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Abstract

Soft set(SS) is one of the soft computing techniques that plays an important role in addressing the hiddenness and uncertainty associated with uncertain data. In other hand the idea of interval-valued neutrosophic soft sets (IVNSSs) is a new generalization of the neutrosophic soft sets to the neutrosophic sets when the authors combine the critical features of IVNS and soft sets (SSs) in one model. Accordingly, this model worked to provide decision-makers with more flexibility in the process of interpreting uncertain information. From a scientific point of view, the process of evaluating this high-performance IVNSS disappears. Therefore, in this paper, we initiated a new approach known as possibility interval-valued neutrosophic soft sets (PIVNSSs) as a new development in a fuzzy soft computing environment. We investigate some fundamental operations on PIVNSSs along with their basic properties. Also, we investigate AND and OR operations between two PIVNSSs as well as several numerical examples to clarify the above fundamental operations. Finally, we have given distance measures (DM) between two PIVNSSs to construct a new algorithm that is used to demonstrate the effectiveness of the method in handling some real-life applications.

Kewords: Neutrosophic sets; neutrosophic soft sets; interval-valued neutrosophic soft sets; possibility interval-valued neutrosophic soft sets.

1 introduction

Some topics in fields of science and social science like engineering, environmental, economics, and management include data that have data with fuzzy, imprecise, and uncertain properties. Classical approaches do not always solve these problems as the features can be quite complex. Recently, several theories were put forward to deal effectively with uncertainty and vagueness. One of these theories is the fuzzy set(FS) that Zadeh[1] in 1965 developed as a mathematical way of describing and dealing with vagueness in daily life. Fuzzy set theory extended the classical set theory's range values from integer 0 and 1 to interval [0,1] for object membership.

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As the years progressed and the nature of life and the nature of thinking in the human mind became more complex, it can be said that the FS structure is insufficient to deal with this tremendous development involving uncertainty. To cover this weakness, Smarandache[2] introduced the concept of the neutrosophic set (NS) when he extended both notions of FS and intuitionistic FS (IFS).[3] Later, for the purpose of increasing flexibility and applicability to NSs, Wang et al.[4] reformed the NSs structures into interval forms when they proposed the notion of interval-valued NSs (IVNSs). These models have been widely applied to deal with various difficulties in daily life see.[5–10] In 1999, the Russian researcher Molodtsov[11] expressed an important opinion, which is that these concepts lack mathematical tools that give a more comprehensive description of the set of an object, and therefore he based it on a new mathematical tool called the soft set (SS). Relying on NS, INS, and SS, researchers around the world present many important works dealing with various fields such as engineering, medicine, administration, etc see.[12–16] Among these researchers, we mention: Deli[17] developed an approach to MCDM in IVNSSs and then he gave an example to illustrate the feasibility of this model. Şahin and Küçük[18] proposed both similarity and entropy on NSSs, and following them, Al-Quran et al.[19,20] proposed these technical aspects on IVNSSs with complex numbers.Quek et al.[21] defined VIKOR and TOPSIS methods on (t, s)-regulated IVNSSs.Broumi et al.[22] worked on generalized IVNSSs and studied their operations. Al-Sharqi et al.[23,24] extend the ideas into IVNSSs environment under complex numbers and they utilize these ideas to solve DM-problems.[25–39] Chinnadurai and Bobin[40] mixed up both IFS and NSSs under interval forms and tested them to solve DM- problems.

All of the models mentioned above lack evaluation scores for their performance. To overcome this crisis, both Alkhazaleh and Salleh[41] resorted to presenting possibility fuzzy soft sets (PFSSs) by establishing a probability score belonging to the interval $[0, 1]$, and works to evaluate the performance of the previous concept (FSSs). This idea helped to develop and increase the efficiency of the work of decision-makers when choosing the best alternatives and the best qualities. Accordingly, many contributions were made to improve the mechanism of operation of these concepts.Zhang and Shu[42] proposed P-multi-FSSs (PMFSSs) then they applied a similarity measure between two PMFSSs. Al-Quran and Hassan[43] discussed possibility of neutrosophic vague soft sets (PNVSSs). Al-Sharqi et al.[44] proposed PNSESs under expert systems for medical diagnosis under uncertainty and other see.[45–48]In this work, we following this path by studied and proposed new hyper-model called possibility of interval-valued neutrosophic soft sets (PIVNSSs) such that each part of the PIVNSS is provided with a fuzzy degree to measure the effectiveness of work performance.Based on these tools, the mechanism of SM is presented and used in building an algorithm to solve one of the decision-making problems. Finely, this article is divided into six parts. In the first and second parts, we provided a full explanation of the basic roots of this concept. In the third part of this article, we presented the mathematical definition of our proposed concept with some illustrative examples. In the fourth and fifth parts, they presented the mathematical definitions of the basic mathematical operations of this concept with some properties supported by illustrative examples. In the last part, we presented an algorithm based on the clustering factor of our proposed concept.

2 Preliminaries

In this section, we recapitulate some of the ideas like FS, NS, IVNS, SS, and IVNSS that are considered beneficial in developing our new concept.

Definition 2.1. [1] Assume that $\hat{\mathbb{T}} = \{\tau_1, \tau_2, \tau_3, ..., \tau_n\}$ be a reference set. Then the FS formed as following structure:

$$\mathcal{Q} = \left\{ \left(\tau, \left\langle \ddot{\wp}^t_Q(\tau_i) \right\rangle \right) \middle| \tau \in \hat{\mathbb{T}} \right\}$$

where $\ddot{\wp}^t_Q(\tau_i)$ refer to true membership of object τ_i in $\hat{\mathbb{T}}$ and persistent as a mapping:$\ddot{\wp}^t_Q : \hat{\mathbb{T}} \to [0,1]$.

Definition 2.2. [1] Let $\mathcal{Q}_1 = \left\{ \left(\tau, \left\langle \ddot{\wp}^t_{Q_1}(\tau_i) \right\rangle \right) \middle| \tau \in \hat{\mathbb{T}} \right\}$ and $\mathcal{Q}_2 = \left\{ \left(\tau, \left\langle \ddot{\wp}^t_{Q_2}(\tau_i) \right\rangle \right) \middle| \tau \in \hat{\mathbb{T}} \right\}$ be two FS on reference set $\hat{\mathbb{T}}$ Then the fundamental operation on FSs defined as following:

1.Union $Q_3 = \left\{ \left(\tau, \left\langle \max\left(\ddot{\wp}^t_{Q_1}(\tau_i), \ddot{\wp}^t_{Q_2}(\tau_i)\right) \right\rangle \right) \middle| \tau \in \hat{\mathbb{T}} \right\}$.

2.Intersection $Q_3 = \left\{ \left(\tau, \left\langle \min\left(\ddot{\wp}^t_{Q_1}(\tau_i), \ddot{\wp}^t_{Q_2}(\tau_i)\right) \right\rangle \right) \middle| \tau \in \hat{\mathbb{T}} \right\}$.

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3.Complement $Q_1^c = \left\{ \left(\tau, \left\langle \left(1 - \ddot{\wp}^t_{Q_1}(\tau_i)\right)\right\rangle\right) \middle| \tau \in \hat{\mathbb{T}} \right\}$.

4.Subset $\mathcal{Q}_1 \subseteq \mathcal{Q}_2$ if $\ddot{\wp}^t_{Q_1}(\tau_i) \leq \ddot{\wp}^t_{Q_2}(\tau_i)$.

Definition 2.3. [2] Assume that $\hat{\mathbb{T}} = \{\tau_1, \tau_2, \tau_3, ..., \tau_n\}$ be a reference set. Then the NS formed as following structure:

$$\hat{\hat{\mathcal{A}}}_{NS} = \left\{ \left(\tau, \left\langle \ddot{\wp}^t_{\hat{\hat{\mathcal{A}}}}(\tau_i), \ddot{\wp}^i_{\hat{\hat{\mathcal{A}}}}(\tau_i), \ddot{\wp}^f_{\hat{\hat{\mathcal{A}}}}(\tau_i)\right\rangle\right) \middle| \tau \in \hat{\mathbb{T}} \right\}$$

where $\ddot{\wp}^t_{\hat{\hat{\mathcal{A}}}}(\tau_i), \ddot{\wp}^i_{\hat{\hat{\mathcal{A}}}}(\tau_i), \ddot{\wp}^f_{\hat{\hat{\mathcal{A}}}}(\tau_i)$ refer to true membership,indeterminacy membership and falsehood membership of object τ_i in $\hat{\mathbb{T}}$ and persistent as a mapping:$\ddot{\wp}^t_{\hat{\hat{\mathcal{A}}}}(\tau_i), \ddot{\wp}^i_{\hat{\hat{\mathcal{A}}}}(\tau_i), \ddot{\wp}^f_{\hat{\hat{\mathcal{A}}}}(\tau_i) : \hat{\mathbb{T}} \to [0,1]$.

Definition 2.4. [4] Assume that $\hat{\mathbb{T}} = \{\tau_1, \tau_2, \tau_3, ..., \tau_n\}$ be a reference set. Then the IVNS formed as following structure:

$$\hat{\hat{\mathcal{A}}}_{IVNS} = \left\{ \left(\tau, \left\langle \ddot{\wp}^t_{\hat{\hat{\mathcal{A}}}}(\tau_i), \ddot{\wp}^i_{\hat{\hat{\mathcal{A}}}}(\tau_i), \ddot{\wp}^f_{\hat{\hat{\mathcal{A}}}}(\tau_i)\right\rangle\right) \middle| \tau \in \hat{\mathbb{T}} \right\}$$

Where the three IVNS memberships represent as a interval real number as following:
$\ddot{\wp}^t_{\mathcal{P}}(\tau_i) = \left[\ddot{\wp}^{t,l}_{\mathcal{P}}(\tau_i), \ddot{\wp}^{t,u}_{\mathcal{P}}(\tau_i)\right]$, $\ddot{\wp}^i_{\mathcal{P}}(\tau_i) = \left[\ddot{\wp}^{i,l}_{\mathcal{P}}(\tau_i), \ddot{\wp}^{i,u}_{\mathcal{P}}(\tau_i)\right]$ and $\ddot{\wp}^f_{\mathcal{P}}(\tau_i) = \left[\ddot{\wp}^{f,l}_{\mathcal{P}}(\tau_i), \ddot{\wp}^{f,u}_{\mathcal{P}}(\tau_i)\right]$ and its refer to true interval membership,indeterminacy interval membership and falsehood interval membership of object τ_i in $\hat{\mathbb{T}}$ and persistent as a mapping:$\ddot{\wp}^t_{\hat{\hat{\mathcal{A}}}}(\tau_i), \ddot{\wp}^i_{\hat{\hat{\mathcal{A}}}}(\tau_i), \ddot{\wp}^f_{\hat{\hat{\mathcal{A}}}}(\tau_i) : \hat{\mathbb{T}} \to [0,1]$.

Definition 2.5. [4] Let $\hat{\hat{\mathcal{A}}}_{IVNS} = \left\{ \left(\tau, \left\langle \ddot{\wp}^t_{\hat{\hat{\mathcal{A}}}}(\tau_i), \ddot{\wp}^i_{\hat{\hat{\mathcal{A}}}}(\tau_i), \ddot{\wp}^f_{\hat{\hat{\mathcal{A}}}}(\tau_i)\right\rangle\right) \middle| \tau \in \hat{\mathbb{T}} \right\}$ and
$\hat{\hat{\mathcal{B}}}_{IVNS} = \left\{ \left(\tau, \left\langle \ddot{\wp}^t_{\hat{\hat{\mathcal{B}}}}(\tau_i), \ddot{\wp}^i_{\hat{\hat{\mathcal{B}}}}(\tau_i), \ddot{\wp}^f_{\hat{\hat{\mathcal{B}}}}(\tau_i)\right\rangle\right) \middle| \tau \in \hat{\mathbb{T}} \right\}$ be two IVNS on reference set $\hat{\mathbb{T}}$ Then the fundamental operation on FSs defined as following:

1.Union
$\hat{\hat{\mathcal{C}}}_{IVNS} = \left\{ \left(\tau, \left\langle \max\left[\ddot{\wp}^{t,l}_{\hat{\hat{\mathcal{A}}}}(\tau_i), \ddot{\wp}^{t,u}_{\hat{\hat{\mathcal{A}}}}(\tau_i)\right], \min\left[\ddot{\wp}^{i,l}_{\hat{\hat{\mathcal{A}}}}(\tau_i), \ddot{\wp}^{i,u}_{\hat{\hat{\mathcal{A}}}}(\tau_i)\right], \min\left[\ddot{\wp}^{f,l}_{\hat{\hat{\mathcal{A}}}}(\tau_i), \ddot{\wp}^{f,u}_{\hat{\hat{\mathcal{A}}}}(\tau_i)\right]\right\rangle\right) | \tau \in T \right\}$.

