse is less then 1 (<1). In the mean time the nurse who is working in between these two are termed as interminancy. Based on the above mentioned conditions we are assigning membership values for the nurses.
For the first week (see Fig:3) we have

$$\begin{aligned}
\text{spec } T(G_1) &= (-1.0815, -0.6430, 0.1471, 0.4489, 1.1235) \\
\text{spec } I(G_1) &= (-0.7445, -0.3680, 0.1364, 0.2093, 0.7667) \\
\text{spec } F(G_1) &= (-2.0789, -1.8700, 0.7305, 0.7700, 2.4484) \\
E(G_1) &= (3.449, 2.2249, 7.8978) \\
\text{Laplace spec } T(G_1) &= (0, 0.6, 0.6405, 1.6256, 2.3340) \\
\text{Laplace spec } I(G_1) &= (0, 0.3746, 0.4460, 0.9626, 1.6169) \\
\text{Laplace spec } F(G_1) &= (0, 1.6684, 1.6931, 4.2316, 4.6069) \\
LE(G_1) &= \langle 5.2001, 3.4, 12.2\rangle \\
\text{Signless Laplacian spec } T((G_1)) &= (0.1336, 0.2243, 0.9657, 1.5224, 2.3541) \\
\text{Signless Laplacian spec } I((G_1)) &= (1.6663, -0.0835, 0.0978, 0.9254, 0.7941) \\
\text{Signless Laplacian spec } F((G_1)) &= (0.4446, 0.4853, 3.1424, 3.2147, 4.9130) \\
E(G_1) &= \langle S(T(G)), S(I(G)), S(F(G))\rangle \\
&= \langle 5.2, 3.4, 12.20\rangle
\end{aligned}$$

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[red,fill=yellow,node$_s$$tyle, label = below : (0.2, 0.6, 1.1)](u02)at(0, 0)u_2$;
[red,fill=yellow,node$_s$$tyle, label = below : (0.5, 1.2, 0.8)](u03)at(4, 0)u_3$;
[ultra
thick,edge$_s$$tyle, above, sloped](u01)edge[red]node(0.2, 0.5, 1.3)(u02); [ultrathick, edge_style, above, sloped](u02)edge[red$

Figure 1: Neutrosophic over topological grphs

[scale=0.8, auto,node distance=5cm,
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[red,fill=yellow,node$_s$$tyle, label = above : (0.3, 0.5, 1.2)](u1)at(10, 7)u_1$;
[red,fill=yellow,node$_s$$tyle, label = right : (1.2, 0.3, 0.8)](u2)at(12, 4)u_2$;
[red,fill=yellow,node$_s$$tyle, label = right : (1.1, 0.4, 0.9)](u3)at(12, 0)u_3$;
[red,fill=yellow,node$_s$$tyle, label = left : (0.5, 0.6, 1.3)](u4)at(8, 0)u_4$;
[red,fill=yellow,node$_s$$tyle, label = left : (0.8, 1.3, 0.6)](u5)at(8, 4)u_5$;
[ultra
thick,edge$_s$$tyle, above, sloped](u1)edge[red]node(0.3, 0.3, 1.2)(u2); [ultrathick, edge_style, above, sloped](u2)edge[red]no$

Figure 2: Nover top graph

[scale=0.85, auto,node distance=5cm,
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[red,fill=yellow,node$_s$$tyle, label = left : (0.6, 0.5, 1.2)](u1)at(4, 0)u_1$;
[red,fill=yellow,node$_s$$tyle, label = left : (0.3, 0.2, 1.1)](u2)at(4, 4)u_2$;
[red,fill=yellow,node$_s$$tyle, label = above : (0.8, 0.9, 1.3)](u3)at(8, 7)u_3$;
[red,fill=yellow,node$_s$$tyle, label = right : (1.4, 0.3, 0.7)](u4)at(12, 4)u_4$;
[red,fill=yellow,node$_s$$tyle, label = right : (0.8, 0.5, 1.2)](u5)at(12, 0)u_5$;
[ultra
thick,edge$_s$$tyle, above, sloped](u1)edge[red]node(0.6, 0.5, 1.3)(u3); [ultrathick, edge_style, above, sloped](u3)edge[red]node$

Figure 3: G_1

[scale=0.85, auto,node distance=5cm,
node$_s$$tyle/.style = circle, draw, ultrathick, edge_style/.style = draw = black]$
[red,fill=yellow,node$_s$$tyle, label = left : (1.5, 0.4, 0.3)](u1)at(4, 0)u_1$;
[red,fill=yellow,node$_s$$tyle, label = left : (1.4, 0.2, 0.8)](u2)at(4, 4)u_2$;
[red,fill=yellow,node$_s$$tyle, label = above : (0.8, 0.7, 1.1)](u3)at(8, 7)u_3$;
[red,fill=yellow,node$_s$$tyle, label = right : (0.4, 0.8, 1.2)](u4)at(12, 4)u_4$;
[red,fill=yellow,node$_s$$tyle, label = right : (0.8, 0.2, 1.3)](u5)at(12, 0)u_5$;
[ultra
thick,edge$_s$$tyle, above, sloped](u1)edge[red]node(0.8, 0.4, 1.1)(u3); [ultrathick, edge_style, above, sloped](u3)edge[red]node$

Figure 4: G_2

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$$\begin{aligned}
\text{spec } T(G_2) &= (-1.2944, -0.8000, 0.2343, 0.4944, 1.3657)\\
\text{spec } I(G_2) &= (-0.9136, -0.3159, 0.0873, 0.2206, 0.9216)\\
\text{spec } F(G_2) &= (-2.0615, -1.8890, 0.6904, 0.8141, 2.4459)\\
E(G_2) &= \langle 4.1888, 2.4589, 7.9009\rangle\\
\text{Laplace spec } T(G_2) &= (0, 0.8, 0.8, 2, 2.800)\\
\text{Laplace spec } I(G_2) &= (0, 0.3149, 0.4530, 0.8482, 1.9840)\\
\text{Laplace spec } F(G_2) &= (0, 1.6339, 1.7281, 4.2625, 4.5755)\\
LE(G_1) &= \langle 6.4, 3.6, 12.2\rangle\\
\text{Signless Laplacian spec } T(G_2) &= (0.1754, 0.2812, 1.2853, 1.8246, 2.8335)\\
\text{Signless Laplacian spec } I(G_2) &= (0.0798, 0.1322, 0.6273, 0.7740, 1.9858)\\
\text{Signless Laplacian spec } F(G_2) &= (0.4481, 0.4811, 3.1065, 3.2607, 4.9036)\\
\text{Signless Laplacian spec } E(G_2) &= \langle 6.400, 3.6, 12.2\rangle
\end{aligned}$$

$$\begin{aligned}
\text{spec } T(G_3) &= (-1.0815, -0.6430, 0.1471, 0.4489, 1.1235)\\
\text{spec } I(G_3) &= (-0.4854, -0.3592, 0.1251, 0.1854, 0.5341)\\
\text{spec } F(G_3) &= (-2.0292, -1.5373, 0.5965, 0.7190, 2.2509)\\
E(G_3) &= \langle 5.4916, 1.6892, 7.1329\rangle\\
\text{Laplace spec } T(G_3) &= (0.0901, 1.0429, 1.4679, 2.5786, 3.9205)\\
\text{Laplace spec } I(G_3) &= (-0.0236, 0.3000, 1.4314, 3.4964, 4.3245)\\
\text{Laplace spec } F(G_3) &= (-0.1098, 1.3575, 1.4314, 3.4964, 4.3245)\\
LE(G_3) &= \langle 9.1, 2.5, 10.5\rangle\\
\text{Signless Laplacian spec } T(G_3) &= (0.2179, 0.6446, 2.0695, 2.2173, 3.9506)\\
\text{Signless Laplacian spec } I(G_3) &= (0.0505, 0.0985, 0.5845, 0.6883, 1.0781)\\
\text{Signless Laplacian spec } F(G_3) &= (0.2033, 0.4224, 2.4739, 2.8325, 4.4679)\\
\text{Signless Laplacian spec } E(G_3) &= \langle 9.1, 2.5, 10.5\rangle
\end{aligned}$$

	$E(G)$	$LE(G)$	$SLE(G)$
I week (G_1)	$\langle 3.449, 2.2249, 7.8978\rangle$	$\langle 5.2001, 3.4, 12.2\rangle$	$\langle 5.2, 3.4, 12.2\rangle$
II week (G_2)	$\langle 4.1888, 2.4589, 7.9009\rangle$	$\langle 6.4, 3.6, 12.2\rangle$	$\langle 6.4, 3.6, 12.2\rangle$
III week (G_3)	$\langle 5.4916, 1.6892, 7.1329\rangle$	$\langle 9.1, 2.5, 10.5\rangle$	$\langle 9.1, 2.5, 10.5\rangle$

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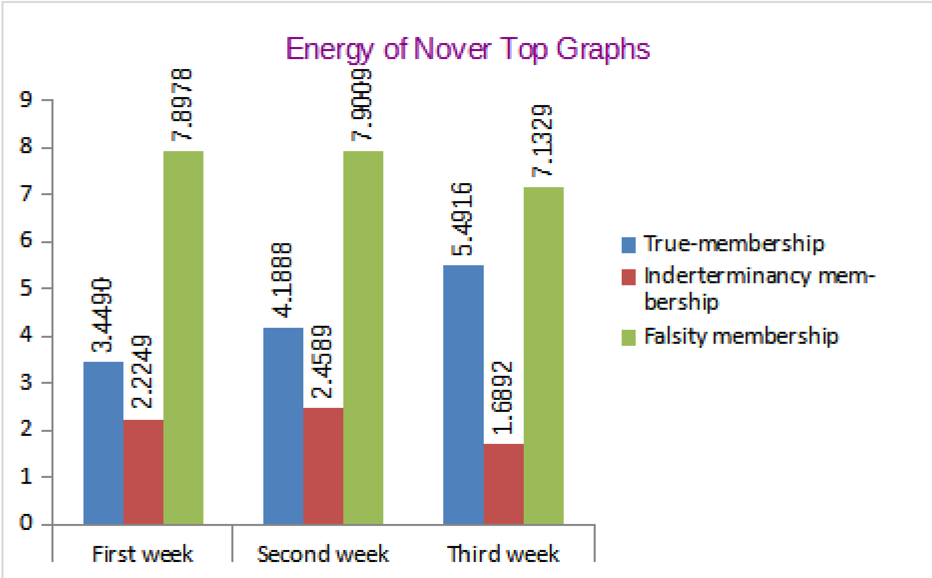

