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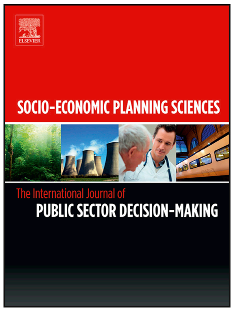

A Neutrosophic WENSLO-ARLON Model for Measuring Sustainable Brand Equity Performance

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PII: S0038-0121(24)00117-4

DOI: https://doi.org/10.1016/j.seps.2024.101918

Reference: SEPS 101918

To appear in: *Socio-Economic Planning Sciences*

Received Date: 18 February 2024

Revised Date: 14 March 2024

Accepted Date: 7 May 2024

Please cite this article as: Kara K, Yalçın GC, Akagün Ergin E, Simic V, Pamucar D, A Neutrosophic WENSLO-ARLON Model for Measuring Sustainable Brand Equity Performance, *Socio-Economic Planning Sciences*, https://doi.org/10.1016/j.seps.2024.101918.

Title Page

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A Neutrosophic WENSLO-ARLON Model for Measuring Sustainable Brand Equity Performance

Abstract

Sustainable brand equity denotes the perceived brand equity by consumers, influenced by companies' non-financial and environmental practices. To cultivate sustainable brand equity, corporations undertake green initiatives and engage in social responsibility endeavors, thereby communicating environmentally oriented messages to consumers. Companies feel the necessity to establish a competitive position in sustainability relative to their counterparts. Hence, the primary objective of this research is to conduct an inquiry aimed at identifying sustainable brand equity to meet this exigency of companies. In this context, the motivation of this study lies in treating the calculation of sustainable brand equity as a decision-making process and developing a decision support system for this purpose. The proposed decision support system for determining sustainable brand equity levels in this research has three fundamental inputs: experts, criteria, and brands. The impact levels of experts on the decision-making process are quantified utilizing type-2 neutrosophic number (T2NN) sets. The weights of criteria are ascertained employing the weight by envelope and slope (WENSLO) method extended with T2NN sets. The ranking of sustainable brand equities is established through the innovative T2NN-based alternative ranking using two-step logarithmic normalization (ARLON) method. These methodologies are amalgamated, and the T2NN-WENSLO-ARLON hybrid model is posited as a decision support system for evaluating sustainable brand equity performance. An algorithm for T2NN-WENSLO-ARLON is formulated and elucidated alongside a case study focusing on companies operating in the cosmetics sector in Turkey. The case study identifies the "green product leadership" criterion as the most influential factor in determining sustainable brand equity. Furthermore, the cosmetic company "Misbahçe Inc." is identified as the brand with the highest sustainable brand equity. Sensitivity and comparative analyses are conducted to ascertain the robustness of the results and the decision model. All scenario outcomes bolster the decision model. The research provides comprehensive insights into both the developed tool for calculating sustainable brand equity and develops implications for the cosmetics industry.

Keywords: Sustainable Brand Equity; Sustainability Performance; Multi-Attribute Group Decision-Making; Type-2 Neutrosophic Numbers; Weight by Envelope and Slope Method; The Alternative Ranking Using Two-Step Logarithmic Normalization Method.

1. Introduction

Brand equity is a term that was first introduced by marketing professionals in the 1980s and it refers to the value of a brand name, which is based on how much consumers are willing to pay for a particular product or service [1]. Brand equity is a value premium that a company generates from a product with a recognizable name when compared to a generic equivalent. Companies can create brand equity for their products by making them memorable, easily recognizable, and superior in quality and reliability [2]. For a brand to survive in the competitive market, it needs to establish strong brand loyalty [3–4]. It is not only about providing excellent products and services but also about giving customers an experience that far exceeds their rising expectations [5]. An essential strategy for fostering customer loyalty entails the provision of high-quality products and services [6]. Nevertheless, the purchasing decision is subject to the customer's perception not only of the product or service itself but also of the company's role as a provider of solutions.

In the era of sustainability, it is important for every company to take responsibility for its actions. Brands need to be aware of their social and environmental impacts and make sure that they are not harming society in any way. Sustainability is the future of building brand equity and a shopping priority for consumers [7]. Once a company knows where it stands regarding sustainability, then it can start making the necessary changes to move closer to the green end of the spectrum. Ameer & Othman [8] found that companies that attend to ecosystems, societies, and environments of the future have a higher financial performance compared to those that do not engage in such practices. With this in mind, a brand must approach sustainability with intention rather than simply as a marketing tool. Too many brands try to "fit in" with shifting cultural norms by drastically altering their identity [9]. Consumers will immediately realize if the attempts are not genuine.

Corporations must consider the sustainability of their products and services to cultivate a beneficial influence on society. The idea of sustainability is often associated with environmentalism or corporate responsibility [10–11]. However, sustainability goes beyond that and includes social responsibility. Consumers are willing to pay more for products from companies they consider are doing social or societal good [12]. That means the more sustainable a business is, the more customers are willing to buy, and they are willing to pay more too [13].

Sustainable brand equity denotes the enduring reservoir of brand equity upheld through conscientious consideration of a brand's social, environmental, and societal ramifications [14].

It fosters the perception among consumers that the brand is environmentally and socially conscious and respectful [15]. In creating sustainable brand equity, companies engage in initiatives that reduce environmental impacts and provide benefits to society [16]. These initiatives aim to minimize environmental impact throughout all stages, from the production of goods and services to their delivery to customers, and even their retrieval. Sustainable brand equity increasingly gains importance in consumer brand preferences as consumer awareness of environmental issues grows [17]. Hence, for enterprises, the establishment and maintenance of sustainable brand equity assume escalating significance, encompassing dimensions of customer loyalty and market competitiveness.

The primary focus of this research is to take a step toward examining brand equity from a sustainable perspective. Despite the acknowledged significance of brand equity and the imperative of sustainability, there remains a dearth of empirical evidence concerning the measurability of sustainable brand equity. Hence, drawing upon fundamental approaches indicative of filling the gaps in the literature [18–19], this study aims to rectify the deficiencies by striving to discern sustainable brand equity and assess the prevailing position thereof vis-à-vis rival firms. Aligned with this objective, the motivation and significance of the study, its objectives, contributions to the literature, and anticipated impact are delineated with precision and concreteness under subheadings.

1.1. Motivation and Significance of the Study

Efforts are being made by brands to develop sustainable brand equity, primarily focusing on making brand equity more environmentally friendly. However, research on determining the performance of sustainable brand equity for companies is limited. Such research initiatives focus on determining firm-level sustainable brand equity. A measurement approach indicating a brand's sustainable brand equity and determining its position relative to competitors has yet to be developed. The primary motivation of this research is to develop a decision support system (DSS) to determine the performance of sustainable brand equities.

The motivation behind this study stems from two perspectives. The first perspective is methodological, while the second is practical applicability. Methodologically, the development of a decision support tool for ascertaining sustainable brand equity will contribute to enriching the literature by employing neutrosophic set theory. From a practical implementation standpoint, obtaining qualitative assessments based on linguistic scales will enhance the applicability of the DSS. The significance of the research thus becomes prominent. By not solely delving into theoretical approaches or methodological explanations, but also by presenting the applicability and robustness through case studies, this research ensures the coherence between theory and methodology, thereby providing both conceptual clarity to the literature and demonstrating its practical feasibility.

The significance of this research lies in the adoption of a multi-attribution group decision-making (MAGDM) approach and leveraging neutrosophic sets to determine sustainable brand equity performance and rank brands. Such a hybrid approach allows for a comprehensive assessment, incorporating diverse factors and considering the complexities inherent in sustainable brand equity evaluation. The reason for this approach is to complete the decision-making process by consulting a group of expert opinions rather than a single expert opinion. This approach also blends with neutrosophic logic to handle expert evaluations more precisely and minimize uncertainty levels. In this context, the type-2 neutrosophic number (T2NN) sets, which provide flexible decision processes [20–21], are utilized. These sets provide an advantage in reducing uncertainties in decision processes. For determining the importance levels of sustainable brand equity performance measurement criteria, the weight by envelope and slope (WENSLO) method is preferred because it is an understandable and easily objective weighting method [22]. Additionally, this method is employed by extending the T2NN sets. The ranking of brands according to their sustainable brand equity performance is achieved through the alternative ranking using two-step logarithmic normalization (ARLON) method [23]. This method is based on a two-step logarithmic normalization process, allowing for more precise normalization procedures. Additionally, the ARLON method is used together with T2NN sets.

1.2. Aims of the Study

Companies with high sustainable brand equity strategically plan their market positioning to gain a competitive advantage. For such strategic planning, companies must understand their current sustainable brand equity and position relative to their competitors. This underscores the aims of this research, which enables companies to ascertain their sustainable brand equities and determine their sustainability positions vis-à-vis their competitors. The robustness and applicability of the proposed DSS facilitate companies across various industries in calculating their sustainable brand equities.

In this research, the T2NN-WENSLO-ARLON hybrid model is developed as a DSS. The individual steps of this hybrid model are presented in the form of an algorithm. To demonstrate its applicability, a case study is conducted on companies in the cosmetics sector, supporting the feasibility of applying this hybrid method. Sensitivity analysis scenarios are also performed to test the robustness of the hybrid method, confirming its resilience and suitability for determining sustainable brand equity.

1.3. Contributions and Impacts of the Study

This research makes a significant contribution to the academic literature by addressing the limited research on determining and comparing sustainable brand equity of companies. The study introduces a novel DSS aimed at evaluating sustainable brand equity, particularly in the context of the cosmetics sector. The key contributions and impacts of this study can be summarized as follows:

(i) *The impacts and contribution of developing a hybrid DSS* – The study proposes a three-stage hybrid method for determining sustainable brand equity. This method integrates T2NN sets, T2NN-WENSLO, and T2NN-ARLON methods to provide a comprehensive DSS. The development of the T2NN-WENSLO-ARLON hybrid method yields a unique impact and offers an innovative contribution.

(ii) *The impacts and contribution of method enhancement* – The research contributes to the advancement of the existing WENSLO method by developing and presenting the T2NN-WENSLO criterion weighting method, thereby addressing the significance of this advancement in accommodating the utilization of the WENSLO method with neutrosophic sets. This extension of the WENSLO method incorporates T2NN sets, adding a layer of sophistication to the criterion weighting process. Similarly, the ARLON method has been enhanced with the development of T2NN sets, resulting in the formulation of the T2NN-ARLON alternative ranking method.

(iii) *The contribution of the cosmetic sector application* – The study applies the developed hybrid method to companies in the cosmetics sector, demonstrating its practicality and effectiveness. The case study results support the applicability of the T2NN-WENSLO-ARLON hybrid method in evaluating sustainable brand equity in a real-world industry context, especially for the cosmetic sector. Besides, this application shows the appropriateness of the novel DSS.

(iv) *Robustness testing and sensitivity analysis* – Rigorous testing of the developed hybrid method is conducted through sensitivity analysis scenarios, establishing its robustness in different situations. The outcomes affirm the reliability and strength of the proposed method in determining sustainable brand equity.

(v) *Recommendations for sustainable brand equity assessment* – The research concludes with insights into sustainable brand equity in the cosmetics sector, offering valuable recommendations. The T2NN-WENSLO-ARLON hybrid method is suggested as an effective tool for companies seeking to assess and enhance their sustainable brand equity.

In summary, this study contributes to the academic literature by not only addressing the gap in research on sustainable brand equity but also by providing a practical and innovative DSS for companies in the cosmetics sector.

1.4. Organization of the Study

This paper consists of five sections. Section 2 introduces the T2NN-WENSLO-ARLON hybrid model. Section 3 represents the literature review. Section 4 illustrates the calculation of sustainable brand equities for companies in the cosmetic sector through a case study. Section 5 discusses the research findings, sensitivity analysis scenarios, and implications. Section 6 is the conclusion, wherein research limitations and recommendations for future studies are presented.

2. Literature Review

Brand equity is the value indicating how a brand is perceived by its customers, competitors, employees, and civil society organizations, reflecting its reputation status, level of trust, and recognition. Brands with high brand equity are more socially acceptable, whereas those with low brand equity have lower levels of acceptability. Brand equity is influenced by various concepts and, in turn, influences many concepts. Moreover, relationships among concepts have regulatory and mediating effects.

In the literature, numerous studies explore the relationships between brand equity and different concepts such as brand engagement [24], brand identity [25], corporate social responsibility [26], corporate reputation [27], brand experience [28], brand loyalty [29], brand activism [30], and brand image [31]. Additionally, the dimensions of brand equity have been approached differently in the literature. Theurer et al. [32] explained that brand equity dimensions were presented in two main ways. Firstly, the dimensions presented by Aaker [33] included brand loyalty, brand associations, perceived quality, name awareness, and other proprietary assets. Secondly, the dimensions presented by Keller [34] were brand awareness and brand image. However, studies in the literature have approached the dimensions of brand equity in various ways. Madadi et al. [35] identified perceived quality, brand association, brand awareness, and brand loyalty as the dimensions of brand equity.

The concept of green brand equity emerged from the consideration of environmental and social aspects of brand equity [36]. Chen [17] primarily defined green brand equity as the creation of brand equity based on environmental and green concerns. Additionally, the main drivers of green brand equity were explained as green brand image, green satisfaction, and green trust. Ng et al. [37] proposed the antecedents of green brand equity as green brand perceived value, brand credibility, and green brand image. Kang & Hur [38] identified green trust, green loyalty, green affect, and green satisfaction as precursors of green brand equity. The concept of sustainable brand equity, while paralleling green brand equity, encompasses the consideration of brand equity within the scope of sustainable competitiveness [39]. In this context, sustainable brand equity was examined by considering sustainable innovation and green brand equity concepts together [40].

In this study, the indicators determining the level of sustainable brand equity are examined by considering the key performance indicators of sustainable brand equity. These indicators include the green brand image, green trust, green satisfaction, green product leadership, green perceived quality, green innovation, environmental benefit, corporate social responsibility, green brand loyalty, green brand name, green wash, and green perceived risk. Green brand image refers to the level of a brand's image as an environmentally friendly brand [41]. Green trust indicates the confidence shown by customers in a brand producing and introducing environmentally friendly products to the market [17]. Green satisfaction signifies the level of satisfaction among customers regarding the environmentally friendly products offered by a brand [42]. Green product leadership involves a brand's development and introduction of products that lead to environmental aspects in the market conditions [43]. Green perceived quality refers to the level of a brand's perceived quality in the environmental perspective by customers [44]. Green innovation is a brand's innovation process focusing on environmental concerns during product development [45]. Environmental benefit involves a brand's implementation of activities for the benefit of the environment and nature. Corporate social responsibility involves a brand's development of social responsibility projects in the process of brand equity creation and spreading brand value in this way [46]. Green brand loyalty involves a brand's creation of intentions for re-purchase and increasing brand loyalty among environmentally conscious customers [47]. Green brand name involves the creation of a brand name in an environmentally friendly manner and creating a green-focused brand image in the minds of customers. Green wash involves a brand's portrayal of a green image despite not being environmentally friendly to motivate customers to repurchase [48]. Green perceived risk involves customers perceiving the risk of a brand not being environmentally friendly [42].

In the literature, there are limited studies addressing multi-criteria decision-making problems related to brand equity. Dinçer et al. [49] utilized fuzzy decision making trial and evaluation laboratory (DEMATEL), fuzzy analytic network process (ANP), fuzzy technique for order preference by similarity to ideal solution (TOPSIS), and fuzzy VIKOR methods for determining brand equity performance. Wang & Tzeng [50] amalgamated DEMATEL, ANP, and VIKOR methods for brand equity creation. Hue & Oanh [51] employed Delphi and analytic hierarchy process (AHP) methods to identify the precursors of green brand equity. Yeğin & Ikram [52] employed AHP and TOPSIS methods for evaluating green furniture brands. Ultimately, no research has been encountered in the literature focusing on determining brand equity performance, particularly in developing DSS for determining sustainable brand equity performance. In this context, this study deeply investigated the process of determining sustainable brand equity performance based on key performance indicators of sustainable brand equity.