2.Intersection
$\hat{\hat{\mathcal{C}}}_{IVNS} = \left\{ \left(\tau, \left\langle \min\left[\ddot{\wp}^{t,l}_{\hat{\hat{\mathcal{A}}}}(\tau_i), \ddot{\wp}^{t,u}_{\hat{\hat{\mathcal{A}}}}(\tau_i)\right], \max\left[\ddot{\wp}^{i,l}_{\hat{\hat{\mathcal{A}}}}(\tau_i), \ddot{\wp}^{i,u}_{\hat{\hat{\mathcal{A}}}}(\tau_i)\right], \max\left[\ddot{\wp}^{f,l}_{\hat{\hat{\mathcal{A}}}}(\tau_i), \ddot{\wp}^{f,u}_{\hat{\hat{\mathcal{A}}}}(\tau_i)\right]\right\rangle\right) | \tau \in T \right\}$.

3.Complement
$\hat{\hat{\mathcal{A}}}^c_{IVNS} = \left\{ \left(\tau, \left\langle \left[\ddot{\wp}^{f,l}_{\hat{\hat{\mathcal{A}}}}(\tau_i), \ddot{\wp}^{f,u}_{\hat{\hat{\mathcal{A}}}}(\tau_i)\right], \left[1 - \ddot{\wp}^{i,u}_{\hat{\hat{\mathcal{A}}}}(\tau_i), 1 - \ddot{\wp}^{i,l}_{\hat{\hat{\mathcal{A}}}}(\tau_i)\right], \left[\ddot{\wp}^{t,l}_{\hat{\hat{\mathcal{A}}}}(\tau_i), \ddot{\wp}^{t,u}_{\hat{\hat{\mathcal{A}}}}(\tau_i)\right]\right\rangle\right) | \tau \in T \right\}$.

4.Subset
$\hat{\hat{\mathcal{A}}}_{IVNS} \subseteq \hat{\hat{\mathcal{B}}}_{IVNS}$ if$\ddot{\wp}^t_{\hat{\hat{\mathcal{A}}}}(\tau_i) \leq \ddot{\wp}^t_{\hat{\hat{\mathcal{B}}}}(\tau_i)$,$\ddot{\wp}^i_{\hat{\hat{\mathcal{A}}}}(\tau_i) \geq \ddot{\wp}^i_{\hat{\hat{\mathcal{B}}}}(\tau_i)$,$\ddot{\wp}^f_{\hat{\hat{\mathcal{A}}}}(\tau_i) \geq \ddot{\wp}^f_{\hat{\hat{\mathcal{B}}}}(\tau_i)$.

Definition 2.6. [1] Let $\hat{\mathbb{T}} = \{\tau_1, \tau_2, \tau_3, ..., \tau_n\}$ and $\hat{\mathbb{E}} = \{z_1, z_2, z_3, ..., z_m\}$ be a reference set and attribute set, respectively. Then a SS over $\hat{\mathbb{T}}$ given as structures as follows:

$$\overset{\leftrightarrow}{S} = \left\{ \left(z, \left\langle \overset{\leftrightarrow}{S}(z_i)\right\rangle\right) \middle| z \in \hat{\mathbb{E}} \right\}$$

where the function $\overset{\leftrightarrow}{S}$ given by following mapping:

$$\overset{\leftrightarrow}{S} = E \to P^{(T)}$$

Here $P^{(T)}$ refer to collection of subsets of reference set $\hat{\mathbb{T}}$.

Definition 2.7. A term $\mathcal{P}^{ivnss}$ is said to be IVNSS on soft reference set $\left(\hat{\mathbb{T}}, \hat{\mathbb{E}}\right)$, where $\mathcal{P}^{ivnss} : \hat{\mathbb{E}} \to IVN(P)^{(T)}$, such that $IVN(P)^{(T)}$ is a collection of all IVNS-subsets over $\hat{\mathbb{T}}$.

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3 Possibility Interval-Valued Neutrosophic Soft Sets (PIVNSSs)

In this section, we introduce the concept of a PIVNSS and define some properties of this model, namely, the null of the PIVNSS, the absolute of the PIVNSS, a subset of the PIVNSS, and the equality of the PIVNSS. Illustrated examples are also given.

Definition 3.1. Let $\hat{\mathbb{T}} = \{\tau_1, \tau_2, \tau_3, ..., \tau_n\}$ and $\hat{\mathbb{E}} = \{z_1, z_2, z_3, ..., z_m\}$ be a reference set and attribute set, respectively. Then a possibility interval valued neutrosophic soft set (PIVNSS) $\mathcal{P}_\Theta$ on $\hat{\mathbb{T}}$ has the structures as follows:

$$\mathcal{P}_\Theta^{ivnss} = \left\{ \left(\tau, \left\langle \ddot{\wp}_\mathcal{P}^t(\tau_i)(z_j), \ddot{\wp}_\mathcal{P}^i(\tau_i)(z_j), \ddot{\wp}_\mathcal{P}^f(\tau_i)(z_j) \right\rangle \Theta(\tau_i)(z_j) \right) \middle| \tau \in \hat{\mathbb{T}}, z \in \hat{\mathbb{E}} \right\}$$

Where the three IVNS memberships represent as a interval real number as following:
$\ddot{\wp}_\mathcal{P}^t(\tau_i)(z_j) = \left[\ddot{\wp}_\mathcal{P}^{t,l}(\tau_i)(z_j), \ddot{\wp}_\mathcal{P}^{t,u}(\tau_i)(z_j)\right]$, $\ddot{\wp}_\mathcal{P}^i(\tau_i)(z_j) = \left[\ddot{\wp}_\mathcal{P}^{i,l}(\tau_i)(z_j), \ddot{\wp}_\mathcal{P}^{i,u}(\tau_i)(z_j)\right]$ and
$\ddot{\wp}_\mathcal{P}^f(\tau_i)(z_j) = \left[\ddot{\wp}_\mathcal{P}^{f,l}(\tau_i)(z_j), \ddot{\wp}_\mathcal{P}^{f,u}(\tau_i)(z_j)\right]$, and $\Theta(\tau_i)(z_j)$ refer to possibility degree of element $\tau_i \in \hat{\mathbb{T}}$ to $\mathcal{P}_\Theta^{ivnss}$ who can denoted by $\mathcal{P}_\Theta = (\Lambda_\mathbb{E}, \Theta)$

Example 3.2. Assume that the married couple, Mr. Xu and Mrs. Xu wants to purchase a house in one of the low-cost residential complexes. In the low-cost residential complexes, there are three houses that represent by reference set $\hat{\mathbb{T}} = \{\tau_1, \tau_2, \tau_3\}$.The two couples in their selection focus on observing the attributes that can be represented by the following attribute set $\hat{\mathbb{E}} = \{z_1, z_2, z_3\}$ such that z_1 =House area,z_2 =House price, and z_3 =The distance of the home from neighboring homes.Now, we use our proposed concept to analyze the married couple's opinions $(IVNS - memberships)$ and the expert's degree$(possibility\ degree)$ of evaluation of this evaluation as a follows:

$$\mathcal{P}_\Theta(z_1) = \left\{ \left(\frac{\tau_1}{\langle[0.2,0.7],[0.3,0.5],[0.1,0.3]\rangle}, 0.3 \right), \left(\frac{\tau_2}{\langle[0.3,0.3],[0.2,0.6],[0.7,0.8]\rangle}, 0.6 \right), \left(\frac{\tau_3}{\langle[0.5,0.8],[0.1,0.1],[0.4,0.6]\rangle}, 0.7 \right) \right\}.$$

$$\mathcal{P}_\Theta(z_2) = \left\{ \left(\frac{\tau_1}{\langle[0.6,0.8],[0.1,0.3],[0.4,0.7]\rangle}, 0.5 \right), \left(\frac{\tau_2}{\langle[0.5,0.8],[0.6,0.8],[0.9,0.9]\rangle}, 0.7 \right), \left(\frac{\tau_3}{\langle[0.7,0.9],[0.3,0.9],[0.2,0.5]\rangle}, 0.4 \right) \right\}.$$

$$\mathcal{P}_\Theta(z_3) = \left\{ \left(\frac{\tau_1}{\langle[0.5,0.5],[0.8,1],[0.3,0.8]\rangle}, 0.2 \right), \left(\frac{\tau_2}{\langle[0.4,0.7],[0.2,0.2],[0.2,0.7]\rangle}, 0.5 \right), \left(\frac{\tau_3}{\langle[0.3,0.8],[0.4,0.7],[0.1,0.4]\rangle}, 0.6 \right) \right\}.$$

Also $\mathcal{P}_\Theta(z_i)$ can represent as a matrix as a following form:
$\mathcal{P}_\Theta =$

$$\begin{pmatrix} (([0.2,0.7],[0.3,0.5],[0.1,0.3])\,0.3) & (([0.3,0.3],[0.2,0.6],[0.7,0.8])\,0.6) & (([0.5,0.8],[0.1,0.1],[0.4,0.6])\,0.7) \\ (([0.6,0.8],[0.1,0.3],[0.4,0.7])\,0.5) & (([0.5,0.8],[0.6,0.8],[0.9,0.9])\,0.7) & (([0.7,0.9],[0.3,0.9],[0.2,0.5])\,0.4) \\ (([0.5,0.5],[0.8,1],[0.3,0.8])\,0.2) & (([0.4,0.7],[0.2,0.2],[0.2,0.7])\,0.5) & (([0.3,0.8],[0.4,0.7],[0.1,0.4])\,0.6) \end{pmatrix}$$

Definition 3.3. (PIVNS-subset): Let $\mathcal{P}_\Theta$ and $\mathcal{G}_\Xi$ be two PIVNS-sets on reference set $\hat{\mathbb{T}}$. Then $\mathcal{P}_\Theta$ is said PIVNS-subset of $\mathcal{G}_\Xi$ and denoted by $\mathcal{P}_\Theta \subseteq \mathcal{G}_\Xi$ if:

1. $\mathcal{P}(z_i)$ is IVNS-subset of $\mathcal{G}(z_i)$, $\forall z_i \in \hat{\mathbb{T}}$.

2. $\Theta(z_i)$ is F-subset of $\Xi(z_i)$, $\forall z_i \in \hat{\mathbb{T}}$.

Example 3.4. Conseder $\mathcal{P}_\Theta(z_i)$ in Example 3.2. above and take

$$\mathcal{G}_\Xi(z_1) = \left\{ \left(\frac{\tau_1}{\langle[0.3,0.9],[0.2,0.4],[0,0.2]\rangle}, 0.4 \right), \left(\frac{\tau_2}{\langle[0.4,0.5],[0.1,0.4],[0.5,0.5]\rangle}, 0.7 \right), \left(\frac{\tau_3}{\langle[0.6,0.8],[0,0],[0.2,0.4]\rangle}, 0.9 \right) \right\}.$$

$$\mathcal{G}_\Xi(z_2) = \left\{ \left(\frac{\tau_1}{\langle[0.8,0.8],[0.1,0.6],[0.5,0.8]\rangle}, 0.6 \right), \left(\frac{\tau_2}{\langle[0.7,0.7],[0.5,0.6],[1,1]\rangle}, 0.9 \right), \left(\frac{\tau_3}{\langle[0.8,0.9],[0.2,0.6],[0.1,0.4]\rangle}, 0.5 \right) \right\}.$$

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$$\mathcal{G}_{\Xi}(z_3) = \left\{ \left(\frac{\tau_1}{\langle [0.6,0.8],[0.9,1],[0.4,0.9] \rangle}, 0.5 \right), \left(\frac{\tau_2}{\langle [0.2,0.5],[0.1,0.2],[0.2,0.4] \rangle}, 0.8 \right), \left(\frac{\tau_3}{\langle [0.4,0.8],[0.5,0.8],[0.2,0.7] \rangle}, 0.9 \right) \right\}.$$

Now, its clear $\mathcal{P}_{\Theta} \subseteq \mathcal{G}_{\Xi}$.

