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A review on history, trends and perspectives of fuzzy linear programming

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Abstract

This paper presents a literature review and analysis of fuzzy linear programming models and methods. The selected bibliography is related to the original linear programming with fuzzy linear constraints models proposed by Hans Jürgen Zimmermann during 1974–76 since they have become a reference for scholars and practitioners due to its ease to implement and understand. To do so, we analyze the timeline of publication of the most important works in the field, evaluate the influence of relevant papers using a time–weighted index based on the amount of citations to provide a perspective about the future directions in fuzzy linear programming and recommend selected books, journals and some recent contributions to readers.

Keywords: Fuzzy linear programming, Fuzzy constraints, Fuzzy optimization, Uncertain optimization

1. Introduction and Motivation

Optimization under uncertainty is nowadays one of the most interesting problems since real world always come with some degree of uncertainty which can be analyzed from different points of view (probabilistic, stochastic, interval, fuzzy, possibilistic, etc.) and solved with different available methods. In fact, Scopus database retrieves > 70000 papers with the terms `optimization (and) uncertainty` which is a considerable amount of publications hard to analyze since such different uncertainty theories are not necessarily correlated or interrelated. As fuzzy sets theory was proposed by Lofti A. Zadeh in 1965, the idea of solving fuzzy optimization and more specifically fuzzy linear programming problems have gained attention in the academic community since the first works in fuzzy optimization have been published in 1974 and > 6700 papers have been published with the terms `fuzzy (and) linear (and) programming` since then.

This way, our focus is the Fuzzy Linear Programming (FLP) model introduced by Zimmermann [199, 200] who proposed a method to solve Linear Programming (LP) problems with fuzzy linear constraints and track the timeline of related works and developments. His contributions have influenced many other works and opened the door to involve fuzzy reasoning in optimization so it is also important to refer to related fuzzy mathematical models/methods. A large sequel of related works, extensions and applications have been published after Zimmermann published his seminal works in 1977. Different scholars including

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mathematicians, economists and engineers have contributed to the development of fuzzy mathematical programming based on the principles given by Bellman and Zadeh [18] who provided a unified framework for fuzzy decision–making. As a consequence, an important amount of applications of FLPs have appeared in the literature (> 4500), so it is important to evaluate its influence over time.

Interesting reviews in FLPs have been published in the literature. Baykasoğlu and Göcken [14] proposed 15 types of fuzzy mathematical programming models according to its fuzzy components and Schryen and Hristova [154] written a survey in duality for FLPs with 31 possible classes of FLPs. More specific reviews were published by Ebrahimnejad and Verdegay [39] who reviewed several solution methods for FLPs according to the classification proposed by Bector and Chandra [16], Hesam Shams and Haji [65] who published a brief review on several FLP models and suggested three classes only; Ghanbari et al. [56] who recently published a extensive review of models/methods for solving FLPs and classified them into five categories depending on fuzzy parameters/variables, and Sotoudeh-Anvari [166] has made an interesting criticism about some drawbacks and mathematical incorrect assumptions in fuzzy OR methods, including FLPs from 2010 to 2020. The main contribution of this paper compared to the aforementioned reviews is to characterize FLPs using two criteria: the amount of fuzzy parameters involved and the kind of obtained solution (fuzzy or crisp), to provide an analysis of the influence of the most cited papers where the influence of a paper is evaluated using two criteria: the amount of non self citations and its publication date (existence over time), to link FLPs to other methods such as interval, chance constrained, stochastic, Type–2 mmodels/methods etc. and to visualize trends, perspectives and new solution methods.

The paper is organized as follows: Section 1 introduces the paper; Section 2 presents some basics on FLPs; Section 3 performs a literature review; Section 4 introduces a classification based on modeling/solution methods, Section 5 shows the influence of the most cited papers; Section 6 introduces some trends, perspectives and recommended topics in FLP to finally present some concluding remarks in Section 7.

2. Basics on fuzzy linear programming

A fuzzy set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$ is characterized by a function $\mu_{\tilde{A}}(x)$ which measures the membership of x regarding a concept/word/label A. $\mathcal{F}(X)$ is the class of all fuzzy sets, $\mathcal{F}(\mathbb{R})$ is the class of all real-valued fuzzy sets, and $\mathcal{F}_1(\mathbb{R})$ is the class of all fuzzy numbers. A fuzzy number (see Klir and Yuan [83], Dubois and Prade [38] and Arana-Jiménez [8]) is defined as follows.

Definition 1. *Let $\tilde{A} : \mathbb{R} \to [0, 1]$ be a fuzzy subset of the reals. Then $\tilde{A} \in \mathcal{F}_1(\mathbb{R})$ is a Fuzzy Number (FN) iff there exists a closed interval $[x_l, x_r] \neq \emptyset$ with a membership function $\mu_{\tilde{A}}(x)$ such that:*

$$\mu_{\tilde{A}}(x) = \begin{cases} c(x) & \text{for } x \in [c_l, c_r], \\ l(x) & \text{for } x \in [-\infty, x_l], \\ r(x) & \text{for } x \in [x_r, \infty], \end{cases} \tag{1}$$

where $c(x) = 1$ for $x \in [c_l, c_r]$, $l : (-\infty, x_l) \to [0, 1]$ is monotonic non-decreasing, continuous from the right, i.e. $l(x) = 0$ for $x < x_l$; $l : (x_r, \infty) \to [0, 1]$ is monotonic non-increasing, continuous from the left, i.e. $r(x) = 0$ for $x > x_r$.

The fully fuzzy linear programming problem referred in this paper is:

$$\tilde{z} = \underset{x}{\text{Max}}\{\tilde{c}'x : \tilde{A}x \precsim \tilde{b}, x \in \mathbb{R}^+\} \tag{2}$$

where $\tilde{c} \in \mathcal{F}_1(\mathbb{R}^n), \tilde{A} \in \mathcal{F}_1(\mathbb{R}^{mn}), \tilde{b} \in \mathcal{F}_1(\mathbb{R}^m)$ and $\precsim$ is the fuzzy max–order binary relation (see Ramík and Řimánek [143]). The seminal FLP problem (fuzzy constraints only) was defined by Zimmermann [199, 200]:

$$\tilde{z} = \underset{x}{\text{Max}}\{c'x : Ax \precsim \tilde{b}, x \in \mathbb{R}^+\} \tag{3}$$

where $c \in \mathbb{R}^n, A \in \mathbb{R}^{mn}, \tilde{b} \in \mathcal{F}_1(\mathbb{R}^m)$.

The main idea is to look for values $\check{z} = \text{Max}_x\{c'x : Ax \leqslant \check{b}, x \in \mathbb{R}^+\}$ and $\hat{z} = \text{Max}_x\{c'x : Ax \leqslant \hat{b}, x \in \mathbb{R}^+\}$ to then solve the conflict of interests between the goal $\tilde{z}$ and its constraints $\tilde{b}$ using the following LP to maximize the satisfaction degree λ between $\tilde{z}$ and $\tilde{b}$ (see Figure 3):

$$\underset{\lambda,x}{\text{Max}}\{\lambda : c'x - \lambda(\hat{z} - \check{z}) = \check{z}; Ax + \lambda(\overline{b} - \underline{b}) \leqslant \overline{b}; x \in \mathbb{R}^+\}. \tag{4}$$

Although model (2) is a general approach, the model (3) is one of the most popular models in academy and industry due to its applicability and ease to solve using the LP model (4) which assumes fuzzy linear constraints only (see Figure 3). Some of its popular names include "flexible constraints", "soft constraints", "fuzzy constraints", "satisfaction degree", etc. so we will refer to all of them as FLPs.

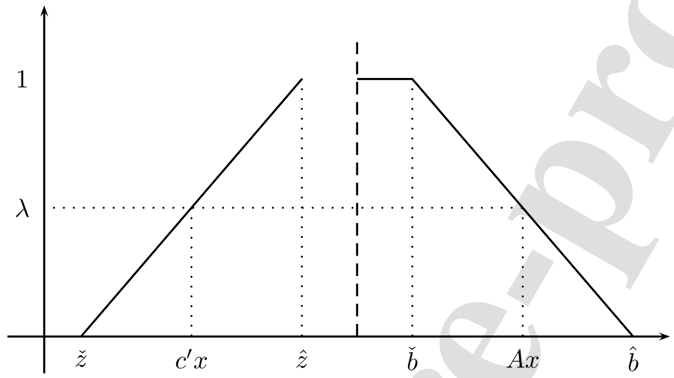

Figure 1: Conflict of interest between $\tilde{z}$ and $\tilde{b}$

3. Literature review

The methodology used to compose a timeline and review of FLPs is by retrieving published papers from Scopus (as of December 22, 2021) with the following features:

- Timeline: 1970 to present
- Terms: `fuzzy (and) linear (and) programming`
- Fields: `article title (and) abstract (and) keywords`
- Topics: Theory, applications and related problems
- Publications: Peer–reviewed journals, conference proceedings and book chapters

Many contributions to fuzzy sets theory/applications appeared after Zadeh [193] proposed fuzzy sets as a branch of mathematics for uncertainty analysis. One of the most influential works in uncertain decision making was later written by Bellman and Zadeh [18] who gave the foundations of decision making and optimization models in a fuzzy environment and opened the door to involve fuzzy sets into optimization models/methods. To do so, we classified fuzzy mathematical programming problems into three categories:

i) Fuzzy linear constrained models: it includes theory, extensions and applications of the Zimmermann FLP model (see Eq. (3)) by computing the set of fuzzy optimal values $\tilde{z}$ from fuzzy parameters:

$$(\tilde{A}, \tilde{b}, \tilde{c}) \overset{lp}{\longrightarrow} \tilde{z} \rightarrow z^* \tag{5}$$

ii) Fuzzy ranked models: it includes theory and applications of the general FLP (see Eq. (2)) using ranking measures (instances) namely $r(\cdot)$ such as centroid, Yager index, median, etc. to then find a set of crisp parameters (A, b, c) and solve a crisp LP problem given such instance (ranked values):

$$(\tilde{A}, \tilde{b}, \tilde{c}) \xrightarrow{r(\cdot)} (A, b, c) \rightarrow z^* \tag{6}$$

iii) Fuzzy general models: this category includes theory and applications of transformations and related methodologies to the FLP shown in Eq. (2) namely $h(\cdot)$ such as chance, possibilistic, interval, etc.

$$(\tilde{A}, \tilde{b}, \tilde{c}) \xrightarrow{h(\cdot)} (A, b, c) \rightarrow z^* \tag{7}$$

Scopus classifies > 6700 publications with the keywords `fuzzy (and) linear (and) programming` from 1975 to present, so we first debugged them into one of the categories (i), (ii), and (iii). Papers focused to fuzzy transforms such as chance constrained, ranking measures, penalty functions and other representations such as intervals, grey/fuzzy sets, intuitionistic fuzzy sets (IFS), interval-valued and Type–2 among others were also included. First, we perform a historical review of the most important papers to then analyze the most influential models/methods in different FLPs and its extensions.

3.1. Timeline of fuzzy linear constrained models (i)

The theory and practice of fuzzy linear constrained models is given in two fronts: fuzzy mathematical programming (see Eq. (2)) and fuzzy linear programming (see Eqs. (3) and (4)). Fuzzy mathematical programming was introduced by Tanaka et al. [175] followed by Orlovsky [135], Verdegay [179] and Zimmermann [201] then Chanas [27] addressed multiobjective FLP models using aspiration levels, Narasimhan [124] introduced the fuzzy goal programming problem and Tanaka and Asai [172], Tanaka and Asai [173] & Wang [183] addressed the problem of fuzzy costs/constraints using λ satisfaction degrees. The resolution of FLPs with fuzzy matrix/constraints was first discussed by Negoiţă [128] as robust/flexible programming and later extended by Zimmermann [202] & Delgado et al. [35] which led to the generalized FLP model (see Eq. (2)), Shaocheng [158] introduced interval/fuzzy number–based LPs and its α–cut representation and Qiu et al. [142] addressed the problem of fuzzy fuzzy nonlinear constraints. Relevant resolution methods for fully FLPs (see Eq. (2)) include Buckley and Feuring [19] who used evolutionary algorithms to solve complex FLPs, Sengupta et al. [157] who solve interval–constrained problems, Jiménez et al. [75] who proposed an interactive resolution method, Jamison and Lodwick [74] & Van-Hop [178] who proposed a penalty function method and Kreinovich and Figueroa-Garcia [87] who proposed an iterative algorithm based in the cumulative membership function of a fuzzy set/number.

Early works and ideas about FLPs were given by Gould [58] and Negoiţă [128] which were later elaborated by Negoiţă and Sularia [130], Negoiţă et al. [129] and Orlovsky [134] until the seminal works of Zimmermann [199, 200] provided unified definitions and methods for solving fuzzy linear constrained problems (see Eq. (3)) using LP methods including duality for FLPs as discussed by Verdegay [180] and interactive decision making for FLPS as proposed by Ebrahimnejad and Verdegay [39] and Rommelfanger [147]. Campos [23] proposed the first approach for solving fuzzy matrix games using FLPs and Rommelfanger [148] presented a review of theoretical advances and applications of FLPs. Extensions to Interval Type–2 fuzzy constraints were proposed by Figueroa-García [44] and Figueroa-García et al. [52] who focused on modeling and solving fuzzy linear constrained problems involving Interval Type–2 fuzzy sets and Figueroa-García [48] who proposed an FLP with Interval Type–2 fuzzy matrix (technological coefficients).

3.2. Timeline of Fuzzy ranked models (ii) and Fuzzy general models (iii)

Mixed random/stochastic/fuzzy methodologies also emerged to cover other kinds of uncertainty. Fuzzy–random linear programming has been proposed by Luhandjula [111] and later extended by Zhong and Guang-yuan [196] & Zhong et al. [197]; Fuzzy chance constrained models have been proposed by Liu and Iwamura [106] and Liu [102]; Fuzzy–random programming models were introduced by Luhandjula [111] by considering fuzzy–random sets which were extended to a fuzzy stochastic framework by Luhandjula and

Gupta [112] in order to make it solvable by using stochastic optimization techniques. Fuzzy–possibilistic linear programming was proposed by Luhandjula [113] and later extended by Dubois [37] from a fuzzy point of view and Wierzchon [188] from a possibilistic distribution point of view. IFS optimization problems were introduced by Angelov [7] and later IFS based LPs were proposed by Parvathi and Malathi [137]. Two new approaches to involve fuzzy uncertainty are Neutrosophic linear/goal programming models proposed by Abdel-Baset et al. [1] which involve truth and belief in fuzzy sets and and Pythagorean FLP models which proposed by Chen [31] (a Pythagorean fuzzy set is an extension of an intuitionistic fuzzy set).

FLP methods/models have been also applied to random related problems: Campos and Verdegay [24] used ranking of fuzzy numbers to solve crisp LPs, Lai and Hwang [95] proposed a method for solving possibilistic LPs and Shih et al. [160] solved a multi-level programming problem using fuzzy sets. Liu and Iwamura [106] & Liu [102] proposed the fuzzy chance constrained model; Maleki et al. [116] solved LPs with fuzzy variables and randomness, Mikhailovv [119] used fuzzy models for deriving priorities in analytic hierarchy processes, Guo and Tanaka [60] proposed a perceptual method for solving fuzzy Data Envelopment Analysis (DEA) problems and Lertworasirikul et al. [96] proposed the possibilistic DEA.

Gen and Yun [55] provided an interesting review of reliability in soft computing techniques including fuzzy optimization, Xu and Chen [190] proposed a method for solving fuzzy multiple attribute group decision making problems using interactive methods, Lin et al. [101] applied IFSs to multicriteria fuzzy decision–making problems, Torabi and Hassini [176] solved multiobjective supply chain planning problems using interactive possibilistic programming models/methods, Li et al. [99] and Li [98] proposed a model/method for for multiattribute group decision making using interval and IFSs and Kumar et al. [90] proposed to solve general fuzzy models using lexicography rules with fuzzy equality constraints.

3.3. Timeline of applications of FLPs, generalized FLPs and other methodologies

Fuzzy goal programming was addressed by Narasimhan [124] who proposed an early fuzzy goal programming method, Hannan [61] & Hannan [62] proposed a sequel on fuzzy goal programming and multiple fuzzy goals, [149] addressed the problem of fuzzy costs using α–cuts, Słowiński [165] solved water supply planning problems using a multicriteria FLP method, Sakawa et al. [151] proposed an interactive fuzzy satisfaction approach for multiobjective LPs based on the original Zimmermann's ideas, Chanas and Kuchta [28] analyze the meaning of optimality in transportation problems with fuzzy costs, Liu and Sahinidis [107] focused in process planning under fuzzy uncertainty using optimization methods and Roy and Maiti [150] approached the fuzzy EOQ model with demand-dependent costs and storage capacity constraints.

Inuiguchi and Ramík [73] proposed a seminal paper in possibilistic LP models/methods applied to portfolio problems, Sakawa and Kubota [152] used genetic algorithms to solve multiobjective fuzzy job shop scheduling problems, Kumar et al. [91] used fuzzy goal programming to solve vendor selection problems in supply chains and Kumar et al. [92] solved vendor selection problems in supply chains using fuzzy programming, Wang and Liang [184] applied fuzzy multiobjective LPs and Wang and Liang [185] used possibilistic LP to solve aggregate production planning problems, Selim et al. [156] solved supply chain collaborative production-distribution planning problems using fuzzy goal programming, Peidro et al. [138] solved supply chain planning problems with fuzzy supplies, demand and process uncertainties, Wang et al. [187] focused to risk evaluation using fuzzy weighted geometric mean, Amin et al. [6] mixed fuzzy SWOT analysis and FLPs to a supplier selection/allocation problem, Shaw et al. [159] solved supplier selection problems for low carbon supply chains using fuzzy multiobjective models and Kannan et al. [79] solved multicriteria supplier selection/allocation for green supply chain problems using multiobjective programming.

3.4. Timeline of Interval Linear Programming

One of the first approaches to involve uncertainties in LPs was supported by interval analysis which declares a parameter/variable as an interval set of possible values which comes from different uncertainty sources. Its relationship to FLPs is very close since the α–cuts representation (aka α–level sets representation) is a popular way to model/solve general FLPs. First ideas about inexact LP problems/methods involving interval uncertainty were given by Gould [58], Soyster [167] who analyzed an LP problem with constraints defined as interval tolerances, then Charnes et al. [30] addressed a more general problem of interval constraints and proposed an algorithm for finding solutions to such problem; Krawczyk [86] addressed the problem of interval decision variables x and matrix A by using interval computations; Beeck [17] and Mráz [121] later defined the general Interval Linear Programming (ILP) problem as the problem of having A, b, c defined as intervals and Mráz [122] addressed ILPs with interval matrix A and constraints b via its supremum/infimum values. Other interesting papers on ILPs include Mráz [123], Chinneck and Ramadan [33] who focused to compute the boundaries of a general class of ILPs (with unbounded decision variables X). Papers focused to the relationship between fuzzy and ILP include Kuchta [88] & Hladík and Černý [68] Hladík [66], Hladík [67], Černý and Hladík [26] & Garajová et al. [54] where the authors solve FLPs using ILPs, and early works in fuzzy quadratic programming were written by Silva et al. [161] & Cruz et al. [34].

Other relevant ILP applications/models include the self–called grey programming which is basically an alternative way to model ILPs whose foundations were proposed by Julong [76] and later extended by Huang and Moore [71]. Some applications of mixed grey/fuzzy models include Huang et al. [69] who used a grey FLP method in waste planning problems, Huang et al. [72], Nie et al. [131] and Huang et al. [70] who mixed interval/fuzzy/stochastic programming models and methods for solving waste management problems.

3.5. Perspectives and future directions in theory and models

As shown before and displayed in Figure 2 FLPs started as a branch of fuzzy decision making and evolved to different approaches involving different uncertainty sources: fuzzy linear constraints, fuzzy matrix $\tilde{A}$ and linear constraints $\tilde{b}$, fuzzy objectives (costs) $\tilde{c}$ and fully fuzzy parameters $\tilde{A}, \tilde{b}, \tilde{c}$ where different authors approached the problem from different perspectives including fuzzy representations (λ satisfaction degree, α–cuts representation, fuzzy extension principle, etc.) and ranking methods/measures to then use well known LP/optimization methods and mixed–uncertainty models (chance–constrained, quadratic, stochastic optimization, fuzzy–interval, etc). Figure 2 displays a timeline of the most influential works.

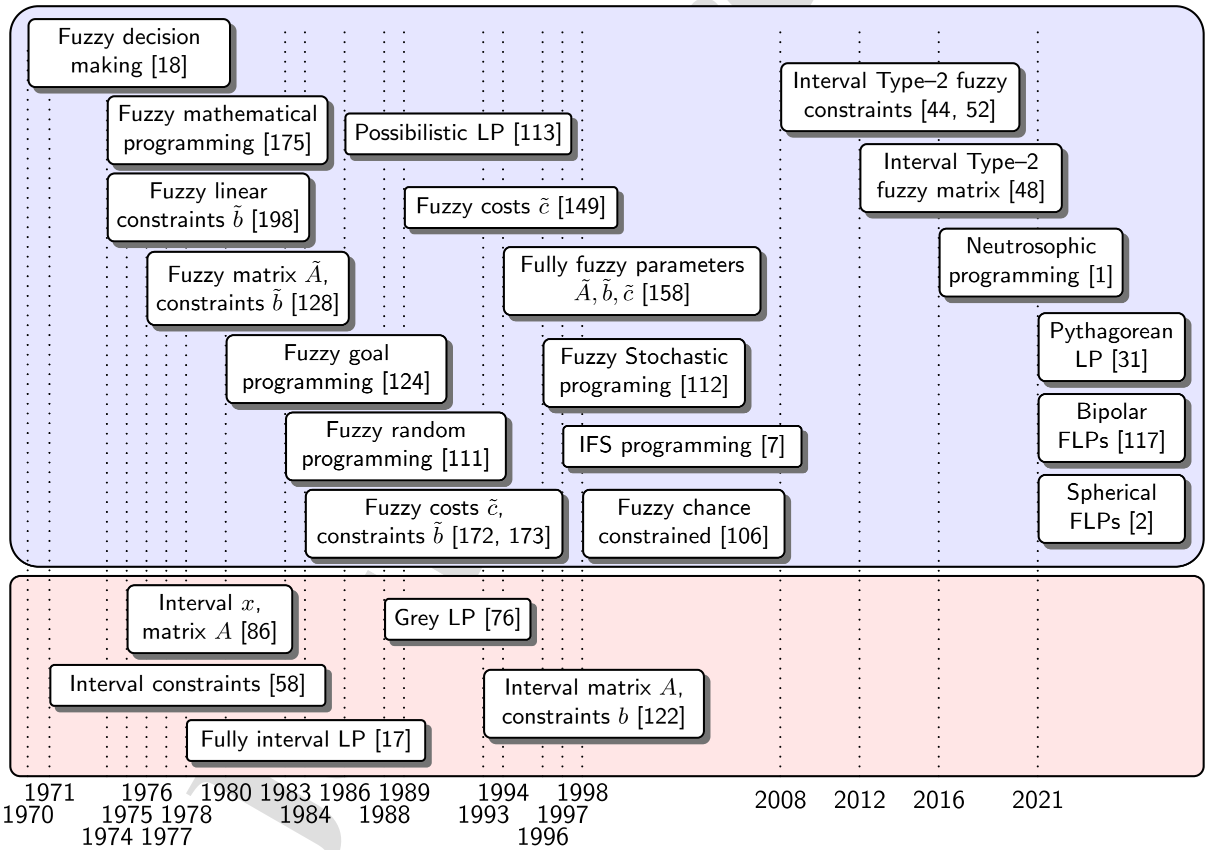

Figure 2: Timeline of fuzzy/interval programming problems

Although Tanaka et al. [175] have defined a fuzzy mathematical programming model in the framework of the fuzzy decision making principle and α–cuts representation, so far the only unified solution method for FLPs is the soft constraints method (see Zimmermann [199, 200] and Eq. (3)). On the other hand, models that involve fuzzy parameters $\tilde{A}, \tilde{b}, \tilde{c}$ are in need for a unified framework/decision criteria since most of recent works are focused to compute ranking measures to then obtain a deterministic solution while fuzzy parameters should lead to compose the set of all possible solutions $\tilde{z}$ as its most important feature. Recently, the use of other uncertainty representations such as IFSs, Interval Type–2 fuzzy sets, Neutrosophic and Pythagorean fuzzy sets and the close relationship between ILPs and FLPs (see Mráz [122], Černý and Hladík [26] and Hladík and Černý [68]) have been opened both new directions and new theoretical problems to solve which are summarized as follows:

- There is a need for a unified approach to model/solve LPs with fully fuzzy parameters $\tilde{A}, \tilde{b}, \tilde{c}$
- How to reduce the computational cost of using the fuzzy extension principle in FLPs?
- The interrelation among the fuzzy extension principle, fuzzy decision making principle and related representations such as possibilistic fuzzy sets, IFSs, Interval Type–2 fuzzy sets, Neutrosophic and Pythagorean fuzzy sets.
- The relationship between interval linear programming models/methods and FLPs

Other fields with the potential to attract theoretical developments and applications are multi–label fuzzy variables and Explainable–AI (XAI). In the first approach a fuzzy variable can be described using multiple labels/attributes so there is a relationship with linguistic reasoning and computing with words while the later is related to a philosophical question around AI and its explainability, more specifically: what is the sense of a fuzzy optimal decision (rather than a solution enclosed into a fuzzy set) and its implications in computing/solving problems under uncertainty?.

4. Solution–based classification

Section 3 classifies papers based on their mathematical representation/model, but researchers and practitioners are often driven to look for the kind of the obtained solution in order to either explore or select a model/method. This way, we can relate the previous three categories i), ii) and iii) to two main types of solutions: a) Fuzzy set–based solutions and b) Deterministic solutions.

4.1. Fuzzy set–based solutions (a)

The scope of using fuzzy sets to handle uncertainty/imprecision is to involve human–like perceptions into optimization models (see Bellman and Zadeh [18]) so the most comprehensive way to extend the computation of deterministic to fuzzy functions is by using the fuzzy extension principle (see Klir and Yuan [84]):

Definition 2. *Any given function namely optimal solution $z^* : \mathbb{R}^{mn} \rightarrow \mathbb{R}$ induces a function*

$$z^* : \mathcal{F}_1(\mathbb{R}^{mn}) \rightarrow \mathcal{F}_1(\mathbb{R}) \tag{8}$$

which is defined by

$$\mu_{\tilde{z}(\tilde{A},\tilde{b},\tilde{c})}(z^*) = \sup_{z^* \in z^{-1}(x^*)} \{\tilde{A}', \tilde{b}', \tilde{c}'\} \tag{9}$$

where $'$ denotes the set of parameters associated to the binding constraints and the optimal solution x^*.

Then, a function z given fuzzy parameters $\tilde{A}, \tilde{b}, \tilde{c}$ and an optimal solution x^* induces a fuzzy set $\tilde{z}$ of optimal solutions which preserves all fuzzy information into but it also requires way more computations which is prohibitive in large–scale problems, nonlinear/combinatorial models/methods, etc. and leads to an important question: which of the solutions contained into $\tilde{z}$ should be recommended to decision makers?.

4.2. Deterministic–based solutions (b)

The NP–hardness nature of the computation of fuzzy functions via the fuzzy extension principle (see Definition 2) have led many researchers to simplify solving FLPs by using ranking measures such as the centroid/Yager index or penalty functions to then solve the problem using classical mathematical programming methods like LP, nonlinear/quadratic programming, Lagrange, gradient–based methods, etc.

A popular approach to overcome the NP–hardness of the fuzzy extension principle in FLPs is to rank fuzzy parameters (specially the matrix $\tilde{A}$) before using one of three main approaches: the Zimermmann FLP, classical LP models or alternative models like stochastic/random programming, chance constrained, grey–fuzzy programming or ILP methods to then find a solution. A solution in such defuzzified conditions is then called an *instance* of the whole FLP since it does not preserve fuzzy information which is lost in the defuzzification process. Having said that, there are few ways to deal with FLPs depending on the uncertainty source, its representation and the desired solution by decision makers which are summarized in Figure 3.

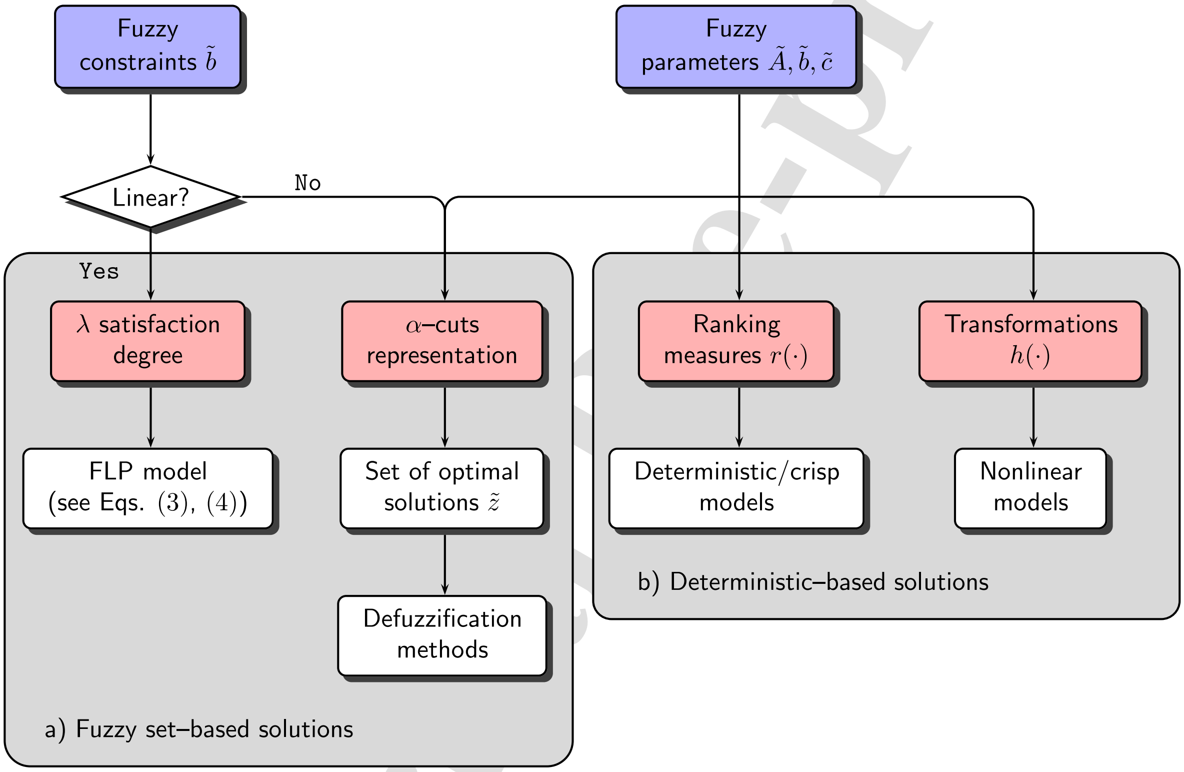

Figure 3: Classification of FLPs based on the obtained solution

Regardless the amount of citations and/or popularity that any work can have, it is important to evaluate if a work is really a fuzzy optimization method/model since most of papers in category b) should not be considered as fuzzy optimization methods/models by a simple reason: while works classified into category a) are proper fuzzy programming methods/models since they preserve fuzzy information either via α–cuts or computing the fuzzy set of optimal values of the problem, papers that fall into category b) simply reduce the whole FLP to a crisp model via ranking and/or transformations which completely loses fuzzy information without any clear/robust interface/interralation to the fuzzy sets where they come from, for the sake of computing a crisp solution. Some highly cited papers that fall into category include Guo and Huang [59], Huang et al. [72], Huang et al. [70], Nie et al. [131], Liu and Iwamura [106], Liu [102] and Wierzchon [188].

4.3. Perspectives in solving FLPs

ILPs have had interesting advances in the definition of a unified approach to find a solution, for instance Ashayerinasab et al. [11] proposed an exact method for finding the set of optimal solutions of an ILP and Batamiz et al. [13] proposed a method for finding efficient solutions in multiobjective ILPs, so their works and ideas can be extended to a fuzzy environment. To do so, there is a need for extending the notion of *Optimal Solution Set* (OSS) to FLPs via α–cuts/ representations, λ satisfaction degrees, fuzzy decision making and the extension principle.

Although the easiest method to solve FLPs is by solving an instance of its fuzzy parameters via classical LPs, the knowledge induced by experts comprised into fuzzy sets is lost so there is a need for solving FLPs while preserving fuzzy information even if the solution is represented by a single value/instance of the whole problem. So far, the only two methodologies known for preserving fuzzy information via λ–satisfaction degree and/or α–cuts are Zimmermann [199, 200], Verdegay [180], Tanaka et al. [175] and Tanaka et al. [175], so there is a need for extending their results to other uncertainty representations and design a method for finding optimal solutions while preserving fuzzy information.

Figure 3 shows a classification of FLPs into two groups: a) fuzzy solutions which are models/methods which lead to a set of possible solutions and keep fuzzy information, and b) deterministic–based models/methods which solve either an instance of an FLP (ranking, defuzzified parameters, etc.) or a uncertain–based deterministic model (chance, fractional, stochastic, interval, etc.). Many of the first works have been dedicated to the first category while most of the recent bibliography focuses to applications, extensions to new uncertainty representations and hybrid modeling. Table 1 presents some recent papers in the categories a) and b) (see Figure 3) which are summarized into theory and applications.

Table 1: Recent advances in theory and applications

Theoretical advances	
Osuna-Gómez et al. [136]	Optimality conditions for FLPs (a)
Ramík [144, 145]	Duality in FLPs (a)
Stefanini and Arana-Jiménez [168]	Fuzzy KKT conditions (a)
Farhadinia [41]	Fuzzy KKT conditions (a)
Mahdavi-Amiri et al. [115]	Fuzzy Farkas Lemma (a)
Nasseri and Ardil [125]	Fuzzy Farkas Lemma (a)
Nasseri and Chitgar [126]	Fuzzy Farkas Lemma (a)
Kasperski and Zieliński [81]	Robust solutions for possibilistic models (b)
Kreinovich and Figueroa-Garcia [87]	Algorithms for fully fuzzy LPs (a)
Hladík and Černý [68]	α–cuts representation (a)
Yang et al. [192]	max-product FLP (b)
Applications and modeling	
Kumar and Dash [89]	Fuzzy marketing (b)
Wang and Watada [186]	Fuzzy stochastic optimization (b)
Singh and Dhiman [163]	Fuzzy forecasting (b)
Zhang and Guo [195]	Fuzzy credibility irrigation model (b)
Jamison and Lodwick [74] & Van-Hop [178]	Fuzzy EOQ models (b)
Roy and Maiti [150]	Penalty methods for FLPs (b)
El-Alaoui [40]	Fully Interval Type–2 FLP (b)
Fathy [42]	IFS solving methods (b)
Fathy and Hassanien [43]	Harmonic mean for multiobjective FLPs (b)
Valipour and Yaghoobi [177]	Linearization of fractional FLPs (b)
Singh and Yadav [164]	Scalarization of fractional FLPs (b)
Wan et al. [181]	Hybrid FLPs for emergency problems (b)
Negarandeh and Tajdin [127]	Health problems (b)
Figueroa-García [46], Figueroa-García and Hernández [51]	Interval Type–2 FLP (a)
Qin et al. [141]	Type–2 fuzzy DEA models (b)
Figueroa-García and Hernández [50]	Type–2 fuzzy transportation models (b)
Qiu et al. [142]	Nonlinear fuzzy constraints (a)

On the other hand, there are some papers focused to solve FLPs by using metaheuristics to solve complex FLPs. This way, most of published works are hybrid methodologies based on evolutive metaheuristics such as

Particle Swarm Optimization (see Atta et al. [12], Schweickardt et al. [155] and Yalaoui et al. [191]), genetic algorithms (see Al-Bazi et al. [4], Molla-Alizadeh-Zavardehi et al. [120]), harmony search (see Memari et al. [118], andCengiz Kahraman [78]), scatter search (see Stojiljković [169]), NSGA-II algorithms (see Li et al. [100]) or comparing the performance of different metaheuristics (see Goodarzian et al. [57]). These works are mostly intended to speed up the search process and to find efficient solutions either via local search or combining different algorithms.

5. Influence of a publication

The influence of a paper can be seen from different perspectives which lead to different criteria to measure influence. In general, we consider a paper to be *influential* if satisfies two basic criteria:

- High amount of non self–citations
- Publication date (existence over time)

Although we do not consider self–citing as a bad practice, it is clear that a paper is more influential if it is read and cited by other authors, so the selected criterion is non self–citations. Regarding the existence of a paper over time, it is also clear that older papers had the chance to be read by more authors than recent papers (under the assumption that old papers have influenced more works in different disciplines than recently published works i.e. inertia over time) so an entropy–like measure is proposed to measure its influence. The methodology used to evaluate the influence of a paper is by retrieving published papers from Scopus (as of December 22, 2021) with the following features:

- Timeline: 1970 to present
- Terms: `fuzzy (and) linear (and) programming`
- Fields: `article title (and) abstract (and) keywords`
- Topics: Theory, applications and fuzzy LP–solvable problems
- Citations: > 20
- Publications: Peer–reviewed journals, conference proceedings and book chapters

After selecting theoretical approaches and applications with > 20 citations we finally retrieved 472 publications from 42 countries (a paper is assumed to come from its corresponding author affiliation's country) which means that just $\approx 7\%$ of the published papers reach > 20 citations. Due to the high amount of papers, only the 50 most cited papers plus some selected topics are referred in this paper while the obtained measures are based on the initial 472 publications.

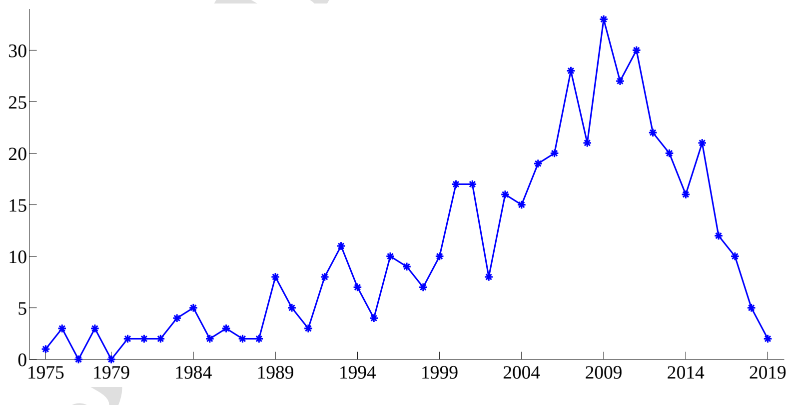

Figure 4: Amount of papers per year (> 20 citations)

Figure 4 shows a timeline of the 472 selected publications. Its growth since 1975 got an expected peak in 2009 (it takes some years to a paper to get cited). It is also interesting to find young papers in this category (three of them with > 90 citations) since a 5 year time interval is short to gain such popularity, but access to information is much easier today than 70-80's, so readers have now immediate access to recent advances.

A common way to measure the influence of a paper is by citation/self–citation counting and ratio per year. While citation counting gives a clear idea of how popular a publication is, its year ratio provides a flat idea of its time–dependant influence and they do not necessarily provide a clear idea of its influence over time since low–cited young papers could equate to highly–cited older papers. Now as highly cited papers tend to be older than young papers, we propose to measure the influence of a paper by weighting citation counting (popularity) from its year of publication (inertia over time) using a Shannon entropy–based measure.

Definition 3. *The Shannon entropy for a probability measure $H(X)$ of a random variable X is:*

$$H(X) = -\sum_i p(x_i) \ln p(x_i). \tag{10}$$

This way, the information provided by a certain amount of years T is:

Definition 4. *The time–based weight of existence $H(T)$ of a paper is:*

$$H(T) = -\sum_{t=1}^{T} \frac{1}{T} \ln \left(\frac{1}{T}\right) = \ln T \tag{11}$$

where T is the amount of years since a paper was published.

Now, given the amount of citations Q of a paper in T years, its time-weighted influence is as follows:

Definition 5. *Let Q_i be the amount of citations of the i_{th} paper in T years and $H_i(T)$ be its time–based weight, the impact index ρ_i of a paper is:*

$$Q_i = \sum_{t=1}^{T} q_{i,t} \tag{12}$$

$$\rho_i = Q_i H_i(T) = Q \ln T \tag{13}$$

where $q_{i,t}$ is the amount of citations of the i_{th} paper in the year t and Q_i is its total amount of citations.

It is important to remark that $H(T)$ uniformly weights each year. This means that $Q_i H_i(T)$ gives more importance to highly cited and older papers than young papers (even highly cited papers). Another way to measure the influence of a paper is by its citations in the last K years, as shown as follows.

Definition 6. *Let $Q_{i,K}$ be the amount of citations of the i_{th} paper in the last K years, the time–weighted index $\rho_{i,K}$ of a paper is:*

$$Q_{i,K} = \sum_{k=0}^{K} q_{i,T-k} \tag{14}$$

$$\rho_{i,K} = Q_{i,K} H_i(T) = Q_{i,K} \ln T \tag{15}$$

where K is the time–lapse in consideration.

The most cited papers (overall and in the last $K = 5$ years with self–citations) are shown in Table 2. It is interesting to note that the top–five of most cited papers keep their positions in the last 5 years rank. Also note that IFSs and possibilistic LPs have gained popularity in the last 10 years (see Li [97] and Inuiguchi and Ramík [73]) and multiobjective approaches became popular in the last 5 years (see Kannan et al. [79] and Shaw et al. [159]) which shows how the interests in FLP models have evolved.

Table 2: Top ten of most cited papers

Citation index Q_i		Non self–citation index Q_i		Self–citation index Q_i	
Zimmermann [200]	2411	Zimmermann [200]	2395	Huang et al. [69]	155
Zimmermann [199]	622	Zimmermann [199]	612	Huang et al. [72]	106
Li [97]	575	Li [97]	544	Huang et al. [70]	72
Inuiguchi and Ramík [73]	565	Inuiguchi and Ramík [73]	542	Nie et al. [131]	71
Kannan et al. [79]	424	Kannan et al. [79]	415	Cai et al. [21]	56
Shaw et al. [159]	411	Shaw et al. [159]	407	Guo and Huang [59]	43
Torabi and Hassini [176]	402	Wang et al. [187]	390	Torabi and Hassini [176]	40
Jiménez et al. [75]	399	Jiménez et al. [75]	384	Mahdavi-A. and Nasseri [114]	32
Wang et al. [187]	395	Torabi and Hassini [176]	362	Li [97]	31
Kumar et al. [91]	359	Kumar et al. [91]	350	Delgado et al. [35]	31
Citation index $Q_{i,K}$		Non self–citation index $Q_{i,K}$		Self–citation index $Q_{i,K}$	
Zimmermann [200]	680	Zimmermann [200]	680	Chandrawat et al. [29]	23
Kannan et al. [79]	327	Kannan et al. [79]	323	Singh and Dhiman [163]	20
Shaw et al. [159]	273	Shaw et al. [159]	271	Aliev et al. [5]	18
Wang et al. [187]	203	Wang et al. [187]	201	Zhang and Guo [195]	17
Torabi and Hassini [176]	200	Torabi and Hassini [176]	187	Zeng et al. [194]	16
Jiménez et al. [75]	185	Jiménez et al. [75]	184	Yang et al. [192]	16
Inuiguchi and Ramík [73]	158	Inuiguchi and Ramík [73]	158	Kundu et al. [93]	15
Li [97]	150	Li [97]	149	Wan and Li [182]	14
Lertworasirikul et al. [96]	128	Lertworasirikul et al. [96]	128	Buşoniu et al. [20]	14
Talaei et al. [170]	125	Talaei et al. [170]	122	Torabi and Hassini [176]	13

Regarding self–citations (counted from all authors of a paper) the rank changes since only two out of the ten most cited papers are into the most self–cited papers. Note that the top three of most self–cited papers is led by Huang G.H. (see Huang et al. [69], Huang et al. [72] and Huang et al. [70]) and the 5 year self–citation index changes to be led by Chandrawat et al. [29], Singh and Dhiman [163] and Aliev et al. [5].

Table 3: Top ten of the time–based most influential papers

Index ρ_i		Non self–citation index ρ_i		Self–citation index ρ_i	
Zimmermann [200]	9011,5	Zimmermann [200]	8951,7	Huang et al. [69]	510,9
Zimmermann [199]	2353,8	Zimmermann [199]	2315,9	Huang et al. [72]	312,1
Inuiguchi and Ramík [73]	1692,6	Inuiguchi and Ramík [73]	1623,7	Huang et al. [70]	231,8
Li [97]	1557,1	Li [97]	1473,2	Nie et al. [131]	182,1
Narasimhan [124]	1261,6	Narasimhan [124]	1254,2	Cai et al. [21]	143,6
Hannan [61]	1146,7	Hannan [61]	1139,4	Delgado et al. [35]	106,5
Lai and Hwang [95]	1029,7	Lai and Hwang [95]	1009,7	Guo and Huang [59]	103,1
Jiménez et al. [75]	1023,4	Jiménez et al. [75]	984,9	Torabi and Hassini [176]	99,4
Torabi and Hassini [176]	998,9	Tanaka and Asai [172]	974,7	Sakawa et al. [151]	90,9
Kumar et al. [91]	995,4	Kumar et al. [91]	970,4	Li [97]	83,9
Index $\rho_{i,K}$		Non self–citation index $\rho_{i,K}$		Self–citation index $\rho_{i,K}$	
Zimmermann [200]	2541,6	Zimmermann [200]	2541,6	Zeng et al. [194]	36,8
Kannan et al. [79]	636,3	Kannan et al. [79]	628,5	Huang et al. [69]	33,0
Shaw et al. [159]	567,7	Shaw et al. [159]	563,5	Torabi and Hassini [176]	32,3
Torabi and Hassini [176]	497,0	Wang et al. [187]	482,0	Buşoniu et al. [20]	32,2
Wang et al. [187]	486,8	Inuiguchi and Ramík [73]	473,3	Singh et al. [162]	30,8
Jiménez et al. [75]	474,5	Jiménez et al. [75]	472,0	Kundu et al. [93]	29,2
Inuiguchi and Ramík [73]	473,3	Torabi and Hassini [176]	464,7	Aliev et al. [5]	29,0
Li [97]	406,2	Li [97]	403,5	Mahdavi-A. and Nasseri [114]	28,2
Lertworasirikul et al. [96]	362,7	Lertworasirikul et al. [96]	362,7	Chandrawat et al. [29]	25,3
Guo and Tanaka [60]	318,0	Guo and Tanaka [60]	318,0	Tan et al. [171]	24,8

The rank of the time–weighted most influential papers (see Definitions 5 and 6) is shown in Table 3. The results of this rank differ to citation index rank (see Table 2) since other seminal papers are now into the best ranked papers. For instance Narasimhan [124] and Hannan [61] who first approached fuzzy goal/multiobjective problems come up to the top ten and Lai and Hwang [95] who proposed one of the first works on possibilistic LPs also appears into the best ranked. This connects to the most cited papers in the last 5 years since fuzzy goal programming and possibilistic LPs are getting interest in the community.

It is important to remark that some authors/papers keep its influence over time e.g. Zimmermann, Inuiguchi, Ramík, Tanaka, Asai, Jiménez while other authors/papers have lost some influence in the last $K = 5$ years e.g. Lai and Hwang, Kummar, Naransimhan and Hannan. This is a clear sign that seminal papers are always in the scope of recent developments/applications and the rank is in constant change.

Regarding self–citations, Huang et al. [69], Huang et al. [72], Huang et al. [70] keeps the first three places in time–weighted self citations while the 5 year time–weighted self citations index changed to include applications of possibilistic and fuzzy multiobjective LPs Zeng et al. [194], Huang et al. [69], Torabi and Hassini [176]. It is a sign that self–citation practices decrease over time and they are more related to real world applications and emergent topics whose dynamics are in constant change as well.

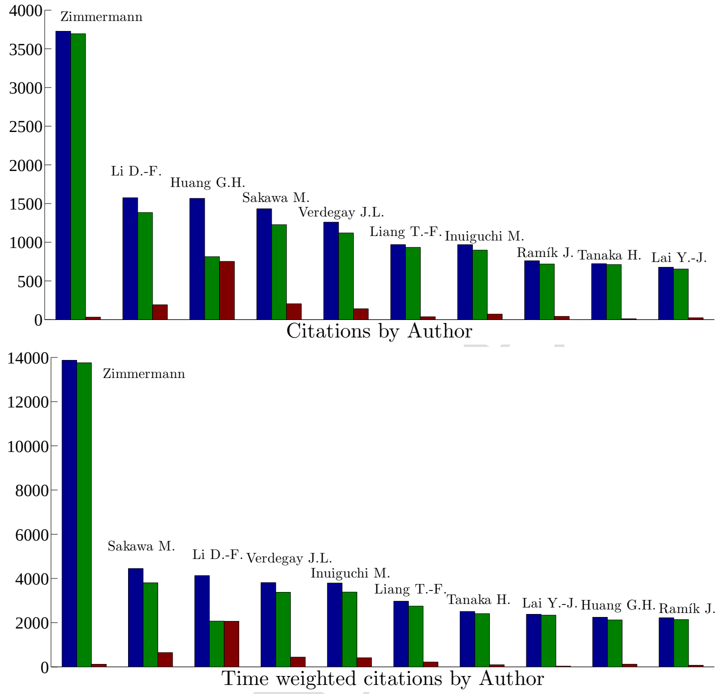

Figure 5: Ranking of authors

Figure 5 shows citations and time–weighted influence by author. Blue bar shows citations, green bar shows non self–citations and red bar shows self citations where H.-J. Zimmermann is the most influential author in FLPs so far, followed by G.H. Huang, D.-F. Li and M. Sakawa. Based on the overall figures, it is clear that self citations is a potential factor to be considered when evaluating the influence of a particular paper/author since authors like G.H Huang and D.-F Li who are highly cited seem to have a smaller time–weighted influence than authors like M. Sakawa and J.L. Verdegay.

5.1. Books and journals

Books, journals and conferences have much more citations due to the range of topics they cover, so we will refer to journals based on the selected 472 publications (we point out that the presented Journal ranking is specific to FLPs) and we also recommend several books that include theory and/or applications of FLPs. Journals can be ranked in a similar way than authors i.e. using the amount of citations and its time–weighted indexes. The obtained results are summarized in Table 4.

Table 4: Top five of the most influential journals

Index Q		Index Q_K		# papers	
Fuzzy Sets Syst	13262	Fuzzy Sets Syst	3669	Fuzzy Sets Syst	116
Eur J Oper Res	3108	Expert Sys Appl	1218	Eur J Oper Res	35
Expert Sys Appl	2177	Eur J Oper Res	947	Inf Sci	24
Inf Sci	2045	Appl. Math. Model.	864	Appl. Math. Model.	22
Appl. Math. Model.	1446	Inf Sci	797	Expert Sys Appl	21
Ln index ρ		Ln index ρ_K		# papers in K years	
Fuzzy Sets Syst	42557,9	Fuzzy Sets Syst	11422,7	Inf Sci	4
Eur J Oper Res	8606,0	Expert Sys Appl	2562,3	J. Clean. Prod.	2
Inf Sci	5071,8	Eur J Oper Res	2498,0	Int J Prod Econ	2
Expert Sys Appl	4742,5	Appl. Math. Model.	1751,8	Int J Computer Integr Manuf	2
Appl. Math. Model.	3043,3	Inf Sci	1703,9	Appl. Math. Model.	1

As expected, high impact journals are the preferred source of information among researchers. Fuzzy Sets and Systems, European Journal of Operations Research, Information Sciences and Expert Systems with Applications journals have been classical references in optimization for many years, so they are for FLPs as well. An interesting journal is Fuzzy Optimization and Decision Making journal which is in the race of the top-5 (probably because it is one of the youngest high impact journals). Such journals with a wider scope like the International Journal of General Systems keeps its influence due to the citations of seminal works like Zimmermann [199] and Applied Mathematical Modelling journal due to the highly cited works of Kumar et al. [90] and Kundu et al. [93].

Among books that include fuzzy optimization topics we recommend the classical books of Lai and Hwang [94], Kacprzyk and Orlovski [77], Klir and Yuan [84] & Zimmermann [203] who provide important both theoretical foundations and some examples of the Zimmermann method. Recommended up-to-date books focused to fuzzy mathematical programming include Lodwick [108] who present recent advances in fuzzy optimization models/methods and its applications, Wang and Watada [186] who present recent concepts/advances in fuzzy stochastic optimization models and Lodwick and Thipwiwatpotjana [110] who present comprehensive and updated concepts of generalized optimization under uncertainty including flexible, interval and fuzzy linear/nonlinear models and methods. Other recommended books include Bector and Chandra [16] who focused to fuzzy game theory models/methods and Kumar and Dash [89] who focused to marketing models under fuzzy uncertainty.

6. Trends, perspectives and new directions

A considerable amount of publications about FLPs has been published from 2018 to 2021 (> 200) so few very recent papers which are out-of-the citation counting (> 20) can lead new trends in the near future. Lexicographic methods for FLPs have been addressed by Pérez-Cañedo et al. [140]; Osuna-Gómez et al. [136] have extended optimality conditions to fuzzy constrained optimization problems; the theory of non–dominance in FLPs was extended by Arana-Jiménez and Sánchez-Gil [10]; Pérez-Cañedo et al. [139] proposed an epsilon method for solving fully multiobjective FLPs and Arana-Jiménez [9] introduces concepts of fuzzy Pareto solutions for fully multiobjective FLPs; Cao et al. [25] solved fuzzy relational linear programming problems and Wu [189] proposed a method for LP with fuzzy decision variables; Krapež et al. [85] used fuzzy quasigroups theory to solve linear equations; Kasperski and Zielinski [80] defined some robustness measures for possibilistic optimization problems; Ranjbar and Effati [146] addressed hesitant/interval FLPs; Pythagorean FLPs were proposed by Chen [32] & Sarkar and Biswas [153], Spherical set–based FLPs were reported by Ahmad and Adhami [2] & Ahmad and Adhami [3], Neutrosophic set–based LP models/methods were reported by Khatter [82] and Bipolar FLPs have been proposed by Mehmood et al. [117]. Another interesting topic that deserves more attention is the problem of solving fuzzy mathematical programming models with fuzzy variables which is a complex nonlinear problem (see Baykasoğlu and Gocken [15] who used particle swarm optimization techniques to solve it, Hashemi et al. [63] and Tanaka et al. [174]).

Some future interesting topics in fuzzy optimization include IFSs, Type-2 fuzzy sets and combinatorial integer/discrete FLPs. Recommended papers on those emergent topics were written by Fathy [42] & Fathy and Hassanien [43] who propose new solution methods for FLPs with IFSs, Herrera and Verdegay [64] who focus to integer FLPs which are very popular in logistics and management, Niksirat [132] addressed a hub location problem with IFSs, Cakır et al. [22] proposed a general model and solution for fuzzy multi–shift vehicle routing problems, Nucci [133] addressed fuzzy vehicle routing problems using chance–constrained models and Niksirat [132] solved vehicle routing problems with fuzzy multi-objective milk-run optimization under time window constraints; Type-2 FLPs have been addressed by Qin et al. [141], Figueroa-García [49], Dinagar and Anbalagan [36] and El-Alaoui [40] while LPs with Interval Type-2 fuzzy constraints have been addressed by Figueroa-García [45] and Figueroa-García [47], Figueroa-García [49], Figueroa-García et al. [53] and Figueroa-García and Hernández [50].

Other topics with an interesting potential to become influential in the future include uncertain programming proposed by Liu [103] and Liu [104] who extended the λ satisfaction degree methodology to the uncertainty theory proposed by Liu [105], and the flexible and generalized uncertainty optimization proposed by Lodwick and Salles-Neto [109] which links fuzzy to possibilistic optimization into a common approach. Although there is a bridge between fuzzy models and other methodologies such as stochastic and interval models/methods, there is still a need for developing a generalized theory/model/method for solving uncertain LPs under any kind of uncertainty source.

7. Concluding Remarks

Mathematical programming and LP models/methods under uncertainty have become popular due its flexibility and ability to deal with different uncertainty sources. More specifically LPs under fuzzy uncertainty have become popular in theory and applications since they involved human language and perceptions into optimization problems via fuzzy sets which are defined by experts. However, so many of the published works are oriented to compute an instance of an FLP via centroid, Yager index, median, etc. before using well known crisp or random–based models (quadratic, chance–constrained, random programming models, etc.) and just a small amount of works are dedicated either to study crisp or fuzzy theoretical optimization ($\approx 11\%$). Given the classifications provided in Sections 3 and 4 there is a need for strengthen the relationship among the fuzzy extension principle, α–cuts, λ satisfaction degree and interval methods to efficiently compute the fuzzy set of optimal solutions $\tilde{z}$ and extend the existent models/methods to other uncertainty representations like IFSs, Type–2, Pythagorean, spherical, bipolar, neutrosophic sets among others.

A comprehensive review of the most important publications in FLPs was provided and time–based indexes to evaluate the influence of papers based in a group of 472 publications with > 20 citations retrieved from Scopus database using the keywords `fuzzy (and) linear (and) programming` from 1975 to present have been presented. The proposed indexes (see Definitions 5 & 6 and Tables 2 & 3) are a reference point to see the influence of a paper in different ways and it can be used/extended to other factors different than the amount of citations. Diverse topics/fields and disciplines can be evaluated at readers will without restrictions.

The most influential journals for FLPs (see Table 4) were almost founded at the same time the original FLP models/methods were proposed i.e. *Fuzzy Sets and Systems* (Est. 1978), the *European Journal of Operational Research* (Est. 1977), *Information Sciences* (Est. 1968) and *Applied Mathematical Modelling* (Est. 1976). Other recent journals like *Expert Systems with Applications* (Est. 1990) and *Fuzzy Optimization and Decision Making* (Est. 2002) have also consolidated as influential sources of information for authors and practitioners.

Finally, the possibility of defining unified criteria for solving LP problems under different uncertainty sources should be a topic to be addressed in the near future since historically many authors have opted for solving FLPs using chance–constrained, stochastic, random and interval programming as two–step methods, so the development of generalized uncertainty theory has opened a door for integrating different uncertainty sources in optimization.

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Highlights

- The timeline of the most important and influential Fuzzy + Linear + Programming (FLP) papers show that many works are focused on solving defuzzified instances of a FLP
- There is a need for unified criteria for solving and modeling FLPs
- Time-weighted influence of a paper is kept in the most influential papers
- Self-citations tend to decrease over time
- Perspectives and trends of fuzzy linear programming and fuzzy optimization are to explore new fuzzy and uncertainty representations

Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: