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ORIGINAL ARTICLE

An intuitionistic fuzzy hypersoft expert set-based robust decision-support framework for human resource management integrated with modified TOPSIS and correlation coefficient

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Abstract

Human resource management is the process of making a company's human resources decisions. In general, these decisions include hiring, firing, training, and developing people according to their positions and the needs of the organization. It includes a variety of policies and strategies designed to recognize the contribution that people make to an organization. The core goal of this article is to depict a novel fuzzy multi-criteria decision-making methodology for selecting employees. The purpose behind selecting employees is to identify and hire individuals who possess the required skills, qualifications, and attributes that align with the organization's goals and job requirements. To reflect an inadequate assessment, ambiguity, and anxiety in making choices, the intuitionistic fuzzy hypersoft expert set is an extension of the intuitionistic fuzzy soft expert and hypersoft sets. It is a novel approach to decisions and intelligent computing in the face of uncertainty. The intuitionistic fuzzy hypersoft expert set has a better ability to handle ambiguous and unclear data. In the research that follows, the ideas and characteristics of the correlation coefficient and the weighted correlation coefficient of the intuitionistic fuzzy hypersoft expert sets are proposed. Under the aegis of intuitionistic fuzzy hypersoft expert sets, a TOPSIS based on correlation coefficients and weighted correlation coefficients is introduced. Aside from that, we also covered aggregation operators, including intuitionistic fuzzy hypersoft weighted geometric operators. The decision-making process is suggested in an intuitionistic fuzzy hypersoft expert environment to resolve uncertain and ambiguous information, relying on the well-established TOPSIS approach and aggregation operators. An illustration of decision-making challenges shows how the suggested algorithm can be used. The efficacy of this strategy is lastly demonstrated by comparing its benefits, effectiveness, flexibility, and numerous current studies.

Keywords Intuitionistic fuzzy soft set · Intuitionistic fuzzy soft expert set · Hypersoft set · Intuitionistic fuzzy hypersoft expert set

1 Introduction

Optimal resource utilization is a crucial aspect in various fields, including computer science, engineering, medical science, and others. In computer science, efficient algorithms and data structures facilitate the maximum utilization of computational resources, ensuring speedy and accurate data processing [1–3]. In engineering, the judicious use of materials and energy is paramount, enabling sustainable designs and minimizing waste production [4–6]. Medical science focuses on resource allocation to enhance patient care, such as optimizing healthier facilities and personnel distribution to achieve equitable access and efficient healthier delivery [7]. In social sciences, understanding and effectively utilizing resources such as data, research methodologies, and human capital contribute to informed decision-making and the development of policies that address societal challenges [8–10]. Each of these fields recognizes the importance of optimal resource utilization,

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applying innovative strategies to achieve their respective goals while minimizing waste and maximizing efficiency. Moreover, the effective management of human resources plays a pivotal role in driving success across these disciplines [11, 12].

Human resource management is the process of managing an organization's human resources in pursuit of organizational objectives [13]. It consists of both legal and administrative aspects, the latter relating to all aspects of the recruitment, selection, placement, compensation, and evaluation of individuals in order to provide all employees with an opportunity for optimal performance, access to training and development opportunities, and ensure their satisfaction. Multi-criteria decision making (MCDM) [14] recognizes that it is not only beneficial but also essential to operationalize decision criteria. The candidates are usually assessed on the basis of a set of criteria based on several soft competencies, such as communication skills or leadership ability, which cannot be expressed numerically. On the other hand, numerous workplaces actually require more than one type of formal qualification for an employee to get a job; this demand often leads to a conflict between soft and hard completeness and may even discourage soft skills from being taken into account as part of an employee's evaluation. When different rating criteria are combined in a single decision rule, they are treated as separate variables in this approach. This approach allows decision-makers to select candidates based on other factors than just their objective scores. In addition, decision-makers who will ultimately make the final selection may give different priorities to the selection criteria. These priorities are rarely expressed through crisp values. For instance, a human resources manager may regard communication skills as "important" or "very important" in the decision-making process. Candidate selection methods, also called hiring criteria, are the procedures used to identify candidates that meet specific requirements; they help decision-makers choose between different applicants. At the same time, they must be flexible enough to accept diversity in their judgments and practices. A lot of research [15] has been conducted on this subject to develop better candidate selection processes. Moreover, any systematic methodology for selecting human resources must cater to these multiple criteria and also give decision-makers the opportunity to express their own subjectivity in a straightforward manner. One class of approaches [16] that deal with subjectivity includes techniques based on the well-known TOPSIS, which reduces complex decisions to a series of pairwise comparisons and synthesizes the results. Multi-attribute decision-making [17] is a particular type of decision-making, where several sources of uncertainty must be taken into account. Its purpose is probably to differentiate the most specific alternative from feasible alternatives. The person making the decision needs to evaluate a given choice through various types of evaluation conditions, such as numbers and intervals. However, it is difficult for one person in several circumstances, since there are various uncertainties within the data, to pick out the suitable one due to a lack of expertise or infraction. In situations where the available knowledge is ambiguous or vague, conventional logic may not always be relevant. A particular class of sets known as a fuzzy set ($\mathcal{FS}$) (started by Zadeh [18]) is thought to be well-suited to handle these kinds of situations. In such a collection, each component of the starting universe is given a corresponding grade between [0, 1]. However, the $\mathcal{FS}$ proved to be ineffective in dealing with increasingly complicated and uncertain real-life circumstances, leading Atanassov [19] to develop a novel model, the intuitionistic fuzzy set ($\mathcal{IFS}$), which is better at handling ambiguity in data. Each initial universe component is given a grade of belonging or non-belonging if and only if their sum falls between [0, 1]. Furthermore, $\mathcal{IFS}$ excel at simulating the supplied data more precisely and logically. The contributions of Deli and Keles [20], Mahmood et al. [21], Unver et al. [22], and Wang and Garg [23] are more noteworthy in terms of the extensions of $\mathcal{FS}$ and $\mathcal{IFS}$. They introduced novel extensions of $\mathcal{IFS}$ and applied them in real-world problems through algorithm-based methods for making decisions. In a recent study by the authors [24], a decision-making approach was developed for selecting a cache placement strategy in content-centric networking (CCN)(approach used to determine the optimal placement of caches or storage nodes in Content-Centric Networking architecture). This approach utilized the combined compromise solution (CoCoSo) method (a method used in MCDM to find a compromise solution among multiple criteria) and incorporated criteria importance analysis through inter-criteria correlation (CRITIC)(approach focuses on determining the relative importance or weights of the criteria in the decision-making process). The study focused on the $\mathcal{IFS}$, which have extensive applications in fuzzy logic, soft computing, and various other domains. However, it was observed that the results obtained from $\mathcal{IFS}$ solely captured the interval-based information due to its consideration of membership and non-membership values. This led to a limitation in reflecting the complete information available. In other words, the $\mathcal{IFS}$ approach only addressed the data intervals with insufficient information. Smarandache [25] proposed the neutrosophic set $\mathcal{NS}$ to address these restrictions after it was found that both $\mathcal{FS}$ and $\mathcal{IFS}$ cannot take the grade of indeterminacy into account. The $\mathcal{NS}$ is better at maintaining the fuzziness in the contents of perceived data and could greatly help rough thinking. Due to taking into account of a neutrality level, the expressive capacity of $\mathcal{NS}$ is higher than that of the traditional $\mathcal{FS}$ and $\mathcal{IFS}$, but its computational cost is also higher than that of $\mathcal{FS}$ and $\mathcal{IFS}$. The previous structures revealed some flaws when the parameterization tool was taken into account. Molodtsov [26] created the soft set ($\mathcal{SS}$) as a brand-new mathematical technique to address this deficiency. In the $\mathcal{SS}$, an estimated mapping is utilized to relate a set of characteristics to the universe's power set. Convexity in a soft set is described by Rahman et al. [27], who also discussed its many results. In order to make use of $\mathcal{SS}$ in uncertain environment, fuzzy soft set($\mathcal{FSS}$) was developed by the Maji et al. [28] with the combination of $\mathcal{SS}$ and $\mathcal{FS}$. Rahman et al. [29] discussed some types of convexity in $\mathcal{FSS}$ and proved its authenticated results. In order to make decisions, Roy et al. [30] introduced a novel approach to object detection from erroneous multi-observer data. Adam and Hassan [31] introduced the Q-$\mathcal{FSS}$ model and explored its characteristics using in-depth computational instances. They presented several processes and used them in various applications in [32]. Basu et al. [33]'s usage of $\mathcal{FSS}$ in medical science by way of the example of a making decision. A$\mathcal{IFSS}$ was created by Maji et al. [34] by combining a $\mathcal{IFS}$ and $\mathcal{SS}$. The concept of a ($\mathcal{SS}$) is commonly employed to handle situations where a single expert's opinion is considered. However, Alkhazaleh and Salleh [35] introduced the notion of a soft expert set ($\mathcal{SES}$) in the literature. This extension was proposed to incorporate multiple experts' perspectives within a single model, thereby enhancing the awareness of diverse expert opinions. Convexity was presented by Ihsan et al. [?] in the context of $\mathcal{SES}$, and they provided some key properties. Alkhazaleh and Saleh [36] created the fuzzy soft expert set $\mathcal{FSES}$ by integrating their research on $\mathcal{SES}$ into a fuzzy environment and applying it to making choices. An addition to Alkhazaleh's approach was created by Broumi et al. [37] by including an intuitionistic fuzzy soft expert set ($\mathcal{IFSES}$). After this effort, the non-membership function issue that developed in the$\mathcal{FSES}$ system has been resolved. Many researchers have made use of $\mathcal{SS}$ as a parameter tool. But they faced problems when parameters were found in the form of sub-parameters having disjoint parametric set values. Smarandache [38] an American mathematician, solved this problem by splitting up the parametric values into the required form. He used this idea in the form of the hypersoft set ($\mathcal{HSS}$) instead of a $\mathcal{SS}$. Saeed et al. [39, 40] made an extension to the basic characteristics and operations of $\mathcal{HSS}$ by discussing numerical cases. In 2020, Rahman et al. [41–43] converted different structures of $\mathcal{SS}$ like complex fuzzy, rough, fuzzy parameterized, and neutrosophic, into $\mathcal{HSS}$. Saeed et al. [44] also converted some structures of $\mathfrak{S}$s like complex multi-fuzzy, into $\mathfrak{h}$ss and discussed mappings in this new structure. Yolcu et al. [45, 46] took an interest in $\mathcal{HSS}$ and made changes to this model by applying membership and non-membership degrees to it and introducing fuzzy as well as intuitionistic fuzzy $\mathcal{HSS}$. Saqlain et al. [47–49] contributed to $\mathcal{HSS}$ by introducing single and multi-valued neutrosophic $\mathcal{HSS}$ and calculating their tangent similarity measures. They also brought about changes in this structure by finding the aggregation operators and TOPSIS method for neutrosophic $\mathcal{HSS}$. All the above-described structures of $\mathcal{HSS}$ have certain drawbacks in the form of the multi-decisive opinions of experts. To see this problem, Ihsan et al. [50] changed the structure of $\mathcal{HSS}$ into the hypersoft expert set ($\mathcal{HSES}$). They successfully [51] did an inclusive study on the fundamentals of $\mathcal{HSES}$. Then, they [52] embedded this structure into a fuzzy environment by introducing a fuzzy hypersoft expert set ($\mathcal{FHSES}$).

1.1 Research gap and motivation

In contrast to most earlier publications, which defined the correlation on other sets, different methods [53] for calculating the correlation coefficient ($\mathbb{CC}$) for fuzzy data were based on mathematical statistics. The result of the algorithm is not only the degree of the $\mathcal{FSS}$ link but also whether they are positively or negatively associated. Gerstenkorn and Mańko [54] developed a coefficient of such a correlation and looked at its features. They defined a function measuring the interrelation of $\mathcal{IFSS}$, the so-called correlation of these sets. In order to calculate the correlation and $\mathbb{CC}$ of intuitionistic fuzzy sets, which are comparable to the cosine of the intersectional angle in finite sets and probability space, respectively, Zeng and Li [55] considered all three parameters describing an intuitionistic fuzzy set from the perspective of the geometrical representation of an intuitionistic fuzzy set. They also examined some of their characteristics and provided three numerical examples to fairly demonstrate the suggested approach. Kumar and Garg [56] provided a paper for rating various product choices using set pair analysis (SPA). They created a connection number, which is a key element of SPA, using the object's comprehensive ideal values and preference values. In accordance with the suggested connection number of SPA, an extension of the TOPSIS approach is added to determine the relative proximity of sets of alternatives that are utilized to provide the ranking order of the alternatives. To illustrate the applicability and reliability of the suggested methodology, a real-world case is used. Wan et al. [57] suggested a brand-new $\mathcal{IFSS}$ risk attitude rating methodology and applied it to a MADM with missing weight data. They used the proximity degree to characterize the amount of information according to the geometrical depiction of an IFS, which was inspired by the TOPSIS. To assess the accuracy of the data, they computed the area of a triangle. Luo and Ren [58] created a new similarity measure of intuitionistic fuzzy sets and gradually employed it in pattern recognition and medical diagnosis to get over-the-counter-intuitive in some situations. They proposed a new approach to the MADM problem that uses an intuitionistic fuzzy set to express attribute values. Hanafy [59] suggested a technique for figuring out the $\mathbb{CC}$ of neutrosophic sets. The results of this method allowed us to determine the degree of relationship and if there is a positive or negative relationship between the neutrosophic sets. Based on the score functions of normal neutrosophic numbers (NNNs), the fundamental components of normal neutrosophic sets (NNSs), Ye [60] proposed two $\mathbb{CC}$ between NNSs and looked into their qualities. Then, using a MADM technique with NNSs in typical neutrosophic environments, he was able to determine the ranking order of alternatives and the best option during a typical neutrosophic decision-making process by comparing the correlation coefficient values between each evaluated NNS and the ideal NNS. The use and viability of the created decision-making process are finally shown by an example illustrating the selection dilemma of investment choices. On the basis of single-valued neutrosophic information measurements, Wu et al. [61] created a MADM technique. Three axiomatic definitions of information measures have been established. Entropy, similarity measure, and cross-entropy were some of them. Then, using the cosine function as a foundation, they created information-measurement formulas. The link between entropy, similarity measure, and cross-entropy, as well as their mutual transformations, were then discussed. On the basis of these data measurement formulas, they also presented a method for single-valued neutrosophic MADM. By expanding the TOPSIS to a single-valued neutrosophic setting, Biswas et al. [62] proposed a novel method for multi-attribute group decision-making situations. Ratings of options in relation to each feature are thought of as single-valued neutrosophic sets that express the judgment of the decision makers given the data presented. To address the issue of risk assessment, Chang et al. [63] suggested a soft TOPSIS technique. Finally, he included a case study of the initial design, production, and development of the notebook module to show the viability and logic of the suggested process even with little data. The $\mathbb{CC}$ was first [64] established for fuzzy sets, and it was then expanded to the fuzzy soft set environment. The generalized normalized correlation efficiency, which is derived from the generalized normalized correlation efficiency, has been used as a measure of the objective weights of the decision-maker's knowledge in multiple-attribute group decision-making (MAGDM) situations from a statistical and cognitive perspective. Under the dual hesitant fuzzy soft set environment, where pairs of membership and non-membership are taken into consideration as vector representations during the formulation and to investigate their properties, Arora and Garg [65] created $\mathbb{CC}$ and weighted $\mathbb{CC}$. They also presented an MCDM based on the proposed $\mathbb{CC}$ in this context. To show the effectiveness of the suggested approach, three numerical examples from the selection process, medical diagnosis, and pattern recognition are used, and their findings are compared to those of several other current approaches. Garg and Arora [66] introduced the TOPSIS for information on intuitionistic fuzzy soft sets and presented $\mathbb{CC}$ for intuitionistic fuzzy soft sets. They also looked at a handful of features discovered by these measures. They developed a method to resolve decision-making issues using the proposed TOPSIS method based on correlation measures in light of these methodologies. Finally, they provided a clear illustration to show how the suggested strategy is suitable. After this Zulqarnain et al. [67] introduced $\mathbb{CC}$ and its characteristics on intuitionistic fuzzy hypersoft set. But this structure lacks expert opinions. The purpose of this article is to address the present problems and limitations associated with traditional employee selection practices in the field of human resource management. By giving a thorough strategy that comprises a modified TOPSIS, correlation analysis, and aggregation procedures, the study seeks to provide a novel fuzzy multi-criteria decision-making methodology for staff selection. This methodology attempts to improve the hiring process' effectiveness, efficiency, and accuracy, which will eventually lead to better hiring decisions and higher organizational performance. Upon reviewing the literature, it is evident that soft set-like structures handle a single expert opinion in a specific manner. However, there are certain scenarios where: (a) Multiple decision-makers's opinions need to be integrated within a single model, such as when using survey responses. (b) Non-overlapping sub-parameter sets are present as specific parameters. (c) Information data exist in both membership and non-membership forms. These circumstances necessitate alternative approaches for managing and incorporating diverse opinions and data formats effectively. The literature analysis also on the MADM mentioned previously offers inspiration (motivation) for employing $\mathcal{IFHSEN}$s to solve MADM problems in addition to creating a $\mathbb{CC}$-based strategy. The research article has the following primary objectives:

(1) To explain the $\mathbb{CC}$ measure and weighted $\mathbb{CC}$ using $\mathcal{IFHSEN}$s, highlighting their significant characteristics.
(2) The second objective of the research article is to extend the MADM approach by incorporating the $\mathbb{CC}$ measure, weighted $\mathbb{CC}$ measure, and aggregation operators.

After considering these scenarios, the literature requires a new framework to close these gaps. As a result, the $\mathbb{CC}$, features have been used to characterize a new structure called the $\mathcal{IFHSE}$-Set, and a suggested approach is given in MADM for a particular application. The technical novelty of the study lies in its integration of several elements to address HRM decision-making challenges. Here are potential technical novelties associated with the study:

(1) A modified version of the TOPSIS approach that has been specially designed for HRM applications is introduced in the paper. In order to address the ambiguity and imprecision present in HRM decision-making, this update may involve the use of intuitionistic fuzzy hypersoft expert sets computing techniques. The improved TOPSIS technique might improve the efficacy and accuracy of HRM decision results.

(2) In the work, intuitionistic fuzzy sets and hypersoft computing are combined in an environment called an intuitionistic fuzzy hypersoft expert system. This setting offers a thorough framework for HRM decision-making that takes into account both an element's degree of membership and non-membership as well as ambiguity and imprecision. The application of this environment specifically to HRM and its potential advantages in addressing difficult HRM decision scenarios are what make it distinctive.

(3) Within the integrated method, the study uses correlation coefficient analysis. Correlation coefficients are useful tools for decision-making because they quantify the connections and interdependencies between variables or criteria. This study's integration of correlation coefficient analysis with other methods in the context of human resource management may be a first.

1.2 Main contributions

This article has the following main features:

(1) In this paper, a mathematical model for the $\mathcal{IFHSES}$-Set is presented. It describes the mathematical $\mathcal{IFHSES}$-Set's set-theoretic procedures.

(2) The $\mathbb{CC}$'s introduction and advantages over the $\mathcal{IFHSES}$-Set.

(3) On the basis of the $\mathbb{CC}$, aggregation operators are being developed, and the TOPSIS approach is being extended.

(4) The suggested process-based algorithm is presented in this study. An example problem-based scenario has been designed to assess its applicability in real-world circumstances.

(5) The structural comparison process has been approached from several angles.

The remainder of the paper has been divided into seven major sections: Sect. 2 provides some background information on some terms related to the article's main body. The specifics of the $\mathcal{IFHSE}$-Set with characteristics are described in Sect. 3. The definition of the $\mathbb{CC}$ and its key features are covered in Sect. 4, and in Sect. 5, the application of this concept is illustrated by the description of a few aggregation operators and a suggested algorithm. The TOPSIS approach has been applied to the decision-making problem in Sect. 6 on the basis of $\mathbb{CC}$, and an example is also provided. A discussion with comparison has been conducted in Sect. 7, and in Sect. 8, a conclusion to the article has been reached by outlining its potential future directions.

2 Preliminary knowledge

This background information offers some fundamental concepts and terms that facilitate comprehension of the primary study. With the power sets $P(\widehat{\Theta})$ and $\widetilde{\aleph}$ for a given set of parameters, $\widehat{\Theta}$ will be used in this article to represent the universe of discourse. The τ is equal to $\widetilde{\aleph} \times \widetilde{\triangledown} \times \widetilde{\Xi}$. Consider the symbol $\widetilde{\triangledown}$ as a group of experts and $\widetilde{\Xi}$ as a group of conclusions.

Definition 2.1 [18] A collection of order pairs is named as a soft set Γ_S over $\widehat{\Theta}$ and is written as $\Gamma_S = \{(y, \gamma_S(y)) : y \in \widetilde{\aleph}, \gamma_S(y) \in P(\widehat{\Theta})\}$, where S is a subset of set of parameters and γ_S is named as an approximate function of Γ_S and is defined from set of parameters to power set of universe. Some necessarily operations of Γ_S can be described as

(1) $(\Psi_{\breve{M}}, \breve{M}) \subseteq (\Psi_{\breve{N}}, \breve{N})$ whenever $\breve{M} \subseteq \breve{N}$ and $\Psi_{\breve{M}}(o) \subseteq \Psi_{\breve{N}}(o)$ for all $o \in \breve{M}$.

(2) $(\Psi_{\breve{M}}, \breve{M}) \sqcup (\Psi_{\breve{N}}, \breve{N}) = (\Psi_{\breve{Q}}, \breve{P})$, where $\breve{P} = \breve{M} \cup \breve{N}$, $\Psi_{\breve{Q}}(\ltimes) = \Psi_{\breve{M}}(\ltimes)$ 0r $\Psi_{\breve{N}}(\ltimes)$.

(3) $(\Psi_{\breve{M}}, \breve{M}) \sqcap (\Psi_{\breve{N}}, \breve{N}) = (\Psi_{\breve{Q}}, \breve{P})$, where $\breve{P} = \breve{M} \cap \breve{N}$, $\Psi_{\breve{Q}}(\ltimes) = \Psi_{\breve{M}}(\ltimes)$ 0r $\Psi_{\breve{N}}(\ltimes)$.

Definition 2.2 [28] A collection of order pairs is named as a soft expert set $\ddot{\Gamma}$ over $\widehat{\Theta}$ and is written as $\ddot{\Gamma} = \{(t, \ddot{\gamma}(t)) : t \in \tau, \ddot{\gamma}(t) \in P(\widehat{\Theta})\}$, where $\ddot{\gamma}$ is named as an approximate function of $\ddot{\Gamma}$ and is defined from τ to power set of universe.

Definition 2.3 [33] Suppose distinct attributes are represented by $r_1, r_2, r_3, \ldots, r_m$, for $m \geq 1$ and $\widetilde{\text{Ⅎ}}_1, \widetilde{\text{Ⅎ}}_2, \widetilde{\text{Ⅎ}}_3, \ldots, \widetilde{\text{Ⅎ}}_m$, with $\widetilde{\text{Ⅎ}}_{\barwedge} \cap \widetilde{\text{Ⅎ}}_{\veebar} = \emptyset$, for $\barwedge \neq \veebar$, and $\barwedge, \veebar$ belong to finite positive integer set are the sets of corresponding attributes values, respectively. Then, a hypersoft set is defined by a mapping $\Psi_{\breve{Ⅎ}} : \widetilde{} \rightarrow P(\widehat{\Theta})$ where $\widetilde{} = \widetilde{Ⅎ}_1 \times \widetilde{Ⅎ}_2 \times \widetilde{Ⅎ}_3 \times \cdots \times \widetilde{Ⅎ}_m$.

Definition 2.4 [50] A hypersoft expert set ϖ is characterized by an approximate function $\Phi_{\varpi} : \ddot{\mathbb{M}} \rightarrow P(\widehat{\Theta})$ which is defined by approximate elements $\Phi_{\varpi}(\hat{v})$ for all members $\hat{v}$ of $\ddot{\mathbb{M}}$ where $\mathbb{M} \subseteq \widetilde{T} = \aleph \times \widetilde{\nabla} \times \widetilde{\Xi}$ such that $\aleph = \widetilde{\curlywedge}_1 \times \widetilde{\curlywedge}_2 \times \widetilde{\curlywedge}_3 \times \cdots \times \widetilde{\curlywedge}_m$.

Definition 2.5 [51] A fuzzy hypersoft expert set ς over $\widehat{\Theta}$ is clarified by as $\omega_{\varsigma} : \widetilde{S} \rightarrow I^{\widehat{\Theta}}$ where $I^{\widehat{\Theta}}$ is collection of all fuzzy subsets of $\widehat{\Theta}$, $\widetilde{S} \subseteq \widetilde{T} = \aleph \times \widetilde{\nabla} \times \widetilde{\Xi}$ such that $\aleph = \widetilde{\curlywedge}_1 \times \widetilde{\curlywedge}_2 \times \widetilde{\curlywedge}_3 \times \cdots \times \widetilde{\curlywedge}_m$.

3 Intuitionistic fuzzy hypersoft expert set ($\mathcal{IFHSE}$-set)

This portion describes the definition of $\mathcal{IFHSE}$-Set with some essential properties. Examples are also given for the understanding of this structure.

Definition 3.1 A pair $(\omega_{\varsigma}, \widetilde{S})$ is named as an $\mathcal{IFHSE}$-Set over $\widehat{\Theta}$ if $\omega_{\varsigma} : \widetilde{S} \rightarrow IF^{\widehat{\Theta}}$ with $IF^{\widehat{\Theta}}$ is collection of all intuitionistic fuzzy subsets of $\widehat{\Theta}$.

Example 3.2 Suppose that Mr. Lee wishes to purchase a computer from a market. There are four types of computer available in market forming the set of discourse $\mathfrak{Z} = \{\widehat{\Upsilon}_1, \widehat{\Upsilon}_2, \widehat{\Upsilon}_3, \widehat{\Upsilon}_4\}$. The choice of computer may be carried out by keeping in mind the following attributes, i.e., γ_1 = price, γ_2 = size, and γ_3 = weight, γ_4 = material. Following are the attribute-valued sets corresponding to these attributes are: $\tau_{\gamma 1} = \{\gamma_{11}, \gamma_{12}\}$, $\tau_{\gamma 2} = \{\gamma_{21}, \gamma_{22}\}$, $\tau_{\gamma 3} = \{\gamma_{31}, \gamma_{32}\}$, then $\tau_{\gamma} = \tau_{\gamma 1} \times \tau_{\gamma 2} \times \tau_{\gamma 3}$

$$\tau_{\gamma} = \left\{ \begin{array}{c} (1 = \{\gamma_{11}, \gamma_{21}, \gamma_{31}\}), (2 = \{\gamma_{11}, \gamma_{21}, \gamma_{32}\}), (3 = \{\gamma_{11}, \gamma_{22}, \gamma_{31}\}), \\ (4 = \{\gamma_{11}, \gamma_{22}, \gamma_{32}\}), (5 = \{\gamma_{12}, \gamma_{21}, \gamma_{31}\}), (6 = \{\gamma_{12}, \gamma_{21}, \gamma_{32}\}), \\ (7 = \{\gamma_{12}, \gamma_{22}, \gamma_{31}\}), (8 = \{\gamma_{12}, \gamma_{22}, \gamma_{32}\}) \end{array} \right\}.$$

Now, $\mathfrak{H} = \tau_{\gamma} \times \widetilde{Y} \times \widetilde{O}$

$$\mathfrak{H} = \left\{ \begin{array}{c} (1, \mathfrak{a}, 0), (1, \mathfrak{a}, 1), (1, \mathfrak{a}, 0), (1, \mathfrak{a}, 1), (1, \mathfrak{a}, 0), (1, \mathfrak{a}, 1), \\ (2, \mathfrak{a}, 0), (2, \mathfrak{a}, 1), (2, \mathfrak{a}, 0), (2, \mathfrak{a}, 1), (2, \mathfrak{a}, 0), (2, \mathfrak{a}, 1), \\ (3, \mathfrak{a}, 0), (3, \mathfrak{a}, 1), (3, \mathfrak{a}, 0), (3, \mathfrak{a}, 1), (3, \mathfrak{a}, 0), (3, \mathfrak{a}, 1), \\ (4, \mathfrak{a}, 0), (4, \mathfrak{a}, 1), (4, \mathfrak{a}, 0), (4, \mathfrak{a}, 1), (4, \mathfrak{a}, 0), (4, \mathfrak{a}, 1), \\ (5, \mathfrak{a}, 0), (5, \mathfrak{a}, 1), (5, \mathfrak{a}, 0), (5, \mathfrak{a}, 1), (5, \mathfrak{a}, 0), (5, \mathfrak{a}, 1), \\ (6, \mathfrak{a}, 0), (6, \mathfrak{a}, 1), (6, \mathfrak{a}, 0), (6, \mathfrak{a}, 1), (6, \mathfrak{a}, 0), (6, \mathfrak{a}, 1), \\ (7, \mathfrak{a}, 0), (7, \mathfrak{a}, 1), (7, \mathfrak{a}, 0), (7, \mathfrak{a}, 1), (7, \mathfrak{a}, 0), (7, \mathfrak{a}, 1), \\ (8, \mathfrak{a}, 0), (8, \mathfrak{a}, 1), (8, \mathfrak{a}, 0), (8, \mathfrak{a}, 1), (8, \mathfrak{a}, 0), (8, \mathfrak{a}, 1) \end{array} \right\}$$

let

$$\widetilde{S} = \left\{ \begin{array}{c} (1, \mathfrak{a}, 0), (1, \mathfrak{a}, 1), (1, \mathfrak{a}, 0), (1, \mathfrak{a}, 1), (1, \mathfrak{a}, 0), (1, \mathfrak{a}, 1), \\ (2, \mathfrak{a}, 0), (2, \mathfrak{a}, 1), (2, \mathfrak{a}, 0), (2, \mathfrak{a}, 1), (2, \mathfrak{a}, 0), (2, \mathfrak{a}, 1) \\ (3, \mathfrak{a}, 0), (3, \mathfrak{a}, 1), (3, \mathfrak{a}, 0), (3, \mathfrak{a}, 1), (3, \mathfrak{a}, 0), (3, \mathfrak{a}, 1), \end{array} \right\}$$

be a subset of $\mathfrak{H}$ and $\widetilde{Y} = \{\mathfrak{a}, \mathfrak{a}, \mathfrak{a},\}$ represents the set of specialists. Following survey depicts choices of three specialists:

$$\Lambda_1 = \Lambda(_1, \alpha, 1) = \left\{ \frac{\widehat{\widehat{\Upsilon}}_1}{<0.2, 0.4>}, \frac{\widehat{\Upsilon}_2}{<0.6, 0.2>}, \frac{\widehat{\Upsilon}_3}{<0.5, 0.4>}, \frac{\widehat{\Upsilon}_4}{<0.1, 0.3>} \right\},$$

$$\Lambda_2 = \Lambda(_1, \alpha, 1) = \left\{ \frac{\widehat{\Upsilon}_1}{<0.3, 0.2>}, \frac{\widehat{\Upsilon}_2}{<0.8, 0.1>}, \frac{\widehat{\Upsilon}_3}{<0.4, 0.5>}, \frac{\widehat{\Upsilon}_4}{<0.2, 0.5>} \right\},$$

$$\Lambda_3 = \Lambda(_1, \alpha, 1) = \left\{ \frac{\widehat{\Upsilon}_1}{<0.6, 0.2>}, \frac{\widehat{\Upsilon}_2}{<0.5, 0.3>}, \frac{\widehat{\Upsilon}_3}{<0.6, 0.3>}, \frac{\widehat{\Upsilon}_4}{<0.3, 0.5>} \right\},$$

$$\Lambda_4 = \Lambda(_2, \alpha, 1) = \left\{ \frac{\widehat{\Upsilon}_1}{<0.8, 0.1>}, \frac{\widehat{\Upsilon}_2}{<0.4, 0.5>}, \frac{\widehat{\Upsilon}_3}{<0.7, 0.2>}, \frac{\widehat{\Upsilon}_4}{<0.3, 0.4>} \right\},$$

$$\Lambda_5 = \Lambda(_2, \alpha, 1) = \left\{ \frac{\widehat{\Upsilon}_1}{<0.3, 0.5>}, \frac{\widehat{\Upsilon}_2}{<0.8, 0.1>}, \frac{\widehat{\Upsilon}_3}{<0.3, 0.6>}, \frac{\widehat{\Upsilon}_4}{<0.2, 0.6>} \right\},$$

$$\Lambda_6 = \Lambda(_2, \alpha, 1) = \left\{ \frac{\widehat{\Upsilon}_1}{<0.5, 0.4>}, \frac{\widehat{\Upsilon}_2}{<0.3, 0.5>}, \frac{\widehat{\Upsilon}_3}{<0.6, 0.2>}, \frac{\widehat{\Upsilon}_4}{<0.8, 0.1>} \right\},$$

$$\Lambda_7 = \Lambda(_3, \alpha, 1) = \left\{ \frac{\widehat{\Upsilon}_1}{<0.2, 0.6>}, \frac{\widehat{\Upsilon}_2}{<0.8, 0.1>}, \frac{\widehat{\Upsilon}_3}{<0.4, 0.5>}, \frac{\widehat{\Upsilon}_4}{<0.5, 0.4>} \right\},$$

$$\Lambda_8 = \Lambda(_3, \alpha, 1) = \left\{ \frac{\widehat{\Upsilon}_1}{<0.4, 0.5>}, \frac{\widehat{\Upsilon}_2}{<0.6, 0.3>}, \frac{\widehat{\Upsilon}_3}{<0.7, 0.2>}, \frac{\widehat{\Upsilon}_4}{<0.9, 0.1>} \right\},$$

$$\Lambda_9 = \Lambda(_3, \alpha, 1) = \left\{ \frac{\widehat{\Upsilon}_1}{<0.7, 0.2>}, \frac{\widehat{\Upsilon}_2}{<0.3, 0.6>}, \frac{\widehat{\Upsilon}_3}{<0.5, 0.4>}, \frac{\widehat{\Upsilon}_4}{<0.2, 0.7>} \right\},$$

$$\Lambda_{10} = \Lambda(_1, \alpha, 0) = \left\{ \frac{\widehat{\Upsilon}_1}{<0.3, 0.5>}, \frac{\widehat{\Upsilon}_2}{<0.2, 0.4>}, \frac{\widehat{\Upsilon}_3}{<0.4, 0.5>}, \frac{\widehat{\Upsilon}_4}{<0.1, 0.8>} \right\},$$

$$\Lambda_{11} = \Lambda(_1, \alpha, 0) = \left\{ \frac{\widehat{\Upsilon}_1}{<0.1, 0.8>}, \frac{\widehat{\Upsilon}_2}{<0.8, 0.1>}, \frac{\widehat{\Upsilon}_3}{<0.6, 0.3>}, \frac{\widehat{\Upsilon}_4}{<0.2, 0.7>} \right\},$$

$$\Lambda_{12} = \Lambda(_1, \alpha, 0) = \left\{ \frac{\widehat{\Upsilon}_1}{<0.2, 0.7>}, \frac{\widehat{\Upsilon}_2}{<0.1, 0.7>}, \frac{\widehat{\Upsilon}_3}{<0.3, 0.5>}, \frac{\widehat{\Upsilon}_4}{<0.5, 0.4>} \right\},$$

$$\Lambda_{13} = \Lambda(_2, \alpha, 0) = \left\{ \frac{\widehat{\Upsilon}_1}{<0.5, 0.1>}, \frac{\widehat{\Upsilon}_2}{<0.3, 0.6>}, \frac{\widehat{\Upsilon}_3}{<0.5, 0.4>}, \frac{\widehat{\Upsilon}_4}{<0.7, 0.2>} \right\},$$

$$\Lambda_{14} = \Lambda(_2, \alpha, 0) = \left\{ \frac{\widehat{\Upsilon}_1}{<0.7, 0.2>}, \frac{\widehat{\Upsilon}_2}{<0.2, 0.5>}, \frac{\widehat{\Upsilon}_3}{<0.9, 0.1>}, \frac{\widehat{\Upsilon}_4}{<0.4, 0.5>} \right\},$$

$$\Lambda_{15} = \Lambda(_2, \alpha, 0) = \left\{ \frac{\widehat{\Upsilon}_1}{<0.6, 0.3>}, \frac{\widehat{\Upsilon}_2}{<0.7, 0.2>}, \frac{\widehat{\Upsilon}_3}{<0.3, 0.5>}, \frac{\widehat{\Upsilon}_4}{<0.2, 0.7>} \right\},$$

$$\Lambda_{16} = \Lambda(_3, \alpha, 0) = \left\{ \frac{\widehat{\Upsilon}_1}{<0.1, 0.6>}, \frac{\widehat{\Upsilon}_2}{<0.4, 0.5>}, \frac{\widehat{\Upsilon}_3}{<0.7, 0.2>}, \frac{\widehat{\Upsilon}_4}{<0.8, 0.2>} \right\},$$

$$\Lambda_{17} = \Lambda(_3, \alpha, 0) = \left\{ \frac{\widehat{\Upsilon}_1}{<0.2, 0.6>}, \frac{\widehat{\Upsilon}_2}{<0.9, 0.1>}, \frac{\widehat{\Upsilon}_3}{<0.8, 0.2>}, \frac{\widehat{\Upsilon}_4}{<0.3, 0.5>} \right\},$$

$$\Lambda_{18} = \Lambda(_3, \alpha, 0) = \left\{ \frac{\widehat{\Upsilon}_1}{<0.5, 0.3>}, \frac{\widehat{\Upsilon}_2}{<0.3, 0.6>}, \frac{\widehat{\Upsilon}_3}{<0.6, 0.3>}, \frac{\widehat{\Upsilon}_4}{<0.1, 0.8>} \right\}.$$

The $\mathcal{IFHSE}$-Set can be described as

$$(\Lambda,\widetilde{S})=\left\{\begin{array}{l}
\left((_1,\alpha,1),\left\{\frac{\widehat{\Upsilon}_1}{<0.2,0.4>},\frac{\widehat{\Upsilon}_2}{<0.6,0.2>},\frac{\widehat{\Upsilon}_3}{<0.5,0.4>},\frac{\widehat{\Upsilon}_4}{<0.1,0.5>}\right\}\right),\\
\left((_1,\alpha,1),\left\{\frac{\widehat{\Upsilon}_1}{<0.3,0.2>},\frac{\widehat{\Upsilon}_2}{<0.8,0.1>},\frac{\widehat{\Upsilon}_3}{<0.4,0.5>},\frac{\widehat{\Upsilon}_4}{<0.2,0.5>}\right\}\right),\\
\left((_1,\alpha,1),\left\{\frac{\widehat{\Upsilon}_1}{<0.6,0.2>},\frac{\widehat{\Upsilon}_2}{<0.5,0.3>},\frac{\widehat{\Upsilon}_3}{<0.6,0.3>},\frac{\widehat{\Upsilon}_4}{<0.3,0.5>}\right\}\right),\\
\left((_2,\alpha,1),\left\{\frac{\widehat{\Upsilon}_1}{<0.8,0.1>},\frac{\widehat{\Upsilon}_2}{<0.4,0.5>},\frac{\widehat{\Upsilon}_3}{<0.7,0.2>},\frac{\widehat{\Upsilon}_4}{<0.3,0.4>}\right\}\right),\\
\left((_2,\alpha,1),\left\{\frac{\widehat{\Upsilon}_1}{<0.3,0.5>},\frac{\widehat{\Upsilon}_2}{<0.8,0.1>},\frac{\widehat{\Upsilon}_3}{<0.3,0.6>},\frac{\widehat{\Upsilon}_4}{<0.2,0,6>}\right\}\right),\\
\left((_2,\alpha,1),\left\{\frac{\widehat{\Upsilon}_1}{<0.5,0.4>},\frac{\widehat{\Upsilon}_2}{<0.3,0.5>},\frac{\widehat{\Upsilon}_3}{<0.6,0.2>},\frac{\widehat{\Upsilon}_4}{<0.8,0.1>}\right\}\right),\\
\left((_3,\alpha,1),\left\{\frac{\widehat{\Upsilon}_1}{<0.2,0.6>},\frac{\widehat{\Upsilon}_2}{<0.8,0.1>},\frac{\widehat{\Upsilon}_3}{<0.4,0.5>},\frac{\widehat{\Upsilon}_4}{<0.5,0.4>}\right\}\right),\\
\left((_3,\alpha,1),\left\{\frac{\widehat{\Upsilon}_1}{<0.4,0.5>},\frac{\widehat{\Upsilon}_2}{<0.6,0.3>},\frac{\widehat{\Upsilon}_3}{<0.7,0.2>},\frac{\widehat{\Upsilon}_4}{<0.9,0.1>}\right\}\right),\\
\left((_3,\alpha,1),\left\{\frac{\widehat{\Upsilon}_1}{<0.7,0.2>},\frac{\widehat{\Upsilon}_2}{<0.3,0.6>},\frac{\widehat{\Upsilon}_3}{<0.5,0.4>},\frac{\widehat{\Upsilon}_4}{<0.2,0.7>}\right\}\right),\\
\left((_1,\alpha,0),\left\{\frac{\widehat{\Upsilon}_1}{<0.3,0.5>},\frac{\widehat{\Upsilon}_2}{<0.2,0.4>},\frac{\widehat{\Upsilon}_3}{<0.4,0.5>},\frac{\widehat{\Upsilon}_4}{<0.1,0.8>}\right\}\right),\\
\left((_1,\alpha,0),\left\{\frac{\widehat{\Upsilon}_1}{<0.1,0.8>},\frac{\widehat{\Upsilon}_2}{<0.8,0.1>},\frac{\widehat{\Upsilon}_3}{<0.6,0.2>},\frac{\widehat{\Upsilon}_4}{<0.2,0.7>}\right\}\right),\\
\left((_1,\alpha,0),\left\{\frac{\widehat{\Upsilon}_1}{<0.2,0.7>},\frac{\widehat{\Upsilon}_2}{<0.1,0.7>},\frac{\widehat{\Upsilon}_3}{<0.3,0.5>},\frac{\widehat{\Upsilon}_4}{<0.5,0.4>}\right\}\right),\\
\left((_2,\alpha,0),\left\{\frac{\widehat{\Upsilon}_1}{<0.5,0.1>},\frac{\widehat{\Upsilon}_2}{<0.3,0.6>},\frac{\widehat{\Upsilon}_3}{<0.5,0.4>},\frac{\widehat{\Upsilon}_4}{<0.7,0.2>}\right\}\right),\\
\left((_2,\alpha,0),\left\{\frac{\widehat{\Upsilon}_1}{<0.7,0.2>},\frac{\widehat{\Upsilon}_2}{<0.2,0.5>},\frac{\widehat{\Upsilon}_3}{<0.9,0.1>},\frac{\widehat{\Upsilon}_4}{<0.4,0.5>}\right\}\right),\\
\left((_2,\alpha,0),\left\{\frac{\widehat{\Upsilon}_1}{<0.6,0.3>},\frac{\widehat{\Upsilon}_2}{<0.7,0.2>},\frac{\widehat{\Upsilon}_3}{<0.3,0.5>},\frac{\widehat{\Upsilon}_4}{<0.2,0.7>}\right\}\right),\\
\left((_3,\alpha,0),\left\{\frac{\widehat{\Upsilon}_1}{<0.1,0.6>},\frac{\widehat{\Upsilon}_2}{<0.4,0.5>},\frac{\widehat{\Upsilon}_3}{<0.7,0.2>},\frac{\widehat{\Upsilon}_4}{<0.8,0.2>}\right\}\right),\\
\left((_3,\alpha,0),\left\{\frac{\widehat{\Upsilon}_1}{<0.2,0.6>},\frac{\widehat{\Upsilon}_2}{<0.9,0.1>},\frac{\widehat{\Upsilon}_3}{<0.8,0.2>},\frac{\widehat{\Upsilon}_4}{<0.3,0.5>}\right\}\right),\\
\left((_3,\alpha,0),\left\{\frac{\widehat{\Upsilon}_1}{<0.5,0.3>},\frac{\widehat{\Upsilon}_2}{<0.3,0.6>},\frac{\widehat{\Upsilon}_3}{<0.6,0.3>},\frac{\widehat{\Upsilon}_4}{<0.1,0.8>}\right\}\right),
\end{array}\right\}.$$

Definition 3.3 The operation union between two $\mathcal{IFHSE}$-Sets $(\Lambda_1,\widetilde{S})$ and $(\Lambda_2,\widetilde{R})$ over $\widehat{\Theta}$ is $(\Lambda_3,\widetilde{L})$ with $\widetilde{L}=\widetilde{S}\cup\widetilde{R}$, defined as

$$\Lambda_3(\theta)=\begin{cases}\Lambda_1(\theta) & ;\ \theta\in\widetilde{S}-\widetilde{R}\\ \Lambda_2(\theta) & ;\ \theta\in\widetilde{R}-\widetilde{S}\\ s(\Lambda_1(\theta),\Lambda_2(\theta)) & ;\ \theta\in\widetilde{S}\cap\widetilde{R}\end{cases}$$

where s is s-norm.

Example 3.4 Reconsideration Example 3.2, having two sets

$\widetilde{A}_1=\{(_1,\alpha,1),(_3,\alpha,0),(_1,\alpha,1),(_3,\alpha,1),\}$,

$A_2=\{(_1,\alpha,1),(_3,\alpha,0),(_3,\alpha,1),(_1,\alpha,1),(_3,\alpha,1),(_1,\alpha,0)\}$.

Suppose $(\Lambda_1,\widetilde{A}_1)$ and $(\Lambda_2,\widetilde{A}_2)$ over $\widehat{\Omega}$ are two $\mathcal{IFHSE}$-Sets such that

$$(\Lambda_1, \widetilde{A}_1) = \left\{\begin{array}{l} \left(({}_1, \mathfrak{a}, 1), \left\{\frac{\widehat{\Upsilon}_1}{<0.2, 0.4>}, \frac{\widehat{\Upsilon}_2}{<0.6, 0.2>}, \frac{\widehat{\Upsilon}_3}{<0.5, 0.4>}, \frac{\widehat{\Upsilon}_4}{<0.1, 0.5>}\right\}\right), \\ \left(({}_1, \mathfrak{a}, 1), \left\{\frac{\widehat{\Upsilon}_1}{<0.3, 0.2>}, \frac{\widehat{\Upsilon}_2}{<0.8, 0.1>}, \frac{\widehat{\Upsilon}_3}{<0.4, 0.5>}, \frac{\widehat{\Upsilon}_4}{<0.1, 0.5>}\right\}\right), \\ \left(({}_3, \mathfrak{a}, 1), \left\{\frac{\widehat{\Upsilon}_1}{<0.4, 0.5>}, \frac{\widehat{\Upsilon}_2}{<0.6, 0.3>}, \frac{\widehat{\Upsilon}_3}{<0.7, 0.2>}, \frac{\widehat{\Upsilon}_4}{<0.9, 0.1>}\right\}\right), \\ \left(({}_3, \mathfrak{a}, 1), \left\{\frac{\widehat{\Upsilon}_1}{<0.7, 0.2>}, \frac{\widehat{\Upsilon}_2}{<0.3, 0.6>}, \frac{\widehat{\Upsilon}_3}{<0.5, 0.4>}, \frac{\widehat{\Upsilon}_4}{<0.2, 0.7>}\right\}\right), \end{array}\right\},$$

$$(\Lambda_2, \widetilde{A}_2) = \left\{\begin{array}{l} \left(({}_1, \mathfrak{a}, 1), \left\{\frac{\widehat{\Upsilon}_1}{<0.2, 0.3>}, \frac{\widehat{\Upsilon}_2}{<0.5, 0.1>}, \frac{\widehat{\Upsilon}_3}{<0.2, 0.1>}, \frac{\widehat{\Upsilon}_4}{<0.1, 0.4>}\right\}\right), \\ \left(({}_1, \mathfrak{a}, 1), \left\{\frac{\widehat{\Upsilon}_1}{<0.2, 0.1>}, \frac{\widehat{\Upsilon}_2}{<0.7, 0.1>}, \frac{\widehat{\Upsilon}_3}{<0.2, 0.3>}, \frac{\widehat{\Upsilon}_4}{<0.1, 0.2>}\right\}\right), \\ \left(({}_3, \mathfrak{a}, 1), \left\{\frac{\widehat{\Upsilon}_1}{<0.1, 0.3>}, \frac{\widehat{\Upsilon}_2}{<0.2, 0.1>}, \frac{\widehat{\Upsilon}_3}{<0.4, 0.3>}, \frac{\widehat{\Upsilon}_4}{<0.1, 0.2>}\right\}\right), \\ \left(({}_3, \mathfrak{a}, 1), \left\{\frac{\widehat{\Upsilon}_1}{<0.4, 0.2>}, \frac{\widehat{\Upsilon}_2}{<0.5, 0.3>}, \frac{\widehat{\Upsilon}_3}{<0.6, 0.1>}, \frac{\widehat{\Upsilon}_4}{<0.8, 0.1>}\right\}\right), \\ \left(({}_3, \mathfrak{a}, 0), \left\{\frac{\widehat{\Upsilon}_1}{<0.2, 0.7>}, \frac{\widehat{\Upsilon}_2}{<0.4, 0.5>}, \frac{\widehat{\Upsilon}_3}{<0.7, 0.2>}, \frac{\widehat{\Upsilon}_4}{<0.8, 0.1>}\right\}\right), \\ \left(({}_3, \mathfrak{a}, 0), \left\{\frac{\widehat{\Upsilon}_1}{<0.2, 0.5>}, \frac{\widehat{\Upsilon}_2}{<0.7, 0.2>}, \frac{\widehat{\Upsilon}_3}{<0.8, 0.2>}, \frac{\widehat{\Upsilon}_4}{<0.3, 0.5>}\right\}\right), \end{array}\right\}.$$

Then, $(\Lambda_1, \widetilde{A}_1) \cup (\Lambda_2, \widetilde{A}_2) = (\Lambda_3, \widetilde{A}_3)$

$$\left\{\begin{array}{l} \left(({}_1, \mathfrak{a}, 1), \left\{\frac{\widehat{\Upsilon}_1}{<0.2, 0.3>}, \frac{\widehat{\Upsilon}_2}{<0.6, 0.1>}, \frac{\widehat{\Upsilon}_3}{<0.5, 0.1>}, \frac{\widehat{\Upsilon}_4}{<0.1, 0.4>}\right\}\right), \\ \left(({}_1, \mathfrak{a}, 1), \left\{\frac{\widehat{\Upsilon}_1}{<0.3, 0.1>}, \frac{\widehat{\Upsilon}_2}{<0.8, 0.1>}, \frac{\widehat{\Upsilon}_3}{<0.4, 0.3>}, \frac{\widehat{\Upsilon}_4}{<0.1, 0.2>}\right\}\right), \\ \left(({}_3, \mathfrak{a}, 1), \left\{\frac{\widehat{\Upsilon}_1}{<0.7, 0.3>}, \frac{\widehat{\Upsilon}_2}{<0.3, 0.1>}, \frac{\widehat{\Upsilon}_3}{<0.5, 0.3>}, \frac{\widehat{\Upsilon}_4}{<0.2, 0.2>}\right\}\right), \\ \left(({}_3, \mathfrak{a}, 1), \left\{\frac{\widehat{\Upsilon}_1}{<0.4, 0.2>}, \frac{\widehat{\Upsilon}_2}{<0.5, 0.3>}, \frac{\widehat{\Upsilon}_3}{<0.7, 0.1>}, \frac{\widehat{\Upsilon}_4}{<0.9, 0.1>}\right\}\right), \\ \left(({}_3, \mathfrak{a}, 0), \left\{\frac{\widehat{\Upsilon}_1}{<0.1, 0.6>}, \frac{\widehat{\Upsilon}_2}{<0.1, 0.5>}, \frac{\widehat{\Upsilon}_3}{<0.7, 0.1>}, \frac{\widehat{\Upsilon}_4}{<0.8, 0.1>}\right\}\right), \\ \left(({}_3, \mathfrak{a}, 0), \left\{\frac{\widehat{\Upsilon}_1}{<0.1, 0.5>}, \frac{\widehat{\Upsilon}_2}{<0.8, 0.1>}, \frac{\widehat{\Upsilon}_3}{<0.8, 0.2>}, \frac{\widehat{\Upsilon}_4}{<0.3, 0.5>}\right\}\right), \end{array}\right\}.$$

Definition 3.5 The intersection between two $\mathcal{IFHSE}$-Sets $(\Lambda_1, \widetilde{S})$ and $(\Lambda_2, \widetilde{R})$ over $\widehat{\Theta}$ is $(\Lambda_3, \widetilde{L})$ with $\widetilde{L} = \widetilde{S} \cap \widetilde{R}$, defined by as

$$\Lambda_3(\theta) = \begin{cases} \Lambda_1(\theta) & ; \theta \in \widetilde{S} - \widetilde{R} \\ \Lambda_2(\theta) & ; \theta \in \widetilde{R} - \widetilde{S} \\ t(\Lambda_1(\theta), \Lambda_2(\theta)) & ; \theta \in \widetilde{S} \cap \widetilde{R} \end{cases}$$

where t is a t-norm.

Example 3.6 Reconsideration Example 3.2, with following two sets

$$\widetilde{A}_1 = \{ ({}_1, \mathfrak{a}, 1), ({}_3, \mathfrak{a}, 0), ({}_1, \mathfrak{a}, 1), ({}_3, \mathfrak{a}, 1), \}$$
$$\widetilde{A}_2 = \{ ({}_1, \mathfrak{a}, 1), ({}_3, \mathfrak{a}, 0), ({}_3, \mathfrak{a}, 1), ({}_1, \mathfrak{a}, 1), ({}_3, \mathfrak{a}, 1), ({}_1, \mathfrak{a}, 1) \}.$$

Suppose $(\Lambda_1, \widetilde{A}_1)$ and $(\Lambda_2, \widetilde{A}_2)$ over $\widehat{\Theta}$ are two $\mathcal{IFHSE}$-Sets such that

$$(\Lambda_1, \widetilde{A}_1) = \left\{ \begin{array}{l} \left((_1, \mathfrak{a}, 1), \left\{ \frac{\widehat{\Upsilon}_1}{<0.2, 0.4>}, \frac{\widehat{\Upsilon}_2}{<0.6, 0.2>}, \frac{\widehat{\Upsilon}_3}{<0.5, 0.4>}, \frac{\widehat{\Upsilon}_4}{<0.1, 0.5>} \right\} \right), \\ \left((_1, \mathfrak{a}, 1), \left\{ \frac{\widehat{\Upsilon}_1}{<0.3, 0.2>}, \frac{\widehat{\Upsilon}_2}{<0.8, 0.1>}, \frac{\widehat{\Upsilon}_3}{<0.4, 0.5>}, \frac{\widehat{\Upsilon}_4}{<0.1, 0.5>} \right\} \right), \\ \left((_3, \mathfrak{a}, 1), \left\{ \frac{\widehat{\Upsilon}_1}{<0.4, 0.5>}, \frac{\widehat{\Upsilon}_2}{<0.6, 0.3>}, \frac{\widehat{\Upsilon}_3}{<0.7, 0.2>}, \frac{\widehat{\Upsilon}_4}{<0.9, 0.1>} \right\} \right), \\ \left((_3, \mathfrak{a}, 1), \left\{ \frac{\widehat{\Upsilon}_1}{<0.7, 0.2>}, \frac{\widehat{\Upsilon}_2}{<0.3, 0.6>}, \frac{\widehat{\Upsilon}_3}{<0.5, 0.4>}, \frac{\widehat{\Upsilon}_4}{<0.2, 0.7>} \right\} \right), \end{array} \right\},$$

$$(\Lambda_2, \widetilde{A}_2) = \left\{ \begin{array}{l} \left((_1, \mathfrak{a}, 1), \left\{ \frac{\widehat{\Upsilon}_1}{<0.2, 0.3>}, \frac{\widehat{\Upsilon}_2}{<0.5, 0.1>}, \frac{\widehat{\Upsilon}_3}{<0.2, 0.1>}, \frac{\widehat{\Upsilon}_4}{<0.1, 0.4>} \right\} \right), \\ \left((_1, \mathfrak{a}, 1), \left\{ \frac{\widehat{\Upsilon}_1}{<0.2, 0.1>}, \frac{\widehat{\Upsilon}_2}{<0.7, 0.1>}, \frac{\widehat{\Upsilon}_3}{<0.2, 0.3>}, \frac{\widehat{\Upsilon}_4}{<0.1, 0.2>} \right\} \right), \\ \left((_3, \mathfrak{a}, 1), \left\{ \frac{\widehat{\Upsilon}_1}{<0.1, 0.3>}, \frac{\widehat{\Upsilon}_2}{<0.2, 0.1>}, \frac{\widehat{\Upsilon}_3}{<0.4, 0.3>}, \frac{\widehat{\Upsilon}_4}{<0.1, 0.2>} \right\} \right), \\ \left((_3, \mathfrak{a}, 1), \left\{ \frac{\widehat{\Upsilon}_1}{<0.4, 0.2>}, \frac{\widehat{\Upsilon}_2}{<0.5, 0.3>}, \frac{\widehat{\Upsilon}_3}{<0.6, 0.1>}, \frac{\widehat{\Upsilon}_4}{<0.8, 0.1>} \right\} \right), \\ \left((_3, \mathfrak{a}, 0), \left\{ \frac{\widehat{\Upsilon}_1}{<0.2, 0.7>}, \frac{\widehat{\Upsilon}_2}{<0.4, 0.5>}, \frac{\widehat{\Upsilon}_3}{<0.7, 0.2>}, \frac{\widehat{\Upsilon}_4}{<0.8, 0.1>} \right\} \right), \\ \left((_3, \mathfrak{a}, 0), \left\{ \frac{\widehat{\Upsilon}_1}{<0.2, 0.5>}, \frac{\widehat{\Upsilon}_2}{<0.7, 0.2>}, \frac{\widehat{\Upsilon}_3}{<0.8, 0.2>}, \frac{\widehat{\Upsilon}_4}{<0.3, 0.5>} \right\} \right), \end{array} \right\}.$$

Then, $(\Lambda_1, \widetilde{A}_1) \cap (\Lambda_2, \widetilde{A}_2) = (\Lambda_3, \widetilde{A}_3)$

$$\left\{ \begin{array}{l} \left((_1, \mathfrak{a}, 1), \left\{ \frac{\widehat{\Upsilon}_1}{<0.2, 0.3>}, \frac{\widehat{\Upsilon}_2}{<0.5, 0.2>}, \frac{\widehat{\Upsilon}_3}{<0.2, 0.4>}, \frac{\widehat{\Upsilon}_4}{<0.1, 0.5>} \right\} \right), \\ \left((_1, \mathfrak{a}, 1), \left\{ \frac{\widehat{\Upsilon}_1}{<0.2, 0.2>}, \frac{\widehat{\Upsilon}_2}{<0.7, 0.1>}, \frac{\widehat{\Upsilon}_3}{<0.2, 0.5>}, \frac{\widehat{\Upsilon}_4}{<0.1, 0.5>} \right\} \right), \\ \left((_3, \mathfrak{a}, 1), \left\{ \frac{\widehat{\Upsilon}_1}{<0.4, 0.5>}, \frac{\widehat{\Upsilon}_2}{<0.5, 0.1>}, \frac{\widehat{\Upsilon}_3}{<0.6, 0.2>}, \frac{\widehat{\Upsilon}_4}{<0.8, 0.1>} \right\} \right), \\ \left((_3, \mathfrak{a}, 1), \left\{ \frac{\widehat{\Upsilon}_1}{<0.3, 0.1>}, \frac{\widehat{\Upsilon}_2}{<0.2, 0.6>}, \frac{\widehat{\Upsilon}_3}{<0.4, 0.4>}, \frac{\widehat{\Upsilon}_4}{<0.1, 0.7>} \right\} \right), \end{array} \right\}.$$

4 Correlation coefficient for $\mathcal{IFHSE}$-set

This portion contains the idea of proposed correlation, $\mathbb{CC}$ and weighted correlation coefficient. Some results related to the correlation are also proved.

Definition 4.1 Consider (G_1, L_1) and (G_1, L_1) two $\mathcal{IFHSE}$-Sets over $\widehat{\Theta}$ in such a way that

$(G_1, L_1) = \{\phi_i, \zeta_{G_{1(\ell_j)}}(\phi_i), \kappa_{G_{1(\ell_j)}}(\phi_i) \mid \phi_i \in \widehat{\Theta}\}$ and $(G_2, L_2) = \{\phi_i, \zeta_{G_{2(\ell_j)}}(\phi_i), \kappa_{G_{2(\ell_j)}}(\phi_i) \mid \phi_i \in \widehat{\Theta}\}$. First, we describe the intuitionistic energies of these two $\mathcal{IFHSE}$-Set (G_1, L_1) and (G_1, L_1) in the following way

$$\mathcal{E}_1(G_1, L_1) = \mathcal{E}_1(G_1, L_1) = \sum_{i=1}^{n} \sum_{j=1}^{m} (((\zeta_{G_{1(\ell_j)}}(\phi_i))^2 + (\kappa_{G_{1(\ell_j)}}(\phi_i))^2)$$

$$\mathcal{E}_2(G_2, L_2) = \mathcal{E}_1(G_2, L_2) = \sum_{i=1}^{n} \sum_{j=1}^{m} (((\zeta_{G_{2(\ell_j)}}(\phi_i))^2 + (\kappa_{G_{2(\ell_j)}}(\phi_i))^2).$$

Definition 4.2 Considering the above two $\mathcal{IFHSE}$-Sets $(G_1, L_1) = \{\phi_i, \zeta_{G_{1(\ell_j)}}(\phi_i), \kappa_{G_{1(\ell_j)}}(\phi_i) \mid \phi_i \in \widehat{\Theta}\}$ and $(G_2, L_2) = \{\phi_i, \zeta_{G_{2(\ell_j)}}(\phi_i), \kappa_{G_{2(\ell_j)}}(\phi_i) \mid \phi_i \in \widehat{\Theta}\}$, then the correlation between these two sets can be defined as

$$Co((G_1,L_1),(G_2,L_2)) = \sum_{i=1}^{n}\sum_{j=1}^{m}((\zeta_{G_{1(\ell_j)}}(\phi_i) \times \zeta_{G_{2(\ell_j)}}(\phi_i) + \kappa_{G_{1(\ell_j)}}(\phi_i) \times \kappa_{G_{2(\ell_j)}}(\phi_i)).$$

Proposition 4.3 *Consider the two $\mathcal{IFHSE}$-Sets* $(G_1,L_1) = (G_1,L_1) = \{\phi_i, \zeta_{G_{1(\ell_j)}}(\phi_i), \kappa_{G_{1_{(\ell_j)}}}(\phi_i) \mid \phi_i \in \widehat{\Theta}\}$ *and* $(G_2,L_2) = \{\phi_i, \zeta_{G_{2(\ell_j)}}(\phi_i), \kappa_{G_{2_{(\ell_j)}}}(\phi_i) \mid \phi_i \in \widehat{\Theta}\}$, *then the following characteristics holds:*

(1) $Co((G_1,L_1),(G_1,L_1)) = \mathcal{E}_1$ = *Intuitionistic energy* of (G_1,L_1)
(2) $Co((G_2,L_2),(G_2,L_2)) = \mathcal{E}_2$ = *Intuitionistic energy* of (G_2,L_2).

Proof

(1) Since we know that $Co((G_1,L_1),(G_2,L_2)) = \sum_{i=1}^{n}\sum_{j=1}^{m}((\zeta_{G_{1(\ell_j)}}(\phi_i) \times \zeta_{G_{2(\ell_j)}}(\phi_i) + \kappa_{G_{1(\ell_j)}}(\phi_i) \times \kappa_{G_{2(\ell_j)}}(\phi_i))$, therefore $Co((G_1,L_1),(G_1,L_1)) = \sum_{i=1}^{n}\sum_{j=1}^{m}((\zeta_{G_{1(\ell_j)}}(\phi_i) \times \zeta_{G_{1(\ell_j)}}(\phi_i) + \kappa_{G_{1(\ell_j)}}(\phi_i) \times \kappa_{G_{1(\ell_j)}}(\phi_i))$. Hence, $Co((G_1,L_1),(G_1,L_1)) = \sum_{i=1}^{n}\sum_{j=1}^{m}(((\zeta_{G_{1(\ell_j)}}(\phi_i))^2 + (\kappa_{G_{1(\ell_j)}}(\phi_i))^2)))$, so the result is proved.
(2) By definition of correlation between two sets $Co((G_1,L_1),(G_2,L_2)) = \sum_{i=1}^{n}\sum_{j=1}^{m}((\zeta_{G_{1(\ell_j)}}(\phi_i) \times \zeta_{G_{1(\ell_j)}}(\phi_i) + \kappa_{G_{1(\ell_j)}}(\phi_i) \times \kappa_{G_{1(\ell_j)}}(\phi_i))$, therefore, $Co((G_2,L_2),(G_2,L_2)) = \sum_{i=1}^{n}\sum_{j=1}^{m}((\zeta_{G_{2(\ell_j)}}(\phi_i) \times \zeta_{G_{2(\ell_j)}}(\phi_i) + \kappa_{G_{2(\ell_j)}}(\phi_i) \times \kappa_{G_{2(\ell_j)}}(\phi_i))$. Hence $E_2(G_2,L_2) = \sum_{i=1}^{n}\sum_{j=1}^{m}(((\zeta_{G_{2(\ell_j)}}(\phi_i))^2 + (\kappa_{G_{2_{(\ell_j)}}}(\phi_i))^2)$.

□

Definition 4.4 Considering the above two $\mathcal{IFHSE}$-Sets $(G_1,L_1) = \{\phi_i, \zeta_{G_{1(\ell_j)}}(\phi_i), \kappa_{G_{1_{(\ell_j)}}}(\phi_i) \mid \phi_i \in \widehat{\Theta}\}$ and $(G_2,L_2) = \{\phi_i, \zeta_{G_{2(\ell_j)}}(\phi_i), \kappa_{G_2}(\phi_i) \mid \phi_i \in \widehat{\Theta}\}$, the $\mathbb{CC}$ between two sets can be expressed as follows:

$$\mathbb{CC}((G_1,L_1),(G_2,L_2)) = \frac{Co((G_1,L_1),(G_2,L_2))}{\sqrt{E_1(G_1,L_1)} \times \sqrt{E_2(G_2,L_2)}}$$

or $\mathbb{CC}((G_1,L_1),(G_2,L_2))$=

$$\frac{\sum_{i=1}^{n}\sum_{j=1}^{m}((\zeta_{G_{1(\ell_j)}}(\phi_i) \times \zeta_{G_{2(\ell_j)}}(\phi_i) + \kappa_{G_{1(\ell_j)}}(\phi_i) \times \kappa_{G_{2(\ell_j)}}(\phi_i))}{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m}(((\zeta_{G_{1(\ell_j)}}(\phi_i))^2 + (\kappa_{G_{1_{(\ell_j)}}}(\phi_i))^2)} \times \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m}(((\zeta_{G_{2(\ell_j)}}(\phi_i))^2 + (\kappa_{G_{2_{(\ell_j)}}}(\phi_i))^2)}}.$$

Proposition 4.5 *Considering the two $\mathcal{IFHSE}$-Sets* $(G_1,L_1) = \{\phi_i, \zeta_{G_{1(\ell_j)}}(\phi_i), \kappa_{G_{1_{(\ell_j)}}}(\phi_i) \mid \phi_i \in \widehat{\Theta}\}$ *and* $(G_2,L_2) = \{\phi_i, \zeta_{G_{2(\ell_j)}}(\phi_i), \kappa_{G_{2_{(\ell_j)}}}(\phi_i) \mid \phi_i \in \widehat{\Theta}\}$, *then the* $\mathbb{CC}$ *satisfies the following characteristics:*

(1) $0 \leq \mathbb{CC}((G_1,L_1),(G_2,L_2)) \leq 1$
(2) $\mathbb{CC}((G_1,L_1),(G_2,L_2)) = \mathbb{CC}((G_2,L_2),(G_1,L_1))$
(3) If $(G_1,L_1) = (G_2,L_2)$ i.e $\zeta_{G_1}(\epsilon_i) = \zeta_{G_2}(\epsilon_i)$, and $\kappa_{G_1}(\epsilon_i) = \kappa_{G_2}(\epsilon_i)$ for all i, then $\mathbb{CC}((G_1,L_1),(G_2,L_2)) = 1$.

Proof

(1) Since $\mathbb{CC}((G_1,L_1),(G_2,L_2)) \geq 0$. Now, we have to prove that $\mathbb{CC}((G_1,L_1),(G_2,L_2)) \leq 1$. Since $\mathbb{CC}((G_1,L_1),(G_2,L_2)) = \sum_{i=1}^{n}\sum_{j=1}^{m}((\zeta_{G_{1(\ell_j)}}(\phi_i) \times \zeta_{G_{2(\ell_j)}}(\phi_i) + \kappa_{G_{1(\ell_j)}}(\phi_i) \times \kappa_{G_{2(\ell_j)}}(\phi_i))$

$$= \sum_{j=1}^{m}((\zeta_{G_{1(\ell_j)}}(\phi_1) \times \zeta_{G_{2(\ell_j)}}(\phi_1) + \kappa_{G_{1(\ell_j)}}(\phi_1) \times \kappa_{G_{2(\ell_j)}}(\phi_1))$$
$$+ \sum_{j=1}^{m}((\zeta_{G_{1(\ell_j)}}(\phi_2) \times \zeta_{G_{2(\ell_j)}}(\phi_2) + \kappa_{G_{1(\ell_j)}}(\phi_2) \times \kappa_{G_{2(\ell_j)}}(\phi_2))$$
$$+ \\ . \\ . \\ . \\ +$$
$$\sum_{j=1}^{m}((\zeta_{G_{1(\ell_j)}}(\phi_n) \times \zeta_{G_{2(\ell_j)}}(\phi_n) + \kappa_{G_{1(\ell_j)}}(\phi_n) \times \kappa_{G_{2(\ell_j)}}(\phi_n))$$

$$\mathbb{CC}((G_1,L_1),(G_2,L_2)) = \left\{\begin{array}{r} ((\zeta_{G_{1(\ell_1)}}(\phi_1)\times\zeta_{G_{2(\ell_1)}}(\phi_1)+\kappa_{G_{1(\ell_1)}}(\phi_1)\times\kappa_{G_{2(\ell_1)}}(\phi_1))+ \\ ((\zeta_{G_{1(\ell_2)}}(\phi_1)\times\zeta_{G_{2(\ell_2)}}(\phi_1)+\kappa_{G_{1(\ell_2)}}(\phi_2)\times\kappa_{G_{2(\ell_2)}}(\phi_2))+ \\ . \\ . \\ . \\ \zeta_{G_{1(\ell_m)}}(\phi_1)\times\zeta_{G_{2(\ell_m)}}(\phi_1)+\kappa_{G_{1(\ell_m)}}(\phi_1)\times\kappa_{G_{2(\ell_m)}}(\phi_1)) \end{array}\right\}$$

$$+\left\{\begin{array}{r} ((\zeta_{G_{1(\ell_1)}}(\phi_2)\times\zeta_{G_{2(\ell_1)}}(\phi_2)+\kappa_{G_{1(\ell_1)}}(\phi_2)\times\kappa_{G_{2(\ell_1)}}(\phi_2))+ \\ ((\zeta_{G_{1(\ell_2)}}(\phi_2)\times\zeta_{G_{2(\ell_2)}}(\phi_2)+\kappa_{G_{1(\ell_2)}}(\phi_2)\times\kappa_{G_{2(\ell_2)}}(\phi_2))+ \\ . \\ . \\ . \\ \zeta_{G_{1(\ell_m)}}(\phi_2)\times\zeta_{G_{2(\ell_m)}}(\phi_2)+\kappa_{G_{1(\ell_m)}}(\phi_2)\times\kappa_{G_{2(\ell_m)}}(\phi_2)) \end{array}\right\}$$

$$+ \; . \; . \; . \; +$$

$$\left\{\begin{array}{r} ((\zeta_{G_{1(\ell_1)}}(\phi_n)\times\zeta_{G_{2(\ell_1)}}(\phi_n)+\kappa_{G_{1(\ell_1)}}(\phi_n)\times\kappa_{G_{2(\ell_1)}}(\phi_n))+ \\ ((\zeta_{G_{1(\ell_2)}}(\phi_n)\times\zeta_{G_{2(\ell_2)}}(\phi_n)+\kappa_{G_{1(\ell_2)}}(\phi_n)\times\kappa_{G_{2(\ell_2)}}(\phi_n))+ \\ . \\ . \\ . \\ \zeta_{G_{1(\ell_m)}}(\phi_n)\times\zeta_{G_{2(\ell_m)}}(\phi_n)+\kappa_{G_{1(\ell_m)}}(\phi_n)\times\kappa_{G_{2(\ell_m)}}(\phi_n)) \end{array}\right\}$$

$$= \sum_{j=1}^{m}\left\{\begin{array}{r} ((\zeta_{G_{1(\ell_m)}}(\phi_1)\times\zeta_{G_{2(\ell_m)}}(\phi_1))+(\zeta_{G_{1(\ell_m)}}(\phi_2)\times\zeta_{G_{2(\ell_m)}}(\phi_2)) \\ +\cdots+(\zeta_{G_{1(\ell_m)}}(\phi_n)\times\zeta_{G_{2(\ell_m)}}(\phi_n)) \end{array}\right\}$$

$$+\sum_{j=1}^{m}\left\{\left(\begin{array}{r} (\kappa_{G_{1(\ell_m)}}(\phi_1)\times\kappa_{G_{2(\ell_m)}}(\phi_1))+(\kappa_{G_{1(\ell_m)}}(\phi_2)\times\kappa_{G_{2(\ell_m)}}(\phi_2)) \\ +\cdots+(\kappa_{G_{1(\ell_m)}}(\phi_n)\times\kappa_{G_{2(\ell_m)}}(\phi_n)) \end{array}\right)\right\}.$$

By making use of Cauchy–Schwarz Inequality, we get

$$(\mathbb{CC}((G_1,L_1),(G_2,L_2)))^2 \le \sum_{j=1}^{m}\left\{\begin{array}{r} \left(\left(\zeta_{G_{1(\ell_m)}}(\phi_1)\right)^2+\left(\zeta_{G_{1(\ell_m)}}(\phi_2)\right)^2+\cdots+\left(\zeta_{G_{1(\ell_m)}}(\phi_n)\right)^2\right)+ \\ \left(\left(\kappa_{G_{1(\ell_m)}}(\phi_1)\right)^2+\left(\kappa_{G_{1(\ell_m)}}(\phi_2)\right)^2+\cdots+\left(\kappa_{G_{1(\ell_m)}}(\phi_n)\right)^2\right) \end{array}\right\}$$

$$\times\sum_{j=1}^{m}\left\{\begin{array}{r} \left(\left(\zeta_{G_{2(\ell_m)}}(\phi_1)\right)^2+\left(\zeta_{G_{2(\ell_m)}}(\phi_2)\right)^2+\cdots+\left(\zeta_{G_{2(\ell_m)}}(\phi_n)\right)^2\right)+ \\ \left(\left(\kappa_{G_{2(\ell_m)}}(\phi_1)\right)^2+\left(\kappa_{G_{2(\ell_m)}}(\phi_2)\right)^2+\cdots+\left(\kappa_{G_{2(\ell_m)}}(\phi_n)\right)^2\right) \end{array}\right\}$$

$$(\mathbb{CC}((G_1,L_1),(G_2,L_2)))^2 \le \sum_{j=1}^{m}\sum_{i=1}^{n}\left\{\left(\left(\zeta_{G_{1(\ell_m)}}(\phi_i)\right)^2+\left(\kappa_{G_{1(\ell_m)}}(\phi_i)\right)^2\right)\right\}$$

$$\times\sum_{j=1}^{m}\sum_{i=1}^{n}\left\{\left(\left(\zeta_{G_{2(\ell_m)}}(\phi_i)\right)^2+\left(\kappa_{G_{2(\ell_m)}}(\phi_i)\right)^2\right)\right\}$$

$$(\mathbb{CC}((G_1,L_1),(G_2,L_2)))^2 \le \mathcal{E}_1(G_1,L_1)\times\mathcal{E}_2(G_2,L_2).$$

By making use of $\mathbb{CC}$ definition $\mathbb{CC}((G_1,L_1),(G_2,L_2))) \leq 1$, So, $0 \leq \mathbb{CC}((G_1,L_1),(G_2,L_2))) \leq 1$; hence, the result is proved.

(2) Since we know that

$$\mathbb{CC}((G_1,L_1),(G_2,L_2)) = \frac{Co((G_1,L_1),(G_2,L_2))}{\sqrt{\mathcal{E}_1(G_1,L_1)} \times \sqrt{\mathcal{E}_2(G_2,L_2)}}$$

or

$$\mathbb{CC}((G_1,L_1),(G_2,L_2)) = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}((\zeta_{G_1(\ell_j)}(\phi_i) \times \zeta_{G_2(\ell_j)}(\phi_i) + \kappa_{G_1(\ell_j)}(\phi_i) \times \kappa_{G_2(\ell_j)}(\phi_i))}{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m}(((\zeta_{G_1(\ell_j)}(\phi_i))^2 + (\kappa_{G_{1(\ell_j)}}(\phi_i))^2)} \times \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m}(((\zeta_{G_2(\ell_j)}(\phi_i))^2 + (\kappa_{G_{2(\ell_j)}}(\phi_i))^2)}$$
$$= \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}((\zeta_{G_1(\ell_j)}(\phi_i) \times \zeta_{G_2(\ell_j)}(\phi_i) + \kappa_{G_1(\ell_j)}(\phi_i) \times \kappa_{G_2(\ell_j)}(\phi_i))}{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m}(((\zeta_{G_2(\ell_j)}(\phi_i))^2 + (\kappa_{G_{2(\ell_j)}}(\phi_i))^2)} \times \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m}(((\zeta_{G_2(\ell_j)}(\phi_i))^2 + (\kappa_{G_{2(\ell_j)}}(\phi_i))^2)}$$
$$= \frac{Co((G_2,L_2),(G_1,L_1))}{\sqrt{\mathcal{E}_2(G_2,L_2)} \times \sqrt{\mathcal{E}_1(G_1,L_1)}}$$

hence proved

$$\mathbb{CC}((G_1,L_1),(G_2,L_2)) = \mathbb{CC}((G_2,L_2),(G_1,L_1)).$$

(3) By definition of $\mathbb{CC}$, we have $\mathbb{CC}((G_1,L_1),(G_2,L_2))$=

$$\frac{\sum_{i=1}^{n}\sum_{j=1}^{m}((\zeta_{G_1(\ell_j)}(\phi_i) \times \zeta_{G_2(\ell_j)}(\phi_i) + \kappa_{G_1(\ell_j)}(\phi_i) \times \kappa_{G_2(\ell_j)}(\phi_i))}{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m}(((\zeta_{G_1(\ell_j)}(\phi_i))^2 + (\kappa_{G_{1(\ell_j)}}(\phi_i))^2)} \times \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m}(((\zeta_{G_2(\ell_j)}(\phi_i))^2 + (\kappa_{G_{2(\ell_j)}}(\phi_i))^2)}$$

since given that $\zeta_{G_1}(\phi_i) = \zeta_{G_2}(\phi_i)$, and $\kappa_{G_1}(\phi_i) = \kappa_{G_2}(\phi_i)$ for all i, therefore $\mathbb{CC}((G_1,L_1),(G_2,L_2))$=

$$\frac{\sum_{i=1}^{n}\sum_{j=1}^{m}((\zeta_{G_2(\ell_j)}(\phi_i))^2 + (\kappa_{G_{2(\ell_j)}}(\phi_i))^2)}{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m}(((\zeta_{G_2(\ell_j)}(\phi_i))^2 + (\kappa_{G_{2(\ell_j)}}(\phi_i))^2)} \times \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m}(((\zeta_{G_2(\ell_j)}(\phi_i))^2 + (\kappa_{G_{2(\ell_j)}}(\phi_i))^2)}$$

$$\mathbb{CC}((G_1,L_1),(G_2,L_2)) = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}((\zeta_{G_2(\ell_j)}(\phi_i))^2 + (\kappa_{G_{2(\ell_j)}}(\phi_i))^2)}{\sum_{i=1}^{n}\sum_{j=1}^{m}(((\zeta_{G_2(\ell_j)}(\phi_i))^2 + (\kappa_{G_{2(\ell_j)}}(\phi_i))^2)}$$

$$\Rightarrow \mathbb{CC}((G_1,L_1),(G_2,L_2)) = 1.$$

□

Definition 4.6 Considering the above two $\mathcal{IFHSE}$-Sets $(G_1,L_1) = \{\phi_i, \zeta_{G_1(\ell_j)}(\phi_i), \kappa_{G_{1(\ell_j)}}(\phi_i) \mid \phi_i \in \widehat{\Theta}\}$ and $(G_2,L_2) = \{\phi_i, \zeta_{G_2(\ell_j)}(\phi_i), \kappa_{G_{2(\ell_j)}}(\phi_i) \mid \phi_i \in \widehat{\Theta}\}$,

then the $\mathbb{CC}$ between them is described by as follows:

$$\gamma_{IFHSES}\mathbb{CC}((G_1,L_1),(G_2,L_2)) = \frac{Co((G_1,L_1),(G_2,L_2))}{\max\{\mathcal{E}_1(G_1,L_1),\mathcal{E}_2(G_2,L_2)\}}$$

$\gamma_{IFHSES}\mathbb{CC}((G_1,L_1),(G_2,L_2))$=

$$\frac{\sum_{i=1}^{n}\sum_{j=1}^{m}((\zeta_{G_1(\ell_j)}(\phi_i) \times \zeta_{G_2(\ell_j)}(\phi_i) + \kappa_{G_1(\ell_j)}(\phi_i) \times \kappa_{G_2(\ell_j)}(\phi_i))}{\max\left\{\begin{array}{l}\sum_{i=1}^{n}\sum_{j=1}^{m}(((\zeta_{G_1(\ell_j)}(\phi_i))^2 + (\kappa_{G_1(\ell_j)}(\phi_i))^2),\\ \sum_{i=1}^{n}\sum_{j=1}^{m}(((\zeta_{G_2(\ell_j)}(\phi_i))^2 + (\kappa_{G_2(\ell_j)}(\phi_i))^2)\end{array}\right\}}.$$

Proposition 4.7 *Considering the above two $\mathcal{IFHSE}$-Sets $(G_1,L_1) = \{\phi_i, \zeta_{G_1(\ell_j)}(\phi_i), \kappa_{G_{1(\ell_j)}}(\phi_i) \mid \phi_i \in \widehat{\Theta}\}$ and $(G_2,L_2) = \{\phi_i, \zeta_{G_2(\ell_j)}(\phi_i), \kappa_{G_{2(\ell_j)}}(\phi_i) \mid \phi_i \in \widehat{\Theta}\}$,* *then the* $\mathbb{CC}$ *between them describe the following properties:*

(1) $0 \le \gamma_{IFHSES}\mathbb{CC}((G_1, L_1), (G_2, L_2)) \le 1$
(2) $\gamma_{IFHSES}\mathbb{CC}((G_1, L_1), (G_2, L_2)) = \gamma_{IFHSES}\mathbb{CC}((G_2, L_2), (G_1, L_1))$
(3) If $(G_1, L_1) = (G_2, L_2)$ i.e $\zeta_{G_1}(\phi_i) = \zeta_{G_2}(\phi_i)$, and $\kappa_{G_1}(\phi_i) = \kappa_{G_2}(\phi_i)$ for all i, then $\gamma_{IFHSES}\mathbb{CC}((G_1, L_1), (G_2, L_2)) = 1$.

Proof Proof is simple. □

Definition 4.8 Suppose the weight vectors for parameters and experts are represented by $W = \{\dot{w}_1, \dot{w}_2, \dot{w}_3, \ldots, \dot{w}_m\}^T$ and $E = \{\breve{e}_1, \breve{e}_2, \breve{e}_3, \ldots, \breve{e}_n\}^T$ such that $\dot{w}_j > 0, \sum_{j=1}^{m} \dot{w}_j = 0$, $\breve{e}_i > 0, \sum_{i=1}^{n} \breve{e}_i = 0$. Consider again the above two $\mathcal{IFHSE}$-Sets $(G_1, L_1) = \{\phi_i, \zeta_{G_{1}(\ell_j)}(\phi_i), \kappa_{G_{1}(\ell_j)}(\phi_i) \mid \phi_i \in \widehat{\Theta}\}$ and $(G_2, L_2) = \{\phi_i, \zeta_{G_{2}(\ell_j)}(\phi_i), \kappa_{G_{2}(\ell_j)}(\phi_i) \mid \phi_i \in \widehat{\Theta}\}$, then the weighted $\mathbb{CC}$ between sets can be defined as follows:

$$Weighted\mathbb{CC}((G_1, L_1), (G_2, L_2)) = \frac{WeightedCo((G_1, L_1), (G_2, L_2))}{\sqrt{Weighted\mathcal{E}_1(G_1, L_1)} \times \sqrt{Weighted\mathcal{E}_2(G_2, L_2)}}$$

or $Weighted\mathbb{CC}((G_1, L_1), (G_2, L_2)) =$

$$\frac{\sum_{i=1}^{n} \dot{w}_j(\sum_{j=1}^{m} \breve{e}_i((\zeta_{G_{1}(\ell_j)}(\phi_i) \times \zeta_{G_{2}(\ell_j)}(\phi_i) + \kappa_{G_{1}(\ell_j)}(\phi_i) \times \kappa_{G_{2}(\ell_j)}(\phi_i))}{\sqrt{\sum_{j=1}^{m} \dot{w}_j(\sum_{i=1}^{n} \breve{e}((\zeta_{G_{1}(\ell_j)}(\phi_i))^2 + (\kappa_{G_{1}(\ell_j)}(\phi_i))^2)} \times \sqrt{\sum_{j=1}^{m} \dot{w}_j(\sum_{i=1}^{n} \breve{e}((\zeta_{G_{2}(\ell_j)}(\phi_i))^2 + (\kappa_{G_{2}(\ell_j)}(\phi_i))^2)}}.$$

Definition 4.9 Consider again the above two $\mathcal{IFHSE}$-Sets$(G_1, L_1) = \{\phi_i, \zeta_{G_{1}(\ell_j)}(\phi_i), \kappa_{G_{1}(\ell_j)}(\phi_i) \mid \phi_i \in \widehat{\Theta}\}$ and $(G_2, L_2) = \{\phi_i, \zeta_{G_{2}(\ell_j)}(\phi_i), \kappa_{G_{2}(\ell_j)}(\phi_i) \mid \phi_i \in \widehat{\Theta}\}$, then the weighted $\mathbb{CC}$ between sets can also be described as:

$$\gamma_{WIFHSES}\mathbb{CC}((G_1, L_1), (G_2, L_2)) = \frac{WeightedCo((G_1, L_1), (G_2, L_2))}{\max\{Weighted\mathcal{E}_1(G_1, L_1), Weighted\mathcal{E}_2(G_2, L_2)\}}$$

$\gamma_{WIFHSES}\mathbb{CC}((G_1, L_1), (G_2, L_2))=$

$$\frac{\sum_{j=1}^{m} \dot{w}_j(\sum_{i=1}^{n} \breve{e}((\zeta_{G_{1}(\ell_j)}(\phi_i) \times \zeta_{G_{2}(\ell_j)}(\phi_i) + \kappa_{G_{1}(\ell_j)}(\phi_i) \times \kappa_{G_{2}(\ell_j)}(\phi_i))}{\max\left\{\begin{array}{l}\sum_{j=1}^{m} \dot{w}_j(\sum_{i=1}^{n} \breve{e}((\zeta_{G_{1}(\ell_j)}(\phi_i))^2 + (\kappa_{G_{1}(\ell_j)}(\phi_i))^2), \\ \sum_{j=1}^{m} \dot{w}_j(\sum_{i=1}^{n} \breve{e}((\zeta_{G_{2}(\ell_j)}(\phi_i))^2 + (\kappa_{G_{2}(\ell_j)}(\phi_i))^2)\end{array}\right\}}.$$

Proposition 4.10 *Consider again the above two* $\mathcal{IFHSE}$-*Sets* $(G_1, L_1) = \{\phi_i, \zeta_{G_{1}(\ell_j)}(\phi_i), \kappa_{G_{1}(\ell_j)}(\phi_i) \mid \phi_i \in \widehat{\Theta}\}$ *and* $(G_2, L_2) = \{\phi_i, \zeta_{G_{2}(\ell_j)}(\phi_i), \kappa_{G_{2}(\ell_j)}(\phi_i) \mid \phi_i \in \widehat{\Theta}\}$, *then the weighted* $\mathbb{CC}$ *satisfies the following characteristics as:*

(1) $0 \le \gamma_{WIFHSES}\mathbb{CC}((G_1, L_1), (G_2, L_2)) \le 1$
(2) $\gamma_{WIFHSES}\mathbb{CC}((G_1, L_1), (G_2, L_2)) = \gamma_{WIFHSES}\mathbb{CC}((G_2, L_2), (G_1, L_1))$
(3) If $(G_1, L_1) = (G_2, L_2)$ i.e $\zeta_{G_1}(\phi_i) = \zeta_{G_2}(\phi_i)$, and $\kappa_{G_1}(\phi_i) = \kappa_{G_2}(\phi_i)$ for all i, then $\gamma_{WIFHSES}\mathbb{CC}((G_1, L_1), (G_2, L_2)) = 1$.

Proof Proof is simple. □

5 Aggregation operators to $\mathcal{IFHSE}$-set

This section contains the average and geometric weighted operators for $\mathcal{IFHSE}$-Set.

Definition 5.1 Suppose $S = (\zeta, \kappa)$, $S_1 = (\zeta_1, \kappa_1)$ and $S_2 = (\zeta_2, \kappa_2)$ be the three $\mathcal{IFHSEN}$s with η positive real number. Then, operational laws can be defined as follows:

(1) $S_1 + S_2 = <\zeta_1 + \zeta_2 - \zeta_1\zeta_2, \kappa_1\kappa_2>$
(2) $S_1 \times S_2 = <\kappa_1\kappa_2, \zeta_1 + \zeta_2 - \kappa_1\kappa_2>$
(3) $\eta S = <[1 - (1 - \zeta)^\eta, (\kappa)^\eta]>$
(4) $S^\eta = <[(\zeta)^\eta, [1 - (1 - \kappa)^\eta]>$.

Definition 5.2 [67] Consider the $\mathcal{IFHSEN}$s $S_{ij} = (\zeta_{ij}, \kappa_{ij})$ and let experts and sub-attribute of some chosen parameters have γ_i, δ_j the weight vectors, respectively, with the given conditions $\gamma_i > 0$, $\sum_{i=1}^{m} \gamma_j = 1$, $\delta_i > 0$, $\sum_{j=1}^{n} \delta_j = 1$. Then, weight average operator is a mapping $WA : \triangle^n \to \triangle$ defined by as

$$WA(S_{11}, S_{12}, S_{13}, \ldots, S_{mn}) = \bigotimes_{j=1}^{n} \delta\left(\bigotimes_{i=1}^{m} \gamma_i S_{ij}\right). \quad (1)$$

Theorem 5.3 *Suppose* $S_{ij} = (\zeta_{ij}, \kappa_{ij})$ *be the* $\mathcal{IFHSEN}$s *with* $i = 1, 2, 3, \ldots m$, $j = 1, 2, 3, \ldots n$, *then its weighted operator is defined by as*

$$WA(S_{11}, S_{12}, S_{13}, \ldots, S_{mn}) = \left\langle 1 - \Pi_{i=1}^{n}(\Pi_{j=1}^{m}(1 - \zeta_{ij})^{\gamma_j})^{\delta_i}, \Pi_{i=1}^{n}(\Pi_{j=1}^{m}(\kappa)^{\gamma_j})^{\delta_i} \right\rangle. \quad (2)$$

Proof This proof is simple □

Table 1 Decision matrix for substitute $\Re_1$

Θ	Θ_1	Θ_2	Θ_3	Θ_4	Θ_5	Θ_6	Θ_7	Θ_8
$(k_1, T_1, 0)$	(0.5, 0.6)	(0.4, 0.7)	(0.2, 0.9)	(0.3, 0.7)	(0.1, 0.9)	(0.4, 0.9)	(0.5, 0.8)	(0.5, 0.7)
$(k_1, T_2, 0)$	(0.4, 0.6)	(0.5, 0.6)	(0.1, 0.6)	(0.5, 0.6)	(0.2, 0.9)	(0.5, 0.6)	(0.1, 0.9)	(0.3, 0.9)
$(k_1, T_3, 0)$	(0.5,, 0.8)	(0.1, 0.6)	(0.5, 0.7)	(0.3, 0.8)	(0.5, 0.7)	(0.2, 0.6)	(0.5, 0.8)	(0.5, 0.6)
$(k_2, T_1, 0)$	(0.2, 0.6)	(0.5, 0.7)	(0.3, 0.6)	(0.5, 0.9)	(0.2, 0.9)	(0.5, 0.9)	(0.2, 0.9)	(0.1, 0.9)

Table 2 Decision matrix for substitute $\Re_2$

Θ	Θ_1	Θ_2	Θ_3	Θ_4	Θ_5	Θ_6	Θ_7	Θ_8
$(k_1, T_1, 0)$	(0.1, 0.6)	(0.2, 0.7)	(0.5, 0.6)	(0.2, 0.7)	(0.3, 0.9)	(0.2, , 0.9)	(0.1, 0.9)	(0.1, 0.9)
$(k_1, T_2, 0)$	(0.5, 0.7)	(0.5, 0.9)	(0.5, 0.8)	(0.3, 0.6)	(0.5, 0.7)	(0.5, 0.9)	(0.5, 0.7)	(0.5, 0.8)
$(k_1, T_3, 0)$	(0.2, 0.6)	(0.3, 0.6)	(0.2, 0.6)	(0.6, 0.8)	(0.5, 0.8)	(0.6, 0.9)	(0.3, 0.8)	(0.4, 0.9)
$(k_2, T_1, 0)$	(0.5, 0.8)	(0.5, 0.8)	(0.3, 0.7)	(0.5, 0.6)	(0.1, 0.9)	(0.5, 0.8)	(0.4, 0.9)	(0.5, 0.6)

Table 3 Decision matrix for substitute $\Re_3$

Θ	Θ_1	Θ_2	Θ_3	Θ_4	Θ_5	Θ_6	Θ_7	Θ_8
$(k_1, T_1, 0)$	(0.2, 0.6)	(0.3, 0.7)	(0.5, 0.9)	(0.1, 0.7)	(0.2, 0.9)	(0.3, 0.9)	(0.2, 0.9)	(0.3, 0.6)
$(k_1, T_2, 0)$	(0.1, 0.9)	(0.8, 0.9)	(0.3, 0.6)	(0.5, 0.6)	(0.5, 0.8)	(0.4, 0.8)	(0.3, 0.8)	(0.5, 0.3)
$(k_1, T_3, 0)$	(0.4, 0.6)	(0.5, 0.7)	(0.5, 0.9)	(0.4, 0.6)	(0.4, 0.9)	(0.5, 0.6)	(0.4, 0.7)	(0.3, 0.9)
$(k_2, T_1, 0)$	(0.5, 0.7)	(0.5, 0.9)	(0.2, 0.6)	(0.6, 0.9)	(0.5, 0.7)	(0.6, 0.7)	(0.5, 0.8)	(0.5, 0.8)

Table 4 Decision matrix for substitute $\Re_4$

Θ	Θ_1	Θ_2	Θ_3	Θ_4	Θ_5	Θ_6	Θ_7	Θ_8
$(k_1, T_1, 0)$	(0.5, 0.6)	(0.1, 0.7)	(0.5, 0.9)	(0.6, 0.7)	(0.5, 0.9)	(0.2, 0.9)	(0.3, 0.9)	(0.3, 0.9)
$(k_1, T_2, 0)$	(0.3, 0.8)	(0.5, 0.8)	(0.4, 0.7)	(0.5, 0.6)	(0.3, 0.8)	(0.5, 0.7)	(0.5, 0.8)	(0.5, 0.8)
$(k_1, T_3, 0)$	(0.5, 0.7)	(0.5, 0.9)	(0.5, 0.6)	(0.5, 0.6)	(0.4, 0.9)	(0.1, 0.9)	(0.3, 0.9)	(0.4, 0.9)
$(k_2, T_1, 0)$	(0.2, 0.6)	(0.3, 0.6)	(0.4, 0.8)	(0.5, 0.6)	(0.5, 0.7)	(0.4, 0.8)	(0.2, 0.6)	(0.4, 0.6)

Example 5.4 Let expert set is denoted by $\widetilde{Y} = \{u_1, u_2, u_3\}$, while weight vector is $\mathbb{W} = (0.10, 0.16, 0.17)^T$. Suppose a task is given to the experts to measure the quality of the National Bank with predefined attributes $\Phi = \{\hbar_1 = Education\ of\ Manager, \hbar_2 = pay\}$. While their corresponding sub-attributes are $\hbar_1 = \{\hbar_{11} = 14years, \hbar_{12} = 16years\}$ and $\hbar_2 = \{\hbar_{21} = 50Dollar, \hbar_{21} = 60Dollar\}$. Then, we get $\tilde{\Phi} = \{\hbar_1 \times \hbar_2\} = \{\hbar_{11}, \hbar_{12}\} \times \{\hbar_{21}, \hbar_{22}\} = \{\kappa_1 = (v_1, \{\hbar_{11}, \hbar_{21}\}), \kappa_2 = (v_2, \{\hbar_{11}, \hbar_{22}\}), \kappa_3 = (v_3, \{\hbar_{12}, \hbar_{21}\}), \kappa_4 = (v_4, \{\hbar_{12}, \hbar_{22}\})\}$. Let $\tilde{\nabla} = \{\kappa_1, \kappa_2, \kappa_3\} \subseteq \tilde{\Phi}$ represents the collection of multi sub-attributes having weights Æ$= \{0.24, 0.55, 0.58\}^T$. Considering $\{\overline{\wedge}_1, \overline{\wedge}_2, \overline{\wedge}_3, \overline{\wedge}_4\}$ as a initial universe. Then, the opinions given by the experts committee after having a deep analysis can be seen as: $(F, \mathcal{C})$ =

Fig. 1 Ranking of substitutes

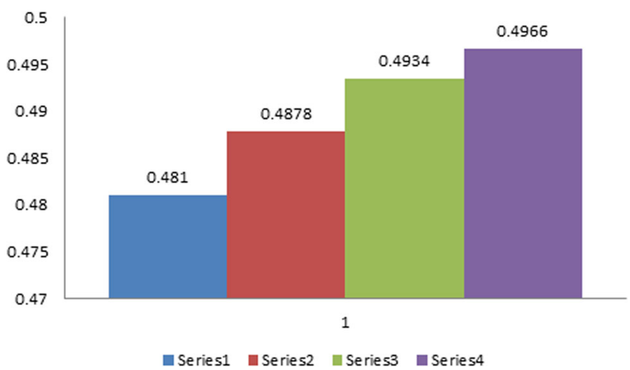

Fig. 2 Ranking of substitutes

$$\mathrm{WA}(S_{11}, S_{12}, S_{13}, \ldots, S_{mn}) = \left\langle 1 - \Pi_{i=1}^{n}(\Pi_{j=1}^{m}(1-\zeta_{ij})^{\gamma_j})^{\delta_i}, \Pi_{i=1}^{n}(\Pi_{j=1}^{m}(\kappa)^{\gamma_j})^{\delta_i} \right\rangle.$$

$$\left\{(\kappa_1, \mathrm{u}_1, 1)\left\{\left(\frac{\bar{\Lambda}_1}{0.7}\right), \left(\frac{\bar{\Lambda}_2}{0.5}\right), \left(\frac{\bar{\Lambda}_3}{0.6}\right), \left(\frac{\bar{\Lambda}_4}{0.3}\right)\right\}\right\}, \left\{(\kappa_2, \mathrm{u}_1, 1)\left\{\left(\frac{\bar{\Lambda}_1}{0.5}\right), \left(\frac{\bar{\Lambda}_2}{0.4}\right), \left(\frac{\bar{\Lambda}_3}{0.3}\right), \left(\frac{\bar{\Lambda}_4}{0.1}\right)\right\}\right\},$$
$$\left\{(\kappa_3, \mathrm{u}_1, 1)\left\{\left(\frac{\bar{\Lambda}_1}{0.6}\right), \left(\frac{\bar{\Lambda}_2}{0.4}\right), \left(\frac{\bar{\Lambda}_3}{0.1}\right), \left(\frac{\bar{\Lambda}_4}{0.2}\right)\right\}\right\}, \left\{(\kappa_1, \mathrm{u}_2, 1)\left\{\left(\frac{\bar{\Lambda}_1}{0.4}\right), \left(\frac{\bar{\Lambda}_2}{0.5}\right), \left(\frac{\bar{\Lambda}_3}{0.2}\right), \left(\frac{\bar{\Lambda}_4}{0.3}\right)\right\}\right\},$$
$$\left\{(\kappa_1, \mathrm{u}_3, 1)\left\{\left(\frac{\bar{\Lambda}_1}{0.9}\right), \left(\frac{\bar{\Lambda}_2}{0.6}\right), \left(\frac{\bar{\Lambda}_3}{0.2}\right), \left(\frac{\bar{\Lambda}_4}{0.7}\right)\right\}\right\}, \left\{(\kappa_2, \mathrm{u}_3, 1)\left\{\left(\frac{\bar{\Lambda}_1}{0.1}\right), \left(\frac{\bar{\Lambda}_2}{0.4}\right), \left(\frac{\bar{\Lambda}_3}{0.3}\right), \left(\frac{\bar{\Lambda}_4}{0.6}\right)\right\}\right\},$$
$$\left\{(\kappa_3, \mathrm{u}_3, 1)\left\{\left(\frac{\bar{\Lambda}_1}{0.8}\right), \left(\frac{\bar{\Lambda}_2}{0.7}\right), \left(\frac{\bar{\Lambda}_3}{0.1}\right), \left(\frac{\bar{\Lambda}_4}{0.4}\right)\right\}\right\}, \left\{(\kappa_1, \mathrm{u}_1, 0)\left\{\left(\frac{\bar{\Lambda}_1}{0.6}\right), \left(\frac{\bar{\Lambda}_2}{0.9}\right), \left(\frac{\bar{\Lambda}_3}{0.2}\right), \left(\frac{\bar{\Lambda}_4}{0.1}\right)\right\}\right\},$$
$$\left\{(\kappa_2, \mathrm{u}_2, 1)\left\{\left(\frac{\bar{\Lambda}_1}{0.2}\right), \left(\frac{\bar{\Lambda}_2}{0.4}\right), \left(\frac{\bar{\Lambda}_3}{0.3}\right), \left(\frac{\bar{\Lambda}_4}{0.9}\right)\right\}\right\}, \left\{(\kappa_3, \mathrm{u}_2, 1)\left\{\left(\frac{\bar{\Lambda}_1}{0.6}\right), \left(\frac{\bar{\Lambda}_2}{0.7}\right), \left(\frac{\bar{\Lambda}_3}{0.1}\right), \left(\frac{\bar{\Lambda}_4}{0.8}\right)\right\}\right\},$$
$$\left\{(\kappa_2, \mathrm{u}_1, 0)\left\{\left(\frac{\bar{\Lambda}_1}{0.4}\right), \left(\frac{\bar{\Lambda}_2}{0.3}\right), \left(\frac{\bar{\Lambda}_3}{0.4}\right), \left(\frac{\bar{\Lambda}_4}{0.2}\right)\right\}\right\}, \left\{(\kappa_3, \mathrm{u}_1, 0)\left\{\left(\frac{\bar{\Lambda}_1}{0.7}\right), \left(\frac{\bar{\Lambda}_2}{0.5}\right), \left(\frac{\bar{\Lambda}_3}{0.2}\right), \left(\frac{\bar{\Lambda}_4}{0.3}\right)\right\}\right\},$$
$$\left\{(\kappa_1, \mathrm{u}_2, 0)\left\{\left(\frac{\bar{\Lambda}_1}{0.8}\right), \left(\frac{\bar{\Lambda}_2}{0.5}\right), \left(\frac{\bar{\Lambda}_3}{0.2}\right), \left(\frac{\bar{\Lambda}_4}{0.9}\right)\right\}\right\}, \left\{(\kappa_2, \mathrm{u}_2, 0)\left\{\left(\frac{\bar{\Lambda}_1}{0.7}\right), \left(\frac{\bar{\Lambda}_2}{0.8}\right), \left(\frac{\bar{\Lambda}_3}{0.2}\right), \left(\frac{\bar{\Lambda}_4}{0.9}\right)\right\}\right\},$$
$$\left\{(\kappa_3, \mathrm{u}_2, 0)\left\{\left(\frac{\bar{\Lambda}_1}{0.4}\right), \left(\frac{\bar{\Lambda}_2}{0.5}\right), \left(\frac{\bar{\Lambda}_3}{0.4}\right), \left(\frac{\bar{\Lambda}_4}{0.8}\right)\right\}\right\}, \left\{(\kappa_1, \mathrm{u}_3, 0)\left\{\left(\frac{\bar{\Lambda}_1}{0.9}\right), \left(\frac{\bar{\Lambda}_2}{0.6}\right), \left(\frac{\bar{\Lambda}_3}{0.2}\right), \left(\frac{\bar{\Lambda}_4}{0.7}\right)\right\}\right\},$$
$$\left\{(\kappa_2, \mathrm{u}_3, 0)\left\{\left(\frac{\bar{\Lambda}_1}{0.1}\right), \left(\frac{\bar{\Lambda}_2}{0.4}\right), \left(\frac{\bar{\Lambda}_3}{0.3}\right), \left(\frac{\bar{\Lambda}_4}{0.6}\right)\right\}\right\}, \left\{(\kappa_3, \mathrm{u}_3, 0)\left\{\left(\frac{\bar{\Lambda}_1}{0.8}\right), \left(\frac{\bar{\Lambda}_2}{0.7}\right), \left(\frac{\bar{\Lambda}_3}{0.1}\right), \left(\frac{\bar{\Lambda}_4}{0.4}\right)\right\}\right\}.$$

Here, $\mathcal{IFHSEN}$s $(\Upsilon, \tilde{\Phi}) = \langle \phi_{ij}, \psi_{ij} \rangle_{3\times 3}$ represents the supposed rating values of the specialists with each sub-attributes can be calculated as

$$(\Upsilon_1, \tilde{\Phi_1}) = \begin{bmatrix} (.7,.6) & (.5,.9) & (.6,.3) \\ (.5,.4) & (.4,.5) & (.3,.7) \\ (.6,.7) & (.4,.6) & (.3,.8) \end{bmatrix},$$

$$(\Upsilon_2, \tilde{\Phi_2}) = \begin{bmatrix} (.3,.4) & (.1,.4) & (.2,.1) \\ (.2,.2) & (.1,.3) & (.1,.4) \\ (.4,.4) & (.2,.4) & (.1,.2) \end{bmatrix}$$

$$(\Upsilon_1, \tilde{\Phi_1}) - (\Upsilon_2, \tilde{\Phi_2}) = \begin{bmatrix} (.4,.2) & (.4,.5) & (.4,.2) \\ (.3,.2) & (.3,.2) & (.2,.3) \\ (.2,.3) & (.2,.2) & (.2,.6) \end{bmatrix}.$$

Making use of eq. 7(2),

$$=$$

$$\left\langle 1-\begin{pmatrix}\left\{(.3)^{.10}(.2)^{.16}(.1)^{.17}\right\}^{0.24}\\ \left\{(.3)^{.10}(.2)^{.16}(.1)^{.17}\right\}^{0.55}\\ \left\{(.3)^{.10}(.1)^{.16}(.1)^{.17}\right\}^{0.58}\end{pmatrix},\begin{pmatrix}\left\{(.2)^{.10}(.2)^{.16}(.3)^{.17}\right\}^{0.24}\\ \left\{(.5)^{.10}(.2)^{.16}(.2)^{.17}\right\}^{0.55}\\ \left\{(.2)^{.10}(.3)^{.16}(.6)^{.17}\right\}^{0.58}\end{pmatrix}\right\rangle.$$

$=\langle .9873, .1940\rangle$.