3. Methodological Framework

For the assessment of sustainable brand equity in this research, the T2NN-WENSLO-ARLON hybrid model is proposed from a methodological perspective (Fig. 1).

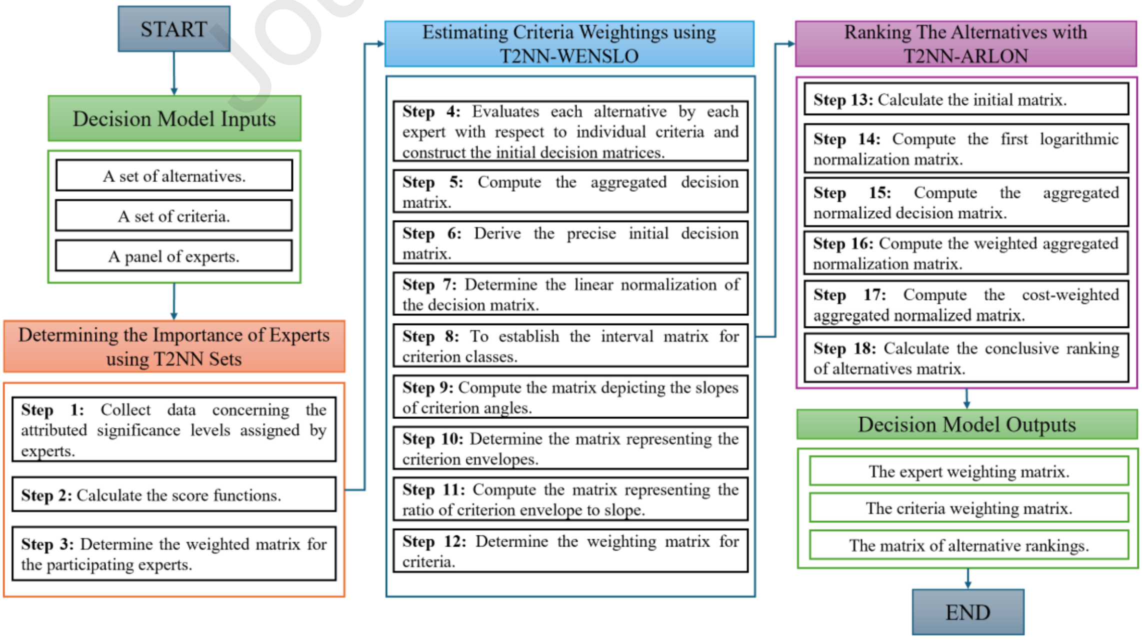

Fig. 1. Methodological framework.

This model is developed as a decision support tool. The hybrid method enables the ranking of brands' sustainable brand equities through a total of three main stages. In this context, the inputs of the model include experts, criteria, and brands. Following the incorporation of model inputs, first, expert weights are calculated using T2NN sets. Second, the importance levels of criteria are determined using T2NN-WENSLO. Third, brands are ranked according to their sustainable brand equities using the T2NN-ARLON method. After these three stages, three main outputs are obtained; i.e. the impact levels of experts in the decision-making process, the importance role of criteria in the decision-making process, and the sustainable brand equities of brands.

3.1. Preliminaries of T2NN Sets

Definition 1. Within the prescribed domain of discourse denoted as $\mathfrak{H}$, the symbol $\tilde{\mathfrak{L}}$, as defined by Eq. (1) in accordance with [53], represents a T2NN functioning within the comprehensive set $\mathfrak{H}$:

$$\tilde{\mathfrak{L}} = \left\{\langle \mathfrak{h}, \tilde{T}_{\tilde{\mathfrak{L}}}(\mathfrak{h}), \tilde{I}_{\tilde{\mathfrak{L}}}(\mathfrak{h}), \tilde{F}_{\tilde{\mathfrak{L}}}(\mathfrak{h}) \mid \mathfrak{h} \in \mathfrak{H} \rangle\right\}. \quad (1)$$

In the context of the presented framework, the mappings $\tilde{T}_{\tilde{\mathfrak{L}}}(\mathfrak{h}): \mathfrak{H} \rightarrow D[0,1]$, $\tilde{I}_{\tilde{\mathfrak{L}}}(\mathfrak{h}): \mathfrak{H} \rightarrow D[0,1]$, and $\tilde{F}_{\tilde{\mathfrak{L}}}(\mathfrak{h}): \mathfrak{H} \rightarrow D[0,1]$, delineate the conceptual domains of truth, indeterminacy, and falsehood, respectively. These mappings are explicitly formulated under the condition that the inequality $0 \leq \tilde{T}_{\tilde{\mathfrak{L}}}(\mathfrak{h})^3 + \tilde{I}_{\tilde{\mathfrak{L}}}(\mathfrak{h})^3 + \tilde{F}_{\tilde{\mathfrak{L}}}(\mathfrak{h})^3 \leq 3$ is satisfied for every element $\mathfrak{h}$ within the set $\mathfrak{H}$. The explicit formulations of these functions are articulated as follows: $\tilde{T}_{\tilde{\mathfrak{L}}}(\mathfrak{h}) = \left(T_{T_{\tilde{\mathfrak{L}}}}(\mathfrak{h}), T_{I_{\tilde{\mathfrak{L}}}}(\mathfrak{h}), T_{F_{\tilde{\mathfrak{L}}}}(\mathfrak{h})\right)$, $\tilde{I}_{\tilde{\mathfrak{L}}}(\mathfrak{h}) = \left(I_{T_{\tilde{\mathfrak{L}}}}(\mathfrak{h}), I_{I_{\tilde{\mathfrak{L}}}}(\mathfrak{h}), I_{F_{\tilde{\mathfrak{L}}}}(\mathfrak{h})\right)$, and $\tilde{F}_{\tilde{\mathfrak{L}}}(\mathfrak{h}) = \left(F_{T_{\tilde{\mathfrak{L}}}}(\mathfrak{h}), F_{I_{\tilde{\mathfrak{L}}}}(\mathfrak{h}), F_{F_{\tilde{\mathfrak{L}}}}(\mathfrak{h})\right)$.

Definition 2. Abdel-Basset et al. [53] exemplified manipulations and methodologies employed in the treatment of T2NN, grounded in the following guiding principles. Consider the scenario where $\tilde{\mathfrak{L}}$, $\tilde{\mathfrak{L}}_1$, and $\tilde{\mathfrak{L}}_2$ are three T2NNs over the universe $\mathfrak{H}$. The interactions and functional relationships among these three T2NNs are delineated through the following operational elucidations:

(i)
$$\tilde{\mathfrak{L}}_1 \oplus \tilde{\mathfrak{L}}_2 = \Big\{\langle\Big(T_{T_{\tilde{\mathfrak{L}}_1}}(\mathfrak{h}) + T_{T_{\tilde{\mathfrak{L}}_2}}(\mathfrak{h}) - T_{T_{\tilde{\mathfrak{L}}_1}}(\mathfrak{h}) * T_{T_{\tilde{\mathfrak{L}}_2}}(\mathfrak{h}), T_{I_{\tilde{\mathfrak{L}}_1}}(\mathfrak{h}) + T_{I_{\tilde{\mathfrak{L}}_2}}(\mathfrak{h}) - T_{I_{\tilde{\mathfrak{L}}_1}}(\mathfrak{h}) * T_{I_{\tilde{\mathfrak{L}}_2}}(\mathfrak{h}), T_{F_{\tilde{\mathfrak{L}}_1}}(\mathfrak{h}) + T_{F_{\tilde{\mathfrak{L}}_2}}(\mathfrak{h}) - T_{F_{\tilde{\mathfrak{L}}_1}}(\mathfrak{h}) * T_{F_{\tilde{\mathfrak{L}}_2}}(\mathfrak{h})\Big), \Big(I_{T_{\tilde{\mathfrak{L}}_1}}(\mathfrak{h}) * I_{T_{\tilde{\mathfrak{L}}_2}}(\mathfrak{h}), I_{I_{\tilde{\mathfrak{L}}_1}}(\mathfrak{h}) * I_{I_{\tilde{\mathfrak{L}}_2}}(\mathfrak{h}), I_{F_{\tilde{\mathfrak{L}}_1}}(\mathfrak{h}) * I_{F_{\tilde{\mathfrak{L}}_2}}(\mathfrak{h})\Big), \Big(F_{T_{\tilde{\mathfrak{L}}_1}}(\mathfrak{h}) * F_{T_{\tilde{\mathfrak{L}}_2}}(\mathfrak{h}), F_{I_{\tilde{\mathfrak{L}}_1}}(\mathfrak{h}) * F_{I_{\tilde{\mathfrak{L}}_2}}(\mathfrak{h}), F_{F_{\tilde{\mathfrak{L}}_1}}(\mathfrak{h}) * F_{F_{\tilde{\mathfrak{L}}_2}}(\mathfrak{h})\Big)\rangle \mid \mathfrak{h} \in \mathfrak{H}\Big\},$$

(ii) $\widetilde{\mathfrak{L}}_1 \otimes \widetilde{\mathfrak{L}}_2 = \Big\{\langle\Big(T_{T_{\widetilde{\mathfrak{L}}_1}}(\mathfrak{h}) * T_{T_{\widetilde{\mathfrak{L}}_2}}(\mathfrak{h}), T_{I_{\widetilde{\mathfrak{L}}_1}}(\mathfrak{h}) * T_{I_{\widetilde{\mathfrak{L}}_2}}(\mathfrak{h}), T_{F_{\widetilde{\mathfrak{L}}_1}}(\mathfrak{h}) * T_{F_{\widetilde{\mathfrak{L}}_2}}(\mathfrak{h})\Big), \Big(I_{T_{\widetilde{\mathfrak{L}}_1}}(\mathfrak{h}) + I_{T_{\widetilde{\mathfrak{L}}_2}}(\mathfrak{h}) - I_{T_{\widetilde{\mathfrak{L}}_1}}(\mathfrak{h}) * I_{T_{\widetilde{\mathfrak{L}}_2}}(\mathfrak{h}), I_{I_{\widetilde{\mathfrak{L}}_1}}(\mathfrak{h}) + I_{I_{\widetilde{\mathfrak{L}}_2}}(\mathfrak{h}) - I_{I_{\widetilde{\mathfrak{L}}_1}}(\mathfrak{h}) * I_{I_{\widetilde{\mathfrak{L}}_2}}(\mathfrak{h}), I_{F_{\widetilde{\mathfrak{L}}_1}}(\mathfrak{h}) + I_{F_{\widetilde{\mathfrak{L}}_2}}(\mathfrak{h}) - I_{F_{\widetilde{\mathfrak{L}}_1}}(\mathfrak{h}) * I_{F_{\widetilde{\mathfrak{L}}_2}}(\mathfrak{h})\Big), \Big(F_{T_{\widetilde{\mathfrak{L}}_1}}(\mathfrak{h}) + F_{T_{\widetilde{\mathfrak{L}}_2}}(\mathfrak{h}) - F_{T_{\widetilde{\mathfrak{L}}_1}}(\mathfrak{h}) * F_{T_{\widetilde{\mathfrak{L}}_2}}(\mathfrak{h}), F_{I_{\widetilde{\mathfrak{L}}_1}}(\mathfrak{h}) + F_{I_{\widetilde{\mathfrak{L}}_2}}(\mathfrak{h}) - F_{I_{\widetilde{\mathfrak{L}}_1}}(\mathfrak{h}) * F_{I_{\widetilde{\mathfrak{L}}_2}}(\mathfrak{h}), F_{F_{\widetilde{\mathfrak{L}}_1}}(\mathfrak{h}) + F_{F_{\widetilde{\mathfrak{L}}_2}}(\mathfrak{h}) - F_{F_{\widetilde{\mathfrak{L}}_1}}(\mathfrak{h}) * F_{F_{\widetilde{\mathfrak{L}}_2}}(\mathfrak{h})\Big)\rangle \mid \mathfrak{h} \in \mathfrak{H}\Big\}$,

(iii) $w\widetilde{\mathfrak{L}} = \Big\{\langle\Big(1-\big(1-T_{T_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\big)^{w}, 1-\big(1-T_{I_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\big)^{w}, 1-\big(1-T_{F_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\big)^{w}\Big), \Big(\big(I_{T_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\big)^{w}, \big(I_{I_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\big)^{w}, \big(I_{F_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\big)^{w}\Big), \Big(\big(F_{T_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\big)^{w}, \big(F_{I_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\big)^{w}, \big(F_{F_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\big)^{w}\Big)\rangle \mid \mathfrak{h} \in \mathfrak{H}\Big\}$ for $w > 0$,

(iv) $\widetilde{\mathfrak{L}}^{w} = \Big\{\langle\Big(\big(T_{T_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\big)^{w}, \big(T_{I_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\big)^{w}, \big(T_{F_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\big)^{w}\Big), \Big(1-\big(1-I_{T_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\big)^{w}, 1-\big(1-I_{I_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\big)^{w}, 1-\big(1-I_{F_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\big)^{w}\Big), \Big(1-\big(1-F_{T_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\big)^{w}, 1-\big(1-F_{I_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\big)^{w}, 1-\big(1-F_{F_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\big)^{w}\Big)\rangle \mid \mathfrak{h} \in \mathfrak{H}\Big\}$ for $w > 0$.

In alignment with the specifications outlined by Abdel-Basset et al. [53], Definition 2 is anticipated to conform to the subsequent criteria:

(i) $\widetilde{\mathfrak{L}}_1 \oplus \widetilde{\mathfrak{L}}_2 = \widetilde{\mathfrak{L}}_2 \oplus \widetilde{\mathfrak{L}}_1$,

(ii) $\widetilde{\mathfrak{L}}_1 \otimes \widetilde{\mathfrak{L}}_2 = \widetilde{\mathfrak{L}}_2 \otimes \widetilde{\mathfrak{L}}_1$,

(iii) $w(\widetilde{\mathfrak{L}}_1 \oplus \widetilde{\mathfrak{L}}_2) = w\widetilde{\mathfrak{L}}_1 \oplus w\widetilde{\mathfrak{L}}_2$ for $w > 0$,

(iv) $(\widetilde{\mathfrak{L}}_1 \otimes \widetilde{\mathfrak{L}}_2)^{w} = \widetilde{\mathfrak{L}}_1^{\,w} \otimes \widetilde{\mathfrak{L}}_2^{\,w}$ for $w > 0$,

(v) $w_1\widetilde{\mathfrak{L}}_1 \oplus w_2\widetilde{\mathfrak{L}}_1 = (w_1 + w_2)\widetilde{\mathfrak{L}}_1$ for $w_1, w_2 > 0$,

(vi) $\widetilde{\mathfrak{L}}_1^{\,w_1} \otimes \widetilde{\mathfrak{L}}_1^{\,w_2} = \widetilde{\mathfrak{L}}_1^{\,(w_1+w_2)}$ for $w_1, w_2 > 0$.

Definition 3. Consider the scenario where $\widetilde{\mathfrak{L}}$ is defined as $\langle\Big(T_{T_{\widetilde{\mathfrak{L}}}}(\mathfrak{h}), T_{I_{\widetilde{\mathfrak{L}}}}(\mathfrak{h}), T_{F_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\Big), \Big(I_{T_{\widetilde{\mathfrak{L}}}}(\mathfrak{h}), I_{I_{\widetilde{\mathfrak{L}}}}(\mathfrak{h}), I_{F_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\Big), \Big(F_{T_{\widetilde{\mathfrak{L}}}}(\mathfrak{h}), F_{I_{\widetilde{\mathfrak{L}}}}(\mathfrak{h}), F_{F_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\Big)\rangle$ representing T2NNs across the universal set $\mathfrak{H}$. The computation of the score function, denoted as $\mathbb{SF}(\widetilde{\mathfrak{L}})$, is performed using Eq. (2), while the accuracy function $\mathbb{AF}(\widetilde{\mathfrak{L}})$ is determined through the application of Eq. (3):

$$\mathbb{SF}(\widetilde{\mathfrak{L}}) = \frac{1}{12}\langle 8 + \Big(T_{T_{\widetilde{\mathfrak{L}}}}(\mathfrak{h}) + 2\Big(T_{I_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\Big) + T_{F_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\Big) - \Big(I_{T_{\widetilde{\mathfrak{L}}}}(\mathfrak{h}) + 2\Big(I_{I_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\Big) + I_{F_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\Big) - \Big(F_{T_{\widetilde{\mathfrak{L}}}}(\mathfrak{h}) + 2\Big(F_{I_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\Big) + F_{F_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\Big)\rangle, \quad (2)$$

$$\mathfrak{A}(\widetilde{\mathfrak{L}})=\frac{1}{4}\langle\left(T_{T_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})+2\left(T_{I_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\right)+T_{F_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\right)-\left(F_{T_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})+2\left(F_{I_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\right)+F_{F_{\widetilde{\mathfrak{L}}}}(\mathfrak{h})\right)\rangle. \quad (3)$$

Definition 4. Consider the instances where $\widetilde{\mathfrak{L}}_1$ and $\widetilde{\mathfrak{L}}_2$ are representing T2NNs across the universal set $\mathfrak{H}$. In the context provided, with $\mathbb{SF}(\widetilde{\mathfrak{L}}_x)$ and $\mathbb{AF}(\widetilde{\mathfrak{L}}_x)$ denoting the score and accuracy functions associated with $\widetilde{\mathfrak{L}}_x(x=1,2)$, hierarchical relationships can be established as follows:

(i) If $\mathbb{SF}(\widetilde{\mathfrak{L}}_1)>\mathbb{SF}(\widetilde{\mathfrak{L}}_2)$, then $\widetilde{\mathfrak{L}}_1$ greater than $\widetilde{\mathfrak{L}}_2$, that is $\widetilde{\mathfrak{L}}_1>\widetilde{\mathfrak{L}}_2$.

(ii) If $\mathbb{SF}(\widetilde{\mathfrak{L}}_1)=\mathbb{SF}(\widetilde{\mathfrak{L}}_2)$, $\mathbb{AF}(\widetilde{\mathfrak{L}}_1)>\mathbb{AF}(\widetilde{\mathfrak{L}}_2)$, then $\widetilde{\mathfrak{L}}_1$ greater than $\widetilde{\mathfrak{L}}_2$, that is $\widetilde{\mathfrak{L}}_1>\widetilde{\mathfrak{L}}_2$.

(iii) If $\mathbb{SF}(\widetilde{\mathfrak{L}}_1)=\mathbb{SF}(\widetilde{\mathfrak{L}}_2)$, $\mathbb{AF}(\widetilde{\mathfrak{L}}_1)=\mathbb{AF}(\widetilde{\mathfrak{L}}_2)$, then $\widetilde{\mathfrak{L}}_1$ is equal to $\widetilde{\mathfrak{L}}_2$, that is $\widetilde{\mathfrak{L}}_1=\widetilde{\mathfrak{L}}_2$.

Definition 5. Examine $\widetilde{\mathfrak{L}}_{\mathfrak{f}}$, expressed as $\widetilde{\mathfrak{L}}_{\mathfrak{f}}=\langle\left(T_{T_{\widetilde{\mathfrak{L}}_{\mathfrak{f}}}}(\mathfrak{h}),T_{I_{\widetilde{\mathfrak{L}}_{\mathfrak{f}}}}(\mathfrak{h}),T_{F_{\widetilde{\mathfrak{L}}_{\mathfrak{f}}}}(\mathfrak{h})\right),\left(I_{T_{\widetilde{\mathfrak{L}}_{\mathfrak{f}}}}(\mathfrak{h}),I_{I_{\widetilde{\mathfrak{L}}_{\mathfrak{f}}}}(\mathfrak{h}),I_{F_{\widetilde{\mathfrak{L}}_{\mathfrak{f}}}}(\mathfrak{h})\right),\left(F_{T_{\widetilde{\mathfrak{L}}_{\mathfrak{f}}}}(\mathfrak{h}),F_{I_{\widetilde{\mathfrak{L}}_{\mathfrak{f}}}}(\mathfrak{h}),F_{F_{\widetilde{\mathfrak{L}}_{\mathfrak{f}}}}(\mathfrak{h})\right)\rangle$, signifying a T2NN sets $\left(\widetilde{\mathfrak{L}}_{\mathfrak{f}}=(\widetilde{\mathfrak{L}}_1,\widetilde{\mathfrak{L}}_2,\dots,\widetilde{\mathfrak{L}}_{\mathcal{F}})\right)$. Simultaneously, introduce the corresponding weight vector $w=(w_1,w_2,\dots,w_{\mathcal{F}})$ where $w_{\mathfrak{f}}\in[0,1]$, subject to the constraint that $\sum_{\mathfrak{f}=1}^{\mathcal{F}}w_{\mathfrak{f}}=1$. In this framework, the T2NNWA aggregation operator is formalized through Eq. (4) [53]:

$$T2NNWA\left(\widetilde{\mathfrak{L}}_1,\widetilde{\mathfrak{L}}_2,\dots,\widetilde{\mathfrak{L}}_{\mathcal{F}}\right)=\oplus_{\mathfrak{f}=1}^{\mathcal{F}}w_{\mathfrak{f}}\widetilde{\mathfrak{L}}_{\mathfrak{f}}=$$

$$\left(\begin{array}{c}\left(1-\prod_{\mathfrak{f}=1}^{\mathcal{F}}\left(1-T_{T_{\widetilde{\mathfrak{L}}_{\mathfrak{f}}}}(\mathfrak{h})\right)^{w_{\mathfrak{f}}}\right),\left(1-\prod_{\mathfrak{f}=1}^{\mathcal{F}}\left(1-T_{I_{\widetilde{\mathfrak{L}}_{\mathfrak{f}}}}(\mathfrak{h})\right)^{w_{\mathfrak{f}}}\right),\left(1-\prod_{\mathfrak{f}=1}^{\mathcal{F}}\left(1-T_{F_{\widetilde{\mathfrak{L}}_{\mathfrak{f}}}}(\mathfrak{h})\right)^{w_{\mathfrak{f}}}\right),\\ \left(\prod_{\mathfrak{f}=1}^{\mathcal{F}}\left(I_{T_{\widetilde{\mathfrak{L}}_{\mathfrak{f}}}}(\mathfrak{h})\right)^{w_{\mathfrak{f}}}\right),\left(\prod_{\mathfrak{f}=1}^{\mathcal{F}}\left(I_{I_{\widetilde{\mathfrak{L}}_{\mathfrak{f}}}}(\mathfrak{h})\right)^{w_{\mathfrak{f}}}\right),\left(\prod_{\mathfrak{f}=1}^{\mathcal{F}}\left(I_{F_{\widetilde{\mathfrak{L}}_{\mathfrak{f}}}}(\mathfrak{h})\right)^{w_{\mathfrak{f}}}\right),\\ \left(\prod_{\mathfrak{f}=1}^{\mathcal{F}}\left(F_{T_{\widetilde{\mathfrak{L}}_{\mathfrak{f}}}}(\mathfrak{h})\right)^{w_{\mathfrak{f}}}\right),\left(\prod_{\mathfrak{f}=1}^{\mathcal{F}}\left(F_{I_{\widetilde{\mathfrak{L}}_{\mathfrak{f}}}}(\mathfrak{h})\right)^{w_{\mathfrak{f}}}\right),\left(\prod_{\mathfrak{f}=1}^{\mathcal{F}}\left(F_{F_{\widetilde{\mathfrak{L}}_{\mathfrak{f}}}}(\mathfrak{h})\right)^{w_{\mathfrak{f}}}\right)\end{array}\right)\mid \mathfrak{h}\in\mathfrak{H}. \quad (4)$$

3.2. The Novel T2NN-WENSLO-ARLON Hybrid Method

The T2NN-WENSLO-ARLON hybrid model has been conceptualized to facilitate the comprehensive evaluation of sustainable brand equity performance. The brand equity set, denoted as $B_{\mathfrak{k}}=\{B_1,B_2,\dots,B_{\mathfrak{K}}\}$ $(\mathfrak{k}=1,2,\dots,\mathfrak{K})$, and the criteria set, denoted as, $\Gamma_{\mathfrak{d}}=\{\Gamma_1,\Gamma_2,\dots,\Gamma_{\mathfrak{D}}\}$ $(\mathfrak{d}=1,2,\dots,\mathfrak{D})$ play integral roles in this evaluative framework. Furthermore, the inclusion of an expert-driven indicator set $H_{\mathfrak{s}}=\{H_1,H_2,\dots,H_{\mathfrak{S}}\}$ $(\mathfrak{s}=1,2,\dots,\mathfrak{S})$ underscores the involvement of experts in the holistic assessment of brand equity. The methodological steps intrinsic to the T2NN-WENSLO-ARLON hybrid approach are elucidated in the subsequent discussion:

- ***Stage 1*** *– Assigning weights to each expert.*

Step 1: The experts' expertise levels are evaluated based on the linguistic variables (LVs) in Table 1. Following this, LVs undergo a transformation into T2NNs, subsequently determining the significance levels attributed to each expert based on these T2NNs.

Step 2: Crisp metrics are determined by employing the score function denoted as $\mathbb{SF}(\widetilde{H}_{\mathfrak{s}})$ to evaluate the relative importance of experts, as delineated in Eq. (5) for the T2NN sets:

$$\mathbb{SF}(\widetilde{H}_{\mathfrak{s}}) = \frac{1}{12}\langle 8 + \left(T_{T_{\widetilde{H}_{\mathfrak{s}}}}(\mathfrak{h}) + 2\left(T_{I_{\widetilde{H}_{\mathfrak{s}}}}(\mathfrak{h})\right) + T_{F_{\widetilde{H}_{\mathfrak{s}}}}(\mathfrak{h})\right) - \left(I_{T_{\widetilde{H}_{\mathfrak{s}}}}(\mathfrak{h}) + 2\left(I_{I_{\widetilde{H}_{\mathfrak{s}}}}(\mathfrak{h})\right) + I_{F_{\widetilde{H}_{\mathfrak{s}}}}(\mathfrak{h})\right) - \left(F_{T_{\widetilde{H}_{\mathfrak{s}}}}(\mathfrak{h}) + 2\left(F_{I_{\widetilde{H}_{\mathfrak{s}}}}(\mathfrak{h})\right) + F_{F_{\widetilde{H}_{\mathfrak{s}}}}(\mathfrak{h})\right)\rangle,\ \mathfrak{s} = 1, 2, \ldots, \mathfrak{S}. \quad (5)$$

Table 1

Variables characterizing the linguistic aspects of expert attributes [54].

Expertise	***T2NNs***
Very-poor (VP)	⟨ (0.20,0.20,0.11), (0.65,0.80,0.85), (0.45,0.80,0.70) ⟩
Poor (P)	⟨ (0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65) ⟩
Medium-poor (MP)	⟨ (0.40,0.30,0.35), (0.50,0.45,0.60), (0.45,0.40,0.60) ⟩
Medium (M)	⟨ (0.50,0.45,0.50), (0.40,0.35,0.50), (0.35,0.30,0.45) ⟩
Medium-good (MG)	⟨ (0.60,0.45,0.50), (0.20,0.15,0.25), (0.10,0.25,0.15) ⟩
Good (G)	⟨ (0.70,0.75,0.80), (0.15,0.20,0.25), (0.10,0.15,0.20) ⟩
Very-good (VG)	⟨ (0.95,0.90,0.95), (0.10,0.10,0.05), (0.05,0.05,0.05) ⟩

Step 3: The formulation of the weighting matrix for the experts ($\mathfrak{w} = [\mathfrak{w}_{\mathfrak{s}}]_{\mathfrak{S}}$) is executed through the application of Eq. (6):

$$\mathfrak{w}_{\mathfrak{s}} = \frac{\mathbb{SF}(\widetilde{H}_{\mathfrak{s}})}{\sum_{\mathfrak{s}=1}^{\mathfrak{S}} \mathbb{SF}(\widetilde{H}_{\mathfrak{s}})},\ \mathfrak{s} = 1, 2, \ldots, \mathfrak{S}. \quad (6)$$

- ***Stage 2*** *– Deriving the weights allocated to each criterion is accomplished through the implementation of the T2NN-WENSLO method.*

Step 4: Formulate the initial decision matrices $\left(\widetilde{\mathfrak{E}}^{(\widetilde{H}_{\mathfrak{s}})} = \left[\widetilde{\mathfrak{E}}^{(\widetilde{H}_{\mathfrak{s}})}{}_{\mathfrak{k}\mathfrak{d}}\right]_{\mathfrak{K}x\mathfrak{D}}\right)$, where $\widetilde{\mathfrak{E}}^{(\widetilde{H}_{\mathfrak{s}})}{}_{\mathfrak{k}\mathfrak{d}}$ ($\mathfrak{k} = 1,2, \ldots, \mathfrak{K};\ \mathfrak{d} = 1,2, \ldots, \mathfrak{D};\ \mathfrak{s} = 1,2, \ldots, \mathfrak{S}$) represent T2NN sets employed for evaluating alternative $B_{\mathfrak{k}}$ concerning criteria $\Gamma_{\mathfrak{d}}$. The assessment is based on expert input from the invited expert $H_{\mathfrak{s}}$. These initial decision matrices are established using a T2NN linguistic scale as delineated in Table 2.

Step 5: Utilize the T2NNWA aggregation operator, as delineated in Eq. (7), to calculate the aggregated decision matrix $\left(\widetilde{\mathfrak{E}} = \left[\widetilde{\mathfrak{E}}_{\mathfrak{k}\mathfrak{d}}\right]_{\mathfrak{K}x\mathfrak{D}}\right)$:

$$T2NNWA\left(\tilde{\mathfrak{E}}^{(1)},\tilde{\mathfrak{E}}^{(2)},\dots,\tilde{\mathfrak{E}}^{(\mathfrak{S})}\right)=\oplus_{\mathfrak{s}=1}^{\mathfrak{S}}\mathfrak{w}_{\mathfrak{s}}\tilde{\mathfrak{E}}^{(\mathfrak{s})}=$$

$$\left\{\left(\begin{array}{c}\left(1-\prod_{\mathfrak{s}=1}^{\mathfrak{S}}\left(1-T_{T_{\tilde{\mathfrak{E}}^{(\mathfrak{s})}}}(\mathfrak{h})\right)^{\mathfrak{w}_{\mathfrak{s}}}\right),\left(1-\prod_{\mathfrak{s}=1}^{\mathfrak{S}}\left(1-T_{I_{\tilde{\mathfrak{E}}^{(\mathfrak{s})}}}(\mathfrak{h})\right)^{\mathfrak{w}_{\mathfrak{s}}}\right),\left(1-\prod_{\mathfrak{s}=1}^{\mathfrak{S}}\left(1-T_{F_{\tilde{\mathfrak{E}}^{(\mathfrak{s})}}}(\mathfrak{h})\right)^{\mathfrak{w}_{\mathfrak{s}}}\right),\\ \left(\prod_{\mathfrak{s}=1}^{\mathfrak{S}}\left(I_{T_{\tilde{\mathfrak{E}}^{(\mathfrak{s})}}}(\mathfrak{h})\right)^{\mathfrak{w}_{\mathfrak{s}}}\right),\left(\prod_{\mathfrak{s}=1}^{\mathfrak{S}}\left(I_{I_{\tilde{\mathfrak{E}}^{(\mathfrak{s})}}}(\mathfrak{h})\right)^{\mathfrak{w}_{\mathfrak{s}}}\right),\left(\prod_{\mathfrak{s}=1}^{\mathfrak{S}}\left(I_{F_{\tilde{\mathfrak{E}}^{(\mathfrak{s})}}}(\mathfrak{h})\right)^{\mathfrak{w}_{\mathfrak{s}}}\right),\\ \left(\prod_{\mathfrak{s}=1}^{\mathfrak{S}}\left(F_{T_{\tilde{\mathfrak{E}}^{(\mathfrak{s})}}}(\mathfrak{h})\right)^{\mathfrak{w}_{\mathfrak{s}}}\right),\left(\prod_{\mathfrak{s}=1}^{\mathfrak{S}}\left(F_{I_{\tilde{\mathfrak{E}}^{(\mathfrak{s})}}}(\mathfrak{h})\right)^{\mathfrak{w}_{\mathfrak{s}}}\right),\left(\prod_{\mathfrak{s}=1}^{\mathfrak{S}}\left(F_{F_{\tilde{\mathfrak{E}}^{(\mathfrak{s})}}}(\mathfrak{h})\right)^{\mathfrak{w}_{\mathfrak{s}}}\right)\end{array}\right)\middle|\ \mathfrak{h}\in\mathfrak{H}\right\}. \tag{7}$$

wherein, the vector $\mathfrak{w}_{\mathfrak{s}}=(\mathfrak{w}_1,\mathfrak{w}_2,\dots,\mathfrak{w}_{\mathfrak{S}})$ is defined, with each $\mathfrak{w}_{\mathfrak{s}}$ belonging to the interval $[0,1]$, subject to the constraint $\sum_{\mathfrak{s}=1}^{\mathfrak{S}}\mathfrak{w}_{\mathfrak{s}}=1$.

Table 2

Linguistic variables employed in the evaluation of alternatives for criteria [54].

Linguistic variable	***T2NNs***
Very-low (VL)	⟨ (0.20, 0.20, 0.10), (0.65, 0.80, 0.85), (0.45, 0.80, 0.70) ⟩
Low (L)	⟨ (0.35, 0.35, 0.10), (0.50, 0.75, 0.80), (0.50, 0.75, 0.65) ⟩
Medium-low (ML)	⟨ (0.40, 0.30, 0.35), (0.50, 0.45, 0.60), (0.45, 0.40, 0.60) ⟩
Medium (M)	⟨ (0.50, 0.45, 0.50), (0.40, 0.35, 0.50), (0.35, 0.30, 0.45) ⟩
Medium-high (MH)	⟨ (0.60, 0.45, 0.50), (0.20, 0.15, 0.25), (0.10, 0.25, 0.15) ⟩
High (H)	⟨ (0.70, 0.75, 0.80), (0.15, 0.20, 0.25), (0.10, 0.15, 0.20) ⟩
Very-high (VH)	⟨ (0.95, 0.90, 0.95), (0.10, 0.10, 0.05), (0.05, 0.05, 0.05) ⟩

Step 6: Derive the crisp initial decision matrix for criteria ($\mathfrak{E}=[\mathfrak{E}_{\mathfrak{k}\mathfrak{d}}]_{\mathfrak{K}x\mathfrak{D}}$) through the computation facilitated by the score function $\left(\mathbb{SF}\left(\tilde{\mathfrak{E}}_{\mathfrak{k}\mathfrak{d}}\right)\right)$ elucidated in Eq. (8):

$$\mathbb{SF}\left(\tilde{\mathfrak{E}}_{\mathfrak{k}\mathfrak{d}}\right)=\frac{1}{12}\langle 8+\left(T_{T_{\tilde{\mathfrak{E}}_{\mathfrak{k}\mathfrak{d}}}}(\mathfrak{h})+2\left(T_{I_{\tilde{\mathfrak{E}}_{\mathfrak{k}\mathfrak{d}}}}(\mathfrak{h})\right)+T_{F_{\tilde{\mathfrak{E}}_{\mathfrak{k}\mathfrak{d}}}}(\mathfrak{h})\right)-\left(I_{T_{\tilde{\mathfrak{E}}_{\mathfrak{k}\mathfrak{d}}}}(\mathfrak{h})+2\left(I_{I_{\tilde{\mathfrak{E}}_{\mathfrak{k}\mathfrak{d}}}}(\mathfrak{h})\right)+ I_{F_{\tilde{\mathfrak{E}}_{\mathfrak{k}\mathfrak{d}}}}(\mathfrak{h})\right)-\left(F_{T_{\tilde{\mathfrak{E}}_{\mathfrak{k}\mathfrak{d}}}}(\mathfrak{h})+2\left(F_{I_{\tilde{\mathfrak{E}}_{\mathfrak{k}\mathfrak{d}}}}(\mathfrak{h})\right)+F_{F_{\tilde{\mathfrak{E}}_{\mathfrak{k}\mathfrak{d}}}}(\mathfrak{h})\right)\rangle,\ \mathfrak{k}=1,2,\dots,\mathfrak{K};\ \mathfrak{d}=1,2,\dots,\mathfrak{D}, \tag{8}$$

wherein $\mathbb{SF}\left(\tilde{\mathfrak{E}}_{\mathfrak{k}\mathfrak{d}}\right)=\mathfrak{E}_{\mathfrak{k}\mathfrak{d}}$ $(\mathfrak{k}=1,2,\dots,\mathfrak{K};\ \mathfrak{d}=1,2,\dots,\mathfrak{D})$.

Step 7: Calculate the normalization of the decision matrix ($\mathfrak{F}=[\mathfrak{F}_{\mathfrak{k}\mathfrak{d}}]_{\mathfrak{K}x\mathfrak{D}}$) employing Eq. (9). During this phase, a linear normalization approach is implemented [53]:

$$\mathfrak{F}_{\mathfrak{k}\mathfrak{d}}=\frac{\mathfrak{E}_{\mathfrak{k}\mathfrak{d}}}{\sum_{\mathfrak{k}=1}^{\mathfrak{K}}\mathfrak{E}_{\mathfrak{k}\mathfrak{d}}},\ \mathfrak{k}=1,2,\dots,\mathfrak{K};\ \mathfrak{d}=1,2,\dots,\mathfrak{D}. \tag{9}$$

Step 8: To determine the interval matrix for criterion classes ($\Delta\mathfrak{F}_{\mathfrak{d}}=[\Delta\mathfrak{F}_{\mathfrak{d}}]_{\mathfrak{D}}$), Eq. (10) is invoked, utilizing Sturges' rule for computation:

$$\Delta\mathfrak{F}_{\mathfrak{d}}=\frac{\max\limits_{\mathfrak{k}=1,2,\dots,\mathfrak{K}}(\mathfrak{F}_{\mathfrak{k}\mathfrak{d}})-\min\limits_{\mathfrak{k}=1,2,\dots,\mathfrak{K}}(\mathfrak{F}_{\mathfrak{k}\mathfrak{d}})}{1+3.322\log(\mathfrak{K})},\ \mathfrak{d}=1,2,\dots,\mathfrak{D}. \tag{10}$$

Step 9: To compute the slope matrix for criterion angles ($\tan\theta_{\mathfrak{d}}=[\tan\theta_{\mathfrak{d}}]_{\mathfrak{D}}$), Eq. (11) is employed for the calculation:

$$\tan\theta_{\mathfrak{d}}=\frac{\sum_{\mathfrak{k}=1}^{\mathfrak{K}}\mathfrak{F}_{\mathfrak{k}\mathfrak{d}}}{(\mathfrak{K}-1)(\Delta\mathfrak{F}_{\mathfrak{d}})},\ \mathfrak{d}=1,2,\dots,\mathfrak{D}. \tag{11}$$

***Step 10**:* To determine the envelope matrix for the criteria ($\mathit{\Xi}_{\mathfrak{d}} = [\mathit{\Xi}_{\mathfrak{d}}]_{\mathfrak{D}}$), Eq. (12) is employed for the computational process:

$$\mathit{\Xi}_{\mathfrak{d}} = \sum_{\mathfrak{k}=1}^{\mathfrak{K}-1} \sqrt{\left(\mathfrak{F}_{\mathfrak{k}+1,\mathfrak{d}} - \mathfrak{F}_{\mathfrak{k}\mathfrak{d}}\right)^2 + (\Delta\mathfrak{F}_{\mathfrak{d}})^2},\ \mathfrak{d} = 1,2,\dots,\mathfrak{D}. \tag{12}$$

***Step 11**:* To compute the matrix representing the ratio of criterion envelope to slope ($\mathit{\Phi}_{\mathfrak{d}} = [\mathit{\Phi}_{\mathfrak{d}}]_{\mathfrak{D}}$), Eq. (13) is employed for the analytical procedure:

$$\mathit{\Phi}_{\mathfrak{d}} = \frac{\mathit{\Xi}_{\mathfrak{d}}}{\tan\theta_{\mathfrak{d}}},\ \mathfrak{d} = 1,2,\dots,\mathfrak{D}. \tag{13}$$

***Step 12**:* To determine the weighting matrix for the criteria ($\mathbb{w}_{\mathfrak{d}} = [\mathbb{w}_{\mathfrak{d}}]_{\mathfrak{D}}$), Eq. (14) is invoked for the calculation process:

$$\mathbb{w}_{\mathfrak{d}} = \frac{\mathit{\Phi}_{\mathfrak{d}}}{\sum_{\mathfrak{d}=1}^{\mathfrak{D}} \mathit{\Phi}_{\mathfrak{d}}},\ \mathfrak{d} = 1,2,\dots,\mathfrak{D}. \tag{14}$$

- ***Stage 3*** *– Establishing the rankings of alternatives through the application of the T2NN-ARLON method.*

***Step 13**:* The decision matrix ($\mathfrak{E} = [\mathfrak{E}_{\mathfrak{k}\mathfrak{d}}]_{\mathfrak{K}x\mathfrak{D}}$) obtained in Step 6 is considered as the initial decision matrix for the alternative ranking process. However, to execute ARLON, values ranging from 0 to 1 need to be transformed into values ranging from 1 to 100. Therefore, the initial matrix for the ARLON method ($\mathcal{E} = [\mathcal{E}_{\mathfrak{k}\mathfrak{d}}]_{\mathfrak{K}x\mathfrak{D}}$) is created using Eq. (15).

$$\mathcal{E}_{\mathfrak{k}\mathfrak{d}} = (\mathfrak{E}_{\mathfrak{k}\mathfrak{d}} * 100),\ \mathfrak{k} = 1,2,\dots,\mathfrak{K};\ \mathfrak{d} = 1,2,\dots,\mathfrak{D}. \tag{15}$$

***Step 14**:* The initial matrix for the ARLON method ($\mathcal{E} = [\mathcal{E}_{\mathfrak{k}\mathfrak{d}}]_{\mathfrak{K}x\mathfrak{D}}$) undergoes two distinct logarithmic normalizations outlined in Eq. (16) [55] and Eq. (17) [56]. Subsequently, the first logarithmic normalization matrix ($\mathfrak{N}^{1st} = [\mathfrak{N}^{1st}{}_{\mathfrak{k}\mathfrak{d}}]_{\mathfrak{K}x\mathfrak{D}}$) and second logarithmic normalization matrix ($\mathfrak{N}^{2nd} = [\mathfrak{N}^{2nd}{}_{\mathfrak{k}\mathfrak{d}}]_{\mathfrak{K}x\mathfrak{D}}$) are obtained:

$$\mathfrak{N}^{1st}{}_{\mathfrak{k}\mathfrak{d}} = \begin{cases} \mathfrak{N}^{1st(+)}{}_{\mathfrak{k}\mathfrak{d}} = \dfrac{\ln(\mathcal{E}_{\mathfrak{k}\mathfrak{d}})}{\ln\left(\prod_{\mathfrak{k}=1}^{\mathfrak{K}} \mathcal{E}_{\mathfrak{k}\mathfrak{d}}\right)}, \mathfrak{d} \in \text{Benefit} \\ \mathfrak{N}^{1st(-)}{}_{\mathfrak{k}\mathfrak{d}} = \dfrac{1 - \dfrac{ln(\mathcal{E}_{\mathfrak{k}\mathfrak{d}})}{ln\left(\prod_{\mathfrak{k}=1}^{\mathfrak{K}} \mathcal{E}_{\mathfrak{k}\mathfrak{d}}\right)}}{\mathfrak{K}-1}, \mathfrak{d} \in \text{Cost} \end{cases},\ \mathfrak{k} = 1,2,\dots,\mathfrak{K};\ \mathfrak{d} = 1,2,\dots,\mathfrak{D}, \tag{16}$$

$$\mathfrak{N}^{2nd}{}_{\mathfrak{k}\mathfrak{d}} = \begin{cases} \mathfrak{N}^{2nd(+)}{}_{\mathfrak{k}\mathfrak{d}} = \dfrac{\log_2(\mathcal{E}_{\mathfrak{k}\mathfrak{d}})}{\sum_{\mathfrak{k}=1}^{K}(log_2(\mathcal{E}_{\mathfrak{k}\mathfrak{d}}))}, \mathfrak{d} \in \text{Benefit} \\ \mathfrak{N}^{2nd(-)}{}_{\mathfrak{k}\mathfrak{d}} = 1 - \dfrac{log_2(\mathcal{E}_{\mathfrak{k}\mathfrak{d}})}{\sum_{\mathfrak{k}=1}^{K}(log_2(\mathcal{E}_{\mathfrak{k}\mathfrak{d}}))}, \mathfrak{d} \in \text{Cost} \end{cases},\ \mathfrak{k} = 1,2,\dots,\mathfrak{K};\ \mathfrak{d} = 1,2,\dots,\mathfrak{D}. \tag{17}$$

***Step 15**:* The first logarithmic normalization and second logarithmic normalization matrices are then combined using the Heron Mean (Eq. (18)). The parameter δ in this context indicates the blending ratio distribution, where the δ parameter falls within the [0,1] interval. Subsequently, the aggregated normalized decision matrix ($\mathfrak{N}^{norm} = [\mathfrak{N}_{\mathfrak{k}\mathfrak{d}}^{norm}]_{\mathfrak{K}x\mathfrak{D}}$) is derived.

$$\mathfrak{N}_{\mathfrak{k}\mathfrak{d}}^{norm}\left(\mathfrak{N}^{1st}{}_{\mathfrak{k}\mathfrak{d}}, \mathfrak{N}^{2nd}{}_{\mathfrak{k}\mathfrak{d}}\right) = \left((1-\delta)\sqrt{(\mathfrak{N}^{1st}{}_{\mathfrak{k}\mathfrak{d}})(\mathfrak{N}^{2nd}{}_{\mathfrak{k}\mathfrak{d}})} + (\delta)\,\frac{(\mathfrak{N}^{1st}{}_{\mathfrak{k}\mathfrak{d}})+(\mathfrak{N}^{2nd}{}_{\mathfrak{k}\mathfrak{d}})}{2}\right), \tag{18}$$

$\mathfrak{k} = 1,2, \dots, \mathfrak{K};\ \mathfrak{d} = 1,2, \dots, \mathfrak{D}.$

Step 16: The calculation of the weighted aggregated normalization matrix $(\mathfrak{P} = [\mathfrak{P}_{\mathfrak{k}\mathfrak{d}}]_{\mathfrak{K}x\mathfrak{D}})$ is performed through the application of Eq. (19), wherein consideration is given to the criterion weights matrix $(\mathbb{w}_{\mathfrak{d}} = [\mathbb{w}_{\mathfrak{d}}]_{\mathfrak{D}})$:

$$\mathfrak{P}_{\mathfrak{k}\mathfrak{d}} = (\mathfrak{N}^{norm}_{\mathfrak{k}\mathfrak{d}} . \mathbb{w}_{\mathfrak{d}}),\ \mathfrak{k} = 1,2, \dots, \mathfrak{K};\ \mathfrak{d} = 1,2, \dots, \mathfrak{D}. \quad (19)$$

Step 17: The summation of the weighted aggregated normalized values for cost criteria and benefit criteria is separately conducted through the utilization of Eq. (20) and Eq. (21), respectively. Thus, the matrices of cost-weighted aggregated normalized values $\left(\mathfrak{P}^- = [\mathfrak{P}^-{}_{\mathfrak{k}}]_{\mathfrak{K}}\right)$ and benefit-weighted aggregated normalized values $\left(\mathfrak{P}^+ = [\mathfrak{P}^+{}_{\mathfrak{k}}]_{\mathfrak{K}}\right)$ are obtained:

$$\mathfrak{P}^-{}_{\mathfrak{k}} = \sum_{\mathfrak{d}\in C} \mathfrak{P}_{\mathfrak{k}\mathfrak{d}},\ \mathfrak{k} = 1,2, \dots, \mathfrak{K}, \quad (20)$$

$$\mathfrak{P}^+{}_{\mathfrak{k}} = \sum_{\mathfrak{d}\in B} \mathfrak{P}_{\mathfrak{k}\mathfrak{d}},\ \mathfrak{k} = 1,2, \dots, \mathfrak{K}. \quad (21)$$

Step 18: The conclusive ranking of the alternatives matrix $(P = [P_{\mathfrak{k}}]_{\mathfrak{K}})$ is ascertained through the application of Eq. (22). The parameter ϕ in this context signifies the distribution of blending ratios, with ϕ constrained to the interval [0,1].

$$P_{\mathfrak{k}} = \left((\mathfrak{P}^+{}_{\mathfrak{k}})^{\phi} + (\mathfrak{P}^-{}_{\mathfrak{k}})^{(1-\phi)}\right),\ \mathfrak{k} = 1,2, \dots, \mathfrak{K}. \quad (22)$$

The Algorithm steps for T2NN-WENSLO-ARLON are outlined in Table 3.

