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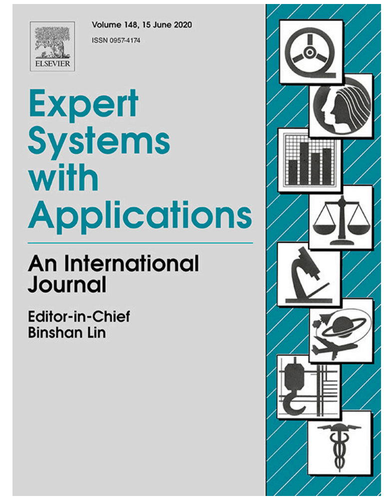

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Multi-attribute group decision-making considering opinion dynamics

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ABSTRACT: Multi-attribute group decision-making (MAGDM) is a complicated cognitive process that involves evaluation of opinion expression, information fusion, and analysis of multi-source uncertainties. In the incipient stage of MAGDM, decision makers (DMs) express their opinions on alternatives for each attribute and tend to interact with others. Despite the opinions of DMs are dynamically evolved, the traditional information fusion techniques are always relying on the data acquired at a static decision-making time point, which leads to the loss of information. In this study, we introduce a novel MAGDM method considering opinion dynamics, which employs the 2-tuple linguistic model for the representation of linguistic judgements and the technique for order preference by similarity to an ideal solution (TOPSIS) as the decision-making framework, to reduce the loss of information from the dimension of opinion formation. Moreover, a modified opinion dynamics model is as well as developed by extending the hypothesis of bounded confidence, where the opinion similarity, the credibility of DMs, and the human bounded rationality are collectively regarded as influential factors within the process of opinion evolution. Subsequently, three simulations are carried out to verify the feasibility of the extended bounded confidence model. And finally, a case study of supplier selection, as a typical MAGDM problem, is implemented and a comparison analysis is conducted to demonstrate the rationality of the proposed method.

Keywords: MAGDM; Opinion dynamics; 2-tuple linguistic; TOPSIS; Bounded rationality; Extended bounded confidence

1. Introduction

Multi-attribute group decision making (MAGDM) is an area of research interest with strong practical significance as its theory and tools are motivated by practical problems encountered in the private and public sectors (Bao, Xie, Long, & Wei, 2017). MAGDM consists of many decision matrices provided by a panel of decision makers (DMs), each of which involves a set of finite alternatives that are depicted by finite attributes (Zhang, Li, Liang, & Wang, 2020). To handle the uncertainty and fuzziness of DMs' judgments, qualitative attributes are described by using fuzzy sets (Zadeh, 1965), triangular fuzzy numbers (Fahmi & Amin, 2019), intuitionistic fuzzy numbers (Atanassov, 1986; Krishankumar et al., 2019), or bipolar soft sets (Shabir, & Naz, 2013). But majority of these approaches require transforming vague variables into numerical values; thus, some fuzzy information may be distorted. To reduce the distortion of information, the 2-tuple linguistic model (TLM), which has the advantage of being continuous in its domain of definition (Wan, Xu, & Dong, 2017; He et al., 2020), is adopted for representation of linguistic judgments in this study.

Aggregation methods are then employed to consolidate the sundry matrices into a synthetic decision matrix before ranking the alternatives. Traditionally, aggregation methods include the ordered weighted averaging (OWA) operator (Qi, Liang, & Zhang, 2015), weighted geometric operator (Khaleie & Fasanghari, 2012), simple additive weighting operator (Chen & Yang, 2011), and symmetric OWA operator (Xia & Xu, 2012). These operators are all based on data acquired only at a certain decision-making time point, that is, the formation of opinions is assumed to be static. However, in many real decision-making scenarios, a group of DMs have rounds of discussion at the initial evaluation stage. During the discussion process, affected by the previous discussions, DMs modify and present their new opinions, which leads to the evolution of their opinions. Therefore, it is imperative to develop a modified MAGDM method that can consider the dynamic interaction process among DMs.

It is worth mentioning at this time that opinion dynamics investigates the fusion process of opinion formation in a group of DMs and is a powerful tool for supporting the management of public opinions (Dong, Ding, Martínez, & Herrera, 2017). In opinion dynamics models, the bounded confidence is recognised as an effective representative fusion rule and has been intensively studied in recent years (Liang, Li, Dong, & Jiang, 2016; Shi, Wang, Palomares, Guo, & Ding, 2018; Zhang, Gao, & Li, 2020). Nevertheless, most extant models only use the opinion similarity to define the confidence threshold and do not consider the bounded rationality of DMs; thus, they cannot be effectively applied to practical problems such as MAGDM. Consequently, based on opinion dynamics, a modified bounded confidence model that can cope with the bounded rationality of DMs in the context of MAGDM should be developed.

In this article, we introduce a dynamical MAGDM method combining opinion dynamics, which employs the TLM for representing the linguistic judgements and the TOPSIS as the decision-making framework, creatively reducing the information loss from the dimension of opinion formation. Concurrently, inspired by the Hegselmann and Krause (HK) model, an extended bounded confidence (EBC) model is proposed by extending the hypothesis of bounded confidence. In the EBC model, not only the opinion similarity but also the credibility of DMs determines the confidence threshold; thus, one DM may have different thresholds to others in the opinion interaction process. Moreover, to be more consistent with the complex context in real MAGDM, the EBC model further considers the human bounded rationality, which is represented as stubborn DMs (SDMs), authority DMs (ADMs), and negative bias (NBs), than the existing opinion dynamics models.

The remainder of this article is organised as follows. In Section 2, a literature review is provided. In Section 3, preliminaries of the TLM and HK model are introduced. Section 4 describes the TOPSIS-based framework of the MAGDM problem considering the opinion dynamics. Section 5 constructs the simulation analysis to illustrate the rationality and feasibility of the EBC model. A case study and comparison analysis are established in Section 6 to elaborate on the effectiveness of the proposed MAGDM framework. Conclusions and future work are provided in Section 7.

2. Literature review

2.1 Multi-attribute group decision-making and information fusion techniques

MAGDM is a decision-making problem involving multiple schemes, multiple attributes, and a group of DMs. Research on MAGDM mainly concentrates on the development of decision-making frameworks, the quantification of qualitative attributes, and the fusion techniques of evaluation information.

The purpose of MAGDM is to obtain the alternative with the highest utility score from a set of alternatives. Thus, a series of alternative ranking methods have been developed, such as TOPSIS (Selvachandran, Quek, Smarandache, & Broumi, 2018), VIKOR (Opricovic & Tzeng, 2004), PROMETHEE (Nassereddine, Azar, Rajabzadeh, & Afsar, 2019), and ELECTRE (Singh, Verma, & Tiwari, 2020). Additionally, the utility score is defined based on the preferences of the DMs and the initial evaluations (Zeng, Hu, Balezentis, & Streimikiene, 2020). To handle the uncertainties have always been a prime concern for the researchers. Many efforts have been constantly made, but the first concrete effort was initiated by Zadeh who introduced the notion of fuzzy sets (Zadeh, 1965). Since then, fuzzy sets have been applied in many directions such as decision making (Ekel, 2002), medical diagnosis (Yao & Yao, 2001) and pattern recognition (Pedrycz, 1990). Keeping in the importance of "fuzzy" of fuzzy sets, many extensions of fuzzy sets have been introduced like rough sets (Pawlak, 1982), soft sets (Molodtsov, 1999), intuitionistic fuzzy sets (Atanassov, 1986), linear diophantine fuzzy sets (Riaz & Hashmi, 2019), bipolar valued fuzzy sets (Lee, 2000), and bipolar soft sets (Shabir & Naz, 2013; Mahmood, 2020), neutrosophic sets (Smarandache, 2000; Smarandache, 2002), spherical and T-spherical sets (Mahmood, Ullah, Khan, & Jan, 2019). However, most of them depend on the defuzzification approach by converting fuzzy variables to numeric values, which causes the distortion of information. In order to maintain the fuzzy information, the 2-tuple linguistic information processing manner, as its superiority of being continuous in its domain of definition, has always been favoured by scholars (He et al., 2020; Wei & Gao, 2020; He, Wei, Wu, & Wei, 2021). As these extensions of fuzzy sets have been extensively used in decision-making problems (Liu, 2014; Ullah, Mahmood, & Jan, 2018; Ashraf & Abdullah, 2019), then how to efficiently aggregate this information seems particularly important.

Aggregation operators for MAGDM have drawn significant attention, and many achievements have been gained in recent years. Among them, several intensively investigated works include the OWA operators, which allow the implementation of concepts of the fuzzy majority, at least half and as much as possible (Geng, Chu, & Zhang, 2010), correlated aggregation operators (Xu, 2010; Wei & Zhao, 2012), generalised aggregation operators (Qi, Liang, & Zhang, 2013), and hybrid aggregation operators (Zhang & Qi, 2012). However, these methods fuse the decision-making information based on static data and predefined weights. Extreme data tends to be normalised and cannot be delivered to the end, which leads to loss of information.

Since decision-making is information-driven, it is necessary to reduce the loss of information. Although preference expression and management in MAGDM are well studied, there is still limited research on information fusion considering opinion dynamics.

2.2 Opinion dynamics in decision-making

Given that bounded rational (Riddalls & Bennett, 2003) people are ubiquitous in real economic, engineering, and social systems, previous studies have introduced the "opinion dynamics" to deal with the uncertainty created because people tend to adjust their opinions (Lorenz, 2005; Castellano, Fortunato, & Loreto, 2009). Extant models of opinion dynamics can be classified into two types, namely, the discrete opinion model (DOM) and continuous opinion model (COM). The DOM defines DM opinion choices as discrete values, such as the Ising model (Sznajd-Weron & Weron, 2002), voter model (Sood & Redner, 2005), and Sznajd model (Bernardes, Stauffer, & Kertesz, 2002). The COM dissimilarly defines continuous variables based on the hypothesis of bounded confidence, which assumes that DMs only consider opinions similar to those of themselves and ignore the sufficiently different ones.

The use of bounded confidence as a fusion rule has become a topic of intensive research in recent years (Liang, Li, Dong, & Jiang, 2016). The classical models mainly include the HK model (Hegselmann & Krause, 2002) and the Deffuant and Weisbuch (DW) model (Weisbuch, Deffuant, & Amblard, 2005). DMs update their opinions by averaging all the opinions in their confidence set in the HK model, whereas they do so by pairwise-sequential communication in the DW model, which determines that the former is more suitable for small-sized groups. However, these models can only be applied to some simple and certain problems.

Researchers have sought to establish relative rules in response to the complexity and uncertainty of objective things and the ambiguity of human thinking; thus, many modified bounded confidence models have been proposed (Shi, Wang, Palomares, Guo, & Ding, 2018; Liang, Li, Jiang, & Dong, 2019; Zhang, Gao, & Li, 2020). In general, these models are extended from three dimensions: (1) opinions of DMs are expressed in the form of exact numbers to linguistic terms; (2) the agent-based homogeneous models are expanded to heterogeneous models, that is, each DM has a respective confidence threshold; and (3) it is assumed that the confidence thresholds change in a certain range as time varies. Scholars select one or more of the three dimensions in their studies. Dong, Chen, Liang, and Li (2016) proposed a linguistic opinion dynamics model in the framework of bounded confidence and the TLM with numerical scales. Liang et al. (2016) introduced the trapezoidal fuzzy number and further considered the third dimension. It is undeniable that these works have solved many practical problems and have been of great enlightening significance.

Nonetheless, in most of the above models, a DM has only one confidence threshold at a certain time, which means that different opinions have equivalent effect intensity. This is an oversimplification of a real-life phenomenon, and especially contrary to the reality that different DMs own different weights in MAGDM. Consequently, the dynamical (relative) influence of the others for a DM should be investigated in the opinion dynamics in the context of MAGDM.

2.3 Influential factors in opinion dynamics

Under the dynamic decision-making environment, the opinions of a DM change and are inevitably influenced by other factors outside of the confidence radius. People tend to accept ideas that are similar to theirs, and simultaneously consider the credibility of the providers, for example, the credibility of the ADMs would be higher, while that of the opinions of common DMs would be lower (Su, Liu, Li, & Ma, 2014). It has also been proven that SDMs may be unable to reach consensus and cause disagreement in a social influence network (Proskurnikov, Tempo, Cao, & Friedkin, 2017; Tian & Wang, 2018). In addition, the NBs are psychological effects; people prefer to perceive and process negative events first in complex information environments (Ito, Larsen, Smith, & Cacioppo, 1998; Rozin & Royzman, 2016), that is, there is a greater impact of negative versus positive stimuli on a DM (Akhtar, Faff, Oliver, & Subrahmanyam, 2011). These are all manifestations of bounded rationality, and it is consequential to propose an effective method to predict the evolution of opinions for a more rational decision result considering the above issues.

Motivated by the above review, the contribution of this study is twofold:

(1) To reduce the loss of information caused by the static nature of traditional information fusion techniques, opinion dynamics is introduced to describe the dynamic interaction and evolution of DMs' opinions in the MAGDM process.

(2) An EBC-modified opinion dynamics model catering to the MAGDM problem is developed, which extends the hypothesis of bounded confidence and simultaneously considers the human bounded rationality. The MAGDM method that employs the EBC model can effectively reflect the reality, thus producing more practical and rational decision-making outcomes.

3. Preliminaries

This section provides preliminaries regarding the TLM and HK model in preparation for the following discussion.

3.1 Two-tuple linguistic representation model

Herrera and Martinez (2000) first introduced the TLM, which consists of a linguistic term and a numeric value. Subsequently, Wan, Xu, and Dong (2017) applied a generalised TLM and interchangeable function between linguistic terms and 2-tuples in their research. In this study, we adopt the generalised TLM to quantify the evaluation of qualitative attributes.

Definition 1. (Wan, Xu, and Dong, 2017) Let $S = \{S_0, S_1, S_2, \ldots, S_g\}$ be a predefined linguistic term set with granularity $g + 1$. The linguistic information will be expressed by means of (S_i, α_i), where $S_i \in S$ represents the linguistic label centre of the information and $\alpha_i \in [-\frac{1}{2g}, \frac{1}{2g})$ indicates the deviation to the central value of S_i. Specifically, a linguistic term S_i can be converted into a 2-tuple linguistic variable $(S_i, 0)$.

Example 1. Let $S = \{S_0, S_1, S_2, S_3, S_4, S_5, S_6\}$ be a linguistic term set. The meanings of its elements are: S_0 = extremely bad (EB), S_1 = very bad (VB), S_2 = bad (B), S_3 = general (GE), S_4 = good (G), S_5 = very good (VG), and S_6 = extremely good (EG). Thus, $(S_4, 0.2)$ means that the real evaluation is better than the term "good" with a degree of 0.2. Besides, "VB-B" means the degree between "very bad" and "bad" which can be expressed as $(S_1, 0.04)$.

Definition 2. (Wan, Xu, and Dong, 2017) Together with the 2-tuple variable (S_i, α_i), there is a numerical value $\beta \in [0, 1]$ representing the result of an aggregation of a set of indices assessed in a linguistic term set S. Correspondingly, a pair of reciprocal translation functions Δ and Δ^{-1} are defined as follows:

$$\Delta(\beta) = (S_i, \alpha_i), \begin{cases} i = round\,(\beta \times g) \\ \alpha = \beta - \frac{i}{g} \quad, \alpha \in \left[-\frac{1}{2g}, \frac{1}{2g} \right) \end{cases} \tag{1}$$

$$\beta = \Delta^{-1}(S_i, \alpha_i) \tag{2}$$

Example 2. The same conditions as in Example 1 suppose the symbolic aggregation operation $\beta = 0.35$. In virtue of Eq. (1), it yields that i = round $(0.35 \times 6) = 2$ and α = $0.35-2/6 = 0.017$. Therefore, the 2-tuple $(S_2, 0.017)$ that expresses the equivalent information to 0.35 is obtained, and a reverse calculation can be obtained by Eq. (2).

Definition 3. (Wan, Xu, and Dong, 2017) A matrix $\mathbf{Z} = (z_{ij})_{m\times n}$ is termed as 2-tuple linguistic matrix if all its elements $z_{ij} = (S_{ij}, \alpha_{ij})$ are defined as Definition 1, where $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

Definition 4. (Wan, Xu, and Dong, 2017) Given a 2-tuple linguistic matrix $\mathbf{Z}$ = $(z_{ij})_{m\times n}$, if $\beta_{ij} = \Delta^{-1}(z_{ij})$, matrix $\Delta^{-1}(\mathbf{Z}) = (\beta_{ij})_{m\times n}$ is called a transformed 2-tuple linguistic matrix of $\mathbf{Z}$. Meanwhile, $\mathbf{Z}$ is called the 2-tuple linguistic matrix of $\Delta^{-1}(\mathbf{Z})$.

3.2 Hegselmann and Krause model

Considering that the size of the decision-making group is normally small, the HK model is suitable to be selected from the family of opinion dynamics models. Its contents are briefly introduced as follows (Hegselmann & Krause, 2002).

Let $D = \{d_1, d_2, ..., d_k\}$ denote a group of k DMs and time $t \in \{0, 1, 2, \ldots, N\}$ as a discrete variable representing the number of rounds of opinion interaction (N is a sufficiently large positive integer). For $\forall d_i \in D$, we denote his numeric opinion at time t by $x_i^t \in [0, 1]$. Accordingly, a column vector $\mathbf{X}^t = (x_1^t, x_2^t, \ldots, x_k^t)^{\mathrm{T}}$ is called the opinion profile at time t.

Regardless of time, d_i only considers opinions that differ not more than a nonnegative confidence threshold ε_i from his own opinion. Therefore, the confidence set, that is, the set of DMs whose opinions are considered by d_i is determined by using

$$I\,(d_i, x^t) = \{d_j \mid Dis\,(x_i^t, x_j^t) \leq \varepsilon_i\},\ i, j = 1, \ldots, k \quad (3),$$

where "Dis" is the Euclidean distance representing the similarity between opinions.

Next, let w_{ij}^t represent the confidence degree that d_i assigns to d_j at time t, and each DM in the confidence set has equal weight.

$$w_{ij}^t = \begin{cases} \frac{1}{\|I\,(\mathrm{d}_i, x^t)\|}, & |x_i^t - x_j^t| \leq \varepsilon_i \\ 0, & |x_i^t - x_j^t| > \varepsilon_i \end{cases} \quad (4)$$

where $\|\cdot\|$ denotes the number of elements in a finite set, $w_{ij}^t \geq 0$, and $\sum_{j=1}^{n} w_{ij}^t = 1$. Finally, the updated opinions are obtained by the weighted arithmetic means of opinions in the confidence sets by using

$$x_i^{t+1} = \sum_{j=1}^{n} w_{ij}^t x_j^t \quad (5).$$

Specifically, if $\varepsilon_i = \varepsilon_j$ for $\forall i, j = 1, 2, ..., k$, we call the HK model homogeneous, otherwise heterogeneous.

The above iteration process is presented in Program I, as included in Table 1.

Table 1. Opinion evolution process for HK model

Steps	Details
1:	Input $\mathbf{X}^t$, k and ε
2:	$t \leftarrow 0$
3:	Calculating the weights w_{ij}^t by using Eq. (3) and (4)
4:	Updating the current opinions x_i^t ($i = 1, 2, \ldots, k$) by weighted arithmetic means of opinions according to Eq. (5)
5:	If $x_i^{t+1} = x_i^t$, then output $\mathbf{X}^t$ and t
6:	Else $t \leftarrow t+1$, go to Step 3
7:	End if

4. Multi-attribute group decision-making framework considering opinion dynamics

For the sake of convenience, the problem description is given by means of the relevant variables and their annotations, which are listed in Table 2. The framework of the proposed MAGDM methodology is demonstrated in Figure 1. As shown in Figure 1, there are three phases of the proposed methodology, namely, Phase I: expression of linguistic judgments based on TLM, Phase II: information fusing considering dynamic evolution of DMs' opinions, and Phase III: order ranking of alternatives by using TOPSIS.

Table 2. Variables and annotations

Variables	annotations
$D = \{d_1, d_2, ..., d_k\}$	k DMs
$C = \{c_1, c_2, \ldots, c_n\}$	n attributes
$A = \{a_1, a_2, \ldots, a_m\}$	m alternatives
$\Omega = \{\omega_1, \omega_2, ..., \omega_k\}$	weights of DMs
$W = \{w_1, w_2, ..., w_n\}$	weights of attributes
$S = \{S_0, S_1, \ldots, S_6\}$	linguistic term set defined in Example 1
G	grade assignment defined in Table 3
$t \in \{0,1, 2, \ldots, N\}$	discrete time variables
$\mathbf{O} = (o_1, o_2, \ldots, o_k)^{\mathrm{T}}$	2-tuple linguistic vector
$\overline{\mathbf{O}^{\mathbf{t}}} = (\beta^t{}_1, \beta^t{}_2, \ldots, \beta^t{}_k)^{\mathrm{T}}$	transformed 2-tuple linguistic vector at time t
$\mathbf{T} = [T_{ij}]_{k\times k}$	trust matrix
$\mathbf{E} = [e_{ij}]_{k\times k}$	intensity matrix
$\hat{u}_{ij}^t$	relative importance of d_j on d_i at time t
ε_i	confidence threshold of the i^{th} DM

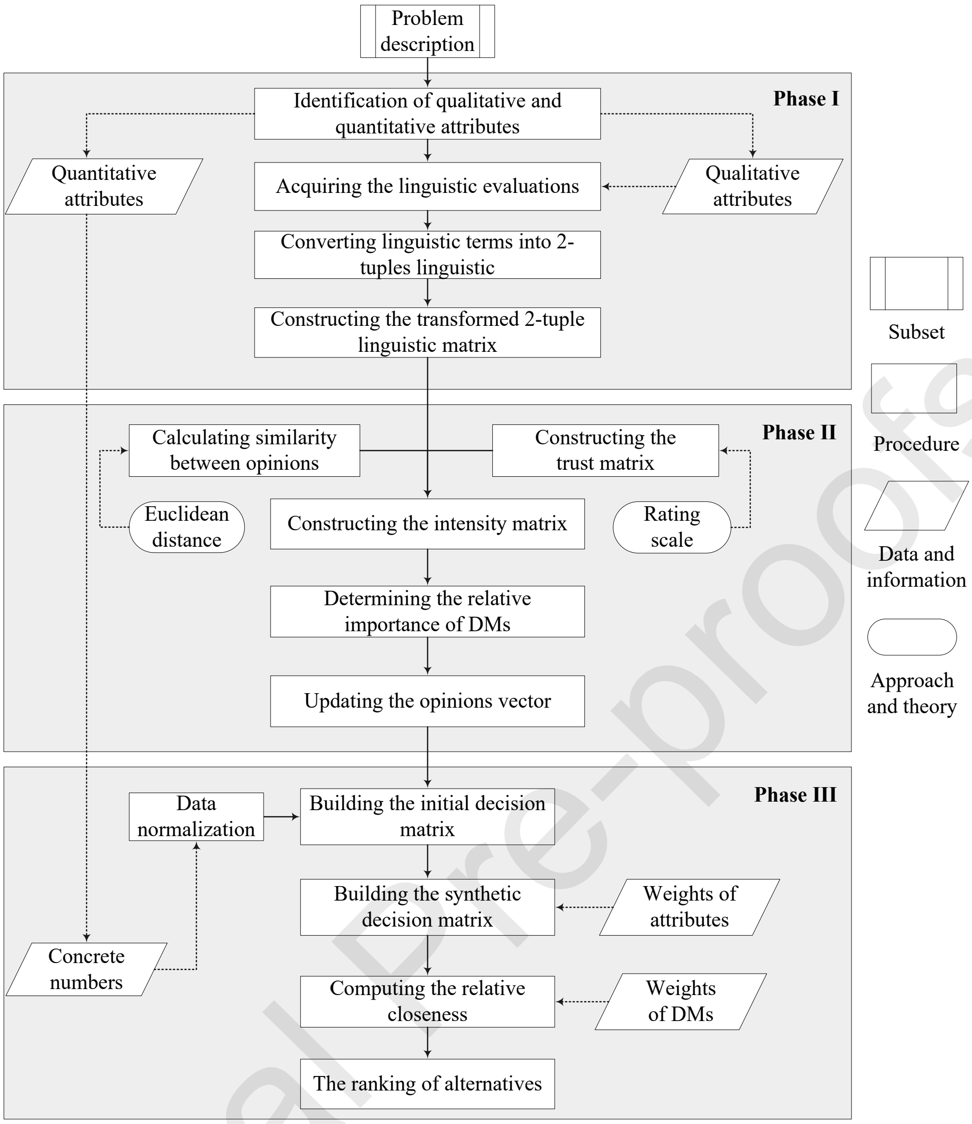

Figure 1. Framework of the proposed MAGDM methodology considering opinion dynamics

4.1 Obtaining evaluation information based on 2-tuple linguistic representation model

At the beginning of decision-making, the original linguistic judgments on qualitative attributes and concrete numbers on quantitative attributes are collected. The linguistic judgments on qualitative attributes are represented as the 2-tuple linguistic variable according to Definition 1 and then transformed into numerical values according to Definition 2. Thereby, the 2-tuple linguistic matrix and the corresponding transformed 2-tuple linguistic matrix can be obtained based on Definition 3 and Definition 4.

4.2 Information fusion considering opinion dynamics

Inspired by the HK model, an EBC model based on opinion dynamics is detailed in this subsection to establish the decision-making information fusion, which can reflect the process of opinion interaction in a real decision-making scenario. The steps are as follows:

4.2.1 Construct the intensity matrix

Define a k-order non-negative matrix $\mathbf{T}$ as the trust matrix, in which each element $T_{ij} \in [0, 1]$ denotes the trust degree from d_i to $d_j \in D$. Especially, when $i = j$, T_{ii} represents the self-confidence degree of d_i. For simplicity, the assessment T_{ij} is presented in the form of an accurate numerical value under a predetermined scale G (Su, Liu, Li, & Ma, 2014). Let G be a uniformly distributed array from 0 to 1 with nine elements, as listed in Table 3.

Table 3. Grade assignment for 9 granularities

Values	0	0.125	0.25	0.375	0.5	0.625	0.75	0.875	1
Degree	extremely distrust	very distrust	distrust	slightly distrust	generally trust	slightly trust	trust	very trust	extremely trust

As introduced in Definition 4, the corresponding transformed 2-tuple linguistic vector $\overline{\mathbf{O}^t}=\Delta^{-1}(\mathbf{O}^t) = (\beta_i^t)_{k\times1}$ can be obtained by virtue of Eq. (2).

Consider that the effect intensity of others on a DM is mainly affected by two aspects: it is directly proportional to the trust degree and inversely proportional to the similarity between their opinions. The intensity matrix can be constructed by using

$$e^t_{ij}= \frac{T_{ij}}{max\ \{Dis(\beta^t{}_i, \beta^t_j), \sigma\}} \tag{6}$$

where e^t_{ij} denotes the effect intensity of d_i on d_j, $Dis(\beta_i^t, \beta_j^t)$ is the Euclidean similarity between their opinions, and σ is a small enough positive number to make the denominator non-zero (Su, Liu, Li, & Ma, 2014). In this study, we take $\sigma = 0.01$ because under the TLM environment, the deviation of opinions is generally no less than 0.01 in terms of numerical values.

4.2.2 Determine the relative importance of DMs

Consistent with the HK model, DMs adjust their opinions by the weighted arithmetic means of the opinions of others whose effect intensity is above a given threshold ε. As there will be no evident change in trust relations between DMs within a short time, it might be assumed that ε does not change over time, that is, $\varepsilon_i^t = \varepsilon_i^{t+1}$, i = 1,2, …, k, t = 1, 2, …, N. Specifically, if $\varepsilon_i = \varepsilon_j$ for $\forall\, i, j$ = 1, 2, ..., k, we call the EBC model homogeneous, otherwise heterogeneous (Hegselmann & Krause, 2002).

Let u_{ij}^t denote the actual influence of d_i on d_j at time t.

$$u_{ij}^t = \begin{cases} e_{ij}^t, & e_{ij}^t \geq \varepsilon^i \text{ or } \beta_i^t > 0.5 \text{ but } \beta_j^t < 0.34 \\ 0\,, & \text{otherwise} \end{cases} \tag{7}$$

The condition "$\beta_i^t > 0.5$ but $\beta_j^t < 0.34$ " is set to take into account the effects of NBs, where a numerical value above 0.5 in TLM indicates the positive evaluations, while values below 0.34 (corresponding to the linguistic term "bad" defined in Definition 1) indicates negative evaluations (Zou, Li, & Huang, 2020). In such a case, the calculated e^t_{ij} tends to be small owing to the large denominator, which subtly matches the reality that although positive DMs will accept negative information, their acceptance degree is low due to a large gap between opinions.

By substituting Eq. (6) into Eq. (7) and applying the appropriate transformation, Eq. (7) can be written as a general bounded confidence rule, that is,

$$u_{ij}^t = \begin{cases} e_{ij}^t, & \max\{Dis(\beta_i^t, \beta_j^t), \sigma\} \leq \frac{T_{ij}}{\varepsilon_i} \text{ or } \beta_i^t > 0.5 \text{ but } \beta_j^t < 0.34 \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

It can be observed from Eq. (8) that T_{ij} and ε_i jointly determine the confidence threshold. Consequently, a DM can have several confidence thresholds for different DMs (including himself), instead of having an identical threshold for all DMs in the HK model, as shown in Figure 2.

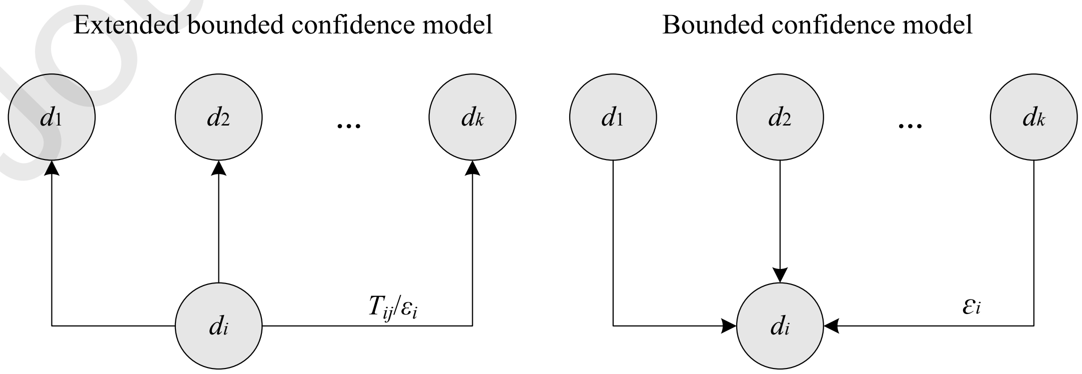

Figure 2. Illustration of the confidence threshold acquisition

4.2.3 Update the opinions vector

By analogy to the confidence set in the HK model, we propose an influence set, that is, the set of DMs whose opinions will be considered by DM d_i as

$$I_i^t = \{d_j \mid u_{ij}^t \neq 0\},\ d_j \in D, \tag{9}$$

$$\hat{u}_{ij}^t = \frac{u_{ij}^t}{\sum_{l=1}^{k} u_{il}^t}. \tag{10}$$

Eq. (10) describes a normalisation process to obtain the relative importance of DMs at a specific time point in the opinion interaction process, which makes $\hat{u}_{ij}^t \geqslant 0$ and $\sum_{j=1}^{n} \hat{u}_{ij}^t = 1$.

After d_i receives the opinions from the other DMs, then will (not) change his/her opinion according to

$$\overline{O_i^{t+1}} = \begin{cases} \sum_{j=1}^{k} \hat{u}_{ij}^t \cdot \beta_j^t, & \|I_i^t\| \neq 0 \\ O_i^t, & \|I_i^t\| = 0 \end{cases} \tag{11},$$

where $\|\cdot\|$ denotes the number of elements in a finite set.

To describe the terminal conditions of opinion interaction, we introduce two definitions regarding the stabilised patterns, consensus, and fragmentation, as Definitions 5–6. The iteration process of the EBC model is presented in Program II, which is included in Table 4.

Definition 5. If the opinions satisfy $|\beta_i^{t+1} - \beta_i^t| \leq \xi$ ($i = 1, 2, \ldots, k$), then the stabilised patterns in opinion formation are formed at time t, where ξ is a small enough number to prevent the evolution process from becoming stuck into an endless loop. We might also set $\xi = 0.01$, as numerical values with a difference of less than 0.01 can be converted into the same linguistic term in the TLM.

Definition 6. If the stabilised patterns in opinion evolution are formed at time t, and the opinions satisfy $|\beta_i^t - \beta_j^t| \leq \xi$ ($i, j = 1, 2, \ldots, k$ and $i \neq j$), then we consider that a consensus among the DMs is reached at time t; otherwise, we consider that fragmentation among DMs is formed at time t.

Table 4. Opinions evolution process for the EBC model

Steps	Details

1: Input $\overline{\mathbf{O}^t}$, $\mathbf{T}$, ξ, k and ε

2: $t \leftarrow 0$

3: Obtaining the effect intensity e^t_{ij} by using Eq. (6)

4: Determining the actual influence u^t_{ij} $(i, j = 1, 2, \ldots, k)$ according to Eq. (7)

5: Updating the opinions vector by virtue of Eqs. (9), (10), (11).

6: If $|\beta_i^{t+1} - \beta_i^t| \leq \xi$, then output $\overline{\mathbf{O}^t}$ and t

7: Else $t \leftarrow t+1$, go to Step 3

8: End if

In conclusion, this model extends the hypothesis of bounded confidence, which further considers the bounded rationality of DMs during information fusion. However, with an increase in the number of DMs, it can be a time-consuming process. This may result in a huge trust matrix and a large number of pairwise comparisons. Consequently, the proposed model is applicable to the communication and discussion process of small-or medium-sized groups.

4.3 Ranking of alternatives using TOPSIS

As a widely applied MADM approach, TOPSIS aims to sort the best alternative, which should be closest to the positive ideal solution (PIS) and farthest from the negative ideal solution (NIS). Considering its simplicity and ability to yield an explicit preference order of alternatives, TOPSIS is employed in this study to rank the alternatives. After the decision-making information fusion process considering the proposed opinion dynamics model, the decision-making matrices of a_r on c_q $(r = 1, 2, \ldots, m;\ q = 1, 2, \ldots, n)$ can be obtained. Then, the main steps of alternative ranking are introduced as follows:

Step 1: The 0–1 standardisation method is used to transform the decision-making matrices into standardised matrices.

Step 2: Fuse the evaluations of n attributes given by k DMs to build the synthetic decision matrix according to

$$\tilde{x}_{rq} = \sum_{i=1}^{k} \omega_i \cdot x^i_{rq} \tag{12}$$

where x^i_{rq} describes the evaluation of the r^{th} alternative with respect to attribute c_q given by the i^{th} DM (i = 1, 2, …, k).

Step 3: Determine the PIS and NIS, and calculate the Euclidean distances d_r^+/ d_r^- of each alternative from the PIS / NIS.

$$d_r^+=\sum_{q=1}^{n} w_q \cdot Dis\left(\tilde{x}_{rq},\mathrm{PIS}\right)$$

$$d_r^-=\sum_{q=1}^{n} w_q \cdot Dis\left(\tilde{x}_{rq},\mathrm{NIS}\right) \quad (13)$$

Step 4: Compute the comprehensive relative closeness (RC) to NIS; then, the alternatives can be ranked according to the ascending order of RC_r.

$$RC_r= d_r^- / (d_r^- + d_r^+) \quad (14)$$

5. Simulation analysis

In this section, three simulations using the MATLAB program are implemented to verify the rationality of the proposed EBC model.

5.1 Simulation 1: process of opinion evolution

It is assumed that the initial opinions are randomly distributed within the interval [0.2, 0.9], k = 50, ε = 0.5/0.3 (according to the left part of Figure 2), in which, $\forall\ T_{ij}$ = 0.5 means all DMs generally trust each other, and 0.3 is a critical threshold determining whether to reach consensus in the homogeneous bounded confidence model (Sparks, So, & Bradley, 2016). The simulation results are obtained according to Program II and are depicted in Figure 3.

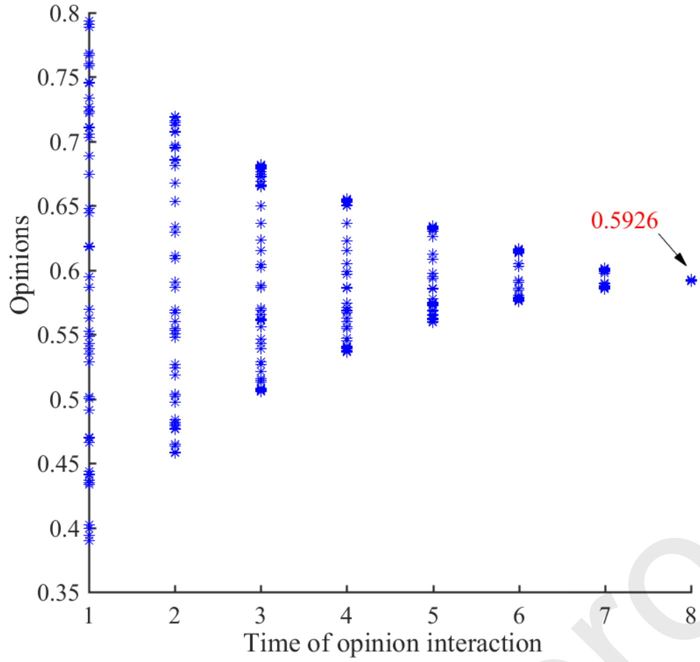

Figure 3. Opinion interaction process for the homogeneous EBC model

As shown in Figure 3, each data point represents the numerical opinion value of a DM at a time point. It can be observed that the opinions of 50 DMs gradually become centralised and finally reach consensus, which illustrates the typical process of opinion evolution.

5.2 Simulation 2: effects of the confidence threshold on reaching stabilisation

To eliminate the interference of other variables, let the current model be a homogeneous model with k = 20. Then, the average stability time t under different confidence thresholds is obtained by Program II 20 times, and the simulation results are depicted in Figure 4.

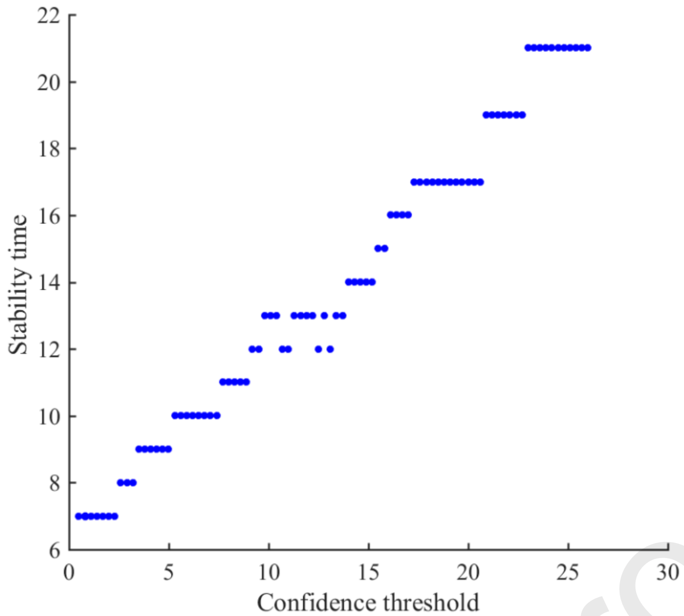

Figure 4. Average stability time under different ε values

From Figure 4, we observe that t is approximately proportional to ε, which implies that the time required to reach consensus increases as the range of the dynamic confidence thresholds increases. This is because a larger ε brings more difficulty for DMs to meet the interaction conditions; thus, it requires a longer time to form a stabilised pattern.

5.3 Simulation 3: reaction to the influential factors

As mentioned in Section 2.3, this model should be able to cope with the bounded rationality of DMs, which will be described in the following simulation.

There is a decision-making group with 20 DMs. Most of them give positive initial evaluations, without being stubborn or sequacious ($\beta_i^t \in [0.5,0.9]$ and $T_{ij} \in [0.4,0.8]$), only two DMs give negative initial evaluations but still hold on to their own ($\beta_i^t < 0.34$ and $T_{ii} \in [0.8,1]$). If there is no interaction or opinion evolution, the information that they want to convey is likely to be buried in the subsequent calculation of aggregation, resulting in loss of information. The same input data are processed by Program II and Program I, and the comparative results are shown in Figure 5.

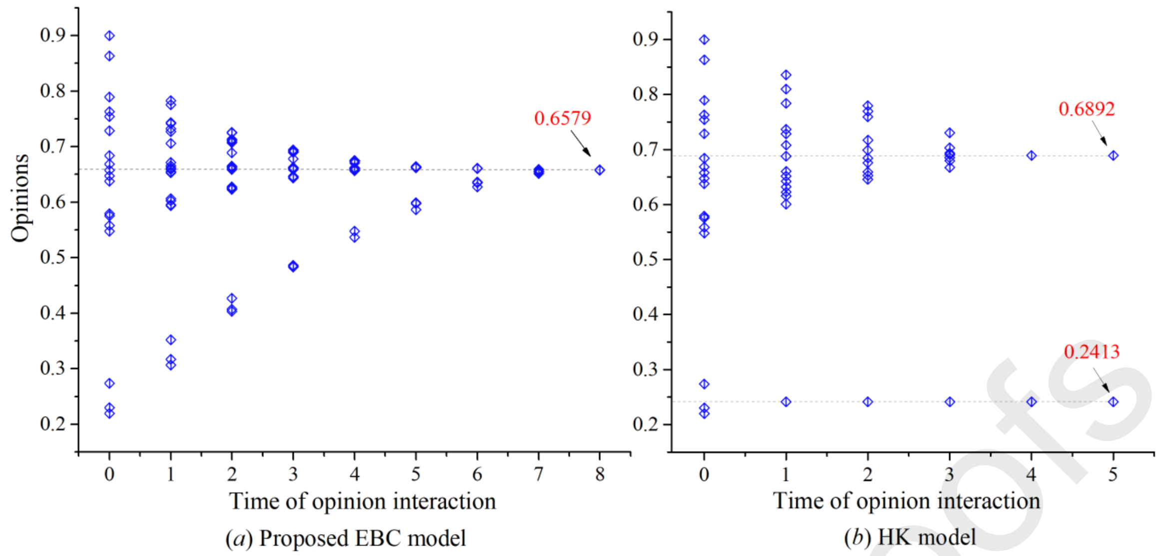

Figure 5. Opinion interaction process of the proposed EBC model and HK model

From Figure 5, we obtain the following observations: the EBC model finally reaches a consensus at 0.6579, while the HK model obtains two completely separated opinion clusters, one of which is the average of positive opinions (0.6892), and the other is the average of negative opinions (0.2413). The above observations are mainly attributed to the following reasons: the proposed model further considers the NBs and the relative importance of DMs, so that the negative information can be conveyed in the interaction and more rational opinions are finally presented. In contrast, the DMs in the HK model only average the opinions with high similarity, which leads to the isolation of negative opinions and loss of information.

Summarising the above simulation results, opinions from DMs will converge after rounds of interactions, and the convergence process will be affected by the confidence threshold of DMs, the relative importance of DMs, and the bounded rationality, including ADMs, SDMs, and NBs.

6. A case study

A case study of supplier selection in a producer of a Bluetooth headsets company is presented to illustrate the effectiveness of the proposed method. By comparison, several significant advantages of the EBC model over the HK model are also discussed.

6.1 Problem description and solution procedures

Due to the requirement of business expansion, M Company plans to select a new supplier. After pre-evaluation, six suppliers have remained as alternatives for further evaluation, and experts from different departments are invited to form a decision-making group for optimal selection from a set of alternatives. Six attributes were chosen as the evaluation attributes, and they were quality reliability (c_1), technology leadership (c_2), exception handling capacity (c_3), business status (c_4), delivery cycle (c_5), and procurement cost (c_6). It is apparent that the first four elements in C are qualitative attributes, and c_5, c_6 are quantitative attributes. $W = \{0.3, 0.1, 0.15, 0.25, 0.15, 0.05\}$ are predefined weights of these attributes, and $\Omega = \{0.06, 0.06, 0.05, 0.08, 0.07, 0.06, 0.06, 0.07, 0.06, 0.07, 0.07, 0.06, 0.08, 0.07, 0.08\}$ includes the weights of 15 DMs. There is a situation where supplier a_1 has good public reputation, but it has a major defect that could affect the quality of delivery, which is only known by d_2 and d_3 owing to the expertise restrictions.

6.1.1 Acquisition of evaluation information

The acquisition processes of the evaluation information of 15 DMs are similar, so only the numerical calculation process of d_2 is provided in the following. The original evaluations of qualitative attributes and concrete data on quantitative attributes are given in Table 5. The 2-tuple linguistic matrix according to Definition 1 and Definition 3 is presented in Table 6. The transformed 2-tuple linguistic matrix of d_2 is constructed in Table 7 by virtue of Eq. (2) and Definition 4.

Table 5. Original evaluations of d_2

	c_1	c_2	c_3	c_4	c_5	c_6
a_1	VB-B	B	VB	VB-B	29.00	385.13
a_2	GE-G	GE-G	G	G-VG	35.00	439.13
a_3	VB-B	G-VG	G-VG	GE	32.00	480.10
a_4	GE-G	GE-G	GE-G	G-VG	33.00	397.90
a_5	G-VG	G	GE-G	B-GE	27.00	423.65
a_6	GE	GE-G	G	GE	30.00	465.00

Table 6. Two-tuple linguistic matrix of d_2

O	c_1	c_2	c_3	c_4	c_5	c_6
a_1	$(S_1,0.04)$	$(S_2,0)$	$(S_1,0)$	$(S_1,0.04)$	29.00	385.13

a_2	(S_3,0.04)	(S_3,0.04)	(S_4,0)	(S_4,0.04)	35.00	439.13
a_3	(S_1,0.04)	(S_4,0.04)	(S_4,0.04)	(S_3,0)	32.00	480.10
a_4	(S_3,0.04)	(S_3,0.04)	(S_3,0.04)	(S_4,0.04)	33.00	397.90
a_5	(S_4,0.04)	(S_4,0)	(S_3,0.04)	(S_2,0.04)	27.00	423.65
a_6	(S_3,0)	(S_3,0.04)	(S_4,0)	(S_3,0)	30.00	465.00

Table 7. Transformed 2-tuple linguistic matrix of d_2

	c_1	c_2	c_3	c_4	c_5	c_6
a_1	0.21	0.33	0.17	0.21	29.00	385.13
a_2	0.54	0.54	0.67	0.71	35.00	439.13
a_3	0.21	0.71	0.71	0.50	32.00	480.10
a_4	0.54	0.54	0.54	0.71	33.00	397.90
a_5	0.71	0.67	0.54	0.37	27.00	423.65
a_6	0.50	0.54	0.67	0.50	30.00	465.00

6.1.2 Dynamic evolution of opinions

As mentioned in Section 4.2, a homogenous EBC model is applied in this case (known ε = 2), and the trust degree between DMs is presented in the form of a numerical value, as presented in Table 3. Based on the principle that DMs with higher prestige would be more convincing and determined, we finally obtain the trust matrix.

Table 8. Trust matrix of DMs

T_{ij}	d_1	d_2	d_3	d_4	…	d_{12}	d_{13}	d_{14}	d_{15}
d_1	0.5	0.875	0.75	0.625	…	0.625	0.5	0.75	0.5
d_2	0.375	1	0.5	0.5	…	0.625	0.5	0.5	0.5
d_3	0.375	0.5	0.875	0.5	…	0.625	0.5	0.5	0.5
d_4	0.5	0.875	0.75	0.75	…	0.75	0.5	0.625	0.5
d_5	0.375	0.875	0.625	0.75	…	0.5	0.75	0.5	0.625
d_6	0.375	0.875	0.5	0.625	…	0.75	0.5	0.5	0.5
d_7	0.375	1	0.875	0.5	…	0.875	0.5	0.625	0.5
d_8	0.5	0.75	0.75	0.5	…	0.75	0.5	0.5	0.5
d_9	0.375	0.75	0.5	0.5	…	0.5	0.5	0.625	0.5
d_{10}	0.5	0.75	0.75	0.5	…	0.625	0.375	0.5	0.5
d_{11}	0.375	0.75	0.875	0.625	…	0.75	0.5	0.5	0.5
d_{12}	0.5	0.5	0.75	0.5	…	0.75	0.375	0.5	0.375
d_{13}	0.5	0.875	0.875	0.5	…	0.75	0.625	0.625	0.5
d_{14}	0.5	0.75	0.875	0.5	…	0.625	0.5	0.625	0.5
d_{15}	0.5	0.875	1	0.75	…	0.625	0.5	0.625	0.625

Input the data in Table 8 to Program II, and the stabilised results (see Definition 5) of evolution can be obtained. For instance, take a_1; the sorted input data are given in Table 9, the converged process is illustrated in Figure 6, and the output is listed in Table 10. Similarly, the rest of the decision matrices are obtained.

Table 9. Initial evaluation matrix of a_1

$\overline{O}$	c_1	c_2	c_3	c_4
d_1	0.54	0.54	0.71	0.67
d_2	0.21	0.33	0.17	0.21
d_3	0.21	0.33	0.33	0.17
d_4	0.54	0.67	0.83	0.67
d_5	0.83	0.71	0.71	0.71
d_6	0.67	0.71	0.67	0.71
d_7	0.54	0.71	0.67	0.67
d_8	0.71	0.67	0.67	0.54
d_9	0.54	0.54	0.71	0.67
d_{10}	0.54	0.71	0.67	0.71
d_{11}	0.67	0.71	0.54	0.67
d_{12}	0.54	0.67	0.54	0.67
d_{13}	0.83	0.71	0.83	0.71
d_{14}	0.67	0.71	0.67	0.67
d_{15}	0.83	0.71	0.67	0.71

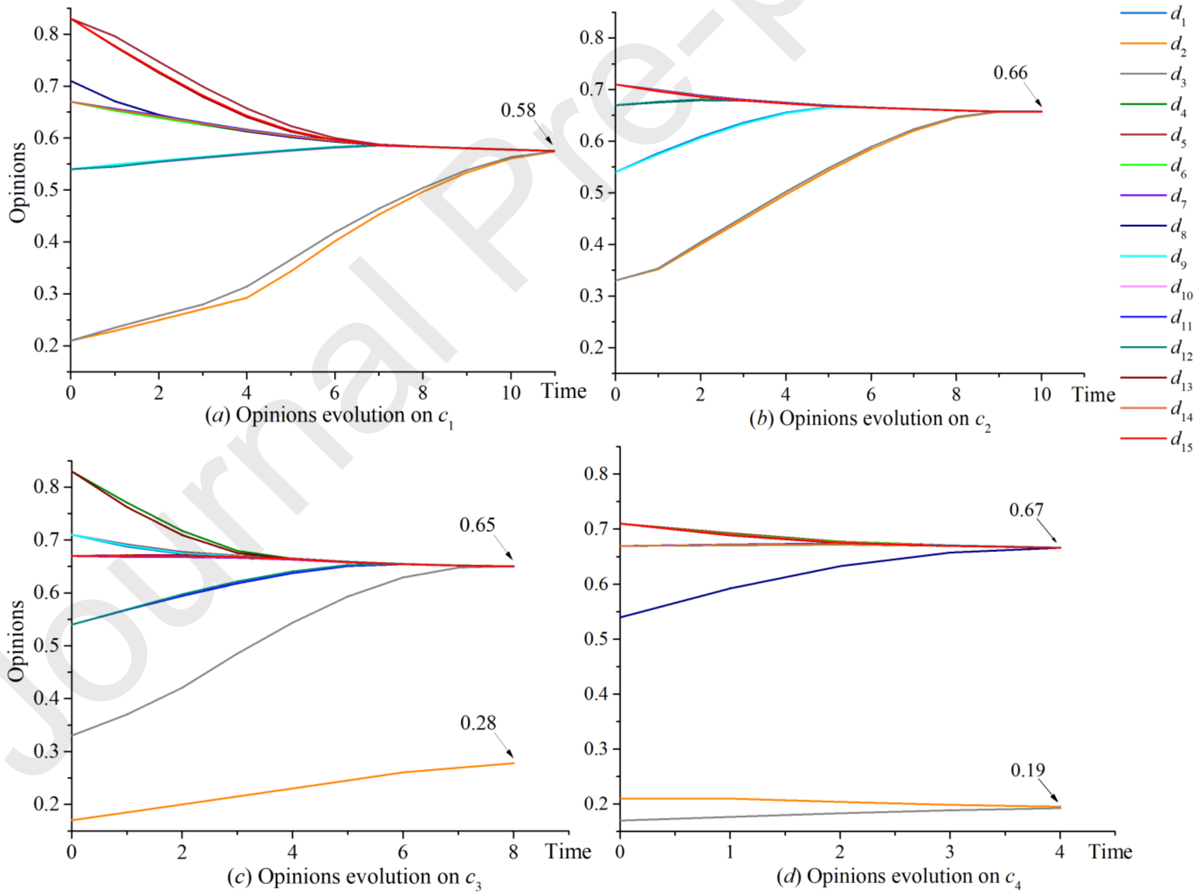

Figure 6. Updating process of opinions on qualitative attributes for a_1

Table 10. Updated decision matrix of a_1

	c_1	c_2	c_3	c_4
d_1	0.58	0.66	0.65	0.67

d_2	0.58	0.66	0.28	0.19
d_3	0.58	0.66	0.65	0.19
d_4	0.58	0.66	0.65	0.67
d_5	0.58	0.66	0.65	0.67
d_6	0.58	0.66	0.65	0.67
d_7	0.58	0.66	0.65	0.67
d_8	0.58	0.66	0.65	0.67
d_9	0.58	0.66	0.65	0.67
d_{10}	0.58	0.66	0.65	0.67
d_{11}	0.58	0.66	0.65	0.67
d_{12}	0.58	0.66	0.65	0.67
d_{13}	0.58	0.66	0.65	0.67
d_{14}	0.58	0.66	0.65	0.67
d_{15}	0.58	0.66	0.65	0.67

6.1.3 Ranking of alternatives

TOPSIS requires initial evaluation matrices as inputs. Following the steps introduced in Section 4.3, the synthetic decision matrix is tabulated in Table 11. Subsequently, the RC_r values between a_r and NIS are calculated for each alternative using Eqs. (13) and (14), and are put in order as 0.7661, 0.2667, 0.3357, 0.4122, 0.7810, 0.5005, respectively. The preference order of suppliers is ranked as $a_5 \succ a_1 \succ a_6 \succ a_4 \succ a_3 \succ a_2$.

Table 11. Synthetic decision matrix

	c_1	c_2	c_3	c_4	c_5	c_6
a_1	0.61	0.64	0.64	0.62	0.75	1.00
a_2	0.54	0.61	0.58	0.58	0.00	0.43
a_3	0.51	0.68	0.60	0.56	0.38	0.00
a_4	0.54	0.53	0.54	0.60	0.25	0.87
a_5	0.70	0.67	0.61	0.44	1.00	0.59
a_6	0.58	0.63	0.63	0.51	0.63	0.16

6.2 Comparison analysis

To demonstrate the rationality of the proposed MAGDM method over traditional methodology, we made a comparison of the decision-making outcomes between the proposed method and two other methods in this case. Method I is the traditional static MAGDM method without considering opinion dynamics, while in Method II, though considering opinion dynamics, it applies the existing bounded confidence model (i.e., HK model). The reason why we choose these two methods is that they precisely correspond to the innovation points of this study. The results obtained by adopting Method I and Method II are presented in Table 12 and Table 13, respectively.

Table 12. Decision-making results applied Method I

	c_1	c_2	c_3	c_4	c_5	c_6	d_r^+	d_r^-	RC_r
a_1	0.61	0.64	0.64	0.62	0.75	1.00	0.05	0.19	0.7918
a_2	0.54	0.61	0.58	0.58	0.00	0.43	0.18	0.06	0.2415
a_3	0.51	0.68	0.60	0.56	0.38	0.00	0.16	0.08	0.3325
a_4	0.54	0.53	0.54	0.60	0.25	0.87	0.14	0.09	0.3858
a_5	0.70	0.67	0.61	0.44	1.00	0.59	0.05	0.18	0.7876
a_6	0.58	0.63	0.63	0.51	0.63	0.16	0.12	0.12	0.4938

Table 13. Decision-making results applied Method II

	c_1	c_2	c_3	c_4	c_5	c_6	d_r^+	d_r^-	RC_r
a_1	0.60	0.63	0.64	0.64	0.75	1.00	0.05	0.19	0.7817
a_2	0.55	0.62	0.59	0.58	0.00	0.43	0.18	0.06	0.2531
a_3	0.51	0.69	0.60	0.55	0.38	0.00	0.16	0.08	0.3214
a_4	0.53	0.53	0.54	0.61	0.25	0.87	0.15	0.09	0.3859
a_5	0.70	0.67	0.61	0.44	1.00	0.59	0.05	0.19	0.7793
a_6	0.59	0.63	0.63	0.51	0.63	0.16	0.12	0.12	0.4897

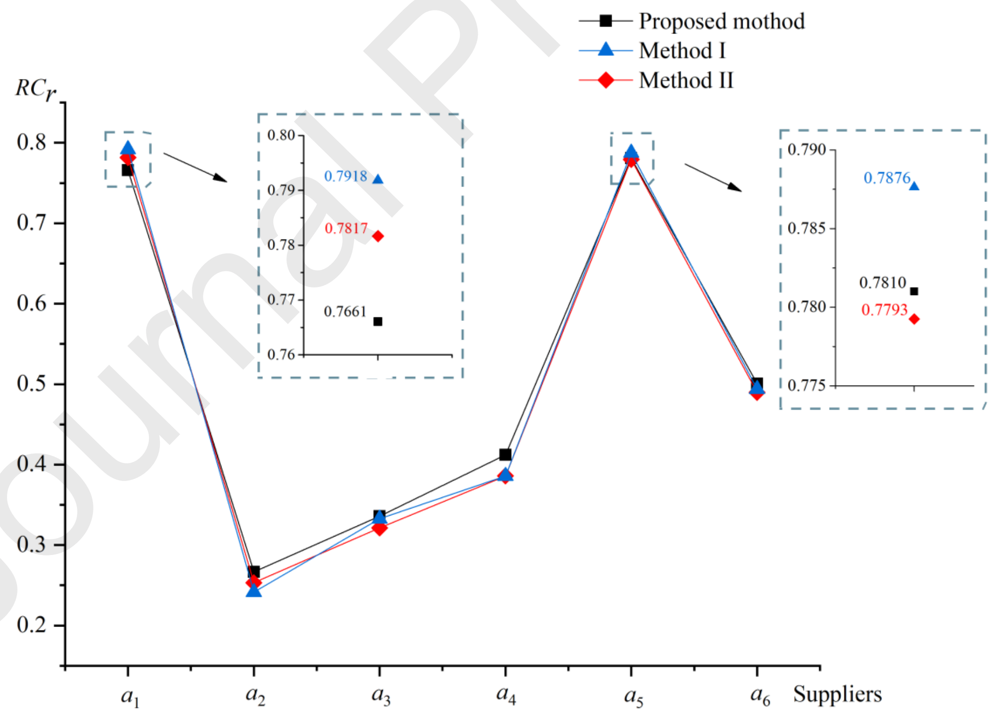

Figure 7. Comparison analysis of decision results

In conjunction with the ranking results of Figure 7, the order of rankings of suppliers yielded by the Method I and Method II is uniform, that is, a_1 is the optimal. Whereas the best supplier obtained by employing the proposed method is a_5. This is because the negative information given by d_2 and d_3 about a_1 in Method I and Method II are neutralized in the phase of information fusion and cannot be retained to the end, which causes the information loss as well as affects the accuracy of the final decision-making outcome. While in proposed method, the negative information representing defects are disseminated effectively, causing a drop in the ranking of a_1.

In general, the comparison results conform to the expected logic, which demonstrates that the proposed method is more practical and effective for dealing with MAGDM problems in engineering reality.

7. Conclusions and future works

MAGDM is a complex decision-making problem concerning multi-attributes, alternatives, and DMs. Traditionally, the evaluation information of different DMs is fused directly when it is collected, which is incompatible with real decision-making scenarios. Consequently, a novel MAGDM method integrating the EBC model is proposed. Conclusions of this study are summarised as follows.

(1) Opinion dynamics is introduced to describe the dynamic interaction and evolution of DMs' opinions in the real MAGDM process, which reduces the loss of information caused by traditional information fusion techniques.

(2) The EBC model extends the hypothesis of bounded confidence, which not only captures the reality that different DMs have different weights, but also considers the impact of human bounded rationality, which is presented as ADMs, SDMs, and NBs, thus producing a more rational decision-making result.

(3) Three simulations and a case study are established to illustrate the rationality and utility of the proposed MAGDM method.

However, due to interpersonal relationships, people may ignore the professionality of DMs, and selectively assign more trust to the familiar DMs, which indirectly affects the decision-making results. Thus, in future work, we will strive to reduce the interference of subjective factors in evaluation. Moreover, the influential factors of bounded confidence could be further explored.

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Credit author statement

Yupeng Li: Conceptualization, Validation, Funding acquisition. **Meng Liu**: Methodology, Software, Formal analysis, Writing- Original draft preparation, Writing- Reviewing and Editing. **Jin Cao**: Investigation. ***Xiaolin Wang****:* Supervision, Validation. **Na Zhang:** Data curation, Resources.

- A novel decision-making method considering opinion dynamics is introduced.
- A modified opinion dynamics model considering the bounded rationality is proposed.
- Applicability of the proposal is demonstrated through 3 simulations and a case study.
- The proposal reduces loss of information caused by traditional static methodology.