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Neutrosophic Orbit Topological Spaces

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Abstract

Neutrosophy is a flourishing arena which conceptualizes the notion of true, falsity and indeterminancy attributes of an event. In the study of dynamical systems, an orbit is a collection of points related by the evolution function of the dynamical system. Hence in this paper we focus on introducing the concept of neutrosophic orbit topological space denoted as (X, τ_{NO}). Also, some of the important characteristics of neutrosophic orbit open sets are discussed with suitable examples.

Keywords: Neutrosophic orbit open set, Neutrosophic orbit topology, Neutrosophic orbit topological spaces

Introduction

Fuzzy concept has invaded almost all branches of Mathematics since its introduction of the concept of fuzzy set by Zadeh [14]. Fuzzy sets have applications in many fields. The idea of fuzzy set is welcomed because it handles uncertainty and vagueness which Cantorian set could not address. However, in reality, it may not always be true that the degree of non-membership of an element in a fuzzy set is equal to 1 minus the membership degree because there may be some hesitation degree. Therefore, a generalization of fuzzy sets was introduced byAtanassov [1] as intuitionistic fuzzy sets (IFS) which incorporated the degree of hesitation called hesitation margin (and is defined as 1 minus the sum of membership and non-membership degrees respectively). As a generalization of intuitionistic fuzzy sets neutrosophic set was formulated by Smarandache [8-10] originally gave the definition of a neutrosophic set and neutrosophic logic. The neutrosophic logic is a formal frame trying to measure the truth, indeterminacy and falsehood. In 2012 Salama and Alblowi [11,12] introduced the concept of neutrosophic topological spaces(NTOP). The concept of orbit function in general metric space was introduced by Devaney [2]. The orbit in mathematics has an important role in the study of dynamical systems. The concept of fuzzy orbit open sets under the mapping $f: X \to X$ in a fuzzy topological space $(X, \tau)$was introduced by Malathi and Uma[4]. The concept of fuzzy orbit topological spaces was introduced by Majeed and El-Sheikh[5]. The concept of intuitionistic fuzzy orbit set and intuitionistic fuzzy orbit topological space was introduced by Priscilla and Irudayam [3]. The concept of neutrosophic orbit set was introduced by Madhumathi *et al*. [6]. The purpose of this paper is to study the collection of all neutrosophic orbit open sets under the mapping $f: X \to X$. we introduce the necessary conditions on the mapping $\boldsymbol{f}$ in order to obtain a fixed orbit of a neutrosophic set (i.e., $\boldsymbol{f}(\boldsymbol{\mu}) = \boldsymbol{\mu}$) for any neutrosophic orbit open set $\boldsymbol{\mu}$ under the mapping $\boldsymbol{f}$. Also, some properties of neutrosophic orbit open sets related with union (intersection) of these sets are introduced. we also prove that the family of all neutrosophic orbit open sets constructs a new neutrosophic topological space. This new space is called neutrosophic orbit topological space (X, τ_{NO}). Furthermore, the concept of neutrosophic orbit interior (closure) is defined. Finally, the category of neutrosophic orbit topological spaces (NOTOP) is defined. And we show this category is isomorphic to a subcategory of the category of NTOP.

Preliminaries

1 Definition [9] Let X be a nonempty set. A neutrosophic set (NS for short) A is an object having the form A = <x,A^T,A^I,A^F> where A^T,A^I,A^F represent the degree of membership , the degree of indeterminacy and the degree of non-membership respectively of each element x∈X to the set A.

2 Definition [9] Let X be a non empty set, A = <x,A^T,A^I,A^F> and B = <x,B^T,B^I,B^F> be neutrosophic sets on X, and let {A_i : i∈ J} be an arbitrary family of neutrosophic sets in X, where A_i = <x,A^T, A^I, A^F>

(i) A ⊆B if and only if $A^T \leq B^T$, $A^I \geq B^I$ and $A^F \geq B^F$

(ii) A = B if and only if A ≤ B and B ≤ A.

(iii) $\overline{A}$= <x,A^F,1-A^I,A^T>

(iv) A∩B=<x,$A^T \wedge B^T$,$A^I \vee B^I$,$A^F \vee B^F$>

(v) A∪B=<x,$A^T \vee B^T$,$A^I \wedge B^I$,$A^F \wedge B^F$>

(vi) ∪A_i = <x,$\vee A_i^T$,$\wedge A_i^I$,$\wedge A_i^F$>

(vii) ∩A_i = <x,$\wedge A_i^T$,$\vee A_i^I$,$\vee A_i^F$>

(viii) A − B = A $\wedge \overline{B}$.

(ix) 0_N = <x,0,1,1>; 1_N = <x,1,0,0>.

3 Definition [11] A neutrosophic topology (NT for short) on a nonempty set X is a family τ of neutrosophic set in X satisfying the following axioms:

(i) 0_N, 1_N∈τ.

(ii) $G_1 \wedge G_2$ ∈ τ for any G_1,G_2∈ τ.

(iii) $\vee G_i$∈ τ for any arbitrary family {G_i :i∈J} ⊆ τ.

In this case the pair (X, τ) is called a neutrosophic topological space (NTS for short) and any neutrosophic set in τ is called a neutrosophic open set(NOS for short) in X. The complement A of a neutrosophic open set A is called a neutrosophic closed set (NCS for short) in X.

4 Definition [11] Let (X, τ) be a neutrosophic topological space and A = <X,A^T,A^I,A^F> be a set in X. Then the closure and interior of A are defined by

Ncl(A) = ∧{K : K is a neutrosophic closed set in X and A ≤ K},

Nint(A) = ∨{G : G is a neutrosophic open set in X and G ≤ A}.

It can be also shown that Ncl(A) is a neutrosophic closed set and Nint(A) is a neutrosophic open set in X, and A is a neutrosophic closed set in X iff Ncl(A) = A; and A is a neutrosophic open set in X iff Nint(A) = A.

5 Definition [2] Orbit of a point x in X under the mapping f is $O_f(x)$={x, f(x), $f^2(x)$,...}

6 Definition [6] Let X be a nonempty set and f : X → X be any mapping. Let α be any neutrosophic set in X. The neutrosophic orbit $O_f(\alpha)$ of α under the mapping f is defined as $O_{fT}(\alpha)$ = {α,$f^1(\alpha)$,$f^2(\alpha)$,...$f^n(\alpha)$}, $O_{fI}(\alpha)$ = {α,$f^1(\alpha)$,$f^2(\alpha)$,...$f^n(\alpha)$} , $O_{fF}(\alpha)$ = {α,$f^1(\alpha)$,$f^2(\alpha)$,...$f^n(\alpha)$} for α∈ X and n ∈ Z^+.

7 Definition [6] Let X be a nonempty set and let f : X→ X be any mapping. The neutrosophic orbit set of α under the mapping f is defined as $NO_f(\alpha)$ = <α,$O_{fT}(\alpha)$,$O_{fI}(\alpha)$,$O_{fF}(\alpha)$> for α∈ X, where $O_{fT}(\alpha)$= {$\alpha \wedge f^1(\alpha) \wedge f^2(\alpha) \wedge ... \wedge f^n(\alpha)$},$O_{fI}(\alpha)$={$\alpha \vee f^1(\alpha) \vee f^2(\alpha) \vee ... \vee f^n(\alpha)$}, $O_{fF}(\alpha)$= {$\alpha \vee f^1(\alpha) \vee f^2(\alpha) \vee ... \vee f^n(\alpha)$}.

8 Definition [6] Let (X, τ) be a neutrosophic topological space. Let f : X → X be any mapping. The neutrosophic orbit set under the mapping f which is in neutrosophic topology τ is called neutrosophic orbit open set under the mapping f. Its complement is called a neutrosophic orbit closed set under the mapping f.