Table 3

Algorithm.

Algorithm	*The procedural steps for determining the performance of sustainable brand equity in the T2NN-WENSLO-ARLON hybrid model are elucidated in this algorithm.*
Input	Within the context of a set of alternatives $B_{\mathfrak{k}} = \{B_1, B_2, \dots, B_{\mathfrak{K}}\}$, criteria $\Gamma_{\mathfrak{d}} = \{\Gamma_1, \Gamma_2, \dots, \Gamma_{\mathfrak{D}}\}$, and a panel of experts $H_{\mathfrak{s}} = \{H_1, H_2, \dots, H_{\mathfrak{S}}\}$, the determination process involves the establishment of the expert's weighting matrix $(\mathfrak{w} = [\mathfrak{w}_{\mathfrak{s}}]_{\mathfrak{S}})$, the criteria weighting matrix $(\mathbb{w}_{\mathfrak{d}} = [\mathbb{w}_{\mathfrak{d}}]_{\mathfrak{D}})$, and the matrix for ranking alternatives $(P = [P_{\mathfrak{k}}]_{\mathfrak{K}})$.
Output	Determining the sustainable brand equity of companies.
Begin	
First stage	***Assigning weights to each expert;***
Step 1	Collect data concerning the attributed significance levels assigned by experts to the LVs delineated in Table 1. Subsequently, transform these LVs into T2NN sets.
Step 2	Calculate the score functions $\mathbb{SF}(\widetilde{H}_{\mathfrak{s}})$ through the utilization of Eq. (5) to ascertain the crisp values.
Step 3	Determine the weighted matrix $(\mathfrak{w} = [\mathfrak{w}_{\mathfrak{s}}]_{\mathfrak{S}})$ for the participating experts by applying Eq. (6).
Second stage	***Deriving weights allocated to each criterion;***
Step 4	Evaluate each alternative by each expert with respect to individual criteria utilizing the LVs delineated in Table 2. Subsequently, transforms the LVs into T2NN sets, followed by the derivation of the initial decision matrices $\left(\widetilde{\mathfrak{E}}^{(\widetilde{H}_{\mathfrak{s}})} = [\widetilde{\mathfrak{E}}^{(\widetilde{H}_{\mathfrak{s}})}{}_{\mathfrak{k}\mathfrak{d}}]_{\mathfrak{K}x\mathfrak{D}}\right)$.
Step 5	Compute the aggregated decision matrix $\left(\widetilde{\mathfrak{E}} = [\widetilde{\mathfrak{E}}_{\mathfrak{k}\mathfrak{d}}]_{\mathfrak{K}x\mathfrak{D}}\right)$ by employing the T2NNWA aggregation operator expressed as $\left(T2NNWA\left(\widetilde{\mathfrak{E}}^{(1)}, \widetilde{\mathfrak{E}}^{(2)}, \dots, \widetilde{\mathfrak{E}}^{(\mathfrak{S})}\right) = \oplus_{\mathfrak{s}=1}^{\mathfrak{S}} \mathfrak{w}_{\mathfrak{s}} \widetilde{\mathfrak{E}}^{(\mathfrak{s})}\right)$ (Eq. (7)).
Step 6	Derive the precise initial decision matrix $(\mathfrak{E} = [\mathfrak{E}_{\mathfrak{k}\mathfrak{d}}]_{\mathfrak{K}x\mathfrak{D}})$ for criteria by calculating the crisp values through the implementation of the score function (Eq. (8)).

Step 7	Determine the linear normalization of the decision matrix ($\mathfrak{F} = [\mathfrak{F}_{\mathfrak{t}\mathfrak{d}}]_{\mathfrak{K}x\mathfrak{D}}$) using Eq. (9).
Step 8	To establish the interval matrix for criterion classes ($\Delta\mathfrak{F}_{\mathfrak{d}} = [\Delta\mathfrak{F}_{\mathfrak{d}}]_{\mathfrak{D}}$), Sturges' rule is employed for computation, as described in Eq. (10).
Step 9	Compute the matrix depicting the slopes of criterion angles ($tan\,\theta_{\mathfrak{d}} = [tan\,\theta_{\mathfrak{d}}]_{\mathfrak{D}}$) through the utilization of Eq. (11).
Step 10	Determine the matrix representing the criterion envelopes ($\Xi_{\mathfrak{d}} = [\Xi_{\mathfrak{d}}]_{\mathfrak{D}}$) through the application of Eq. (12).
Step 11	Compute the matrix representing the ratio of criterion envelope to slope ($\Phi_{\mathfrak{d}} = [\Phi_{\mathfrak{d}}]_{\mathfrak{D}}$) through the utilization of Eq. (13).
Step 12	Determine the weighting matrix for the criteria ($\mathbb{w}_{\mathfrak{d}} = [\mathbb{w}_{\mathfrak{d}}]_{\mathfrak{D}}$) through the application of Eq. (14).
Third stage	***Establishing the rankings of each alternative;***
Step 13	Calculate the initial matrix for the ARLON method ($\mathcal{E} = [\mathcal{E}_{\mathfrak{t}\mathfrak{d}}]_{\mathfrak{K}x\mathfrak{D}}$) using Eq. (15).
Step 14	Compute the first logarithmic normalization matrix ($\mathfrak{N}^{1st} = [\mathfrak{N}^{1st}{}_{\mathfrak{t}\mathfrak{d}}]_{\mathfrak{K}x\mathfrak{D}}$) and second logarithmic normalization matrix ($\mathfrak{N}^{2nd} = [\mathfrak{N}^{2nd}{}_{\mathfrak{t}\mathfrak{d}}]_{\mathfrak{K}x\mathfrak{D}}$) using Eq. (16) and Eq. (17), respectively.
Step 15	Compute the aggregated normalized decision matrix ($\mathfrak{N}^{norm} = [\mathfrak{N}^{norm}_{\mathfrak{t}\mathfrak{d}}]_{\mathfrak{K}x\mathfrak{D}}$) using Heron Mean (Eq. (18)) ($\delta \in [0,1]$).
Step 16	Compute the weighted aggregated normalization matrix ($\mathfrak{P} = [\mathfrak{P}_{\mathfrak{t}\mathfrak{d}}]_{\mathfrak{K}x\mathfrak{D}}$) using Eq. (19).
Step 17	Compute the cost-weighted aggregated normalized matrix $\left(\mathfrak{P}^{-} = [\mathfrak{P}^{-}{}_{\mathfrak{t}}]_{\mathfrak{K}}\right)$ and benefit-weighted aggregated normalized matrix $\left(\mathfrak{P}^{+} = [\mathfrak{P}^{+}{}_{\mathfrak{t}}]_{\mathfrak{K}}\right)$ using Eq. (20) and Eq. (21), respectively.
Step 18	Calculate the conclusive ranking of the alternative matrix ($P = [P_{\mathfrak{t}}]_{\mathfrak{K}}$) using Eq. (22) ($\phi \in [0,1]$).
End.	

4. Case Study

The understanding of sustainability in the cosmetics sector is relatively more significant compared to other industries because the sustainability approach must be adopted throughout all production processes to prevent damage to the environment and nature caused by the production of cosmetic products. Particularly, harmful chemicals used in this sector can adversely affect human health and safety. Therefore, cosmetic companies need to fulfill their social responsibilities and engage in production within the framework of sustainability. Moreover, customers in the cosmetics sector are increasingly inclined towards products that pose fewer risks to human health. Hence, cosmetic brands strive to attain sustainable brand equity. In this regard, the implementation of DSS in the cosmetics sector not only addresses sectoral needs but also enhances the generalizability of the case study's success. Hence, the cosmetics industry was chosen for the case study.

The T2NN-WENSLO-ARLON hybrid model is proposed as the DSS for evaluating the sustainable brand equity of cosmetic brands. A case study was conducted to demonstrate the feasibility of this hybrid method. The objective of this case study was to rank the sustainable brand equity of cosmetic companies manufacturing in Turkey. To this end, 12 criteria were identified to determine sustainable brand equity. These criteria were derived from a literature review and validated through expert opinions. Ten cosmetic brands were selected for the case study, chosen from cosmetic companies operating in Turkey and international markets. An expert group of eight individuals consisting of cosmetic company managers, cosmetic brand experts, cosmetic sector specialists, and academics was identified to execute DSS. Details regarding the identified experts, criteria, and cosmetic brands are provided in the respective subsections. Subsequently, within the scope of the case study, the steps of the T2NN-WENSLO-ARLON hybrid model were demonstrated individually, and the findings were shared.

4.1. Experts, Criteria, and Brands

4.1.1. Identification of Experts

To assess the sustainable brand equity of ten cosmetic brands operating in Turkey, eight experts proficient in sustainable brand evaluation were identified. These experts possess in-depth knowledge about the brands operating in the cosmetic sector. Furthermore, they are competent in evaluating brands based on the sustainable brand assessment criteria established for the case study. The expertise levels and professions of these experts are presented in Table 4. Evaluations of the cosmetic brands based on the criteria were conducted according to linguistic expressions through consultations with these experts. The obtained dataset was utilized to conduct the case study.

Table 4

The expert panel.

Experts	*Significance level*	*Professions*
H_1	Very-good (VG)	Brand manager in a cosmetics company
H_2	Good (G)	Cosmetic Industry Specialists
H_3	Good (G)	Marketing Experts in a cosmetics company
H_4	Good (G)	Brand manager in a cosmetics company
H_5	Very-good (VG)	Cosmetic Industry Specialists
H_6	Very-good (VG)	Brand manager in a cosmetics company
H_7	Very-good (VG)	General manager in a cosmetics company
H_8	Good (G)	Academist conducting research on brand equity

4.1.2. Identification of Criteria

Twelve criteria were identified for the assessment of sustainable brand equity for the cosmetic companies operating in Turkey. These criteria serve as the precursors and assessment parameters for sustainable brand equity. The process of determining these criteria involved a thorough review of the literature and consultation with the experts. Detailed explanations of these criteria are provided as follows:

- *Green brand image* (Γ_1) − The criterion of "green brand image" signifies the perception and reputation a brand engenders regarding its environmental sustainability [14]. It is desired for companies to produce goods and services that are valuable in terms of sustainability for environmentally conscious consumers [57–58]. Thus, consumers attach importance to the sustainability images created by brands. It is crucial for brands to conduct their production processes, waste management processes, and supply chain processes with environmentally friendly practices to attain a green brand image [41]. Particularly, energy efficiency and product lifecycle provide clues about the product sustainability of the brand. Furthermore, the brand's engagement in joint ventures with environmental organizations and possession of environmental management certifications shape customers' perceptions of the brand. Moreover, brands with eco-friendly products play an effective role in ensuring the loyalty of environmentally conscious customers [59]. Increasing brand awareness through various social responsibility projects also plays a significant role in the formation of the green brand image [60]. In this context, this criterion is considered among the selection criteria in the sustainable brand equity assessment process.

- *Green trust* (Γ_2) – The criterion of "green trust" indicates the confidence consumers have in brands being environmentally friendly and sustainable [61]. Brands fulfilling their environmental commitments, visibly recognized by customers, positively influence the brand's level of green trust [62]. Transparency and honesty in a brand's environmental practices foster consumer confidence in the brand's eco-friendliness. Moreover, brands operating with a sustainability-focused approach in their operational activities are embraced by customers as consistent and reliable [42]. Additionally, collaborative efforts with stakeholders to ensure environmental sustainability contribute to the formation of green trust [14]. Brands meeting consumers' environmental expectations within the realm of corporate social responsibility create a greater sense of trust, thereby gaining a competitive advantage over their counterparts. Considering the contribution of green trust to sustainable brand equity formation, this criterion is included in the decision model.
- *Green satisfaction* (Γ_3) – The criterion of "green satisfaction" indicates the sense of fulfillment consumers experience regarding a brand's environmental stance [17]. The emphasis on environmental focus in consumers' evaluations of the brand is crucial for its sustainability. The prominence of environmental approaches in product and service quality directly contributes to increasing consumers' green satisfaction. Another factor influencing the level of satisfaction is the brand's adherence to ethical standards [63]. Brands operating in accordance with environmental ethical standards are expected to have high levels of green satisfaction. Additionally, initiatives aimed at extending the product lifecycle through

post-purchase services and the retrieval of end-of-life products through various recycling programs also enhance green satisfaction. In this context, the inclusion of this criterion in the decision-making process is evaluated based on its impact on sustainable brand equity.

- *Green product leadership* (Γ_4) – The "green product leadership" criterion pertains to the production and initial introduction of environmentally friendly and innovative products by brands. Green products can be elucidated as brands' development of products that minimize harm to the environment [64]. Brands that allocate research and development budgets towards developing eco-friendly products gain a competitive advantage [65]. These brands implement market differentiation strategies with an environmental focus, thus dominating the market as leaders in green product innovation. Moreover, by embracing a green approach across all supply chain processes, brands attract attention to their focus on green product production [66], thereby shaping the perception of the brand as environmentally conscious. Given the contribution of this criterion to the formation of sustainable green equity, it is evaluated as a criterion in the decision model.
- *Green perceived quality* (Γ_5) – The "green perceived quality" criterion primarily signifies the level of environmental quality associated with a brand's products [67]. Brands perceived to possess environmental quality are viewed as eco-friendly and trustworthy. By constructing their products and services in recyclable formats, brands can enhance their green qualities, thereby fostering perceptions of dependability and reliability [68]. This perception, in turn, encourages environmentally conscious consumers to gravitate towards higher-priced and eco-friendly products. Furthermore, from a brand reputation standpoint, being perceived as having green quality confers a competitive advantage, supporting the formation of sustainable brand equity. Consequently, this criterion is acknowledged as a determinant in the process of determining sustainable brand equity.
- *Green innovation* (Γ_6) – The "green innovation" criterion signifies brands' environmentally focused approach in their innovative endeavors [69]. Green-oriented approaches in product and process innovations support the development of environmentally friendly products and processes [70]. Particularly, the implementation of innovative ideas aimed at reducing waste production and energy consumption supports brands in producing more environmentally friendly and durable products [71]. Additionally, establishing green partnerships with other stakeholders in research and development activities accelerates the formation of green innovation [27]. Brands can achieve sustainable goals and meet

customer expectations through these approaches, thereby creating sustainable brand equity. For these reasons, this criterion is included in the decision model.

