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Multi-criteria decision making of water resource management problem (in Agriculture field, Purulia district) based on possibility measures under generalized single valued non-linear bipolar neutrosophic environment

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ABSTRACT

Purulia district is a drought-prone area in West Bengal, India. Naturally, the farmers of the Purulia district could not plant the crops due to the absence of water resources. Water resource management (WRM) has the potential mechanism to address the available water resources. This paper developed a multi-criterion water resource management technique for defining the available water resources in the agriculture field. We have introduced a novel ranking method of generalized single-valued non-linear bipolar neutrosophic numbers based on possibility measures. Additionally, we have first defined the possibilistic mean of generalized single-valued non-linear bipolar neutrosophic number. Using the possibilistic mean, we have invented the positive and negative rank expositors (S_{μ^+}, S_{μ^-}) for authenticity, hesitate, and falsity membership function. In the present situation, water resource management in agriculture has become a big problem worldwide. Multi-criteria decision-making technique is necessary for developing this situation. Using the proposed multi-criteria decision-making method, we have solved a real water resource management problem in the Purulia district under GSVnTbN-environment. In this decision-making problem, we have considered the GSVnTbN-environment. Because during the summertime, water demand is very high, and at the end of summer, when monsoon season comes, water demand is deficient in the agriculture field of Purulia district. Sometimes the monsoon season, rainwater does not occur entirely. At that time, monsoon crop planting was tough in the agriculture field. So, this district's water scarcity nature is a non-linear and uncertain type of nature. Therefore, we proposed an MCDM method for water resource management problems under GSVnTbN-environment.

1. Introduction

Purulia district is a drought-prone area in West Bengal, India. Purulia district has only a minimal number of rivers. During the summertime, the different seasonal water substances dry up, and also, the river flow is not helpful (Das et al., 2019) this time. In this present situation, water is inevitable, and it is very uncertain about make a proper crop in the agriculture field of this district. Due to the absence of water, the farmers of the Purulia district are unable to produce crops. Many agricultural areas of the Purulia district are destroyed due to the unavailable water. The crop pattern was also damaged, and the farmers of this district were unemployed during this summertime. The rural area people of this district are very conscious of water conservation. They have followed various strategies and techniques to cope with the water crisis. But due to the proper management, they cannot handle this situation. The management skills are necessary for the available water resources. Decision-making technique is prescribed for making favourable decisions on water resources management. Multi-criterion decision making (MCDM) system is a workable tool that provides the proper guidance to find the best water resources technique (alternatives).

Nowadays, decision-making technique is challenging employ tools in water resource management problem. Many researchers are working on decision-making (DM) methods to cope with the water resource management problem. A choice-based multi-criteria decision-making technique for water resource management problems was introduced by Tecle (1988). Stanescu and Filip (2010) studied an environmental multi-attribute decision making (MADM) to access the water resource in arid regions of Romania. Mimi and Smith (1999) proposed a criteria evaluation technique for water resource management problems. Yilmaz and Harmancioglu (2010) presented a WRM model for Gediz

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River Basin, Turkey; then, it was solved by the MCDM technique. The fuzzy MADM technique on water resource projects in Zayanderud Basin, Iran, was developed by Zarghaami et al. (2009). Bender and Simonovic (2000) presented a fuzzy compromise approach to the water resource system with uncertainty. Hajkowicz and Collins (2007) successfully implemented the multi-criteria analysis (MCA) technique on the WRM problem. Bauri et al. (2020) studied water conservation and management in Purulia district, West Bengal. An MCA mechanism for evaluating the WRM strategies was developed by Alamanos et al. (2018). Alamanos et al. (2017) formulated an integrated WRM problem of Lake Karla Basin, Greece using linear programming.

Uncertainty is a prevalent factor in every real problem. To overcome this situation, the first time Zadeh (1965) introduced the concept of fuzzy sets (FSs), which are measured by the degree of membership function, and the membership function takes the value within [0, 1]. Smarandache (1998) proposed the concept of neutrosophic sets, which is a proper extension of FSs and is measured by the three membership functions, which are truth, indeterminacy, and falsity. Wang et al. (2010) define the concept of single-valued neutrosophic sets. All kinds of membership functions in the FSs take the value within [0, 1]. But in the bipolar fuzzy sets, the membership function contains both positive and negative regions in [0, 1] and [−1, 0]. Zhang (1994) initiated the concept of bipolar fuzzy sets. The comparison analysis of bipolar LR fuzzy numbers was proposed by Ghanbari et al. (2018). Sarwar and Akram (2017a, 2017b) discussed the application of bipolar fuzzy sets and applied them to decision-making problems. After that, Deli et al. (2015) introduced the bipolar neutrosophic sets, which are an extension of bipolar fuzzy sets. The disjunctive representation of bipolar neutrosophic number was presented by Chakraborty et al. (2019).

The possibility is one uncertainty theory in FSs, which was proposed by Zadeh (1978). Its theoretical concept and actual application are significant in FSs theory. The data information in many real decision-making problems is imprecise in nature due to time pressure. Therefore, Dubois and Prade (1987) defined the fuzzy expectation of interval fuzzy numbers using the possibility measure. Carlsson and Fuller (2001) proposed the possibility mean and variance of a fuzzy number. Fuller and Majlender (2003) defined the concept of the weighted upper possibilistic mean and lower possibilistic mean. Firstly, Wan et al. (2013a) proposed the possibility mean and variance of triangular intuitionistic fuzzy numbers. Garai et al. (2018) presented the notion of possibility mean, variance, and covariance of a generalized intuitionistic fuzzy number. A ranking method of neutrosophic numbers based on the possibility mean was proposed by Garai et al. (2020b). Recently, Garai et al. (2020a) defined the possibilistic mean, variance, and covariance of generalized neutrosophic numbers. Wan et al. (2013b) investigated multiple attribute group decision-making (MAGDM) problems with triangular intuitionistic fuzzy numbers (TrIFN). The possibility variance of TrIFN and its application to MADM was proposed by Wan (2013). Wan and Dong (2014) developed a new ranking method of TIFNs and application to MAGDM. Wan and Zhu (2016) developed some Bonferroni harmonic operators of TIFN and applied them to the MAGDM problem under TIFNs. The weighted possibility means of TIFNs are defined by Dong and Wan (2016a). Recently many researchers worked in this area like as: Wan et al. (2015), Wan and Yi (2016), Wan et al. (2016), Dong and Wan (2016b), Dong et al. (2016), Wan et al. (2017), Wang et al. (2021b, 2021a), Tian and Peng (2020), Tian et al. (2019).

MCDM system is a procedure to choose the best alternative from a set of alternatives. MCDM systems are discriminated in terms of multiple quarrelling criteria. A heterogeneous MADM emergency model via the optimized relative entropy method was proposed by Chen et al. (2021). Zavadskas et al. (2015) developed a multi-criteria selection procedure of deep-water port in the Eastern Baltic Sea. Mohtashami (2021) presented a novel MCDM method considering fuzzy pairwise comparisons. An MCDM technique based on Bonferroni mean under interval type-2 fuzzy number was developed by Chiao (2021). Fan et al. (2019) considered a multi-criteria group decision-making problem with intuitionistic fuzzy rough numbers. Chen et al. (2020) solved an MCDM problem using the Choquet integral-based fuzzy logic approach under an uncertain environment. Garg and Nancy (2018) developed an MCDM method based on a prioritized Muirhead mean aggregation operator with a neutrosophic set number. Using the distance operator and bipolar operator, Riaz et al. (2021) have proposed an MCDM method. Agheli et al. (2021) defined a similarity measure of the Pythagorean fuzzy number and applied it to the MCDM problem. Golfaml et al. (2019) developed an MCDM method based on VIKOR and FOWA and applied in the climate-change adaptation of water supply management. Alghamdi et al. (2018) developed an MCDM method in a bipolar fuzzy environment. Dey et al. (2016) solved a MADM problem with a bipolar neutrosophic environment using the TOPSIS technique. In Table 1, we have presented some differences between our method and other existing methods.

However, there is no one investigating the multi-criteria decision-making method for water resource management problems under a neutrosophic environment. The main focus of this paper is to develop a multi-criteria decision-making method for water resource management problems, and it is solved under a generalized single-valued non-linear bipolar neutrosophic environment. We have introduced the rank expositor of the generalized non-linear bipolar neutrosophic numbers. We also demonstrated the concept of possibilistic mean of generalized single-valued non-linear bipolar neutrosophic numbers. For the first time, we developed a multi-criteria decision-making method for water resource management problems using the possibility measures. Utilizing this novel technique, we solved one real water resource management problem of the Purulia district in the agriculture field. Our main aim is to find the best water resource technique according to three important criteria. A sensitivity analysis was presented concerning the significant parameter μ of rank expositor (S_{μ^+}, S_{μ^-}).

1.1. Motivation of the paper

Purulia district is a drought-prone area in West Bengal, India. The farmers of the Purulia district are unable to propagate crops in the non-appearance of water. Water is essential for the planting of good crops in the agriculture field. Farming is the main occupation for the rural area peoples of the Purulia district. Due to the water scarcity, the rural area peoples reduce the job opportunities. There are four main issues in the unable of water resources, which are (i) the available water is low, but demands are very high, (ii) the huge amount of water withdrawal from subsurface and surface area, (iii) infrastructure issues as per demand is increasing, (iv) unable to use proper water conservation techniques. However, the farmer of this district has followed different strategies and techniques to cover up the water crisis. But the people cannot handle this situation. There is one crucial question that arises in the researcher's mind. How can we control the water crisis situation in the agriculture field?

Some real decision-making problem depends on positive and negative human thought. Also, the nature of the decision-making problem is not linear type, i.e., non-linear type. For example, water demand is very high during the summertime for agriculture purposes. At the end of summer, when monsoon season comes, water demand is deficient in the rural area of the Purulia district. Sometimes the monsoon season, rainwater does not occur entirely. At that time, monsoon crop planting was very difficult in agriculture. So, this district's water scarcity nature is a non-linear and uncertain type of nature. Management technique is necessary for the available water resource for this district. Now, if the researchers can apply the decision-making technique to the water resource management problem, then some question arises in the researcher's mind. How can we find decision criteria for water resource management problems? Which multi-criteria decision-making is suitable? Which environment is appropriate for this problem? Suppose this problem is solved under a bipolar neutrosophic environment, then what will be the benefit get? Linear type of bipolar neutrosophic

Table 1
Different between our and other studies.

Author's	Methods	Environment	Application
Wan (2013)	Possibility variance	TIFN	MADM
Wan and Dong (2014)	Possibility method	TIFN	MAGDM
Dong and Wan (2016a)	Possibility mean	TIFN	MAGDM
Dong and Wan (2016b)	possibility mean and standard deviation	TIFN	MAGDM
Dong et al. (2016)	possibility attitudinal expected	Atanassov's(TIFN)	MAGDM
Wan and Zhu (2016)	Bonferroni harmonic mean	TIFN	MAGDM
Deli et al. (2015)	Score, certainty and accuracy functions	Bipolar sets	MCDM
Deli and Subas (2017)	Values and ambiguities	SVN-number	MADM
Chakraborty et al. (2019)	De-Bipolarization	Bipolar sets	MCDM
Our method	Ranking expositors	GSVnTbN	WRM

environment is this suitable for WRM problem? The information of some real decision-making problems is not linear type neutrosophic. Therefore, a non-linear type environment is needed for this problem. To find out the answer to these questions. Here, we have proposed our research work of MCDM on water resource management with a non-linear bipolar neutrosophic environment.

1.2. Novelty of the paper

Previously, some research works had been published in the area of decision-making analysis of water resource management problems. Most of them considered decision-making for water resource management problems in a crisp environment. However, many interesting decision-making techniques for water resource management problem is still unknown. Our aim tried to develop these strange results, which are described as follows:

(i) Based on possibility measures, we have developed a Multi-criteria decision-making technique for water resource management problems.
(ii) The possibilistic mean of generalized single-valued non-linear bipolar neutrosophic number has been invented here.
(iii) We have introduced the positive and negative rank expositor (S_{μ^+}, S_{μ^-}) of generalized single-valued non-linear bipolar neutrosophic (GSVnbN) number.
(iv) We have defined the water resource management problem of the agriculture field in the Purulia district.
(v) Water resource management problem solving under the generalized non-linear bipolar neutrosophic environment.
(vi) Sensitivity analysis is performed according to parameter μ and discusses the ranking order for different values of μ.

1.3. Structure of the paper

The rest of the paper is organized as follows: Section 2 introduces the concept of generalized single-valued non-linear bipolar neutrosophic numbers and some basic definitions. In Section 3, we define the possibilistic mean of generalized single-valued non-linear bipolar neutrosophic number and one novel lexicographic ranking method. Multi-criteria decision-making with a generalized non-linear bipolar neutrosophic environment has been discussed in Section 4. In Section 5, we proposed one real water resource management problem and solved it under a generalized non-linear bipolar neutrosophic environment. In Section 6, we summarize the sensitivity analysis with respect to the significant parameter μ. Finally, the conclusion and chance for future work plans consider in Section 7.

2. Basic preliminaries

Definition 2.1. A neutrosophic set (NS) $\tilde{\mathcal{A}}$ over the universal set X is denoted by $\tilde{\mathcal{A}} = \{\langle x, (T_{\mathcal{A}}(x), I_{\tilde{\mathcal{A}}}(x), F_{\tilde{\mathcal{A}}}(x))\rangle : x \in X\}$, where $T_{\tilde{\mathcal{A}}}$ is the truth membership function, $I_{\tilde{\mathcal{A}}}$ is the indeterminacy membership (Garai et al., 2020a) function and $F_{\tilde{\mathcal{A}}}$ is the falsity membership function of the NS $\tilde{\mathcal{A}}$ over the universal set X. The range of the all functions are respectively follows as:

$$T_{\tilde{\mathcal{A}}} : X \to]0^-, 1^+[, \quad I_{\tilde{\mathcal{A}}} : X \to]0^-, 1^+[, \quad F_{\tilde{\mathcal{A}}} : X \to]0^-, 1^+[$$

such that $0^- \le T_{\tilde{\mathcal{A}}}(x) + I_{\tilde{\mathcal{A}}}(x) + F_{\tilde{\mathcal{A}}}(x) \le 3^+$.

Definition 2.2. Let X be a universal set. A single valued neutroshopic (SVN) set $\tilde{\mathcal{A}}^n$ over (Garai et al., 2020a) the universal set X is a neutroshopic set over the set X. But $T_{\tilde{\mathcal{A}}^n}(x)$, $I_{\tilde{\mathcal{A}}^n}(x)$ and $F_{\tilde{\mathcal{A}}^n}(x)$ are defined as follow

$$T_{\tilde{\mathcal{A}}^n} : X \to [0,1], \quad I_{\tilde{\mathcal{A}}^n} : X \to [0,1], \quad F_{\tilde{\mathcal{A}}^n} : X \to [0,1]$$

such that $0 \le T_{\tilde{\mathcal{A}}^n}(x) + I_{\tilde{\mathcal{A}}^n}(x) + F_{\tilde{\mathcal{A}}^n}(x) \le 3$.

Definition 2.3. A generalized single valued (Garai et al., 2020a) neutroshopic (SVN) number is a special case of NS on $\mathbb{R}$, which is defined as $\tilde{a}_N = \langle (a_1, a_2, a_3, a_4; w_{\tilde{a}_N}), (b_1, b_2, b_3, b_4; u_{\tilde{b}_N}), (c_1, c_2, c_3, c_4; y_{\tilde{c}_N})\rangle$ where $w_{\tilde{a}_N}, u_{\tilde{b}_N}, y_{\tilde{c}_N} \in [0,1]$ be any three real numbers. The truth membership $T_{\tilde{a}_N} : \mathbb{R} \to [0,1]$, indeterminacy membership, $I_{\tilde{a}_N} : \mathbb{R} \to [0,1]$ and falsity membership $F_{\tilde{a}_N} : \mathbb{R} \to [0,1]$ functions are defined as:

$$T_{\tilde{a}_N}(x) = \begin{cases} T_N^l(x) w_{\tilde{a}_N}, & \text{if } a_1 \le x < a_2 \\ w_{\tilde{a}_N}, & \text{if } a_2 \le x \le a_3 \\ T_n^r(x) w_{\tilde{a}_N}, & \text{if } a_3 < x \le a_4 \\ 0 & \text{otherwise} \end{cases}$$

$$I_{\tilde{a}_N}(x) = \begin{cases} I_N^l(x) w_{\tilde{b}_N}, & \text{if } b_1 \le x < b_2 \\ u_{\tilde{b}_N}, & \text{if } b_2 \le x \le b_3 \\ I_N^r(x) w_{\tilde{b}_N}, & \text{if } b_3 < x \le b_4 \\ 1 & \text{otherwise} \end{cases}$$

$$F_{\tilde{a}_N}(x) = \begin{cases} F_n^l(x) y_{\tilde{c}_N}, & \text{if } c_1 \le x < c_2 \\ y_{\tilde{c}_N}, & \text{if } c_2 \le x \le c_3 \\ F_N^r(x) y_{\tilde{c}_N}, & \text{if } c_3 < x \le c_4 \\ 0 & \text{otherwise} \end{cases}$$

Definition 2.4. Let X be a universal set. A bipolar (Garai et al., 2020a) neutrosophic set $\tilde{A}_{bN}$ is a extension of neutrosophic set on $\mathbb{R}$, which is defined as $\tilde{A}_{bN} = \langle (x; T^+_{\tilde{a}_{bN}}(x), T^-_{\tilde{a}_{bN}}(x), I^+_{\tilde{a}_{bN}}(x), I^-_{\tilde{a}_{bN}}(x), F^+_{\tilde{a}_{bN}}(x), F^-_{\tilde{a}_{bN}}(x)) : x \in X\rangle$, where $T^+_{\tilde{a}_{bN}} : \mathbb{R} \to [0,1], T^-_{\tilde{a}_{bN}} : \mathbb{R} \to [-1,0]$ represents the degree of confidence, $I^+_{\tilde{a}_{bN}} : \mathbb{R} \to [0,1], I^-_{\tilde{a}_{bN}} : \mathbb{R} \to [-1,0]$ represents the degree of hesitation and $F^+_{\tilde{a}_{bN}} : \mathbb{R} \to [0,1], F^-_{\tilde{a}_{bN}} : \mathbb{R} \to [-1,0]$ represents the degree of falseness of the decision.

2.5. Generalized single valued non-linear triangular bipolar neutrosophic number

Definition 2.6. A generalized single valued non-linear triangular bipolar neutrosophic (GSVnTbN) number $\tilde{a}_{bN} = \langle (a_1, a_2, a_3; w_{\tilde{a}_{bN}}, p_1, p_2)$ $(b_1, b_2, b_3; u_{\tilde{b}_{bn}}, q_1, q_2)(c_1, c_2, c_3; y_{\tilde{c}_{bn}}, r_1, r_2)\rangle$ is a special kind of neutrosophic set on $\mathbb{R}$, whose positive and negative parts of authenticity,

Table 2
Difference between GSVnTbN and other existing fuzzy numbers.

Different Fuzzy Number	Satisfaction grade (Membership degree)	Abstinence grade (Neutral Membership degree)	Dissatisfaction grade (Non-Membership degree)	Bipolarity	Nonlinearity
Triangular Fuzzy Number	Applicable	Not Applicable	Not Applicable	Not Applicable	Not Applicable
Triangular Intuitionistic Fuzzy Number	Applicable	Not Applicable	Applicable	Not Applicable	Not Applicable
Single valued Triangular Neutrosophic Number	Applicable	Applicable	Applicable	Not Applicable	Not Applicable
Generalized Single Valued Nonlinear Triangular Bipolar Neutrosophic Number (GSVnTbN)	Applicable	Applicable	Applicable	Applicable	Applicable

hesitation and falsity membership functions (cf. Fig. 1) are defined as follows:

$$T^{+}_{\tilde{a}_{bN}}(x)=\begin{cases}\left(\dfrac{x-a_1}{a_2-a_1}\right)^{p_1} w_{\tilde{a}_{bN}}, & \text{if } a_1\le x<a_2\\ w_{\tilde{a}_{bN}}, & \text{if } x=a_2\\ \left(\dfrac{a_3-x}{a_3-a_2}\right)^{p_1} w_{\tilde{a}_{bN}}, & \text{if } a_2<x\le a_3\\ 0 & \text{otherwise}\end{cases}$$

$$T^{-}_{\tilde{a}_{bN}}(x)=\begin{cases}\left(\dfrac{a_2-x}{a_2-a_1}\right)^{p_2} w_{\tilde{a}_{bN}}, & \text{if } a_1\le x<a_2\\ -w_{\tilde{a}_{bN}}, & \text{if } x=a_2\\ \left(\dfrac{x-a_3}{a_3-a_2}\right)^{p_2} w_{\tilde{a}_{bN}}, & \text{if } a_2<x\le a_3\\ 0 & \text{otherwise}\end{cases}$$

$$I^{+}_{\tilde{a}_{bN}}(x)=\begin{cases}\left(\dfrac{b_2-x}{b_2-b_1}\right)^{q_1} u_{\tilde{b}_{bN}}, & \text{if } b_1\le x<b_2\\ u_{\tilde{b}_{bN}}, & \text{if } x=b_2\\ \left(\dfrac{x-b_3}{b_3-b_2}\right)^{q_1} u_{\tilde{b}_{bN}}, & \text{if } b_2<x\le b_3\\ 1 & \text{otherwise}\end{cases}$$

$$I^{-}_{\tilde{a}_{bN}}(x)=\begin{cases}\left(\dfrac{x-b_2}{b_2-b_1}\right)^{q_2} u_{\tilde{b}_{bN}}, & \text{if } b_1\le x<b_2\\ -u_{\tilde{b}_{bN}}, & \text{if } x=b_2\\ \left(\dfrac{b_3-x}{b_3-b_2}\right)^{q_2} u_{\tilde{b}_{bN}}, & \text{if } b_2<x\le b_3\\ -1 & \text{otherwise}\end{cases}$$

$$F^{+}_{\tilde{a}_{bN}}(x)=\begin{cases}\left(\dfrac{c_2-x}{c_2-c_1}\right)^{r_1} y_{\tilde{c}_{bN}}, & \text{if } c_1\le x<c_2\\ y_{\tilde{c}_{bN}}, & \text{if } x=c_2\\ \left(\dfrac{x-c_3}{c_3-c_2}\right)^{r_1} y_{\tilde{c}_{bN}}, & \text{if } c_2<x\le c_3\\ 1 & \text{otherwise}\end{cases}$$

$$F^{-}_{\tilde{a}_{bN}}(x)=\begin{cases}\left(\dfrac{x-c_2}{c_2-c_1}\right)^{r_2} y_{\tilde{c}_{bN}}, & \text{if } c_1\le x<c_2\\ -y_{\tilde{c}_{bN}}, & \text{if } x=c_2\\ \left(\dfrac{c_3-x}{c_3-c_2}\right)^{r_2} y_{\tilde{c}_{bN}}, & \text{if } c_2<x\le c_3\\ -1 & \text{otherwise}\end{cases}$$

where $-1\le T_{\tilde{a}_{bN}}(x)+I_{\tilde{a}_{bN}}(x)+F_{\tilde{a}_{bN}}(x)\le 1$, for all $x\in\tilde{A}_{bN}$. The difference between GSVnTbN and other existing fuzzy numbers are presented in Table 2.

Definition 2.7. Let $\tilde{a}_{bN}$ be a generalized single valued non-linear bipolar neutrosophic (GSVnbN) number. Then the positive and negative $\langle\alpha,\beta,\gamma\rangle$-cut sets of the GSVnbN number $\tilde{a}_{bN}$, denoted by $(\tilde{a}_{bN})^{+}_{\alpha,\beta,\gamma}$ and $(\tilde{a}_{bN})^{-}_{\alpha,\beta,\gamma}$ which is defined as

$$(\tilde{a}_{bN})^{+}_{\alpha,\beta,\gamma}=\{x: T^{+}_{\tilde{a}}\ge\alpha, I^{+}_{\tilde{a}}\le\beta, F^{+}_{\tilde{a}}\le\gamma, x\in\mathbb{R}\}$$

$$(\tilde{a}_{bN})^{-}_{\alpha,\beta,\gamma}=\{x: T^{-}_{\tilde{a}}\le-\alpha, I^{-}_{\tilde{a}}\ge-\beta, F^{-}_{\tilde{a}}\ge-\gamma, x\in\mathbb{R}\}$$

where, α,β,γ lies in the regions $-1\le\alpha\le w_{\tilde{a}_{bN}}$, $-u_{\tilde{b}_{bn}}\le\beta\le 1$, $-y_{\tilde{c}_{bN}}\le\gamma\le 1$ and $-1\le\alpha+\beta+\gamma\le 1$.

Example 2.1. Let $\tilde{a}_{bN}=\langle(a_1,a_2,a_3;w_{\tilde{a}_{bN}},p_1,p_2)(b_1,b_2,b_3;u_{\tilde{b}_{bn}},q_1,q_2)(c_1,c_2,c_3;y_{\tilde{c}_{bn}},r_1,r_2)\rangle$ be a GSVnTbN number. From the above Definition 2.7 the positive and negative α-cut, β-cut and γ-cut sets of GSVnTbN number $\tilde{a}_{bN}$ formulated as:

$$\begin{aligned}(\tilde{a}_{bN})^{+}_{\alpha}&=[(a_{bN})^{l+}_{\alpha},(a_{bN})^{r+}_{\alpha}]\\&=\left[a_1+\left(\frac{\alpha}{w_{\tilde{a}_{bN}}}\right)^{\frac{1}{p_1}}(a_2-a_1),\ a_3-\left(\frac{\alpha}{w_{\tilde{a}_{bN}}}\right)^{\frac{1}{p_1}}(a_3-a_2)\right]\end{aligned}\tag{1}$$

$$\begin{aligned}(\tilde{a}_{bN})^{-}_{\alpha}&=[(a_{bN})^{l-}_{\alpha},(a_{bN})^{r-}_{\alpha}]\\&=\left[a_2-\left(\frac{-\alpha}{w_{\tilde{a}_{bN}}}\right)^{\frac{1}{p_2}}(a_2-a_1),\ a_3+\left(\frac{-\alpha}{w_{\tilde{a}_{bN}}}\right)^{\frac{1}{p_2}}(a_3-a_2)\right]\end{aligned}\tag{2}$$

$$\begin{aligned}(\tilde{a}_{bN})^{+}_{\beta}&=[(a_{bN})^{l+}_{\beta},(a_{bN})^{r+}_{\beta}]\\&=\left[b_2-\left(\frac{\beta}{u_{\tilde{b}_{bN}}}\right)^{\frac{1}{q_1}}(b_2-b_1),\ b_3+\left(\frac{\beta}{u_{\tilde{b}_{bN}}}\right)^{\frac{1}{q_1}}(b_3-b_2)\right]\end{aligned}\tag{3}$$

$$\begin{aligned}(\tilde{a}_{bN})^{-}_{\beta}&=[(a_{bN})^{l-}_{\beta},(a_{bN})^{r-}_{\beta}]\\&=\left[b_2+\left(\frac{-\beta}{u_{\tilde{b}_{bN}}}\right)^{\frac{1}{q_2}}(b_2-b_1),\ b_3-\left(\frac{-\beta}{u_{\tilde{b}_{bN}}}\right)^{\frac{1}{q_2}}(b_3-b_2)\right]\end{aligned}\tag{4}$$

$$\begin{aligned}(\tilde{a}_{bN})^{+}_{\gamma}&=[(a_{bN})^{l+}_{\gamma},(a_{bN})^{r+}_{\gamma}]\\&=\left[c_2-\left(\frac{\gamma}{y_{\tilde{c}_{bN}}}\right)^{\frac{1}{r_1}}(c_2-c_1),\ c_3+\left(\frac{\beta}{y_{\tilde{c}_{bN}}}\right)^{\frac{1}{r_1}}(c_3-c_2)\right]\end{aligned}\tag{5}$$

$$\begin{aligned}(\tilde{a}_{bN})^{-}_{\gamma}&=[(a_{bN})^{l-}_{\gamma},(a_{bN})^{r-}_{\gamma}]\\&=\left[c_2+\left(\frac{-\gamma}{y_{\tilde{c}_{bN}}}\right)^{\frac{1}{r_2}}(c_2-c_1),\ c_3-\left(\frac{-\gamma}{u_{\tilde{c}_{bN}}}\right)^{\frac{1}{r_2}}(c_3-c_2)\right]\end{aligned}\tag{6}$$

Definition 2.8. Let $\tilde{a}_{bN}=\langle(a_1,a_2,a_3;w_{\tilde{a}_{bN}},p_1,p_2)(b_1,b_2,b_3;u_{\tilde{b}_{bN}},q_1,q_2)(c_1,c_2,c_3;y_{\tilde{c}_{bN}},r_1,r_2)\rangle$ and $\tilde{a}'_{bN}=\langle(a'_1,a'_2,a'_3;w_{\tilde{a}'_{bN}},p'_1,p'_2)(b'_1,b'_2,b'_3;u_{\tilde{b}'_{bn}},q'_1,q'_2)(c'_1,c'_2,c'_3;y_{\tilde{c}'_{bN}},r'_1,r'_2)\rangle$ be two GSVnTbN numbers and $k\neq 0$ be any real number. Then for $p_1=p'_1$, $p_2=p'_2$, $q_1=q'_1$, $q_2=q'_2$, $r_1=r'_1$, $r_2=r'_2$ and $a_1<c_3$, $a'_1<c'_3$ we have

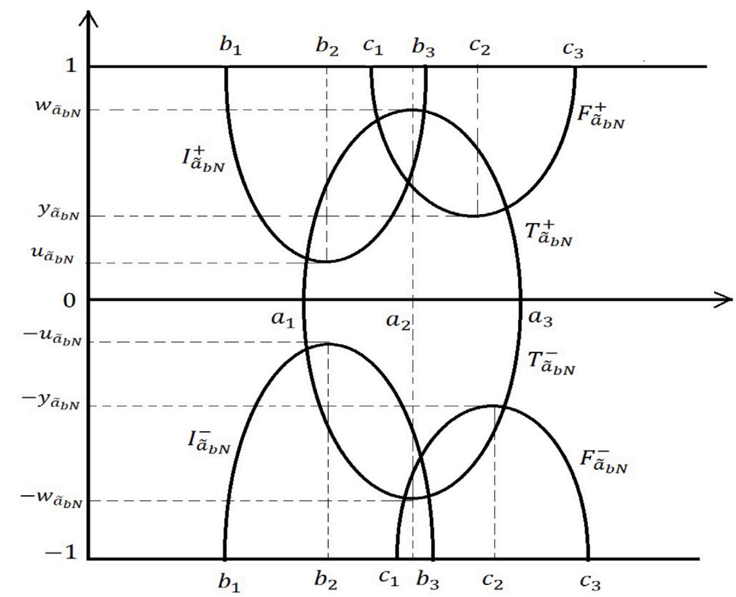

Fig. 1. Graphical representation of GSVnTbN-number.

(i) $\tilde{a}_{bN}+\tilde{a}'_{bN} = \langle (a_1+a'_1, a_2+a'_2, a_3+a'_3; w_{\tilde{a}'_{bN}} \wedge w_{\tilde{a}_{bN}}, p_1, p_2)(b_1+b'_1, b_2+b'_2, b_3+b'_3; u_{\tilde{b}_{bn}} \vee u_{\tilde{b}'_{bN}}, q_1, q_2)(c_1+c'_1, c_2+c'_2, c_3+c'_3; y_{\tilde{c}_{bn}} \vee y_{\tilde{c}'_{bN}}, r_1, r_2)\rangle$

(ii) $$\tilde{a}_{bN}\tilde{a}'_{bN} = \begin{cases} \langle (a_1a'_1, a_2a'_2, a_3a'_3; w_{\tilde{a}'_{bN}} \wedge w_{\tilde{a}_{bN}}, p_1, p_2) \\ (b_1b'_1, b_2b'_2, b_3b'_3; u_{\tilde{b}_{bN}} \vee u_{\tilde{b}'_{bN}}, q_1, q_2) \\ (c_1c'_1, c_2c'_2, c_3c'_3; y_{\tilde{c}_{bn}} \vee y_{\tilde{c}'_{bN}}, r_1, r_2)\rangle & for \quad c_3>0, c'_3>0 \\ \langle (a_1c'_3, a_2c'_2, a_3c'_1; w_{\tilde{a}_{bN}} \wedge w_{\tilde{a}'_{bN}}, p_1, p_2) \\ (b_1b'_3, b_2b'_2, b_3b'_1; u_{\tilde{b}_{bN}} \vee u_{\tilde{b}'_{bN}}, q_1, q_2) \\ (a_1c'_3, a_2c'_2, a_3c'_1; y_{\tilde{c}_{bN}} \vee y_{\tilde{c}'_{bN}}, r_1, r_2)\rangle & for \quad c_3>0, c'_3<0 \\ \langle (c_3c'_3, c_2c'_2, c_1c'_1; y_{\tilde{c}_{bN}} \wedge y_{\tilde{c}'_{bN}}, p_1, p_2) \\ (b_3b'_3, b_2b'_2, b_1b'_1; u_{\tilde{b}_{bN}} \vee u_{\tilde{b}'_{bN}}, q_1, q_2) \\ (a_3a'_3, a_2a'_2, a_1a'_1; w_{\tilde{a}_{bN}} \vee w_{\tilde{a}'_{bN}}, r_1, r_2)\rangle & for \quad c_3<0, c'_3<0 \end{cases}$$

Here we assume that $a_1 < c_3$, $a'_1 < c'_3$, so if we considered $c_3 > 0$, then $\tilde{a}_{bN} > 0$. Again, if we considered $c_3 < 0$, then $\tilde{a}_{bN} < 0$. Therefore the above results holds for different cases of c_3, c'_3.

(iii) $$k\tilde{a}_{bN} = \begin{cases} \langle (ka_1, ka_2, ka_3; w_{\tilde{a}_{bN}}, p_1, p_2) \\ (kb_1, kb_2, kb_3; u_{\tilde{b}_{bN}}, q_1, q_2)(kc_1, kc_2, kc_3; y_{\tilde{c}_{bN}}, r_1, r_2)\rangle & k>0 \\ \langle (ka_3, ka_2, ka_1; w_{\tilde{a}_{bN}}, p_1, p_2) \\ (kb_3, kb_2, kb_1; u_{\tilde{b}_{bN}}, q_1, q_2)(kc_3, kc_2, kc_1; y_{\tilde{c}_{bN}}, r_1, r_2)\rangle & k<0 \end{cases}$$

where $\vee$*=Max,* $\wedge$*=Min.*

3. Possibilistic mean of generalized single valued non-linear triangular bipolar neutrosophic number

Here, we have proposed the possibilistic mean generalized single valued non-linear triangular bipolar neutrosophic number. Using the possibilistic mean, we invented a new ranking method of GSVnTbN-numbers. In this conjunction, we have develop the some definition and ethos with GSVnTbN-numbers.

3.1. Possibilistic mean of GSVnTbN-numbers

Definition 3.2. Let $(\tilde{a}_{bN})^+_\alpha = [(a_{bN})^{l+}_\alpha, (a_{bN})^{r+}_\alpha]$ be the positive α-cut set of GSVnTbN-number $\tilde{a}_{bN}$ with $0 \le \alpha \le w_{\tilde{a}_{bN}}$ and $(\tilde{a}_{bN})^-_\alpha = [(a_{bN})^{l-}_\alpha, (a_{bN})^{r-}_\alpha]$ be the negative α-cut set of GSVnTbN-numbers $\tilde{a}_{bN}$ with $-w_{\tilde{a}_{bN}} \le \alpha \le 0$. The f-weighted lower possibilistic means ($\underline{M}_{T^+}$) and f-weighted upper possibilistic means ($\overline{M}_{T^+}$) for positive authenticity function of a GSVnTbN-number $\tilde{a}_{bN}$ are respectively defined as:

$$\underline{M}_{T^+}(\tilde{a}_{bN}) = \int_0^{w_{\tilde{a}_{bN}}} f(Pos[\tilde{a}_{bN} \le (a_{bN})^{l+}_\alpha])(a_{bN})^{l+}_\alpha d\alpha$$

$$= \int_0^{w_{\tilde{a}_{bN}}} f(\alpha)(a_{bN})^{l+}_\alpha d\alpha \tag{7}$$

$$\overline{M}_{T^+}(\tilde{a}_{bN}) = \int_0^{w_{\tilde{a}_{bN}}} f(Pos[\tilde{a}_{bN} \ge (a_{bN})^{r+}_\alpha])(a_{bN})^{r+}_\alpha d\alpha$$

$$= \int_0^{w_{\tilde{a}_{bN}}} f(\alpha)(a_{bN})^{r+}_\alpha d\alpha \tag{8}$$

where $f : [0, w_{\tilde{a}_{bN}}] \to \mathbb{R}$ is a monotonic increasing and non-negative weighted function on $[0, w_{\tilde{a}_{bN}}]$ and $f(\alpha) \in [0,1]$ for $\alpha \in [0, w_{\tilde{a}_{bN}}]$ and satisfies that $\int_0^{w_{\tilde{a}_{bN}}} f(\alpha)d\alpha = x \in [0,1]$.

$$Pos[\tilde{e}_{bN} \le e^{l+}_\alpha] = \sup_{x \le (a_{bN})^{l+}_\alpha} \{T^+_{\tilde{a}_{bN}}(x)\} = \alpha$$

$$Pos[\tilde{e}_{bN} \ge (a_{bN})^{r+}_\alpha] = \sup_{x \ge (a_{bN})^{r+}_\alpha} \{T^+_{\tilde{a}_{bN}}(x)\} = \alpha$$

The f-weighted lower possibilistic means ($\underline{M}_{T^+}$) and f-weighted upper possibilistic means ($\overline{M}_{T^+}$) for negative authenticity function of a GSVnTbN-number $\tilde{a}_{bN}$ are respectively defined as:

$$\underline{M}_{T^-}(\tilde{a}_{bN}) = \int_{-w_{\tilde{a}_{bN}}}^0 f(Pos[\tilde{a}_{bN} \ge (a_{bN})^{l-}_\alpha])(a_{bN})^{l-}_\alpha d\alpha$$

$$= \int_{-w_{\tilde{a}_{bN}}}^0 f(-\alpha)(a_{bN})^{l-}_\alpha d\alpha \tag{9}$$

$$\overline{M}_{T^-}(\tilde{a}_{bN}) = \int_{-w_{\tilde{a}_{bN}}}^0 f(Pos[\tilde{a}_{bN} \le (a_{bN})^{r-}_\alpha])(a_{bN})^{r-}_\alpha d\alpha$$

$$= \int_{-w_{\tilde{a}_{bN}}}^0 f(-\alpha)(a_{bN})^{r-}_\alpha d\alpha \tag{10}$$

where $f : [-w_{\tilde{a}_{bN}}, 0] \to \mathbb{R}$ is an monotonic increasing and non-positive weighted function on $[-w_{\tilde{a}_{bN}}, 0]$ and $f(\alpha) \in [-1,0]$ for $\alpha \in [-w_{\tilde{a}_{bN}}, 0]$ and satisfies that $\int_{-w_{\tilde{a}_{bN}}}^0 f(\alpha)d\alpha = x \in [-1,0]$.

$$Pos[\tilde{a}_{bN} \ge (a_{bN})^{l-}_\alpha] = \inf_{x \ge (a_{bN})^{l-}_\alpha} \{T^-_{\tilde{a}_{bN}}(x)\} = -\alpha$$

$$Pos[\tilde{a}_{bN} \le (a_{bN})^{r-}_\alpha] = \inf_{x \le (a_{bN})^{r+}_\alpha} \{T^-_{\tilde{a}_{bN}}(x)\} = -\alpha$$

Definition 3.3. Let $(\tilde{a}_{bN})^+_\beta = [(a_{bN})^{l+}_\beta, (a_{bN})^{r+}_\beta]$ be the positive β-cut set of GSVnTbN-number $\tilde{a}_{bN}$ with $u_{\tilde{a}_{bN}} \le \beta \le 1$ and $(\tilde{a}_{bN})^-_\beta = [(a_{bN})^{l-}_\beta, (a_{bN})^{r-}_\beta]$ be the negative β-cut set of GSVnTbN-number $\tilde{a}_{bN}$

with $-1 \leq \beta \leq -u_{\tilde{a}_{bN}}$. The g-weighted lower possibility means ($\underline{M}_{I^+}$) and g-weighted upper possibility means ($\overline{M}_{I^+}$) for hesitate function of a GSVnTbN-number $\tilde{a}_{bN}$ are respectively defined as:

$$\underline{M}_{I^+}(\tilde{a}_{bN}) = \int_{u_{\tilde{b}_{bN}}}^{1} g(Pos[\tilde{a}_{bN} \leq (a_{bN})_{\beta}^{l+}])(a_{bN})_{\beta}^{l+} d\beta = \int_{u_{\tilde{b}_{bN}}}^{1} g(\beta)(a_{bN})_{\beta}^{l+} d\beta \tag{11}$$

$$\overline{M}_{I^+}(\tilde{a}_{bN}) = \int_{u_{\tilde{b}_{bN}}}^{1} g(Pos[\tilde{a}_{bN} \geq (a_{bN})_{\beta}^{r+}])(a_{bN})_{\beta}^{r+} d\beta = \int_{u_{\tilde{b}_{bN}}}^{1} g(\beta)(a_{bN})_{\beta}^{r+} d\beta \tag{12}$$

where $g : [u_{\tilde{a}_{bN}}, 1] \to \mathbb{R}$ is an monotonic increasing and non-negative weighted function on $[u_{\tilde{a}_{bN}}, 1]$ and $g(\beta) \in [0,1]$ for $\beta \in [u_{\tilde{a}_{bN}}, 1]$ and satisfies that $\int_{u_{\tilde{a}_{bN}}}^{1} g(\beta)d\beta = x \in [0,1]$.

$$Pos[\tilde{a}_{bN} \leq (a_{bN})_{\beta}^{l+}] = \sup_{x \leq (a_{bN})_{\beta}^{l+}} \{I_{\tilde{a}_{bN}}^{+}(x)\} = \beta$$

$$Pos[\tilde{a}_{bN} \geq (a_{bN})_{\beta}^{r+}] = \sup_{x \geq (a_{bN})_{\beta}^{r+}} \{I_{\tilde{a}_{bN}}^{+}(x)\} = \beta$$

The g-weighted lower possibility means ($\underline{M}_{I_+}$) and g-weighted upper possibility means ($\overline{M}_{I_+}$) for hesitate function of a GSVnTbN-number $\tilde{a}_{bN}$ are respectively defined as:

$$\underline{M}_{I^-}(\tilde{b}_{bN}) = \int_{-1}^{-u_{\tilde{b}_{bN}}} g(Pos[\tilde{a}_{bN} \geq (a_{bN})_{\alpha}^{l-}])(a_{bN})_{\alpha}^{l-} d\beta = \int_{-1}^{-u_{\tilde{b}_{bN}}} g(-\beta)(a_{bN})_{\alpha}^{l-} d\beta \tag{13}$$

$$\overline{M}_{I^-}(\tilde{a}_{bN}) = \int_{-1}^{-u_{\tilde{b}_{bN}}} g(Pos[\tilde{a}_{bN} \leq (a_{bN})_{\beta}^{r-}])(a_{bN})_{\beta}^{r-} d\beta = \int_{-1}^{-u_{\tilde{b}_{bN}}} g(-\beta)(a_{bN})_{\beta}^{r-} d\beta \tag{14}$$

where $g : [-1, -u_{\tilde{b}_{bN}}] \to \mathbb{R}$ is an increasing and non-positive weighted function on $[-1, -u_{\tilde{a}_{bN}}]$ and $g(\beta) \in [-1, 0]$ for $\beta \in [-1, -u_{\tilde{a}_{bN}}]$ and satisfies that $\int_{-1}^{-u_{\tilde{a}_{bN}}} g(\beta)d\beta = x \in [-1, 0]$.

$$Pos[\tilde{a}_{bN} \geq (a_{bN})_{\beta}^{l-}] = \inf_{x \geq (a_{bN})_{\beta}^{l-}} \{I_{\tilde{a}_{bN}}^{-}(x)\} = -\beta$$

$$Pos[\tilde{a}_{bN} \leq (a_{bN})_{\beta}^{r-}] = \inf_{x \leq (a_{bN})_{\beta}^{r+}} \{I_{\tilde{a}_{bN}}^{-}(x)\} = -\beta$$

Definition 3.4. Let $(\tilde{a}_{bN})_{\gamma}^{+} = [(a_{bN})_{\gamma}^{l+}, (a_{bN})_{\gamma}^{r+}]$ be the positive γ-cut set of GSVnTbN-number $\tilde{a}_{bN}$ with $y_{\tilde{c}_{bN}} \leq \beta \leq 1$ and $(\tilde{a}_{bN})_{\gamma}^{-} = [(a_{bN})_{\gamma}^{l-}, (a_{bN})_{\gamma}^{r-}]$ be the negative γ-cut set of GSVnTbN-number $\tilde{a}_{bN}$ with $-1 \leq \gamma \leq -u_{\tilde{a}_{bN}}$. The h-weighted lower possibility means ($\underline{M}_{F^+}$) and h-weighted upper possibility means ($\overline{M}_{F^+}$) for hesitate function of a GSVnTbN-number $\tilde{a}_{bN}$ are respectively defined as:

$$\underline{M}_{F^+}(\tilde{a}_{bN}) = \int_{y_{\tilde{a}_{bN}}}^{1} h(Pos[\tilde{a}_{bN} \leq (a_{bN})_{\gamma}^{l+}])(a_{bN})_{\gamma}^{l+} d\gamma = \int_{y_{\tilde{c}_{bN}}}^{1} g(\gamma)(a_{bN})_{\gamma}^{l+} d\gamma \tag{15}$$

$$\overline{M}_{F^+}(\tilde{a}_{bN}) = \int_{y_{\tilde{c}_{bN}}}^{1} h(Pos[\tilde{a}_{bN} \geq (a_{bN})_{\gamma}^{r+}])(a_{bN})_{\gamma}^{r+} d\gamma = \int_{y_{\tilde{c}_{bN}}}^{1} h(\gamma)(a_{bN})_{\gamma}^{r+} d\gamma \tag{16}$$

where $h : [y_{\tilde{a}_{bN}}, 1] \to \mathbb{R}$ is an increasing and non-negative weighted function on $[y_{\tilde{a}_{bN}}, 1]$ and $h(\gamma) \in [0,1]$ for $\gamma \in [y_{\tilde{a}_{bN}}, 1]$ and satisfies that $\int_{y_{\tilde{a}_{bN}}}^{1} h(\gamma)d\gamma = x \in [0,1]$.

$$Pos[\tilde{a}_{bN} \leq (a_{bN})_{\gamma}^{l+}] = \sup_{x \leq (a_{bN})_{\gamma}^{l+}} \{F_{\tilde{a}_{bN}}^{+}(x)\} = \gamma$$

$$Pos[\tilde{a}_{bN} \geq (a_{bN})_{\gamma}^{r+}] = \sup_{x \geq (a_{bN})_{\gamma}^{r+}} \{F_{\tilde{a}_{bN}}^{+}(x)\} = \gamma$$

The g-weighted lower possibility means ($\underline{M}_{F_+}$) and g-weighted upper possibility means ($\overline{M}_{F_+}$) for hesitate function of a GSVnTbN-number $\tilde{a}_{bN}$ are respectively defined as:

$$\underline{M}_{F^-}(\tilde{a}_{bN}) = \int_{-1}^{-y_{\tilde{c}_{bN}}} h(Pos[\tilde{a}_{bN} \geq (a_{bN})_{\gamma}^{l-}])(a_{bN})_{\gamma}^{l-} d\gamma = \int_{-1}^{-y_{\tilde{c}_{bN}}} h(-\gamma)(a_{bN})_{\gamma}^{l-} d\gamma \tag{17}$$

$$\overline{M}_{F^-}(\tilde{a}_{bN}) = \int_{-1}^{-y_{\tilde{c}_{bN}}} h(Pos[\tilde{a}_{bN} \leq (a_{bN})_{\gamma}^{r-}])(a_{bN})_{\gamma}^{r-} d\gamma = \int_{-1}^{-y_{\tilde{c}_{bN}}} h(-\gamma)(a_{bN})_{\gamma}^{r-} d\gamma \tag{18}$$

where $h : [-1, -y_{\tilde{c}_{bN}}] \to \mathbb{R}$ is an increasing and non-positive weighted function on $[-1, -y_{\tilde{a}_{bN}}]$ and $h(\gamma) \in [-1, 0]$ for $\gamma \in [-1, -y_{\tilde{a}_{bN}}]$ and satisfies that $\int_{-1}^{-y_{\tilde{a}_{bN}}} h(\gamma)d\gamma = x \in [-1, 0]$.

$$Pos[\tilde{a}_{bN} \geq (a_{bN})_{\gamma}^{l-}] = \inf_{x \geq (a_{bN})_{\gamma}^{l-}} \{F_{\tilde{a}_{bN}}^{-}(x)\} = -\gamma$$

$$Pos[\tilde{a}_{bN} \leq (a_{bN})_{\gamma}^{r-}] = \inf_{x \leq (a_{bN})_{\gamma}^{r+}} \{F_{\tilde{a}_{bN}}^{-}(x)\} = -\gamma$$

Definition 3.5. Let $\tilde{a}_{bN}$ be a GSVnTbN-number, the f-weighted possibilistic mean for positive and negative authenticity membership function, g-weighted possibilistic mean for positive and negative hesitate membership function and h-weighted possibilistic mean for positive and negative falsity membership function are respectively defined as

$$M_{T^+}(\tilde{a}_{bN}) = \frac{\underline{M}_{T^+}(\tilde{a}_{bN}) + \overline{M}_{T^+}(\tilde{a}_{bN})}{2}, \quad M_{T^-}(\tilde{a}_{bN}) = \frac{\underline{M}_{T^-}(\tilde{a}_{bN}) + \overline{M}_{T^-}(\tilde{a}_{bN})}{2} \tag{19}$$

$$M_{I^+}(\tilde{a}_{bN}) = \frac{\underline{M}_{I^+}(\tilde{a}_{bN}) + \overline{M}_{I^+}(\tilde{a}_{bN})}{2}, \quad M_{I^-}(\tilde{a}_{bN}) = \frac{\underline{M}_{I^-}(\tilde{a}_{bN}) + \overline{M}_{I^-}(\tilde{e}_{bN})}{2} \tag{20}$$

and

$$M_{F^+}(\tilde{a}_{bN}) = \frac{\underline{M}_{F^+}(\tilde{a}_{bN}) + \overline{M}_{F^+}(\tilde{a}_{bN})}{2}, \quad M_{F^-}(\tilde{a}_{bN}) = \frac{\underline{M}_{F^-}(\tilde{a}_{bN}) + \overline{M}_{F^-}(\tilde{a}_{bN})}{2} \tag{21}$$

Example 3.1. Let $\tilde{a}_{bN} = \langle(a_1, a_2, a_3; w_{\tilde{a}_{bN}}, p_1, p_2)(b_1, b_2, b_3; u_{\tilde{b}_{bN}}, q_1, q_2)(c_1, c_2, c_3; y_{\tilde{c}_{bN}}, r_1, r_2)\rangle$ be a generalized single valued non-linear triangular bipolar neutroshopic number. Then the possibilistic means of positive and negative authenticity, positive and negative hesitate, positive and negative falsity membership functions are respectively formulated as follows:

The α-cut set of GSVnTbN-number $\tilde{a}_{bN} = \langle(a_1, a_2, a_3; w_{\tilde{a}_{bN}}, p_1, p_2)(b_1, b_2, b_3; u_{\tilde{b}_{bN}}, q_1, q_2)(c_1, c_2, c_3; y_{\tilde{c}_{bN}}, r_1, r_2)\rangle$ defined as

$$(\tilde{a}_{bN})_{\alpha}^{+} = [(a_{bN})_{\alpha}^{l+}, (a_{bN})_{\alpha}^{r+}] = \left[a_1 + \left(\frac{\alpha}{w_{\tilde{a}_{bN}}}\right)^{\frac{1}{p_1}} (a_2 - a_1),\ a_3 - \left(\frac{\alpha}{w_{\tilde{a}_{bN}}}\right)^{\frac{1}{p_1}} (a_3 - a_2)\right]$$

$$(\tilde{a}_{bN})_{\alpha}^{-} = [(a_{bN})_{\alpha}^{l-}, (a_{bN})_{\alpha}^{r-}]$$

$$= \left[a_2 - \left(\frac{-\alpha}{w_{\tilde{a}_{bN}}} \right)^{\frac{1}{p_2}} (a_2 - a_1),\ a_3 + \left(\frac{-\alpha}{w_{\tilde{a}_{bN}}} \right)^{\frac{1}{p_2}} (a_3 - a_2) \right]$$

By Eqs. (7) and (8), *if we assume that* $f(\alpha) = 2\left(\frac{\alpha}{w_{\tilde{a}_{bN}}}\right)^{1/p_1}$ *then the lower and upper possibilistic mean for positive authenticity function of GSVnTbN-number* $\tilde{a}_{bN}$ *are calculated as*

$$\underline{M}_{T^+}(\tilde{a}_{bN}) = \int_0^{w_{\tilde{c}_{bN}}} f(Pos[\tilde{a}_{bN} \le (a_{bN})_\alpha^{l+}])(a_{bN})_\alpha^{l+} d\alpha$$
$$= \int_0^{w_{\tilde{a}_{bN}}} f(\alpha)(a_{bN})_\alpha^{l+} d\alpha = \frac{2a_2(p_1^2+p_1)+2a_1p_1}{p_1^2+3p_1+2} w_{\tilde{a}_{bN}}$$

and

$$\overline{M}_{T^+}(\tilde{a}_{bN}) = \int_0^{w_{\tilde{a}_{bN}}} f(Pos[\tilde{a}_{bN} \ge (a_{bN})_\alpha^{r+}])(a_{bN})_\alpha^{r+} d\alpha$$
$$= \int_0^{w_{\tilde{a}_{bN}}} f(\alpha)(a_{bN})_\alpha^{r+} d\alpha = \frac{2a_2(p_1^2+p_1)+2a_2p_1}{p_1^2+3p_1+2} w_{\tilde{a}_{bN}}$$

The possibilistic mean of positive authenticity function is

$$M_{T^+}(\tilde{a}_{bN}) = \frac{\underline{M}_{T^+}(\tilde{a}_{bN}) + \overline{M}_{T^+}(\tilde{a}_{bN})}{2} = \frac{2a_2(p_1^2+p_1)+(a_1+a_2)p_1}{p_1^2+3p_1+2} w_{\tilde{a}_{bN}} \tag{22}$$

By Eqs. (9) and (10), *if we assume that* $f(\alpha) = 2\left(\frac{\alpha}{w_{\tilde{a}_{bN}}}\right)^{1/p_2}$ *then the lower and upper possibilistic mean for negative authenticity function of GSVnTbN-number* $\tilde{a}_{bN}$ *are calculated as*

$$\underline{M}_{T^-}(\tilde{a}_{bN}) = \int_{-w_{\tilde{a}_{bN}}}^0 f(Pos[\tilde{a}_{bN} \ge (a_{bN})_\alpha^{l-}])(a_{bN})_\alpha^{l-} d\alpha$$
$$= \int_{-w_{\tilde{a}_{bN}}}^0 f(-\alpha)(a_{bN})_\alpha^{l-} d\alpha = -\left(\frac{2a_1p_2+2a_2(p_2^2+p_2)}{p_2^2+3p_2+2}\right) w_{\tilde{a}_{bN}}$$

and

$$\overline{M}_{T^-}(\tilde{a}_{bN}) = \int_{-w_{\tilde{a}_{bN}}}^0 f(Pos[\tilde{a}_{bN} \le (a_{bN})_\alpha^{r-}])(a_{bN})_\alpha^{r-} d\alpha$$
$$= \int_{-w_{\tilde{a}_{bN}}}^0 f(-\alpha)(a_{bN})_\alpha^{r-} d\alpha = \frac{2a_2(p_2^2+p_2)-2a_3(2p_2^2+3)}{p_2^2+3p_2+2} w_{\tilde{a}_{bN}}$$

The possibilistic mean of negative authenticity function

$$M_{T^-}(\tilde{a}_{bN}) = \frac{\underline{M}_{T^-}(\tilde{a}_{bN}) + \overline{M}_{T^-}(\tilde{a}_{bN})}{2} = -\left(\frac{a_1p_2+a_3(2p_2^2+3)}{p_2^2+3p_2+2}\right) w_{\tilde{a}_{bN}} \tag{23}$$

The positive and negative β*-cut set of GSVnTbN-number* $\tilde{a}_{bN} = \langle(a_1,a_2,a_3; w_{\tilde{a}_{bN}}, p_1, p_2)(b_1,b_2,b_3; u_{\tilde{b}_{bN}}, q_1, q_2)$ $(c_1,c_2,c_3; y_{\tilde{c}_{bN}}, r_1, r_2)\rangle$ *defined as*

$$(\tilde{a}_{bN})_\beta^+ = [(a_{bN})_\beta^{l+}, (a_{bN})_\beta^{r+}] = \left[b_2 - \left(\frac{\beta}{u_{\tilde{b}_{bN}}}\right)^{\frac{1}{q_1}} (b_2-b_1),\ b_3 + \left(\frac{\beta}{u_{\tilde{b}_{bN}}}\right)^{\frac{1}{q_1}} (b_3-b_2) \right]$$

$$(\tilde{a}_{bN})_\beta^- = [(a_{bN})_\beta^{l-}, (a_{bN})_\beta^{r-}] = \left[b_2 + \left(\frac{-\beta}{u_{\tilde{b}_{bN}}}\right)^{\frac{1}{q_2}} (b_2-b_1),\ b_3 - \left(\frac{-\beta}{u_{\tilde{b}_{bN}}}\right)^{\frac{1}{q_2}} (b_3-b_2) \right]$$

By Eqs. (11) and (12), *if we assume that* $g(\beta) = 2\left(\frac{\beta}{u_{\tilde{b}_{bN}}}\right)^{1/q_1}$ *then the lower and upper possibilistic mean for positive hesitate function of GSVnTbN-number* $\tilde{a}_{bN} = \langle(a_1,a_2,a_3; w_{\tilde{a}_{bN}}, p_1, p_2)(b_1,b_2,b_3; u_{\tilde{b}_{bN}}, q_1, q_2)$ $(c_1,c_2,c_3; y_{\tilde{c}_{bN}}, r_1, r_2)\rangle$ *are calculated as*

$$\underline{M}_{I^+}(\tilde{a}_{bN}) = \int_{u_{\tilde{b}_{bN}}}^1 g(Pos[\tilde{a}_{bN} \le (a_{bN})_\beta^{l+}])(a_{bN})_\beta^{l+} d\beta$$
$$= \int_{u_{\tilde{b}_{bN}}}^1 g(\beta)(a_{bN})_\beta^{l+} d\beta$$
$$= \frac{2b_2(q_1^2+2q_1)(u_{\tilde{b}_{bN}}^{\frac{1}{q_1}} - u_{\tilde{b}_{bN}}^{\frac{2}{q_1}+1}) - 2(b_2-b_1)(q_1^2+q_1)(1-(u_{\tilde{b}_{bN}})^{\frac{2}{q_1}+1})}{(q_1^2+3q_1+2)(u_{\tilde{b}_{bN}})^{\frac{2}{q_1}}}$$

and

$$\overline{M}_{I^+}(\tilde{a}_{bN}) = \int_{u_{\tilde{b}_{bN}}}^1 g(Pos[\tilde{a}_{bN} \ge (a_{bN})_\beta^{r+}])(a_{bN})_\beta^{r+} d\beta$$
$$= \int_{u_{\tilde{b}_{bN}}}^1 g(\beta)(a_{bN})_\beta^{r+} d\beta$$
$$= \frac{2b_3(q_1^2+2q_1)(u_{\tilde{b}_{bN}}^{\frac{1}{q_1}} - u_{\tilde{b}_{bN}}^{\frac{2}{q_1}+1}) + 2(b_3-b_2)(q_1^2+q_1)(1-(u_{\tilde{b}_{bN}})^{\frac{2}{q_2}+1})}{(q_1^2+3q_1+2)(u_{\tilde{b}_{bN}})^{\frac{2}{q_1}}}$$

The possibilistic mean of positive hesitate function

$$M_{I^+}(\tilde{a}_{bN}) = \frac{\underline{M}_{I^+}(\tilde{a}_{bN}) + \overline{M}_{I^+}(\tilde{a}_{bN})}{2}$$
$$= \frac{(b_2+b_3)(q_1^2+2q_1)(u_{\tilde{b}_{bN}}^{\frac{1}{q_1}} - u_{\tilde{b}_{bN}}^{\frac{2}{q_1}+1}) + (b_3-2b_2+b_1)(q_1^2+q_1)(1-(u_{\tilde{b}_{bN}})^{\frac{2}{q_1}+1})}{(q_1^2+3q_1+2)(u_{\tilde{b}_{bN}})^{\frac{2}{q_1}}} \tag{24}$$