Definition 3.5. (Equality of PIVNSS): Let $\mathcal{P}_{\Theta}$ and $\mathcal{G}_{\Xi}$ be two PIVNS-sets on reference set $\hat{\mathbb{T}}$. Then $\mathcal{P}_{\Theta}$ is called equal of $\mathcal{G}_{\Xi}$ and denoted by $\mathcal{P}_{\Theta} = \mathcal{G}_{\Xi}$ if:

1. $\mathcal{P}(z_i)$ is IVNS-subset of $\mathcal{G}(z_i)$ and $\mathcal{G}(z_i)$ is IVNS-subset of $\mathcal{P}(z_i)$, $\forall z_i \in \hat{\mathbb{T}}$.

2. $\Theta(z_i)$ is F-subset of $\Xi(z_i)$ and $\Xi(z_i)$ is F-subset of $\Theta(z_i)$, $\forall z_i \in \hat{\mathbb{T}}$.

Example 3.6. Conseder $\mathcal{P}_{\Theta}(z_i)$ in Example 3.2. above and take $\mathcal{P}_{\Theta} =$

$$\begin{pmatrix} (([0.2,0.7],[0.3,0.5],[0.1,0.3])\,0.3) & (([0.3,0.3],[0.2,0.6],[0.7,0.8])\,0.6) & (([0.5,0.8],[0.1,0.1],[0.4,0.6])\,0.7) \\ (([0.6,0.8],[0.1,0.3],[0.4,0.7])\,0.5) & (([0.5,0.8],[0.6,0.8],[0.9,0.9])\,0.7) & (([0.7,0.9],[0.3,0.9],[0.2,0.5])\,0.4) \\ (([0.5,0.5],[0.8,1],[0.3,0.8])\,0.2) & (([0.4,0.7],[0.2,0.2],[0.2,0.7])\,0.5) & (([0.3,0.8],[0.4,0.7],[0.1,0.4])\,0.6) \end{pmatrix}$$

and
$\mathcal{G}_{\Xi} =$

$$\begin{pmatrix} (([0.3,0.9],[0.2,0.4],[0,0.2])\,0.4) & (([0.4,0.5],[0.1,0.4],[0.5,0.5])\,0.7) & (([0.6,0.8],[0,0],[0.2,0.4])\,0.9) \\ (([0.8,0.8],[0.1,0.6],[0.5,0.8])\,0.6) & (([0.7,0.7],[0.5,0.6],[1,1])\,0.9) & (([0.8,0.9],[0.2,0.6],[0.1,0.4])\,0.5) \\ (([0.6,0.8],[0.9,1],[0.4,0.9])\,0.5) & (([0.2,0.5],[0.1,0.2],[0.2,0.4])\,0.8) & (([0.4,0.8],[0.5,0.8],[0.2,0.7])\,0.9) \end{pmatrix}$$

and
$\mathcal{H}_{\Upsilon} =$

$$\begin{pmatrix} (([0.2,0.7],[0.3,0.5],[0.1,0.3])\,0.3) & (([0.3,0.3],[0.2,0.6],[0.7,0.8])\,0.6) & (([0.5,0.8],[0.1,0.1],[0.4,0.6])\,0.7) \\ (([0.6,0.8],[0.1,0.3],[0.4,0.7])\,0.5) & (([0.5,0.8],[0.6,0.8],[0.9,0.9])\,0.7) & (([0.7,0.9],[0.3,0.9],[0.2,0.5])\,0.4) \\ (([0.5,0.5],[0.8,1],[0.3,0.8])\,0.2) & (([0.4,0.7],[0.2,0.2],[0.2,0.7])\,0.5) & (([0.3,0.8],[0.4,0.7],[0.1,0.4])\,0.6) \end{pmatrix}$$

Then, its clear $\mathcal{P}_{\Theta} = \mathcal{H}_{\Upsilon}$ and $\mathcal{P}_{\Theta} \neq \mathcal{G}_{\Xi}$.

Definition 3.7. (Possibility null IVNS-set): Let $\mathcal{P}_{\Theta}$ be PIVNS-set on reference set $\hat{\mathbb{T}}$. Then we say that $\mathcal{P}_{\Theta}$ is "possibility null IVNS-set" and denoted by $\widehat{\Phi}_{(0)}$ if $\mathcal{P}(z_i) = ([0,0],[1,1],[1,1])$ and $\Theta(z_i) = 0, \forall z_i \in \hat{\mathbb{T}}$.

Example 3.8. Taking into account the matrix notation of $\mathcal{P}_{\Theta}$ as an Example 3.2, it can be observed that we possess.

$$\widehat{\Phi}_{(0)} = \begin{pmatrix} (([0,0],[1,1],[1,1])\,0) & (([0,0],[1,1],[1,1])\,0) & (([0,0],[1,1],[1,1])\,0) \\ (([0,0],[1,1],[1,1])\,0) & (([0,0],[1,1],[1,1])\,0) & (([0,0],[1,1],[1,1])\,0) \\ (([0,0],[1,1],[1,1])\,0) & (([0,0],[1,1],[1,1])\,0) & (([0,0],[1,1],[1,1])\,0) \end{pmatrix}$$

Definition 3.9. (Possibility absolute IVNS-set): Let $\mathcal{P}_{\Theta}$ be PIVNS-set on reference set $\hat{\mathbb{T}}$. Then we say that $\mathcal{P}_{\Theta}$ is "possibility absolute IVNS-set" and denoted by $\widehat{\Omega}_{(1)}$ if $\mathcal{P}(z_i) = ([1,1],[0,0],[0,0])$ and $\Theta(z_i) = 1, \forall z_i \in \hat{\mathbb{T}}$.

Example 3.10. Taking into account the matrix notation of $\mathcal{P}_{\Theta}$ as an Example 3.2, it can be observed that we possess.

$$\widehat{\Omega}_{(1)} = \begin{pmatrix} (([1,1],[0,0],[0,0])\,1) & (([1,1],[0,0],[0,0])\,1) & (([1,1],[0,0],[0,0])\,1) \\ (([1,1],[0,0],[0,0])\,1) & (([1,1],[0,0],[0,0])\,1) & (([1,1],[0,0],[0,0])\,1) \\ (([1,1],[0,0],[0,0])\,1) & (([1,1],[0,0],[0,0])\,1) & (([1,1],[0,0],[0,0])\,1) \end{pmatrix}$$

Definition 3.11. (Complement operation of PIVNS-set): Let $\mathcal{P}_{\Theta}$ be PIVNS-set on reference set $\hat{\mathbb{T}}$ and defied as follows

$$\mathcal{P}_{\Theta} = \left\{ z_j \left(\left[\ddot{\wp}_P^{t,l}(\tau_i)(z_j), \ddot{\wp}_P^{t,u}(\tau_i)(z_j) \right], \left[\ddot{\wp}_P^{i,l}(\tau_i)(z_j), \ddot{\wp}_P^{i,u}(\tau_i)(z_j) \right], \left[\ddot{\wp}_P^{f,l}(\tau_i)(z_j), \ddot{\wp}_P^{f,u}(\tau_i)(z_j) \right] \Theta(\tau_i)(z_j) \right) \middle| \tau_i \in \hat{\mathbb{T}} \right\}$$

Then,complement operation of PIVNS-set defined as follows

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$\mathcal{P}^c_\Theta = \Big\{ z_j \Big(\Big[\ddot{\wp}^{f,l}_{\mathcal{P}}(\tau_i)(z_j), \ddot{\wp}^{f,u}_{\mathcal{P}}(\tau_i)(z_j) \Big], \Big[1 - \ddot{\wp}^{i,u}_{\mathcal{P}}(\tau_i)(z_j), 1 - \ddot{\wp}^{i,l}_{\mathcal{P}}(\tau_i)(z_j) \Big],$
$\Big[\ddot{\wp}^{t,l}_{\mathcal{P}}(\tau_i)(z_j), \ddot{\wp}^{t,u}_{\mathcal{P}}(\tau_i)(z_j) \Big] (1 - \Theta(\tau_i)(z_j)) \Big) \Big| \tau_i \in \hat{\mathbb{T}}, z_j \in \hat{\mathbb{E}} \Big\}$

Here: we follow the complement role of both IVNS and FS complement.

Example 3.12. Taking into account the matrix notation of $\mathcal{P}_\Theta$ as an Example 3.2, it can be observed that we possess.

$\mathcal{P}^c_\Theta =$

$$\begin{pmatrix} (([0.1,0.3],[0.5,0.7],[0.2,0.7])\,0.7) & (([0.7,0.8],[0.4,0.8],[0.3,0.3])\,0.4) \\ (([0.4,0.7],[0.7,0.9],[0.6,0.8])\,0.5) & (([0.9,0.9],[0.2,0.4],[0.5,0.8])\,0.7) \\ (([0.3,0.8],[0,0.2],[0.5,0.5])\,0.8) & (([0.2,0.7],[0.2,0.2],[0.4,0.7])\,0.5) \end{pmatrix}$$

4 The set-theoretic operations pertaining to PIVNS-sets.

Now, in this section, we introduce the set-theoretic operations pertaining to PIVNS-sets as well as some properties and numerical examples that illustrate how these tools work in algebraic environments.

Definition 4.1. (Union of PIVNS-sets) Let
$\mathcal{P}^{ivnss}_\Theta = \Big\{ \Big(\tau, \Big\langle \ddot{\wp}^t_{\mathcal{P}}(\tau_i)(z_j), \ddot{\wp}^i_{\mathcal{P}}(\tau_i)(z_j), \ddot{\wp}^f_{\mathcal{P}}(\tau_i)(z_j) \Big\rangle \Theta(\tau_i)(z_j) \Big) \Big| \tau \in \hat{\mathbb{T}}, z \in \hat{\mathbb{E}} \Big\}$ and
$\mathcal{G}^{ivnss}_\Xi = \Big\{ \Big(\tau, \Big\langle \ddot{\wp}^t_{\mathcal{G}}(\tau_i)(z_j), \ddot{\wp}^i_{\mathcal{G}}(\tau_i)(z_j), \ddot{\wp}^f_{\mathcal{G}}(\tau_i)(z_j) \Big\rangle \Xi(\tau_i)(z_j) \Big) \Big| \tau \in \hat{\mathbb{T}}, z \in \hat{\mathbb{E}} \Big\}$ be two PIVNS-sets on reference set $\hat{\mathbb{T}}$. Then,the union of PIVNS-sets denoted by $\mathcal{P}^{ivnss}_\Theta \hat{\cup} \mathcal{G}^{ivnss}_\Xi$ and defiend as following:

1. $\mathcal{D}(z_i) = \mathcal{P}(z_i)\hat{\cup}\mathcal{G}(z_i)$, where $\hat{\cup}$ denotes IVNS-union.

2. $\Psi(z_i) = \max\big(\Theta(z_i)\hat{\cup}\Xi(z_i)\big)$ following FS-union.