9 Example Let X={a, b, c}=Y. Define τ = {0_N, 1_N, α, γ}where$\alpha^T, \gamma^T: X \to \left]{}^{-}0, 1^{+}\right[$

$\alpha^I, \gamma^I : X \to \left]{}^{-}0, 1^{+}\right[$

$\alpha^F, \gamma^F : X \to \left]{}^{-}0, 1^{+}\right[$

are defined as

$\alpha^T(a) = 0.3, \alpha^I(a) = 0.5, \alpha^F(a) = 0.6, \alpha^T(b) = 0.4, \alpha^I(b) = 0.6, \alpha(b) = 0.8, \alpha^T(c) = 0.1,$

$\alpha^I(c) = 0.3, \alpha^F(c) = 0.7$

$\gamma^T(a) = 0.1, \gamma^I(a) = 0.6, \gamma^F(a) = 0.8, \gamma^T(b) = 0.1, \gamma^I(b) = 0.6, \gamma^F(b) = 0.8,$

$\gamma^T = 0.1, \gamma^I(c) = 0.6, \gamma^F(c) = 0.8.$

Define f : X→ X as f(a)=c, f(b)=a, f(c)=b. The neutrosophic orbit set of α under the mapping f is defined as $NO_f(\alpha) = \alpha \cap f^1(\alpha) \cap f^2(\alpha) \cap ... \cap f^n(\alpha)$, $NO_f(\alpha)=\gamma$. Then γ is a neutrosophic orbit open set under the mapping f.
From the above definition its clear that every neutrosophic orbit open set under the mapping f is an open neutrosophic set in X. But the converse is not true, in this example the neutrosohic set α is an open neutrosophic set, however it is not neutrosophic orbit open set under the mapping f, because there is not exists a neutrosophic set $\vartheta \in I^X$ such that $NO_f(\vartheta) = \alpha$.
10 Definition [13] A mapping f :$(X, \tau) \rightarrow (Y, \tau)$ is called neutrosophic continuous if the inverse image of every closed set in Y is neutrosophic closed in X.

Some properties of neutrosophic orbit open sets

In our work we consider X as a nonempty countable set,we give the conditions on a mapping $f: X \rightarrow X$,to obtain a fixed neutrosophic orbit open set(i.e.,$f(\mu) = \mu$) for any neutrosophic orbit open set μ, and study some properties of these sets.
Theorem 3.1:
Let (X, τ) be a NTOP and $f: X \rightarrow X$ be any bijective mapping.Then $f(\mu) = \mu$ for any neutrosophic orbit open set μunder the mappingf.
Proof:
Let (X, τ) be a NTOPand $f: X \rightarrow X$ be a bijective mapping. Then we have 3 cases:
Case 1:
If $f(x_i) = x_j; x_i, x_j \in X$ and $i \neq j$ for all $i, j \in \wedge$ .Suppose $X = \{x_1, x_2\}$ and $f: X \rightarrow X$ defined as
$f(x_1) = x_2, f(x_2) = x_1$. Let μ be a neutrosophic orbit open set under the mapping f.
Then there exists a neutrosophic set $\lambda \in I^X$ such that $NO_f(\lambda) = \lambda \cap f(\lambda) \cap f^2(\lambda) ... = \mu$.
Let $\lambda = \{(x_1, u_1, v_1, w_1), (x_2, u_2, v_2, w_2); x_1, x_2 \in X, u_1, u_2, v_1, v_2, w_1, w_2 \in I\}$. This implies
$f(\lambda) = \{(x_1, u_2, v_2, w_2), (x_2, u_1, v_1, w_1)\}, f^2(\lambda) = \{(x_1, u_1, v_1, w_1), (x_2, u_2, v_2, w_2)\}, ...$
$\therefore NO_f(\lambda) = \{(x_1, [\inf(u_1, u_2, u_1, ...), \sup(v_1, v_2, v_1, ...), \sup(w_1, w_2, w_1, ...)],$
$(x_2, [\inf(u_2, u_1, u_2, ...), \sup(v_2, v_1, v_2, ...), \sup(w_2, w_1, w_2, ...)]\}$
$= \{(x_1, [\min\{u_1, u_2\}, \max\{v_1, v_2\}, \max\{w_1, w_2\}],$
$(x_2, [\min\{u_1, u_2\}, \max\{v_1, v_2\}, \max\{w_1, w_2\})\}]\} = \mu$
In general, if $X = \{x_1, x_2, ...\}$ and μ be a neutrosophic orbit open set under the mapping f, then there exists a neutrosophic set
$\lambda = \{(x_1, u_1, v_1, w_1), (x_2, u_2, v_2, w_2), (x_3, u_3, v_3, w_3) ... ...\} = \{(x_i, u_i, v_i, w_i); x_i \in X, u_i, v_i, w_i \in I, i \in \wedge\}$
Such that $NO_f(\lambda) = \mu$. That means
$NO_f(\lambda) = \{(x_i, [\inf\{u_i\}, \sup\{v_i\}, \sup\{w_i\}]); x_i \in X, u_i, v_i, w_i \in I, i \in \Lambda\}$
$= \{(x_i, [r, s, t]); x_i \in X, \{r = \inf\{u_i\}, s = \sup\{v_i\}, t = \sup\{w_i\}; u_i, v_i, w_i \in I\}, i = \mu$
Now for each $x_j \in X$, we have

$$f(\mu)(x_j) = \begin{cases} \bigcup_{f(x_i)=x_j} \mu(x_i) & if\ f^{-1}(x_j) \neq \emptyset \\ (0,1,1) & if\ f^{-1}(x_j) = \emptyset \end{cases}$$

From the hypothesis and the definition of f,we get $f(\mu)(x_j) = \mu(x_i) = r, s, t$ for all $x_j \in X$. Hence $f(\mu) = \mu$.
Case 2:
If $f(x_i) = x_j; x_i, x_j \in X$ and $i = j$ for some $i, j \in \wedge$. In this case the least number of elements in X must be 3 elements. So,Suppose that if $X = \{x_1, x_2, x_3\}$,then from the hypothesis and the definition of f, the mapping $f: X \rightarrow X$ can be defined as $f(x_1) = x_1, f(x_2) = x_3$ and $f(x_3) = x_2$(i.e., $f(x_i) = x_j$ when $i = j = 1$ and $f(x_i) = x_j, i \neq j$ when $i, j \in \{2,3\}$). Let μ be a neutrosophic orbit open set under the mapping f. Then there exists a neutrosophic set $\lambda \in I^X$ such that $NO_f(\lambda) = \lambda \cap f(\lambda) \cap f^2(\lambda) ... = \mu$.
Let $\lambda = \{(x_1, u_1, v_1, w_1), (x_2, u_2, v_2, w_2), (x_3, u_3, v_3, w_3); x_i \in X, u_i, v_i, w_i \in I, i = 1,2,3\}$.Then from the definition of f, we get
$f(\lambda) = \{(x_1, u_1, v_1, w_1), (x_2, u_3, v_3, w_3), (x_3, u_2, v_2, w_2)\},$
$f^2(\lambda) = \{(x_1, u_1, v_1, w_1), (x_2, u_2, v_2, w_2), (x_3, u_3, v_3, w_3)\} ...$
$\therefore NO_f(\lambda) = (x_1, u_1, v_1, w_1), (x_2, [\inf\{u_2, u_3, u_2 ...\}, \sup\{v_2, v_3, v_2 ...\}, \sup\{w_2, w_3, w_2 ...\}],$
$(x_3, [\inf\{u_3, u_2, u_3 ...\}, \sup\{v_3, v_2, v_3 ...\}, \sup\{w_3, w_2, w_3 ...\}]$
$= (x_1, u_1, v_1, w_1), (x_2, [\min\{u_2, u_3\}, \max\{v_2, v_3\}, \max\{w_2, w_3\}],$

$(x_3, [\min\{u_3, u_2\}, \max\{v_3, v_2\}, \max\{w_3, w_2\}]$
$= \mu$
In general, if $X = \{x_1, x_2, \dots.\}$ and μ be a neutrosophic orbit open set under the mappingf, then there exists a neutrosophic set
$\lambda = \{(x_1, u_1, v_1, w_1), (x_2, u_2, v_2, w_2), (x_3, u_3, v_3, w_3) \dots\} = \{(x_i, u_i, v_i, w_i); x_i \in X, u_i, v_i, w_i \in I, i \in \wedge\}$
Such that $NO_f(\lambda) = \mu$. This implies

$$NO_f(\lambda) = \begin{cases} (x_i, (u_i, v_i, w_i)); f(x_i) = x_j, i = j, i \in \wedge \\ (x_i, [\inf\{u_i\}, \sup\{v_i\}, \sup\{w_i\}],); f(x_i) = x_j, i \neq j, i \in \wedge \end{cases}$$

$$= \begin{cases} (x_i, (u_i, v_i, w_i)); f(x_i) = x_j, i = j, i \in \wedge \\ (x_i, (r, s, t)); r = \inf\{u_i\}, s = \sup\{v_i\}, t = \sup\{w_i\}; f(x_i) = x_j, i \neq j, i \in \wedge \end{cases}$$

$= \mu$
Now for each $x_j \in X$, we have

$$f(\mu)(x_j) = \mu(x_i) = \begin{cases} \bigcup_{f(x_i)=x_j} \mu(x_i) & \text{if } f^{-1}(x_j) \neq \emptyset \\ (0,1,1) & \text{if } f^{-1}(x_j) = \emptyset \end{cases}$$

From the hypothesis and the definition of f,we get for all $x_j \in X$.

$$f(\mu)(x_j) = \begin{cases} (u_i, v_i, w_i) & \text{if } i = j \\ (r, s, t), & \text{if } i \neq j \end{cases}$$

Hence $f(\mu) = \mu$.
Case 3:
If f is the identity mapping. In this case, every open neutrosophic set in X is neutrosophic orbit open set under the mapping fandf$(\mu) = \mu$ for every neutrosophic set $\mu \in I^X$.Thus the proof is obtained.

Theorem 3.2:
Let (X, τ) be a NTOPand $f: X \to X$ be any constant mapping. Then $f(\mu) = \mu$ for any neutrosophic orbit open set μ under the mapping f.
Proof:
Let (X, τ) be a neutrosophic topological space.
Let μ be a neutrosophic orbit open set under the mapping f.
Then, from Definition, there exists a neutrosophic set $\lambda = \{(x_i, u_i, v_i, w_i); x_i \in X, u_i, v_i, w_i \in I, i \in \wedge\}$ such that $NO_f(\lambda) = \mu$.
Since f is constant mapping, this implies there exists a fixed element $x_k \in X$ such that $f(x_i) = x_k$ for all $x_i \in X$ and $i \in \wedge$.
Now form, the definition of $f(\lambda)$for all $x_j \in X$, we have

$$f(\lambda)(x_j) = \begin{cases} \bigcup_{f(x_i)=x_j} \lambda(x_i) & \text{if } f^{-1}(x_j) \neq \emptyset \\ (0,1,1) & \text{otherwise} \end{cases}$$

Thus

$$f(\lambda)(x_j) = \begin{cases} [sup_{i\in\wedge}\{\lambda(x_i)\}, inf_{i\in\wedge}\{\lambda(x_i)\}, inf_{i\in\wedge}\{\lambda(x_i)\}] & \text{if } x_j = x_k, \\ (0,1,1) & \text{if } x_j \neq x_k \end{cases}$$

Therefore, $f(\lambda) = \{(x_k, sup_{i\in\wedge}\{\lambda(x_i)\}, inf_{i\in\wedge}\{\lambda(x_i)\}, inf_{i\in\wedge}\{\lambda(x_i)\})\}$.
This means $f(\lambda)$ is a neutrosophic point in X with support
x_k and degree $\sup_{i\in\wedge}\{\lambda(x_i)\}$, degree $\inf_{i\in\wedge}\{\lambda(x_i)\}$, degree $\inf_{i\in\wedge}\{\lambda(x_i)\}$.
By the same way we have
$f^2(\lambda) = \{(x_k, sup_{i\in\wedge}\{\lambda(x_i)\}, inf_{i\in\wedge}\{\lambda(x_i)\}, inf_{i\in\wedge}\{\lambda(x_i)\})\}$,
$f^3(\lambda) = \{(x_k, sup_{i\in\wedge}\{\lambda(x_i)\}, inf_{i\in\wedge}\{\lambda(x_i)\}), inf_{i\in\wedge}\{\lambda(x_i)\}\}, \dots$
For more clearing we have the following:
$\lambda = \{(x_1, u_1, v_1, w_1), (x_2, u_2, v_2, w_2), \dots, (x_k, u_k, v_k, w_k), \dots\}$
$f(\lambda) = \{(x_1, 0,1,1), (x_2, 0,1,1), \dots, (x_k, sup_{i\in\wedge}\{\lambda(x_i)\}, inf_{i\in\wedge}\{\lambda(x_i)\}, inf_{i\in\wedge}\{\lambda(x_i)\}), \dots\}$
$f^2(\lambda) = \{(x_1, 0,1,1), (x_2, 0,1,1), \dots, (x_k, sup_{i\in\wedge}\{\lambda(x_i)\}, inf_{i\in\wedge}\{\lambda(x_i)\}, inf_{i\in\wedge}\{\lambda(x_i)\}), \dots\}$
$f^3(\lambda) = \{(x_1, 0,1,1), (x_2, 0,1,1), \dots, (x_k, sup_{i\in\wedge}\{\lambda(x_i)\}, inf_{i\in\wedge}\{\lambda(x_i)\}, inf_{i\in\wedge}\{\lambda(x_i)\}\}), \dots\}$

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Thus $NO_f(\lambda) = \lambda \cap f(\lambda) \cap f^2(\lambda) \ldots$

$NO_f(\lambda)$
$= [(x_1,(0,1,1)),(x_2,(0,1,1)) \ldots (x_k, \min\{u_k, sup_{i\in\wedge}\{\lambda(x_i)\}\}, \max\{v_k, inf_{i\in\wedge}\{\lambda(x_i)\}\}, \max\{w_k, inf_{i\in\wedge}\{\lambda(x_i)\}\})]$

$$= \begin{cases} (x_i, 0,1,1) & if\ i \neq k \\ (x_k, \min\{u_k, sup_{i\in\wedge}\{\lambda(x_i)\}\}, \max\{v_k, inf_{i\in\wedge}\{\lambda(x_i)\}\}, \max\{w_k, inf_{i\in\wedge}\{\lambda(x_i)\}\}) & if\ i = k \end{cases}$$
$$= \mu$$

This yield $NO_f(\lambda) = \mu$ is a neutrosophic point in X with support x and degree min $\{u_k, \sup_{i\in\wedge}\{(x_i)\}\}$,degree $\max\{v_k, inf_{i\in\wedge}\{\lambda(x_i)\}\}$, degree $\max\{w_k, inf_{i\in\wedge}\{\lambda(x_i)\}\}$.
Hence, from the definition of f, we get $f(\mu) = \mu$.

Remark 3.3:
The condition to be f:$X \to X$ is bijective or constant is necessary condition to obtain fixed neutrosophic orbit open sets for any neutrosophic orbit open set μ under the mapping f.
For more explain, we give an example for a neutrosophic topological space (X,) and f:$X \to X$ not bijective, we show that $(\mu) \neq \mu$ for some neutrosophic orbit open set μ under the mapping f.

Example 3.4:
Let $X = \{x_1, x_2, x_3, x_4, x_5\}$.Define $\tau = \{\overline{0}, \overline{1}, \mu\}$where $\mu \in I^X$ defined as
$\mu = \{(x_1,(0,1,1)),(x_2,(0.1,0.4,0.9)),(x_3,(0,0.8,1)),(x_4,(0.6,0.2,0.4)),(x_5,(0.6,0.2,0.4))\}$
Define $f: X \to X$ as $f(x_1) = x_2, f(x_2) = x_1, f(x_3) = x_2, f(x_4) = x_5, f(x_5) = x_4$. It is clear that f is not bijective mapping(i.e., f is not one to one and not onto).Let $\lambda \in I^X$ defined as follows:
$\lambda = \{(x_1, 0.1,0.3,0.9),(x_2, 0.2,0.4,0.8),(x_3, 0,0.8,1),(x_4, 0.6,0.2,0.4),(x_5, 0.7,0.1,0.3)\}$.
Then the neutrosophic orbit of λ are $NO_f(\lambda) = \lambda \cap f(\lambda) \cap f^2(\lambda) \ldots = \mu$. which is
$f(\lambda) = \{(x_1, 0,1,1),(x_2, 0.1,0.3,0.9),(x_3, 0.2,0.4,0.8),(x_4, 0.7,0.1,0.3),(x_5, 0.6,0.2,0.4)\}$
$f^2(\lambda) = \{(x_1, 0,1,1),(x_2, 0.2,0.4,0.8),(x_3, 0.1,0.3,0.9),(x_4, 0.6,0.2,0.4),(x_5, 0.7,0.1,0.3)\}$
$f^3(\lambda) = \{(x_1, 0,1,1),(x_2, 0.1,0.3,0.9),(x_3, 0.2,0.4,0.8),(x_4, 0.7,0.1,0.3),(x_5, 0.6,0.2,0.4)\}$
Therefore, the neutrosophic orbit set of λ is $NO_f(\lambda) = \lambda \cap f(\lambda) \cap f^2(\lambda) \ldots = \mu$.
$NO_f(\lambda) = \{(x_1, [\inf\{0.1,0,0,0 \ldots\}, \sup\{0.3,1,1, \ldots\}, \sup\{0.9,1,1, \ldots\}]$,
$(x_2, [\inf\{0.2,0.1,0.2,0.1, \ldots\}, \sup\{0.4,0.3,0.4 \ldots\}, \sup\{0.8,0.9,0.8 \ldots\}]$,
$(x_3, [\inf\{0,0.2,0.1,0.2, \ldots\}, \sup\{0.8,0.4,0.3,0.4, \ldots\}, \sup\{1,0.8,0.9,0.8, \ldots\}]$,
$(x_4, [\inf\{0.6,0.7,0.6,0.7, \ldots\}, \sup\{0.2,0.1,0.2,0.1, \ldots\}, \sup\{0.4,0.3,0.4,0.3, \ldots\}]$,
$(x_5, [\inf\{0.7,0.6,0.7,0.6, \ldots\}, \sup\{0.1,0.2,0.1,0.2, \ldots\}, \sup\{0.3,0.4,0.3,0.4..\}]$,
$=$
$\{(x_1,(0,1,1)),(x_2,(0.1,0.4,0.9)),(x_3,(0,0.8,1)),(x_4,(0.6,0.2,0.4)),(x_5,(0.6,0.2,0.4))\} = \mu$

Thus, the open neutrosophic set μ is neutrosophic orbit open set under the mapping f. But $f(\mu) \neq \mu$.
From Theorem 3.1 and 3.2 we obtain the following result

Result 3.5:
Let (X,τ) be a NTOPand $f: X \to X$ be any mapping such that either f is bijective mapping or f is constant mapping and μ is a neutrosophic orbit open set under the mapping f, then $f(\mu) = \mu$.
In our work, we consider the mapping f:$X \to X$ that satisfies the conditions this Result.

Proposition 3.6:
Let (X, τ) be a NTOPand f:$X \to X$ be any mapping. If μ is a neutrosophic orbit open set under the mapping f, then $NO_f(\mu) = \mu$.
Proof:
The proof follows directly from the definition of $NO_f(\mu)$ and Result 3.5. i.e.,
$NO_f(\mu) = \mu \cap f(\mu) \cap f^2(\mu) \cap \ldots$
From Result 3.5, we have $f(\mu) = \mu$, this implies $f^2(\mu) = f(f(\mu)) = \mu$, $f^3(\mu) = f(f^2(\mu)) = \mu$,... Hence, $NO_f(\mu) = \mu$.

Theorem 3.7:
Let (X,τ) be a neutrosophic topological space and f:$X \to X$ be a mapping.
If μ_1 and μ_2 are neutrosophic orbit open sets under the mapping f, then $NO_f(\mu_1 \cap \mu_2) = NO_f(\mu_1) \cap NO_f(\mu_2)$
Proof:
First we prove the theorem if f is bijective mapping. From Theorem 3.1, we have 3 cases.
We prove the theorem in case 1.

The proof of theorem in case 2 is similar to case 1, and the prove of theorem in case 3 is easy.
Case 1:
Suppose that f is bijective mapping and $f(x_i) = x_j; x_i, x_j \in X$ and $i \neq j$ for all $i,j \in \Lambda$.
Let μ_1 and μ_2 are neutrosophic orbit open sets under the mapping f.
Then, there exist $\lambda_1, \lambda_2 \in I^X$ defined as $\lambda_1 = \{(x_i, u_i, v_i, w_i); x_i \in X,\ u_i, v_i, w_i \in I, i \in \Lambda\}$ and $\lambda_2 = \{(x_i, r_i, s_i, t_i);\ x_i \in X,\ r_i, s_i, t_i \in I, i \in \Lambda\}$ such that $NO_f(\lambda_1) = \mu_1$ and $NO_f(\lambda_2) = \mu_2$.
From Theorem 3.1 case 1, we have $NO_f(\lambda_1) = \{(x_i, (u, v, w)); u = \inf\{u_i, i \in \wedge\} = \mu_1, v = \sup\{v_i, i \in \wedge\}\} = \mu_1, w = \sup\{w_i, i \in \wedge\}\} = \mu_1$
And $NO_f(\lambda_2) = \{(x_i, (r, s, t); r = \inf\{r_i, i \in \wedge\} = \mu_2, s = \sup\{s_i, i \in \wedge\}\} = \mu_2, t = \sup\{t_i, i \in \wedge\} = \mu_2$
Thus, $\mu_1 \cap \mu_2 = \{(x_i, [\min\{u, r\}, \max\{v, s\}, \max\{w, t\}]); x_i \in X, i \in \wedge\}$.
Let $a = \min\{u, r\}, b = \max\{v, s\}, c = \max\{w, t\}$.
Now for all $x_j \in X, j \in \wedge$.

$$f(\mu_1 \cap \mu_2)(x_j) = \begin{cases} \bigcup_{f(x_i)=x_j} (\mu_1 \cap \mu_2)(x_i) & if\ f^{-1}(x_j) \neq \emptyset \\ (0,1,1) & if\ \ f^{-1}(x_j) = \emptyset \end{cases}$$

$= (a, b, c)$
Hence $f(\mu_1 \cap \mu_2) = \mu_1 \cap \mu_2$. This implies $f^2(\mu_1 \cap \mu_2) = \mu_1 \cap \mu_2, f^3(\mu_1 \cap \mu_2) = \mu_1 \cap \mu_2$ ...
Therefore, from the definition of $NO_f(\mu_1 \cap \mu_2)$ and Theorem 3.1 we get
$NO_f(\mu_1 \cap \mu_2) = \mu_1 \cap \mu_2 = NO_f(\mu_1) \cap NO_f(\mu_2)$.
Case 2:
Suppose that f is bijective mapping and $f(x_i) = x_j; x_i, x_j \in X$ and $i=j$ for all $i,j \in \Lambda$.
Let μ_1 and μ_2 are neutrosophic orbit open sets under the mapping f.
Then, there exist $\lambda_1, \lambda_2 \in I^X$ defined as $\lambda_1 = \{(x_i, u_i, v_i, w_i); x_i \in X, u_i, v_i, w_i \in I, i \in \Lambda\}$ and $\lambda_2 = \{(r_i, s_i, t_i); x_i \in X, r_i, s_i, t_i, \in I, i \in \Lambda\}$ such that $NO_f(\lambda_1) = \mu_1$ and $NO_f(\lambda_2) = \mu_2$.
From Theorem 3.1 case 2, we have
$NO_f(\lambda_1) = \{(x_i, (u_i, v_i, w_i)); f(x_i) = x_j, i = j, (x_i, [u = \inf\{u_i, i \in \wedge\}, v = \sup\{v_i, i \in \wedge\}, w = \sup\{w_i, i \in \wedge\}]); f(x_i) = x_j, i \neq j\} = \mu_1$ and
$NO_f(\lambda_2) = \{(x_i, (r_i, s_i, t_i))\ ; f(x_i) = x_j, i = j, (x_i, [r = \inf\{r_i, i \in \wedge\}, s = \sup\{s_i, i \in \wedge\}, t = \sup\{t_i, i \in \wedge\}];\ f(x_i) = x_j, i \neq j\} = \mu_2$.

$$\text{Thus } \mu_1 \cap \mu_2 = \begin{Bmatrix} (x_i, [\min\{u_i, r_i\}, \max\{v_i, s_i\}, \max\{w_i, t_i\}]); & f(x_i) = x_j, i = j, \\ (x_i, [\min\{u, r\}, \max\{v, s\}, \max\{w, t\}]); & f(x_i) = x_j, i \neq j \end{Bmatrix}.$$

Let $k = \min\{m_i, s_i\}, t = max\{n_i, r_i\}$. Now for all $x_j \in X, j \in \wedge$

$$f(\mu_1 \cap \mu_2)(x_j) = \begin{cases} \bigcup_{f(x_i)=x_j} (\mu_1 \cap \mu_2)(x_i) & if f^{-1}(x_j) \neq \emptyset \\ (0,1,1) & if\ f^{-1}(x_j) = \emptyset \end{cases}$$

$$= \begin{cases} \{[\min\{u_i, r_i\}, \max\{v_i, s_i\}, \max\{w_i, t_i\}]\}, & if\ f(x_i) = x_j, i = j \\ \{[\min\{u, r\}, \max\{v, s\}, \max\{w, t\}]\}, & if\ f(x_i) = x_j, i \neq j \end{cases}$$

Hence $f(\mu_1 \cap \mu_2) = \mu_1 \cap \mu_2$.
This implies $f^2(\mu_1 \cap \mu_2) = \mu_1 \cap \mu_2, f^3(\mu_1 \cap \mu_2) = \mu_1 \cap \mu_2$ ...
Therefore, from the definition of $NO_f(\mu_1 \cap \mu_2)$ and Theorem 3.1 we get
$NO_f(\mu_1 \cap \mu_2) = \mu_1 \cap \mu_2 = NO_f(\mu_1) \cap NO_f(\mu_2)$.
Case 3:
Now, if f is constant mapping, let μ_1 and μ_2 are neutrosophic orbit open sets under the mapping f.
Then, there exist $\lambda_1, \lambda_2 \in I^X$ defined as $\lambda_1 = \{(x_i, u_i, v_i, w_i);\ x_i \in X, u_i, v_i, w_i \in I, i \in \Lambda\}$ and $\lambda_2 = \{(x_i, r_i, s_i, t_i);\ x_i \in X, r_i, s_i, t_i \in I, i \in \Lambda\}$ such that $NO_f(\lambda_1) = \mu_1 and\ NO_f(\lambda_2) = \mu_2$.
From Theorem 3.2, we have

$$NO_f(\lambda_1) = \begin{cases} (x_i, 0,1,1) & if\ i \neq k \\ (x_k, [\min\{(u_k), sup_{i\in\wedge}\{\lambda_1(x_i)\}\}, \max\{(v_k), inf_{i\in\wedge}\{\lambda_1(x_i)\}\}, \max\{(w_k), inf_{i\in\wedge}\{\lambda_1(x_i)\}\}]]\ ) & if\ i = k \end{cases}$$

$= \mu_1$

$$NO_f(\lambda_2) = \begin{cases} (x_i, 0,0,0) & if\ i \neq k \\ (x_k, [\min\{(r_k), sup_{i\in\wedge}\{\lambda_1(x_i)\}\}, max\{(s_k), inf_{i\in\wedge}\{\lambda_1(x_i)\}\}, max\{(t_k), inf_{i\in\wedge}\{\lambda_1(x_i)\}\}]]) & if\ i = k \end{cases}$$

$= \mu_2$
Thus
$\mu_1 \cap \mu_2$

$$= \begin{cases} (x_i, 0,0,0) & if\ i \neq k \\ \left(x_k, min\left[\min\{(u_k), sup_{i\in\wedge}\{\lambda_1(x_i)\}\}, \min\{(r_k), sup_{i\in\wedge}\{\lambda_1(x_i)\}\}\right], max\left[\max\{(v_k), inf_{i\in\wedge}\{\lambda_1(x_i)\}\}\right], \max\{(s_k), inf_{i\in\wedge}\{\lambda_1(x_i)\}\}]] \atop \max\left[max\{(w_k), inf_{i\in\wedge}\{\lambda_1(x_i)\}\}\right], max\{(t_k), inf_{i\in\wedge}\{\lambda_1(x_i)\}\}]]\right) & if\ i = k \end{cases}$$

Thismeans that $\mu_1 \cap \mu_2$is a neutrosophic point in X with support x_k and degree $[\min[\min\{(u_k), \sup_{i\in\wedge}\{\lambda_1(x_i)\}\}]\ , [\min\{(r_k), \sup_{i\in\wedge}\{\lambda_2(x_i)\}\}]]$,
$[\max\ [\ \max\{(v_k), \inf_{i\in\wedge}\{\lambda_1(x_i)\}\}, \max\{(s_k), \inf_{i\in\wedge}\{\lambda_1(x_i)\}\}]\ , [\max\ [\ \max\{(w_k), \inf_{i\in\wedge}\{\lambda_1(x_i)\}\}]\ , \max\{(t_k), \inf_{i\in\wedge}\{\lambda_2(x_i)\}\}]]$
Hence from the definition of f we get$f(\mu_1 \cap \mu_2) = \mu_1 \cap \mu_2$. This implies $f^2(\mu_1 \cap \mu_2) = \mu_1 \cap \mu_2$,
$f^3(\mu_1 \cap \mu_2) = \mu_1 \cap \mu_2 \ldots$
Therefore, from the definition of $NO_f(\mu_1 \cap \mu_2)$ and Theorem 3.2 we get
$NO_f(\mu_1 \cap \mu_2) = \mu_1 \cap \mu_2 = NO_f(\mu_1) \cap NO_f(\mu_2)$.
Hence the proof

Theorem 3.8:
Let (X, τ) be a NTOPand $f: X \to X$ be a mapping.Let $\{\mu_\alpha\}_{\alpha\in\Delta}$ be any family of neutrosophic orbit open sets under the mapping f,then $NO_f(\cup_{\alpha\in\Delta} \mu_\alpha) = \cup_{\alpha\in\Delta} NO_f(\mu_\alpha)$
Proof:
The outline of proofing this theorem is proceeds in a way similar to Theorem 3.7. As in Theorem 3.7, we consider 3 cases:
Case 1:
Suppose that f is bijective mapping
and$f(x_i) = x_j; x_i, x_j \in X$ and $i \neq j$ for all $i, j \in\wedge$.
Let $\{\mu_\alpha\}_{\alpha\in\Delta}$ be any family of neutrosophic orbit open sets under the mapping f,then there exists$\lambda_\alpha \in I^X, \alpha \in \Delta$ defined as $\lambda_\alpha = \{(x_i, u_{i_\alpha}, v_{i_\alpha}, w_{i_\alpha}); x_i \in X, u_{i_\alpha}, v_{i_\alpha}, w_{i_\alpha} \in I, i \in\wedge\}$such that $NO_f(\lambda_\alpha) = \mu_\alpha$ for all $\alpha \in \Delta$. From Theorem 3.1 case 1, we have
$NO_f(\lambda_\alpha) = \{(x_i, (u_\alpha, v_\alpha, v_\alpha)); u_\alpha = \inf\{u_{i_\alpha}, i \in\wedge\}, v_\alpha = \sup\{v_{i_\alpha}, i \in\wedge\}, w_\alpha = \sup\{w_{i_\alpha}, i \in\wedge\}\} = \mu_\alpha$
Thus $(\cup_{\alpha\in\Delta} \mu_\alpha) = \{(x_i, [\sup_{\alpha\in\Delta}(u_\alpha)], \inf_{\alpha\in\Delta}[(v_\alpha)], \inf_{\alpha\in\Delta}[(w_\alpha);\ x_i \in X, i\in\wedge\}$.
Let a$= \sup_{\alpha\in\Delta}(u_\alpha), b = \inf_{\alpha\in\Delta}(v_\alpha), c = \inf_{\alpha\in\Delta}(w_\alpha)$. Now for all $x_j \in X, j \in\wedge$.

$$f(\cup_{\alpha\in\Delta} \mu_\alpha)(x_j) = \begin{cases} \cup_{f(x_i)=x_j} (\cup_{f(x_i)=x_j} \mu_\alpha)(x_i) & \text{if } f^{-1}(x_j) \neq \emptyset \\ (0,1,1) & \text{if } f^{-1}(x_j) = \emptyset \end{cases}$$

$= (a, b, c)$
Hence $f(\cup_{\alpha\in\Delta} \mu_\alpha) = (\cup_{\alpha\in\Delta} \mu_\alpha)$. This implies $f^2(\cup_{\alpha\in\Delta} \mu_\alpha) = (\cup_{\alpha\in\Delta} \mu_\alpha), f^3(\cup_{\alpha\in\Delta} \mu_\alpha) = (\cup_{\alpha\in\Delta} \mu_\alpha), \ldots$.
Therefore from the definition of $NO_f(\cup_{\alpha\in\Delta} \mu_\alpha)$ and Theorem 3.1 we get
$NO_f(\cup_{\alpha\in\Delta} \mu_\alpha) = (\cup_{\alpha\in\Delta} \mu_\alpha) = \cup_{\alpha\in\Delta} NO_f(\mu_\alpha)$
Case 2:
Suppose that f is bijective mapping and $f(x_i) = x_j; x_i, x_j \in X\ and\ i = j\ for\ all\ i, j \in\wedge$.
Let $\{\mu_\alpha\}_{\alpha\in\Delta}$ be any family of neutrosophic orbit open sets under the mapping f, then there exists $\lambda_\alpha \in I^X, \alpha \in \Delta$ defined as $\lambda_\alpha = \{(x_i, u_{i_\alpha}, v_{i_\alpha}, w_{i_\alpha}); x_i \in X, u_{i_\alpha}, v_{i_\alpha}, w_{i_\alpha} \in I, i \in\wedge\}$ such that $NO_f(\lambda_\alpha) = \mu_\alpha$ for all $\alpha \in \Delta$. From Theorem 3.1 case 2 we have
$NO_f(\lambda_\alpha) = \{(x_i, (u_{i_\alpha}, v_{i_\alpha}, w_{i_\alpha}))\ ; f(x_i) = x_j, i = j, (x_i, [\inf\{u_{i_\alpha}\}, \sup\{v_{i_\alpha}\}, \sup\{w_{i_\alpha}\}]$
put $u_\alpha = \inf\{u_{i_\alpha}, i \in\wedge\}, v_\alpha = \sup\{v_{i_\alpha}, i \in\wedge\}, w_\alpha = \sup\{w_{i_\alpha}, i \in\wedge\}$,it follows:
$(\cup_{\alpha\in\Delta} \mu_\alpha)\ = \{(x_i, \sup_{\alpha\in\Delta}(u_{i_\alpha}), \inf_{\alpha\in\Delta}(v_{i_\alpha}), \inf_{\alpha\in\Delta}(w_{i_\alpha}))\ ; f(x_i) = x_j, i = j,$
$(x_i, \sup_{\alpha\in\Delta}(u_\alpha), \inf_{\alpha\in\Delta}(v_\alpha), \inf_{\alpha\in\Delta}(w_\alpha)); i \in\wedge); f(x_i) = x_j, i \neq j\}$.
Let $k = \sup_{\alpha\in\Delta}(m_{i_\alpha}, n_{i_\alpha}), t = \inf_{\alpha\in\Delta}[1 - (m_{i_\alpha}, n_{i_\alpha})], p = \sup_{\alpha\in\Delta}[\inf\{m_\alpha\}, \sup\{n_\alpha\}]$,
$q = \inf_{\alpha\in\Delta}[1 - [\inf\{m_\alpha\}, \sup\{n_\alpha\}]]$
Now for all $x_j \in X, j \in\wedge$
$f(\cup_{\alpha\in\Delta} \mu_\alpha)(x_j)$

$$= \begin{cases} \cup_{f(x_i)=x_j} (\cup_{f(x_i)=x_j} \mu_\alpha)(x_i) & \text{if } f^{-1}(x_j) \neq \emptyset \\ (0,1,1) & \text{if } f^{-1}(x_j) = \emptyset \end{cases}$$

$= \begin{cases} \sup_{\alpha\in\Delta}(u_{i_\alpha}), \inf_{\alpha\in\Delta}(v_{i_\alpha}), \inf_{\alpha\in\Delta}(w_{i_\alpha}) & \text{if } f(x_i) = x_j, i = j, \\ \sup_{\alpha\in\Delta}(u_\alpha), \inf_{\alpha\in\Delta}(v_\alpha), \inf_{\alpha\in\Delta}(w_\alpha) & \text{if } f(x_i) = x_j, i \neq j \end{cases}$

Hence $f(\cup_{\alpha\in\Delta} \mu_\alpha) = (\cup_{\alpha\in\Delta} \mu_\alpha)$. This implies $f^2(\cup_{\alpha\in\Delta} \mu_\alpha) = (\cup_{\alpha\in\Delta} \mu_\alpha), f^3(\cup_{\alpha\in\Delta} \mu_\alpha) = (\cup_{\alpha\in\Delta} \mu_\alpha), \ldots$. Therefore from the definition of $NO_f(\cup_{\alpha\in\Delta} \mu_\alpha)$ and Theorem 3.1 we getf $NO_f(\cup_{\alpha\in\Delta} \mu_\alpha) = (\cup_{\alpha\in\Delta} \mu_\alpha) = \cup_{\alpha\in\Delta} NO_f(\mu_\alpha)$

Case 3:

Now if f is any constant mapping,

Let $\{\mu_\alpha\}_{\alpha\in\Delta}$ be any family of neutrosophic orbit open sets under the mapping f, then there exists $\lambda_\alpha \in I^X, \alpha \in \Delta$ defined as $\lambda_\alpha = \{(x_i, u_{i_\alpha}, v_{i_\alpha}, w_{i_\alpha}); x_i \in X, u_{i_\alpha}, v_{i_\alpha}, w_{i_\alpha} \in I, i \in \wedge\}$ such that $NO_f(\lambda_\alpha) = \mu_\alpha$ for all $\alpha \in \Delta$.

From Theorem 3.2 we get,

$NO_f(\lambda_\alpha)$

$= \begin{cases} (x_i, 0,1,1) & \text{if } i \neq k \\ (x_k, \min\{u_{k_\alpha}, \sup_{i\in\wedge}\{\lambda_\alpha(x_i)\}\}, \max\{v_{k_\alpha}, \inf_{i\in\wedge}\lambda_\alpha(x_i)\}\}, \max\{w_{k_\alpha}, \inf_{i\in\wedge}\{\lambda_\alpha(x_i)\}\}) & \text{if } i = k \end{cases}$

Thus,

$(\cup_{\alpha\in\Delta} \mu_\alpha)$

$= \begin{cases} (x_i, 0,1,1) & \text{if } i \neq k \\ (x_k, \sup_{\alpha\epsilon\Delta}\{\min\{u_{k_\alpha}, \sup_{i\in\wedge}\{\lambda_\alpha(x_i)\}\}, \inf_{\alpha\epsilon\Delta}\{\max\{v_{k_\alpha}, \inf_{i\in\wedge}\lambda_\alpha(x_i)\}\}, \inf_{\alpha\epsilon\Delta}\{\max\{w_{k_\alpha}, \inf_{i\in\wedge}\{\lambda_\alpha(x_i)\}\}\}) & \text{if } i = k \end{cases}$

This means $(\cup_{\alpha\in\Delta} \mu_\alpha)$ is a neutrosophic point in X with support x_k and degree $[\sup_{\alpha\in\Delta}\{\min\{(m_{k_\alpha}), \sup_{i\in\wedge}\{\lambda_\alpha(x_i)\}\}\},\{\max\{(n_{k_\alpha}), \inf_{i\in\wedge}\{\lambda_\alpha(x_i)\}\}\}]$,

Hence from the definition of f we get $f(\cup_{\alpha\in\Delta} \mu_\alpha) = (\cup_{\alpha\in\Delta} \mu_\alpha)$. This implies $f^2(\cup_{\alpha\in\Delta} \mu_\alpha) = (\cup_{\alpha\in\Delta} \mu_\alpha)$, $f^3(\cup_{\alpha\in\Delta} \mu_\alpha)(\cup_{\alpha\in\Delta} \mu_\alpha), \ldots$. Therefore from the definition of $NO_f(\cup_{\alpha\in\Delta} \mu_\alpha)$ and Theorem 3.2 we get $NO_f(\cup_{\alpha\in\Delta} \mu_\alpha) = (\cup_{\alpha\in\Delta} \mu_\alpha) = \cup_{\alpha\in\Delta} NO_f(\mu_\alpha)$

Hence the proof.

Neutrosophic orbit topological spaces

In this section we show that the family of all neutrosophic orbit open sets under the mapping f constrict a neutrosophic orbit topology on X, denoted by τ_{NO} which is coarser than τ.

Theorem 4.1:

Let (X, τ) be a NTS and $f: X \longrightarrow X$ be a mapping.Let τ_{NO} denote to the family of all neutrosophic orbit open sets under the mapping f.Then, τ_{NO} is a neutrosophic topology on X coaser than τ.

Proof:

We must show τ_{NO} satisfies the 3 axioms of the definition of neutrosophic topology. It is clear that $\overline{0}$ and $\overline{1}$ are neutrosophic orbit open sets because there exist $\lambda = \overline{0}$ and $\nu = \overline{1}$such that $NO_f(\lambda) = \overline{0} \in \tau$ and $NO_f(\nu) = \overline{1} \in \tau$. Thus, $\overline{0} \in \tau_{NO}$ and $\overline{1} \in \tau_{NO}$.

Let $\mu_1 and \mu_2$be neutrosophic orbit open sets under the mapping f.To show $\mu_1 \cap \mu_2$ is a neutrosophic orbit open sets under the mapping f, we must find a neutrosophic set $\lambda \in I^X$ such that

$$NO_f(\lambda) = \mu_1 \cap \mu_2 \in \tau$$

If we choose $\lambda = \mu_1 \cap \mu_2$then from Theorem 3.3 and Proposition 3.6 we have

$NO_f(\lambda) = NO_f(\mu_1 \cap \mu_2) = NO_f(\mu_1) \cap NO_f(\mu_2) = \mu_1 \cap \mu_2$

On the other hand, since every neutrosophic orbit open set is an open neutrosophic set in X, then $\mu_1 \cap \mu_2 \in \tau$

Hence, the result.

Let $\{\mu_\alpha\}_{\alpha\in\Delta}$ be any family of neutrosophic orbit open sets under the mapping f. Let $\lambda = (\cup_{\alpha\in\Delta} \mu_\alpha)$.

Then from Theorem 3.9, $NO_f(\lambda) = NO_f(\vee_{\alpha\in\Delta} \mu_\alpha) = \cup_{\alpha\in\Delta} NO_f(\mu_\alpha) = (\cup_{\alpha\in\Delta} \mu_\alpha)$.And $(\cup_{\alpha\in\Delta} \mu_\alpha) \in \tau$.

Thus, τ_{NO} is a neutrosophic topology on X.

Furthermore, $\tau_{NO} \subset \tau$ since every neutrosophic orbit open set is an open neutrosophic set in X.

Definition 4.2:

Let (X,τ) be a NTOPand $f: X \rightarrow X$ be a mapping. The pair (X,τ_{NO}) is called neutrosophic orbit topological space (NOT) associated with (X,τ) if it satisfies the following axioms

(i) $\overline{0} \in \tau_{NO}$ and $\overline{1} \in \tau_{NO}$

(ii) $G_1 \cap G_2 \in \tau_{NO}, for any G_1, G_2 \in \tau_{NO}$

(iii) $\cup\, G_i \in \tau_{NO}, for any arbitrary family \{G_i, G_i \in \tau_{NO}, i \in I\}$

Example 4.3:

1. For any nonempty countable set X, τ_{NO}^{O}={$\overline{0}$, $\overline{1}$} is a neutrosophic orbit topology on X, and is called the indiscrete neutrosophic orbit topology.
2. For any nonempty countable set X, if f:$X \to X$ is the identity mapping, then $\tau_{NO} = \tau$

Next the notion of neutrosophic orbit closure (resp. interior) of a neutrosophic set is introduced.

Definition 4.4:

Let (X,τ_{NO}) be a neutrosophic orbit topological space and $\lambda \in I^X$. The neutrosophic orbit closure of λ, denoted by $cl_{NO}(\lambda)$, is the intersection of all neutrosophic orbit closed supersets under the mapping f of λ. i.e.,

$cl_{NO}(\lambda) = \cap \{\rho \in I^X / \rho \supseteq \lambda, \overline{1} - \rho \in \tau_{NO}\}$

And, the neutrosophic orbit interior of λ, denoted by $int_{NO}(\lambda)$, is the union of all neutrosophic orbit open subsets under the mapping f of λ i.e.,

$int_{NO}(\lambda) = \cup \{\rho \in I^X / \rho \subseteq \lambda, p \in \tau_{NO}\}$

Clearly, $cl_{NO}(\lambda)$ (resp., $int_{NO}(\lambda)$) is the smallest (resp., largest)neutrosophic orbit closed (resp., open) set under the mapping f which contains (resp., contained in) λ.

Proposition 4.5:

Let (X,τ_{NO}) be a neutrosophic orbit topological space and $\lambda \in I^X$. Then

$int_{NO}(\lambda) \subseteq int(\lambda) \subseteq \lambda \subseteq cl(\lambda) \subseteq cl_{NO}(\lambda)$

Proof:

The proof follows directly from the fact that every neutrosophic orbit closed (resp., open) set under the mapping f is closed (resp., open) neutrosophic set.

Proposition 4.6:

Let (X, τ_{NO}) be a neutrosophic orbit topological space and $\lambda, \mu \in I^X$.Then

1. $cl_{NO}(\overline{0}) = \overline{0}$ and $cl_{NO}(\overline{1}) = \overline{1}$.
2. $\lambda \subseteq cl_{NO}(\lambda)$
3. $cl_{NO}(\lambda \cup \mu) = cl_{NO}(\lambda) \cup cl_{NO}(\mu)$
4. if $\lambda \subseteq \mu, then cl_{NO}(\lambda) \subseteq cl_{NO}(\mu)$
5. $cl_{NO}(cl_{NO}(\lambda)) = cl_{NO}(\lambda)$
6. λisaneutrosophicorbitclosedsetunderthemappingfiff$\lambda = \mathrm{cl_{NO}}(\lambda)$.

Proof: Straightforward

Proposition 4.7:

Let (X, τ_{NO}) be a neutrosophic orbit topological space and $\lambda, \mu \in I^X$.Then

1. $int_{NO}(\overline{0}) = \overline{0}$ and $int_{NO}(\overline{1}) = \overline{1}$.
2. $int_{NO}(\lambda) \subseteq \lambda$.
3. $int_{NO}(\lambda \cup \mu) = int_{NO}(\lambda) \cup int_{NO}(\mu)$
4. if $\lambda \subseteq \mu, then int_{NO}(\lambda) \subseteq int_{NO}(\mu)$
5. $int_{NO}(int_{NO}(\lambda)) = int_{NO}(\lambda)$
6. λisaneutrosophicorbitopensetunderthemappingfiff
   $\lambda = \mathrm{int_{NO}}(\lambda)$.

Proof: Straightforward

Theorem 4.8:

Let (X, τ_{NO}) be a neutrosophic orbit topological space and $\lambda \in I^X$. Then,

1. $\overline{1} - int_{NO}(\lambda) = cl_{NO}(\overline{1} - \lambda)$.
2. $\overline{1} - cl_{NO}(\lambda) = int_{NO}(\overline{1} - \lambda)$.

Proof:

We prove that part 1 and by the similar way one can prove part 2.From Proposition 4.7 part 2 $int_{NO}(\lambda) \subseteq \lambda$ so by taking the complement we have $\overline{1} - \lambda \subseteq \overline{1} - int_{NO}(\lambda)$.Since $\overline{1} - int_{NO}(\lambda)$ is a neutrosophic orbit closed set and by Proposition 4.6 part 4,$cl_{NO}(\overline{1} - \lambda) \subseteq cl_{NO}\left(\overline{1} - int_{NO}(\lambda)\right) = \overline{1} - int_{NO}(\lambda)$. Hence $cl_{NO}(\overline{1} - \lambda) \subseteq \overline{1} - int_{NO}(\lambda)$.

Conversely, by Proposition 4.6 part 2, $\overline{1} - \lambda \subseteq cl_{NO}(\overline{1} - \lambda)$.By taking the complement $\overline{1} - cl_{NO}(\overline{1} - \lambda) \subseteq \lambda$.Since $cl_{NO}(\overline{1} - \lambda)$ is a neutrosophic orbit closed set.Then $\overline{1} - cl_{NO}(\overline{1} - \lambda)$ is a neutrosophic orbit open set and by Proposition 4.7 part 6,we have $\overline{1} - cl_{NO}(\overline{1} - \lambda) \subseteq int_{NO}(\lambda)$,again by taking the complement we obtain,

$\overline{1} - int_{NO}(\lambda) \subseteq cl_{NO}(\overline{1} - \lambda)$.

Theorem 4.9:

Let $f: (X, \tau_{NO}) \rightarrow (Y, \tau'_{NO})$ and $g: (Y, \tau_{NO}) \rightarrow (Z, \tau''_{NO})$ be any 2 mappings.Then $g \circ f$ is any neutrosophic continuous mapping if f and g are neutrosophic continuous.

Proof:

Let V be a orbit open set in (Z, τ''_{NO}). Then g^{-1} (V) is neutrosophic orbit open in (Y, τ_{NO})(since g is neutrosophic continuous), and hence $f^{-1}(g^{-1}(v))$ is neutrosophic orbit open in (X, τ_{NO})(since f is neutrosophic continuous). But $f^{-1}(g^{-1}(v)) = (g \circ f)^{-1}$(V). Thus the composition function $g \circ f$ is neutrosophic continuous.

Category of neutrosophic orbit topological spaces

In this section, we will construct the category of NOTOP, and study its relation with the category of NTOP.

Definition5.1:

Let NOTOPbe the collection of all NOTOP(X,τ_{NO}), (Y,τ'_{NO}), … associated with (X,τ),(Y,τ'), … respectively. For each pair of objects (X,τ_{NO}), (Y,τ'_{NO}) ofNOTOP, define Mor((X,τ_{NO}), (Y,τ'_{NO})) to bethe set of all neutrosophic continuous mapping f with respect to τ_{NO} and τ'_{NO}. Composition of 2 morphisms f: (X,τ_{NO}) → (Y,τ'_{NO}),g: (Y,τ'_{NO}) → (Z,τ''_{NO}) is defined by gof:(X,τ_{NO}) → (Z, τ''_{NO}).

Theorem 5.2:

NOTOPis a category.

Proof:

First, from Theorem 4.9, the composition of neutrosophic continuous mappings between NOTOP is also neutrosophiccontinuous, hence the composition of morphisms is well defined and associative. Second, to each object (X,τ_{NO}) in NOTOP define the identity morphism $1_{(X,\tau_{NO})}: (X, \tau_{NO}) \rightarrow (X, \tau_{NO})$ by the idtity set mapping. Thus, we get to the required result.

Remark 5.3:

NOTOPis not a subcategory of NTOP, because if f is intuitionistic fuzzy continuous from (X,τ_{NO}) to (Y,τ'_{NO}), then f need not to be neutrosophic continuous from (X,)to (Y,τ'). That is mean Mor((X,τ_{NO}), (Y,τ'_{NO})) ⊈ Mor((X,τ),(Y,τ')). We give an example to explain that.

Example 5.4:

Let $X = \{x_1, x_2, x_3\}$and$Y = \{y_1, y_2, y_3\}$. Define$\tau = \{\overline{0}, \overline{1}, \lambda\}and\tau' = \{\overline{0}, \overline{1}, \mu_1, \mu_2\}$

where$\lambda \in I^X$and$\mu_1, \mu_2 \in I^Y$suchthat$\lambda = \{(x_1, 0.2,0.4,0.8), (x_2, 0.3,0.5,0.7), (x_3, 0.3,0.5,0.7)\}$,

$\mu_1 = \{(y_1, 0.2,0.4,0.8), (y_2, 0.3,0.5,0.7), (y_3, 0.3,0.5,0.7)\}$ andμ_2

$= \{(y_1, 0.6,0.3,0.4), (y_2, 0.5,0.5,0.5), (y_3, 0.7,0.5,0.3)\}$

clearly$(X, \tau), (Y, \tau')$ areNTOP.

Define$f: (X, \tau) \rightarrow (Y, \tau'), f_1: X \rightarrow Xandf_2: Y \rightarrow Yasf(x_1) = y_1, f(x_2) = y_3, f(x_3) = y_2$,

$f_1(x_1) = x_1, f_1(x_2) = x_3, f_1(x_3) = x_2andf_2(x_1) = x_1, f_2(x_2) = x_3, f_2(x_3) = x_2$.

Then$\tau_{NO} = \{\overline{0}, \overline{1}, \lambda\}and\tau'_{NO} = \{\overline{0}, \overline{1}, \mu_1\}$. Itisclearthatfisaneutrosophiccontinuous withrespecttoτ_{NO}andτ'_{NO}.

Butfisnotaneutrosophiccontinuouswithrespecttoτandτ' , since$\mu_2$isanopen neutrosophicsetinY, howeverf$^{-1}(\mu_2)$ isnotaneutrosophicopensetinX.

Theorem 5.5:

NOTOP isomorphic to a subcategory of NTOP.

Proof:

Let NOTOPω be a collection {(X,I_X)} of objects in NTOP, such that I_X is the indiscrete neutrosophic topology on X. For any pair of objects (X,I_X),(Y,I_Y) of NOTOPω, take Mor((X,I_X),(Y,I_Y)) (in NTOP) as the set of morphisms in NOTOPω. Then it is clear NOTOPω is a subcategory of NTOP. Now define F: NOTOP → NOTOPω by F((X,τ_{NO})) = (X,I_X) and for each morphism f :(X,τ_{NO}) → (Y,τ'_{NO}) define $F(f) = f$:(X,I_X) → (Y,I_Y). It can be verified that F is indeed a bijective functor. Thus NOTOP isomorphic to NTOP.

Remark 5.6: From the above theorem, we can say that NOTOP is embedded in NTOP as a subcategory.

Conclusions

In this paper, we study the collection of neutrosophic orbit open sets under the mapping $f:X \to X$. We give the necessary conditions on the mapping f in order to obtain a fixed orbit of a neutrosophic set for any neutrosophic orbit open set under the mapping f. As a main result, we prove the family of all neutrosophic orbit open sets constructs a neutrosophic orbit topological space.In addition, the category of NOTOPand neutrosophic continuous mappings NOTOP is defined. And we show this category is isomorphic to a subcategory of the category of NTOP.

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