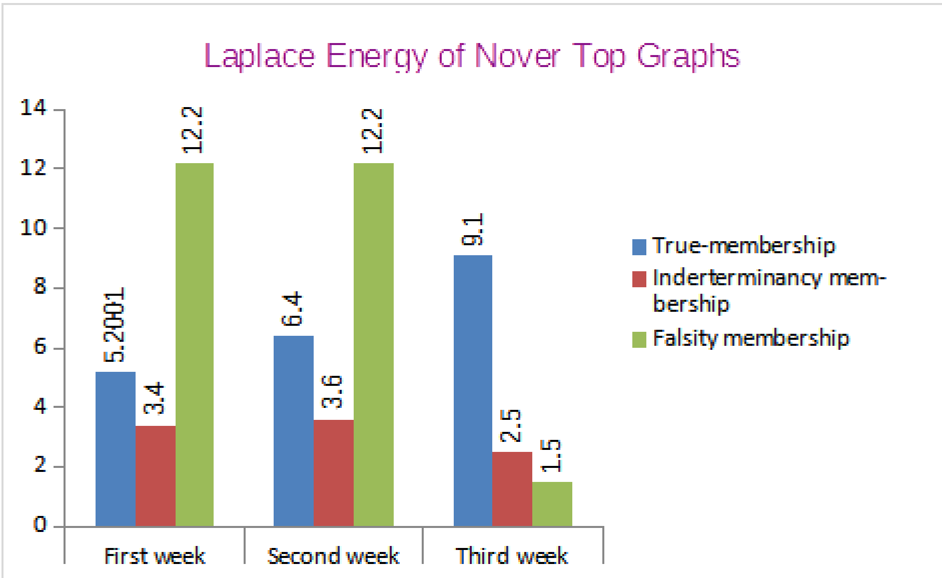

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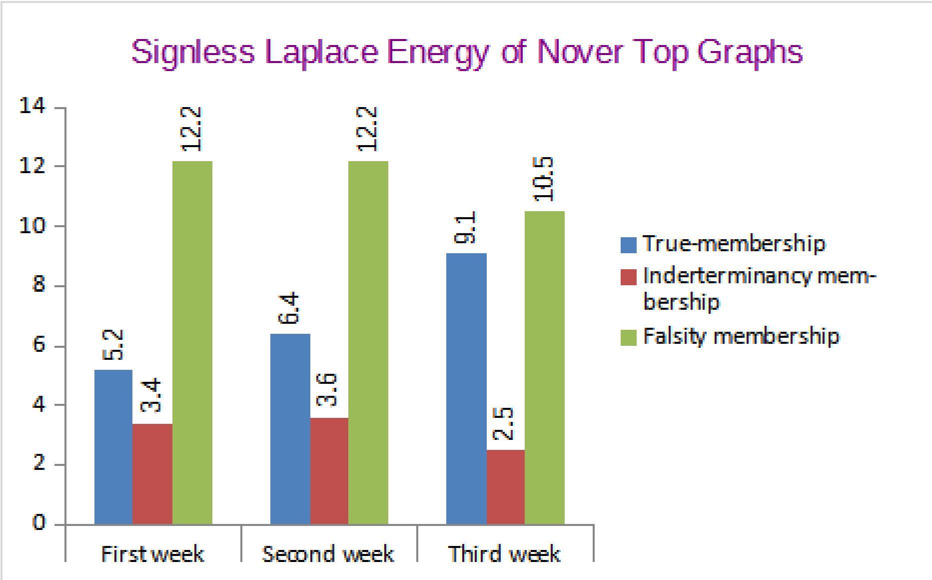

The bar diagram, shown in figuers: 3,4,5 represent the Energy,Laplacian energy and signless Laplcian energy of six nurses for the 3 week corresponding to the truth-membership, indeterminancy -membership and falsity-membership values. From the above Nover top graphs, Energy, Laplacian energy and Signless laplacian energy of truth membership for the third week is high compared to other weeks, the energy, Laplacian energy and Signless laplacian energy of Inderterminancy-membership for the second week is high and the Energy, Laplacian energy and Signless laplacian energy of falsity-membership for the period first and second week is high. Based on the above mentioned details we say that the frequency is high in the third week.

8 conclusion

In this paper, we have introduced the concepts energy, Laplacian energy and Signless laplacian energy of Nover top graphs. we have desired the lower and upper bounds for the energy, Laplacian energy and Signless laplacian energy of a Nover top graphs. We have obtained the relations and proprties of energy, Laplacian energy and Signless laplacian energy of Nover top graphs. These concepts are also illustrated with real-time examples. we are planning to extend our research work to Neutrosophic off/under top graphs.

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[red,fill=yellow,node$_{s}tyle, label = left : (0.5, 0.3, 1.1)](u2)at(4, 4)u_2$;
[red,fill=yellow,node$_{s}tyle, label = above : (1.2, 0.5, 0.9)](u3)at(8, 7)u_3$;
[red,fill=yellow,node$_{s}tyle, label = right : (0.8, 0.3, 1.3)](u4)at(12, 4)u_4$;
[red,fill=yellow,node$_{s}tyle, label = right : (1.5, 0.2, 0.7)](u5)at(12, 0)u_5$;
[ultra
thick,edge$_{s}tyle, above, sloped](u3)edge[red]node$ $(1.2, 0.3, 0.9)$ $(u1)[ultrathick, edge_{s}tyle, above, sloped](u3)edge[red]node$

Figure 5: G_3

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