Example 4.2. Taking into account the PIVNSS $\mathcal{P}_\Theta$ as an Example 3.2, and $\mathcal{G}_\Xi$ given in an Example 3.4, then,the union of PIVNS-sets can be possess as following :

$$\mathcal{D}_\Psi(z_1) = \Big\{ \Big(\tfrac{\tau_1}{\langle[0.3,0.9],[0.2,0.4],[0,0.2]\rangle}, 0.4 \Big), \Big(\tfrac{\tau_2}{\langle[0.4,0.5],[0.1,0.4],[0.5,0.5]\rangle}, 0.7 \Big), \Big(\tfrac{\tau_3}{\langle[0.6,0.8],[0,0],[0.2,0.4]\rangle}, 0.9 \Big) \Big\}.$$

$$\mathcal{D}_\Psi(z_2) = \Big\{ \Big(\tfrac{\tau_1}{\langle[0.8,0.8],[0.1,0.3],[0.4,0.7]\rangle}, 0.6 \Big), \Big(\tfrac{\tau_2}{\langle[0.7,0.7],[0.5,0.6],[0.9,0.9]\rangle}, 0.9 \Big), \Big(\tfrac{\tau_3}{\langle[0.8,0.9],[0.2,0.6],[0.1,0.4]\rangle}, 0.5 \Big) \Big\}.$$

$$\mathcal{D}_\Psi(z_3) = \Big\{ \Big(\tfrac{\tau_1}{\langle[0.6,0.8],[0.8,1],[0.3,0.8]\rangle}, 0.5 \Big), \Big(\tfrac{\tau_2}{\langle[0.4,0.7],[0.1,0.2],[0.2,0.4]\rangle}, 0.8 \Big), \Big(\tfrac{\tau_3}{\langle[0.4,0.8],[0.4,0.7],[0.1,0.4]\rangle}, 0.9 \Big) \Big\}.$$

Also $\mathcal{D}_\Psi(z_i)$ can represent as a matrix as a following form:
$\mathcal{D}_\Psi =$

$$\begin{pmatrix} (([0.3,0.9],[0.2,0.4],[0,0.2])\,0.4) & (([0.4,0.5],[0.1,0.4],[0.5,0.5])\,0.7) & (([0.5,0.8],[0.1,0.1],[0.4,0.6])\,0.7) \\ (([0.6,0.8],[0.1,0.3],[0.4,0.7])\,0.5) & (([0.5,0.8],[0.6,0.8],[0.9,0.9])\,0.7) & (([0.7,0.9],[0.3,0.9],[0.2,0.5])\,0.4) \\ (([0.5,0.5],[0.8,1],[0.3,0.8])\,0.2) & (([0.4,0.7],[0.2,0.2],[0.2,0.7])\,0.5) & (([0.3,0.8],[0.4,0.7],[0.1,0.4])\,0.6) \end{pmatrix}$$

Definition 4.3. (Intersection of PIVNS-sets) Let
$\mathcal{P}^{ivnss}_\Theta = \Big\{ \Big(\tau, \Big\langle \ddot{\wp}^t_{\mathcal{P}}(\tau_i)(z_j), \ddot{\wp}^i_{\mathcal{P}}(\tau_i)(z_j), \ddot{\wp}^f_{\mathcal{P}}(\tau_i)(z_j) \Big\rangle \Theta(\tau_i)(z_j) \Big) \Big| \tau \in \hat{\mathbb{T}}, z \in \hat{\mathbb{E}} \Big\}$ and
$\mathcal{G}^{ivnss}_\Xi = \Big\{ \Big(\tau, \Big\langle \ddot{\wp}^t_{\mathcal{G}}(\tau_i)(z_j), \ddot{\wp}^i_{\mathcal{G}}(\tau_i)(z_j), \ddot{\wp}^f_{\mathcal{G}}(\tau_i)(z_j) \Big\rangle \Xi(\tau_i)(z_j) \Big) \Big| \tau \in \hat{\mathbb{T}}, z \in \hat{\mathbb{E}} \Big\}$ be two PIVNS-sets on reference set $\hat{\mathbb{T}}$. Then,the intersection of PIVNS-sets denoted by $\mathcal{P}^{ivnss}_\Theta \hat{\cap} \mathcal{G}^{ivnss}_\Xi$ and defiend as following:

1. $\mathcal{C}(z_i) = \mathcal{P}(z_i)\hat{\cap}\mathcal{G}(z_i)$, where $\hat{\cap}$ denotes IVNS-intersection.

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2. $\Upsilon(z_i) = \max\left(\Theta(z_i)\hat{\cap}\Xi(z_i)\right)$ following FS-intersection.

Example 4.4. Taking into account the PIVNSS $\mathcal{P}_\Theta$ as an Example 3.2, and $\mathcal{G}_\Xi$ given in an Example 3.4, then,the intersection of PIVNS-sets can be possess as following :

$$\mathcal{C}_\Upsilon(z_1) = \left\{\left(\frac{\tau_1}{\langle[0.2,0.7],[0.3,0.5],[0.1,0.3]\rangle}, 0.3\right), \left(\frac{\tau_2}{\langle[0.3,0.3],[0.2,0.6],[0.7,0.8]\rangle}, 0.6\right), \left(\frac{\tau_3}{\langle[0.5,0.8],[0.1,0.1],[0.4,0.6]\rangle}, 0.7\right)\right\}.$$

$$\mathcal{C}_\Upsilon(z_2) = \left\{\left(\frac{\tau_1}{\langle[0.6,0.8],[0.1,0.3],[0.4,0.7]\rangle}, 0.5\right), \left(\frac{\tau_2}{\langle[0.5,0.8],[0.6,0.8],[0.9,0.9]\rangle}, 0.7\right), \left(\frac{\tau_3}{\langle[0.7,0.9],[0.3,0.9],[0.2,0.5]\rangle}, 0.4\right)\right\}.$$

$$\mathcal{C}_\Upsilon(z_3) = \left\{\left(\frac{\tau_1}{\langle[0.5,0.5],[0.8,1],[0.3,0.8]\rangle}, 0.2\right), \left(\frac{\tau_2}{\langle[0.4,0.7],[0.2,0.2],[0.2,0.7]\rangle}, 0.5\right), \left(\frac{\tau_3}{\langle[0.3,0.8],[0.4,0.7],[0.1,0.4]\rangle}, 0.6\right)\right\}.$$

Also $\mathcal{C}_\Upsilon(z_i)$ can represent as a matrix as a following form:
$\mathcal{C}_\Upsilon =$

$$\begin{pmatrix} (([0.2,0.7],[0.3,0.5],[0.1,0.3])\,0.3) & (([0.3,0.3],[0.2,0.6],[0.7,0.8])\,0.6) & (([0.5,0.8],[0.1,0.1],[0.4,0.6])\,0.7) \\ (([0.6,0.8],[0.1,0.3],[0.4,0.7])\,0.5) & (([0.5,0.8],[0.6,0.8],[0.9,0.9])\,0.7) & (([0.7,0.9],[0.3,0.9],[0.2,0.5])\,0.4) \\ (([0.5,0.5],[0.8,1],[0.3,0.8])\,0.2) & (([0.4,0.7],[0.2,0.2],[0.2,0.7])\,0.5) & (([0.3,0.8],[0.4,0.7],[0.1,0.4])\,0.6) \end{pmatrix}$$

Proposition 4.5. *Let* $\mathcal{P}_\Theta^{ivnss} = \left\{\left(\tau, \left\langle \ddot{\wp}_{\mathcal{P}}^t(\tau_i)(z_j), \ddot{\wp}_{\mathcal{P}}^i(\tau_i)(z_j), \ddot{\wp}_{\mathcal{P}}^f(\tau_i)(z_j)\right\rangle \Theta(\tau_i)(z_j)\right) \middle| \tau \in \hat{\mathbb{T}}, z \in \hat{\mathbb{E}}\right\}$ *be a PIVNS-set on reference set* $\hat{\mathbb{T}}$. *Then,the following statements hold:*

1. $\mathcal{P}_\Theta \hat{\cup} \mathcal{P}_\Theta = \mathcal{P}_\Theta$.
2. $\mathcal{P}_\Theta \hat{\cap} \mathcal{P}_\Theta = \mathcal{P}_\Theta$.
3. $\mathcal{P}_\Theta \hat{\cup} \widehat{\Phi}_{(0)} = \mathcal{P}_\Theta$.
4. $\mathcal{P}_\Theta \hat{\cap} \widehat{\Phi}_{(0)} = \widehat{\Phi}_{(0)}$.
5. $\mathcal{P}_\Theta \hat{\cup} \widehat{\Omega}_{(1)} = \widehat{\Omega}_{(1)}$.
6. $\mathcal{P}_\Theta \hat{\cap} \widehat{\Omega}_{(1)} = \mathcal{P}_\Theta$.

Proposition 4.6. *Let*
$\mathcal{P}_\Theta^{ivnss} = \left\{\left(\tau, \left\langle \ddot{\wp}_{\mathcal{P}}^t(\tau_i)(z_j), \ddot{\wp}_{\mathcal{P}}^i(\tau_i)(z_j), \ddot{\wp}_{\mathcal{P}}^f(\tau_i)(z_j)\right\rangle \Theta(\tau_i)(z_j)\right) \middle| \tau \in \hat{\mathbb{T}}, z \in \hat{\mathbb{E}}\right\}$,
$\mathcal{G}_\Xi^{ivnss} = \left\{\left(\tau, \left\langle \ddot{\wp}_{\mathcal{G}}^t(\tau_i)(z_j), \ddot{\wp}_{\mathcal{G}}^i(\tau_i)(z_j), \ddot{\wp}_{\mathcal{G}}^f(\tau_i)(z_j)\right\rangle \Xi(\tau_i)(z_j)\right) \middle| \tau \in \hat{\mathbb{T}}, z \in \hat{\mathbb{E}}\right\}$ *and*
$\mathcal{C}_\Upsilon^{ivnss} = \left\{\left(\tau, \left\langle \ddot{\wp}_{\mathcal{C}}^t(\tau_i)(z_j), \ddot{\wp}_{\mathcal{C}}^i(\tau_i)(z_j), \ddot{\wp}_{\mathcal{C}}^f(\tau_i)(z_j)\right\rangle \Upsilon(\tau_i)(z_j)\right) \middle| \tau \in \hat{\mathbb{T}}, z \in \hat{\mathbb{E}}\right\}$ *be three PIVNS-sets on reference set* $\hat{\mathbb{T}}$. *Then,the following statements hold:*

1. $\mathcal{P}_\Theta \hat{\cup} \mathcal{G}_\Xi = \mathcal{G}_\Xi \hat{\cup} \mathcal{P}_\Theta$.
2. $\mathcal{P}_\Theta \hat{\cap} \mathcal{G}_\Xi = \mathcal{G}_\Xi \hat{\cap} \mathcal{P}_\Theta$.
3. $\mathcal{P}_\Theta \hat{\cup} \left(\mathcal{G}_\Xi \hat{\cup} \mathcal{C}_\Upsilon\right) = \left(\mathcal{P}_\Theta \hat{\cup} \mathcal{G}_\Xi\right) \hat{\cup} \mathcal{C}_\Upsilon$.
4. $\mathcal{P}_\Theta \hat{\cap} \left(\mathcal{G}_\Xi \hat{\cap} \mathcal{C}_\Upsilon\right) = \left(\mathcal{P}_\Theta \hat{\cap} \mathcal{G}_\Xi\right) \hat{\cap} \mathcal{C}_\Upsilon$.
5. $\mathcal{P}_\Theta \hat{\cup} \left(\mathcal{G}_\Xi \hat{\cap} \mathcal{C}_\Upsilon\right) = \left(\mathcal{P}_\Theta \hat{\cup} \mathcal{G}_\Xi\right) \hat{\cap} \left(\mathcal{P}_\Theta \hat{\cup} \mathcal{C}_\Upsilon\right)$.
6. $\mathcal{P}_\Theta \hat{\cap} \left(\mathcal{G}_\Xi \hat{\cup} \mathcal{C}_\Upsilon\right) = \left(\mathcal{P}_\Theta \hat{\cap} \mathcal{G}_\Xi\right) \hat{\cup} \left(\mathcal{P}_\Theta \hat{\cap} \mathcal{C}_\Upsilon\right)$.

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Proof. **1.** We take the left side $\mathcal{P}_\Theta \hat{\cup} \mathcal{G}_\Xi =$
$\left\{\left(\tau, \left\langle \ddot{\wp}^t_{\mathcal{P}}(\tau_i)(z_j), \ddot{\wp}^i_{\mathcal{P}}(\tau_i)(z_j), \ddot{\wp}^f_{\mathcal{P}}(\tau_i)(z_j)\right\rangle \Theta(\tau_i)(z_j)\right) \middle| \tau \in \hat{\mathbb{T}}, z \in \hat{\mathbb{E}}\right\} \cup$
$\left\{\left(\tau, \left\langle \ddot{\wp}^t_{\mathcal{G}}(\tau_i)(z_j), \ddot{\wp}^i_{\mathcal{G}}(\tau_i)(z_j), \ddot{\wp}^f_{\mathcal{G}}(\tau_i)(z_j)\right\rangle \Xi(\tau_i)(z_j)\right) \middle| \tau \in \hat{\mathbb{T}}, z \in \hat{\mathbb{E}}\right\}$
$= \left\{ z_j \left(\left[\ddot{\wp}^{t,l}_P(\tau_i)(z_j), \ddot{\wp}^{t,u}_P(\tau_i)(z_j)\right] \cup \left[\ddot{\wp}^{t,l}_G(\tau_i)(z_j), \ddot{\wp}^{t,u}_G(\tau_i)(z_j)\right] \right.\right.$
$\left[\ddot{\wp}^{i,l}_P(\tau_i)(z_j), \ddot{\wp}^{i,u}_P(\tau_i)(z_j)\right] \cup \left[\ddot{\wp}^{i,l}_G(\tau_i)(z_j), \ddot{\wp}^{i,u}_G(\tau_i)(z_j)\right],$
$\left.\left.\left[\ddot{\wp}^{f,l}_P(\tau_i)(z_j), \ddot{\wp}^{f,u}_P(\tau_i)(z_j)\right] \cup \left[\ddot{\wp}^{f,l}_P(\tau_i)(z_j), \ddot{\wp}^{f,u}_P(\tau_i)(z_j)\right]\right) |\tau_i \in T, z_j \in E \right\}$
$= \left\{ z_j \left(\max\left[\ddot{\wp}^{t,l}_P(\tau_i)(z_j), \ddot{\wp}^{t,l}_G(\tau_i)(z_j)\right], \max\left[\ddot{\wp}^{t,u}_P(\tau_i)(z_j), \ddot{\wp}^{t,u}_G(\tau_i)(z_j)\right] \right.\right.$
$\min\left[\ddot{\wp}^{i,l}_P(\tau_i)(z_j), \ddot{\wp}^{i,l}_G(\tau_i)(z_j)\right], \min\left[\ddot{\wp}^{i,u}_P(\tau_i)(z_j), \ddot{\wp}^{i,u}_G(\tau_i)(z_j)\right],$
$\left.\left.\min\left[\ddot{\wp}^{f,l}_P(\tau_i)(z_j), \ddot{\wp}^{f,l}_G(\tau_i)(z_j)\right], \min\left[\ddot{\wp}^{f,u}_P(\tau_i)(z_j), \ddot{\wp}^{f,u}_G(\tau_i)(z_j)\right]\right) |\tau_i \in T, z_j \in E \right\}$
$= \left\{ z_j \left(\max\left[\ddot{\wp}^{t,l}_G(\tau_i)(z_j), \ddot{\wp}^{t,l}_P(\tau_i)(z_j)\right], \max\left[\ddot{\wp}^{t,u}_G(\tau_i)(z_j), \ddot{\wp}^{t,u}_P(\tau_i)(z_j)\right] \right.\right.$
$\min\left[\ddot{\wp}^{i,l}_G(\tau_i)(z_j), \ddot{\wp}^{i,l}_P(\tau_i)(z_j)\right], \min\left[\ddot{\wp}^{i,u}_G(\tau_i)(z_j), \ddot{\wp}^{i,u}_P(\tau_i)(z_j)\right],$
$\left.\left.\min\left[\ddot{\wp}^{f,l}_G(\tau_i)(z_j), \ddot{\wp}^{f,l}_P(\tau_i)(z_j)\right], \min\left[\ddot{\wp}^{f,u}_G(\tau_i)(z_j), \ddot{\wp}^{f,u}_P(\tau_i)(z_j)\right]\right) |\tau_i \in T, z_j \in E \right\}$
$= \left\{ z_j \left(\left[\ddot{\wp}^{t,l}_G(\tau_i)(z_j), \ddot{\wp}^{t,u}_G(\tau_i)(z_j)\right] \cup \left[\ddot{\wp}^{t,l}_P(\tau_i)(z_j), \ddot{\wp}^{t,u}_P(\tau_i)(z_j)\right] \right.\right.$
$\left[\ddot{\wp}^{i,l}_G(\tau_i)(z_j), \ddot{\wp}^{i,u}_G(\tau_i)(z_j)\right] \cup \left[\ddot{\wp}^{i,l}_P(\tau_i)(z_j), \ddot{\wp}^{i,u}_P(\tau_i)(z_j)\right],$
$\left.\left.\left[\ddot{\wp}^{f,l}_G(\tau_i)(z_j), \ddot{\wp}^{f,u}_G(\tau_i)(z_j)\right] \cup \left[\ddot{\wp}^{f,l}_P(\tau_i)(z_j), \ddot{\wp}^{f,u}_P(\tau_i)(z_j)\right]\right) |\tau_i \in T, z_j \in E \right\}$
$= \mathcal{G} \hat{\cup} \mathcal{P}$
and for fuzzy possibility degree it essay to show $\Theta \cup \Xi = \Xi \cup \Theta$,then we get $= \mathcal{G}_\Xi \hat{\cup} \mathcal{P}_\Theta$ □

Note: *The rest of the proof is similar to proof method 1*

Proposition 4.7. *Let*
$\mathcal{P}^{ivnss}_\Theta = \left\{\left(\tau, \left\langle \ddot{\wp}^t_{\mathcal{P}}(\tau_i)(z_j), \ddot{\wp}^i_{\mathcal{P}}(\tau_i)(z_j), \ddot{\wp}^f_{\mathcal{P}}(\tau_i)(z_j)\right\rangle \Theta(\tau_i)(z_j)\right) \middle| \tau \in \hat{\mathbb{T}}, z \in \hat{\mathbb{E}}\right\}$, *and*
$\mathcal{G}^{ivnss}_\Xi = \left\{\left(\tau, \left\langle \ddot{\wp}^t_{\mathcal{G}}(\tau_i)(z_j), \ddot{\wp}^i_{\mathcal{G}}(\tau_i)(z_j), \ddot{\wp}^f_{\mathcal{G}}(\tau_i)(z_j)\right\rangle \Xi(\tau_i)(z_j)\right) \middle| \tau \in \hat{\mathbb{T}}, z \in \hat{\mathbb{E}}\right\}$ *be two PIVNS-sets on reference set* $\hat{\mathbb{T}}$. *Then,the following statements hold:*

1. $({\mathcal{P}_\Theta}^c)^c = \mathcal{P}_\Theta$.
2. $\left(\mathcal{P}_\Theta \hat{\cup} \mathcal{G}_\Xi\right)^c = {\mathcal{P}_\Theta}^c \hat{\cap} {\mathcal{G}_\Xi}^c$.
3. $\left(\mathcal{P}_\Theta \hat{\cap} \mathcal{G}_\Xi\right)^c = {\mathcal{P}_\Theta}^c \hat{\cup} {\mathcal{G}_\Xi}^c$.

Here, paragraphs 2 and 3 refer to De Morgan's law.

Proof. **1.** $\mathcal{P}_\Theta = \left\{\left(\tau, \left\langle \ddot{\wp}^t_{\mathcal{P}}(\tau_i)(z_j), \ddot{\wp}^i_{\mathcal{P}}(\tau_i)(z_j), \ddot{\wp}^f_{\mathcal{P}}(\tau_i)(z_j)\right\rangle \Theta(\tau_i)(z_j)\right) \middle| \tau \in \hat{\mathbb{T}}, z \in \hat{\mathbb{E}}\right\}$
$= \left\{ z_j \left(\left[\ddot{\wp}^{t,l}_{\mathcal{P}}(\tau_i)(z_j), \ddot{\wp}^{t,u}_{\mathcal{P}}(\tau_i)(z_j)\right], \left[\ddot{\wp}^{i,u}_{\mathcal{P}}(\tau_i)(z_j), \ddot{\wp}^{i,l}_{\mathcal{P}}(\tau_i)(z_j)\right], \right.\right.$
$\left.\left.\left[\ddot{\wp}^{f,l}_{\mathcal{P}}(\tau_i)(z_j), \ddot{\wp}^{f,u}_{\mathcal{P}}(\tau_i)(z_j)\right] (\Theta(\tau_i)(z_j))\right) \middle| \tau_i \in \hat{\mathbb{T}}, z \in \hat{\mathbb{E}} \right\}$
$\mathcal{P}^c_\Theta = \left\{ z_j \left(\left[\ddot{\wp}^{f,l}_{\mathcal{P}}(\tau_i)(z_j), \ddot{\wp}^{f,u}_{\mathcal{P}}(\tau_i)(z_j)\right], \left[1 - \ddot{\wp}^{i,u}_{\mathcal{P}}(\tau_i)(z_j), 1 - \ddot{\wp}^{i,l}_{\mathcal{P}}(\tau_i)(z_j)\right], \right.\right.$
$\left.\left.\left[\ddot{\wp}^{t,l}_{\mathcal{P}}(\tau_i)(z_j), \ddot{\wp}^{t,u}_{\mathcal{P}}(\tau_i)(z_j)\right] (1 - \Theta(\tau_i)(z_j))\right) \middle| \tau_i \in \hat{\mathbb{T}}, z \in \hat{\mathbb{E}} \right\}$
$(\mathcal{P}^c_\Theta)^c = \left\{ z_j \left(\left[\ddot{\wp}^{t,l}_{\mathcal{P}}(\tau_i)(z_j), \ddot{\wp}^{t,u}_{\mathcal{P}}(\tau_i)(z_j)\right], \left[1 - \left(1 - \ddot{\wp}^{i,u}_{\mathcal{P}}(\tau_i)(z_j)\right), 1 - \left(1 - \ddot{\wp}^{i,l}_{\mathcal{P}}(\tau_i)(z_j)\right)\right], \right.\right.$
$\left.\left.\left[\ddot{\wp}^{f,l}_{\mathcal{P}}(\tau_i)(z_j), \ddot{\wp}^{f,u}_{\mathcal{P}}(\tau_i)(z_j)\right] (1 - (1 - \Theta(\tau_i)(z_j)))\right) \middle| \tau_i \in \hat{\mathbb{T}}, z \in \hat{\mathbb{E}} \right\}$
$= \left\{ z_j \left(\left[\ddot{\wp}^{t,l}_{\mathcal{P}}(\tau_i)(z_j), \ddot{\wp}^{t,u}_{\mathcal{P}}(\tau_i)(z_j)\right], \left[\ddot{\wp}^{i,u}_{\mathcal{P}}(\tau_i)(z_j), \ddot{\wp}^{i,l}_{\mathcal{P}}(\tau_i)(z_j)\right], \right.\right.$
$\left.\left.\left[\ddot{\wp}^{f,l}_{\mathcal{P}}(\tau_i)(z_j), \ddot{\wp}^{f,u}_{\mathcal{P}}(\tau_i)(z_j)\right] \Theta(\tau_i)(z_j)\right) \middle| \tau_i \in \hat{\mathbb{T}}, z \in \hat{\mathbb{E}} \right\}$
$= \mathcal{P}_\Theta$

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Note: The rest of proof is similar to proof method 1 □

Definition 4.8. (***AND* of PIVNS-sets):** Let $\mathcal{P}_\Theta$and$\mathcal{G}_\Xi$ be two PIVNS-sets on reference set $\hat{\mathbb{T}}$. Then the "*AND*" operation of both $\mathcal{P}_\Theta$ and $\mathcal{G}_\Xi$ defined as $\mathcal{P}_\Theta \wedge \mathcal{G}_\Xi = \mathcal{R}_\Upsilon$ such that $\mathcal{R}_\Upsilon(\tau_i, \tau_j)(z_i) = \mathcal{P}_\Theta(\tau_j)(z_i) \cap \mathcal{G}_\Xi(\tau_j)(z_i)$ and for fuzzy possibility degree $\Upsilon(\tau_i, \tau_j)(z_i) = min(\Theta(\tau_j)(z_i), \Xi(\tau_j)(z_i))$,where $\forall (z_i, z_i) \in \hat{\mathbb{E}} \times \hat{\mathbb{E}}, \tau_j \times \tau_j \in \hat{\mathbb{T}}$. Here $\cap$ refer to the intersection of IVNS.

Example 4.9. Taking into account the PIVNSS $\mathcal{P}_\Theta$ as an Example 3.2, and $\mathcal{G}_\Xi$ given in an Example 3.4, then,the intersection of PIVNS-sets can be possess as following :

$$\mathcal{R}_\Sigma(z_1 \times z_1) = \left\{ \left(\frac{\tau_1}{\langle [0.2,0.7],[0.3,0.5],[0.1,0.3] \rangle}, 0.3 \right), \left(\frac{\tau_2}{\langle [0.3,0.3],[0.1,0.4],[0.5,0.5] \rangle}, 0.6 \right), \left(\frac{\tau_3}{\langle [0.5,0.8],[0.1,0.1],[0.4,0.6] \rangle}, 0.7 \right) \right\}.$$
$$\mathcal{R}_\Sigma(z_1 \times z_2) = \left\{ \left(\frac{\tau_1}{\langle [0.2,0.7],[0.3,0.5],[0.5,0.8] \rangle}, 0.3 \right), \left(\frac{\tau_2}{\langle [0.3,0.3],[0.5,0.6],[1,1] \rangle}, 0.6 \right), \left(\frac{\tau_3}{\langle [0.5,0.8],[0.2,0.6],[0.4,0.6] \rangle}, 0.5 \right) \right\}.$$
$$\mathcal{R}_\Sigma(z_1 \times z_3) = \left\{ \left(\frac{\tau_1}{\langle [0.2,0.7],[0.9,1],[0.4,0.9] \rangle}, 0.3 \right), \left(\frac{\tau_2}{\langle [0.2,0.3],[0.2,0.6],[0.7,0.8] \rangle}, 0.6 \right), \left(\frac{\tau_3}{\langle [0.4,0.8],[0.5,0.8],[0.4,0.7] \rangle}, 0.7 \right) \right\}.$$

$$\mathcal{R}_\Sigma(z_2 \times z_1) = \left\{ \left(\frac{\tau_1}{\langle [0.3,0.8],[0.1,0.3],[0.4,0.7] \rangle}, 0.4 \right), \left(\frac{\tau_2}{\langle [0.4,0.5],[0.6,0.8],[0.9,0.9] \rangle}, 0.7 \right), \left(\frac{\tau_3}{\langle [0.6,0.8],[0.3,0.9],[0.2,0.5] \rangle}, 0.4 \right) \right\}.$$
$$\mathcal{R}_\Sigma(z_2 \times z_2) = \left\{ \left(\frac{\tau_1}{\langle [0.6,0.8],[0.1,0.6],[0.5,0.8] \rangle}, 0.5 \right), \left(\frac{\tau_2}{\langle [0.5,0.7],[0.5,0.6],[1,1] \rangle}, 0.9 \right), \left(\frac{\tau_3}{\langle [0.7,0.9],[0.3,0.9],[0.2,0.5] \rangle}, 0.4 \right) \right\}.$$
$$\mathcal{R}_\Sigma(z_2 \times z_3) = \left\{ \left(\frac{\tau_1}{\langle [0.6,0.8],[0.9,1],[0.4,0.9] \rangle}, 0.5 \right), \left(\frac{\tau_2}{\langle [0.2,0.5],[0.6,0.8],[0.9,0.9] \rangle}, 0.7 \right), \left(\frac{\tau_3}{\langle [0.4,0.8],[0.5,0.9],[0.2,0.7] \rangle}, 0.4 \right) \right\}.$$

$$\mathcal{R}_\Sigma(z_3 \times z_1) = \left\{ \left(\frac{\tau_1}{\langle [0.3,0.5],[0.8,0.1],[0.3,0.8] \rangle}, 0.2 \right), \left(\frac{\tau_2}{\langle [0.4,0.5],[0.1,0.2],[0.2,0.5] \rangle}, 0.5 \right), \left(\frac{\tau_3}{\langle [0.3,0.8],[0.4,0.7],[0.2,0.4] \rangle}, 0.6 \right) \right\}.$$
$$\mathcal{R}_\Sigma(z_2 \times z_2) = \left\{ \left(\frac{\tau_1}{\langle [0.5,0.5],[0.8,1],[0.3,0.8] \rangle}, 0.2 \right), \left(\frac{\tau_2}{\langle [0.4,0.7],[0.5,0.6],[1,1] \rangle}, 0.5 \right), \left(\frac{\tau_3}{\langle [0.3,0.8],[0.4,0.7],[0.1,0.4] \rangle}, 0.5 \right) \right\}.$$
$$\mathcal{R}_\Sigma(z_3 \times z_3) = \left\{ \left(\frac{\tau_1}{\langle [0.5,0.5],[0.9,1],[0.4,0.9] \rangle}, 0.2 \right), \left(\frac{\tau_2}{\langle [0.2,0.5],[0.2,0.2],[0.2,0.7] \rangle}, 0.5 \right), \left(\frac{\tau_3}{\langle [0.3,0.8],[0.5,0.8],[0.2,0.7] \rangle}, 0.6 \right) \right\}.$$

Also $\mathcal{R}_\Sigma(z_i)$ can represent as a matrix as a following form:
$\mathcal{R}_\Sigma =$

$$\begin{pmatrix}
(([0.2,0.7],[0.3,0.5],[0.1,0.3])\,0.3) & (([0.3,0.3],[0.1,0.4],[0.5,0.5])\,0.6) & (([0.5,0.8],[0.1,0.1],[0.4,0.6])\,0.7) \\
(([0.2,0.7],[0.3,0.5],[0.5,0.8])\,0.3) & (([0.3,0.3],[0.5,0.6],[1,1])\,0.6) & (([0.5,0.8],[0.2,0.6],[0.4,0.6])\,0.5) \\
(([0.2,0.7],[0.9,1],[0.4,0.9])\,0.3) & (([0.2,0.3],[0.2,0.6],[0.7,0.8])\,0.6) & (([0.4,0.8],[0.5,0.8],[0.4,0.7])\,0.7) \\
(([0.3,0.8],[0.1,0.3],[0.4,0.7])\,0.4) & (([0.4,0.5],[0.6,0.8],[0.9,0.9])\,0.7) & (([0.6,0.8],[0.3,0.9],[0.2,0.5])\,0.4) \\
(([0.6,0.8],[0.1,0.6],[0.5,0.8])\,0.5) & (([0.5,0.7],[0.5,0.6],[0.9,0.9])\,0.7) & (([0.4,0.8],[0.5,0.9],[0.2,0.7])\,0.4) \\
(([0.3,0.5],[0.8,1],[0.3,0.8])\,0.2) & (([0.4,0.5],[0.1,0.2],[0.2,0.5])\,0.5) & (([0.3,0.8],[0.4,0.7],[0.1,0.4])\,0.5) \\
(([0.5,0.5],[0.9,1],[0.4,0.9])\,0.2) & (([0.2,0.5],[0.2,0.2],[0.2,0.7])\,0.5) & (([0.3,0.8],[0.5,0.8],[0.2,0.7])\,0.6) \\
(([0.5,0.5],[0.9,1],[0.4,0.9])\,0.2) & (([0.2,0.5],[0.2,0.2],[0.9,0.7])\,0.5) & (([0.3,0.8],[0.5,0.8],[0.2,0.7])\,0.7) \\
(([0.5,0.5],[0.9,1],[0.4,0.9])\,0.2) & (([0.2,0.5],[0.2,0.2],[0.9,0.7])\,0.5) & (([0.3,0.8],[0.5,0.8],[0.2,0.7])\,0.6)
\end{pmatrix}$$

Definition 4.10. (***OR* of PIVNS-sets):** Let $\mathcal{P}_\Theta$and$\mathcal{G}_\Xi$ be two PIVNS-sets on reference set $\hat{\mathbb{T}}$. Then the "*OR*" operation of both $\mathcal{P}_\Theta$ and $\mathcal{G}_\Xi$ defined as $\mathcal{P}_\Theta \vee \mathcal{G}_\Xi = \mathcal{Q}_\Sigma$ such that $\mathcal{R}_\Sigma(\tau_i, \tau_j)(z_i) = \mathcal{P}_\Theta(\tau_j)(z_i) \cup \mathcal{G}_\Xi(\tau_j)(z_i)$and for fuzzy possibility degree $\Sigma(\tau_i, \tau_j)(z_i) = max(\Theta(\tau_j)(z_i), \Xi(\tau_j)(z_i))$,where $\forall (z_i, z_i) \in \hat{\mathbb{E}} \times \hat{\mathbb{E}}, \tau_j \times \tau_j \in \hat{\mathbb{T}}$. Here $\cup$ refer to the union of IVNS.

Example 4.11. Taking into account the PIVNSS $\mathcal{P}_\Theta$ as an Example 3.2, and $\mathcal{G}_\Xi$ given in an Example 3.4, then,the union of PIVNS-sets can be possess as following :

$$\mathcal{R}_\Sigma(z_1 \times z_1) = \left\{ \left(\frac{\tau_1}{\langle [0.3,0.9],[0.2,0.4],[0,0.2] \rangle}, 0.4 \right), \left(\frac{\tau_2}{\langle [0.4,0.5],[0.1,0.4],[0.5,0.5] \rangle}, 0.7 \right), \left(\frac{\tau_3}{\langle [0.6,0.8],[0,0],[0.2,0.4] \rangle}, 0.9 \right) \right\}.$$
$$\mathcal{R}_\Sigma(z_1 \times z_2) = \left\{ \left(\frac{\tau_1}{\langle [0.8,0.8],[0.1,0.5],[0.1,0.3] \rangle}, 0.6 \right), \left(\frac{\tau_2}{\langle [0.7,0.7],[0.2,0.6],[0.7,0.8] \rangle}, 0.9 \right), \left(\frac{\tau_3}{\langle [0.8,0.9],[0.1,0.1],[0.1,0.4] \rangle}, 0.7 \right) \right\}.$$
$$\mathcal{R}_\Sigma(z_1 \times z_3) = \left\{ \left(\frac{\tau_1}{\langle [0.6,0.8],[0.3,0.5],[0.1,0.4] \rangle}, 0.5 \right), \left(\frac{\tau_2}{\langle [0.3,0.5],[0.1,0.2],[0.2,0.4] \rangle}, 0.8 \right), \left(\frac{\tau_3}{\langle [0.5,0.8],[0.1,0.1],[0.2,0.6] \rangle}, 0.9 \right) \right\}.$$

$$\mathcal{R}_\Sigma(z_2 \times z_1) = \left\{ \left(\frac{\tau_1}{\langle [0.6,0.9],[0.1,0.3],[0,0.2] \rangle}, 0.5 \right), \left(\frac{\tau_2}{\langle [0.5,0.8],[0.1,0.4],[0.5,0.5] \rangle}, 0.7 \right), \left(\frac{\tau_3}{\langle [0.7,0.9],[0,0],[0.2,0.4] \rangle}, 0.9 \right) \right\}.$$
$$\mathcal{R}_\Sigma(z_2 \times z_2) = \left\{ \left(\frac{\tau_1}{\langle [0.8,0.8],[0.1,0.3],[0.4,0.7] \rangle}, 0.6 \right), \left(\frac{\tau_2}{\langle [0.7,0.7],[0.5,0.6],[0.9,0.9] \rangle}, 0.9 \right), \left(\frac{\tau_3}{\langle [0.8,0.9],[0.2,0.6],[0.1,0.4] \rangle}, 0.5 \right) \right\}.$$

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$\mathcal{R}_{\Sigma}\left(z_2 \times z_3\right)=\left\{\left(\frac{\tau_1}{\langle[0.6,0.8],[0.1,0.3],[0.4,0.7]\rangle}, 0.5\right),\left(\frac{\tau_2}{\langle[0.5,0.8],[0.1,0.2],[0.2,0.4]\rangle}, 0.7\right),\left(\frac{\tau_3}{\langle[0.7,0.9],[0.3,0.8],[0.2,0.5]\rangle}, 0.9\right)\right\}.$

$\mathcal{R}_{\Sigma}\left(z_3 \times z_1\right)=\left\{\left(\frac{\tau_1}{\langle[0.5,0.9],[0.2,0.4],[0,0.2]\rangle}, 0.4\right),\left(\frac{\tau_2}{\langle[0.4,0.7],[0.1,0.2],[0.2,0.5]\rangle}, 0.7\right),\left(\frac{\tau_3}{\langle[0.6,0.8],[0,0],[0.2,0.4]\rangle}, 0.9\right)\right\}.$
$\mathcal{R}_{\Sigma}\left(z_3 \times z_2\right)=\left\{\left(\frac{\tau_1}{\langle[0.8,0.8],[0.1,0.6],[0.3,0.8]\rangle}, 0.6\right),\left(\frac{\tau_2}{\langle[0.7,0.7],[0.2,0.2],[0.2,0.7]\rangle}, 0.9\right),\left(\frac{\tau_3}{\langle[0.8,0.9],[0.2,0.6],[0.1,0.4]\rangle}, 0.6\right)\right\}.$
$\mathcal{R}_{\Sigma}\left(z_3 \times z_3\right)=\left\{\left(\frac{\tau_1}{\langle[0.6,0.8],[0.8,1],[0.3,0.8]\rangle}, 0.5\right),\left(\frac{\tau_2}{\langle[0.4,0.7],[0.1,0.2],[0.2,0.4]\rangle}, 0.8\right),\left(\frac{\tau_3}{\langle[0.4,0.8],[0.4,0.7],[0.1,0.4]\rangle}, 0.9\right)\right\}.$

Also $\mathcal{R}_{\Sigma}\left(z_i\right)$ can represent as a matrix as a following form:
$\mathcal{R}_{\Sigma}=$

$$\begin{pmatrix}
(([0.3,0.9],[0.2,0.4],[0,0.2])\,0.4) & (([0.4,0.5],[0.1,0.4],[0.5,0.5])\,0.7) & (([0.6,0.8],[0,0],[0.2,0.4])\,0.9)\\
(([0.8,0.8],[0.1,0.5],[0.1,0.3])\,0.6) & (([0.7,0.7],[0.2,0.6],[0.7,0.8])\,0.9) & (([0.8,0.9],[0.1,0.1],[0.1,0.4])\,0.7)\\
(([0.6,0.8],[0.3,0.5],[0.1,0.4])\,0.5) & (([0.3,0.5],[0.1,0.2],[0.2,0.4])\,0.8) & (([0.5,0.8],[0.1,0.1],[0.2,0.6])\,0.9)\\
(([0.6,0.9],[0.1,0.3],[0,0.2])\,0.5) & (([0.5,0.8],[0.1,0.4],[0.5,0.5])\,0.7) & (([0.7,0.9],[0,0.],[0.2,0.4])\,0.9)\\
(([0.8,0.8],[0.1,0.3],[0.4,0.7])\,0.6) & (([0.7,0.7],[0.5,0.5],[0.9,0.9])\,0.9) & (([0.8,0.9],[0.2,0.6],[0.1,0.4])\,0.5)\\
(([0.6,0.8],[0.1,0.3],[0.4,0.7])\,0.5) & (([0.5,0.8],[0.1,0.2],[0.2,0.4])\,0.7) & (([0.7,0.9],[0.3,0.8],[0.2,0.5])\,0.9)\\
(([0.5,0.9],[0.2,0.4],[0,0.2])\,0.4) & (([0.4,0.7],[0.1,0.2],[0.2,0.5])\,0.7) & (([0.6,0.8],[0,0],[0.2,0.4])\,0.9)\\
(([0.8,0.8],[0.1,0.6],[0.3,0.8])\,0.6) & (([0.7,0.7],[0.2,0.2],[0.2,0.7])\,0.9) & (([0.8,0.9],[0.2,0.6],[0.1,0.4])\,0.6)\\
(([0.6,0.8],[0.8,1],[0.3,0.8])\,0.5) & (([0.4,0.7],[0.1,0.2],[0.2,0.4])\,0.8) & (([0.4,0.8],[0.4,0.7],[0.1,0.4])\,0.9)
\end{pmatrix}$$

5 Distance Measure on PIVNSS

In this part, we introduce and study the distance measure (DM) of PIVNSSs in order to calculate the ratio of DM between two PIVNSSs. After that, we will employ these DMs in one application in DM problem by proposing an algorithm shown in Figure1.

Definition 5.1. Let
$\mathcal{P}_{\Theta}^{ivnss}=\left\{\left(\tau,\left\langle\ddot{\wp}_{\mathcal{P}}^{t}\left(\tau_i\right)\left(z_j\right), \ddot{\wp}_{\mathcal{P}}^{i}\left(\tau_i\right)\left(z_j\right), \ddot{\wp}_{\mathcal{P}}^{f}\left(\tau_i\right)\left(z_j\right)\right\rangle \Theta\left(\tau_i\right)\left(z_j\right)\right) \middle| \tau \in \hat{\mathbb{T}}, z \in \hat{\mathbb{E}}\right\}$,
$\mathcal{G}_{\Xi}^{ivnss}=\left\{\left(\tau,\left\langle\ddot{\wp}_{\mathcal{G}}^{t}\left(\tau_i\right)\left(z_j\right), \ddot{\wp}_{\mathcal{G}}^{i}\left(\tau_i\right)\left(z_j\right), \ddot{\wp}_{\mathcal{G}}^{f}\left(\tau_i\right)\left(z_j\right)\right\rangle \Xi\left(\tau_i\right)\left(z_j\right)\right) \middle| \tau \in \hat{\mathbb{T}}, z \in \hat{\mathbb{E}}\right\}$ and
$\mathcal{C}_{\Upsilon}^{ivnss}=\left\{\left(\tau,\left\langle\ddot{\wp}_{\mathcal{C}}^{t}\left(\tau_i\right)\left(z_j\right), \ddot{\wp}_{\mathcal{C}}^{i}\left(\tau_i\right)\left(z_j\right), \ddot{\wp}_{\mathcal{C}}^{f}\left(\tau_i\right)\left(z_j\right)\right\rangle \Upsilon\left(\tau_i\right)\left(z_j\right)\right) \middle| \tau \in \hat{\mathbb{T}}, z \in \hat{\mathbb{E}}\right\}$ be three PIVNS-sets on reference set $\hat{\mathbb{T}}$. Then, a function $\mathbb{D}$:PIVNSS×PIVNSS $\rightarrow [0,1]$ is called distance measure (PIVNSS($\hat{\mathbb{T}}$)) if $\mathbb{D}$ fulfilled the following notes:

i. $\mathbb{D}(\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}) \geq 0$, and $\mathbb{D}(\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}) = 0$, iff both $\mathcal{P}_{\Theta}=\mathcal{G}_{\Xi}$.

ii. $\mathbb{D}(\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}) = \mathbb{D}(\mathcal{G}_{\Xi}, \mathcal{P}_{\Theta})$

iii. $\mathbb{D}\left(\mathcal{P}_{\Theta}, \mathcal{C}_{\Upsilon}\right) \leq \mathbb{D}\left(\mathcal{P}_{\Theta}, \mathcal{C}_{\Upsilon}\right)+\mathbb{D}\left(\mathcal{G}_{\Xi}, \mathcal{C}_{\Upsilon}\right)$ (triangle inequality).

Now based on definition 5.1, we will define the following distance measures:

1.Hamming distance
$\mathbb{D}_{PIVNSS}^{H}\left(\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}\right)=\frac{1}{6} \sum_{i}^{n} \sum_{j}^{m} \frac{\left[\left|\Delta_{ij} \ddot{\wp}_{\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}}^{t,l}\right|+\left|\Delta_{ij} \ddot{\wp}_{\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}}^{t,u}\right|+\left|\Delta_{ij} \ddot{\wp}_{\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}}^{i,l}\right|+\left|\Delta_{ij} \ddot{\wp}_{\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}}^{i,u}\right|+\left|\Delta_{ij} \ddot{\wp}_{\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}}^{f,l}\right|+\left|\Delta_{ij} \ddot{\wp}_{\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}}^{f,u}\right|\right]}{6}$
where

$$\Delta_{ij} \ddot{\wp}_{\mathcal{P}, \mathcal{G}_{\Xi}}^{t,l}=\left(\ddot{\wp}_{\mathcal{P}_{\Theta}}^{t,l}-\ddot{\wp}_{\mathcal{P}_{\Xi}}^{t,l}\right) \times(\Theta-\Xi), \Delta_{ij} \ddot{\wp}_{\mathcal{P}, \mathcal{G}_{\Xi}}^{t,u}=\left(\ddot{\wp}_{\mathcal{P}_{\Theta}}^{t,u}-\ddot{\wp}_{\mathcal{G}_{\Xi}}^{t,l}\right) \times(\Theta-\Xi),$$
$$\Delta_{ij} \ddot{\wp}_{\mathcal{P}, \mathcal{G}_{\Xi}}^{i,l}=\left(\ddot{\wp}_{\mathcal{P}_{\Theta}}^{i,l}-\ddot{\wp}_{\mathcal{P}_{\Xi}}^{i,l}\right) \times(\Theta-\Xi), \Delta_{ij} \ddot{\wp}_{\mathcal{P}, \mathcal{G}_{\Xi}}^{i,u}=\left(\ddot{\wp}_{\mathcal{P}_{\Theta}}^{i,u}-\ddot{\wp}_{\mathcal{G}_{\Xi}}^{i,l}\right) \times(\Theta-\Xi),$$
$$\Delta_{ij} \ddot{\wp}_{\mathcal{P}, \mathcal{G}_{\Xi}}^{f,l}=\left(\ddot{\wp}_{\mathcal{P}_{\Theta}}^{f,l}-\ddot{\wp}_{\mathcal{P}_{\Xi}}^{f,l}\right) \times(\Theta-\Xi), \Delta_{ij} \ddot{\wp}_{\mathcal{P}, \mathcal{G}_{\Xi}}^{f,u}=\left(\ddot{\wp}_{\mathcal{P}_{\Theta}}^{f,u}-\ddot{\wp}_{\mathcal{G}_{\Xi}}^{f,l}\right) \times(\Theta-\Xi),$$

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2.Normalized Hamming distance

$\mathbb{D}_{PIVNSS}^{NH}\left(\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}\right)=\frac{D_{PIVNSS}^{H}\left(\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}\right)}{mn}.$

3. Euclidean distance

$$D_{PIVNSS}^{E}\left(\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}\right)=\sqrt{\frac{1}{6} \sum_{i}^{n} \sum_{j}^{m} \frac{\left[\left|\Delta_{ij} \ddot{\wp}_{\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}}^{t,l}\right|+\left|\Delta_{ij} \ddot{\wp}_{\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}}^{t,u}\right|+\left|\Delta_{ij} \ddot{\wp}_{\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}}^{i,l}\right|+\left|\Delta_{ij} \ddot{\wp}_{\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}}^{i,u}\right|+\left|\Delta_{ij} \ddot{\wp}_{\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}}^{f,l}\right|+\left|\Delta_{ij} \ddot{\wp}_{\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}}^{f,u}\right|\right]}{6}}$$

where

$$\begin{gathered}
\Delta_{ij} \ddot{\wp}_{\mathcal{P}, \mathcal{G}_{\Xi}}^{t,l}=\left(\ddot{\wp}_{\mathcal{P}_{\Theta}}^{t,l}-\ddot{\wp}_{\mathcal{P}_{\Xi}}^{t,l}\right) \times(\Theta-\Xi), \Delta_{ij} \ddot{\wp}_{\mathcal{P}, \mathcal{G}_{\Xi}}^{t,u}=\left(\ddot{\wp}_{\mathcal{P}_{\Theta}}^{t,u}-\ddot{\wp}_{\mathcal{G}_{\Xi}}^{t,l}\right) \times(\Theta-\Xi), \\
\Delta_{ij} \ddot{\wp}_{\mathcal{P}, \mathcal{G}_{\Xi}}^{i,l}=\left(\ddot{\wp}_{\mathcal{P}_{\Theta}}^{i,l}-\ddot{\wp}_{\mathcal{P}_{\Xi}}^{i,l}\right) \times(\Theta-\Xi), \Delta_{ij} \ddot{\wp}_{\mathcal{P}, \mathcal{G}_{\Xi}}^{i,u}=\left(\ddot{\wp}_{\mathcal{P}_{\Theta}}^{i,u}-\ddot{\wp}_{\mathcal{G}_{\Xi}}^{i,l}\right) \times(\Theta-\Xi), \\
\Delta_{ij} \ddot{\wp}_{\mathcal{P}, \mathcal{G}_{\Xi}}^{f,l}=\left(\ddot{\wp}_{\mathcal{P}_{\Theta}}^{f,l}-\ddot{\wp}_{\mathcal{P}_{\Xi}}^{f,l}\right) \times(\Theta-\Xi), \Delta_{ij} \ddot{\wp}_{\mathcal{P}, \mathcal{G}_{\Xi}}^{f,u}=\left(\ddot{\wp}_{\mathcal{P}_{\Theta}}^{f,u}-\ddot{\wp}_{\mathcal{G}_{\Xi}}^{f,l}\right) \times(\Theta-\Xi).
\end{gathered}$$

4. Normalized Euclidean distance

$\mathbb{D}_{PIVNSS}^{NE}\left(\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}\right)=\frac{D_{PIVNSS}^{E}\left(\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}\right)}{\sqrt{mn}}.$

Theorem 5.2. *The function* $\mathbb{D}_{PIVNSS}^{H}\left(\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}\right), \mathbb{D}_{PIVNSS}^{NH}\left(\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}\right), D_{PIVNSS}^{E}\left(\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}\right)$ *and* $\mathbb{D}_{PIVNSS}^{NE}\left(\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}\right)$ *:PIVNSS*$\left(\hat{\mathbb{T}}\right) \rightarrow R^{+}$ *that given by Definition 5.1.respectively are metrics, where* R^{+} *is the set all non-negative real-numbers.*

Proof. The proof is straightforward. □

Sometimes users resort to adjusting the resulting values when using the tools given above, using weight values that are specified by the user. Now we will redefine the above tools, but combined with weight values.

Now based on definition 5.1, we will define the following distance measures:

1. Weighted Hamming distance:

$$D_{PIVNSS}^{wH}\left(\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}\right)=$$
$$\left\{\frac{1}{6} \sum_{i}^{n} \sum_{j}^{m} w_{i}\left[\left|\Delta_{ij} \ddot{\wp}_{\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}}^{t,l}\right|+\left|\Delta_{ij} \ddot{\wp}_{\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}}^{t,u}\right|+\left|\Delta_{ij} \ddot{\wp}_{\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}}^{i,l}\right|+\left|\Delta_{ij} \ddot{\wp}_{\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}}^{i,u}\right|+\left|\Delta_{ij} \ddot{\wp}_{\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}}^{f,l}\right|+\left|\Delta_{ij} \ddot{\wp}_{\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}}^{f,u}\right|\right]^{\gamma}\right\}^{\frac{1}{\gamma}}$$

where

$$\begin{gathered}
\Delta_{ij} \ddot{\wp}_{P_{\Theta}, G_{\Xi}}^{t,l}=\left|\ddot{\wp}_{P_{\Theta}}^{t,l}-\ddot{\wp}_{G_{\Xi}}^{t,l}\right|^{\gamma} \times|\Theta-\Xi|^{\gamma}, \Delta_{ij} \ddot{\wp}_{P_{\Theta}, G_{\Xi}}^{t,u}=\left|\ddot{\wp}_{P_{\Theta}}^{t,u}-\ddot{\wp}_{G_{\Xi}}^{t,u}\right|^{\gamma} \times|\Theta-\Xi|^{\gamma} \\
\Delta_{ij} \ddot{\wp}_{P_{\Theta}, G_{\Xi}}^{i,l}=\left|\ddot{\wp}_{P_{\Theta}}^{t,l}-\ddot{\wp}_{G_{\Xi}}^{i,l}\right|^{\gamma} \times|\Theta-\Xi|^{\gamma}, \Delta_{ij} \ddot{\wp}_{P_{\Theta}, G_{\Xi}}^{i,u}=\left|\ddot{\wp}_{P_{\Theta}}^{i,u}-\ddot{\wp}_{G_{\Xi}}^{i,u}\right|^{\gamma} \times|\Theta-\Xi|^{\gamma} \\
\Delta_{ij} \ddot{\wp}_{P_{\Theta}, G_{\Xi}}^{f,l}=\left|\ddot{\wp}_{P_{\Theta}}^{f,l}-\ddot{\wp}_{G_{\Xi}}^{f,l}\right|^{\gamma} \times|\Theta-\Xi|^{\gamma}, \Delta_{ij} \ddot{\wp}_{P_{\Theta}, G_{\Xi}}^{f,u}=\left|\ddot{\wp}_{P_{\Theta}}^{f,u}-\ddot{\wp}_{G_{\Xi}}^{f,u}\right|^{\gamma} \times|\Theta-\Xi|^{\gamma}
\end{gathered}$$

2. Weighted Normalized Hamming distance:

$\mathbb{D}_{PIVNSS}^{wNH}\left(\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}\right)=\frac{D_{PIVNSS}^{wH}\left(\mathcal{P}_{\Theta}, \mathcal{G}_{\Xi}\right)}{mn}.$

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6 Application of PIVNS-sets in real-life situations based on DM

In this section of the current research, we will create a new algorithm based on the tools presented in this work to solve one of the decision-making problems (to help a couple choose a new home in one of the residential complexes). This algorithm will present its steps in Figure 1 as following:

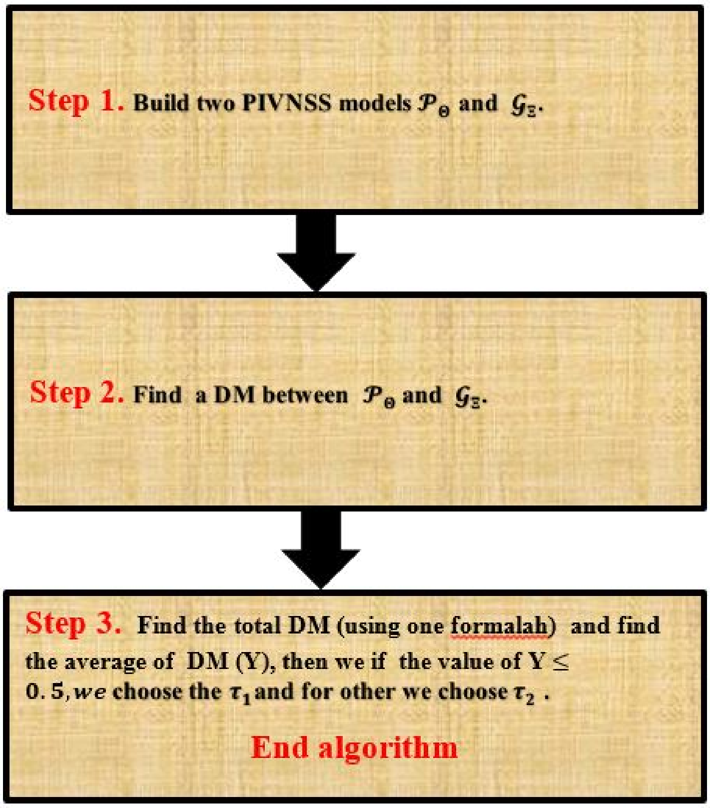

Figure 1: The propose algorithm based on SM technical

6.1 Statement of the problem

Assume that the married couple, Mr. Xu and Mrs. Xu wants to purchase a house in one of the low-cost residential complexes. In the low-cost residential complexes, there are two houses that represent by reference set $\hat{\mathbb{T}} = \{\tau_1, \tau_2\}$.The two couples in their selection focus on observing the attributes that can be represented by the following attribute set $\hat{\mathbb{E}} = \{z_1, z_2, z_3, z_4, z_5\}$ such that z_1 =House area,z_2 =House price, and z_3 =The distance of the home from neighboring homesz_4 = Materials used in building the house, z_5 = Number of rooms in the homes .In this scenario, each couple was given an opinion about the three houses based on the characteristics (criteria) taken into consideration. Here we will analyze the couple's opinions by building two models ($IVNS - memberships$) from our proposed approach in this work and the expert's degree($possibility\ degree$) of evaluation of this evaluation as a follows:

Step 1. Build two PIVNSS models $\mathcal{P}_\Theta$ for Mr. Xu, and $\mathcal{G}_\Xi$ for Mrs. Xu:

$$\mathcal{P}_\Theta =$$
$$\left\{\mathcal{P}_\Theta(z_1) = \left\{\left(\frac{\tau_1}{\langle[0.2,0.7],[0.3,0.5],[0.1,0.3]\rangle}, 0.3\right), \left(\frac{\tau_2}{\langle[0.3,0.3],[0.2,0.6],[0.7,0.8]\rangle}, 0.6\right)\right\}\right.$$

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$\mathcal{P}_{\Theta}(z_2)=\left\{\left(\frac{\tau_1}{\langle[0.6,0.8],[0.1,0.3],[0.4,0.7]\rangle},0.5\right),\left(\frac{\tau_2}{\langle[0.5,0.8],[0.6,0.8],[0.9,0.9]\rangle},0.7\right)\right\}$
$\mathcal{P}_{\Theta}(z_3)=\left\{\left(\frac{\tau_1}{\langle[0.5,0.5],[0.8,1],[0.3,0.8]\rangle},0.2\right),\left(\frac{\tau_2}{\langle[0.4,0.7],[0.2,0.2],[0.2,0.7]\rangle},0.5\right)\right\}$
$\mathcal{P}_{\Theta}(z_4)=\left\{\left(\frac{\tau_1}{\langle[0.6,0.8],[0.1,0.3],[0.4,0.7]\rangle},0.5\right),\left(\frac{\tau_2}{\langle[0.5,0.8],[0.6,0.8],[0.9,0.9]\rangle},0.7\right)\right\}$
$\mathcal{P}_{\Theta}(z_5)=\left\{\left(\frac{\tau_1}{\langle[0.2,0.4],[0.9,0.9],[0.5,0.7]\rangle},0.5\right),\left(\frac{\tau_2}{\langle[0.3,0.6],[0.7,0.8],[0.1,0.3]\rangle},0.4\right)\right\}\Big\}$

$\mathcal{G}_{\Xi}=$
$\Big\{\mathcal{G}_{\Xi}(z_1)=\left\{\left(\frac{\tau_1}{\langle[0.4,0.5],[0.5,0.8],[0.2,0.6]\rangle},0.6\right),\left(\frac{\tau_2}{\langle[0.4,0.5],[0.2,0.7],[0.4,0.9]\rangle},0.5\right)\right\}$
$\mathcal{G}_{\Xi}(z_2)=\left\{\left(\frac{\tau_1}{\langle[0.4,0.5],[0.2,0.6],[0.4,0.7]\rangle},0.3\right),\left(\frac{\tau_2}{\langle[0.9,1],[0.4,0.8],[0.2,0.4]\rangle},0.8\right)\right\}$
$\mathcal{G}_{\Xi}(z_3)=\left\{\left(\frac{\tau_1}{\langle[0.2,0.5],[0.8,1],[0.4,0.8]\rangle},0.7\right),\left(\frac{\tau_2}{\langle[0.2,0.7],[0.1,0.2],[0.2,0.7]\rangle},0.5\right)\right\}$
$\mathcal{G}_{\Xi}(z_4)=\left\{\left(\frac{\tau_1}{\langle[0.6,0.8],[0.1,0.3],[0.4,0.7]\rangle},0.5\right),\left(\frac{\tau_2}{\langle[0.5,0.8],[0.6,0.8],[0.9,0.9]\rangle},0.7\right)\right\}$
$\mathcal{G}_{\Xi}(z_5)=\left\{\left(\frac{\tau_1}{\langle[0.2,0.4],[0.9,0.9],[0.5,0.7]\rangle},0.5\right),\left(\frac{\tau_2}{\langle[0.3,0.6],[0.7,0.8],[0.1,0.3]\rangle},0.4\right)\right\}\Big\}$

Step 2. Find the DM between $\mathcal{P}_{\Theta}$ and $\mathcal{G}_{\Xi}$ according definition 5.1 in part 5, **(the DM value presented in Table 1.)** .

Table 1: Valudes of DM Between $\mathcal{P}(\tau_i),\mathcal{G}(\tau_i)$

$\ddot{D}_k(\mathcal{P}(\tau_i),\mathcal{G}(\tau_i))$	**Degree of DM**
$\ddot{D}_1^H(\mathcal{P}(\tau_1),\mathcal{G}(\tau_2))$	0.574
$\ddot{D}_2^H(\mathcal{P}(\tau_2),\mathcal{G}(\tau_2))$	0.731
$\ddot{D}_3^H(\mathcal{P}(\tau_3),\mathcal{G}(\tau_3))$	0.628
$\ddot{D}_4^H(\mathcal{P}(\tau_4),\mathcal{G}(\tau_4))$	0.435
$\ddot{D}_5^H(\mathcal{P}(\tau_5),\mathcal{G}(\tau_5))$	0.485
Total of $\hat{\hat{\mathbb{D}}}^H(\mathcal{P}_{\Theta},\mathcal{G}_{\Xi})=$	2.853

Step 3. The value of Y is 0.5706, therefor the two couple will choose house τ_2.

7 Conclusion

In this work, the notion of a PIVNSS as an extension to IVNSS is introduced. The basic properties on this model namely null, absolute, subset, equality and complement are presented. Also the basic set theory like union, intersection, OR and AND operations as well as some properties on PIVNSSs are discussed. Finely, we presented a decision making method based on PIVNSS, and gave an application of this method to solve a decision making problem.For future research work, users can combine these tools with other fuzzy algebraic tools see[49–55]

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