- *Environmental benefit* (Γ_7) – The "environmental benefit" criterion demonstrates the tangible steps taken by a brand to contribute positively to the environment. These steps reflect the brand's initiatives aimed at promoting environmental sustainability. Efforts to reduce pollution and conserve resources support effective resource management by facilitating waste control. Particularly, an environmental benefit-focused approach [72] towards preserving biodiversity and reducing carbon footprint directly contributes to brands' formation of sustainable brand equity. Additionally, organizing educational programs and seminars for environmental benefit serves as a tangible indicator of the brand's commitment to environmental benefit. Considering its contribution to the formation of sustainable brand equity, this criterion is included in the decision-making process.
- *Corporate social responsibility (CSR) initiatives* (Γ_8) – The "CSR initiatives" criterion primarily aims to address environmental and social concerns collaboratively with stakeholders and develop solutions to these concerns. Companies that support CSR initiatives take steps toward addressing environmental and social issues by incorporating them into their agendas [73]. These steps are not only taken at the firm level but also involve collaborative efforts at the entire supply chain level [74]. Particularly, the development of a comprehensive environmental management system contributes to environmental management and resource conservation. Additionally, brands take the lead in CSR initiatives to manage consumer perceptions and create awareness [75]. Thus, the formation of sustainable brand equity is facilitated. For this reason, in this research, this criterion is among the criteria for determining sustainable brand equity.
- *Green brand loyalty* (Γ_9) – The "green brand loyalty" criterion illustrates brands' ability to create loyalty by generating green brand equity [14]. In fostering brand loyalty, brands aim to emphasize their environmental aspect to encourage customer loyalty [76]. These brands prioritize their success in fulfilling their environmental responsibilities to manage consumer perceptions and attitudes effectively. Consequently, customers are encouraged to develop repeat purchase behaviors with environmentally friendly products [77]. Moreover, brands that create green brand loyalty see their customers acting as advocates, promoting the brand through word-of-mouth communication [78]. Furthermore, this approach enhances the level of trust between the customer and the brand by fostering customer-brand relationships, ultimately positioning the brand as preferred. Considering

the contribution of strong green brand loyalty to green brand equity, this criterion is included in the research.

- *Green brand name* (Γ_{10}) – The "green brand name" criterion directly pertains to the reputation and perception of a brand regarding its approach to environmental sustainability [66]. Brands with a strong green brand name are perceived as trustworthy by consumers due to their commitment to environmental responsibilities [79]. The green brand name reflects the level of association with environmental initiatives. Additionally, brands that consider environmental issues can establish sustainable brand equity through the creation of their brand name, while the subsequent association with environmentalism further promotes green branding. At this juncture, a green brand name can provide differentiation benefits in the formation of sustainable brand equity, influencing customers. Therefore, this criterion is included in the decision model.
- *Green wash* (Γ_{11}) – The "green wash" criterion refers to the practice whereby brands make misleading or deceptive claims to support environmental sustainability [80]. Brands may exaggerate their environmental initiatives to appear more environmentally friendly, leading to a decrease in consumer trust in the brand's level of commitment to green initiatives. Additionally, brands may misrepresent their promotional efforts to appear environmentally friendly, and they may also tend to conceal environmentally harmful activities. Brands exhibiting greenwashing tendencies typically have lower sustainable brand equities [81]. Therefore, this criterion plays an influential role in determining sustainable brand equity and should be included in the decision model as a negative factor.
- *Green perceived risk* (Γ_{12}) – The "green perceived risk" criterion entails consumers' assessment of the environmental risks associated with a brand [82]. This criterion indicates the environmental risks perceived by consumers stemming from the brand's activities. Skeptical attitudes arising from consumers' environmental concerns negatively impact the processes involved in forming a brand's sustainable brand equity. Particularly, brands that fail to engage in environmental activities or transparently disclose the environmental risks associated with their products face consumers' green perceived risk. Moreover, companies that do not adhere to or sufficiently demonstrate compliance with environmental ethical standards are vulnerable to this risk. Therefore, this criterion is regarded as a cost-based factor in the evaluation process of sustainable brand equity.

4.1.3. Market Equities as Alternatives

In this case study, the comparison of sustainable brand equities of cosmetic products operating in Turkey was conducted using the proposed hybrid method. In this context, 10 well-known cosmetic brands in the sector were identified. The information regarding these brands is as follows:

- *Golden Rose* (B_1) – Golden Rose is a cosmetic company based in Turkey, established in 1983. The brand is known for offering a wide range of products including makeup items, skincare products, and accessories. The brand prioritizes customer satisfaction and has achieved a successful position in the cosmetic sector with its high-quality products and innovative approaches. Golden Rose's products are popular not only in Turkey but also in international markets, being sold in many countries [83].
- *Pastel* (B_2) – Pastel Cosmetics is a Turkey-based cosmetic company established in 1990, offering a wide range of products including makeup, skincare, and personal care items. Pastel aims to provide quality products at affordable prices and endeavors to develop innovative products by staying abreast of trends. Its makeup line includes foundations, eyeshadows, lipsticks, mascaras, etc. Pastel is recognized as one of the leading brands in the Turkish cosmetic industry, with its products available in numerous stores and online platforms [84].
- *Farmasi* (B_3) – Farmasi is a cosmetic company founded in 1950 by Dr. Cevdet Tuna. It is headquartered in Istanbul, Turkey. Farmasi operates in the fields of cosmetics, skincare, personal care, and health products. Its wide range of products includes makeup supplies, perfumes, skincare products, hair care products, personal care items, and dietary supplements. Farmasi's products are sold in various countries through a direct sales model, reaching customers directly. The company prioritizes products made from natural ingredients and places importance on research and development efforts and innovative product development projects. Farmasi has established itself as a cosmetic company committed to customer satisfaction and providing quality products [85].
- *Eyüp Sabri Tuncer* (B_4) – Eyüp Sabri Tuncer Cosmetics Inc. was founded by Eyüp Sabri Tuncer in 1923, and it is a Turkey-based cosmetics company. Its headquarters is located in the Sarıyer district of Istanbul, while its factory is situated in Lalahan, Ankara. Engin Tuncer serves as the Chairman of the Board. The company offers a wide range of products including personal care, home cosmetics, textiles, and spa items, which are marketed both in Turkey and internationally. Under the umbrella of Eyüp Sabri Tuncer, brands such as

Gizli Bahçe and Perfume Jewels specialize in the production and sale of natural and certified organic cosmetic products [86].

- *Cecile* (B_5) – Özsoy Kozmetik was established in 1980 with a commitment to high-quality production and competitive pricing, positioning itself among the prominent organizations in the global cosmetics industry. Cecile, introduced in 2005, is a brand under the umbrella of Özsoy Kozmetik, specializing in colored cosmetics and fragrances. Initially recognized in the Turkish market for its perfumes, the brand later gained prominence with its cosmetic products. With over 200 products in its colored cosmetics and perfume categories, Özsoy Kozmetik is a wholly Turkish-owned company manufacturing nail polish, lipstick, eyeshadow, blush, foundation, lip gloss, as well as skincare and haircare products [87].
- *The Purest Solutions* (B_6) – It was founded by Alim Ozan Evliyaoğlu and Hazal Evliyaoğlu in 2020. Currently, the brand exports to more than 30 countries with 20 different products. It is characterized by its respect for nature, clean ingredients, awareness of social responsibility, and value for humanity. The company manufactures and sells its developed cosmetic products in both the domestic and international cosmetics markets [88].
- *Rosense* (B_7) – The Rosense brand, which has been operating in Turkey since 1954, is a subsidiary of GÜLBİRLİK. Its focus is on the production, sales, and marketing of cosmetic and food products based on roses. Rosense-branded cosmetic products, enriched with rose oil, are sold both domestically and internationally. Rosense produces and offers approximately 120 different cosmetic products to the market [89].
- *Ashley Joy* (B_8) – The Ashley Joy brand was created by Aslı Şen and operates in the cosmetics sector in Turkey. Its product range includes cosmetics such as shampoo, hair masks, hair conditioners, hair styling products, hair serums, and hair tonics. Ashley Joy specializes in producing natural and organic cosmetic products tailored to address various hair problems and types [90].
- *Agarta* (B_9) – Agarta Cosmetics is a hundred percent locally owned cosmetics brand that produces personal care products with herbal and natural ingredients. The establishment objective of Agarta Cosmetics is to provide healthy products in the cosmetics field in Turkey. Founded in Ankara, Agarta Herbal Cosmetics holds a significant market share both domestically and internationally. Herbal cosmetics utilize natural plant-based ingredients in the production of cosmetic products. Agarta Cosmetics conducts production using environmentally friendly methods [91].

- *Misbahçe* (B_{10}) – In 2012, Serra Göney (designer) and her mother Ufuk Göney (chemical engineer) laid the foundations of Misbahçe in Ayvalık, also known as the olive paradise of Turkey. The company aims to produce natural and clean cosmetic products free from chemical and synthetic ingredients. Utilizing local resources and harnessing solar energy, Misbahçe is dedicated to manufacturing cosmetic products. The packaging materials, including glass containers and paper, are recyclable or made from recycled materials. Operating in the Turkish market, this company operates in the cosmetics sector with a focus on sustainability [92].

The criteria and brands are illustrated in Fig. 2.

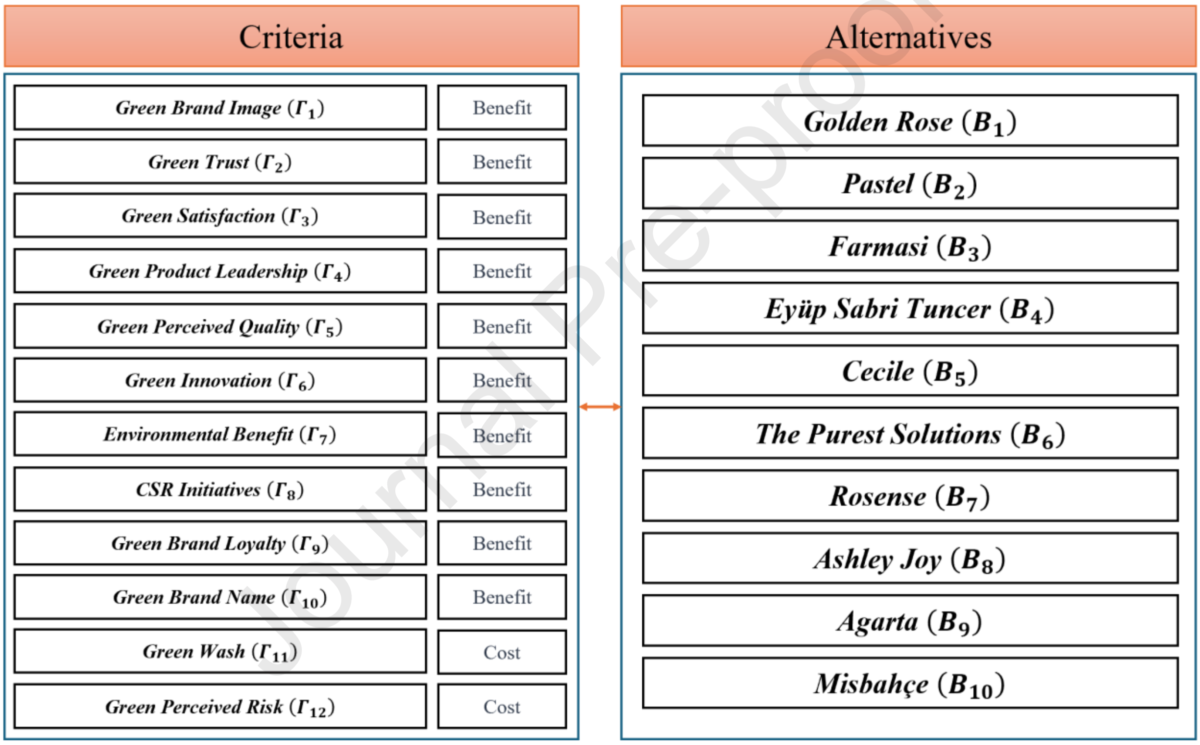

Fig. 2. Criteria and cosmetic brands.

4.2. Evaluation of Sustainable Brand Equity with T2NN-WENSLO-ARLON

The case study created for the evaluation of sustainable brand equity of cosmetic brands was sequentially implemented according to the steps of the T2NN-WENSLO-ARLON hybrid method.

- ***Stage 1*** *– Assigning weights to each expert.*

Step 1: To assess the contribution levels of experts to the sustainable brand equity assessment process, the expertise levels of experts were determined using LVs illustrated in Table 1. Subsequently, LVs were transformed into T2NN sets. LVs and T2NNs corresponding to the proficiency levels of the experts are presented in Table S.1.

Step 2: To obtain crisp values, the score functions $\mathbb{SF}(\widetilde{H}_{\mathfrak{s}})$ indicated in Eq. (5) were computed. The resulting values of the score functions are documented in Table S.1.

Step 3: The calculation of the experts' weighting matrix ($\mathfrak{w} = [\mathfrak{w}_{\mathfrak{s}}]_{\mathfrak{S}}$) (Table 5) was performed through the application of Eq. (6).

Table 5

The matrix representing the weights assigned by the experts.

Experts	H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8
$\mathfrak{w}_{\mathfrak{s}}$	0.1343	0.1157	0.1157	0.1157	0.1343	0.1343	0.1343	0.1157

- ***Stage 2** – Deriving the weights allocated to each criterion using T2NN-WENSLO.*

Step 4: Each expert evaluated the sustainable brand equity of each brand based on LVs presented in Table 2. The assessments of experts based on LVs are provided in Table S.2. Subsequently, LVs were transformed into T2NN sets, leading to the derivation of the initial decision matrices $\left(\widetilde{\mathfrak{E}}^{(\widetilde{H}_{\mathfrak{s}})} = \left[\widetilde{\mathfrak{E}}^{(\widetilde{H}_{\mathfrak{s}})}{}_{\mathfrak{k}\mathfrak{d}}\right]_{\mathfrak{K}x\mathfrak{D}}\right)$. These matrices are presented in Table S.3.

Step 5: The computation of the aggregated decision matrix $\left(\widetilde{\mathfrak{E}} = \left[\widetilde{\mathfrak{E}}_{\mathfrak{k}\mathfrak{d}}\right]_{\mathfrak{K}x\mathfrak{D}}\right)$ (Table S.4) was conducted using the T2NNWA operator as described in Eq. (7).

Step 6: The computation of the initial decision matrix for the criteria in crisp form ($\mathfrak{E} = [\mathfrak{E}_{\mathfrak{k}\mathfrak{d}}]_{\mathfrak{K}x\mathfrak{D}}$) (Table S.5) was performed through the utilization of Eq. (8) involving the score functions.

Step 7: The linear normalization of the decision matrix ($\mathfrak{F} = [\mathfrak{F}_{\mathfrak{k}\mathfrak{d}}]_{\mathfrak{K}x\mathfrak{D}}$) was computed employing Eq. (9). The outcome is depicted in Table S.6.

Step 8: The computation of the criterion class interval matrix ($\Delta\mathfrak{F}_{\mathfrak{d}} = [\Delta\mathfrak{F}_{\mathfrak{d}}]_{\mathfrak{D}}$) was conducted through the application of Eq. (10). The results are outlined in Table S.7.

Step 9: The computation of the criterion class slope matrix ($tan\,\theta_{\mathfrak{d}} = [tan\,\theta_{\mathfrak{d}}]_{\mathfrak{D}}$) was executed through the utilization of Eq. (11). The outcomes are detailed in Table S.7.

Step 10: The computation of the criterion envelope matrix ($\Xi_{\mathfrak{d}} = [\Xi_{\mathfrak{d}}]_{\mathfrak{D}}$) was performed through the application of Eq. (12). The results are provided in Table S.7.

Step 11: The computation of the criterion envelope–slope ratio matrix ($\Phi_{\mathfrak{d}} = [\Phi_{\mathfrak{d}}]_{\mathfrak{D}}$) (Table S.7) was conducted through the application of Eq. (13).

Step 12: Utilizing Eq. (14), the computation of the criterion weighting matrix ($\mathbb{w}_{\mathfrak{d}} = [\mathbb{w}_{\mathfrak{d}}]_{\mathfrak{D}}$) was performed. The outcomes are illustrated in Table 6.

Table 6

The matrix representing the weights assigned to the criteria.

Criteria	Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6	Γ_7	Γ_8	Γ_9	Γ_{10}	Γ_{11}	Γ_{12}
$\mathbb{w}_{\mathfrak{d}}$	0.0737	0.0893	0.0920	0.1015	0.0896	0.0980	0.0887	0.0817	0.0947	0.0947	0.0428	0.0535

- ***Stage 3*** *– Establishing the rankings of the alternatives through T2NN-ARLON.*

Step 13*:* Using Eq. (15), the initial matrix for the ARLON method ($\mathcal{E} = [\mathcal{E}_{\mathfrak{k}\mathfrak{d}}]_{\mathfrak{K}x\mathfrak{D}}$) was calculated. The outcomes are detailed in Table S.8.

Step 14*:* Using Eq. (16) and Eq. (17), the first logarithmic normalization matrix ($\mathfrak{N}^{1st} = [\mathfrak{N}^{1st}{}_{\mathfrak{k}\mathfrak{d}}]_{\mathfrak{K}x\mathfrak{D}}$) (Table S.9) and second logarithmic normalization matrix ($\mathfrak{N}^{2nd} = [\mathfrak{N}^{2nd}{}_{\mathfrak{k}\mathfrak{d}}]_{\mathfrak{K}x\mathfrak{D}}$) (Table S.10) were calculated, respectively.

Step 15*:* Using Heron Mean (Eq. (18)), the aggregated normalized decision matrix ($\mathfrak{N}^{norm} = [\mathfrak{N}^{norm}_{\mathfrak{k}\mathfrak{d}}]_{\mathfrak{K}x\mathfrak{D}}$) was calculated ($\delta = 0.5$). The outcomes are detailed in Table S.11.

Step 16*:* Utilizing Eq. (19), the weighted aggregated normalization matrix ($\mathfrak{P} = [\mathfrak{P}_{\mathfrak{k}\mathfrak{d}}]_{\mathfrak{K}x\mathfrak{D}}$) was calculated. The results are provided in Table S.12.

Step 17*:* Using Eq. (20) and Eq. (21), the cost-weighted aggregated normalized matrix $\left(\mathfrak{P}^- = \left[\mathfrak{P}^-{}_{\mathfrak{k}}\right]_{\mathfrak{K}}\right)$ and benefit-weighted aggregated normalized matrix $\left(\mathfrak{P}^+ = \left[\mathfrak{P}^+{}_{\mathfrak{k}}\right]_{\mathfrak{K}}\right)$ were computed, respectively. The results are provided in Table S.13.

Step 18*:* Utilizing Eq. (22), the conclusive ranking scores of the alternative matrix ($P = [P_{\mathfrak{k}}]_{\mathfrak{K}}$) was computed ($\phi = 0.833$). Since there were 10 benefit criteria and a total of 12 criteria, the ϕ parameter was considered as 10 divided by 12, resulting in 0.833. The results are provided in Table 7.

Table 7

The matrix representing the conclusive ranking scores assigned to the alternatives.

Alternatives	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}
$P_{\mathfrak{k}}$	0.7203	0.7049	0.7161	0.7282	0.6938	0.7239	0.7026	0.7154	0.7258	0.7286

5. Results and Implications

In the conducted case study evaluating the sustainable brand equity of 10 cosmetic brands in the Turkish cosmetic sector, the T2NN-WENSLO-ARLON hybrid method was applied. Each step shown in the algorithm was implemented individually and sequentially. The findings obtained because of the application, namely the algorithm outputs, are grouped under two main headings. The first finding is the importance weights of the criteria included in the decision model. The second finding is the ranking of the brands' sustainable brand equities.

The findings obtained as a result of the T2NN-WENSLO-ARLON application are:

(i) The ranking of the importance levels of the criteria included in the decision model is as follows: "green product leadership" (Γ_4) > "green innovation" (Γ_6) > "green brand loyalty" (Γ_9) = "green brand name" (Γ_{10}) > "green satisfaction" (Γ_3) > "green perceived quality" (Γ_5) > "green trust" (Γ_2) > "environmental benefit" (Γ_7) > "CSR initiatives" (Γ_8) > "green brand image" (Γ_1) > "green perceived risk" (Γ_{12}) > "green wash" (Γ_{11})" According to this ranking, the most important criterion in the evaluation of sustainable brand equity is "green product leadership" (Γ_4). In this case, the most important factor that brands need to focus on for sustainable brand equity is to develop environmentally friendly and green products and dominate the market with these products to be in leadership positions. The second-ranked criterion is "green innovation" (Γ_6). At this point, companies operating in the cosmetic sector should focus on green innovation in product development processes for a sustainable environment. Especially in product design, production processes, and green innovative product manufacturing, a sustainability approach should be adopted, and research and development activities should be carried out accordingly. The striking finding is that the cost-focused criteria, namely "green wash" (Γ_{11}) and "green perceived risk" (Γ_{12}), have the lowest importance levels. In this case, it can be inferred that brands should pay less attention to the negative outcomes created by these criteria compared to other benefit criteria.

(ii) The ranking of the 10 cosmetic companies in Turkey in terms of sustainable brand equity is as follows: "Misbahçe" (B_{10}) > "Eyüp Sabri Tuncer" (B_4) > "Agarta" (B_9) > "The Purest Solutions" (B_6) > "Golden Rose" (B_1) > "Farmasi" (B_3) > "Ashley Joy" (B_8) > "Pastel" (B_2) > "Rosense" (B_7) > "Cecile" (B_5). Considering the expert evaluations and criterion importance levels, the rankings of brands' sustainable value are as stated.

In the context of the two primary findings presented above, green product leadership and green innovation parameters concerning production processes in the cosmetics sector emerge as the two most significant factors in shaping and enhancing companies' sustainable brand equity. Particularly, it is evident that cosmetic companies need to focus on environmentally friendly production processes to introduce new and reliable products in the cosmetics industry. Subsequently, the product must possess sustainable credibility and loyalty. This dimension alludes to the social aspect of sustainable branding. Customers' perceptions of cosmetic products indicate whether the cosmetic company is sustainable or not. On the other hand, perceived risks and greenwashing, which negatively affect sustainable brand equity, do not have the expected impact level on the formation of sustainable brand equity. In this regard, the full perception of risks and greenwashing is considered inadequate. Based on these evaluations, among the cosmetic brands included in the decision model, the alternative "Misbahçe" emerges as the brand with the highest sustainable brand equity. In this context, according to expert opinions, "Misbahçe" is discussed as having the most successful sustainable brand equity in the market among the cosmetic industry brands included in the decision model. This situation can be attributed to Misbahçe successfully conducting the process of creating sustainable brand equity. It is evident that other brands need to develop various strategies to create sustainable brand equity.

5.1. Sensitivity Analysis

Four sensitivity analysis scenarios (SAS) were developed to support the robustness of the findings obtained for evaluating the sustainable brand equity of cosmetic companies through T2NN-WENSLO-ARLON. The main objective of these scenarios was to compare the scenario results approached with different methods with the results obtained in the research as well as interpret the consistency levels. The identified SAS scenarios are as follows:

(i) *SAS-1* – If the importance weights of the experts are considered equal, how would the rankings of brands' sustainable brand equities change?

(ii) *SAS-2* – If each criterion is removed from the decision model and the algorithm is rerun, what would the results be?

(iii) *SAS-3* – If each brand is removed from the decision model one by one, would there be any changes in the rankings of brands' sustainable brand equities?

(iv) *SAS-4* – When there is a change in the importance levels of logarithmic normalizations (i.e. δ parameter values) due to the union of two logarithmic normalizations with Heron mean, how would the rankings of brands' sustainable brand equities change?

For SAS-1, the importance levels of the experts were assumed to be equal. No change in brand rankings was observed.

For SAS-2, each criterion was successively removed from the decision model and the algorithm was rerun. Table 8 shows the weights of the criteria. No significant change in criterion weights was observed. The fourth criterion was seen as the best criterion in all sub-scenarios. The sixth criterion became the best criterion only in the sub-scenario where the fourth criterion was excluded.

Table 8

The prioritization of criteria (SAS-2).

SAS-1	***Ranks***	***Best criterion***
SAS-2a	$\Gamma_4 > \Gamma_6 > \Gamma_9 = \Gamma_{10} > \Gamma_3 > \Gamma_5 > \Gamma_2 > \Gamma_7 > \Gamma_8 > \Gamma_{12} > \Gamma_{11}$	*Green product leadership* (Γ_4)
SAS-2b	$\Gamma_4 > \Gamma_6 > \Gamma_9 = \Gamma_{10} > \Gamma_3 > \Gamma_5 > \Gamma_7 > \Gamma_8 > \Gamma_1 > \Gamma_{12} > \Gamma_{11}$	*Green product leadership* (Γ_4)
SAS-2c	$\Gamma_4 > \Gamma_6 > \Gamma_9 = \Gamma_{10} > \Gamma_5 > \Gamma_2 > \Gamma_7 > \Gamma_8 > \Gamma_1 > \Gamma_{12} > \Gamma_{11}$	*Green product leadership* (Γ_4)
SAS-2d	$\Gamma_6 > \Gamma_9 = \Gamma_{10} > \Gamma_3 > \Gamma_5 > \Gamma_2 > \Gamma_7 > \Gamma_8 > \Gamma_1 > \Gamma_{12} > \Gamma_{11}$	*Green innovation* (Γ_6)
SAS-2e	$\Gamma_4 > \Gamma_6 > \Gamma_9 = \Gamma_{10} > \Gamma_3 > \Gamma_2 > \Gamma_7 > \Gamma_8 > \Gamma_1 > \Gamma_{12} > \Gamma_{11}$	*Green product leadership* (Γ_4)
SAS-2f	$\Gamma_4 > \Gamma_9 = \Gamma_{10} > \Gamma_3 > \Gamma_5 > \Gamma_2 > \Gamma_7 > \Gamma_8 > \Gamma_1 > \Gamma_{12} > \Gamma_{11}$	*Green product leadership* (Γ_4)
SAS-2g	$\Gamma_4 > \Gamma_6 > \Gamma_9 = \Gamma_{10} > \Gamma_3 > \Gamma_5 > \Gamma_2 > \Gamma_8 > \Gamma_1 > \Gamma_{12} > \Gamma_{11}$	*Green product leadership* (Γ_4)
SAS-2h	$\Gamma_4 > \Gamma_6 > \Gamma_9 = \Gamma_{10} > \Gamma_3 > \Gamma_5 > \Gamma_2 > \Gamma_7 > \Gamma_1 > \Gamma_{12} > \Gamma_{11}$	*Green product leadership* (Γ_4)
SAS-2i	$\Gamma_4 > \Gamma_6 > \Gamma_{10} > \Gamma_3 > \Gamma_5 > \Gamma_2 > \Gamma_7 > \Gamma_8 > \Gamma_1 > \Gamma_{12} > \Gamma_{11}$	*Green product leadership* (Γ_4)
SAS-2j	$\Gamma_4 > \Gamma_6 > \Gamma_9 > \Gamma_3 > \Gamma_5 > \Gamma_2 > \Gamma_7 > \Gamma_8 > \Gamma_1 > \Gamma_{12} > \Gamma_{11}$	*Green product leadership* (Γ_4)
SAS-2k	$\Gamma_4 > \Gamma_6 > \Gamma_9 = \Gamma_{10} > \Gamma_3 > \Gamma_5 > \Gamma_2 > \Gamma_7 > \Gamma_8 > \Gamma_1 > \Gamma_{12}$	*Green product leadership* (Γ_4)
SAS-2l	$\Gamma_4 > \Gamma_6 > \Gamma_9 = \Gamma_{10} > \Gamma_3 > \Gamma_5 > \Gamma_2 > \Gamma_7 > \Gamma_8 > \Gamma_1 > \Gamma_{11}$	*Green product leadership* (Γ_4)

According to the sub-scenarios from SAS-2, the rankings of brands' sustainable brand equities are shown in Fig. 3. No change in brand rankings was observed. The fifth brand (i.e. "Misbahçe") had the best sustainable brand equity.

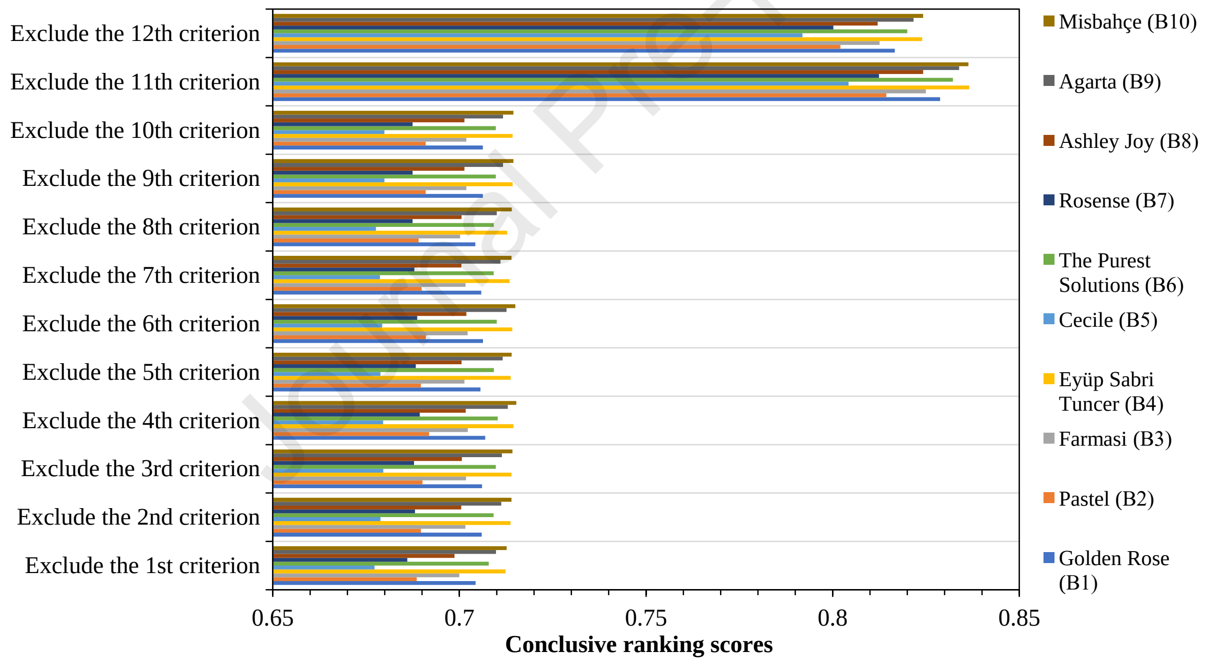

Fig. 3. SAS-2 findings for sustainable brand equity's ranking

For SAS-3, each brand was successively removed from the decision model and the algorithm was reapplied. The rankings of brands are shown in Fig. 4. No change in brand rankings was observed despite the removal of each brand.

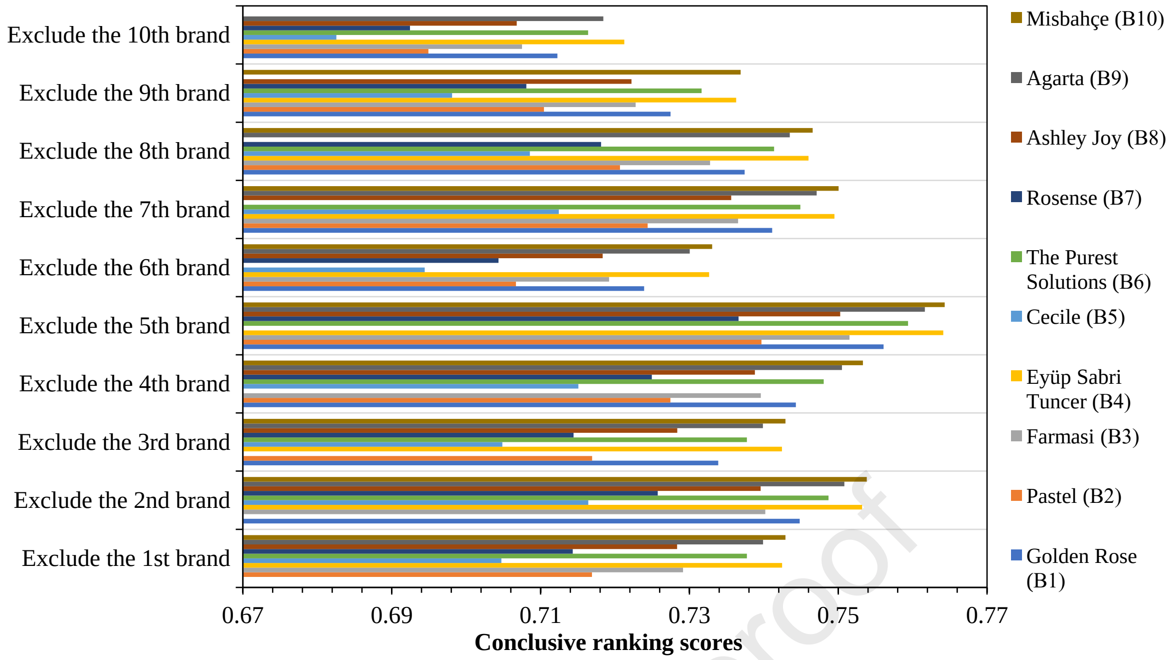

Fig. 4. SAS-3 findings for sustainable brand equity's ranking.

For SAS-4, δ parameter values were set between 0.1 and 0.9. As shown in the radar chart in Fig. 5 regarding the rankings of brands, there was no change in brand rankings.

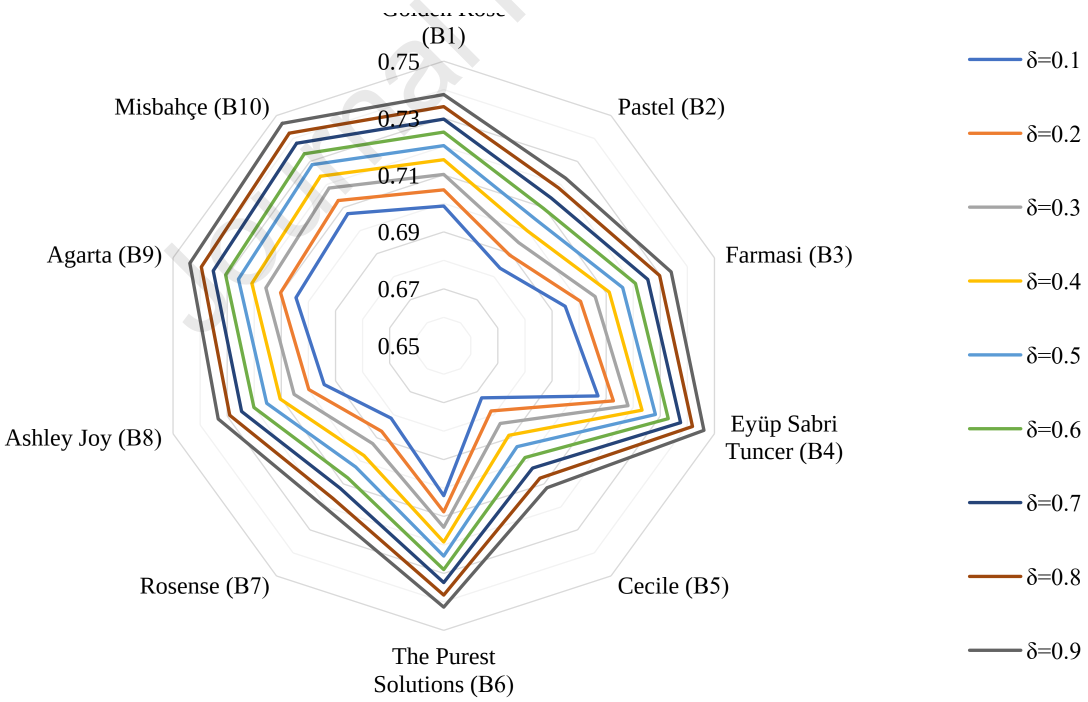

Fig. 5. SAS-4 findings for sustainable brand equity's ranking.

Based on the results of all these SAS scenarios, it is concluded that the T2NN-WENSLO-ARLON hybrid method and algorithm support the research findings and are robust. This hybrid method can be reliably used as an effective DSS for determining the sustainable brand equities of cosmetic companies.

5.2. Comparative Analysis

In the evaluation of sustainable brand equity, the T2NN-WENSLO-ARLON hybrid model has been chosen as DSS. Particularly, the ARLON method has been preferred for alternative ranking. This method is primarily based on two-step logarithmic normalization. The hybrid method has been applied in the case study. The robustness of the obtained results has been demonstrated through SAS analyses. To test the consistency of the obtained ranking, comparison analysis scenarios (CAS) were developed. In this scenario, different methods, including alternative ranking technique based on adaptive standardized intervals (ARTASI), multi-attributive border approximation area comparison (MABAC), measurement of alternatives and ranking according to compromise solution (MARCOS), simple additive weighting (SAW), and weighted aggregated sum product assessment (WASPAS) alternative ranking methods, were applied to the same case study. The obtained brand rankings are presented in Table 9. It is found that the alternative rankings are the same. Thus, the best alternative is identified as "Misbahçe". The consistency of the alternative rankings indicates that the T2NN-WENSLO-ARLON hybrid model provides robust and consistent results.

When comparing the ARLON method with other methods, the following fundamental differences are observed:

(i) In the ARTASI method [93], alternative ranking is performed considering the utility degrees based on ideal and anti-ideal values of alternatives. However, in ARLON, utility degrees are not considered. Instead, the total scores of alternatives are calculated based on proportional values of benefit and cost according to the ratio of the number of benefit criteria to the total number of criteria.

(ii) The MABAC method [94] primarily adopts the border approximation approach and determines alternative ranking based on this border. In contrast, the ARLON method relies on the proportional sum of the two-step logarithmic normalization values of cost and benefit criteria.

(iii) The MARCOS method [95] is based on comparative scoring for alternative ranking. Thus, the comparative performance of each alternative is determined. However, ARLON does not involve comparative scoring of alternatives. Instead, it determines the total score of each alternative based on the two-step logarithmic normalization, considering the benefit and cost criteria.

(iv) The SAW method determines the ranking of alternatives based on their weighted sums. In ARLON, these operations are conducted based on logarithmic normalization.

(v) The WASPAS method presents alternative ranking based on their weighted products. In contrast, ARLON is based on the two-step logarithmic normalization of the weighted decision matrix.

Table 9

Comparative analysis results.

CAS	***Ranks***	***Best sustainable brand***
ARLON	$B_{10} > B_4 > B_9 > B_6 > B_1 > B_3 > B_8 > B_2 > B_7 > B_5$	*Misbahçe* (B_{10})
ARTASI	$B_{10} > B_4 > B_9 > B_6 > B_1 > B_3 > B_8 > B_2 > B_7 > B_5$	*Misbahçe* (B_{10})
MABAC	$B_{10} > B_4 > B_9 > B_6 > B_1 > B_3 > B_8 > B_2 > B_7 > B_5$	*Misbahçe* (B_{10})
MARCOS	$B_{10} > B_4 > B_9 > B_6 > B_1 > B_3 > B_8 > B_2 > B_7 > B_5$	*Misbahçe* (B_{10})
SAW	$B_{10} > B_4 > B_9 > B_6 > B_1 > B_3 > B_8 > B_2 > B_7 > B_5$	*Misbahçe* (B_{10})
WASPAS	$B_{10} > B_4 > B_9 > B_6 > B_1 > B_3 > B_8 > B_2 > B_7 > B_5$	*Misbahçe* (B_{10})

5.3. Research Implications

This study holds several academic implications for the field of sustainable brand equity assessment and DSSs, contributing to both theory and methodology. The following implications highlight the significance of this research:

(i) *Advancement in sustainable brand equity assessment* – The research introduces a novel approach, the T2NN-WENSLO-ARLON hybrid method, for assessing sustainable brand equity.

(ii) *Integration of MAGDM and linguistic scales* – The incorporation of MAGDM with linguistic scales and T2NN sets provides a robust foundation for determining sustainable brand equity. This integration enhances the comprehensiveness of the assessment model, addressing the limitations of previous research.

(iii) *Development of the T2NN-WENSLO method* – The study extends the WENSLO criterion weighting method by integrating T2NN sets, resulting in the development of the T2NN-WENSLO method. This enhancement offers a more nuanced approach to determining the importance levels and weights of criteria, contributing to the methodological advancement in DSSs.

(iv) *Application in the cosmetics sector* – The case study application of the T2NN-WENSLO-ARLON hybrid method within the cosmetics sector provides a real-world validation of its effectiveness. This application enhances the practical relevance of the research findings and demonstrates the adaptability of the proposed model across industry contexts.

(v) *Sensitivity analysis and robustness testing* – The inclusion of sensitivity analysis scenarios and robustness testing strengthens the credibility and reliability of the proposed hybrid method. This contributes to the methodological rigor of DSSs in the assessment of sustainable brand equity.

5.4. Managerial Implications

The findings of this research offer valuable insights and practical implications for managers and decision-makers in the realm of sustainable brand equity assessment. The following managerial implications elucidate the significance of the study for practitioners:

(i) *Strategic decision-making for brand management* – The developed T2NN-WENSLO-ARLON hybrid method provides a structured framework for managers to strategically assess and manage sustainable brand equity. This model aids decision-makers in understanding the key factors influencing brand perception, enabling them to make informed and strategic decisions.

(ii) *Customized DSS for companies* – The three-stage hybrid approach, integrating T2NN sets, WENSLO, and ARLON methods, offers a customizable DSS. Companies can tailor the model to their specific industry, allowing for a nuanced evaluation of sustainable brand equities relevant to their unique context.

(iii) *Upholding environmental and non-financial responsibility* – T2NN-WENSLO-ARLON not only considers non-financial aspects but also incorporates environmental dimensions in brand equity assessment. Managers can use this approach to reinforce their commitment to sustainable practices, aligning brand equities with environmental responsibility.

(iv) *Enhanced stakeholder communication* – The developed model facilitates a comprehensive evaluation that goes beyond traditional brand assessments. Managers can use the insights gained to enhance communication with stakeholders, showcasing the company's commitment to sustainability and transparency in brand management.

(v) *Identification of key performance indicators* – T2NN-WENSLO-ARLON assists managers in identifying and prioritizing key performance indicators related to sustainable brand equity. This enables companies to focus on specific areas that significantly impact brand perception and value.

(vi) *Benchmarking and competitive positioning* – Through the case study application in the cosmetics sector, companies can benchmark their sustainable brand equity against industry peers. This competitive positioning allows managers to identify strengths, weaknesses, and areas for improvement, fostering continuous enhancement of brand equities.

(vii) *Risk mitigation and adaptability* – The sensitivity analysis and robustness testing incorporated into the model contribute to risk mitigation. Managers can assess the resilience of their brand equities under different scenarios, allowing for adaptability and proactive measures to mitigate potential risks.

(viii) *Integration into CSR practices* – Companies with a focus on CSR can integrate T2NN-WENSLO-ARLON into their CSR practices. This ensures that sustainability is not only a part of operational activities but also a core component of brand identity and perception.

6. Conclusion

This study significantly contributed to both academic understanding and practical application in the assessment of sustainable brand equity and DSSs. By introducing the T2NN-WENSLO-ARLON hybrid method, this research advanced the theoretical framework for evaluating sustainable brand equity, providing a comprehensive model that incorporated both non-financial and environmental dimensions. The integration of MAGDM with linguistic scales and T2NN sets bolstered the methodological rigor of the assessment model, offering a robust foundation for decision-making in brand management.

The findings of this research regarding sustainable brand equity assessment are noteworthy. Among these, the most significant result is the applicability and robustness of the proposed DSS, with the decision model identifying "Misbahçe" as the top alternative among others. This indicates "Misbahçe" as the brand with the highest sustainable brand equity. Another noteworthy finding is that "green product leadership" emerges as the most critical criterion for creating sustainable brand equity in the cosmetic sector. This underscores the necessity for cosmetic firms to prioritize environmentally friendly and recyclable products in their product development processes.

Furthermore, all sensitivity and comparison analysis scenarios developed in the research supported the robustness of the novel decision model. Additionally, the development of a DSS based on linguistic assessments in the sustainable brand equity assessment process facilitates the measurability of sustainable brand equity. The enhancement and hybrid application of WENSLO and ARLON methods with neutrosophic sets indicate the advanced level of methodology in this research. Particularly, the usage of these methods based on group decision-making also opens avenues for their application alongside the MAGDM approach.

The managerial implications derived from the research findings emphasize the practical usefulness of the developed DSS for practitioners across various industries. The T2NN-WENSLO-ARLON hybrid method provides a structured framework for strategic decision-making in brand management, enabling companies to align their brand equities with environmental responsibility while making informed decisions. Moreover, the customizable nature of the model allows companies to adapt the assessment to their specific industry context.

Additionally, the incorporation of sensitivity analysis and robustness testing facilitates risk mitigation and adaptability, empowering companies to proactively address potential challenges and uncertainties. Embracing the T2NN-WENSLO-ARLON hybrid method not only enhances companies' brand image and competitive positioning but also reinforces their commitment to sustainability and CSR.

6.1. Limitations of the Study

Although this research contributed significantly to the field of sustainable brand equity assessment and DSSs, it is essential to acknowledge several limitations that might impact the generalizability and interpretation of the findings:

- *A limited sample size of expert opinions* – The reliance on expert opinions for developing DSS introduced a potential constraint due to the limited number of experts involved. The findings might lack diversity and might not fully represent the spectrum of perspectives within the industry.
- *Focus on the cosmetics sector in the case study* – The application of the developed hybrid method in the cosmetics sector for the case study provided valuable insights. However, the exclusive focus on this industry might limit the transferability of findings to other sectors, where factors influencing sustainable brand equity could differ.
- *Subjectivity in linguistic scale-based data* – The utilization of linguistic scales to gather expert evaluations might introduce subjectivity into the data collection process. Despite efforts to mitigate uncertainty using T2NN sets, inherent biases in expert judgments might influence the robustness of the assessment model.
- *Specificity of methodological approach* – While the T2NN-WENSLO-ARLON hybrid method offered a comprehensive framework for sustainable brand equity assessment, its applicability might be constrained by the specificity of the methodology. Alternative approaches or combinations of methods could yield different results, warranting further exploration.

6.2. Recommendations for Future Research

Acknowledging the limitations is crucial for contextualizing the research findings and understanding their implications. Future studies could address the outlined limitations by incorporating larger and more diverse expert panels, extending case study applications to multiple industries, refining methodologies to minimize subjectivity, and exploring alternative methodological approaches. Here are some suggestions for future research directions:

- *Validation across diverse industries* – While the developed T2NN-WENSLO-ARLON hybrid method shows promise in assessing sustainable brand equity within the cosmetics sector, future research could validate its effectiveness across a broader range of industries. Comparing results across diverse sectors would enhance the generalizability and applicability of DSS.
- *Cross-cultural analysis* – Investigating how cultural differences influence perceptions of sustainable brand equity could be an interesting avenue for future research. Comparative studies across different cultural contexts would shed light on the cultural factors shaping consumer attitudes and preferences toward sustainable brands.
- *Exploration of stakeholder engagement strategies* – Investigating effective strategies for engaging stakeholders in the assessment and management of sustainable brand equity could be beneficial. Research could explore stakeholder perceptions, preferences, and expectations to inform more inclusive and participatory decision-making processes.
- *Impact of external factors* – Examining the impact of external factors such as regulatory changes, technological advancements, and market dynamics on sustainable brand equity would be valuable. Understanding how external forces shape brand perceptions and influence consumer behavior could help companies develop more adaptive and resilient brand strategies.

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Highlights

> The sustainable brand equity assessment problem is defined from the MCGDM perspective.

> The neutrosophic DSS is proposed for brand equity assessment across industry contexts.

> T2NN-based WENSLO-ARLON is a robust DSS for benchmarking and competitive positioning.

> Effectiveness is validated by a case study application within the cosmetics sector.

> Implications for stakeholders in the realm of brand equity assessment are given.

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Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐ The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: