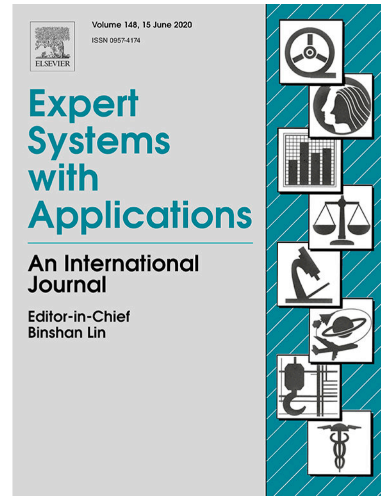

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Reliability Allocation Method Based on Linguistic Neutrosophic Numbers Weight Muirhead Mean Operator

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Abstract—This article presents a reliability allocation method based on Linguistic Neutrosophic Numbers Weight Muirhead mean operator. In order to solve the problem of insufficient consideration of influencing factors in traditional reliability allocation methods, six influencing factors are proposed in the proposed method. Using Linguistic Neutrosophic Numbers, the fuzzy information in the process of reliability allocation can be evaluated completely. The Linguistic Neutrosophic Numbers Weight Muirhead mean operator proposed by this article can process the scoring results by considering the correlation and weight of influencing factors. With the introduction of the safety factor, the exact values of the allocation results can be transformed into interval values. Finally, an example is given to verify the superiority of the proposed method.

Keywords: Reliability allocation; Linguistic Neutrosophic Numbers; Linguistic Neutrosophic Numbers Weight Muirhead mean operator; Reliability allocation factors.

1. Introduction

This article firstly combines Linguistic Neutrosophic Number(LNN) and Muirhead mean(MM) operator to propose Linguistic Neutrosophic Numbers Muirhead mean (LNNMM) and Linguistic Neutrosophic Numbers Weight Muirhead mean (LNNWMM) operators, and a new reliability allocation method based on LNNWMM operator is proposed. Now, the developments of reliability allocation methods, LNN and MM operator are shown as follows:

1.1 Research motivation

Reliability is an important attribute of the product. Product reliability design can effectively improve product reliability. Reliability design includes reliability allocation(Yadav and Zhuang, 2014, Yu *et al.* 2019), reliability evaluation(Hund and Schroeder, 2019), remaining useful life prediction(Zhou *et al.* 2020, Liu *et al.* 2021) and so on. As an important method in reliability design, whether the reliability allocation result is reasonable or not is directly related to the reliability of the product. So how to carry out the reasonable and effective reliability allocation of the product has become a research hotspot of scholars(Joon-Eon *et al.* 1999, Pregelj *et al.* 2006, Pietrantuono *et al.* 2010, Liang *et al.* 2015, Zhang and Chen, 2016).

1.2 Literature review

Traditional reliability allocation methods include Aeronautical Radio Inc. (ARINC)(Alven, 1964), the integrated factor method, the feasibility of objectives (FOO) technique(Anderson, 1976, Chang *et al.* 2009) and so on. These methods are easy to operate, but reasonable and accurate allocation results are often difficult to obtain in the face of increasingly complex products.

In 1999, Joon-Eon (1999) applied genetic algorithm to the reliability allocation problem of a typical pressurised water reactor. In 2017, Pranab K Muhuri (2017) solved the multiobjective reliability redundancy allocation problem by using the non-dominated sorting genetic algorithm-II. Although these methods can ensure the maximum benefit under the constraint conditions, the results of reliability allocation will have a large deviation due to the difficulty of accurately obtaining the functional relationship between subsystem cost and reliability. Therefore, these methods have small application scope.

In addition, the subsystem can also be evaluated through the experience judgment of team members to realise the reliability allocation of the product. In 2001, Y Wang (2001) proposed a reliability allocation method for CNC lathes. In 2013, Kim (2013) proposed a method that determined the weight of reliability allocation based on the subsystem failure severity and its relative frequency. In 2009, Yung-chia Chang (2009) proposed a reliability allocation method based on the maximum entropy ordered weighted averaging (ME-OWA) operator. In 2016, Chang (2016) integrated the ordered weighted averaging tree and soft set approach with regard to make allocation product reliability more flexible.

All the above methods have improved the traditional reliability allocation methods, but they are generally unable to deal with fuzzy information. To solve this problem, in 2014, V. Sriramdas (2014) introduced trapezoidal fuzzy numbers into reliability allocation. Cheng (2019) combined trapezoidal fuzzy numbers with ME-OWA operators in 2019 and proposed a reliability allocation method. These methods can deal with fuzzy information, but they cannot express the hesitant information between the membership degree and the non-membership degree. Thus, the ability of these methods to deal with fuzzy information are limited. In order to more completely express the fuzzy information in the process of reliability allocation, this article introduces a Linguistic Neutrosophic Number (LNN) term set to describe fuzzy information.

In 2017, Fang (2017) firstly proposed LNN, which can describe the membership degree, non-membership degree and hesitation information at the same time. Therefore, compared with trapezoidal fuzzy number, LNN can describe fuzzy information more comprehensively(Luo *et al.* 2019). Many scholars use LNN to deal with decision problems containing fuzzy information (Tan *et al.* 2017, Fan and Ye, 2018, Liu *et al.* 2018, Wang and Liu, 2018). For example, Pamucar (2018) modified the combinative range-based assessment method by using LNN and solved the problem of optimal power generation technology in Libya. Liang (2017) combined LNN with technique for order preference by similarity to an ideal solution to solve the risk of investment in metallic mining projects. Wang (2018) presented a distance formula of the interval linguistic neutrosophic numbers power average operator. Liang (2018) applied the Hamacher aggregation operator based on LNN extension to select an optimal land reclamation scheme. Shi (2017) extended the cosine similarity measures by using multiple-attribute group decision-making method based on the cosine similarity measures under an LNN environment. In the above studies, LNN shows good fuzzy information processing capability. Considering that a large amount of fuzzy information is generated in the process of reliability allocation, the introduction of LNN into reliability allocation will also produce a good effect.

To reduce the difficulty of reliability allocation, the influencing factors of reliability allocation are generally assumed to be independent of each other and have no correlation. However, in engineering practice, a certain correlation exists between reliability influencing factors. For example, a more complex structure corresponds to a high manufacturing cost and more difficult maintenance. If the correlation between factors is not considered, then the accuracy of the reliability allocation result will inevitably be reduced. Therefore, the Muirhead mean (MM) operator is introduced to process the information of influencing factors. The MM operator was firstly proposed by Muirhead(1902), and its outstanding feature is that it can capture the overall interrelation between multiple input parameters. Therefore, the MM operator has attracted the attention of many researchers (Liu and Li, 2017, Liu and You, 2017, Peide and Fei, 2018, Wang *et al.* 2019). For example, Qin (2016) took 2-tuple linguistic to extend the MM operator and applied it to the multiple-attribute decision making problem. Yuan (2018) combined picture fuzzy set with MM operator and proposed a new method to solve the multiple-attribute decision making problem. Zhu (2018) combined a Pythagorean fuzzy set with the MM operator and applied it in enterprise resource selection.

1.3 Necessity of the research

With the continuous development of technology, products are developing towards the direction of precision, complexity and intelligence, and a large amount of fuzzy information will be encountered in the reliability allocation of products. Moreover, with the increase in the influencing factors, the correlation between influencing factors will have a greater impact on the reliability allocation results. Due to the shortcomings of the traditional reliability allocation methods, such as insufficient consideration of factors, weak ability to deal with fuzzy information and failure to consider the correlation between influencing factors, it is difficult to get reasonable reliability allocation results. Therefore, this article proposes a reliability allocation method based on LNNWMM operator to solve these shortcomings.

1.4 Novelty and main contributions

In order to solve the problem of insufficient consideration of influencing factors in traditional reliability allocation methods, six influencing factors are proposed. LNN can solve the problem of inaccurate fuzzy information evaluation in the process of reliability allocation. The LNNWMM operator proposed by us can aggregate the evaluation information on the premise of considering the correlation and weight between influencing factors. The combination of these two points greatly improves the rationality and validity of reliability allocation results. By using the safety factor, the exact value in the allocation result can be converted into interval number, and the flexible allocation of system reliability can be realized.

1.5 Organization and structure of the article.

The rest of this article is organised as follows: The second section reviews basic theories such as the FOO method, LNN and MM operator. The third section proposes the reliability allocation method based on the LNNWMM operator. The fourth section takes machining center as an example to compare and analyzes the proposed method, FOO method and fuzzy allocation method. The fifth section summarises the article.

The last section looks forward to the future research direction.

2. Basic theory

This section describes the basic concepts of the FOO method, LNN, and MM operator.

2.1 Reliability allocation based on FOO method.

In 1976, the FOO method was introduced into the U.S. military standard MIL-HDBK-338B to solve the reliability allocation problem. The reliability allocation process of the FOO method is as follows: Firstly, the product is divided into several subsystems. Then, the complexity (I), technical level (S), working time (P) and working environment (E) of each subsystem are scored by using the expert scoring method, and the four factors are multiplied to obtain the comprehensive score, namely, $ISPE = I * S * P * E$. Afterwards, the complexity of the subsystem is calculated. Finally, the reliability is allocated according to the complexity of each subsystem.

Suppose a system contains N subsystems, the failure rate of the system is λ_s, the failure rate allocated by subsystem k is $\bar{\lambda}_k$, the reliability design goal of the system is R, the running time is T, the complexity of subsystem k is expressed as C'_k, the overall score of subsystem k is w'_k, the sum of the comprehensive scores of all subsystems is W' and the score of subsystem k on factor i is r'_{ik}, where $\forall i \in \{I, S, P, E\}$. According to the FOO (Anderson, 1976):

$$\lambda_s = \frac{-\ln(R)}{T} \quad (1)$$

$$\lambda_s T = \bar{\lambda}_k T \tag{2}$$

$$\bar{\lambda}_k = C_k^{'} \lambda_s, \forall k \tag{3}$$

$$C_k^{'} = \frac{w_k^{'}}{W^{'}}, \forall k \tag{4}$$

$$w_k^{'} = r_{Ik}^{'} \times r_{Sk}^{'} \times r_{Pk}^{'} \times r_{Ek}^{'}, \forall k \tag{5}$$

$$W^{'} = \sum_{k=1}^{N} w_k^{'} \tag{6}$$

2.2 LNNs

LNNS (Linguistic Neutrosophic Numbers) was first proposed by Fang and Ye to deal with consistent, indeterminate and inconsistent linguistic information. Its definitions are as follows:

Definition 1. (Liang *et al.* 2018) Suppose there are a group of linguistic variables $t_i\,(i = 0,1,\ldots,2h)$. For all $i,j = 0,1,\ldots,2h$, the followings are true: (1) When $i < j$, then $t_i < t_j$; (2) when $i = j$, then $t_i = t_j$; (3)when $i > j$, then $t_i > t_j$. And the operational rules of linguistic variables are: $t_i \oplus t_j = t_{i+j}$and $r * t_i = t_{ri}\,(r \geq 0)$. When a collection of linguistic values exist, a discrete linguistic term set is denoted as $T^* = \{t_i \mid i = 0, 1,\ldots, 2h\}\,(h > 0)$, and a continuous linguistic term set is expressed as $T = \{t_i \mid i \in [0, 2s]\}\,(s > 0)$.

Definition 2. (Liang *et al.* 2018) A LNN (linguistic neutrosophic number) is denoted as $a = (t_{T_a}, t_{I_a}, t_{F_a})$, where t_{T_a}, t_{I_a} and t_{F_a} are three independent linguistic terms, which respectively represent true, hesitant and false membership degrees.

Definition 3. (Liang *et al.* 2018) If $T=\{t_i \mid i\in[0,2s]\}$ $(s>0)$ is a continuous linguistic term set and $a=(t_{T_a},t_{I_a},t_{F_a})$ is a LNN, then the score function of a is $U(a)=(4s+T_a-I_a-F_a)/6s$, and the accuracy function of a is $V(a)=(T_a-F_a)/2s$.

Definition 4. (Liang *et al.* 2018) Let $b=(t_{T_b},t_{I_b},t_{F_b})$ and $c=(t_{T_c},t_{I_c},t_{F_c})$ be two LNNs, their order relationship can be described as follows:

(1) if $U(b)>U(c)$, then $b\succ c$;

(2) if $U(b)=U(c)$, then $\begin{cases} V(b)>V(c)\Rightarrow b\succ c \\ V(b)=V(c)\Rightarrow b\cong c \end{cases}$.

2.3 MM operator

Muirhead (Muirhead and F., 1902) proposed the Muirhead mean (MM) operator.

Definition 5. Assume that $x_j(j=1,2,\ldots,n)$ is a set of non-negative real numbers, and let $[p]=(p_1,p_2,\ldots,p_n)\in R^n$ be a vector of parameters. If $MM^{[p]}(x_1,x_2,\ldots,x_n)=\left(\frac{1}{n!}\sum_{\sigma\in S_n}\prod_{j=1}^{n}x_{\sigma(j)}^{p_j}\right)^{(1/\sum_{j=1}^{n}p_j)}$, then we call $MM^{[p]}$ the Muirhead mean (MM) operator, where $\sigma(j)(j=1,2,\ldots,n)$ is any permutation of (1,2,...,n}, and S_n is the set of all permutations of (1,2,...,n).

3. Reliability allocation method based on LNNWMM operator

This section proposes LNNMM and LNNWMM operators and gives the basic definitions of them. And a new reliability allocation method based on LNNWMM operator is proposed.

3.1 LNNMM and LNNWMM operators

Combining LNN with MM operator, LNNMM and LNNWMM operators are proposed.

Definition 6. Given a continuous linguistic term set $T = \{t_i | i \in [0,2s]\}$ and a set of LNNs—α_i (i = 1,2,…,n). Assume a conversion function is $g(t_i) = i/2s$,and the corresponding converse function is $g^{-1}(i) = t_{2s \times i}$, then

$$LNNMM^{[p]}(a_1, a_2, ..., a_n) = \begin{pmatrix} g^{-1}[(\frac{1}{n!}\sum_{\sigma \in S_n}\prod_{j=1}^{n} g(t_{T_{a_j}})_{\sigma(j)}^{p_j})^{\frac{1}{\sum_{j=1}^{n} p_j}}], \\ g^{-1}[(\frac{1}{n!}\sum_{\sigma \in S_n}\prod_{j=1}^{n} g(t_{I_{a_j}})_{\sigma(j)}^{p_j})^{\frac{1}{\sum_{j=1}^{n} p_j}}], \\ g^{-1}[(\frac{1}{n!}\sum_{\sigma \in S_n}\prod_{j=1}^{n} g(t_{F_{a_j}})_{\sigma(j)}^{p_j})^{\frac{1}{\sum_{j=1}^{n} p_j}}] \end{pmatrix} \quad (7)$$

LNNMM$^{[p]}$is called a LNNMM operator, where $[p] = (p_1,p_2,...,p_n) \in R^n$ is a vector of parameters, $\sigma(j)(j = 1,2,...,n)$ is any permutation of (1,2,...,n), and S_n is the set of all permutations of (1, 2,..., n).

Definition 7. Suppose $LNNMM^{[p]}(a_1,a_2,...,a_n) = (t,i,f)$ is a LNNMM operator, then the score function is $U = (4s + t - i - f)/6s$, and the accuracy function is $V = (t - f)/2s$.

Definition 8. Suppose $A_i = (t_{T_i},t_{I_i},t_{F_i}),(i = 1,2,...,n)$ is a set of LNNs, and the weight vector is $W = (w_1,w_2,...,w_n)^T$, where $w_i \in [0,1]$ and$\sum_{i=1}^{n} w_i = 1$. Assume a conversion function is $g(t_i) = i/2s$, and the corresponding converse function is g^{-1} $(i) = t_{2s \times i}$. Then

$$LNNWMM_{nW}^{P}(A_1, A_2, \ldots, A_n) = \begin{pmatrix} g^{-1}[2s(1-\prod_{\sigma\in S_n}(1-\prod_{j=1}^{n}(1-(1-\frac{g(t_{T_{\sigma(j)}})}{2s})^{nw_{\sigma(j)}})^{P_j})^{\frac{1}{n!}})^{\frac{1}{\Sigma_{j=1}^{n}p_j}}], \\ g^{-1}[2s(1-(1-\prod_{\sigma\in S_n}(1-\prod_{j=1}^{n}(1-(\frac{g(t_{I_{\sigma(j)}})}{2s})^{nw_{\sigma(j)}})^{P_j})^{\frac{1}{n!}})^{\frac{1}{\Sigma_{j=1}^{n}p_j}})], \\ g^{-1}[2s(1-(1-\prod_{\sigma\in S_n}(1-\prod_{j=1}^{n}(1-(\frac{g(t_{F_{\sigma(j)}})}{2s})^{nw_{\sigma(j)}})^{P_j})^{\frac{1}{n!}})^{\frac{1}{\Sigma_{j=1}^{n}p_j}})] \end{pmatrix} \quad (8)$$

LNNWMM_{nW}^{P} is called a LNNWMM operator, where $[p] = (p_1,p_2,\ldots,p_n) \in R^n$ is a vector of parameters, $\sigma(j)$ $(j = 1,2,\ldots,n)$ is any permutation of (1,2,...,n), and S_n is the set of all permutations of (1,2,...,n).

Definition 9. Suppose $LNNWMM^{[p]}(a_1,a_2,\ldots,a_n) = (t,i,f)$ is a LNNWMM opera, then the score function is $U = (4s + t - i - f)/6s$, and the accuracy function is $V = (t - f)/2s$.

3.2 Reliability allocation method based on LNNWMM operator

Traditional reliability allocation methods usually have some shortcomings such as inaccurate evaluation, serious information loss and not considering the correlation between influencing factors. The LNNWMM operator is introduced into the reliability allocation to solve the above problems, and its specific implementation steps are as follows:

Step 1: The overall system reliability allocation index is determined;

Step 2: The safety factor is determined, and the actual reliability allocation index is calculated;

$$R_s = a \times R \quad (9)$$

where R is the overall reliability allocation index of the system determined in Step 1, R_s is the actual reliability allocation index, and a is the safety factor, In general $a \in [1.1, 1.5]$.

Step 3: The subsystems are divided;

Step 4: The reliability formula of subsystem to system is determined;

Step 5: The influencing factors and their weights of reliability allocation are determined;

Six influencing factors are selected to improve the rationality and effectiveness of reliability allocation: complexity (C), maintainability (M), cost (Co), environmental condition (E), technical level (S) and running time (T). In the actual operation process, the designer can add or delete the influencing factors according to the actual product. At the initial stage of product design, fewer influencing factors can be selected. As the product advances from the initial stage of design to the end of design, more and more influencing factors can be selected. However, we suggest to select 4 to 8 factors. Because too few factors will reduce the integrity of the description of the subsystem, and too many factors will lead to the increase of computing cost.

Step 6: Experts are invited to evaluate each subsystem according to influencing factors, and the weight of each expert is determined. In the evaluation process, LNN is used for evaluation by all experts. Suppose the system consists of n subsystems. $U_i(i = 1,2,\ldots,n)$, the reliability allocation evaluation team consists of m members $TM_j(j = 1,2,\ldots,m)$, then $\tilde{k}_{ij} = (k_{T_{ij}}, k_{I_{ij}}, k_{F_{ij}})$, $\tilde{m}_{ij} = (m_{T_{ij}}, m_{I_{ij}}, m_{F_{ij}})$, $\tilde{co}_{ij}$

$= (co_{T_{ij}}, co_{I_{ij}}, co_{F_{ij}})$, $\tilde{e}_{ij} = (e_{T_{ij}}, e_{I_{ij}}, e_{F_{ij}})$, $\tilde{s}_{ij} = (s_{T_{ij}}, s_{I_{ij}}, s_{F_{ij}})$ and $\tilde{t}_{ij} = (t_{T_{ij}}, t_{I_{ij}}, t_{F_{ij}})$ are the fuzzy rating given by the *j*th member of the evaluation team to the *i*th subsystem using LNN. $h_j (j = 1,2,...,m)$ is the weight coefficient of the *j*th member in the evaluation team, and it satisfies $\sum_{j=1}^{m} h_j = 1$ and $h_j > 0$, $j = 1, 2,..., m$.

Step 7: The influencing factors of reliability allocation usually include two types of criteria: benefit and cost. Thus, they are changed into the same type of criteria for the convenience of data analysis. The failure rate is used as the allocation index of reliability, which is why the revenue-type standard is transformed into the cost-type standard. The transformation equation is as follows:

$$r_{ij} = \begin{cases} (t_{Ta_{ij}}, t_{Ia_{ij}}, t_{Fa_{ij}}) & for \quad benefit \quad type \quad \beta_y \\ (t_{2s-Ta_{ij}}, t_{Ia_{ij}}, t_{2s-Fa_{ij}}) & for \quad \cos t \quad type \quad \beta_y \end{cases} \tag{10}$$

The fuzzy score after conversion is: $\tilde{C}_{ij} = (C_{T_{ij}}, C_{I_{ij}}, C_{F_{ij}})$, $\tilde{M}_{ij} = (M_{T_{ij}}, M_{I_{ij}}, M_{F_{ij}})$, $\widetilde{Co}_{ij} = (Co_{T_{ij}}, Co_{I_{ij}}, Co_{F_{ij}})$, $\tilde{E}_{ij} = (E_{T_{ij}}, E_{I_{ij}}, E_{F_{ij}})$, $\tilde{S}_{ij} = (S_{T_{ij}}, S_{I_{ij}}, S_{F_{ij}})$ and $\tilde{T}_{ij} = (T_{T_{ij}}, T_{I_{ij}}, T_{F_{ij}})$. On the basis of the above conditions, the overall score of team members on each evaluation factor of subsystem *i* can be calculated according to Equations (11)–(16).

$$\widetilde{C}_i = (\sum_{j=1}^{m} h_j C_{T_{ij}}, \sum_{j=1}^{m} h_j C_{I_{ij}}, \sum_{j=1}^{m} h_j C_{F_{ij}}) \tag{11}$$

$$\widetilde{M}_i = (\sum_{j=1}^{m} h_j M_{T_{ij}}, \sum_{j=1}^{m} h_j M_{I_{ij}}, \sum_{j=1}^{m} h_j M_{F_{ij}}) \tag{12}$$

$$\widetilde{Co}_i = (\sum_{j=1}^{m} h_j Co_{T_{ij}}, \sum_{j=1}^{m} h_j Co_{I_{ij}}, \sum_{j=1}^{m} h_j Co_{F_{ij}}) \tag{13}$$

$$\widetilde{E}_{ij}=(\sum_{J=1}^{m}h_jE_{T_{ij}},\sum_{J=1}^{m}h_jE_{I_{ij}},\sum_{J=1}^{m}h_jE_{F_{ij}}) \tag{14}$$

$$\widetilde{S}_{ij}=(\sum_{j=1}^{m}h_jS_{T_{ij}},\sum_{j=1}^{m}h_jS_{I_{ij}},\sum_{j=1}^{m}h_jS_{F_{ij}}) \tag{15}$$

$$\widetilde{T}_{ij}=(\sum_{j=1}^{m}h_jT_{T_{ij}},\sum_{j=1}^{m}h_jT_{I_{ij}},\sum_{j=1}^{m}h_jT_{F_{ij}}) \tag{16}$$

Step 8: The LNNWMM operator is used to aggregate the evaluation results of experts, and the comprehensive score of each subsystem is calculated. According to Equation (8), $\tilde{C}_i$,$\tilde{M}_i$,$\widetilde{Co}_i$,$\tilde{E}_i$,$\tilde{S}_i$ and $\tilde{T}_i$ are aggregated to obtain the comprehensive score $\tilde{X}_i$ of subsystem *i*. For the convenience of understanding, the case explanation is given. Suppose subsystem *i* has three evaluation factors $\tilde{A}_i=(s_1,s_2,s_3)$, $\tilde{B}_i=(s_4,s_5,s_6)$ and $\tilde{C}_i=(s_7,s_8,s_9)$, it has a weight of $W=(0.2,0.3,0.5)$, $s=5$, $P=(0.1,0.4,0.5)$, then it can be obtained by Equation (8):

$$\widetilde{X}_i = LNNWMM^{(0.1,0.4,0.5)}_{(0.2,0.3,0.5)}\begin{pmatrix}(s_1,s_2,s_3),\\ (s_4,s_5,s_6),\\ (s_7,s_8,s_9)\end{pmatrix}$$

$$= \begin{pmatrix} g^{-1}[2s(1-\prod\limits_{\sigma\in S_n}(1-\prod\limits_{j=1}^{n}(1-(1-\frac{g(t_{T\sigma(j)})}{2s})^{nw_{\sigma(j)}})^{P_j})^{\frac{1}{n!}})^{\frac{1}{\sum_{j=1}^{n}p_j}}], \\ g^{-1}[2s(1-(1-\prod\limits_{\sigma\in S_n}(1-\prod\limits_{j=1}^{n}(1-(\frac{g(t_{I\sigma(j)})}{2s})^{nw_{\sigma(j)}})^{P_j})^{\frac{1}{n!}})^{\frac{1}{\sum_{j=1}^{n}p_j}})], \\ g^{-1}[2s(1-(1-\prod\limits_{\sigma\in S_n}(1-\prod\limits_{j=1}^{n}(1-(\frac{g(t_{F\sigma(j)})}{2s})^{nw_{\sigma(j)}})^{P_j})^{\frac{1}{n!}})^{\frac{1}{\sum_{j=1}^{n}p_j}})] \end{pmatrix}$$

$$= \begin{pmatrix}
g^{-1}\left(10\times\left(1-\begin{pmatrix}(1-(1-(1-\frac{1}{10})^{0.6})^{0.1}\times(1-(1-\frac{4}{10})^{0.9})^{0.4}\times(1-(1-\frac{7}{10})^{1.5})^{0.5})\\ \times(1-(1-(1-\frac{1}{10})^{0.6})^{0.1}\times(1-(1-\frac{7}{10})^{1.5})^{0.4}\times(1-(1-\frac{4}{10})^{0.9})^{0.5})\\ \times(1-(1-(1-\frac{4}{10})^{0.9})^{0.1}\times(1-(1-\frac{1}{10})^{0.6})^{0.4}\times(1-(1-\frac{7}{10})^{1.5})^{0.5})\\ \times(1-(1-(1-\frac{4}{10})^{0.9})^{0.1}\times(1-(1-\frac{7}{10})^{1.5})^{0.4}\times(1-(1-\frac{1}{10})^{0.6})^{0.5})\\ \times(1-(1-(1-\frac{7}{10})^{1.5})^{0.1}\times(1-(1-\frac{4}{10})^{0.9})^{0.4}\times(1-(1-\frac{1}{10})^{0.6})^{0.5})\\ \times(1-(1-(1-\frac{7}{10})^{1.5})^{0.1}\times(1-(1-\frac{1}{10})^{0.6})^{0.4}\times(1-(1-\frac{4}{10})^{0.9})^{0.5})\end{pmatrix}^{\frac{1}{3!}}\right)^{\frac{1}{0.1+0.4+0.5}}\right), \\
g^{-1}\left(10\times\left(1-\left(1-\begin{pmatrix}(1-(1-(\frac{2}{10})^{0.6})^{0.1}\times(1-(\frac{5}{10})^{0.9})^{0.4}\times(1-(\frac{8}{10})^{1.5})^{0.5})\\ \times(1-(1-(\frac{2}{10})^{0.6})^{0.1}\times(1-(\frac{8}{10})^{1.5})^{0.4}\times(1-(\frac{5}{10})^{0.9})^{0.5})\\ \times(1-(1-(\frac{5}{10})^{0.9})^{0.1}\times(1-(\frac{2}{10})^{0.6})^{0.4}\times(1-(\frac{8}{10})^{1.5})^{0.5})\\ \times(1-(1-(\frac{5}{10})^{0.9})^{0.1}\times(1-(\frac{8}{10})^{1.5})^{0.4}\times(1-(\frac{2}{10})^{0.6})^{0.5})\\ \times(1-(1-(\frac{8}{10})^{1.5})^{0.1}\times(1-(\frac{5}{10})^{0.9})^{0.4}\times(1-(\frac{2}{10})^{0.6})^{0.5})\\ \times(1-(1-(\frac{8}{10})^{1.5})^{0.1}\times(1-(\frac{2}{10})^{0.6})^{0.4}\times(1-(\frac{5}{10})^{0.9})^{0.5})\end{pmatrix}^{\frac{1}{3!}}\right)^{\frac{1}{0.1+0.4+0.5}}\right)\right), \\
g^{-1}\left(10\times\left(1-\left(1-\begin{pmatrix}(1-(1-(\frac{3}{10})^{0.6})^{0.1}\times(1-(\frac{6}{10})^{0.9})^{0.4}\times(1-(\frac{9}{10})^{1.5})^{0.5})\\ \times(1-(1-(\frac{3}{10})^{0.6})^{0.1}\times(1-(\frac{9}{10})^{1.5})^{0.4}\times(1-(\frac{6}{10})^{0.9})^{0.5})\\ \times(1-(1-(\frac{6}{10})^{0.9})^{0.1}\times(1-(\frac{3}{10})^{0.6})^{0.4}\times(1-(\frac{9}{10})^{1.5})^{0.5})\\ \times(1-(1-(\frac{6}{10})^{0.9})^{0.1}\times(1-(\frac{9}{10})^{1.5})^{0.4}\times(1-(\frac{3}{10})^{0.6})^{0.5})\\ \times(1-(1-(\frac{9}{10})^{1.5})^{0.1}\times(1-(\frac{6}{10})^{0.9})^{0.4}\times(1-(\frac{3}{10})^{0.6})^{0.5})\\ \times(1-(1-(\frac{9}{10})^{1.5})^{0.1}\times(1-(\frac{3}{10})^{0.6})^{0.4}\times(1-(\frac{6}{10})^{0.9})^{0.5})\end{pmatrix}^{\frac{1}{3!}}\right)^{\frac{1}{0.1+0.4+0.5}}\right)\right)
\end{pmatrix}$$

$$= (s_{1.7478}, s_{5.2760}, s_{8.2740})$$

Step 9: The allocation result of each subsystem is obtained according to the comprehensive score result of the subsystem, and the reliability allocation interval of each subsystem is obtained according to the safety factor. The reliability allocation result can be transformed into an interval result $[\tilde{R}_i, R_i]$ because of the existence of the safety factor. Where R_i is the reliability allocation result of the ith subsystem, $\tilde{R}_i = R_i/a$ is the lowest reliability allocation target of the ith subsystem.

To sum up, the reliability allocation flow chart based on LNNWMM operator is shown in Figure 1.

[Figure 1 near here]

4. Example

In this section, a certain type of machining centre is taken as an example to verify the superiority of the proposed method by comparing it with the FOO method and the fuzzy number method.

A machining centre system is usually composed of 10 subsystems: spindle system (SP), feed system (FE), CNC system (CNC), electrical system (EL), automatic tool change system (ATC), pneumatic system (PN), chip removal system (CR), lubrication system (LU), cooling system (CO) and protection system (PR). Six factors that affect reliability are considered:

Complexity (C): The complexity of a subsystem is usually proportional to the number of major components of the subsystem. The more major components there are, the higher complexity the subsystem is. A high complexity of the subsystem corresponds to a high probability of failure. Therefore, with a higher complexity of the subsystem, lower reliability should be allocated.

Maintainability (M): Maintainability refers to the maintenance cost and maintenance time of the subsystem. Higher maintainability means lower maintenance costs and less maintenance time of the subsystem. Therefore, higher reliability should be allocated to the subsystems with poor maintainability to reduce the maintenance cost.

Cost (CO): In the reliability allocation of the machining centre, it is necessary to reduce the cost as much as possible under the premise of ensuring the reliability of machine centre. Therefore, cost sensitivity (cost increase/reliability increase) should be considered in reliability allocation. A lower cost sensitivity corresponds to a lower cost of improving the reliability of the subsystem. Therefore, a higher cost sensitivity of the subsystem, lower reliability should be allocated.

Environmental condition (E): Environmental condition refers to the general term of adverse factors such as temperature, humidity, vibration, electromagnetic, oil pollution and extrusion to which subsystems are subjected in their work. Under a poor working environment of the subsystem, guaranteeing its reliability becomes more difficult. Therefore, lower reliability should be allocated to subsystems with lower environmental condition.

Technical level (S): Technical level refers to the manufacturing technology of subsystems. Subsystems with a high technical level usually achieve high reliability easily. Therefore, the subsystem with higher technical level should be allocated higher reliability.

Running time (T): In machining centre work, not all subsystems work at the same time. Different subsystems have different working time. When the subsystem is working for a long time, the probability of failure increases and its reliability is reduced. Thus, when the subsystem is running for a long time, lower reliability should be allocated.

The benefit and cost information of influencing factors are shown in Table 1.

[Table 1 near here]

We calculated the MTBF of 8 machining centers of a certain type in one year by counting the fault situation and operation time. The MTBF of the machining center is about 1600h and the average failure rate of the whole machine is $\lambda_s = 1/1600 = 6.25 \times 10^{-4}$. The fault data of each subsystem and the calculated MTBF are shown in Table 2.

[Table 2 near here]

In the example, we set MTBF=1600h as the reliability target according to the machining center of this model. The reliability allocation team consists of four experts, namely, TM1, TM2, TM3 and TM4, whose relative weights are 0.25, 0.35, 0.25 and 0.15, respectively. Each expert uses LNN to score 6 influencing factors of 10 subsystems according to the actual situation of the machining center of this model. The term set adopted by the experts is $T = \{t_i | i \in [0,10]\}$, where t_0 = worst, t_1 = very poor, t_2 = moderately poor, t_3 = slightly poor, t_4 = poor, t_5 = medium, t_6 = good, t_7 = slightly good, t_8 = moderately good, t_9 = very good, t_{10} = best.

The scoring results are shown in Table 3. The grading results after transformation according to Formula (10) are shown in Table 4.

[Table 3 near here]

[Table 4 near here]

4.1 Reliability allocation based on Foo method

The FOO method considers only four factors, namely, complexity, technical level, working time and environmental condition, and it adopts numerical score, which is why it can only represent the membership degree relationship of the subsystem to the influencing factors. Therefore, the true membership information of complexity, technical level, working time and environmental condition in Table 4 is taken as the evaluation information of the FOO method, and the scoring result of FOO is obtained through the conversion formula. The conversion formula is as follows:

$$G(t_i) = 10 \times \frac{i}{2s} = \frac{5i}{s} \tag{17}$$

The scoring results of FOO method are shown in Table 5.

[Table 5 near here]

The average failure rate of FOO method is $\lambda_s = 1/MTBF = 1/1600 = 6.25 \times 10^{-4}$.

According to Table 5, Formulas (2) to (6) are used to calculate the complexity, comprehensive score, failure rate and MTBF of each subsystem *K*, and the results are shown in Table 6.

[Table 6 near here]

4.2 Reliability allocation method based on fuzzy number

In 2014, V. Sriramdas (2014) introduced trapezoidal fuzzy number into reliability allocation, which gave the reliability allocation method the ability to deal with fuzzy information. The conversion mode of language information and trapezoidal fuzzy number is shown in Table 7.

[Table 7 near here]

The language information of this method can only describe the membership degree of influencing factors. Thus, only the membership degree information in the LNN rating table corresponds to the trapezoidal fuzzy number, and the corresponding relationship is shown in Table 8. The transformed evaluation information is shown in Table 9.

[Table 8 near here]

[Table 9 near here]

Subjective opinions of team members on the influencing factors of the *i*th subsystem are summarized according to Formula $\tilde{x}_i = \sum_{j=1}^{m} h_j \tilde{x}_{ij} = (\sum_{j=1}^{m} h_j x_{ijL}, \sum_{j=1}^{m} h_j x_{ijM1}, \sum_{j=1}^{m} h_j x_{ijM2}, \sum_{j=1}^{m} h_j x_{ijU})$, and the results are shown in Table 10, where

$h_j(j = 1,2,3,4)$ is the weight coefficient of the jth member in the evaluation team, and $\tilde{x}_i$ is the influencing factor of the ith subsystem.

[Table 10 near here]

The comprehensive score of each subsystem is calculated. Among the six selected evaluation factors, complexity, cost, maintainability and working time are directly proportional to the failure rate of the subsystem, while technical level and environmental condition are inversely proportional to the failure rate of the subsystem. Therefore, the overall score of the subsystems is

$$FZ_i = \frac{C \times Co \times M \times T}{S \times E} \tag{18}$$

According to the calculation method proposed by V. Sriramdas, the results are shown in Table 11.

[Table 11 near here]

The evaluation information is defuzzified by calculating the centroid of trapezoidal fuzzy number in Table 11, and the results are shown in Table 12. The center of mass of the trapezoidal fuzzy number $\tilde{A} = (a,b,c,d)$ can be solved by Formula (19).

$$C_{\bar{A}} = \frac{c^2 + d^2 + cd - a^2 - b^2 - ab}{3(c + d - a - b)} \tag{19}$$

According to Formulas (3), (4) and (6), the weight of each subsystem and the allocated failure rate are respectively calculated, and the results are shown in Table 12.

[Table 12 near here]

4.3 Reliability allocation based on LNNWMM operator

Step 1: The overall system reliability allocation index is determined. According to the design requirements and operation environment of the system, the reliability design index MTBF of the machining centre is set to 1600 h.

Step 2: The safety factor is determined, and the actual reliability allocation index is calculated. The safety factor $a = 1.2$ is selected, then the actual MTBF allocation index of the machining centre is 1600 × 1.2 = 1920 h. Then, the average failure rate of the whole machine is $\lambda_s = 1/MTBF = 1/1920 \approx 5.21 \times 10^{-4}$.

Step 3: The subsystems are divided. The machining centre generally contains 10 subsystems: spindle system, feed system, CNC system, electrical system, automatic tool change system, pneumatic system, chip removal system, lubrication system, cooling system and protection system.

Step 4: The reliability formula of subsystem to system is determined. A series relationship exists between the eight subsystems of the machining centre, which is why the system is assumed to be in a period of working time and the failure of each unit is independent of each other. Then, the reliability of the system is

$$R_s = \prod_{i=1}^{n} R_i \tag{20}$$

$$\lambda_s = \sum_{i=1}^{n} \lambda_i \tag{21}$$

Where, R_s——The reliability of the system at working time T;

R_i——The reliability of the ith series unit;

n——Number of series system units;

λ_s——System failure rate;

λ_i——The failure rate of subsystem i.

Step 5: The influencing factors and their weights of reliability allocation are determined. The weight vector of complexity (C), maintainability (M), cost (Co), environmental condition (E), technical level (S) and running time (T) is W = [0.2, 0.25, 0.1, 0.15, 0.1, 0.2].

Step 6: Experts are invited to evaluate each subsystem according to influencing factors, and the weight of each expert is determined. Four experts are invited for evaluation, and the weight vector is w = [0.25, 0.35, 0.25, 0.15]. In the evaluation process, LNN is used for evaluation by all experts. The term set adopted by the experts is $T = \{t_i | i \in [0,10]\}$, where t_0 = worst, t_1 = very poor, t_2 = moderately poor, t_3 = slightly poor, t_4 = poor, t_5 = medium, t_6 = good, t_7 = slightly good, t_8 = moderately good, t_9 = very good, t_{10} = best. The evaluation results are shown in Table 3.

Step 7: The influencing factors of reliability allocation usually include two types of criteria, namely, benefit and cost, which are changed into the same type of criteria for the convenience of data analysis. The failure rate is used as the allocation index of reliability. Thus, the benefit-type standard is transformed into the cost-type standard.

Among the influencing factors selected for reliability allocation in this study, environmental conditions and technical level are the criteria for benefit-type. Therefore, the information of these influencing factors is converted according to Formula (10), and the converted evaluation information is shown in Table 4.

Step 8: The LNNWMM operator is used to aggregate the evaluation results of experts, and the comprehensive score of each subsystem is calculated. Firstly, the evaluation information is preliminarily aggregated according to the weight of experts, and the results are shown in Table 13. Then, the LNNWMM operator is used to aggregate the information in Table 13, and the comprehensive score of each subsystem is obtained. P = [1,1,1,1,1,1], and the result is shown in Table 14.

Step 9: The allocation result of each subsystem is obtained according to the comprehensive score result of the subsystem, and the reliability allocation interval of each subsystem is obtained according to the safety factor.

The subsystem reliability evaluation index of machining center is usually MTBF, and the lowest reliability index of each subsystem can be calculated according to the safety factor:

$$mtbf_i = \frac{MTBF_i}{a} \tag{22}$$

Where: $MTBF_i$ is the MTBF result of the ith subsystem

$mtbf_i$ is the lowest MTBF of the ith subsystem.

Thus, the MTBF allocation range of the ith subsystem is $[mtbf_i, MTBF_i]$, and the specific allocation results are shown in Table 14.

[Table 13 near here]

[Table 14 near here]

4.4 Comparison and analysis

The superiority of the proposed reliability allocation method can be illustrated by comparing it with the FOO method and the fuzzy number method. Through Table 2, Table 6, Table 12 and Table 14, we obtain the comparison results of the three methods, as shown in Table 15 to 17. For convenience of viewing, the median of the interval result of the proposed method is compared with the results of the FOO method and the fuzzy number method, as shown in Figure 2. The result of the proposed method is compared with the actual statistical result, as shown in Figure 3.

[Table 15 near here]

[Figure 2 near here]

[Figure 3 near here]

[Table 16 near here]

[Table 17 near here]

The comparison shows that the proposed method has the following advantages:

(1) As can be seen from Table 16, when all the three methods take exact values as the allocation results (the result of the proposed method takes the minimum value of the allocation interval), the mean deviation between the three methods and the actual statistical results is proposed method < fuzzy

number method < FOO method. Therefore, the allocation result of the proposed method is more in line with the actual situation and has higher rationality. The maximum deviation of the proposed method is 13.40%, which is far less than that of FOO method (178.52%) and fuzzy number method (109.20%). The results show that the proposed method is more effective and has more guiding significance for product reliability design.

(2) As can be seen from Figure 3, when the safety factor is introduced, the allocation results of the proposed method change from exact values to interval number. The actual statistical results of six out of ten subsystems are within the range of the results calculated by the proposed method. Therefore, the proposed method not only improves the flexibility of reliability allocation, but also increases the effectiveness of reliability allocation.

(3) Compared with the FOO method, the proposed method considers more comprehensive influencing factors. The FOO method takes into account only four factors, namely, complexity, technical level, running time and environmental condition, while the proposed method also considers the maintainability and cost. In the production, the manufacturing cost is the first factor to consider. For the machining centre, the different subsystems have different levels of maintenance difficulty. Therefore, the cost and the maintainability need to be considered.

(4) The proposed method can better deal with fuzzy information. The FOO method can only express certain information, and it ignores a large amount of

fuzzy information. Although fuzzy number method has the ability to express fuzzy information, it cannot express hesitations between membership and non-membership. For machining centres, their complexity determines that fuzzy information will be the main part in the evaluation of machining centres. Thus, the FOO method cannot obtain reasonable results, and obtaining good results by using the fuzzy number method is difficult. The proposed method uses LNN for information evaluation, which can well describe the fuzzy information and hesitant information, and the reliability allocation results are more accurate and reasonable.

(5) The correlation and weight of influencing factors are considered in the proposed method. The FOO method and the fuzzy number method simply multiply or divide the scores of influencing factors without taking into account the correlation and weight of each influencing factor (which is also a common problem in current reliability allocation methods). Ignoring these relationships will inevitably lead to inaccurate allocation results. The LNNWMM operator adopted in this article can deal with the correlation and weight among the influencing factors. Thus, the proposed method can obtain more accurate reliability allocation results.

(6) The proposed method is more flexible. Once the evaluation results of the FOO method and fuzzy number method are determined, the reliability allocation results will be determined accordingly, which cannot be changed according to the actual situation and has low flexibility. The designer can

change their values according to the situation because of the existence of P and weight vectors W and w in the proposed method. Therefore, the proposed method is suitable for a variety of situations. And the safety factor can change the exact value of the allocation results into the interval number, which greatly increases the flexibility of the proposed method.

In sum, although the FOO method is simple to operate, it does not consider the influencing factors comprehensively, ignores the correlations and weights between the influencing factors, and does not have the ability to deal with fuzzy information. Compared with the FOO method, the fuzzy number method has more comprehensive considerations and the ability to deal with fuzzy information, but it cannot express hesitant information and ignores the correlation and weight among influencing factors. The proposed method considers the influencing factors more comprehensively, can express the fuzzy information completely, and can aggregate the evaluation information under the condition of considering the correlation and weight of the influencing factors. A comparison of the three methods shows that the proposed method has the better rationality, validity and flexibility.

5. Conclusion

This section summarizes the contributions and innovations of the proposed method.

In view of the shortcomings of traditional reliability allocation methods, which do not consider the correlation and weight relationship among influencing factors, and have poor ability to deal with fuzzy information, this article proposes a new reliability allocation method based on LNNWMM operator. By using LNN, experts can completely and accurately express fuzzy information encountered in the process of reliability allocation by using their familiar terminology sets, which greatly improves the ability of reliability allocation method to evaluate fuzzy information. We combine LNN with MM operator and propose LNNMM and LNNWMM operators. A reliability allocation method based on LNNWMM operator is proposed. LNNWMM operator can aggregate evaluation results well under the condition of considering correlation and weight of influencing factors. It overcomes the shortcoming of traditional reliability allocation method that does not consider the correlation and weight of influencing factors. Converting accurate reliability allocation results into interval values through safety factor can help practitioners to be more flexible in selecting subsystem reliability indicators and greatly improve the flexibility of reliability allocation. Through the comparison results of example analysis, it can be seen that the proposed method has better rationality, effectiveness and flexibility than Foo method and fuzzy number method.

6. Prospect

Compared with the traditional methods, the proposed method increases the amount of calculation, but obtains more reasonable allocation results. At present, the proposed method is mainly applied to series systems, but it is not universal enough to be applied to all systems. Therefore, in the future research, we will mainly study how to reduce the computational difficulty of the proposed method, and try to apply the proposed method to parallel systems and series parallel systems.

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Table 1. Benefit/cost information for influencing factors

Influencing factors	benefit/cost
Complexity	cost
Maintainability	cost
Cost	cost
Environmental condition	benefit
Technical level	benefit

Running time	cost

Table 2. Fault data and MTBF of 10 subsystems of the machining center

Subsystems	The fault number	Fault ratio of each subsystem	Failure rate (10^-5)	MTBF (h)
SP	6	0.1429	0.893	11197
FE	5	0.1191	0.744	13434
CNC	5	0.1191	0.744	13434
EL	4	0.0952	0.595	16807
ATC	5	0.1191	0.744	13434
PN	4	0.0952	0.595	16807
CR	4	0.0952	0.595	16807
LU	3	0.0714	0.446	22409
CO	3	0.0714	0.446	22409
PR	3	0.0714	0.446	22409
aggregate	42	1	6.25	—

Table 3. Four experts rate six influencing factors

Subsystems	Experts	C	S	T	E	M	Co	Expert weight
SP	TM1	(t_8,t_1,t_1)	(t_2,t_2,t_8)	(t_8,t_1,t_1)	(t_3,t_1,t_8)	(t_3,t_1,t_8)	(t_8,t_1,t_1)	0.25
	TM2	(t_7,t_1,t_2)	(t_3,t_1,t_8)	(t_9,t_1,t_1)	(t_2,t_1,t_9)	(t_4,t_3,t_9)	(t_9,t_1,t_1)	0.35
	TM3	(t_7,t_1,t_2)	(t_2,t_1,t_9)	(t_9,t_1,t_1)	(t_3,t_1,t_8)	(t_4,t_2,t_8)	(t_8,t_1,t_1)	0.25
	TM4	(t_9,t_1,t_1)	(t_2,t_1,t_9)	(t_8,t_1,t_1)	(t_4,t_2,t_8)	(t_5,t_3,t_8)	(t_7,t_2,t_1)	0.15
FE	TM1	(t_8,t_1,t_1)	(t_3,t_2,t_9)	(t_9,t_1,t_1)	(t_6,t_1,t_7)	(t_3,t_1,t_8)	(t_6,t_1,t_2)	0.25
	TM2	(t_6,t_1,t_2)	(t_3,t_1,t_8)	(t_8,t_1,t_1)	(t_3,t_2,t_9)	(t_2,t_1,t_9)	(t_7,t_1,t_2)	0.35
	TM3	(t_6,t_1,t_2)	(t_4,t_2,t_9)	(t_8,t_1,t_1)	(t_4,t_1,t_8)	(t_5,t_2,t_9)	(t_6,t_2,t_1)	0.25
	TM4	(t_8,t_1,t_1)	(t_5,t_2,t_9)	(t_9,t_1,t_1)	(t_2,t_1,t_9)	(t_4,t_1,t_8)	(t_5,t_2,t_1)	0.15
CNC	TM1	(t_9,t_1,t_1)	(t_2,t_1,t_9)	(t_9,t_1,t_1)	(t_6,t_3,t_7)	(t_2,t_1,t_9)	(t_5,t_2,t_3)	0.25
	TM2	(t_6,t_2,t_2)	(t_3,t_2,t_9)	(t_9,t_1,t_1)	(t_5,t_1,t_7)	(t_4,t_2,t_8)	(t_4,t_2,t_4)	0.35
	TM3	(t_7,t_2,t_1)	(t_1,t_1,t_9)	(t_8,t_1,t_1)	(t_3,t_1,t_8)	(t_4,t_2,t_8)	(t_6,t_2,t_2)	0.25
	TM4	(t_8,t_1,t_1)	(t_4,t_2,t_8)	(t_7,t_2,t_1)	(t_5,t_2,t_7)	(t_2,t_1,t_9)	(t_5,t_2,t_3)	0.15
EL	TM1	(t_7,t_3,t_3)	(t_4,t_3,t_5)	(t_9,t_3,t_2)	(t_8,t_7,t_4)	(t_5,t_5,t_6)	(t_6,t_5,t_3)	0.25
	TM2	(t_6,t_4,t_2)	(t_3,t_3,t_8)	(t_7,t_2,t_3)	(t_6,t_6,t_8)	(t_4,t_2,t_6)	(t_5,t_3,t_4)	0.35
	TM3	(t_8,t_3,t_3)	(t_3,t_4,t_7)	(t_8,t_2,t_4)	(t_5,t_4,t_5)	(t_6,t_3,t_3)	(t_7,t_2,t_5)	0.25
	TM4	(t_4,t_5,t_5)	(t_2,t_2,t_6)	(t_9,t_2,t_3)	(t_4,t_4,t_7)	(t_4,t_3,t_6)	(t_6,t_2,t_6)	0.15
ATC	TM1	(t_7,t_1,t_2)	(t_1,t_1,t_9)	(t_8,t_1,t_1)	(t_2,t_1,t_9)	(t_4,t_1,t_7)	(t_4,t_2,t_4)	0.25
	TM2	(t_9,t_1,t_1)	(t_2,t_1,t_9)	(t_7,t_2,t_1)	(t_1,t_1,t_9)	(t_5,t_1,t_6)	(t_4,t_2,t_4)	0.35
	TM3	(t_8,t_1,t_1)	(t_1,t_1,t_9)	(t_9,t_1,t_1)	(t_3,t_1,t_8)	(t_6,t_3,t_7)	(t_3,t_4,t_3)	0.25
	TM4	(t_9,t_1,t_1)	(t_1,t_1,t_9)	(t_8,t_1,t_1)	(t_2,t_1,t_9)	(t_5,t_2,t_7)	(t_3,t_3,t_4)	0.15
PN	TM1	(t_4,t_2,t_7)	(t_2,t_1,t_7)	(t_8,t_3,t_1)	(t_4,t_2,t_7)	(t_7,t_5,t_3)	(t_3,t_4,t_7)	0.25
	TM2	(t_5,t_4,t_5)	(t_4,t_4,t_6)	(t_6,t_4,t_4)	(t_5,t_4,t_5)	(t_8,t_3,t_2)	(t_4,t_3,t_7)	0.35
	TM3	(t_6,t_3,t_4)	(t_3,t_2,t_6)	(t_7,t_4,t_2)	(t_4,t_4,t_7)	(t_6,t_3,t_4)	(t_5,t_4,t_4)	0.25
	TM4	(t_5,t_3,t_5)	(t_2,t_2,t_8)	(t_9,t_1,t_2)	(t_5,t_3,t_7)	(t_7,t_3,t_4)	(t_3,t_3,t_6)	0.15
CR	TM1	(t_4,t_6,t_6)	(t_6,t_5,t_3)	(t_7,t_3,t_4)	(t_4,t_4,t_5)	(t_6,t_5,t_4)	(t_5,t_4,t_6)	0.25
	TM2	(t_4,t_6,t_5)	(t_5,t_7,t_7)	(t_6,t_6,t_3)	(t_5,t_4,t_4)	(t_4,t_2,t_3)	(t_6,t_3,t_6)	0.35
	TM3	(t_5,t_5,t_5)	(t_4,t_4,t_5)	(t_8,t_4,t_3)	(t_6,t_7,t_6)	(t_3,t_6,t_8)	(t_5,t_7,t_3)	0.25
	TM4	(t_3,t_5,t_7)	(t_7,t_3,t_1)	(t_8,t_4,t_3)	(t_4,t_2,t_5)	(t_6,t_6,t_6)	(t_8,t_3,t_3)	0.15
LU	TM1	(t_5,t_6,t_6)	(t_6,t_6,t_2)	(t_8,t_6,t_6)	(t_4,t_6,t_5)	(t_6,t_5,t_3)	(t_3,t_5,t_8)	0.25
	TM2	(t_3,t_6,t_7)	(t_8,t_6,t_2)	(t_6,t_5,t_6)	(t_7,t_6,t_3)	(t_7,t_6,t_3)	(t_4,t_6,t_6)	0.35
	TM3	(t_4,t_6,t_8)	(t_5,t_5,t_3)	(t_7,t_4,t_5)	(t_5,t_5,t_3)	(t_7,t_6,t_2)	(t_4,t_5,t_7)	0.25
	TM4	(t_4,t_6,t_8)	(t_5,t_6,t_4)	(t_8,t_4,t_5)	(t_4,t_5,t_4)	(t_8,t_5,t_1)	(t_2,t_5,t_9)	0.15
CO	TM1	(t_3,t_5,t_7)	(t_4,t_5,t_5)	(t_8,t_4,t_6)	(t_6,t_5,t_3)	(t_7,t_5,t_2)	(t_3,t_5,t_7)	0.25
	TM2	(t_5,t_5,t_6)	(t_5,t_6,t_5)	(t_9,t_3,t_3)	(t_4,t_5,t_5)	(t_7,t_6,t_2)	(t_5,t_6,t_6)	0.35
	TM3	(t_4,t_7,t_7)	(t_3,t_4,t_5)	(t_8,t_4,t_4)	(t_5,t_7,t_4)	(t_6,t_6,t_4)	(t_4,t_6,t_6)	0.25
	TM4	(t_5,t_6,t_7)	(t_7,t_6,t_1)	(t_7,t_5,t_5)	(t_7,t_5,t_2)	(t_7,t_6,t_2)	(t_2,t_6,t_9)	0.15

PR	TM1	(t_4,t_5,t_8)	(t_5,t_5,t_4)	(t_5,t_5,t_6)	(t_3,t_6,t_5)	(t_7,t_5,t_3)	(t_2,t_6,t_9)	0.25
	TM2	(t_4,t_5,t_8)	(t_4,t_5,t_4)	(t_7,t_4,t_5)	(t_6,t_6,t_4)	(t_6,t_6,t_4)	(t_1,t_4,t_9)	0.35
	TM3	(t_5,t_5,t_6)	(t_5,t_5,t_3)	(t_6,t_5,t_6)	(t_5,t_5,t_4)	(t_8,t_6,t_1)	(t_3,t_6,t_8)	0.25
	TM4	(t_6,t_7,t_6)	(t_5,t_6,t_3)	(t_9,t_3,t_3)	(t_4,t_6,t_5)	(t_7,t_6,t_2)	(t_4,t_5,t_7)	0.15

Table 4. After benefit/cost transformation, four experts rating results

Subsystems	Experts	C	S	T	E	M	Co	Expert weight
SP	TM1	(t_8,t_1,t_1)	(t_8,t_2,t_2)	(t_8,t_1,t_1)	(t_7,t_1,t_2)	(t_3,t_1,t_8)	(t_8,t_1,t_1)	0.25
	TM2	(t_7,t_1,t_2)	(t_7,t_1,t_2)	(t_9,t_1,t_1)	(t_8,t_1,t_1)	(t_4,t_3,t_9)	(t_9,t_1,t_1)	0.35
	TM3	(t_7,t_1,t_2)	(t_8,t_1,t_1)	(t_9,t_1,t_1)	(t_7,t_1,t_2)	(t_4,t_2,t_8)	(t_8,t_1,t_1)	0.25
	TM4	(t_9,t_1,t_1)	(t_8,t_1,t_1)	(t_8,t_1,t_1)	(t_6,t_2,t_2)	(t_5,t_3,t_8)	(t_7,t_2,t_1)	0.15
FE	TM1	(t_8,t_1,t_1)	(t_7,t_2,t_1)	(t_9,t_1,t_1)	(t_4,t_1,t_3)	(t_3,t_1,t_8)	(t_6,t_1,t_2)	0.25
	TM2	(t_6,t_1,t_2)	(t_7,t_1,t_2)	(t_8,t_1,t_1)	(t_7,t_2,t_1)	(t_2,t_1,t_9)	(t_7,t_1,t_2)	0.35
	TM3	(t_6,t_1,t_2)	(t_6,t_2,t_1)	(t_8,t_1,t_1)	(t_6,t_1,t_2)	(t_5,t_2,t_9)	(t_6,t_2,t_1)	0.25
	TM4	(t_8,t_1,t_1)	(t_5,t_2,t_1)	(t_9,t_1,t_1)	(t_8,t_1,t_1)	(t_4,t_1,t_8)	(t_5,t_2,t_1)	0.15
CNC	TM1	(t_9,t_1,t_1)	(t_8,t_1,t_1)	(t_9,t_1,t_1)	(t_4,t_3,t_3)	(t_2,t_1,t_9)	(t_5,t_2,t_3)	0.25
	TM2	(t_6,t_2,t_2)	(t_7,t_2,t_1)	(t_9,t_1,t_1)	(t_5,t_1,t_3)	(t_4,t_2,t_8)	(t_4,t_2,t_4)	0.35
	TM3	(t_7,t_2,t_1)	(t_9,t_1,t_1)	(t_8,t_1,t_1)	(t_7,t_1,t_2)	(t_4,t_2,t_8)	(t_6,t_2,t_2)	0.25
	TM4	(t_8,t_1,t_1)	(t_6,t_2,t_2)	(t_7,t_2,t_1)	(t_5,t_2,t_3)	(t_2,t_1,t_9)	(t_5,t_2,t_3)	0.15
EL	TM1	(t_7,t_3,t_3)	(t_6,t_3,t_5)	(t_9,t_3,t_2)	(t_2,t_7,t_6)	(t_5,t_5,t_6)	(t_6,t_5,t_3)	0.25
	TM2	(t_6,t_4,t_2)	(t_7,t_3,t_2)	(t_7,t_2,t_3)	(t_4,t_6,t_2)	(t_4,t_2,t_6)	(t_5,t_3,t_4)	0.35
	TM3	(t_8,t_3,t_3)	(t_7,t_4,t_3)	(t_8,t_2,t_4)	(t_5,t_4,t_5)	(t_6,t_3,t_3)	(t_7,t_2,t_5)	0.25
	TM4	(t_4,t_5,t_5)	(t_8,t_2,t_4)	(t_9,t_2,t_3)	(t_6,t_4,t_3)	(t_4,t_3,t_6)	(t_6,t_2,t_6)	0.15
ATC	TM1	(t_7,t_1,t_2)	(t_9,t_1,t_1)	(t_8,t_1,t_1)	(t_8,t_1,t_1)	(t_4,t_1,t_7)	(t_4,t_2,t_4)	0.25
	TM2	(t_9,t_1,t_1)	(t_8,t_1,t_1)	(t_7,t_2,t_1)	(t_9,t_1,t_1)	(t_5,t_1,t_6)	(t_4,t_2,t_4)	0.35
	TM3	(t_8,t_1,t_1)	(t_9,t_1,t_1)	(t_9,t_1,t_1)	(t_7,t_1,t_2)	(t_6,t_3,t_7)	(t_3,t_4,t_3)	0.25
	TM4	(t_9,t_1,t_1)	(t_9,t_1,t_1)	(t_8,t_1,t_1)	(t_8,t_1,t_1)	(t_5,t_2,t_7)	(t_3,t_3,t_4)	0.15
PN	TM1	(t_4,t_2,t_7)	(t_8,t_1,t_3)	(t_8,t_3,t_1)	(t_6,t_2,t_3)	(t_7,t_5,t_3)	(t_3,t_4,t_7)	0.25
	TM2	(t_5,t_4,t_5)	(t_6,t_4,t_4)	(t_6,t_4,t_4)	(t_5,t_4,t_5)	(t_8,t_3,t_2)	(t_4,t_3,t_7)	0.35
	TM3	(t_6,t_3,t_4)	(t_7,t_2,t_4)	(t_7,t_4,t_2)	(t_6,t_4,t_3)	(t_6,t_3,t_4)	(t_5,t_4,t_4)	0.25
	TM4	(t_5,t_3,t_5)	(t_8,t_2,t_2)	(t_9,t_1,t_2)	(t_5,t_3,t_3)	(t_7,t_3,t_4)	(t_3,t_3,t_6)	0.15
CR	TM1	(t_4,t_6,t_6)	(t_4,t_5,t_7)	(t_7,t_3,t_4)	(t_6,t_4,t_5)	(t_6,t_5,t_4)	(t_5,t_4,t_6)	0.25
	TM2	(t_4,t_6,t_5)	(t_5,t_7,t_3)	(t_6,t_6,t_3)	(t_5,t_4,t_6)	(t_4,t_2,t_3)	(t_6,t_3,t_6)	0.35
	TM3	(t_5,t_5,t_5)	(t_6,t_4,t_5)	(t_8,t_4,t_3)	(t_4,t_7,t_4)	(t_3,t_6,t_8)	(t_5,t_7,t_3)	0.25
	TM4	(t_3,t_5,t_7)	(t_3,t_3,t_9)	(t_8,t_4,t_3)	(t_6,t_2,t_5)	(t_6,t_6,t_6)	(t_8,t_3,t_3)	0.15
LU	TM1	(t_5,t_6,t_6)	(t_4,t_6,t_8)	(t_8,t_6,t_6)	(t_6,t_6,t_5)	(t_6,t_5,t_3)	(t_3,t_5,t_8)	0.25
	TM2	(t_3,t_6,t_7)	(t_2,t_6,t_8)	(t_6,t_5,t_6)	(t_3,t_6,t_7)	(t_7,t_6,t_3)	(t_4,t_6,t_6)	0.35
	TM3	(t_4,t_6,t_8)	(t_5,t_5,t_7)	(t_7,t_4,t_5)	(t_5,t_5,t_7)	(t_7,t_6,t_2)	(t_4,t_5,t_7)	0.25
	TM4	(t_4,t_6,t_8)	(t_5,t_6,t_6)	(t_8,t_4,t_5)	(t_6,t_5,t_6)	(t_8,t_5,t_1)	(t_2,t_5,t_9)	0.15
CO	TM1	(t_3,t_5,t_7)	(t_6,t_5,t_5)	(t_8,t_4,t_6)	(t_4,t_5,t_7)	(t_7,t_5,t_2)	(t_3,t_5,t_7)	0.25
	TM2	(t_5,t_5,t_6)	(t_5,t_6,t_5)	(t_9,t_3,t_3)	(t_6,t_5,t_5)	(t_7,t_6,t_2)	(t_5,t_6,t_6)	0.35
	TM3	(t_4,t_7,t_7)	(t_7,t_4,t_5)	(t_8,t_4,t_4)	(t_5,t_7,t_6)	(t_6,t_6,t_4)	(t_4,t_6,t_6)	0.25
	TM4	(t_5,t_6,t_7)	(t_3,t_6,t_9)	(t_7,t_5,t_5)	(t_3,t_5,t_8)	(t_7,t_6,t_2)	(t_2,t_6,t_9)	0.15
PR	TM1	(t_4,t_5,t_8)	(t_5,t_5,t_6)	(t_5,t_5,t_6)	(t_7,t_6,t_5)	(t_7,t_5,t_3)	(t_2,t_6,t_9)	0.25
	TM2	(t_4,t_5,t_8)	(t_6,t_5,t_6)	(t_7,t_4,t_5)	(t_4,t_6,t_6)	(t_6,t_6,t_4)	(t_1,t_4,t_9)	0.35
	TM3	(t_5,t_5,t_6)	(t_5,t_5,t_7)	(t_6,t_5,t_6)	(t_5,t_5,t_6)	(t_8,t_6,t_1)	(t_3,t_6,t_8)	0.25
	TM4	(t_6,t_7,t_6)	(t_5,t_6,t_7)	(t_9,t_3,t_3)	(t_6,t_6,t_5)	(t_7,t_6,t_2)	(t_4,t_5,t_7)	0.15

Table 5. Scoring results of four experts after conversion by Formula (17)

Subsystems	Experts	C	S	T	E	Expert weight
SP	TM1	8	6	8	7	0.25
	TM2	7	7	9	8	0.35
	TM3	7	8	9	7	0.25
	TM4	9	8	8	6	0.15
FE	TM1	8	7	9	4	0.25

	TM2	6	7	8	7	0.35
	TM3	6	6	8	6	0.25
	TM4	8	5	9	8	0.15
CNC	TM1	9	8	9	4	0.25
	TM2	6	7	9	5	0.35
	TM3	7	9	8	7	0.25
	TM4	8	6	7	5	0.15
EL	TM1	7	6	9	2	0.25
	TM2	6	7	7	4	0.35
	TM3	8	7	8	5	0.25
	TM4	4	8	9	6	0.15
ATC	TM1	7	9	8	8	0.25
	TM2	9	8	7	9	0.35
	TM3	8	9	9	7	0.25
	TM4	9	9	8	8	0.15
PN	TM1	4	8	8	6	0.25
	TM2	5	6	6	5	0.35
	TM3	6	7	7	6	0.25
	TM4	5	8	9	5	0.15
CR	TM1	4	4	7	6	0.25
	TM2	4	5	6	5	0.35
	TM3	5	6	8	4	0.25
	TM4	3	3	8	6	0.15
LU	TM1	5	4	8	6	0.25
	TM2	3	2	6	3	0.35
	TM3	4	5	7	5	0.25
	TM4	4	5	8	6	0.15
CO	TM1	3	6	8	4	0.25
	TM2	5	5	9	6	0.35
	TM3	4	7	8	5	0.25
	TM4	5	3	7	3	0.15
PR	TM1	4	5	5	7	0.25
	TM2	4	6	7	4	0.35
	TM3	5	5	6	5	0.25
	TM4	6	5	9	6	0.15

Table 6. Reliability allocation results of FOO method

Subsystems	C	S	T	E	Comprehensive score	Complexity	Failure rate(10^-4)	MTBF(h)
SP	7.55	7.65	8.6	7.2	3576.3	0.1917	1.198	8344
FE	6.8	6.45	8.4	6.15	2265.8	0.1215	0.759	13171
CNC	7.3	7.6	8.45	5.25	2461.2	0.132	0.825	12125
EL	6.45	6.9	8.05	4.05	1451.0	0.0778	0.486	20567
ATC	8.25	8.65	7.9	8.1	4566.5	0.2448	1.530	6535
PN	5	7.05	7.2	5.5	1395.9	0.0748	0.468	21378
CR	4.1	4.7	7.05	5.15	699.6	0.0375	0.234	42653
LU	3.9	3.7	7.05	4.7	478.1	0.0256	0.160	62413
CO	4.25	5.45	8.2	4.8	911.7	0.0489	0.306	32733
PR	4.55	5.35	6.55	5.3	845.1	0.0453	0.283	35314

Table 7. Fuzzy evaluation of reliability allocation coefficient

Factors/scale	C	S	T	E	M	Co
(7,8,9,10)	very high	very high	very long	very good	very good	Very high
(5,6,7,8)	high	high	long	good	good	high
(3,4,5,6)	medium	medium	medium	medium	medium	medium
(1,2,3,4)	low	low	short	poor	poor	low

Table 8. Correspondence between LNNs and trapezoidal fuzzy numbers

Factors/scale	LNN
(7,8,9,10)	t_9,t_{10}
(5,6,7,8)	t_7,t_8
(3,4,5,6)	t_4,t_5,t_6
(1,2,3,4)	t_0,t_1,t_2,t_3

Table 9. Evaluation results of four experts after transformation of trapezoidal fuzzy number

Subsystems	Experts	C	S	T	E	M	Co	Expert weight
SP	TM1	(5,6,7,8)	(1,2,3,4)	(5,6,7,8)	(1,2,3,4)	(1,2,3,4)	(5,6,7,8)	0.25
	TM2	(5,6,7,8)	(1,2,3,4)	(7,8,9,10)	(1,2,3,4)	(3,4,5,6)	(7,8,9,10)	0.35
	TM3	(5,6,7,8)	(1,2,3,4)	(7,8,9,10)	(1,2,3,4)	(3,4,5,6)	(5,6,7,8)	0.25
	TM4	(7,8,9,10)	(1,2,3,4)	(5,6,7,8)	(3,4,5,6)	(3,4,5,6)	(5,6,7,8)	0.15
FE	TM1	(5,6,7,8)	(1,2,3,4)	(7,8,9,10)	(3,4,5,6)	(1,2,3,4)	(3,4,5,6)	0.25
	TM2	(3,4,5,6)	(1,2,3,4)	(5,6,7,8)	(1,2,3,4)	(1,2,3,4)	(5,6,7,8)	0.35
	TM3	(3,4,5,6)	(3,4,5,6)	(5,6,7,8)	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	0.25
	TM4	(5,6,7,8)	(3,4,5,6)	(7,8,9,10)	(1,2,3,4)	(3,4,5,6)	(3,4,5,6)	0.15
CNC	TM1	(7,8,9,10)	(1,2,3,4)	(7,8,9,10)	(3,4,5,6)	(1,2,3,4)	(3,4,5,6)	0.25
	TM2	(3,4,5,6)	(1,2,3,4)	(7,8,9,10)	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	0.35
	TM3	(5,6,7,8)	(1,2,3,4)	(5,6,7,8)	(1,2,3,4)	(3,4,5,6)	(3,4,5,6)	0.25
	TM4	(5,6,7,8)	(3,4,5,6)	(5,6,7,8)	(3,4,5,6)	(1,2,3,4)	(3,4,5,6)	0.15
EL	TM1	(5,6,7,8)	(3,4,5,6)	(7,8,9,10)	(5,6,7,8)	(3,4,5,6)	(3,4,5,6)	0.25
	TM2	(3,4,5,6)	(1,2,3,4)	(5,6,7,8)	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	0.35
	TM3	(5,6,7,8)	(1,2,3,4)	(5,6,7,8)	(3,4,5,6)	(3,4,5,6)	(5,6,7,8)	0.25
	TM4	(3,4,5,6)	(1,2,3,4)	(7,8,9,10)	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	0.15
ATC	TM1	(5,6,7,8)	(1,2,3,4)	(5,6,7,8)	(1,2,3,4)	(3,4,5,6)	(3,4,5,6)	0.25
	TM2	(7,8,9,10)	(1,2,3,4)	(5,6,7,8)	(1,2,3,4)	(3,4,5,6)	(3,4,5,6)	0.35
	TM3	(5,6,7,8)	(1,2,3,4)	(7,8,9,10)	(1,2,3,4)	(3,4,5,6)	(1,2,3,4)	0.25
	TM4	(7,8,9,10)	(1,2,3,4)	(5,6,7,8)	(1,2,3,4)	(3,4,5,6)	(1,2,3,4)	0.15
PN	TM1	(3,4,5,6)	(1,2,3,4)	(5,6,7,8)	(3,4,5,6)	(5,6,7,8)	(1,2,3,4)	0.25
	TM2	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	(5,6,7,8)	(3,4,5,6)	0.35
	TM3	(3,4,5,6)	(1,2,3,4)	(5,6,7,8)	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	0.25
	TM4	(3,4,5,6)	(1,2,3,4)	(7,8,9,10)	(3,4,5,6)	(5,6,7,8)	(1,2,3,4)	0.15
CR	TM1	(3,4,5,6)	(3,4,5,6)	(5,6,7,8)	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	0.25
	TM2	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	0.35
	TM3	(3,4,5,6)	(3,4,5,6)	(5,6,7,8)	(3,4,5,6)	(1,2,3,4)	(3,4,5,6)	0.25
	TM4	(1,2,3,4)	(5,6,7,8)	(5,6,7,8)	(3,4,5,6)	(3,4,5,6)	(5,6,7,8)	0.15
LU	TM1	(3,4,5,6)	(3,4,5,6)	(5,6,7,8)	(3,4,5,6)	(3,4,5,6)	(1,2,3,4)	0.25
	TM2	(1,2,3,4)	(5,6,7,8)	(3,4,5,6)	(5,6,7,8)	(5,6,7,8)	(3,4,5,6)	0.35
	TM3	(3,4,5,6)	(3,4,5,6)	(5,6,7,8)	(3,4,5,6)	(5,6,7,8)	(3,4,5,6)	0.25
	TM4	(3,4,5,6)	(3,4,5,6)	(5,6,7,8)	(3,4,5,6)	(5,6,7,8)	(1,2,3,4)	0.15
CO	TM1	(1,2,3,4)	(3,4,5,6)	(5,6,7,8)	(3,4,5,6)	(5,6,7,8)	(1,2,3,4)	0.25
	TM2	(3,4,5,6)	(3,4,5,6)	(7,8,9,10)	(3,4,5,6)	(5,6,7,8)	(3,4,5,6)	0.35
	TM3	(3,4,5,6)	(1,2,3,4)	(5,6,7,8)	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	0.25
	TM4	(3,4,5,6)	(5,6,7,8)	(5,6,7,8)	(5,6,7,8)	(5,6,7,8)	(1,2,3,4)	0.15
PR	TM1	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	(1,2,3,4)	(5,6,7,8)	(1,2,3,4)	0.25
	TM2	(3,4,5,6)	(3,4,5,6)	(5,6,7,8)	(3,4,5,6)	(3,4,5,6)	(1,2,3,4)	0.35
	TM3	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	(5,6,7,8)	(1,2,3,4)	0.25
	TM4	(3,4,5,6)	(3,4,5,6)	(7,8,9,10)	(3,4,5,6)	(5,6,7,8)	(3,4,5,6)	0.15

Table 10. Aggregation results of evaluation information from four experts under fuzzy number method

Subsystems	C	S	T	E	M	Co
SP	(5.3,6.3,7.3,8.3)	(1.5,2.5,3.5,4.5)	(6.2,7.2,8.2,9.2)	(1.3,2.3,3.3,4.3)	(2.5,3.5,4.5,5.5)	(5.7,6.7,7.7,8.7)
FE	(3.8,4.8,5.8,6.8)	(1.8,2.8,3.8,4.8)	(5.8,6.8,7.8,8.8)	(2.0,3.0,4.0,5.0)	(1.8,2.8,3.8,4.8)	(3.7,4.7,5.7,6.7)
CNC	(4.8,5.8,6.8,7.8)	(1.3,2.3,3.3,4.3)	(6.2,7.2,8.2,9.2)	(3.0,4.0,5.0,6.0)	(2.2,3.2,4.2,5.2)	(3.0,4.0,5.0,6.0)
EL	(4.0,5.0,6.0,7.0)	(4.5,5.5,6.5,7.5)	(5.8,6.8,7.8,8.8)	(2.5,3.5,4.5,5.5)	(3.0,4.0,5.0,6.0)	(3.5,4.5,5.5,6.5)
ATC	(6.0,7.0,8.0,9.0)	(1.0,2.0,3.0,4.0)	(5.5,6.5,7.5,8.5)	(1.0,2.0,3.0,4.0)	(3.0,4.0,5.0,6.0)	(2.2,3.2,4.2,5.2)
PN	(3.0,4.0,5.0,6.0)	(1.7,2.7,3.7,4.7)	(4.6,5.6,6.6,7.6)	(3.0,4.0,5.0,6.0)	(4.5,5.5,6.5,7.5)	(2.2,3.2,4.2,5.2)
CR	(2.7,3.7,4.7,5.7)	(2.7,3.7,4.7,5.7)	(4.3,5.3,6.3,7.3)	(3.0,4.0,5.0,6.0)	(2.5,3.5,4.5,5.5)	(3.3,4.3,5.3,6.3)
LU	(2.3,3.3,4.3,5.3)	(3.7,4.7,5.7,6.7)	(4.3,5.3,6.3,7.3)	(3.7,4.7,5.7,6.7)	(4.5,5.5,6.5,7.5)	(2.2,3.2,4.2,5.2)
CO	(2.5,3.5,4.5,5.5)	(2.8,3.8,4.8,5.8)	(5.7,6.7,7.7,8.7)	(3.3,4.3,5.3,6.3)	(4.5,5.5,6.5,7.5)	(2.2,3.2,4.2,5.2)
PR	(3.0,4.0,5.0,6.0)	(3.0,4.0,5.0,6.0)	(4.3,5.3,6.3,7.3)	(2.5,3.5,4.5,5.5)	(4.3,5.3,6.3,7.3)	(1.3,2.3,3.3,4.3)

Table 11. Result of ten subsystem evaluation information processing in fuzzy number method

Subsystems	C×Co×M×T	S×E	FZ
SP	(468.3, 1063.7, 2074.1, 4913.8)	(1.3, 4.6, 9.9, 17.2)	(50.8, 107.4, 450.9, 598.5)
FE	(146.8, 429.5, 979.9, 2786.2)	(3.6, 8.4, 15.2, 24)	(0, 28.3, 116.7, 269.7)
CNC	(196.4, 534.5, 1171.0, 3358.4)	(3.3, 8.1, 14.9, 23.7)	(0, 36.0, 145.5, 327.4)
EL	(243.6, 612.0, 1287.0, 3511.2)	(5.3, 11.3, 19.3, 29.3)	(0, 31.8, 114.4, 289.3)
ATC	(217.8, 582.4, 1260.0, 3763.8)	(1.0, 4.0, 9.0, 16.0)	(24.1, 64.7, 315, 497.7)
PN	(136.6, 394.2, 900.9, 2804.4)	(5.1, 10.8, 18.5, 28.2)	(0, 21.3, 83.4, 244.9)
CR	(95.8, 295.1, 706.2, 2128.4)	(9.9, 17.2, 26.5, 37.8)	(0, 11.1, 41.1, 152.5)
LU	(97.9, 307.8, 739.6, 2379.4)	(13.7, 22.1, 32.5, 44.9)	(0, 9.5, 33.5, 155.0)
CO	(141.1, 412.7, 945.9, 2942.8)	(9.2, 16.3, 25.4, 36.5)	(0, 16.2, 57.9, 217.1)
PR	(72.1, 258.4, 654.9, 2334.1)	(7.5, 14.0, 22.5, 33.0)	(0, 11.5, 46.8, 189.0)

Table 12. Results of reliability allocation in fuzzy number method

Subsystems	Defuzzification (FZ)	Weight	Failure rate 10^-3	MTBF/h
SP	303.65	0.2433	0.1521	6576
FE	108.93	0.0873	0.0545	18332
CNC	133.29	0.1068	0.0667	14981
EL	115.49	0.0925	0.0578	17290
ATC	229.02	0.1835	0.1147	8719
PN	94.37	0.0756	0.0473	21159
CR	56.79	0.0455	0.0284	35161
LU	56.32	0.0451	0.0282	35452
CO	80.87	0.0648	0.0408	24691
PR	69.29	0.0555	0.0347	28818

Table 13. Aggregation results of the evaluation information of influencing factors in the proposed method

Influencing factors	C	S	T	E	M	Co
SP	$(t_{7.55},t_1,t_{1.6})$	$(t_{7.65},t_{1.25},t_{1.6})$	$(t_{8.6},t_1,t_1)$	$(t_{7.2},t_{1.15},t_{1.65})$	$(t_{3.9},t_{2.25},t_{6.85})$	$(t_{8.2},t_{1.15},t_1)$
FE	$(t_{6.8},t_1,t_{1.6})$	$(t_{6.45},t_{1.65},t_{1.35})$	$(t_{8.4},t_1,t_1)$	$(t_{6.15},t_{1.35},t_{1.75})$	$(t_{3.3},t_{1.25},t_{8.6})$	$(t_{6.2},t_{1.4},t_{1.6})$

CNC	$(t_{7.3},t_{1.6},t_{1.35})$	$(t_{7.6},t_{1.5},t_{1.15})$	$(t_{8.45},t_{1.15},t_{1})$	$(t_{5.25},t_{1.65},t_{2.75})$	$(t_{3.2},t_{1.6},t_{8.4})$	$(t_{4.9},t_{2},t_{3.1})$
EL	$(t_{6.45},t_{3.65},t_{2.95})$	$(t_{6.9},t_{3.1},t_{3.3})$	$(t_{8.05},t_{2.25},t_{3})$	$(t_{4.05},t_{5.45},t_{3.9})$	$(t_{4.75},t_{3.15},t_{5.25})$	$(t_{5.9},t_{3.1},t_{4.3})$
ATC	$(t_{8.25},t_{1},t_{1.25})$	$(t_{8.65},t_{1},t_{1})$	$(t_{7.9},t_{1.35},t_{1})$	$(t_{8.1},t_{1},t_{1.25})$	$(t_{5},t_{1.65},t_{6.65})$	$(t_{3.6},t_{2.65},t_{3.75})$
PN	$(t_{5},t_{3.1},t_{5.25})$	$(t_{7.05},t_{2.45},t_{3.45})$	$(t_{7.2},t_{3.3},t_{2.45})$	$(t_{5.5},t_{3.35},t_{3.7})$	$(t_{7.1},t_{3.5},t_{3.05})$	$(t_{3.85},t_{3.5},t_{6.1})$
CR	$(t_{4.1},t_{5.6},t_{5.55})$	$(t_{4.7},t_{5.15},t_{5.4})$	$(t_{7.05},t_{4.45},t_{3.25})$	$(t_{5.15},t_{4.45},t_{5.1})$	$(t_{4.55},t_{4.35},t_{4.95})$	$(t_{5.8},t_{4.25},t_{4.8})$
LU	$(t_{3.9},t_{6},t_{7.15})$	$(t_{3.7},t_{5.75},t_{7.45})$	$(t_{7.05},t_{4.85},t_{5.6})$	$(t_{4.7},t_{5.6},t_{6.35})$	$(t_{6.9},t_{5.6},t_{2.45})$	$(t_{3.45},t_{5.35},t_{7.2})$
CO	$(t_{4.25},t_{5.65},t_{6.65})$	$(t_{5.45},t_{5.25},t_{5.6})$	$(t_{8.2},t_{3.8},t_{4.3})$	$(t_{4.8},t_{5.5},t_{6.2})$	$(t_{6.75},t_{5.75},t_{2.5})$	$(t_{3.8},t_{5.75},t_{6.7})$
PR	$(t_{4.55},t_{5.3},t_{7.2})$	$(t_{5.35},t_{5.15},t_{6.4})$	$(t_{6.55},t_{4.35},t_{5.2})$	$(t_{5.3},t_{5.75},t_{5.6})$	$(t_{6.9},t_{5.75},t_{2.7})$	$(t_{2.2},t_{5.15},t_{8.45})$

Table 14. Comprehensive score and reliability allocation results of subsystems in the proposed method

Subsystems	Comprehensive evaluation information	Comprehensive score	Failure rate (10^-5)	MTBF interval (h)
SP	$(t_{6.5136},t_{1.7531},t_{3.2434})$	0.7172	6.563	[12697,15236]
FE	$(t_{5.6676},t_{1.5940},t_{4.1883})$	0.6628	6.066	[13739,16486]
CNC	$(t_{5.4537},t_{1.9187},t_{4.3163})$	0.6406	5.862	[14215,17058]
EL	$(t_{5.5406},t_{3.8420},t_{4.2170})$	0.5827	5.332	[15628,18753]
ATC	$(t_{6.1747},t_{1.8550},t_{3.3749})$	0.6982	6.389	[13044,15652]
PN	$(t_{5.5767},t_{3.6183},t_{4.3367})$	0.5874	5.375	[15503,18604]
CR	$(t_{4.9173},t_{4.9865},t_{5.1065})$	0.4941	4.522	[18429,22115]
LU	$(t_{4.6539},t_{5.7685},t_{6.4031})$	0.4161	3.807	[21887,26264]
CO	$(t_{5.2351},t_{5.5543},t_{5.6547})$	0.4675	4.278	[19478,23373]
PR	$(t_{4.6942},t_{5.5241},t_{6.4247})$	0.4248	3.888	[21435,25722]

Table 15. Comparison of three reliability allocation methods with actual statistics

Subsystems	Proposed method (h)	FOO (h)	Fuzzy number method (h)	Actual statistical (h)
SP	[12697,15236]	8344	6576	11197
FE	[13739,16486]	13171	18332	13434
CNC	[14215,17058]	12125	14981	13434
EL	[15628,18753]	20567	17290	16807
ATC	[13044,15652]	6535	8719	13434
PN	[15503,18604]	21378	21159	16807
CR	[18429,22115]	42653	35161	16807
LU	[21887,26264]	62413	35452	22409
CO	[19478,23373]	32733	24691	22409
PR	[21435,25722]	35314	28818	22409

Table 16. The deviation between the three methods and the actual statistical results

Subsystems	Proposed method	FOO	Fuzzy number method
SP	13.40%	25.48%	41.27%
FE	2.27%	1.96%	36.46%
CNC	5.81%	9.74%	11.52%
EL	7.01%	22.37%	2.87%
ATC	2.90%	51.35%	35.10%
PN	7.76%	27.20%	25.89%
CR	9.65%	153.78%	109.20%

LU	2.33%	178.52%	58.20%
CO	13.08%	46.07%	10.18%
PR	4.35%	57.59%	28.60%
maximum	13.40%	178.52%	109.20%
mean value	6.86%	57.41%	35.93%

Table 17. The main difference between the three methods

Method selection	Consider factor			
	Order weight	Fuzzy information	Change the fuzzy set	Situation parameter
FOO method	NO	NO	NO	NO
Fuzzy number method	NO	YES	NO	NO
Proposed method	YES	YES	YES	YES

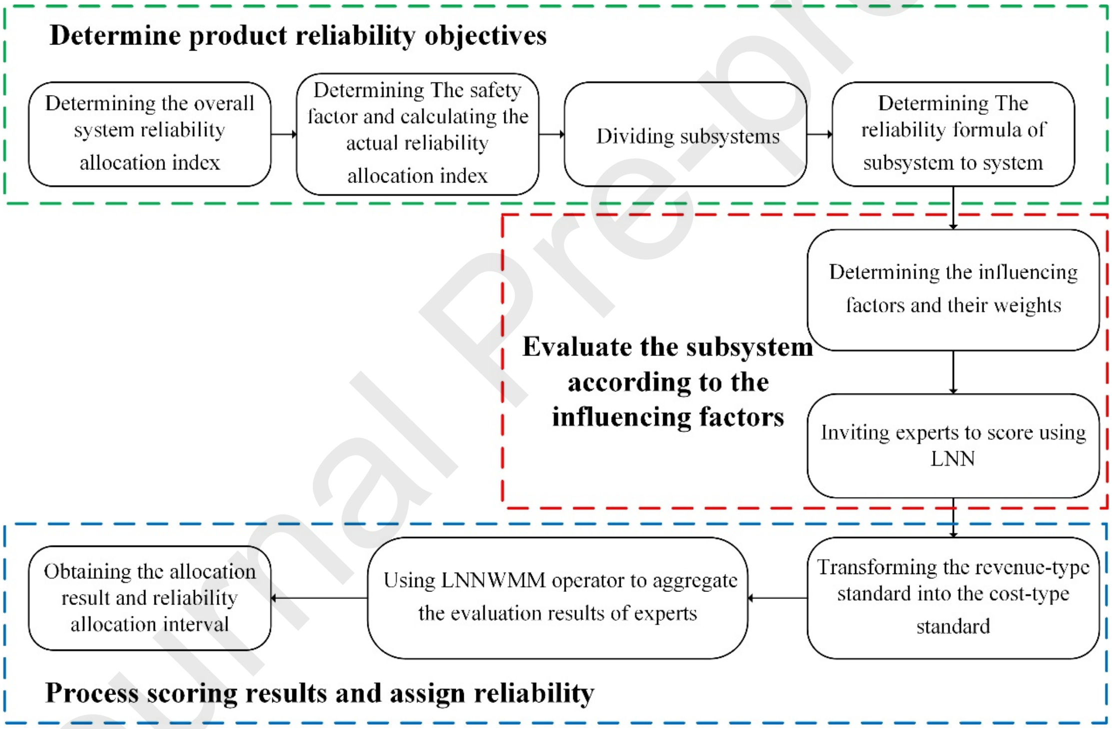


Figure 1. The flow chart of the proposed method

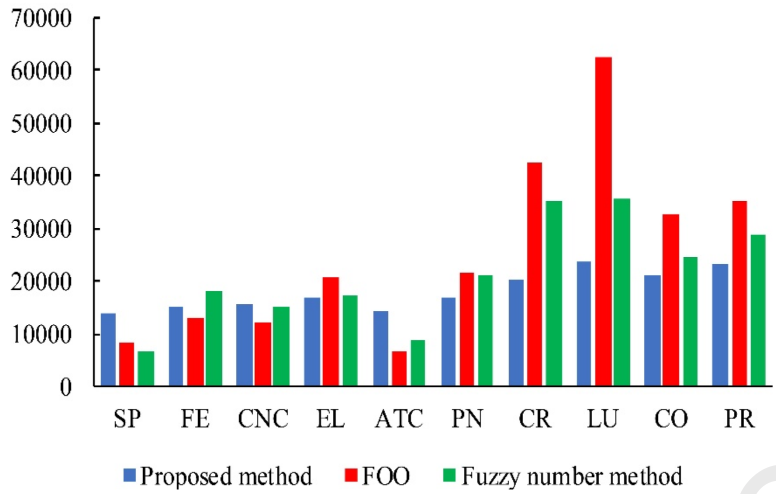


Figure 2. Reliability allocation results of the three methods

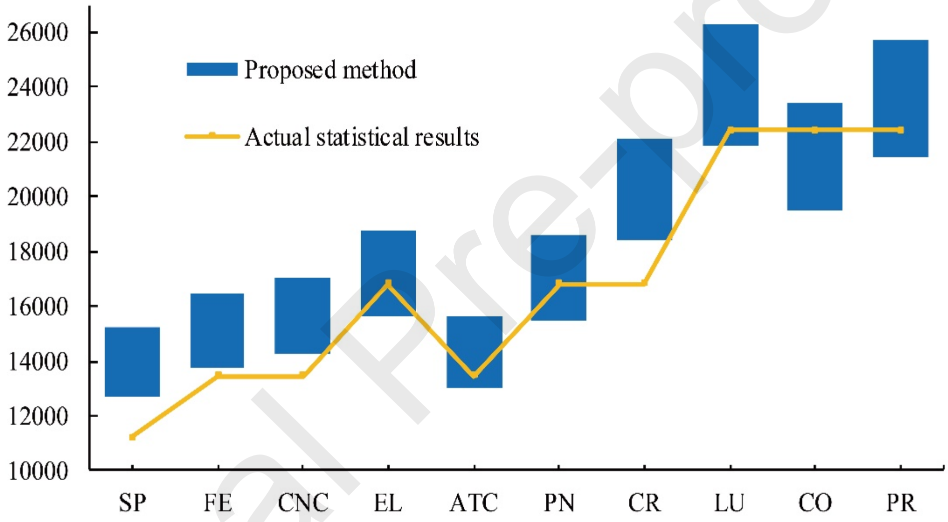


Figure 3. The proposed method results and the actual statistical results

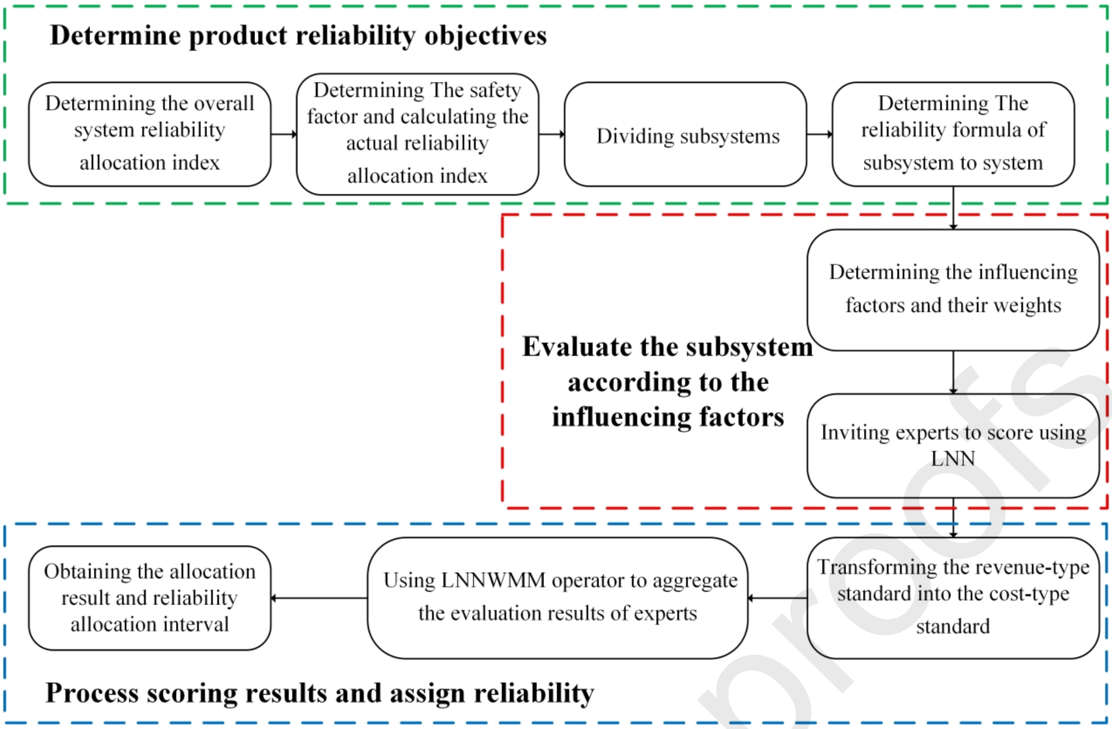


Figure 1. The flow chart of the proposed method

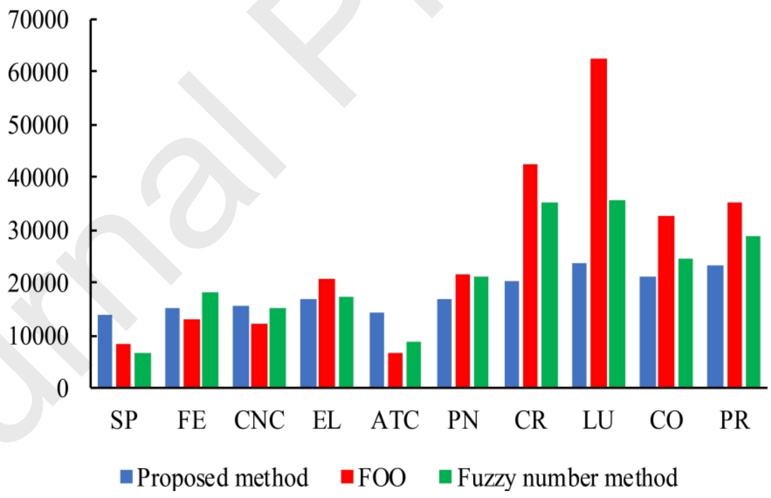


Figure 2. Reliability allocation results of the three methods

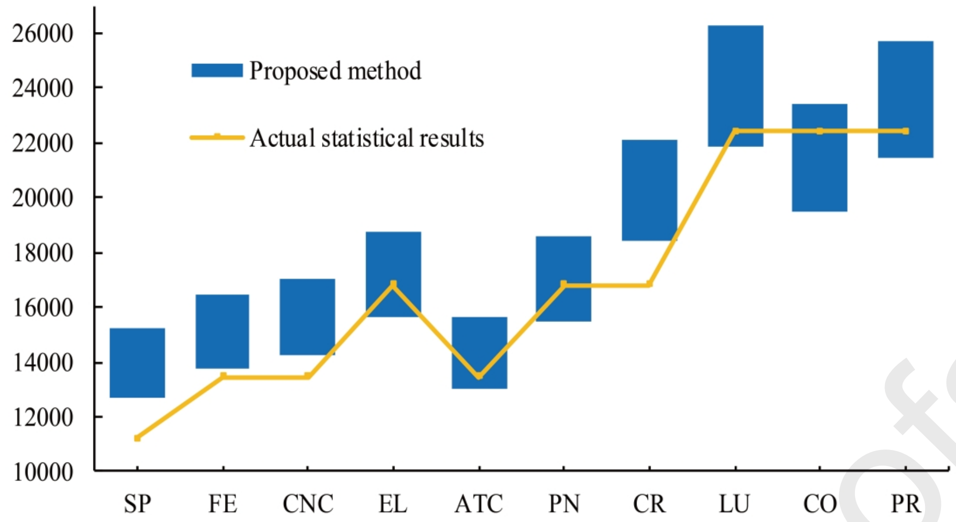
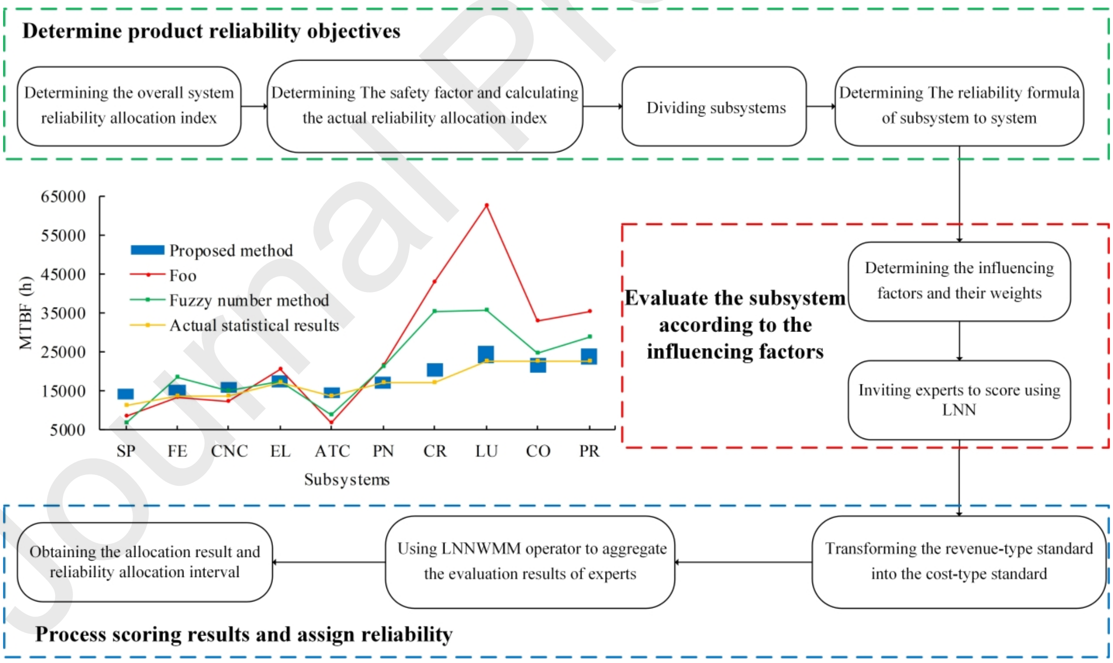


Figure 3. The proposed method results and the actual statistical results



Highlights

- The fuzzy information is expressed in an intuitive and complete way.

- A new reliability allocation method is proposed based on LNNWMM operator.
- The correlation between influencing factors is considered for the first time.
- A case study is given to illustrate the application in engineering field.
- More reasonable and flexible allocation of reliability indicators.

Table 1. Benefit/cost information for influencing factors

Influencing factors	benefit/cost
Complexity	cost
Maintainability	cost
Cost	cost
Environmental condition	benefit
Technical level	benefit
Running time	cost

Table 2. Fault data and MTBF of 10 subsystems of the machining center

Subsystems	The fault number	Fault ratio of each subsystem	Failure rate (10^-5)	MTBF (h)
SP	6	0.1429	0.893	11197
FE	5	0.1191	0.744	13434
CNC	5	0.1191	0.744	13434
EL	4	0.0952	0.595	16807
ATC	5	0.1191	0.744	13434
PN	4	0.0952	0.595	16807
CR	4	0.0952	0.595	16807
LU	3	0.0714	0.446	22409
CO	3	0.0714	0.446	22409
PR	3	0.0714	0.446	22409
aggregate	42	1	6.25	—

Table 3. Four experts rate six influencing factors

Subsystems	Experts	C	S	T	E	M	Co	Expert weight
SP	TM1	(t_8,t_1,t_1)	(t_2,t_2,t_8)	(t_8,t_1,t_1)	(t_3,t_1,t_8)	(t_3,t_1,t_8)	(t_8,t_1,t_1)	0.25
	TM2	(t_7,t_1,t_2)	(t_3,t_1,t_8)	(t_9,t_1,t_1)	(t_2,t_1,t_9)	(t_4,t_3,t_9)	(t_9,t_1,t_1)	0.35
	TM3	(t_7,t_1,t_2)	(t_2,t_1,t_9)	(t_9,t_1,t_1)	(t_3,t_1,t_8)	(t_4,t_2,t_8)	(t_8,t_1,t_1)	0.25
	TM4	(t_9,t_1,t_1)	(t_2,t_1,t_9)	(t_8,t_1,t_1)	(t_4,t_2,t_8)	(t_5,t_3,t_8)	(t_7,t_2,t_1)	0.15
FE	TM1	(t_8,t_1,t_1)	(t_3,t_2,t_9)	(t_9,t_1,t_1)	(t_6,t_1,t_7)	(t_3,t_1,t_8)	(t_6,t_1,t_2)	0.25
	TM2	(t_6,t_1,t_2)	(t_3,t_1,t_8)	(t_8,t_1,t_1)	(t_3,t_2,t_9)	(t_2,t_1,t_9)	(t_7,t_1,t_2)	0.35
	TM3	(t_6,t_1,t_2)	(t_4,t_2,t_9)	(t_8,t_1,t_1)	(t_4,t_1,t_8)	(t_5,t_2,t_9)	(t_6,t_2,t_1)	0.25
	TM4	(t_8,t_1,t_1)	(t_5,t_2,t_9)	(t_9,t_1,t_1)	(t_2,t_1,t_9)	(t_4,t_1,t_8)	(t_5,t_2,t_1)	0.15
CNC	TM1	(t_9,t_1,t_1)	(t_2,t_1,t_9)	(t_9,t_1,t_1)	(t_6,t_3,t_7)	(t_2,t_1,t_9)	(t_5,t_2,t_3)	0.25
	TM2	(t_6,t_2,t_2)	(t_3,t_2,t_9)	(t_9,t_1,t_1)	(t_5,t_1,t_7)	(t_4,t_2,t_8)	(t_4,t_2,t_4)	0.35
	TM3	(t_7,t_2,t_1)	(t_1,t_1,t_9)	(t_8,t_1,t_1)	(t_3,t_1,t_8)	(t_4,t_2,t_8)	(t_6,t_2,t_2)	0.25
	TM4	(t_8,t_1,t_1)	(t_4,t_2,t_8)	(t_7,t_2,t_1)	(t_5,t_2,t_7)	(t_2,t_1,t_9)	(t_5,t_2,t_3)	0.15
EL	TM1	(t_7,t_3,t_3)	(t_4,t_3,t_5)	(t_9,t_3,t_2)	(t_8,t_7,t_4)	(t_5,t_5,t_6)	(t_6,t_5,t_3)	0.25
	TM2	(t_6,t_4,t_2)	(t_3,t_3,t_8)	(t_7,t_2,t_3)	(t_6,t_6,t_8)	(t_4,t_2,t_6)	(t_5,t_3,t_4)	0.35
	TM3	(t_8,t_3,t_3)	(t_3,t_4,t_7)	(t_8,t_2,t_4)	(t_5,t_4,t_5)	(t_6,t_3,t_3)	(t_7,t_2,t_5)	0.25
	TM4	(t_4,t_5,t_5)	(t_2,t_2,t_6)	(t_9,t_2,t_3)	(t_4,t_4,t_7)	(t_4,t_3,t_6)	(t_6,t_2,t_6)	0.15

ATC	TM1	(t_7,t_1,t_2)	(t_1,t_1,t_9)	(t_8,t_1,t_1)	(t_2,t_1,t_9)	(t_4,t_1,t_7)	(t_4,t_2,t_4)	0.25
	TM2	(t_9,t_1,t_1)	(t_2,t_1,t_9)	(t_7,t_2,t_1)	(t_1,t_1,t_9)	(t_5,t_1,t_6)	(t_4,t_2,t_4)	0.35
	TM3	(t_8,t_1,t_1)	(t_1,t_1,t_9)	(t_9,t_1,t_1)	(t_3,t_1,t_8)	(t_6,t_3,t_7)	(t_3,t_4,t_3)	0.25
	TM4	(t_9,t_1,t_1)	(t_1,t_1,t_9)	(t_8,t_1,t_1)	(t_2,t_1,t_9)	(t_5,t_2,t_7)	(t_3,t_3,t_4)	0.15
PN	TM1	(t_4,t_2,t_7)	(t_2,t_1,t_7)	(t_8,t_3,t_1)	(t_4,t_2,t_7)	(t_7,t_5,t_3)	(t_3,t_4,t_7)	0.25
	TM2	(t_5,t_4,t_5)	(t_4,t_4,t_6)	(t_6,t_4,t_4)	(t_5,t_4,t_5)	(t_8,t_3,t_2)	(t_4,t_3,t_7)	0.35
	TM3	(t_6,t_3,t_4)	(t_3,t_2,t_6)	(t_7,t_4,t_2)	(t_4,t_4,t_7)	(t_6,t_3,t_4)	(t_5,t_4,t_4)	0.25
	TM4	(t_5,t_3,t_5)	(t_2,t_2,t_8)	(t_9,t_1,t_2)	(t_5,t_3,t_7)	(t_7,t_3,t_4)	(t_3,t_3,t_6)	0.15
CR	TM1	(t_4,t_6,t_6)	(t_6,t_5,t_3)	(t_7,t_3,t_4)	(t_4,t_4,t_5)	(t_6,t_5,t_4)	(t_5,t_4,t_6)	0.25
	TM2	(t_4,t_6,t_5)	(t_5,t_7,t_7)	(t_6,t_6,t_3)	(t_5,t_4,t_4)	(t_4,t_2,t_3)	(t_6,t_3,t_6)	0.35
	TM3	(t_5,t_5,t_5)	(t_4,t_4,t_5)	(t_8,t_4,t_3)	(t_6,t_7,t_6)	(t_3,t_6,t_8)	(t_5,t_7,t_3)	0.25
	TM4	(t_3,t_5,t_7)	(t_7,t_3,t_1)	(t_8,t_4,t_3)	(t_4,t_2,t_5)	(t_6,t_6,t_6)	(t_8,t_3,t_3)	0.15
LU	TM1	(t_5,t_6,t_6)	(t_6,t_6,t_2)	(t_8,t_6,t_6)	(t_4,t_6,t_5)	(t_6,t_5,t_3)	(t_3,t_5,t_8)	0.25
	TM2	(t_3,t_6,t_7)	(t_8,t_6,t_2)	(t_6,t_5,t_6)	(t_7,t_6,t_3)	(t_7,t_6,t_3)	(t_4,t_6,t_6)	0.35
	TM3	(t_4,t_6,t_8)	(t_5,t_5,t_3)	(t_7,t_4,t_5)	(t_5,t_5,t_3)	(t_7,t_6,t_2)	(t_4,t_5,t_7)	0.25
	TM4	(t_4,t_6,t_8)	(t_5,t_6,t_4)	(t_8,t_4,t_5)	(t_4,t_5,t_4)	(t_8,t_5,t_1)	(t_2,t_5,t_9)	0.15
CO	TM1	(t_3,t_5,t_7)	(t_4,t_5,t_5)	(t_8,t_4,t_6)	(t_6,t_5,t_3)	(t_7,t_5,t_2)	(t_3,t_5,t_7)	0.25
	TM2	(t_5,t_5,t_6)	(t_5,t_6,t_5)	(t_9,t_3,t_3)	(t_4,t_5,t_5)	(t_7,t_6,t_2)	(t_5,t_6,t_6)	0.35
	TM3	(t_4,t_7,t_7)	(t_3,t_4,t_5)	(t_8,t_4,t_4)	(t_5,t_7,t_4)	(t_6,t_6,t_4)	(t_4,t_6,t_6)	0.25
	TM4	(t_5,t_6,t_7)	(t_7,t_6,t_1)	(t_7,t_5,t_5)	(t_7,t_5,t_2)	(t_7,t_6,t_2)	(t_2,t_6,t_9)	0.15
PR	TM1	(t_4,t_5,t_8)	(t_5,t_5,t_4)	(t_5,t_5,t_6)	(t_3,t_6,t_5)	(t_7,t_5,t_3)	(t_2,t_6,t_9)	0.25
	TM2	(t_4,t_5,t_8)	(t_4,t_5,t_4)	(t_7,t_4,t_5)	(t_6,t_6,t_4)	(t_6,t_6,t_4)	(t_1,t_4,t_9)	0.35
	TM3	(t_5,t_5,t_6)	(t_5,t_5,t_3)	(t_6,t_5,t_6)	(t_5,t_5,t_4)	(t_8,t_6,t_1)	(t_3,t_6,t_8)	0.25
	TM4	(t_6,t_7,t_6)	(t_5,t_6,t_3)	(t_9,t_3,t_3)	(t_4,t_6,t_5)	(t_7,t_6,t_2)	(t_4,t_5,t_7)	0.15

Table 4. After benefit/cost transformation, four experts rating results

Subsystems	Experts	C	S	T	E	M	Co	Expert weight
SP	TM1	(t_8,t_1,t_1)	(t_8,t_2,t_2)	(t_8,t_1,t_1)	(t_7,t_1,t_2)	(t_3,t_1,t_8)	(t_8,t_1,t_1)	0.25
	TM2	(t_7,t_1,t_2)	(t_7,t_1,t_2)	(t_9,t_1,t_1)	(t_8,t_1,t_1)	(t_4,t_3,t_9)	(t_9,t_1,t_1)	0.35
	TM3	(t_7,t_1,t_2)	(t_8,t_1,t_1)	(t_9,t_1,t_1)	(t_7,t_1,t_2)	(t_4,t_2,t_8)	(t_8,t_1,t_1)	0.25
	TM4	(t_9,t_1,t_1)	(t_8,t_1,t_1)	(t_8,t_1,t_1)	(t_6,t_2,t_2)	(t_5,t_3,t_8)	(t_7,t_2,t_1)	0.15
FE	TM1	(t_8,t_1,t_1)	(t_7,t_2,t_1)	(t_9,t_1,t_1)	(t_4,t_1,t_3)	(t_3,t_1,t_8)	(t_6,t_1,t_2)	0.25
	TM2	(t_6,t_1,t_2)	(t_7,t_1,t_2)	(t_8,t_1,t_1)	(t_7,t_2,t_1)	(t_2,t_1,t_9)	(t_7,t_1,t_2)	0.35
	TM3	(t_6,t_1,t_2)	(t_6,t_2,t_1)	(t_8,t_1,t_1)	(t_6,t_1,t_2)	(t_5,t_2,t_9)	(t_6,t_2,t_1)	0.25
	TM4	(t_8,t_1,t_1)	(t_5,t_2,t_1)	(t_9,t_1,t_1)	(t_8,t_1,t_1)	(t_4,t_1,t_8)	(t_5,t_2,t_1)	0.15
CNC	TM1	(t_9,t_1,t_1)	(t_8,t_1,t_1)	(t_9,t_1,t_1)	(t_4,t_3,t_3)	(t_2,t_1,t_9)	(t_5,t_2,t_3)	0.25
	TM2	(t_6,t_2,t_2)	(t_7,t_2,t_1)	(t_9,t_1,t_1)	(t_5,t_1,t_3)	(t_4,t_2,t_8)	(t_4,t_2,t_4)	0.35
	TM3	(t_7,t_2,t_1)	(t_9,t_1,t_1)	(t_8,t_1,t_1)	(t_7,t_1,t_2)	(t_4,t_2,t_8)	(t_6,t_2,t_2)	0.25
	TM4	(t_8,t_1,t_1)	(t_6,t_2,t_2)	(t_7,t_2,t_1)	(t_5,t_2,t_3)	(t_2,t_1,t_9)	(t_5,t_2,t_3)	0.15
EL	TM1	(t_7,t_3,t_3)	(t_6,t_3,t_5)	(t_9,t_3,t_2)	(t_2,t_7,t_6)	(t_5,t_5,t_6)	(t_6,t_5,t_3)	0.25
	TM2	(t_6,t_4,t_2)	(t_7,t_3,t_2)	(t_7,t_2,t_3)	(t_4,t_6,t_2)	(t_4,t_2,t_6)	(t_5,t_3,t_4)	0.35
	TM3	(t_8,t_3,t_3)	(t_7,t_4,t_3)	(t_8,t_2,t_4)	(t_5,t_4,t_5)	(t_6,t_3,t_3)	(t_7,t_2,t_5)	0.25
	TM4	(t_4,t_5,t_5)	(t_8,t_2,t_4)	(t_9,t_2,t_3)	(t_6,t_4,t_3)	(t_4,t_3,t_6)	(t_6,t_2,t_6)	0.15
ATC	TM1	(t_7,t_1,t_2)	(t_9,t_1,t_1)	(t_8,t_1,t_1)	(t_8,t_1,t_1)	(t_4,t_1,t_7)	(t_4,t_2,t_4)	0.25
	TM2	(t_9,t_1,t_1)	(t_8,t_1,t_1)	(t_7,t_2,t_1)	(t_9,t_1,t_1)	(t_5,t_1,t_6)	(t_4,t_2,t_4)	0.35
	TM3	(t_8,t_1,t_1)	(t_9,t_1,t_1)	(t_9,t_1,t_1)	(t_7,t_1,t_2)	(t_6,t_3,t_7)	(t_3,t_4,t_3)	0.25
	TM4	(t_9,t_1,t_1)	(t_9,t_1,t_1)	(t_8,t_1,t_1)	(t_8,t_1,t_1)	(t_5,t_2,t_7)	(t_3,t_3,t_4)	0.15
PN	TM1	(t_4,t_2,t_7)	(t_8,t_1,t_3)	(t_8,t_3,t_1)	(t_6,t_2,t_3)	(t_7,t_5,t_3)	(t_3,t_4,t_7)	0.25
	TM2	(t_5,t_4,t_5)	(t_6,t_4,t_4)	(t_6,t_4,t_4)	(t_5,t_4,t_5)	(t_8,t_3,t_2)	(t_4,t_3,t_7)	0.35
	TM3	(t_6,t_3,t_4)	(t_7,t_2,t_4)	(t_7,t_4,t_2)	(t_6,t_4,t_3)	(t_6,t_3,t_4)	(t_5,t_4,t_4)	0.25
	TM4	(t_5,t_3,t_5)	(t_8,t_2,t_2)	(t_9,t_1,t_2)	(t_5,t_3,t_3)	(t_7,t_3,t_4)	(t_3,t_3,t_6)	0.15
CR	TM1	(t_4,t_6,t_6)	(t_4,t_5,t_7)	(t_7,t_3,t_4)	(t_6,t_4,t_5)	(t_6,t_5,t_4)	(t_5,t_4,t_6)	0.25
	TM2	(t_4,t_6,t_5)	(t_5,t_7,t_3)	(t_6,t_6,t_3)	(t_5,t_4,t_6)	(t_4,t_2,t_3)	(t_6,t_3,t_6)	0.35
	TM3	(t_5,t_5,t_5)	(t_6,t_4,t_5)	(t_8,t_4,t_3)	(t_4,t_7,t_4)	(t_3,t_6,t_8)	(t_5,t_7,t_3)	0.25
	TM4	(t_3,t_5,t_7)	(t_3,t_3,t_9)	(t_8,t_4,t_3)	(t_6,t_2,t_5)	(t_6,t_6,t_6)	(t_8,t_3,t_3)	0.15

LU	TM1	(t_5,t_6,t_6)	(t_4,t_6,t_8)	(t_8,t_6,t_6)	(t_6,t_6,t_5)	(t_6,t_5,t_3)	(t_3,t_5,t_8)	0.25
	TM2	(t_3,t_6,t_7)	(t_2,t_6,t_8)	(t_6,t_5,t_6)	(t_3,t_6,t_7)	(t_7,t_6,t_3)	(t_4,t_6,t_6)	0.35
	TM3	(t_4,t_6,t_8)	(t_5,t_5,t_7)	(t_7,t_4,t_5)	(t_5,t_5,t_7)	(t_7,t_6,t_2)	(t_4,t_5,t_7)	0.25
	TM4	(t_4,t_6,t_8)	(t_5,t_6,t_6)	(t_8,t_4,t_5)	(t_6,t_5,t_6)	(t_8,t_5,t_1)	(t_2,t_5,t_9)	0.15
CO	TM1	(t_3,t_5,t_7)	(t_6,t_5,t_5)	(t_8,t_4,t_6)	(t_4,t_5,t_7)	(t_7,t_5,t_2)	(t_3,t_5,t_7)	0.25
	TM2	(t_5,t_5,t_6)	(t_5,t_6,t_5)	(t_9,t_3,t_3)	(t_6,t_5,t_5)	(t_7,t_6,t_2)	(t_5,t_6,t_6)	0.35
	TM3	(t_4,t_7,t_7)	(t_7,t_4,t_5)	(t_8,t_4,t_4)	(t_5,t_7,t_6)	(t_6,t_6,t_4)	(t_4,t_6,t_6)	0.25
	TM4	(t_5,t_6,t_7)	(t_3,t_6,t_9)	(t_7,t_5,t_5)	(t_3,t_5,t_8)	(t_7,t_6,t_2)	(t_2,t_6,t_9)	0.15
PR	TM1	(t_4,t_5,t_8)	(t_5,t_5,t_6)	(t_5,t_5,t_6)	(t_7,t_6,t_5)	(t_7,t_5,t_3)	(t_2,t_6,t_9)	0.25
	TM2	(t_4,t_5,t_8)	(t_6,t_5,t_6)	(t_7,t_4,t_5)	(t_4,t_6,t_6)	(t_6,t_6,t_4)	(t_1,t_4,t_9)	0.35
	TM3	(t_5,t_5,t_6)	(t_5,t_5,t_7)	(t_6,t_5,t_6)	(t_5,t_5,t_6)	(t_8,t_6,t_1)	(t_3,t_6,t_8)	0.25
	TM4	(t_6,t_7,t_6)	(t_5,t_6,t_7)	(t_9,t_3,t_3)	(t_6,t_6,t_5)	(t_7,t_6,t_2)	(t_4,t_5,t_7)	0.15

Table 5. Scoring results of four experts after conversion by Formula (17)

Subsystems	Experts	C	S	T	E	Expert weight
SP	TM1	8	6	8	7	0.25
	TM2	7	7	9	8	0.35
	TM3	7	8	9	7	0.25
	TM4	9	8	8	6	0.15
FE	TM1	8	7	9	4	0.25
	TM2	6	7	8	7	0.35
	TM3	6	6	8	6	0.25
	TM4	8	5	9	8	0.15
CNC	TM1	9	8	9	4	0.25
	TM2	6	7	9	5	0.35
	TM3	7	9	8	7	0.25
	TM4	8	6	7	5	0.15
EL	TM1	7	6	9	2	0.25
	TM2	6	7	7	4	0.35
	TM3	8	7	8	5	0.25
	TM4	4	8	9	6	0.15
ATC	TM1	7	9	8	8	0.25
	TM2	9	8	7	9	0.35
	TM3	8	9	9	7	0.25
	TM4	9	9	8	8	0.15
PN	TM1	4	8	8	6	0.25
	TM2	5	6	6	5	0.35
	TM3	6	7	7	6	0.25
	TM4	5	8	9	5	0.15
CR	TM1	4	4	7	6	0.25
	TM2	4	5	6	5	0.35
	TM3	5	6	8	4	0.25
	TM4	3	3	8	6	0.15
LU	TM1	5	4	8	6	0.25
	TM2	3	2	6	3	0.35
	TM3	4	5	7	5	0.25
	TM4	4	5	8	6	0.15
CO	TM1	3	6	8	4	0.25
	TM2	5	5	9	6	0.35
	TM3	4	7	8	5	0.25
	TM4	5	3	7	3	0.15
PR	TM1	4	5	5	7	0.25
	TM2	4	6	7	4	0.35
	TM3	5	5	6	5	0.25
	TM4	6	5	9	6	0.15

Table 6. Reliability allocation results of FOO method

Subsystems	C	S	T	E	Comprehensive score	Complexity	Failure rate(10^-4)	MTBF(h)
SP	7.55	7.65	8.6	7.2	3576.3	0.1917	1.198	8344
FE	6.8	6.45	8.4	6.15	2265.8	0.1215	0.759	13171
CNC	7.3	7.6	8.45	5.25	2461.2	0.132	0.825	12125
EL	6.45	6.9	8.05	4.05	1451.0	0.0778	0.486	20567
ATC	8.25	8.65	7.9	8.1	4566.5	0.2448	1.530	6535
PN	5	7.05	7.2	5.5	1395.9	0.0748	0.468	21378
CR	4.1	4.7	7.05	5.15	699.6	0.0375	0.234	42653
LU	3.9	3.7	7.05	4.7	478.1	0.0256	0.160	62413
CO	4.25	5.45	8.2	4.8	911.7	0.0489	0.306	32733
PR	4.55	5.35	6.55	5.3	845.1	0.0453	0.283	35314

Table 7. Fuzzy evaluation of reliability allocation coefficient

Factors/scale	C	S	T	E	M	Co
(7,8,9,10)	very high	very high	very long	very good	very good	Very high
(5,6,7,8)	high	high	long	good	good	high
(3,4,5,6)	medium	medium	medium	medium	medium	medium
(1,2,3,4)	low	low	short	poor	poor	low

Table 8. Correspondence between LNNs and trapezoidal fuzzy numbers

Factors/scale	LNN
(7,8,9,10)	t_9,t_{10}
(5,6,7,8)	t_7,t_8
(3,4,5,6)	t_4,t_5,t_6
(1,2,3,4)	t_0,t_1,t_2,t_3

Table 9. Evaluation results of four experts after transformation of trapezoidal fuzzy number

Subsystems	Experts	C	S	T	E	M	Co	Expert weight
SP	TM1	(5,6,7,8)	(1,2,3,4)	(5,6,7,8)	(1,2,3,4)	(1,2,3,4)	(5,6,7,8)	0.25
	TM2	(5,6,7,8)	(1,2,3,4)	(7,8,9,10)	(1,2,3,4)	(3,4,5,6)	(7,8,9,10)	0.35
	TM3	(5,6,7,8)	(1,2,3,4)	(7,8,9,10)	(1,2,3,4)	(3,4,5,6)	(5,6,7,8)	0.25
	TM4	(7,8,9,10)	(1,2,3,4)	(5,6,7,8)	(3,4,5,6)	(3,4,5,6)	(5,6,7,8)	0.15
FE	TM1	(5,6,7,8)	(1,2,3,4)	(7,8,9,10)	(3,4,5,6)	(1,2,3,4)	(3,4,5,6)	0.25
	TM2	(3,4,5,6)	(1,2,3,4)	(5,6,7,8)	(1,2,3,4)	(1,2,3,4)	(5,6,7,8)	0.35
	TM3	(3,4,5,6)	(3,4,5,6)	(5,6,7,8)	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	0.25
	TM4	(5,6,7,8)	(3,4,5,6)	(7,8,9,10)	(1,2,3,4)	(3,4,5,6)	(3,4,5,6)	0.15
CNC	TM1	(7,8,9,10)	(1,2,3,4)	(7,8,9,10)	(3,4,5,6)	(1,2,3,4)	(3,4,5,6)	0.25
	TM2	(3,4,5,6)	(1,2,3,4)	(7,8,9,10)	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	0.35
	TM3	(5,6,7,8)	(1,2,3,4)	(5,6,7,8)	(1,2,3,4)	(3,4,5,6)	(3,4,5,6)	0.25
	TM4	(5,6,7,8)	(3,4,5,6)	(5,6,7,8)	(3,4,5,6)	(1,2,3,4)	(3,4,5,6)	0.15
EL	TM1	(5,6,7,8)	(3,4,5,6)	(7,8,9,10)	(5,6,7,8)	(3,4,5,6)	(3,4,5,6)	0.25
	TM2	(3,4,5,6)	(1,2,3,4)	(5,6,7,8)	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	0.35
	TM3	(5,6,7,8)	(1,2,3,4)	(5,6,7,8)	(3,4,5,6)	(3,4,5,6)	(5,6,7,8)	0.25
	TM4	(3,4,5,6)	(1,2,3,4)	(7,8,9,10)	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	0.15
ATC	TM1	(5,6,7,8)	(1,2,3,4)	(5,6,7,8)	(1,2,3,4)	(3,4,5,6)	(3,4,5,6)	0.25
	TM2	(7,8,9,10)	(1,2,3,4)	(5,6,7,8)	(1,2,3,4)	(3,4,5,6)	(3,4,5,6)	0.35
	TM3	(5,6,7,8)	(1,2,3,4)	(7,8,9,10)	(1,2,3,4)	(3,4,5,6)	(1,2,3,4)	0.25
	TM4	(7,8,9,10)	(1,2,3,4)	(5,6,7,8)	(1,2,3,4)	(3,4,5,6)	(1,2,3,4)	0.15

PN	TM1	(3,4,5,6)	(1,2,3,4)	(5,6,7,8)	(3,4,5,6)	(5,6,7,8)	(1,2,3,4)	0.25
	TM2	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	(5,6,7,8)	(3,4,5,6)	0.35
	TM3	(3,4,5,6)	(1,2,3,4)	(5,6,7,8)	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	0.25
	TM4	(3,4,5,6)	(1,2,3,4)	(7,8,9,10)	(3,4,5,6)	(5,6,7,8)	(1,2,3,4)	0.15
CR	TM1	(3,4,5,6)	(3,4,5,6)	(5,6,7,8)	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	0.25
	TM2	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	0.35
	TM3	(3,4,5,6)	(3,4,5,6)	(5,6,7,8)	(3,4,5,6)	(1,2,3,4)	(3,4,5,6)	0.25
	TM4	(1,2,3,4)	(5,6,7,8)	(5,6,7,8)	(3,4,5,6)	(3,4,5,6)	(5,6,7,8)	0.15
LU	TM1	(3,4,5,6)	(3,4,5,6)	(5,6,7,8)	(3,4,5,6)	(3,4,5,6)	(1,2,3,4)	0.25
	TM2	(1,2,3,4)	(5,6,7,8)	(3,4,5,6)	(5,6,7,8)	(5,6,7,8)	(3,4,5,6)	0.35
	TM3	(3,4,5,6)	(3,4,5,6)	(5,6,7,8)	(3,4,5,6)	(5,6,7,8)	(3,4,5,6)	0.25
	TM4	(3,4,5,6)	(3,4,5,6)	(5,6,7,8)	(3,4,5,6)	(5,6,7,8)	(1,2,3,4)	0.15
CO	TM1	(1,2,3,4)	(3,4,5,6)	(5,6,7,8)	(3,4,5,6)	(5,6,7,8)	(1,2,3,4)	0.25
	TM2	(3,4,5,6)	(3,4,5,6)	(7,8,9,10)	(3,4,5,6)	(5,6,7,8)	(3,4,5,6)	0.35
	TM3	(3,4,5,6)	(1,2,3,4)	(5,6,7,8)	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	0.25
	TM4	(3,4,5,6)	(5,6,7,8)	(5,6,7,8)	(5,6,7,8)	(5,6,7,8)	(1,2,3,4)	0.15
PR	TM1	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	(1,2,3,4)	(5,6,7,8)	(1,2,3,4)	0.25
	TM2	(3,4,5,6)	(3,4,5,6)	(5,6,7,8)	(3,4,5,6)	(3,4,5,6)	(1,2,3,4)	0.35
	TM3	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)	(5,6,7,8)	(1,2,3,4)	0.25
	TM4	(3,4,5,6)	(3,4,5,6)	(7,8,9,10)	(3,4,5,6)	(5,6,7,8)	(3,4,5,6)	0.15

Table 10. Aggregation results of evaluation information from four experts under fuzzy number method

Subsystems	C	S	T	E	M	Co
SP	(5.3,6.3,7.3,8.3)	(1.5,2.5,3.5,4.5)	(6.2,7.2,8.2,9.2)	(1.3,2.3,3.3,4.3)	(2.5,3.5,4.5,5.5)	(5.7,6.7,7.7,8.7)
FE	(3.8,4.8,5.8,6.8)	(1.8,2.8,3.8,4.8)	(5.8,6.8,7.8,8.8)	(2.0,3.0,4.0,5.0)	(1.8,2.8,3.8,4.8)	(3.7,4.7,5.7,6.7)
CNC	(4.8,5.8,6.8,7.8)	(1.3,2.3,3.3,4.3)	(6.2,7.2,8.2,9.2)	(3.0,4.0,5.0,6.0)	(2.2,3.2,4.2,5.2)	(3.0,4.0,5.0,6.0)
EL	(4.0,5.0,6.0,7.0)	(4.5,5.5,6.5,7.5)	(5.8,6.8,7.8,8.8)	(2.5,3.5,4.5,5.5)	(3.0,4.0,5.0,6.0)	(3.5,4.5,5.5,6.5)
ATC	(6.0,7.0,8.0,9.0)	(1.0,2.0,3.0,4.0)	(5.5,6.5,7.5,8.5)	(1.0,2.0,3.0,4.0)	(3.0,4.0,5.0,6.0)	(2.2,3.2,4.2,5.2)
PN	(3.0,4.0,5.0,6.0)	(1.7,2.7,3.7,4.7)	(4.6,5.6,6.6,7.6)	(3.0,4.0,5.0,6.0)	(4.5,5.5,6.5,7.5)	(2.2,3.2,4.2,5.2)
CR	(2.7,3.7,4.7,5.7)	(2.7,3.7,4.7,5.7)	(4.3,5.3,6.3,7.3)	(3.0,4.0,5.0,6.0)	(2.5,3.5,4.5,5.5)	(3.3,4.3,5.3,6.3)
LU	(2.3,3.3,4.3,5.3)	(3.7,4.7,5.7,6.7)	(4.3,5.3,6.3,7.3)	(3.7,4.7,5.7,6.7)	(4.5,5.5,6.5,7.5)	(2.2,3.2,4.2,5.2)
CO	(2.5,3.5,4.5,5.5)	(2.8,3.8,4.8,5.8)	(5.7,6.7,7.7,8.7)	(3.3,4.3,5.3,6.3)	(4.5,5.5,6.5,7.5)	(2.2,3.2,4.2,5.2)
PR	(3.0,4.0,5.0,6.0)	(3.0,4.0,5.0,6.0)	(4.3,5.3,6.3,7.3)	(2.5,3.5,4.5,5.5)	(4.3,5.3,6.3,7.3)	(1.3,2.3,3.3,4.3)

Table 11. Result of ten subsystem evaluation information processing in fuzzy number method

Subsystems	C×Co×M×T	S×E	FZ
SP	(468.3, 1063.7, 2074.1, 4913.8)	(1.3, 4.6, 9.9, 17.2)	(50.8, 107.4, 450.9, 598.5)
FE	(146.8, 429.5, 979.9, 2786.2)	(3.6, 8.4, 15.2, 24)	(0, 28.3, 116.7, 269.7)
CNC	(196.4, 534.5, 1171.0, 3358.4)	(3.3, 8.1, 14.9, 23.7)	(0, 36.0, 145.5, 327.4)
EL	(243.6, 612.0, 1287.0, 3511.2)	(5.3, 11.3, 19.3, 29.3)	(0, 31.8, 114.4, 289.3)
ATC	(217.8, 582.4, 1260.0, 3763.8)	(1.0, 4.0, 9.0, 16.0)	(24.1, 64.7, 315, 497.7)
PN	(136.6, 394.2, 900.9, 2804.4)	(5.1, 10.8, 18.5, 28.2)	(0, 21.3, 83.4, 244.9)
CR	(95.8, 295.1, 706.2, 2128.4)	(9.9, 17.2, 26.5, 37.8)	(0, 11.1, 41.1, 152.5)
LU	(97.9, 307.8, 739.6, 2379.4)	(13.7, 22.1, 32.5, 44.9)	(0, 9.5, 33.5, 155.0)
CO	(141.1, 412.7, 945.9, 2942.8)	(9.2, 16.3, 25.4, 36.5)	(0, 16.2, 57.9, 217.1)
PR	(72.1, 258.4, 654.9, 2334.1)	(7.5, 14.0, 22.5, 33.0)	(0, 11.5, 46.8, 189.0)

Table 12. Results of reliability allocation in fuzzy number method

Subsystems	Defuzzification (FZ)	Weight	Failure rate 10^-3	MTBF/h
SP	303.65	0.2433	0.1521	6576
FE	108.93	0.0873	0.0545	18332
CNC	133.29	0.1068	0.0667	14981
EL	115.49	0.0925	0.0578	17290
ATC	229.02	0.1835	0.1147	8719
PN	94.37	0.0756	0.0473	21159
CR	56.79	0.0455	0.0284	35161
LU	56.32	0.0451	0.0282	35452
CO	80.87	0.0648	0.0408	24691
PR	69.29	0.0555	0.0347	28818

Table 13. Aggregation results of the evaluation information of influencing factors in the proposed method

Influencing factors	C	S	T	E	M	Co
SP	$(t_{7.55},t_{1},t_{1.6})$	$(t_{7.65},t_{1.25},t_{1.6})$	$(t_{8.6},t_{1},t_{1})$	$(t_{7.2},t_{1.15},t_{1.65})$	$(t_{3.9},t_{2.25},t_{6.85})$	$(t_{8.2},t_{1.15},t_{1})$
FE	$(t_{6.8},t_{1},t_{1.6})$	$(t_{6.45},t_{1.65},t_{1.35})$	$(t_{8.4},t_{1},t_{1})$	$(t_{6.15},t_{1.35},t_{1.75})$	$(t_{3.3},t_{1.25},t_{8.6})$	$(t_{6.2},t_{1.4},t_{1.6})$
CNC	$(t_{7.3},t_{1.6},t_{1.35})$	$(t_{7.6},t_{1.5},t_{1.15})$	$(t_{8.45},t_{1.15},t_{1})$	$(t_{5.25},t_{1.65},t_{2.75})$	$(t_{3.2},t_{1.6},t_{8.4})$	$(t_{4.9},t_{2},t_{3.1})$
EL	$(t_{6.45},t_{3.65},t_{2.95})$	$(t_{6.9},t_{3.1},t_{3.3})$	$(t_{8.05},t_{2.25},t_{3})$	$(t_{4.05},t_{5.45},t_{3.9})$	$(t_{4.75},t_{3.15},t_{5.25})$	$(t_{5.9},t_{3.1},t_{4.3})$
ATC	$(t_{8.25},t_{1},t_{1.25})$	$(t_{8.65},t_{1},t_{1})$	$(t_{7.9},t_{1.35},t_{1})$	$(t_{8.1},t_{1},t_{1.25})$	$(t_{5},t_{1.65},t_{6.65})$	$(t_{3.6},t_{2.65},t_{3.75})$
PN	$(t_{5},t_{3.1},t_{5.25})$	$(t_{7.05},t_{2.45},t_{3.45})$	$(t_{7.2},t_{3.3},t_{2.45})$	$(t_{5.5},t_{3.35},t_{3.7})$	$(t_{7.1},t_{3.5},t_{3.05})$	$(t_{3.85},t_{3.5},t_{6.1})$
CR	$(t_{4.1},t_{5.6},t_{5.55})$	$(t_{4.7},t_{5.15},t_{5.4})$	$(t_{7.05},t_{4.45},t_{3.25})$	$(t_{5.15},t_{4.45},t_{5.1})$	$(t_{4.55},t_{4.35},t_{4.95})$	$(t_{5.8},t_{4.25},t_{4.8})$
LU	$(t_{3.9},t_{6},t_{7.15})$	$(t_{3.7},t_{5.75},t_{7.45})$	$(t_{7.05},t_{4.85},t_{5.6})$	$(t_{4.7},t_{5.6},t_{6.35})$	$(t_{6.9},t_{5.6},t_{2.45})$	$(t_{3.45},t_{5.35},t_{7.2})$
CO	$(t_{4.25},t_{5.65},t_{6.65})$	$(t_{5.45},t_{5.25},t_{5.6})$	$(t_{8.2},t_{3.8},t_{4.3})$	$(t_{4.8},t_{5.5},t_{6.2})$	$(t_{6.75},t_{5.75},t_{2.5})$	$(t_{3.8},t_{5.75},t_{6.7})$
PR	$(t_{4.55},t_{5.3},t_{7.2})$	$(t_{5.35},t_{5.15},t_{6.4})$	$(t_{6.55},t_{4.35},t_{5.2})$	$(t_{5.3},t_{5.75},t_{5.6})$	$(t_{6.9},t_{5.75},t_{2.7})$	$(t_{2.2},t_{5.15},t_{8.45})$

Table 14. Comprehensive score and reliability allocation results of subsystems in the proposed method

Subsystems	Comprehensive evaluation information	Comprehensive score	Failure rate (10^-5)	MTBF interval (h)
SP	$(t_{6.5136},t_{1.7531},t_{3.2434})$	0.7172	6.563	[12697,15236]
FE	$(t_{5.6676},t_{1.5940},t_{4.1883})$	0.6628	6.066	[13739,16486]
CNC	$(t_{5.4537},t_{1.9187},t_{4.3163})$	0.6406	5.862	[14215,17058]
EL	$(t_{5.5406},t_{3.8420},t_{4.2170})$	0.5827	5.332	[15628,18753]
ATC	$(t_{6.1747},t_{1.8550},t_{3.3749})$	0.6982	6.389	[13044,15652]
PN	$(t_{5.5767},t_{3.6183},t_{4.3367})$	0.5874	5.375	[15503,18604]
CR	$(t_{4.9173},t_{4.9865},t_{5.1065})$	0.4941	4.522	[18429,22115]
LU	$(t_{4.6539},t_{5.7685},t_{6.4031})$	0.4161	3.807	[21887,26264]
CO	$(t_{5.2351},t_{5.5543},t_{5.6547})$	0.4675	4.278	[19478,23373]
PR	$(t_{4.6942},t_{5.5241},t_{6.4247})$	0.4248	3.888	[21435,25722]

Table 15. Comparison of three reliability allocation methods with actual statistics

Subsystems	Proposed method (h)	FOO (h)	Fuzzy number method (h)	Actual statistical (h)
SP	[12697,15236]	8344	6576	11197
FE	[13739,16486]	13171	18332	13434
CNC	[14215,17058]	12125	14981	13434
EL	[15628,18753]	20567	17290	16807
ATC	[13044,15652]	6535	8719	13434
PN	[15503,18604]	21378	21159	16807
CR	[18429,22115]	42653	35161	16807
LU	[21887,26264]	62413	35452	22409
CO	[19478,23373]	32733	24691	22409
PR	[21435,25722]	35314	28818	22409

Table 16. The deviation between the three methods and the actual statistical results

Subsystems	Proposed method	FOO	Fuzzy number method
SP	13.40%	25.48%	41.27%
FE	2.27%	1.96%	36.46%
CNC	5.81%	9.74%	11.52%
EL	7.01%	22.37%	2.87%
ATC	2.90%	51.35%	35.10%
PN	7.76%	27.20%	25.89%
CR	9.65%	153.78%	109.20%
LU	2.33%	178.52%	58.20%
CO	13.08%	46.07%	10.18%
PR	4.35%	57.59%	28.60%
maximum	13.40%	178.52%	109.20%
mean value	6.86%	57.41%	35.93%

Table 17. The main difference between the three methods

Method selection	Consider factor			
	Order weight	Fuzzy information	Change the fuzzy set	Situation parameter
FOO method	NO	NO	NO	NO
Fuzzy number method	NO	YES	NO	NO
Proposed method	YES	YES	YES	YES