Definition 5.5 [67] Suppose $\Delta_{ij}=\langle\zeta_{ij},\eta_{ij}\rangle$ is an $\mathcal{IFHSEN}$ and consider Λ_i, Υ_j as a weight vectors using for experts and selected sub attributes, respectively, satisfying the prescribed condition $\Lambda_i>0$ such that $\sum_{i=1}^{p}\Lambda_i=1$, $\Upsilon_j>0$ such that $\sum_{j=1}^{q}\Upsilon_j=1$, then $\mathcal{IFHSEWG}$ operator can be stated as

$$WG(\Delta_{11},\Delta_{12},\Delta_{13},\ldots,\Delta_{pq})=\bigodot_{j=1}^{q}\left(\bigodot_{i=1}^{p}\Delta_{ij}^{\Lambda_i}\right)^{\Upsilon_j}. \tag{3}$$

Theorem 5.6 *Making use of $\mathcal{IFHSEWG}$ operator, the getting value is $\mathcal{IFHSEN}$ and can be calculated as*

$$WG(\Delta_{11},\Delta_{12},\Delta_{13},\ldots,\Delta_{pq})=\left\langle\prod_j^p\left(\prod_i^q(\zeta_{ij})^{\Lambda_i}\right)^{\Upsilon_j},1-\prod_j^p\left(1-\prod_i^q(\zeta_{ij})^{\Lambda_i}\right)^{\Upsilon_j}\right\rangle. \tag{4}$$

Proof This proof is simple. □

Definition 5.7 [67] Suppose $\Delta_{ij}=\langle\zeta_{ij},\eta_{ij}\rangle$ is an $\mathcal{IFHSEN}$, then its score function using $\mathcal{IFHSE}$ model can be defined as

$$\mathbb{SF}=\Lambda_i+\Upsilon_j\div 2. \tag{5}$$

Example 5.8 Keeping in view Example 5.4.

$$(\Upsilon,\tilde{\Phi}) == \begin{bmatrix}(.4,.2) & (.4,.5) & (.4,.2)\\ (.3,.2) & (.3,.2) & (.2,.3)\\ (.2,.3) & (.2,.2) & (.2,.6)\end{bmatrix}$$

Making use of equation(4), we get
$WG(\Delta_{11},\Delta_{12},\Delta_{13},\ldots,\Delta_{pq})=\Big\langle\coprod_j^p(\coprod_i^q(\zeta_{ij})^{\Lambda_i})^{\Upsilon_j}$, $1-\coprod_j^p(1-\coprod_i^q(\zeta_{ij})^{\Lambda_i})^{\Upsilon_j}\rangle$.

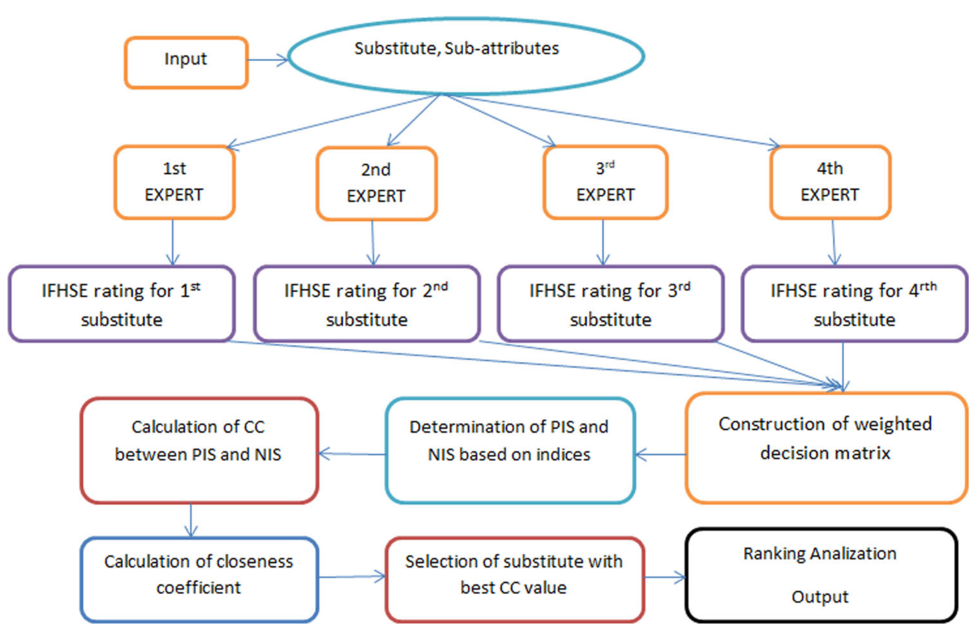

Fig. 3 Flowchart for TOPSIS model

$$\left\langle \begin{pmatrix} \left\{\left\{(.4)^{.10}(.3)^{.16}(.2)^{.17}\right\}^{0.24}\right\}\left\{\left\{(.4)^{.10}(.3)^{.16}(.1)^{.17}\right\}^{0.55}\right\} \\ \left\{(.4)^{.10}\left\{(.1)^{.16}\right\}(.1)^{.17}\right\}^{0.58} \end{pmatrix}, \begin{pmatrix} 1-\left\{(.1)^{.10}(.1)^{.16}(.3)^{.17}\right\}^{0.24}\left\{(.4)^{.10}(.1)^{.16}(.1)^{.17}\right\}^{0.55} \\ \left\{(.1)^{.10}(.2)^{.16}(.5)^{.17}\right\}^{0.58} \end{pmatrix} \right\rangle.$$

$$= \langle .3681, .6367 \rangle.$$

5.1 Decision-making methodology constructed on suggested operators

An MADM methodology is offered constructed on the premeditated operators and termed the mathematical instances for presenting their effectiveness.

Suppose $\Re = \{T^1, T^2, T^3, \ldots, T^q\}$ and $\mathfrak{W} = \{\Re_1, \Re_1, \Re_1, \ldots, \Re_r\}$ represent the set of experts and substitutes, respectively, and weights for experts are $\Theta = (\Theta_1, \Theta_2, \Theta_3, \ldots, \Theta_p)^T$ with $\Theta_i > 0$ and $\sum_{i=1}^{p} \Theta_i = 1$.

5.1.1 Formation of decision-making algorithm

Selection descriptions have been made through the following algorithm.

5.2 Selection of best substitute by using $\mathcal{IFHSEWA}$ operator

Step 1 Find the $\mathcal{IFHSEN}$s with the help of experts, dealing with four alternatives and sub-attributes and their scores are shown in Tables 1, 2, 3 and 4.

Step 2 By making use of Eq. 2, we can find the opinions of different experts and can be potted as: $\Psi_1 = <.7623, .2136>$, $\Psi_2 = <.7234, .2238>$, $\Psi_3 = <.7429, .2310>$, $\Psi_4 = <.7533, .2279>$.

Step 3: In this step, scores values have been found for each substitute with the help of Eq. 5. $= 0.4880, \bigvee_2 = 0.4736, \bigvee_3 = 0.4870, \bigvee_4 = 0.4906$.

Step 4: In this step, we can find the ranking of the substitutes as follows $\bigvee_4 > \bigvee_1 > \bigvee_3 > \bigvee_2$. Therefore, $\Re^4 > \Re^1 > \Re^3 > \Re^2$, so, the substitute $\Re^4$ is the best choice. These ranking is shown in Fig. 1.

5.3 Selection of best substitute by using $\mathcal{IFHSEWG}$ operator

Step 1 Find the $\mathcal{IFHSEN}$s with the help of experts, dealing with four substitutes and sub-attributes and their scores are shown in Tables 1, 2, 3 and 4.

Step 2 By making use of Eq. 1, we can find the opinions of different experts and can be potted as: $\Psi_1 = <.2637, .7123>$, $\Psi_2 = <.2523, .7234>$, $\Psi_3 = <.2445, .7423>$, $\Psi_4 = <.2578, .7354>$.

Step 3 In this step, scores values have been found for each substitute with the help of Eq. 4. $\bigvee_1 = 0.4810, \bigvee_2 = 0.4878, \bigvee_3 = 0.4934, \bigvee_4 = 0.4966$.

▷ **Start**:

▷ **Input**:

1. Considering $\{\bar{\wedge}_1, \bar{\wedge}_2, \bar{\wedge}_3\}$ as a initial universe.

2. Taking set of attributes $\Phi = \{\hbar_1, \hbar_2\}$ with their corresponding sub-attributes $\hbar_1 = \{\hbar_{11}, \hbar_{12}\}$ and $\hbar_2 = \{\hbar_{21}, \hbar_{21}\}$.

3. Expert set $\widetilde{Y} = \{\mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{u}_3\}$.

▷ **Construction**:

———4. Construction of $\mathcal{IFHSE}$-Set for each Substitute.

▷ **Computation**:

———5. Calculation of $\mathcal{IFHSEN}$s for each substitute with the help of $\mathcal{IFHSEWA}$ or $\mathcal{IFHSEWG}$ operators..

———6. Determination of scores values for substitute with the help of eq.5.

———7. Selection of best substitute using best score.

▷ **Output**:

———8. Determination of ranking.

▷ **End**:

==

Table 5 Weighted decision matrix for substitute $\Re_1$

T^1	κ_1	κ_2	κ_3	κ_4	κ_5	κ_6	κ_7	κ_8
$\Re_1$	(0.53, 0.64)	(0.42, 0.73)	(0.21, 0.91)	(0.34, 0.78)	(0.13, 0.99)	(0.43, 0.97)	(0.52, 0.89)	(0.54, 0.71)
$\Re_2$	(0.42, 0.62)	(0.53, 0.64)	(0.13, 0.64)	(0.57, 0.63)	(0.24, 0.97)	(0.54, 0.68)	(0.13, 0.92)	(0.39, 0.93)
$\Re_3$	(0.52, 0.86)	(0.14, 0.66)	(0.57, 0.78)	(0.34, 0.82)	(0.57, 0.72)	(0.22, 0.69)	(0.57, 0.83)	(0.57, 0.61)
$\Re_4$	(0.26, 0.67)	(0.58, 0.73)	(0.34, 0.64)	(0.57, 0.98)	(0.24, 0.92)	(0.51, 0.92)	(0.28, 0.91)	(0.13, 0.94)

Table 6 Weighted decision matrix for substitute $\Re_2$

T^2	κ_1	κ_2	κ_3	κ_4	κ_5	κ_6	κ_7	κ_8
$\Re_1$	(0.14, 0.65)	(0.24, 0.78)	(0.56, 0.62)	(0.25, 0.78)	(0.37, 0.94)	(0.28, 0.98)	(0.81, 0.9)	(0.31, 0.89)
$\Re_1$	(0.54, 0.75)	(0.54, 0.96)	(0.57, 0.82)	(0.37, 0.67)	(0.51, 0.72)	(0.59, 0.47)	(0.95, 0.7)	(0.55, 0.38)
$\Re_1$	(0.23, 0.61)	(0.36, 0.64)	(0.25, 0.63)	(0.66, 0.86)	(0.52, 0.84)	(0.67, 0.65)	(0.63, 0.8)	(0.64, 0.19)
$\Re_1$	(0.54, 0.82)	(0.57, 0.87)	(0.33, 0.71)	(0.54, 0.64)	(0.13, 0.93)	(0.56, 0.82)	(0.54, 0.9)	(0.65, 0.46)

Table 7 Weighted decision matrix for substitute $\Re_3$

T^3	κ_1	κ_2	κ_3	κ_4	κ_5	κ_6	κ_7	κ_8
$\Re_1$	(0.42, 0.56)	(0.43, 0.47)	(0.15, 0.49)	(0.41, 0.87)	(0.32, 0.89)	(0.13, 0.69)	(0.32, 0.59)	(0.23, 0.56)
$\Re_1$	(0.51, 0.19)	(0.58, 0.29)	(0.33, 0.56)	(0.65, 0.56)	(0.25, 0.58)	(0.34, 0.78)	(0.33, 0.68)	(0.15, 0.53)
$\Re_1$	(0.74, 0.26)	(0.65, 0.27)	(0.55, 0.69)	(0.74, 0.56)	(0.54, 0.39)	(0.45, 0.96)	(0.24, 0.67)	(0.33, 0.39)
$\Re_1$	(0.75, 0.37)	(0.25, 0.49)	(0.32, 0.76)	(0.36, 0.39)	(0.75, 0.27)	(0.16, 0.57)	(0.45, 0.38)	(0.25, 0.28)

Table 8 Weighted decision matrix for substitute $\Re_4$

T^4	κ_1	κ_2	κ_3	κ_4	κ_5	κ_6	κ_7	κ_8
$\Re_1$	(0.15, 0.26)	(0.21, 0.57)	(0.85, 0.79)	(0.86, 0.47)	(0.55, 0.29)	(0.52, 0.89)	(0.23, 0.59)	(0.43, 0.29)
$\Re_2$	(0.23, 0.48)	(0.45, 0.68)	(0.44, 0.97)	(0.45, 0.26)	(0.23, 0.28)	(0.75, 0.97)	(0.45, 0.58)	(0.55, 0.58)
$\Re_3$	(0.35, 0.27)	(0.55, 0.79)	(0.55, 0.56)	(0.15, 0.46)	(0.44, 0.39)	(0.41, 0.09)	(0.23, 0.39)	(0.34, 0.89)
$\Re_4$	(0.42, 0.56)	(0.33, 0.46)	(0.34, 0.38)	(0.35, 0.26)	(0.55, 0.47)	(0.34, 0.48)	(0.22, 0.36)	(0.84, 0.26)

Step 4 In this step, we can find the ranking of the substitutes as follows $\bigvee_4 > \bigvee_3 > \bigvee_1 > \bigvee_2$. Therefore, $\Re^4 > \Re^3 > \Re^1 > \Re^2$, so the substitute $\Re^4$ is the best choice. Their ranking is presented in Fig. 2.

6 TOPSIS approach for solving DM problems based on correlation coefficient

This part introduces a strategy to address DM (Decision Making) issues using the proposed $\mathbb{CC}$, which depends upon the TOPSIS technique for the $\mathcal{IFHSE}$-Set. Initially introduced by Hwang and Yoon [68], the TOPSIS strategy emphasizes the ranking of evaluation objects based on their

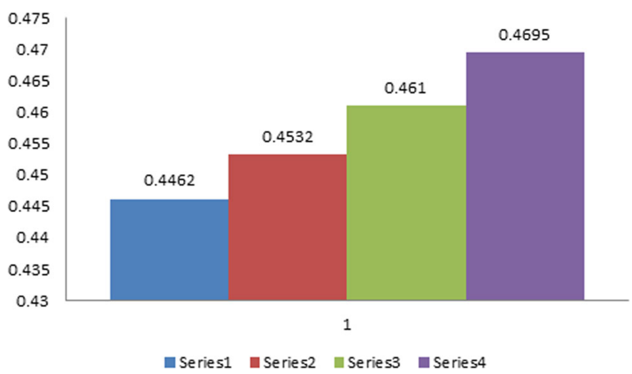

Fig. 4 Ranking of substitutes

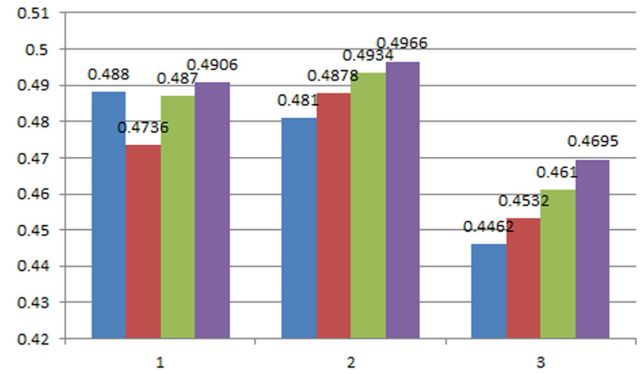

Fig. 5 Ranking of substitutes

proximity to positive and negative ideal solutions for DM issues. This strategy is based on the concept that the most ideal decision should have the smallest distance from the positive ideal solution and the greatest distance from the negative ideal solution. A modified TOPSIS procedure is presented, where the $\mathbb{CC}$ is employed to assess the positive and negative values and determine the rankings of the decisions. In most previous TOPSIS procedures, distinct distance and similarity measures were used to calculate the proximity coefficient. However, if close items are connected, it implies that less related but distant items should also exhibit some level of connection, reflecting their integration or segregation within the data analysis process. The advantage of using the $\mathbb{CC}$ instead of the distance or similarity measure in the TOPSIS strategy is that it preserves the direct relationship among the variables being considered. The algorithm for selecting the best decision using the TOPSIS technique after making a little modification in algorithm [67], based on the proposed $\mathbb{CC}$, is described as follows:

6.1 Proposed approach

Consider a collection of substitutes $M = \{M^1, M^2, M^3, \ldots, M^c\}$. The evaluations of the substitutes are given by a group of specialists $S = \{l_1, l_2, \ldots, l_n\}$ along with weights $\kappa = \{\pi_1, \pi_2, \ldots, \pi_n\}^A$ having property that $\sum_{i=1}^{n}(\pi_i) = 1$ and every π_i is positive. Let $B = \{B_1, B_2, \ldots, B_m\}$ be the set of attributes with weight vector $\varpi = \{\varpi_1, \varpi_2, \ldots, \varpi_e\}^E$ having property that $\sum_{j=1}^{e}(\varpi_j) = 1$ and every ϖ_j is positive. The assessment of substitutes is portrayed as $\mathcal{IFHSEN}$s which is expressed as $M_{ij}^t = (v_i, \chi_{M_t}^t(v_i), \varphi_{M_t}^t(v_i))$ such that $0 \le \chi_{M_t}^t(v_i), \varphi_{M_t}^t(v_i) \le 1$ and $0 \le \chi_{M_t}^t(v_i) + \varphi_{M_t}^t(v_i) \le 1$ separately. Then, at that point, the method for picking the best substitute by using the proposed administrators is summed up in the accompanying advances:

Step 1 Gather the information connected with every substitute and summed up in a matrix as follows:

$$(M^{(t)}, R) = \begin{pmatrix} \left(\chi^{(t)}{}_{11}, \varphi^{(t)}{}_{11}\right) & \left(\chi^{(t)}{}_{12}, \varphi^{(t)}{}_{12}\right) & \cdots & \left(\chi^{(t)}{}_{1q}, \varphi^{(t)}{}_{1q}\right) \\ \left(\chi^{(t)}{}_{21}, \varphi^{(t)}{}_{21}\right) & \left(\chi^{(t)}{}_{22}, \varphi^{(t)}{}_{23}\right) & \cdots & \left(\chi^{(t)}{}_{2q}, \varphi^{(t)}{}_{2q}\right) \\ \vdots & & \ddots & \vdots \\ \left(\chi^{(t)}{}_{p1}, \varphi^{(t)}{}_{p1}\right) & \left(\chi^{(t)}{}_{p2}, \varphi^{(t)}{}_{p2}\right) & \cdots & \left(\chi^{(t)}{}_{pq}, \varphi^{(t)}{}_{pq}\right) \end{pmatrix}.$$

Step 2 Construction of weight decision matrix $M^{-(t)} = (M_{ij}^{-(t)})$, while

$$M_{ij}^{-(t)} = \varpi_i \kappa_j M_{ij}^{-(t)} = 1 - ((1 - \chi_{ij}^{(t)})^{\kappa_i})^{\varpi_j}, ((\varphi_{ij}^{(t)})^{\kappa_i})^{\varpi_j}) = \left(\chi_{ij}^{-(t)}, \varphi_{ij}^{-(t)}\right) \tag{6}$$

Here, κ_i and ϖ_j are representing ith, jth weight vectors for experts and attributes, respectively.

Table 9 Substitute final scores using the suggested methods

Substitute	IFSEWA operator	IFSEWG operator	TOPSIS approach
$\Re_1$	0.4880	0.4810	0.4462
$\Re_2$	0.4736	0.4878	0.4532
$\Re_3$	0.4870	0.4934	0.4610
$\Re_4$	0.4906	0.4966	0.4695

Table 10 Comparison analysis

Features	Model	Truth	Falsity	Parameters	Sub-para	mdo
Zadeh [18]	$\mathfrak{F}$s	✓	×	✓	×	×
Maji et al. [28]	$\mathfrak{F}$ss	✓	×	✓	×	×
Alkhazaleh and Salleh [35]	$\mathfrak{S}$es	✓	×	✓	×	✓
Alkhazaleh and Salleh [36]	$\mathfrak{F}$ses	✓	×	✓	✓	×
Maji et al. [34]	$\mathfrak{I}$fss	✓	×	✓	✓	×
Broumi et al. [37]	$\mathfrak{I}$fses	✓	×	✓	✓	✓
Yolcu et al. [46]	$\mathfrak{I}$fhss	✓	×	✓	✓	×
Proposed model	$\mathcal{IFHSE}$-Set	✓	✓	✓	✓	✓

Step 3 The $\mathbb{CC}$ for every $M_{ij}^{-(t)}$ using complete +ve ideal $d^+ = (0, 1)$ is calculated as:

$$\mathbb{CC}_1(M_{ij}^{-(t)}, d^+) = \frac{M_{IFHSES}(M_{ij}^{-(t)}, d^+)}{\sqrt{\mathcal{E}_{IFHSES}(M_{ij}^{-(t)}).\mathcal{E}_{IFHSES}(M_{ij}^{-(t)})}} \tag{7}$$

or

$$\mathbb{CC}_2(M_{ij}^{-(t)}, d^+) = \frac{M_{IFHSES}(M_{ij}^{-(t)}, d^+)}{\max\left\{\mathcal{E}_{IFHSES}(M_{ij}^{-(t)}).\mathcal{E}_{IFHSES}(M_{ij}^{-(t)})\right\}} \tag{8}$$

After using these relations, we get $\mathbb{CC}$ matrix which is shown as $\varphi = (\varphi_{ij}^{(t)})_{p\times q}$. *Step 4*Calculation of b_{ij}, d_{ij} using $\mathbb{CC}$ matrix φ such as b_{ij}= $\operatorname{argmax}\{\varphi_{ij}^{(t)}\}$ and d_{ij}= $\operatorname{argmin}\{\varphi_{ij}^{(t)}\}$. Depending upon these values, calculate the +ve ideal substitute (PIS)Ψ^+ and -ve ideal substitute (NIS)Ψ^- as

$$\Psi^+ = (\chi^+, \varphi^+)_{p\times q} = \left(\chi_{ij}^{-(t)}, \varphi_{ij}^{-(t)}\right) \tag{9}$$

$$\Psi^- = (\chi^-, \varphi^-)_{p\times q} = \left(\chi_{ij}^{-(t)}, \varphi_{ij}^{-(t)}\right) \tag{10}$$

Step 5 Calculation of $\mathbb{CC}$ between $M^{-(t)}$(weight decision matrix)and Ψ^+(+ve ideal substitute) by using either $\mathbb{CC}_1$ or $\mathbb{CC}_2$ as

$$\mathcal{M}_1 = CC_1\ (M^{-(t)}, \Psi^+) = \frac{M_{IFHSES}(M_{ij}^{-(t)}, \Psi^+)}{\sqrt{\mathcal{E}_{IFHSES}(M_{ij}^{-(t)}).\mathcal{E}_{IFHSES}(\Psi^+)}} \tag{11}$$

$$= \frac{\sum_{i=1}^n \sum_{j=1}^m ((\chi_{ij}^{-(t)}\chi^+ + \varphi_{ij}^{-(t)}\varphi^+))}{\sqrt{\sum_{i=1}^n \sum_{j=1}^m ((\chi_{ij}{}^{-(t)})^2 + (\varphi_{ij}{}^{-(t)})^2)}\sqrt{\sum_{i=1}^n \sum_{j=1}^m ((\chi^{+(t)})^2 + (\varphi^{+(t)})^2)}}$$

or

$$\mathcal{M}_1 = CC_2\,(M^{-(t)}, \Psi^+) = \frac{M_{IFHSES}(M_{ij}^{-(t)}, \Psi^+)}{\max\left\{\mathcal{E}_{IFHSES}(M_{ij}^{-(t)}).\mathcal{E}_{IFHSES}(\Psi^+)\right\}}$$

$$= \frac{\sum_{i=1}^n \sum_{j=1}^m ((\chi^{-(t)}{}_{ij}\chi^+ + \varphi^{-(t)}{}_{ij}\varphi^+))}{\max\left\{\sum_{i=1}^n \sum_{j=1}^m ((\chi_{ij}{}^{-(t)})^2 + (\varphi_{ij}{}^{-(t)})^2), \sum_{i=1}^n \sum_{j=1}^m ((\chi^{+(t)})^2 + (\varphi^{+(t)})^2)\right\}}.$$

Step 6 Calculation of $\mathbb{CC}$ between $M^{-(t)}$(weight decision matrix)and Ψ^-(-ve ideal substitute) by using either $\mathbb{CC}_1$ or $\mathbb{CC}_2$ as

$$\mathcal{M}_2 = CC_1\ (M^{-(t)}, \Psi^-) = \frac{M_{IFHSES}(M_{ij}^{-(t)}, \Psi^-)}{\sqrt{\mathcal{E}_{IFHSES}(M_{ij}^{-(t)}).\mathcal{E}_{IFHSES}(\Psi^-)}} \tag{12}$$

$$= \frac{\sum_{i=1}^n \sum_{j=1}^m ((\chi_{ij}^{-(t)}\chi^- + \varphi_{ij}^{-(t)}\varphi^-))}{\sqrt{\sum_{i=1}^n \sum_{j=1}^m ((\chi_{ij}{}^{-(t)})^2 + (\varphi_{ij}{}^{-(t)})^2)}\sqrt{\sum_{i=1}^n \sum_{j=1}^m ((\chi^{-(t)})^2 + (\varphi^{-(t)})^2)}}$$

or

$$\mathcal{M}_2 = CC_2\,(M^{-(t)}, \Psi^-)$$

$$= \frac{M_{IFHSES}(M_{ij}^{-(t)}, \Psi^-)}{\max\left\{\mathcal{E}_{IFHSES}(M_{ij}^{-(t)}).\mathcal{E}_{IFHSES}(\Psi^-)\right\}}$$

$$= \frac{\sum_{i=1}^n \sum_{j=1}^m ((\chi^{-(t)}{}_{ij}\chi^- - \varphi^{-(t)}{}_{ij}\varphi^-))}{\max\left\{\sum_{i=1}^n \sum_{j=1}^m ((\chi_{ij}{}^{-(t)})^2 + (\varphi_{ij}{}^{-(t)})^2), \sum_{i=1}^n \sum_{j=1}^m ((\chi^{-(t)})^2 + (\varphi^{-(t)})^2)\right\}}.$$

Step 7 Calculation of closeness coefficient for every substitute $M^{(t)}, (t = 1, 2, \ldots, o)$ such as:

$$\mathcal{K}^{()} = \frac{R\left(M^{-(t)}, \Psi^{-(t)}\right)}{\left\{R\left(M^{-(t)}, \Psi^{+(t)}\right) + R\left(M^{-(t)}, \Psi^{-(t)}\right)\right\}} \tag{13}$$

Here, $R\left(M^{-(t)}, \Psi^{+(t)}\right) = 1 - \mathcal{M}_1^t$ and $R\left(M^{-(t)}, \Psi^{-(t)}\right) = 1 - \mathcal{M}_2^t$.

Step 8 Allot ranking to the substitute $M^{(t)}, (t = 1, 2, \ldots, 0)$ by using the lower to higher values of $\mathcal{K}^{()}$, after this obtain the required one.

Graphical presentation of flowchart for TOPSIS model is shown in Fig. 3.

6.2 Application of TOPSIS model

Suppose a bank needs a manager to fill a vacant post. After the initial assessment, there are still four candidates $\mathfrak{R} = \{T^1, T^2, T^3, \ldots, T^q\}$ that need further evaluation before we can make a decision. These candidates have made it through the initial screening process, and we are now in the process of further evaluating them. The details of each option, including an executive summary as well as an overview of all components that make up this option, are provided below along with a cost estimate for each option. The president has assembled a team of four experts $\mathfrak{W} = \{\Re_1, \Re_2, \Re_3, \Re_4\}$ with weights $\Theta = \{0.2, 0.5, 0.6, 0.7\}^T$ with in the field of his bank to evaluate the remaining alternatives. The criteria for the selection of manager should be based on the following points age ($\Im_1$), education($\Im_2$), relevant job experience ($\Im_3$) as an attributes having sub-attributes $\Im_1 = \{æ_{11} = 30years, æ_{12} = 35years, \Im_2 = \{æ_{21} = 16years, æ_{12} = 18years$ and $\Im_3 = \{æ_{11} = 5years, æ_{12} = 7years\}$. Let $\tilde{\nabla} = \Im_1 \times \Im_2 \times \Im_3$ be the Cartesian product of attributes and $\tilde{\nabla} = \{\kappa_1, \kappa_2, \kappa_3, \kappa_4, \kappa_5, \kappa_6, \kappa_7, \kappa_8\}$ represents the collection of multi sub-attributes having weights Æ$= \{0.25, 0.54, 0.59, 0.34, 0.45, 0.91, 0.41, 0.11\}^T$. Then, the selection of best substitute can be carried out by the opinions given by the experts committee after having a deep analysis can be seen as:

Step-1 Using the opinions of experts, we construct the decision matrices for every substitute in terms of $\mathcal{IFHSEN}$s.

Step-2 Using Eq. 6, we construct the weighted decision matrices (WDM) for every substitute and it is given in Tables 5, 6, 7 and 8.

Step-3 Using equations 9 and 10, we can find the PIS and NIS as follows:

$$\psi^+ = \begin{bmatrix} (.45,.53)(.25,.26)(.65,.74)(.66,.98)(.82,.98)(.50,.97)(.80,.98)(.67,.97) \\ (.68,.99)(.56,.95)(.47,.93)(.68,.97)(.52,.68)(.30,.43)(.20,.35)(.57,.93) \\ (.38,.79)(.26,.35)(.23,.47)(.45,.67)(.16,.38)(.20,.37)(.30,.48)(.27,.57) \\ (.28,.59)(.36,.56)(.12,.26)(.66,.92)(.22,.31)(.20,.77)(.10,.28)(.67,.78) \end{bmatrix}$$

$$\psi^- = \begin{bmatrix} (.23,.99)(.26,.45)(.27,.67)(.26,.78)(.32,.68)(.50,.62)(.40,.58)(.34,.87) \\ (.38,.49)(.46,.75)(.37,.57)(.26,.58)(.32,.35)(.50,.97)(.80,.98)(.67,.97) \\ (.46,.56)(.13,.25)(.25,.36)(.37,.48)(.23,.56)(.10,.27)(.30,.28)(.12,.27) \\ (.27,.39)(.11,.25)(.37,.46)(.42,.67)(.82,.98)(.20,.47)(.22,.36)(.27,.38) \end{bmatrix}$$

Step-4 Calculation of $\mathbb{CC}$ between $M-(t)$ (weight decision matrix)and $+\Psi$ (+ve ideal substitute) by using Eq. 11 as $\Lambda^1 = .7623$, $\Lambda^2 = .7564$, $\Lambda^3 = .7606$, $\Lambda^4 = .7763$.

Step-5 Calculation of $\mathbb{CC}$ between $M-(t)$ (weight decision matrix)and $+\Psi$ (+ve ideal substitute) by using Eq. 12 as $\Upsilon^1 = .7529$, $\Upsilon^2 = .7641$, $\Upsilon^3 = .7512$, $\Upsilon^4 = .7665$.

Step-6 Using Eq. 13, we can find closeness coefficient given by as Œ$^1 = .4462$, Œ$^2 = .4532$, Œ$^3 = .4610$, Œ$^4 = .4695$.

Step-7 Selection of best substitute by taking the maximum closeness coefficient Œ$^4 = .4695$, so best option for substitute is $\Re_4$.

Step-8 By applying the ranking to the substitutes on the basis of closeness coefficient, so we have, $\Re_4 \geq \Re_3 \geq \Re_2 \geq \Re_1$. The obtained ranking between substitutes is shown in Fig. 4.

The final scores of each substitute is collectively represented in Table 9, and its graphical image is shown in Fig. 5.

7 Comparative analysis and discussion

The performance of the $\mathcal{IFHSE}$-structure outperforms all other existing models. This model is best for decision-making problems. The proof of this model can be checked by comparing $\mathcal{IFHSE}$-Set with different structures such as $\mathfrak{F}$s and $\Im$fhss. This proposed model is more useful to others as it contains the multi-argument approximate function, which is highly effective in decision-making problems. The suggested methods work well and can be used with any type of provided information. Here, we present two innovative $\mathcal{IFHSE}$-Set-based algorithms: TOPSIS and $\mathcal{IFHSEWA}$/$\mathcal{IFHSEWG}$. The two approaches are doable and can deliver the greatest outcomes for MADM issues.

The suggested methods are clear and understandable, can increase knowledge, and can be used for a wide range of options and metrics. Both methods are adaptable and simple to modify to fit many circumstances, inputs, and outputs. Because various methods have distinct ranking systems, so they can be cheap based on their considerations, there are slight variations between the rankings of the recommended ways. In fuzzy soft sets, membership gives useful information for a substitute, but it fails to account for the non-membership value of the substitute. For this purpose, intuitionistic fuzzy soft is a handy tool. But these two structures, i.e., fuzzy soft and intuitionistic fuzzy soft, become useless when parameters are in the form of sub-parameters (sub-para.) with disjoint attribute-set values. So for this purpose, intuitionistic fuzzy hypersoft sets are handy tools. For the analysis of multi-decisive opinions (mdo), an intuitionistic fuzzy hypersoft expert set is compared with the previous structure. The Zadeh [18], and Maji et al. [19] techniques allow us to deal with the real information of the alternatives, but they are not capable of handling false objects or many sub-para. The models presented by Alkhazaleh and Salleh [35] are only useful as a parameterization tool and not for truth and falsity grades. They cannot also handle the situation of sub-para. The model of Alkhazaleh and Salleh [36] is also only for truth values and for parameters but fails in the case of sub-parameters and falsity membership. The structure of Maji et al. [34] is not a handy tool for sub-para and mdo and a similar situation is with the model of Broumi et al. [38]. The structure of Yolcu et al. [46] is much better than previously discussed because it has the ability to deal with parameters, sub-parameters, and both values of truth and falsity. But it does not deal with mdo situation. All the above structures have at least one property missing. But these challenges can be simply overcome by our suggested approach, which will also produce more fruitful outcomes for the MADM problem. As a substitute, our proposed method is a sophisticated method that can handle alternatives with data from several sub-parameters. The approach we have developed, on the other hand, addresses the truthiness and falsity of alternatives with several sub-para. As a result, in contrast with previous methodologies, the method we devised is more effective and can offer decision-makers better outcomes through a range of data. A comparison analysis is shown in Table 10. For Yes and No, the following symbols have been used ✔ and ×, respectively.

8 Conclusions

An innovative idea is the intuitionistic fuzzy hypersoft expert set, which is both an extension and generalization of the intuitionistic fuzzy soft set and hypersoft set. In this work, we looked at some fundamental ideas and created several fundamental operations for $\mathcal{IFHSE}$-Set and their attributes. In the research that follows, we suggested the union and intersection operations with their desirable qualities in the $\mathcal{IFHSE}$-Set environment. It also presents the concept of operations that are necessary and possible and have a variety of qualities under the $\mathcal{IFHSE}$-Set. With its decision-making technique, this research also establishes the notions of correlation, $\mathbb{CC}$, weighted correlation, and weighted $\mathbb{CC}$. To verify the accuracy and application of the created decision-making approach, we employed it to resolve decision-making issues. To confirm the accuracy and illustrate the proposed method, a comparison study is also offered. According to the findings, it is finally determined that the proposed approaches demonstrated greater consistency and applicability for decision-makers during the decision-making process. The proposed model has drawbacks in some situations, such as when data or information is in the form of intervals and or when the data is of a periodic nature. Anybody may in the future expand the $\mathcal{IFHSE}$-Set to include interval-valued $\mathcal{IFHSE}$-Set and Pythagorean fuzzy $\mathcal{HSE}$-Set, aggregation operators, and TOPSIS approaches.

Data availability Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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