By Eqs. (13) and (14), *if we assume that* $g(\beta) = 2\left(\frac{\beta}{u_{\tilde{b}_{bN}}}\right)^{1/q_2}$ *then the lower and upper possibilistic mean for negative hesitate function of GSVnTbN-number* $\tilde{a}_{bN}$ *are calculated as*

$$\underline{M}_{I^-}(\tilde{a}_{bN}) = \int_{-1}^{-u_{\tilde{b}_{bN}}} g(Pos[\tilde{a}_{bN} \ge (a_{bN})_\alpha^{l-}])(a_{bN})_\alpha^{l-} d\beta$$
$$= \int_{-1}^{-u_{\tilde{b}_{bN}}} g(-\beta)(a_{bN})_\alpha^{l-} d\beta$$
$$= \frac{2b_2(q_2^2+2q_2)(u_{\tilde{b}_{bN}}^{\frac{2}{q_2}+1} - u_{\tilde{b}_{bN}}^{\frac{1}{q_2}}) + 2(b_2-b_1)(q_2^2+q_2)((u_{\tilde{b}_{bN}})^{\frac{2}{q_2}+1}-1)}{(q_2^2+3q_2+2)(u_{\tilde{b}_{bN}})^{\frac{2}{q_2}}}$$

and

$$\overline{M}_{I^-}(\tilde{a}_{bN}) = \int_{-1}^{-u_{\tilde{b}_{bN}}} g(Pos[\tilde{a}_{bN} \le (a_{bN})_\beta^{r-}])(a_{bN})_\beta^{r-} d\beta$$
$$= \int_{-1}^{-u_{\tilde{b}_{bN}}} g(-\beta)(a_{bN})_\beta^{r-} d\beta$$
$$= \frac{2b_3(q_2^2+2q_2)(u_{\tilde{b}_{bN}}^{\frac{2}{q_2}+1} - u_{\tilde{b}_{bN}}^{\frac{1}{q_2}}) - 2(b_3-b_2)(q_2^2+q_2)((u_{\tilde{b}_{bN}})^{\frac{2}{q_2}+1}-1)}{(q_2^2+3q_2+2)(u_{\tilde{b}_{bN}})^{\frac{2}{q_2}}}$$

The possibilistic mean of negative hesitate function

$$M_{I^-}(\tilde{a}_{bN}) = \frac{\underline{M}_{I^-}(\tilde{a}_{bN}) + \overline{M}_{I^-}(\tilde{e}_{bN})}{2}$$
$$= -\frac{(b_2+b_3)(q_2^2+2q_2)(u_{\tilde{b}_{bN}}^{\frac{1}{q_2}} - u_{\tilde{b}_{bN}}^{\frac{2}{q_2}+1}) + (2b_2-b_1-b_3)(q_2^2+q_2)(1-(u_{\tilde{b}_{bN}})^{\frac{2}{q_2}+1})}{(q_2^2+3q_2+2)(u_{\tilde{b}_{bN}})^{\frac{2}{q_2}}} \tag{25}$$

The γ*-cut set of GSVnTbN-number* $\tilde{a}_{bN} = \langle(a_1,a_2,a_3; w_{\tilde{a}_{bN}}, p_1, p_2)$ $(b_1,b_2,b_3; u_{\tilde{b}_{bN}}, q_1, q_2)(c_1,c_2,c_3; y_{\tilde{c}_{bN}}, r_1, r_2)\rangle$ *defined as*

$$(\tilde{a}_{bN})_\gamma^+ = [(a_{bN})_\gamma^{l+}, (a_{bN})_\gamma^{r+}]$$

$$= \left[c_2 - \left(\frac{\gamma}{y_{\tilde{c}_{bN}}}\right)^{\frac{1}{r_1}}(c_2-c_1),\ c_3 + \left(\frac{\beta}{y_{\tilde{c}_{bN}}}\right)^{\frac{1}{r_1}}(c_3-c_2)\right]$$

$$(\tilde{a}_{bN})^-_\gamma = [(a_{bN})^{l-}_\gamma, (a_{bN})^{r-}_\gamma]$$
$$= \left[c_2 + \left(\frac{-\gamma}{y_{\tilde{c}_{bN}}}\right)^{\frac{1}{r_2}}(c_2-c_1),\ c_3 - \left(\frac{-\gamma}{y_{\tilde{c}_{bN}}}\right)^{\frac{1}{r_2}}(c_3-c_2)\right]$$

By Eqs. (15) and (16), *if we assume that* $h(\gamma) = 2\left(\frac{\gamma}{y_{\tilde{c}_{bN}}}\right)^{1/r_1}$ *then the lower and upper possibilistic mean for positive falsity function of GSVnTbN-number* $\tilde{a}_{bN}$ *are calculated as*

$$\underline{M}_{F^+}(\tilde{a}_{bN}) = \int_{y_{\tilde{c}_{bN}}}^1 h(Pos[\tilde{a}_{bN} \le (a_{bN})^{l+}_\gamma])(a_{bN})^{l+}_\gamma d\gamma$$
$$= \int_{y_{\tilde{c}_{bN}}}^1 g(\gamma)(a_{bN})^{l+}_\gamma d\gamma$$
$$= \frac{2c_2(r_1^2+2r_1)(y_{\tilde{c}_{bN}}^{\frac{1}{r_1}} - y_{\tilde{c}_{bN}}^{\frac{2}{r_1}+1}) - 2(c_2-c_1)(r_1^2+r_1)(1-(y_{\tilde{c}_{bN}})^{\frac{2}{r}{}_1+1})}{(r_1^2+3r_1+2)(y_{\tilde{c}_{bN}})^{\frac{2}{r_1}}}$$

and

$$\overline{M}_{F^+}(\tilde{a}_{bN}) = \int_{y_{\tilde{c}_{bN}}}^1 h(Pos[\tilde{a}_{bN} \ge (a_{bN})^{r+}_\gamma])(a_{bN})^{r+}_\gamma d\gamma$$
$$= \int_{y_{\tilde{c}_{bN}}}^1 g(\gamma)(a_{bN})^{r+}_\gamma d\gamma$$
$$= \frac{2c_3(r_1^2+2r_1)(y_{\tilde{c}_{bN}}^{\frac{1}{r_1}} - y_{\tilde{c}_{bN}}^{\frac{2}{r_1}+1}) + 2(c_3-c_2)(r_1^2+r_1)(1-(y_{\tilde{c}_{bN}})^{\frac{2}{r_2}+1})}{(r_1^2+3r_1+2)(y_{\tilde{c}_{bN}})^{\frac{2}{r_1}}}$$

The possibilistic mean of positive falsity function is

$$M_{F^+}(\tilde{a}_{bN}) = \frac{\underline{M}_{F^+}(\tilde{a}_{bN}) + \overline{M}_{F^+}(\tilde{a}_{bN})}{2}$$
$$= \frac{(c_2+c_3)(r_1^2+2r_1)(y_{\tilde{c}_{bN}}^{\frac{1}{r_1}} - y_{\tilde{c}_{bN}}^{\frac{2}{r_1}+1}) + (c_3-2c_2+c_1)(r_1^2+r_1)(1-(y_{\tilde{c}_{bN}})^{\frac{2}{r_1}+1})}{(r_1^2+3r_1+2)(y_{\tilde{c}_{bN}})^{\frac{2}{r_1}}} \tag{26}$$

By Eqs. (17) and (18), *if we assume that* $h(\gamma) = 2\left(\frac{\gamma}{y_{\tilde{c}_{bN}}}\right)^{1/r_2}$ *then the lower and upper possibilistic mean for negative falsity function of GSVnTbN-number* $\tilde{a}_{bN}$ *are calculated as*

$$\underline{M}_{F^-}(\tilde{a}_{bN}) = \int_{-1}^{-y_{\tilde{c}_{bN}}} h(Pos[\tilde{a}_{bN} \ge (a_{bN})^{l-}_\gamma])(a_{bN})^{l-}_\gamma d\gamma$$
$$= \int_{-1}^{-y_{\tilde{c}_{bN}}} h(-\gamma)(a_{bN})^{l-}_\gamma d\gamma$$
$$= \frac{2c_2(r_2^2+2r_2)(y_{\tilde{c}_{bN}}^{\frac{2}{r_2}+1} - y_{\tilde{c}_{bN}}^{\frac{1}{r_2}}) + 2(c_2-c_1)(r_2^2+r_2)((y_{\tilde{c}_{bN}})^{\frac{2}{r_2}+1}-1)}{(r_2^2+3r_2+2)(y_{\tilde{c}_{bN}})^{\frac{2}{r_2}}}$$

and

$$\overline{M}_{F^-}(\tilde{a}_{bN}) = \int_{-1}^{-y_{\tilde{c}_{bN}}} h(Pos[\tilde{a}_{bN} \le (a_{bN})^{r-}_\gamma])(a_{bN})^{r-}_\gamma d\gamma$$
$$= \int_{-1}^{-y_{\tilde{c}_{bN}}} h(-\gamma)(a_{bN})^{r-}_\gamma d\gamma$$
$$= \frac{2c_3(r_2^2+2r_2)(y_{\tilde{c}_{bN}}^{\frac{2}{r_2}+1} - y_{\tilde{c}_{bN}}^{\frac{1}{r_2}}) - 2(c_3-c_2)(r_2^2+r_2)((y_{\tilde{c}_{bN}})^{\frac{2}{r_2}+1}-1)}{(r_2^2+3r_2+2)(y_{\tilde{c}_{bN}})^{\frac{2}{r_2}}}$$

The possibilistic mean of negative falsity function

$$M_{F^-}(\tilde{a}_{bN}) = \frac{\underline{M}_{F^-}(\tilde{a}_{bN}) + \overline{M}_{F^-}(\tilde{a}_{bN})}{2}$$
$$= -\frac{(c_2+c_3)(r_2^2+2r_2)(y_{\tilde{c}_{bN}}^{\frac{1}{r_2}} - y_{\tilde{c}_{bN}}^{\frac{2}{r_2}+1}) + (2c_2-c_1-c_3)(r_2^2+r_2)((1-y_{\tilde{c}_{bN}})^{\frac{2}{r_2}+1})}{(r_2^2+3r_2+2)(y_{\tilde{c}_{bN}})^{\frac{2}{r_2}}} \tag{27}$$

Theorem 3.1. *Let $\tilde{a}_{bN}$ and $\tilde{a}_{bN'}$ be any two generalized single valued non-linear bipolar neutrosophic numbers. Then for any two real number k_1 & k_2, the following equalities are hold*

$$\begin{aligned} M_{T^+}(k_1\tilde{a}_{bN} + k_2\tilde{a}'_{bN}) &= k_1M_{T^+}(\tilde{a}_{bN}) + k_2M_{T^+}(\tilde{a}'_{bN}),\\ M_{T^-}(k_1\tilde{a}_{bN} + k_2\tilde{a}'_{bN}) &= k_1M_{T^-}(\tilde{a}_{bN}) + k_2M_{T^-}(\tilde{a}'_{bN}) \end{aligned} \tag{28}$$

$$\begin{aligned} M_{I^+}(k_1\tilde{a}_{bN} + k_2\tilde{a}'_{bN}) &= k_1M_{I^+}(\tilde{a}_{bN}) + k_2M_{I^+}(\tilde{a}'_{bN}),\\ M_{I^-}(k_1\tilde{a}_{bN} + k_2\tilde{a}'_{bN}) &= k_1M_{I^-}(\tilde{a}_{bN}) + k_2M_{I^-}(\tilde{a}'_{bN}) \end{aligned} \tag{29}$$

$$\begin{aligned} M_{F^+}(k_1\tilde{a}_{bN} + k_2\tilde{a}'_{bN}) &= k_1M_{F^+}(\tilde{a}_{bN}) + k_2M_{F^+}(\tilde{a}'_{bN}),\\ M_{F^-}(k_1\tilde{a}_{bN} + k_2\tilde{a}'_{bN}) &= k_1M_{F^-}(\tilde{a}_{bN}) + k_2M_{F^-}(\tilde{a}'_{bN}) \end{aligned} \tag{30}$$

Proof. Let us assume $k_1, k_2 > 0$.

From the definition 6, we easily calculated the positive α-cut for the authenticity function of GSVnbN-number $k_1\tilde{a}_{bN} + k_2\tilde{a}'_{bN}$ is $(k_1\tilde{a}_{bN} + k_2\tilde{a}'_{bN})^+_\alpha = [k_1(a_{bN})^{l+}_\alpha + k_2(a'_{bN})^{l+}_\alpha,\ k_1(a_{bN})^{r+}_\alpha + k_2(a'_{bN})^{r+}_\alpha]$. By Eq. (19), we obtain

$$\begin{aligned} M_{T^+}(k_1\tilde{a}_{bN} + k_2\tilde{a}'_{bN}) &= \frac{1}{2}\int_0^{w_{\tilde{a}_{bN}} \wedge w_{\tilde{a}'_{bN}}} (k_1(a_{bN})^{l+}_\alpha + k_2(a'_{bN})^{l+}_\alpha \\ &\quad + k_1(a_{bN})^{r+}_\alpha + k_2(a'_{bN})^{r+}_\alpha)f(\alpha)d\alpha \\ &= \frac{1}{2}\int_0^{w_{\tilde{a}_{bN}}} k_1((a_{bN})^{l+}_\alpha + (a_{bN})^{r+}_\alpha)f(\alpha)d\alpha \\ &\quad + \frac{1}{2}\int_0^{w_{\tilde{a}'_{bN}}} (k_2(a'_{bN})^{l+}_\alpha + (a'_{bN})^{r+}_\alpha)f(\alpha)d\alpha \\ &= k_1M_{T^+}(\tilde{a}_{bN}) + k_2M_{T^+}(\tilde{a}'_{bN}) \end{aligned}$$

Again from the definition 6, we easily calculated the negative α-cut for the authenticity function of GSVnbN-number $k_1\tilde{a}_{bN} + k_2\tilde{a}'_{bN}$ is $(k_1\tilde{a}_{bN} + k_2\tilde{a}'_{bN})^-_\alpha = [k_1(a_{bN})^{l-}_\alpha + k_2(a'_{bN})^{l-}_\alpha,\ k_1(a_{bN})^{r-}_\alpha + k_2(a'_{bN})^{r-}_\alpha]$. By Eq. (19), we get

$$\begin{aligned} M_{T^-}(k_1\tilde{a}_{bN} + k_2\tilde{a}'_{bN}) &= \frac{1}{2}\int_{-(w_{\tilde{a}_{bN}} \wedge w_{\tilde{a}'_{bN}})}^0 (k_1(a_{bN})^{l-}_\alpha \\ &\quad + k_2(a'_{bN})^{l-}_\alpha + k_1(a_{bN})^{r-}_\alpha + k_2(a'_{bN})^{r-}_\alpha)f(\alpha)d\alpha \\ &= \frac{1}{2}\int_{-(w_{\tilde{a}_{bN}})}^0 k_1((a_{bN})^{l-}_\alpha + (a_{bN})^{r-}_\alpha)f(\alpha)d\alpha \\ &\quad + \frac{1}{2}\int_{-w_{\tilde{a}'_{bN}}}^0 (k_2(a'_{bN})^{l-}_\alpha + (a'_{bN})^{r-}_\alpha)f(\alpha)d\alpha \\ &= k_1M_{T^-}(\tilde{a}_{bN}) + k_2M_{T^-}(\tilde{a}'_{bN}) \end{aligned}$$

From the definition 6, we easily calculated the positive β-cut for the hesitate function of GSVnbN-number $k_1\tilde{a}_{bN} + k_2\tilde{a}'_{bN}$ is $(k_1\tilde{a}_{bN} + k_2\tilde{a}'_{bN})^+_\beta = [k_1(a_{bN})^{l+}_\beta + k_2(a'_{bN})^{l+}_\beta,\ k_1(a_{bN})^{r+}_\beta + k_2(a'_{bN})^{r+}_\beta]$. By Eq. (20), we obtain

$$\begin{aligned} M_{I^+}(k_1\tilde{a}_{bN} + k_2\tilde{a}'_{bN}) &= \frac{1}{2}\int_{u_{\tilde{b}_{bN}} \vee u_{\tilde{b}'_{bN}}}^1 (k_1(a_{bN})^{l+}_\beta + k_2(a'_{bN})^{l+}_\beta \\ &\quad + k_1(a_{bN})^{r+}_\beta + k_2(a'_{bN})^{r+}_\beta)g(\beta)d\beta \\ &= \frac{1}{2}\int_{u_{\tilde{b}_{bN}}}^1 k_1((a_{bN})^{l+}_\beta + (a_{bN})^{r+}_\beta)g(\beta)d\beta \\ &\quad + \frac{1}{2}\int_{u_{\tilde{b}'_{bN}}}^1 (k_2(a'_{bN})^{l+}_\beta + (a'_{bN})^{r+}_\beta)g(\beta)d\beta \\ &= k_1M_{I^+}(\tilde{a}_{bN}) + k_2M_{I^+}(\tilde{a}'_{bN}) \end{aligned}$$

Again from the definition 6, we easily calculated the negative β-cut for the hesitate function of GSVnbN-number $k_1\tilde{a}_{bN} + k_2\tilde{a}'_{bN}$ is $(k_1\tilde{a}_{bN} +$

$k_2\tilde{a}'_{bN})^-_\beta = [k_1(a_{bN})^{l-}_\beta + k_2(a'_{bN})^{l-}_\beta,\ k_1(a_{bN})^{r-}_\beta + k_2(a'_{bN})^{r-}_\beta]$. By Eq. (20), we get

$$\begin{aligned} M_{I^-}(k_1\tilde{a}_{bN} + k_2\tilde{a}'_{bN}) &= \frac{1}{2}\int_{-(u_{\tilde{a}_{bN}} \vee u_{\tilde{a}'_{bN}})}^{1} (k_1(a_{bN})^{l-}_\beta + k_2(a'_{bN})^{l-}_\beta \\ &\quad + k_1(a_{bN})^{r-}_\beta + k_2(a'_{bN})^{r-}_\beta) g(\beta) d\beta \\ &= \frac{1}{2}\int_{-u_{\tilde{a}_{bN}}}^{1} k_1((a_{bN})^{l-}_\beta + (a_{bN})^{r-}_\beta) g(\beta) d\beta \\ &\quad + \frac{1}{2}\int_{-u_{\tilde{b}'_{bN}}}^{1} (k_2(a'_{bN})^{l-}_\beta + (a'_{bN})^{r-}_\beta) g(\beta) d\beta \\ &= k_1 M_{I^-}(\tilde{a}_{bN}) + k_2 M_{I^-}(\tilde{a}'_{bN}) \end{aligned}$$

From the definition 6, we easily calculated the positive γ-cut for the hesitate function of GSVnbN-number $k_1\tilde{a}_{bN} + k_2\tilde{a}'_{bN}$ is $(k_1\tilde{a}_{bN} + k_2\tilde{a}'_{bN})^+_\gamma = [k_1(a_{bN})^{l+}_\gamma + k_2(a'_{bN})^{l+}_\gamma,\ k_1(a_{bN})^{r+}_\gamma + k_2(a'_{bN})^{r+}_\gamma]$. By Eq. (21), we obtain

$$\begin{aligned} M_{F^+}(k_1\tilde{a}_{bN} + k_2\tilde{a}'_{bN}) &= \frac{1}{2}\int_{y_{\tilde{b}_{bN}} \vee y_{\tilde{c}'_{bN}}}^{1} (k_1(a_{bN})^{l+}_\gamma + k_2(a'_{bN})^{l+}_\gamma \\ &\quad + k_1(a_{bN})^{r+}_\gamma + k_2(a'_{bN})^{r+}_\gamma) h(\gamma) d\gamma \\ &= \frac{1}{2}\int_{y_{\tilde{c}_{bN}}}^{1} k_1((a_{bN})^{l+}_\gamma + (a_{bN})^{r+}_\gamma) h(\gamma) d\gamma \\ &\quad + \frac{1}{2}\int_{y_{\tilde{c}'_{bN}}}^{1} (k_2(a'_{bN})^{l+}_\gamma + (a'_{bN})^{r+}_\gamma) h(\gamma) d\gamma \\ &= k_1 M_{F^+}(\tilde{a}_{bN}) + k_2 M_{F^+}(\tilde{a}'_{bN}) \end{aligned}$$

Again from the definition 6, we easily calculated the negative γ-cut for the falsity function of GSVnbN-number $k_1\tilde{a}_{bN} + k_2\tilde{a}'_{bN}$ is $(k_1\tilde{a}_{bN} + k_2\tilde{a}'_{bN})^-_\gamma = [k_1(a_{bN})^{l-}_\gamma + k_2(a'_{bN})^{l-}_\gamma,\ k_1(a_{bN})^{r-}_\gamma + k_2(a'_{bN})^{r-}_\gamma]$. By Eq. (21), we get

$$\begin{aligned} M_{F^-}(k_1\tilde{a}_{bN} + k_2\tilde{a}'_{bN}) &= \frac{1}{2}\int_{-(y_{\tilde{c}_{bN}} \vee y_{\tilde{c}'_{bN}})}^{1} (k_1(a_{bN})^{l-}_\gamma + k_2(a'_{bN})^{l-}_\gamma \\ &\quad + k_1(a_{bN})^{r-}_\gamma + k_2(a'_{bN})^{r-}_\gamma) g(\beta) d\beta \\ &= \frac{1}{2}\int_{-y_{\tilde{c}_{bN}}}^{1} k_1((a_{bN})^{l-}_\gamma + (a_{bN})^{r-}_\gamma) h(\gamma) d\gamma \\ &\quad + \frac{1}{2}\int_{-y_{\tilde{c}'_{bN}}}^{1} (k_2(a'_{bN})^{l-}_\gamma + (a'_{bN})^{r-}_\gamma) h(\gamma) d\gamma \\ &= k_1 M_{F^-}(\tilde{a}_{bN}) + k_2 M_{F^-}(\tilde{a}'_{bN}) \end{aligned}$$

Using the Eq. (19), (20) & (21), we can also perform the theorem result for $k_1 > 0, k_2 < 0; k_1 < 0, k_2 > 0; k_1 < 0, k_2 < 0$.

Remark 3.1. It is notated that if $k_1 = 1$ and $k_2 = 1$, then by Theorem 1, we have

$$\begin{aligned} M_{T^+}(\tilde{a}_{bN} + \tilde{a}'_{bN}) &= M_{T^+}(\tilde{a}_{bN}) + M_{T^+}(\tilde{a}'_{bN}), \\ M_{T^-}(\tilde{a}_{bN} + \tilde{a}'_{bN}) &= M_{T^-}(\tilde{a}_{bN}) + M_{T^-}(\tilde{a}'_{bN}) \end{aligned} \tag{31}$$

$$\begin{aligned} M_{I^+}(\tilde{a}_{bN} + \tilde{a}'_{bN}) &= M_{I^+}(\tilde{a}_{bN}) + M_{I^+}(\tilde{a}'_{bN}), \\ M_{I^-}(\tilde{a}_{bN} + \tilde{a}'_{bN}) &= M_{I^-}(\tilde{a}_{bN}) + M_{I^-}(\tilde{a}'_{bN}) \end{aligned} \tag{32}$$

$$\begin{aligned} M_{F^+}(\tilde{a}_{bN} + \tilde{a}'_{bN}) &= M_{F^+}(\tilde{a}_{bN}) + M_{F^+}(\tilde{a}'_{bN}), \\ M_{F^-}(\tilde{a}_{bN} + \tilde{a}'_{bN}) &= M_{F^-}(\tilde{a}_{bN}) + M_{F^-}(\tilde{a}'_{bN}) \end{aligned} \tag{33}$$

Definition 3.6. Let M_{T^+}, M_{I^+} and M_{F^+} be the positive authenticity, positive hesitate and positive falsity function of GSVnbN-number $\tilde{a}_{bN}$ and M_{T^-}, M_{I^-} and M_{F^-} be the negative authenticity, negative hesitate and negative falsity function of GSVnbN-number $\tilde{a}_{bN}$ respectively. Then the positive and negative rank expositor of the GSVnbN-number $\tilde{a}_{bN}$ can be defined as:

$$S_{\mu^+}(\tilde{a}_{bN}) = \frac{M_{T^+}(\tilde{a}_{bN}) + (1-\mu)M_{I^+}(\tilde{a}_{bN}) + (1-\mu)M_{F^+}(\tilde{a}_{bN})}{3} \tag{34}$$

and

$$S_{\mu^-}(\tilde{a}_{bN}) = \frac{M_{T^-}(\tilde{a}_{bN}) + (1-\mu)M_{I^-}(\tilde{a}_{bN}) + (1-\mu)M_{F^-}(\tilde{a}_{bN})}{3} \tag{35}$$

where the parameter μ plays the risk attitude of the decision maker $\mu \in [0, 0.5)$ display that the decision maker is risk-flat; $\mu = 0.5$ display that the decision maker is risk-impartial; $\mu \in (0.5, 1)$ shows that the decision maker is risk-apathetic.

3.7. A novel lexicographic ranking method of GSVnbN-numbers considering possibilistic mean

Let $S_{\mu^+}(\tilde{a}_{bN})$ and $S_{\mu^-}(\tilde{a}_{bN})$ be the rank expositor for positive and negative membership functions of the GSVnbN-number $\tilde{a}_{bN}$ respectively. Let $S_{\mu^+}(\tilde{a}'_{bN})$ and $S_{\mu^-}(\tilde{a}'_{bN})$ be the rank expositor for positive and negative membership functions of the GSVnbN-number $\tilde{a}'_{bN}$ respectively. A novel lexicographic ranking method of GSVnbN-numbers $\tilde{a}_{bN}$ and $\tilde{a}'_{bN}$ can be summarized as follows:

If $S_{\mu^+}(\tilde{a}_{bN}) < S_{\mu^+}(\tilde{a}'_{bN})$, then $\tilde{a}_{bN}$ is smaller than $\tilde{a}'_{bN}$ noted by $\tilde{a}_{bN} \prec \tilde{a}'_{bN}$.

If $S_{\mu^+}(\tilde{a}_{bN}) > S_{\mu^+}(\tilde{a}'_{bN})$, then $\tilde{a}_{bN}$ is bigger than $\tilde{a}'_{bN}$ noted by $\tilde{a}_{bN} \succ \tilde{a}'_{bN}$.

If $S_{\mu^+}(\tilde{a}_{bN}) = S_{\mu^+}(\tilde{a}'_{bN})$, then

(i) If $S_{\mu^-}(\tilde{a}_{bN}) < S_{\mu^-}(\tilde{a}'_{bN})$, then $\tilde{a}_{bN}$ is bigger than $\tilde{a}'_{bN}$ noted by $\tilde{a}_{bN} \succ \tilde{a}'_{bN}$.

(ii) If $S_{\mu^-}(\tilde{a}_{bN}) > S_{\mu^-}(\tilde{a}'_{bN})$, then $\tilde{a}_{bN}$ is smaller than $\tilde{a}'_{bN}$ noted by $\tilde{a}_{bN} \prec \tilde{a}'_{bN}$.

If $S_{\mu^+}(\tilde{a}_{bN}) = S_{\mu^+}(\tilde{a}'_{bN})$ and $S_{\mu^-}(\tilde{a}_{bN}) = S_{\mu^-}(\tilde{a}'_{bN})$, then $\tilde{a}_{bN}$ and $\tilde{a}'_{bN}$ represent the same information, noted by $\tilde{a}_{bN} \approx \tilde{a}'_{bN}$.

4. Multi-criteria decision making method under generalized single valued non-linear triangular bipolar neutrosophic environment

This section developed a novel MCDM method based on the proposed rank expositor. Generally, using the MCDM technique, we find one or more than one alternatives according to the significant criteria. With the help of the above-ranking method, we have presented the MCDM method under GSVnTbN environment. Let $\{A_1, A_2, \ldots, A_m\}$ be the set of alternatives and $\{C_1, C_2, \ldots, C_n\}$ bet the set of criteria. The weight vector of the criteria is taken as $\upsilon = (\upsilon_1, \upsilon_2, \ldots, \upsilon_n)^T$, where each $\upsilon_j \geq 0$ and $\sum_{j=1}^{n} \upsilon_j = 1$. Assume that an expert has evaluated each alternative under the different criteria and provided their rating values for GSVnTbN-numbers $(\tilde{a}_{bN})_{ij} = \langle (a_1, a_2, a_3; w_{\tilde{a}_{bN}}, p_1, p_2)(b_1, b_2, b_3; u_{\tilde{b}_{bN}}, q_1, q_2)(c_1, c_2, c_3; y_{\tilde{c}_{bN}}, r_1, r_2) \rangle_{ij}$. Based on his/her preference, a decision matrix is formulated as $\tilde{A} = ((\tilde{a}_{bN})_{ij})_{m\times n}$, $i = 1, 2, \ldots, m$ & $j = 1, 2, \ldots, n$. The objective of the problem is to find the best alternative among the given ones. To solve this MCDM problem, we summarized the following steps of the proposed approach under the GSVnTbN environment.

Step 1: Constructed the generalized single-valued non-linear bipolar neutrosophic decision matrix $\tilde{A} = (\tilde{a}_{ij})_{m\times n}$, $i = 1, 2, \ldots, m$ & $j = 1, 2, \ldots, n$, where each $\tilde{a}_{ij}$ is a GSVnTbN-number, which illustrate the evaluation information of the criteria $C_j \in C$ w.r.t the alternatives $A_i \in A$. Then the decision matrix for the proposed method can be represented by

$$\tilde{A} = [(\tilde{a}_{bN})_{ij}]_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{pmatrix} (\tilde{a}_{bN})_{11} & (\tilde{a}_{bN})_{12} & \cdots & (\tilde{a}_{bN})_{1n} \\ (\tilde{a}_{bN})_{21} & (\tilde{a}_{bN})_{22} & \cdots & (\tilde{a}_{bN})_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ (\tilde{a}_{bN})_{m1} & (\tilde{a}_{bN})_{m2} & \cdots & (\tilde{a}_{bN})_{mn} \end{pmatrix} \end{array} \tag{36}$$

Step 2: Usually, a criterion can be signified in one benefit type criterion and another cost type criterion. Now we calculate the normalized

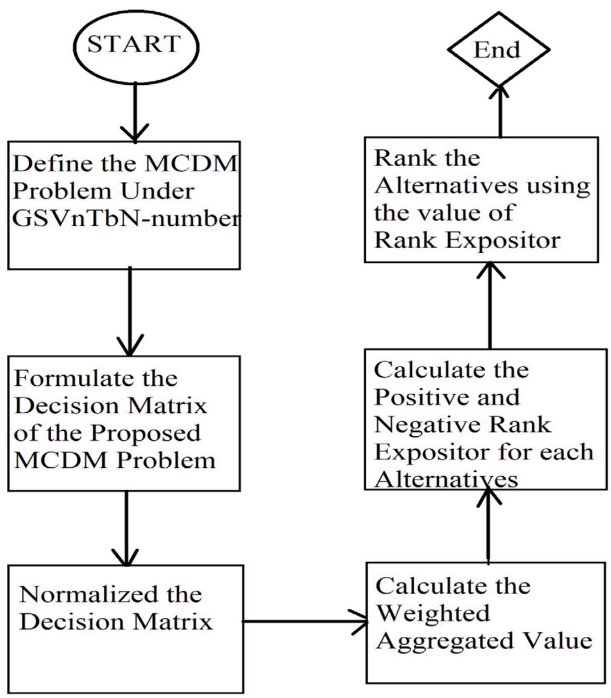

Fig. 2. Flowchart of the proposed Algorithm.

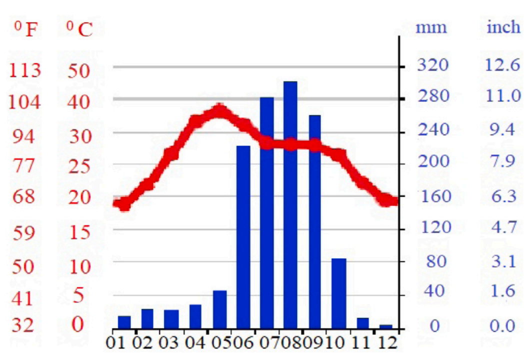

Fig. 3. Average temperatures in the Purulia District in the Year 2018.

Fig. 4. Average rain fall in the West Bengal in the year 2018.

decision matrix. For the normalization of the decision matrix, we have to consider the following formula

$$(\tilde{r}_{bN})_{ij} = \begin{cases} \dfrac{(\tilde{a}_{bN})_{ij}}{\sqrt{\sum_{i=1}^{m}(\tilde{a}_{bN})_{ij}^2}}, & \text{if } C_j \text{ are benefit type criterion} \\ \dfrac{\dfrac{1}{(\tilde{a}_{bN})_{ij}}}{\sqrt{\sum_{i=1}^{m}\left(\dfrac{1}{(\tilde{a}_{bN})_{ij}}\right)^2}}, & \text{if } C_j \text{ are cost type criterion} \end{cases} \tag{37}$$

The normalized decision matrix can be represented by

$$\tilde{R} = [(\tilde{r}_{bN})_{ij}]_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{pmatrix} (\tilde{r}_{bN})_{11} & (\tilde{r}_{bN})_{12} & \cdots & (\tilde{r}_{bN})_{1n} \\ (\tilde{r}_{bN})_{21} & (\tilde{r}_{bN})_{22} & \cdots & (\tilde{r}_{bN})_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ (\tilde{r}_{bN})_{m1} & (\tilde{r}_{bN})_{m2} & \cdots & (\tilde{r}_{bN})_{mn} \end{pmatrix} \end{array} \tag{38}$$

Step 3: If the information about the criteria weights is known as priors, then we can utilize the same. Otherwise, if the information is known as a partially, denoted as H, then we can formulate a model to find the optimal weights for the decision-making problems. In general, in MCDM problems, different criteria may not have equal importance, and hence it is necessary to pay attention to them to determine their suitable value. Further, the importance of the criteria weight depends on the rating of the alternatives given by the experts. To address it completely, we construct an optimization model by maximizing the positive rank expositor of the membership and simultaneously minimizing the negative rank expositor of the membership of the GSVnbN-number $(\tilde{r}_{bN})_{ij}$ for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$, as follows.

$$\begin{aligned} &\text{Maxmize } S = \sum_{j=1}^{n}\sum_{i=1}^{m} \upsilon_j S_{ij} \\ &\text{subject to } 0 \le \upsilon_j \le 1 \\ &\qquad \sum_{j=1}^{n} \upsilon_j = 1 \\ &\qquad \upsilon \in H \\ &\qquad S_{ij} = S_{\mu^+}((\tilde{r}_{bN})_{ij}) - S_{\mu^-}((\tilde{r}_{bN})_{ij}) \end{aligned} \tag{39}$$

Where $\upsilon = \{\upsilon_1, \upsilon_2, \ldots, \upsilon_n\}$ is the set of the criteria weights and H is the partial information about the criteria weights. After solving this model, we get the optimal weight of the criteria.

Based on these weight vectors, we constructed a weighted decision matrix $[\tilde{E}]_{m\times 1}$ by multiplying each column of the decision matrix $\tilde{R} = [(\tilde{r}_{bN})_{ij}]_{m\times n}$ with its associated weight $\{\upsilon_j\}$, i.e., $[\tilde{E}]_{m\times 1} = [(\tilde{r}_{bN})_{ij}]_{m\times n} \bullet [\upsilon_j]_{n\times 1}$, where $\bullet$ noted the matrix multiplication. The formulated weighted decision matrix is summarized as

$$[\tilde{E}]_{m\times 1} = \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_m \end{array} \begin{pmatrix} (\tilde{e}_{bN})_{11} \\ (\tilde{e}_{bN})_{21} \\ (\tilde{e}_{bN})_{31} \\ \vdots \\ (\tilde{e}_{bN})_{m1} \end{pmatrix}$$

Step 4: Computed the positive and negative rank expositor S_{μ^+} and S_{μ^-} of each alternatives using Eq. (34) & (35) respectively.

Step 5: Finally, rank the alternatives A_i, $i = 1, 2, \ldots, m$ according to the value of the rank expositor. The flowchart of the proposed algorithm is presented in Fig. 2.

Fig. 5. Dryness of agriculture field in the Purulia District.

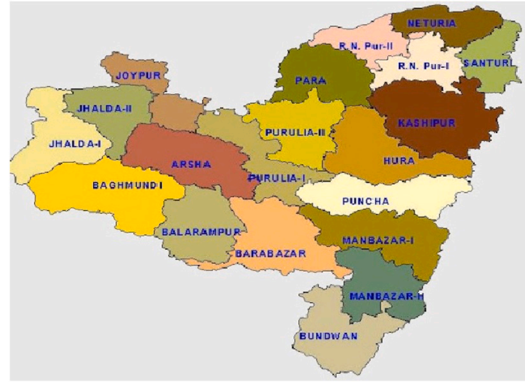

Fig. 6. Drought block area in the Purulia District.

5. Numerical illustration

In this section, we have discussed agriculture productivity is hampered due to the fewer water resources in the Purulia district, West Bengal, India. This district is a drought-prone area in West Bengal (cf. Fig. 3). The Purulia district has only a few rivers, and during the summertime, the rivers of this district disclose a lot of water scarcity. The major crop yield of this district depends on rainwater. Every year, the major repeated disaster is drought, which affects the community life and livelihood of the Purulia district. Due to climate change, the winter temperature is rising, and the variability of rainfall is also increasing (cf. Fig. 4). During the years (2017–2019) citep-Baurietal2020, the Kashipur (Shiulibari, Baykara, Sirjam), Purulia II (Jahajpur), and Manbazar-II (Bari) block areas of the Purulia district are facing the maximum and minimum temperatures (cf. Fig. 6). Rainfall has decreased during the pre-monsoon and post-monsoon time in this block area of the Purulia district. The crop production of the entire Purulia district depends on the rainfall due to the absence of any viable irrigation (surface/subsurface) facility. For the variability of rainfall, crop production is decreasing day by day. However, agriculture is the primary source of income in these block areas. Therefore, much more is needed to develop viable irrigation (surface/subsurface) for agriculture. Otherwise, in the future, the vulnerability of the agriculture sector will be exacerbated in these areas. In the summertime, the Purulia district crop yield is very uncertain compared to other West Bengal districts. The per month crop water demand has been estimated (Irrigation water requirement and adequate rain) for different crops (Aus, Aman, Boro, Mustard, Wheat, Potato, etc.) in the block areas of this district. Also, agriculture productivity is frustrated due to dry weather and less rainfall (cf. Fig. 5). The farmer of this district followed the different techniques and strategies to cope with the natural animosity. These techniques manage the water resources for agricultural purposes.

5.1. Decision making techniques (alternatives) for mange the water resources of agriculture purpose

Technique-1(A_1) **:** Water conservation and storage by digging pit at agriculture land

During the period November to February, a small amount of water is required for the production of the Rabi crop. Due to inadequate rainwater, in this period, crop fields suffered from drought situations. To overcome this situation, the local area people of Purulia district follow the alternative technique, which is called happa (cf. Fig. 7) (happa is a square formed pit in the lower agricultural land with 5–6 ft. depth and 1/6 area of total land).

Technique-2(A_2) **:** Rain water conservation by Khal

Purulia district is a drought-prone area in West Bengal. With every possible technique, this district people try to conserve the rainwater

Fig. 7. Water conservation and storage by digging pit at agriculture land (happa).

Fig. 8. Rain water conservation by Khal.

and use it for agricultural purposes. They dig a pit near the farming land called Khal, a 50 sq. ft. area and a ten ft. depth pit. During heavy rainfall or monsoon time, rainwater from the rooftop is directed to the pit through the open earthen drainage. In the spring or summertime, the farmers used this Khal (cf. Fig. 8) water for agriculture purposes.

Technique-3(A_3) **:** Embankment on river

In the summertime, when all water sources gradually decrease that time farmer faces a big problem. Due to the drought-prone, the summer crops affect drastically. The peoples of Purulia chose the alternative way, which is the embanking of the river. This district people handled the situation using the embanking (cf. Fig. 9) process. To make the river's embankment, the people of Purulia choose the lower region of the river. During the summertime, when the water availability is

Fig. 9. Embankment on river.

Fig. 10. Drip watering system by bamboo reed.

reduced, this time embank region's water fill-up. Using this water, Purulia district people continue day-to-day agriculture activities.

Technique-4(A_4) **:** Drip watering system by bamboo reed

The rural peoples of Purulia make a pipe structure with the whole bamboo of the desired length to remove its inner joints. This natural pipe is suitable for carrying the water from the water source (waterfalls) to agricultural land. Using this procedure, water is poured into the natural pipe and released into the crop field(cf. Fig. 10). This type of watering irrigation system is trendy in the Purulia district.

5.2. Decision making criteria

C_1**: Water availability in the water resources techniques**

Purulia district is a drought-prone area in West Bengal. Therefore, during the summertime, water availability has great importance regarding the demand for water for agricultural purposes. So this is an important criterion in the possible water resource techniques.

C_2**: Cost implementation of the techniques**

The economic benefit is an essential issue in implementing the proposed technique. The management of the plan is motive to design the water resource with low cost for getting the maximum chance benefit of cost. Therefore, these criteria are reasonable for the implementation of the techniques.

C_3**: Feasibility of the techniques**

Feasibility is an effective criterion for implementing the technique concerning the required time, human resources, facilities, skills, etc.

Now, the problem is to find the best water resource technique from the possible methods. So we try to find the best alternative from the set of four alternatives (A_1, A_2, A_3, A_4) on the basis of three criteria (C_1, C_2, C_3). We try to evaluate the best water resource technique from the possible four methods (alternative) based on the three criteria. The alternative information's considered by GSVnTbN-numbers. We have solved the above decision-making problem using the proposed ranking method. Finally, we have summarized the following steps for the proposed ranking method.

Step 1: For the formulation of the decision matrix, we have expressed the available information of the four water resource techniques in terms of GSVnTbN-numbers based on the three criteria. The representation of the decision matrix is given in (40) by Eq. (36) (see Box I).

Step 2: Determine the normalized decision matrix using Eq. (37). Here we assume that all criteria are benefit type. Then we formulate a normalized decision matrix $[\tilde{A}]_{4\times 3}$ to $[\tilde{R}]_{4\times 3}$ according to Eq. (37) & (38) (see Box II).

Step 3: Assume that the information about criteria weight υ_j is partly known, and the partial weight information is given as follows: $H = \{\upsilon_1 \geq 0.36, 0.28 \leq \upsilon_2 \leq 0.34, 0.30 \leq \upsilon_3 \leq 0.33\}$. By utilizing the ratings given in Eq. (41), we formulated an optimization model according to Eq. (39) to determine the criteria weights

$$\begin{aligned} &\text{Maxmize } S = \sum_{j=1}^{3}\sum_{i=1}^{4} \upsilon_j S_{ij} \\ &\text{subject to } 0 \leq \upsilon_j \leq 1 \\ &\qquad \sum_{j=1}^{3} \upsilon_j = 1 \\ &\qquad \upsilon \in H \\ &\qquad S_{ij} = S_{\mu^+}((\tilde{r}_{bN})_{ij}) - S_{\mu^-}((\tilde{r}_{bN})_{ij}) \end{aligned} \tag{42}$$

To address this Eq. (42), we first compute $S_{ij} = S_{\mu^+}((\tilde{r}_{bN})_{ij}) - S_{\mu^-}((\tilde{r}_{bN})_{ij})$ for $i = 1, 2, 3, 4$, $j = 1, 2, 3$ from the ratings given in Eq. (41) by using the expressions of S_{μ^+} and S_{μ^-} as mentioned in Eqs. (34), (35) respectively. The values of the matrix $S = [S_{ij}]_{4\times 3}$ are computed as

$$S = [S_{ij}]_{4\times 3} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{pmatrix} 0.45 & 0.65 & 0.54 \\ 0.48 & 0.52 & 0.50 \\ 0.54 & 0.49 & 0.62 \\ 0.53 & 0.50 & 0.60 \end{pmatrix} \end{array}$$

Hence, the optimization model (42) becomes

$$\begin{aligned} &\text{Maxmize } S = 2.00\upsilon_1 + 2.16\upsilon_2 + 2.26\upsilon_3 \\ &\text{subject to } 0 \leq \upsilon_j \leq 1 \\ &\qquad \sum_{j=1}^{3} \upsilon_j = 1 \\ &\qquad \upsilon_1 \geq 0.36, 0.28 \leq \upsilon_2 \leq 0.34, 0.30 \leq \upsilon_3 \leq 0.33 \end{aligned} \tag{43}$$

After solving this linear model, we obtain the solution as $\upsilon_1 = 0.36, \upsilon_2 = 0.31, \upsilon_3 = 0.33$. Thus, the weight of the criteria $\{C_1, C_2, C_3\}$ are $\upsilon = \{0.36, 0.31, 0.33\}$.

Based on this weighted information and rating of alternatives (given in Eq, (41)), we formulated a weighted decision matrix $\tilde{E}$ by multiplying the column element of $[\tilde{R}]_{4\times 3}$ with associated weighted vector $\upsilon = (0.36, 0.31, 0.33)^T$. The values of the column matrix $\tilde{E}$ are obtained as

$$[\tilde{E}]_{4\times 1} = \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{pmatrix} \langle(.36, .58, .82; .9, 3)(.42, .55, .75; .6, 2)(.35, .53, .82; .5, 2)\rangle \\ \langle(.34, .54, .81; .9, 3)(.43, .57, .74; .6, 2)(.42, .57, .80; .5, 2)\rangle \\ \langle(.40, .59, .73; .9, 3)(.41, .52, .76; .6, 2)(.43, .56, .78; .5, 2)\rangle \\ \langle(.47, .59, .71; .9, 3)(.47, .56, .68; .6, 2)(.44, .52, .73; .5, 2)\rangle \end{pmatrix} \tag{44}$$

Step 4: Computed the rank expositor S_{μ^+} and S_{μ^-} of positive and negative membership function for all the alternatives A_1, A_2, A_3 and A_4

$$[\tilde{A}]_{4\times 3} = \begin{array}{c} \\ A_1 \\ \\ A_2 \\ \\ A_3 \\ \\ A_4 \end{array} \overset{\begin{array}{ccc} C_1 \qquad\qquad\qquad\qquad & C_2 \qquad\qquad\qquad\qquad & C_3 \end{array}}{\begin{pmatrix} \left\langle \begin{matrix} (3,5,8;0.8,3)(2.5,4,7;0.7,2) \\ (4.5,7,9;0.5,2) \end{matrix} \right\rangle & \left\langle \begin{matrix} (4,6,8;0.9,3)(7,8,9;0.6,2) \\ (6,8,12;0.6,2) \end{matrix} \right\rangle & \left\langle \begin{matrix} (5,8,10;0.8,3)(7,9,11;0.6,2) \\ (5.5,9,14;0.5,2) \end{matrix} \right\rangle \\ \left\langle \begin{matrix} (4,6,8;0.7,3)(5,7,9;0.6,2) \\ (6,8,10;0.7,2) \end{matrix} \right\rangle & \left\langle \begin{matrix} (5,6,9;0.9,3)(6,7,9;0.6,2) \\ (7,9,12;0.6,2) \end{matrix} \right\rangle & \left\langle \begin{matrix} (3.5,6,8;0.9,3)(5,7,9;0.6,2) \\ (6,8,11;0.5,2) \end{matrix} \right\rangle \\ \left\langle \begin{matrix} (5,7,9;0.8,3)(6,8,10;0.8,2) \\ (7,9,11;0.6,2) \end{matrix} \right\rangle & \left\langle \begin{matrix} (6,8,9;0.9,3)(6,7,10;0.6,2) \\ (8,10,14;0.5,2) \end{matrix} \right\rangle & \left\langle \begin{matrix} (5,8,10;0.7,3)(6,8,12;0.8,2) \\ (6,9,14;0.6,2) \end{matrix} \right\rangle \\ \left\langle \begin{matrix} (4,6,8;0.8,3)(6,7,9;0.6,2) \\ (7,8,11;0.5,2) \end{matrix} \right\rangle & \left\langle \begin{matrix} (7,8,9;0.9,3)(7,10,12;0.7,2) \\ (8,10,14;0.7,2) \end{matrix} \right\rangle & \left\langle \begin{matrix} (8,9,10;0.7,3)(9,10,12;0.8,2) \\ (9,12,15;0.7,2) \end{matrix} \right\rangle \end{pmatrix}} \quad (40)$$

Box I.

$$[\,\tilde{R}]_{4\times 3} = \begin{array}{c} \\ A_1 \\ \\ A_2 \\ \\ A_3 \\ \\ A_4 \end{array} \overset{\begin{array}{ccc} C_1 \qquad\qquad\qquad\qquad & C_2 \qquad\qquad\qquad\qquad & C_3 \end{array}}{\begin{pmatrix} \left\langle \begin{matrix} (.33,.55,.88;.8,3)(.31,.5,.87;.7,2) \\ (.37,.58,.75;.5,2) \end{matrix} \right\rangle & \left\langle \begin{matrix} (.4,.6,.8;.9,3)(.53,.61,.69;.6,2) \\ (.4,.53,.92;.6,2) \end{matrix} \right\rangle & \left\langle \begin{matrix} (.38,.61,.76;.8,3)(.46,.6,.73;.6,2) \\ (.32,.52,.82;.5,2) \end{matrix} \right\rangle \\ \left\langle \begin{matrix} (.4,.6,.8;.7,3)(.41,.58,.75;.6,2) \\ (.42,.57,.71;.7,2) \end{matrix} \right\rangle & \left\langle \begin{matrix} (.45,.54,.81;.9,3)(.5,.57,.76;.6,2) \\ (.43,.69,.92;.6,2) \end{matrix} \right\rangle & \left\langle \begin{matrix} (.31,.54,.72;.9,3)(.41,.58,.75;.6,2) \\ (.4,.53,.73;.5,2) \end{matrix} \right\rangle \\ \left\langle \begin{matrix} (.41,.58,.75;.8,3)(.42,.57,.71;.8,2) \\ (.46,.6,.73;.6,2) \end{matrix} \right\rangle & \left\langle \begin{matrix} (.46,.61,.69;.9,3)(.46,.53,.76;.6,2) \\ (.44,.55,.77;.5,2) \end{matrix} \right\rangle & \left\langle \begin{matrix} (.38,.61,.76;.7,3)(.4,.53,.8;.8,2) \\ (.35,.52,.82;.6,2) \end{matrix} \right\rangle \\ \left\langle \begin{matrix} (.4,.6,.8;.8,3)(.5,.58,.75;.6,2) \\ (.46,.53,.73;.5,2) \end{matrix} \right\rangle & \left\langle \begin{matrix} (.53,.61,.69;.9,3)(.41,.58,.7;.7,2) \\ (.44,.55,.77;.7,2) \end{matrix} \right\rangle & \left\langle \begin{matrix} (.53,.6,.66;.7,3)(.5,.55,.66;.8,2) \\ (.42,.57,.71;.7,2) \end{matrix} \right\rangle \end{pmatrix}} \quad (41)$$

Box II.

according to the values of $\mu \in [0, 0.5)$, $\mu = 0.5$ and $\mu \in (0.5, 1)$. First, we calculated the possibilistic mean value of authenticity, hesitate, and falsity membership functions which are given below

$M_{T^+}(A_1) = 0.70, \qquad M_{T^+}(A_2) = 0.72,$
$M_{T^+}(A_3) = 0.72, \qquad M_{T^+}(A_4) = 0.77$
$M_{T^-}(A_1) = -0.75, \qquad M_{T^-}(A_2) = -0.69,$
$M_{T^-}(A_3) = -0.70, \qquad M_{T^-}(A_4) = -0.74$
$M_{I^+}(A_1) = 0.78, \qquad M_{I^+}(A_2) = 0.74,$
$M_{I^+}(A_3) = 1.14, \qquad M_{I^+}(A_4) = 0.85$
$M_{I^-}(A_1) = -0.87, \qquad M_{I^-}(A_2) = -0.84,$
$M_{I^-}(A_3) = -1.38, \qquad M_{I^-}(A_4) = -0.83$
$M_{F^+}(A_1) = 1.08, \qquad M_{F^+}(A_2) = 0.87,$
$M_{F^+}(A_3) = 1.24, \qquad M_{F^+}(A_4) = 0.81$
$M_{F^-}(A_1) = -0.98, \qquad M_{F^-}(A_2) = -0.78,$
$M_{F^-}(A_3) = -0.74, \qquad M_{F^-}(A_4) = -0.69$

Now, we computed the rank expositor S_{μ^+} and S_{μ^-} with the help of possibilistic mean values of A_1, A_2, A_3 and A_4 for different value of $\mu \in [0, 0.5)$, $\mu = 0.5$ and $\mu \in (0.5, 1)$. For $\mu = 0.5$, the positive rank expositor

$S_{\mu^+}(A_1) = 0.59, \qquad S_{\mu^+}(A_2) = 0.52,$
$S_{\mu^+}(A_3) = 0.66, \qquad S_{\mu^+}(A_4) = 0.56$

and the negative rank expositor

$S_{\mu^-}(A_1) = -0.53, \qquad S_{\mu^-}(A_2) = -0.60,$
$S_{\mu^-}(A_3) = -0.51, \qquad S_{\mu^+}(A_4) = -0.57$

However, for $\mu \in [0, 0.5)$ and $\mu \in (0.5, 1)$, we have calculated the rank expositor values which have been shown in Table 3.

Step 5: Now we have ranked the alternatives according to the value of the rank expositor. From Table 3, it is clear that the decision-maker $\mu \in [0, 0.5)$ display that the decision-maker is risk-flat; $\mu = 0.5$ display that the decision-maker is risk-impartial; $\mu \in (0.5, 1)$ shows that the decision-maker is risk-apathetic. So for $\mu \in [0, 0.5]$ we get the best ranking order of alternatives A_1, A_2, A_3 and A_4. Hence according to positive rank expositor (S_{μ^+}) and negative rank expositor (S_{μ^-}) value, the ranking order of the alternatives is $A_3 > A_1 > A_4 > A_2$. So the best alternative is A_3 and worst one is A_2.

6. Discussion

In West Bengal, the Purulia district is a drought-prone area. Most of the time a year, the Purulia district's farmers face a water crisis. So the agriculture fields of this district are nonsuitable for the crops due to the unavailable water. Therefore, water conservation technology development has been needed in that area. Additionally, proper management of the available water is essential during the whole year. In the proposed problem, we have considered four alternatives which are A_1(water conservation and storage by digging a pit on agricultural land), A_2 (rainwater conservation by Khal), A_3 (embankment on the river), A_4 (Drip watering system by bamboo reed) with three criteria C_1 (water availability in the water resources techniques), C_2 (cost implementation of the techniques), C_3 (feasibility of the techniques). According to the value of positive rank expositor (S_{μ^+}) and negative rank expositor (S_{μ^-}), we got the ranking order of alternatives is $A_3 > A_1 > A_4 > A_2$. The graphical representation of ranking order is presented in Fig. 11 according to different values of the rank expositor. From Table 3, it is clear that the best alternative is A_3, and the worst one is A_2 for

Table 3
Sensitivity analysis for different values of μ.

μ	Rank Expositor	A_1	A_2	A_3	A_4	Ranking Order
0.0	S_{μ^+}	0.86	0.76	1.04	0.82	$A_3 > A_1 > A_4 > A_2$
	S_{μ^-}	−0.85	−0.74	−0.94	−0.76	
0.10	S_{μ^+}	0.80	0.71	0.99	0.78	$A_3 > A_1 > A_4 > A_2$
	S_{μ^-}	−0.82	−0.71	−0.90	−0.74	
0.20	S_{μ^+}	0.75	0.69	0.96	0.72	$A_3 > A_1 > A_4 > A_2$
	S_{μ^-}	−0.87	−0.64	−0.88	−0.67	
0.30	S_{μ^+}	0.76	0.63	0.93	0.68	$A_3 > A_1 > A_4 > A_2$
	S_{μ^-}	−0.81	−0.63	−0.84	−0.68	
0.40	S_{μ^+}	0.69	0.62	0.88	0.64	$A_3 > A_1 > A_4 > A_2$
	S_{μ^-}	−0.56	−0.58	−0.54	−0.57	
0.50	S_{μ^+}	0.59	0.52	0.66	0.56	$A_3 > A_1 > A_4 > A_2$
	S_{μ^-}	−0.53	−0.60	−0.51	−0.57	
0.60	S_{μ^+}	0.46	0.46	0.58	0.48	$A_4 > A_2 > A_1 > A_3$
	S_{μ^-}	−0.49	−0.44	−0.52	−0.42	
0.70	S_{μ^+}	0.40	0.44	0.52	0.48	$A_3 > A_4 > A_2 > A_1$
	S_{μ^-}	−0.39	−0.41	−0.48	−0.40	
0.80	S_{μ^+}	0.39	0.41	0.46	0.43	$A_3 > A_4 > A_2 > A_1$
	S_{μ^-}	−0.41	−0.38	−0.45	−0.31	
0.90	S_{μ^+}	0.36	0.38	0.42	0.39	$A_3 > A_4 > A_2 > A_1$
	S_{μ^-}	−0.36	−0.35	−0.42	−0.29	

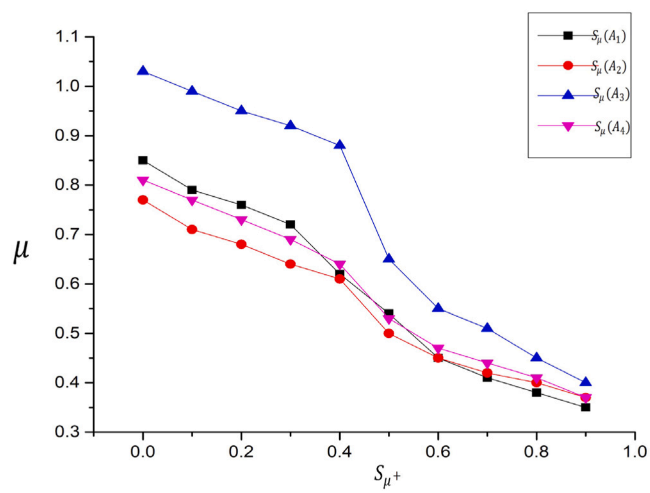

Fig. 11. Graphical representation of raking orders with respect to rank expositor value.

$\mu \in [0, 0.5]$. And for $\mu \in (0.5, 1)$ decision-maker is risk-apathetic according to the value of rank expositor S_{μ^+} and S_{μ^-}. When $\mu = 0.6$, the positive rank expositor S_{μ^+} value are same for the alternatives A_1 and A_2, that time we moved into the negative rank expositor S_{μ^-} values for all alternatives. In this case, the best alternative is A_4. Therefore positive and negative rank expositors (S_{μ^+}, S_{μ^-}) have great importance in the ranking of bipolar numbers. So, according to our observation, 'embankment on the river' is the best water resource technique for balancing the water crisis in the agriculture field of the Purulia district.

6.1. Comparative analysis

In this section, we have presented the comparative analysis of our proposed method with other existing methods. Also, we have demonstrated the effectiveness and feasibility of the proposed method. Our proposed ranking expositor method is compared with other existing methods like Deli and Subas (2017) (Values and Ambiguities), Chakraborty et al. (2019) (De-Bipolarization), Riaz et al. (2021) (Distance measures). The comparative analysis results of our proposed method are given in Table 4. Using the Deli and Subas (2017) method, the best alternative is A_1, and the worst one is A_4. By the Chakraborty et al. (2019) method, the best alternative is A_2, and the worst one is A_1. Using the Riaz et al. (2021) method, the best alternative is A_1, and the worst one is A_4. But, according to our proposed method, the best alternative is A_3, and the worst one is A_2. Again, we compared with Golfaml et al. (2019) VIKOR and FOWA MCDM techniques. In this method, the best alternative A_1 and the worst one is A_3. By the Alghamdi et al. (2018) BF-MCDM technique, the best alternative A_2 and the worst one is A_1. In this problem, Alghamdi et al. (2018) method is not easy to handle to find the best water resource technique. For the complex type of decision-making problem, the rating of the alternative is not a linear type of fuzzy nature. Our proposed method is straightforward to handle for this type of decision-making problem. For this water resource management, our proposed method gives the best alternative is A_3. According to the Purulia district environment, the alternative A_3 is the suitable path. Therefore, for this WRM, our proposed method gives a better result than the other existing methods.

7. Conclusion and future work plan

Nowadays, water resources have become more sparse in the agriculture field of the Purulia district. Management mastery is necessary to recognize the dynamic water resource management's intention. Multi-criteria decision-making techniques may lead to the water resource management problem in the Purulia district. A significant contribution of this paper is trying to develop an MCDM technique for water resource management problems in the different block areas of the Purulia district. This paper has invented a novel ranking method of generalized single-valued non-linear bipolar neutrosophic numbers based on possibility measures. With the help of a possibilistic means, we introduced a positive and negative rank expositor (S^+_μ, S^-_μ). This positive and negative rank expositor (S^+_μ, S^-_μ) has a great important role in the proposed MCDM technique. Using our proposed MCDM method, we have solved one real water resource management problem and found the best water conservation technique from the fourth technique (alternatives). According to the different values of $\mu \in [0, 1)$, we have presented the sensitivity analysis in Table 3. From Table 3, it is clear that for $\mu \in [0, 1]$, we got the best alternative is A_3, i.e., The embankment of the river is the best water resource technique from the selective four techniques. So, the proposed MCDM technique can provide a better water conservation technique than the particular techniques.

In the future, the proposed technique can apply to various real problems such as sustainable water resource management problems, case studies of water resource management problems, multi-objective optimization problems, multi-criteria group decision-making problems, medical diagnoses problems, etc. Further, there is much scope to improve the possibilistic concept in different fuzzy numbers such as

Table 4
Comparative studies.

Methods	Methods	Ranking order	Best alternative
Deli and Subas (2017)	Values and Ambiguities	$A_1 > A_3 > A_2 > A_4$	A_3
Chakraborty et al. (2019)	De-Bipolarization	$A_2 > A_3 > A_4 > A_1$	A_2
Riaz et al. (2021)	Distance measures	$A_1 > A_2 > A_3 > A_4$	A_4
Golfaml et al. (2019)	VIKOR and FOWA MCDM	$A_1 > A_4 > A_3 > A_3$	A_1
Alghamdi et al. (2018)	BF-MCDM technique	$A_2 > A_3 > A_4 > A_1$	A_2
Our Method	Ranking Expositor	$A_3 > A_1 > A_4 > A_2$	A_3

non-linear fuzzy, Fuzzy rough set, Pythagorean set, etc. Moreover, various researchers can establish bipolar neutrosophic sets in many fields such as mathematical modelling, social sciences, engineering problem, artificial intelligence, and many other areas.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Ethical approval: This article does not contain any studies with animals performed by any of the authors.

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