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ABSTRACT

This study presents a new algorithm for group decision-making solutions using Pythagorean Fuzzy Soft Matrices (PFSMs) and confident weight is given by experts. Pythagorean Fuzzy Set (PFS) is a generalization of the intuitionistic fuzzy set (IFS). Therefore, in real-life problems for uncertainty, the decision-making mechanism in PFSs outcomes better than IFS decision-making. Pythagorean Fuzzy Soft Set (PFSS) is deriving from the combination of PFS and Soft Set. PFSM is also the matrix representation of PFSSs. Based on the cardinalities of the PFSS, experts have been given a new method that assigns confident weight. Confident weight is given according to the experience and knowledge of each expert. For this process, the choice matrix and the combined choice matrix are created first. PFSMs and choice matrices given for each expert are multiplied and the matrices obtained are summed. Pythagorean distance measurements were used to check the accuracy of the results obtained by applying the algorithm. A medical case was studied to see if the proposed method for group decision-making is feasible. In the section of medical case, infectious diseases that were common before COVID-19 were selected. The newly given algorithm was applied to the opinions of physicians about these diseases. According to the Hamming Distance values, the results of three out of four physicians are the same; In the values obtained with Euclidean distance, it was seen that the opinions of all physicians were the same. It has been revealed that the newly proposed algorithm has increased the reliability of the results from the group decision analysis.

1. Introduction

Vagueness has been defined and expressed for centuries only as an element of probability theory. In these periods, vagueness was used synonymously with randomness. By the 1960s, this perspective changed with the development of theories that characterize uncertainty with different dimensions, apart from probability theory. With the newly proposed theories, vagueness has started to be considered as a multidimensional concept and it has been accepted that randomness is only a sub-dimension of the concept of vagueness. Today, it is accepted that the basis of the concept of vagueness is the deficiency and inadequacy of the information level in the system. Many limitations such as technological inadequacies, systems that change and transform depending on time, and limitations in the biological sensory system of humanity cause vague systems to exist in all areas.

The concept of the Fuzzy Set (FS), developed by Zadeh (1965), has been accepted as an effective tool to overcome ambiguity and vagueness and has been successfully applied in many different fields such as economics, engineering, and management. In the past few decades, FS theory has been expanded as different approaches with distinct additions by numerous researchers. Among these, Intuitionistic Fuzzy Set Theory (IFS), which is accepted in the literature and has applications in many fields, was developed by Atanassov (1986). Studies have shown that IFS is more effective than FS theory in overcoming uncertainty. While FS theory is modeled to show only the degree of membership defined in the [0,1], in IFS theory, in addition to the membership degree, the degree of non-membership is also defined. In IFS theory, both membership and non-membership degrees are in the range of [0,1]. When evaluated from this point of view, the sum

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of membership degree and non-membership degree is calculated as 1 in FS theory. However, in the IFS theory, the sum of these two parameters does not have to be 1. Atanassov defined a third parameter called hesitancy degree to complete this sum to 1.

Classical methods cannot be used successfully to solve complex problems in the fields of economics, engineering, and the environment due to various uncertainties specific to these problems. There are three theories for solving these problems: probability theory, fuzzy sets theory, and interval mathematics, which we can consider as mathematical tools for dealing with uncertainties. But all these theories have their own difficulties. Molodtsov (1999) suggested that one reason for these difficulties may be the inadequacy of the theory's parameterization tool. To overcome these difficulties, Molodtsov introduced the concept of soft set (SS) theory as a new mathematical tool for dealing with the difficulties-free uncertainties that plague usual theoretical approaches. Maji et al. (2002) gave the first practical application of SSs in decision-making(DM) problems. Maji et al. (2003) defined and studied several basic notions of the SS theory. Maji et al. also extended crisp soft sets to fuzzy soft sets(FSSs) (Maji et al., 2001a). Some developments in SSs, FSs, and applications in DM can be found in Ali et al. (2009), Feng et al. (2017) and Kirişci (2019, 2020a,b). In 2018 Smarandache (2018) generalized the Soft Set to the HyperSoft Set by transforming the classical uni-argument function F into a multi-argument function.

Yager and Abbasov (2013) and Yager (2014) developed the Pythagorean fuzzy set (PFS) specifically to handle the situations where the IFS method falls short. PFS is an extension of IFS. The PFS extension improves both the flexibility and applicability of IFS. PFS is able to show not only the extent of the agreement between experts but also the fuzziness of that extent. In Peng and Yang (2015), the properties such as boundedness, idempotency, and monotonicity related to the PF aggregation operators are investigated. Further, to solve uncertainty, multiple attribute group DM(MAGDM) problems PF superiority and inferiority ranking method was developed in Peng and Yang (2015). Peng et al. (2015), defined the PFSS and investigated its properties. Guleria and Bajaj (2018) proposed PF soft matrix (PFSM) and its diverse feasible types. Additionally, the PFSMs have been well-considered for recommending a new algorithm for DM by using a choice matrix (CHMX) and weighted CHMX.

Statistical methods and in recent years methods such as artificial networks and machine learning are frequently used in medical diagnosis. The knowledge of the group of physicians can be used to achieve a final result in medical diagnosis. In groups of physicians, the interaction of physicians with each other leads to a final conclusion, and this interaction is a group decision-making (GDM) process. As decision-makers (DMKR), although each of the physicians has different opinions than the others, the physicians reach the final result with a common decision to choose the best alternative from among all the alternatives, and the diagnosis will be much more accurate. Here, physicians examine all alternatives and classify these alternatives from best to worst based on available information. The influential continuum that ensures the most convenient solution to real-life decision-making scripts by considering and combining the expert ideas of more than one person on a problem is called Group Decision Making. The collective view of a group of experts in the decision-making continuum improves the reliability of outcomes. Group decision-making is an operation in which more than one person interacts meanwhile, examines problems, appreciates the likely existing options, is characterized by more than one conflicting criteria, and selects an appropriate option solution to the problem.

Wei et al. (2013) presented power aggregation operators to solve the group DM problem. Chen and Chang (2016) presented geometric aggregation operators by using the transformation technique to solve the GDM problems. Zhang and Xu (2014) suggested the concept of Pythagorean fuzzy numbers (PFNs) and presented a TOPSIS method to solve the GDM problem. In Garg (2020), some new neutral addition and scalar multiplication operational laws are defined by combining the properties of the neutral character of the MS degrees of the set and the sum of probability. Furthermore, in Garg (2020), some new PF weighted, ordered weighted, and hybrid neutral averaging aggregation operators were defined as can neutrally treat the MS and non-membership (NMS) degrees. In Zeng et al. (2019), a MAGDM based on PFS has been studied for business decisions. PF confidence induced ordered weighted averaging and PF confidence induced hybrid weighted averaging operators are given depending on the newly defined operators are their ability to take into account both the evaluation data and their corresponding confidence levels. Khan et al. (2019) proposed a GDM process using the PFSs based on a prioritization relationship on attributes and DMKRs. PF Einstein prioritized weighted average and PF Einstein prioritized weighted geometric operators are defined for the new process. Garg and Arora (2018) have extended the concept of Generalized IFSS to Group-based Generalized IFSS.

A primary assignment of medicine is to diagnose diseases. When more than one physician comes together for medical diagnosis, the work turns into a GDM process. However, it is known that diagnosing diseases is not a simple task. Because no matter how much information they have about symptoms, diagnosis of the disease contains uncertainty. When the diagnosis is made on time and closest to the accuracy, the patient's health result will be positive. Because clinical decision making will be tailored to a correct understanding of the patient's health problem. The accurate diagnosis and the most effective treatment appropriate for this diagnosis will also positively affect public policy. Thus, extravagance in health payments, which has an important place in public finance, will be avoided, allocated resources will be used accurately, and research and development opportunities will increase. Medical decision making can be considered as a continuum, assembling both analytical cognition and intuition. It includes reasoning within complicated causal models of multiple notions, generally characterized by uncertain, imprecise, and/or incomplete info. FS theory and generated theories from this theory such as IFS, PFS, Neutrosophic Space (NS) (Smarandache, 2003) has a number of properties that make it appropriate for solemnizing the uncertain info upon which medical diagnosis and treatment is generally based. In Junhua et al. (2019) a similarity measure was defined for IFSSs, and a method based on similarity was proposed by calculating the weight of experts/physicians for group medical diagnosis. Similarly, numerous methods related to FS, IFS, PFS, NS have been used into the medical applications (Ejegwa, 2018; Ejegwa and Onasanya, 2018; Guleria and Bajaj, 2018; Hashim et al., 2020; Kirisci et al., 2019; Kirişci, 2020a,b, 2021; Pratib et al., 2020; Saeed et al., 2020; Srivasta, 2019; Thao et al., 2019; Yasser et al., 2020).

In this study, group DM, a very important method in assisting medical diagnosis, will be used with PFSSs. For the facility of calculation, PFSMs will be used to help physicians make accurate decisions with symptom values in hospital information system. Thanks to the GDS for PFSSs, it has been known that better results are obtained by obtaining the opinions of more than one expert instead of the opinion of a single expert. In the new aggregation operators, score and accuracy functions, as well as expectation function, are included in the process. Thus, weights are calculated with the Expectation Function. In this article, a new algorithmic approach for the group DM method is proposed. PFSS was used in these algorithms. This algorithm consists of the solving procedure section with two cases and the validation section. Multiple

Table 1
PFSs and IFSs.

IFSs	PFSs
$m+n \leqslant 1$	$m+n \leqslant 1$ or $m+n \geqslant 1$
$0 \leqslant m+n \leqslant 1$	$0 \leqslant m^2+n^2 \leqslant 1$
$p = 1-(m+n)$	$p = \sqrt{1-[m^2+n^2]}$
$m+n+p = 1$	$m^2+n^2+p^2 = 1$

experts' opinions have been shown to have the power to reduce errors in the DM process. The proposed method can solve complicated medical diagnosis problems with the GDM method.

2. Preliminaries

Throughout the paper, the initial universe, parameters sets will denote $\mathcal{U}, \mathcal{P}$, respectively.

2.1. Pythagorean Fuzzy Sets (PFSs)

The theory of IFS was developed by Atanassov (1986) and is a natural generalization of the theory of FS. The significant qualification of IFS is that it appoints to each element MS and NMS degrees ($MS + NMS \leqslant 1$). However, in some DM processes, the sum of the MS and NMS values obtained may be greater than 1. In this case, we can take the sum of the squares of these MS and NMS values obtained, which will be less than or equal to 1. As an original idea, PFSs were created by Yager (2013).

For PFS, $MS + NMS \leqslant 1$ or $MS + NMS \geqslant 1$, $0 \leqslant MS^2 + NMS^2 \leq 1, \pi = \sqrt{1-(MS^2+NMS^2)}$ and $\pi^2 + MS^2 + NMS^2 = 1$.

The function $u_A(x) : \mathcal{U} \to [0,1]$ is called FS on $\mathcal{U}$. The FS can be indicated by $A = \{(a_i, u_A(a_i)) : u_A(a_i) \in [0,1]; \forall a_i \in \mathcal{U}\}$.

Choose the MS function $u_B : \mathcal{U} \to [0,1]$ and the NMS function $v_B : \mathcal{U} \to [0,1]$. Let's assume that the condition $0 \leqslant u_B(a) + v_B(a) \leqslant 1$ for any $a \in \mathcal{U}$ is satisfy. Then, the set $B = \{(a, u_B(a), v_B(a)) : a \in \mathcal{U}\}$ is said to be an IFS B on $\mathcal{U}$. In this case, the condition $\pi_B = 1 - u_B(x) - v_B(x)$ is hold Atanassov, 1986.

Suppose that condition $0 \leqslant [u_C(x)]^2 + [v_C(x)]^2 \leqslant 1$ satisfies for $u_C : \mathcal{U} \to [0,1]$ and $v_C : \mathcal{U} \to [0,1]$. Then, an PFS C in $\mathcal{U}$ is defined by $C = \{(x, u_C(x), v_C(x)) : x \in \mathcal{U}\}$. In this case, the condition $\pi_C = \sqrt{1-[u_C(x)]^2 - [v_C(x)]^2}$ is hold (Yager, 2013, 2014; Yager and Abbasov, 2013). Let C be a PFS over Σ. The set of all PFS on $\mathcal{U}$ will be denoted by $\Omega(\mathcal{U})$. For $D \subseteq \Sigma$, consider a set-valued mapping $F : D \to \omega(\mathcal{U})$, where the power set of $\mathcal{U}$ is showed by $\omega(\mathcal{U})$. Therefore, a pair (F, D) is called a SS on $\mathcal{U}$. For $E \subseteq \Sigma$, choose $F : E \to \omega(\mathcal{U})$, where the set of all PFSs over $\mathcal{U}$ is indicated by $\omega(\mathcal{U})$. Then, a pair (F, E) is called a PFSS on $\omega(\mathcal{U})$ (Peng et al., 2015).

Choose the Pythagorean fuzzy numbers (PFNs) $R = (u_R, v_R)$ (Zhang and Xu, 2014). For three PFNs $\theta = (u, v), \theta_1 = (u_1, v_1)$, $\theta_2 = (u_2, v_2)$, some fundamental operations as follows (Yager, 2013; Yager and Abbasov, 2013): For $\alpha > 0, \bar{\theta} = (v, u), \theta_1 \oplus \theta_2 = (\sqrt{u_1^2 + u_2^2 - u_1^2 u_2^2},\ v_1 v_2), \theta_1 \otimes \theta_2 = (u_1 u_2,\ \sqrt{v_1^2 + v_2 2 - v_1^2 v_2^2}),\ \theta_1 \wedge \theta_2 = (\min\{u_1, u_2\}, \max\{v_1, v_2\}), \theta_1 \vee \theta_2 = (\max\{u_1, u_2\}, \min\{v_1, v_2\}), \alpha.\theta = (\sqrt{1-(1-u^2)^\alpha},\ v^\alpha), \theta^\alpha = (u^\alpha,\ \sqrt{1-(1-v^2)^\alpha})$.

The PFS defined by MS and NMS which satisfies the condition $MS^2 + NMS^2 \leqslant 1$ is initiated by Yager (2013). A new PFN formula consist of the strength of commitment r_p and the direction of commitment d_p was also given by Yager (2013). The new PFN is denoted by $P = (r_p, d_p)$, where $r_p \in [0,1]$. Yager (2013) proposed another PFN formulation, namely $P = (r_p, d_p)$, where r_p defined the strength of commitment and $r_p \in [0,1]$. The stronger commitment is defined by the larger the value of r_p. In this case the lower the uncertainty of the commitment. r_p and d_p are characterized by $u_p = r_p \cos(\theta_p), v_p = r_p \sin(\theta_p)$, i.e. these values are related to MS and NMS grades. Here θ_p is expressed in radian and calculate $\theta_p = (1-d_p)(\pi/2)$. If $P = (r_p, d_p)$ is defined in terms of polar coordinates, it is denoted as (r_p, θ_p). In this case d_p has a formula in the form of $d_p = 1 - (2.\theta_p/\pi)$. It can then be said that PFNs consists of $u_p, v_p, \pi_p, r_p, d_p$ and θ_p parameters. If $u_p, v_p \in [0,1]$ then it is clear that $u_p^2 \leqslant u_p, v_p^2 \leqslant v_p$. IFS, offered by Atanassov (1986) is an extension of FS Theory Zadeh, 1965. IFS is characterized by a MD and a ND and therefore can indicate the fuzzy character of data in more detail comprehensively. The prominent characteristic of IFS is that it assigns to each element an MD and an ND with their sum equal to or less than 1. However, in some practical DM processes, the sum of the MD and the ND to which an alternative satisfying a criterion provided by a DMKR may be bigger than 1, but their square sum is equal to or less than 1. Table 1 explains the difference between PFSs and IFSs. Therefore, Yager (2013) proposed PFS characterized by an MD and an ND, which satisfies the condition that the square sum of its MD and ND is less than or equal to 1. Yager and Abbasov (2013) gave an example to state this situation: a DMKR gives his support for MS of an alternative is $\frac{\sqrt{3}}{2}$ and his against MS is $\frac{1}{2}$. Owing to the sum of two values is bigger than 1, they are not available for IFS, but they are available for PFS since $(\frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 \leqslant 1$. Obviously, PFS is more capable than IFS to model the vagueness in the practical MADM problems. The main difference between PFNs and IFNs is their corresponding constraint conditions, which can be easily shown in Fig. 1. Here, we observe that intuitionistic MS grades are all points under the line $m+n \leqslant 1$ and the Pythagorean MS grades are all points with $m^2 + n^2 \leqslant 1$.

2.2. Distance measurement for PFSs

The Hamming, Euclidean distance measures for PFSs are given in this section. Some properties of distance measures are discussed.

In Ejegwa (2018), the distance between PFSs are investigated as follows:

Let $\mathcal{A}$ and $\mathcal{B}$ be two PFSs on $\mathcal{U}$. Then the normalized Hamming distance and the normalized Euclidean distance between $\mathcal{A}$ and $\mathcal{B}$ are defined as

$$D_H(\mathcal{A}, \mathcal{B}) = \frac{1}{2n} \sum_{i=1}^{n} \{|m_{\mathcal{A}}(x_i) - m_{\mathcal{B}}(x_i)| + |n_{\mathcal{A}}(x_i) - n_{\mathcal{B}}(x_i)| + |p_{\mathcal{A}}(x_i) - p_{\mathcal{B}}(x_i)|\} \quad (1)$$

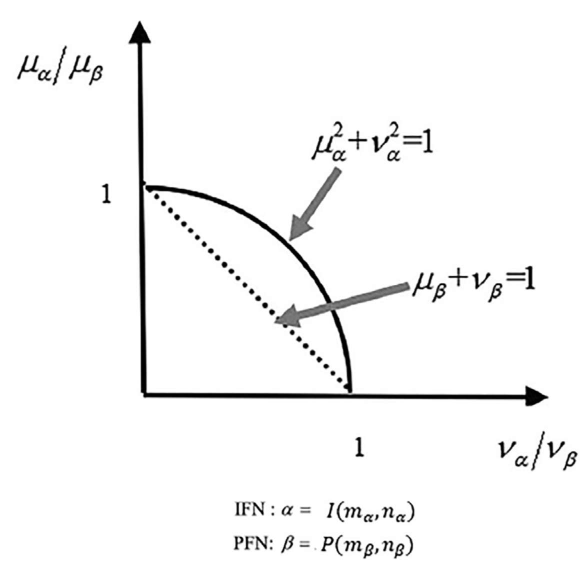

Fig. 1. The PFNs and the IFNs.

$$D_E(\mathcal{A},\mathcal{B}) = \sqrt{\frac{1}{2n}\sum_{i=1}^{n}\left\{(m_{\mathcal{A}}(x_i) - m_{\mathcal{B}}(x_i))^2 + (n_{\mathcal{A}}(x_i) - n_{\mathcal{B}}(x_i))^2 + (p_{\mathcal{A}}(x_i) - p_{\mathcal{B}}(x_i))^2\right\}} \qquad (2)$$

respectively. Taking into account

$$p_{\mathcal{A}}(x_i) = \sqrt{1 - \left[(m_{\mathcal{A}}(x_i))^2 + (n_{\mathcal{A}}(x_i))^2\right]} \quad and \quad p_{\mathcal{B}}(x_i) = \sqrt{1 - \left[(m_{\mathcal{B}}(x_i))^2 + (n_{\mathcal{B}}(x_i))^2\right]}$$

we have

$$\begin{aligned} |p_{\mathcal{A}}(x_i) - p_{\mathcal{B}}(x_i)| &= \left|\sqrt{1 - \left[(m_{\mathcal{A}}(x_i))^2 + (n_{\mathcal{A}}(x_i))^2\right]} - \sqrt{1 - \left[(m_{\mathcal{B}}(x_i))^2 + (n_{\mathcal{B}}(x_i))^2\right]}\right|, \\ \left|(p_{\mathcal{A}}(x_i))^2 - (p_{\mathcal{B}}(x_i))^2\right| &= \left|-\left[(m_{\mathcal{A}}(x_i))^2 + (n_{\mathcal{A}}(x_i))^2\right] + \left[(m_{\mathcal{B}}(x_i))^2 + (n_{\mathcal{B}}(x_i))^2\right]\right| \\ &\leqslant \left|(m_{\mathcal{A}}(x_i))^2 - (m_{\mathcal{B}}(x_i))^2\right| + \left|(n_{\mathcal{A}}(x_i))^2 - (n_{\mathcal{B}}(x_i))^2\right|. \end{aligned}$$

Then

$$|p_{\mathcal{A}}(x_i) - p_{\mathcal{B}}(x_i)| \leqslant \frac{\left|(m_{\mathcal{A}}(x_i))^2 - (m_{\mathcal{B}}(x_i))^2\right| + \left|(n_{\mathcal{A}}(x_i))^2 - (n_{\mathcal{B}}(x_i))^2\right|}{\sqrt{1 - \left[(m_{\mathcal{A}}(x_i))^2 + (n_{\mathcal{A}}(x_i))^2\right]} + \sqrt{1 - \left[(m_{\mathcal{B}}(x_i))^2 + (n_{\mathcal{B}}(x_i))^2\right]}} \qquad (3)$$

It is understood from (3) that the third parameter p cannot be neglected in (1). Let us verify the effect of omitting the third parameter p, from (2), that is

$$\begin{aligned} (p_{\mathcal{A}}(x_i) - p_{\mathcal{B}}(x_i))^2 &= \left(\sqrt{1 - \left[(m_{\mathcal{B}}(x_i))^2 + (n_{\mathcal{B}}(x_i))^2\right]} + \sqrt{1 - \left[(m_{\mathcal{B}}(x_i))^2 + (n_{\mathcal{B}}(x_i))^2\right]}\right)^2 \\ &= 2 - \left[(m_{\mathcal{A}}(x_i))^2 + (n_{\mathcal{A}}(x_i))^2\right] - \left[(m_{\mathcal{B}}(x_i))^2 + (n_{\mathcal{B}}(x_i))^2\right] - \Lambda \end{aligned} \qquad (4)$$

where
$$\Lambda = 2\sqrt{\left(1 - \left[(m_{\mathcal{A}}(x_i))^2 + (n_{\mathcal{A}}(x_i))^2\right]\right)\left(1 - \left[(m_{\mathcal{B}}(x_i))^2 + (n_{\mathcal{B}}(x_i))^2\right]\right)}.$$
Then we say that, taking into account p when calculating the Euclidean distance for PFS does have an influence on the final result.

2.3. Soft Set Theory

A SS was introduced by Molodtsov (1999) as a generic mathematical tool for dealing with uncertain problems which cannot be handled using traditional mathematical tools. The SS is free from such difficulties which can be used for the approximate description of objects without any restriction. As a definite outcome, SS theory has emerged as a convenient and easily applicable tool in practice. In this subsection, basic information about the SS will be given.

Take the power set of $\mathcal{U}$ and $G \subseteq \mathcal{P}$ as $K(\mathcal{U})$. SF_G is called a SS on $\mathcal{U}$, where $SF_G : G \to K(\mathcal{U})$ such that $SF_{G(e)} = \varnothing$ if $e \notin G$ (Molodtsov, 1999).

Choose a set of parameters which are fuzzy words or sentences involving fuzzy words as $\mathcal{P}$. Let $KF(\mathcal{U})$ denotes the set of all FSs of $\mathcal{U}$ and $G \subset \mathcal{P}$. SFS_G is called a fuzzy soft set (FSS) on $\mathcal{U}$, where SFS_G is mapping given by $SFS_G : G \to KF(\mathcal{U})$ such that $SFS_{G(e)} = \varnothing$ if $e \notin G$, where $\varnothing$ is null fuzzy set (Maji et al., 2002).

Let $IK(\mathcal{U})$ denotes the set of all IFSs of $\mathcal{U}$ and $G \subset \mathcal{P}$. $ISFS_G$ is called a intuitionistic fuzzy soft set (IFSS) on $\mathcal{U}$,where $ISFS_G$ is mapping given by $ISFS_G : G \to IK(\mathcal{U})$ such that $ISFS_{G(e)} = \varnothing$ if $e \notin G$, where $\varnothing$ is null IFS (Maji et al., 2001b).

The power set of $\mathcal{U}$ is denoted by $KP(\mathcal{U})$. $\mathcal{A}_{\mathcal{P}}$ is called Pythagorean fuzzy soft set(PFSS) on $\mathcal{U}$, if $C_{\mathcal{U}}$ is mapping given by $C_{\mathcal{U}} : \mathcal{U} \to KP(\mathcal{U})$ (Peng et al., 2015).

2.4. Pythagorean Fuzzy Soft Matrix (PFSM)

In this subsection, the PFSM concept is introduced and the operations of addition and multiplication of PFSMs are given.

If $C_{\mathcal{U}}(\mathcal{P})$ be a PFSS on $\mathcal{U}$, then the subset $\mathcal{U} \times \mathcal{P}$ is uniquely defined by $\Re_A = \{(x,e), e \in A, x \in C_{\mathcal{U}}(e)\}$. The $\Re_A$ can be described by its MS function $m_{\Re_A} : \mathcal{U} \times \mathcal{P} \to [0,1]$ and its NMS function $n_{\Re_A} : \mathcal{U} \times \mathcal{P} \to [0,1]$ Guleria and Bajaj, 2018. So $\Re_A$ is presented as

$$\Re_A = \left\{(m_A(x,e), n_A(x,e)) : \quad 0 \leqslant (m_A)^2 + (n_A)^2 \leqslant 1, \quad (x,e) \in \mathcal{U} \times \mathcal{P}\right\}.$$

Choose $(m_{ij}, n_{ij}) = (m_{\Re_A}(x_i, e_j), m_{\Re_A}(x_i, e_j))$. Then, it can be written the PFSM $P = [p_{ij}]_{m \times n}$ as follows:

$$P = [p_{ij}]_{m\times n} = \begin{bmatrix} (m_{11}, n_{11}) & (m_{12}, n_{12}) & \cdots & (m_{1n}, n_{1n}) \\ (m_{21}, n_{21}) & (m_{22}, n_{22}) & \cdots & (m_{2n}, n_{2n}) \\ \vdots & \vdots & \vdots & \vdots \\ (m_{m1}, n_{m1}) & (m_{m2}, n_{m2}) & \cdots & (m_{mn}, n_{mn}) \end{bmatrix}$$

Some operations of PFSMs can be given as follows: $P^c = [(n_{ij}^P, m_{ij}^P)], \quad (\forall i,j)$. Take two PFSM P and Q such that the number of columns of P is equal to the number of rows of Q. Then,

$$P \times Q = \left[\left\{\max\left(\min_j(m_{ij}^P, m_{jk}^Q)\right), \min\left(\max_j(n_{ij}^P, n_{jk}^Q)\right)\right\}\right]_{m\times p}$$

where $P * Q = [h_{ij}]_{m\times p}$ is called max–min product of PFSM for $P = [a_{ij}]_{mxn}$ and $Q = [b_{jk}]_{nxp}, (\forall i,j,k)$ (Guleria and Bajaj, 2018). For two PFSM P and Q (same order),

$$P + Q = \left[\max(m_{ij}^P, m_{ij}^Q), \min(n_{ij}^P, n_{jk}^Q)\right]_{m\times n}.$$

2.5. Choice matrix

The choice matrices represent the choice parameters of the DMKRs and also help us to solve the PFSS (or PFSM) based decision-making problems with the least computational complexity. In this subsection, the definition of CHMX is given and explained with an example. The CHMX is a square matrix whose rows and columns both indicate parameters which are Pythagorean words or sentences involving Pythagorean words. A CHMX of type $m \times n$ can be defined as follows:

$$CM = \begin{bmatrix} (1,0), & \text{if } i^{th} \text{ and } j^{th} \text{ both parameters are the choice parameters of DMs} \\ (0,1), & \text{if at least one of the } i^{th} \text{ or } j^{th} \text{ parameters be not under choice of the DMs} \end{bmatrix}$$

If there is more than one DMKR, the given CHMX is called the combined CHMX. In combined CHMX, while rows represent choice parameters of the single DM, columns represent the combined choice parameters of the other DMKRs. The choice parameters of the other DMKRs obtain with the intersection of parameter sets.

3. New method

In this section, a new DM method based on PFSS will be defined and the algorithm of this method will be given. This algorithm is built from two cases such as for non-normalized PFSM and for normalized PFSM. Also, there is a validation of results in the algorithm. The flowchart of this algorithm is shown in Fig. 2. This flowchart is provided to easily see the operation of the algorithm. This flowchart, which shows the operation of the stages of the algorithm, explains how Case 1 and Case 2 work, where they merge and what is done in the stages up to the validation section. This flowchart shows how fluent, practical, and easy the working process of the algorithm is.

3.1. An approach for decision-making problem using PFSS

The cardinal set (CS) of PFSS is given as follows:

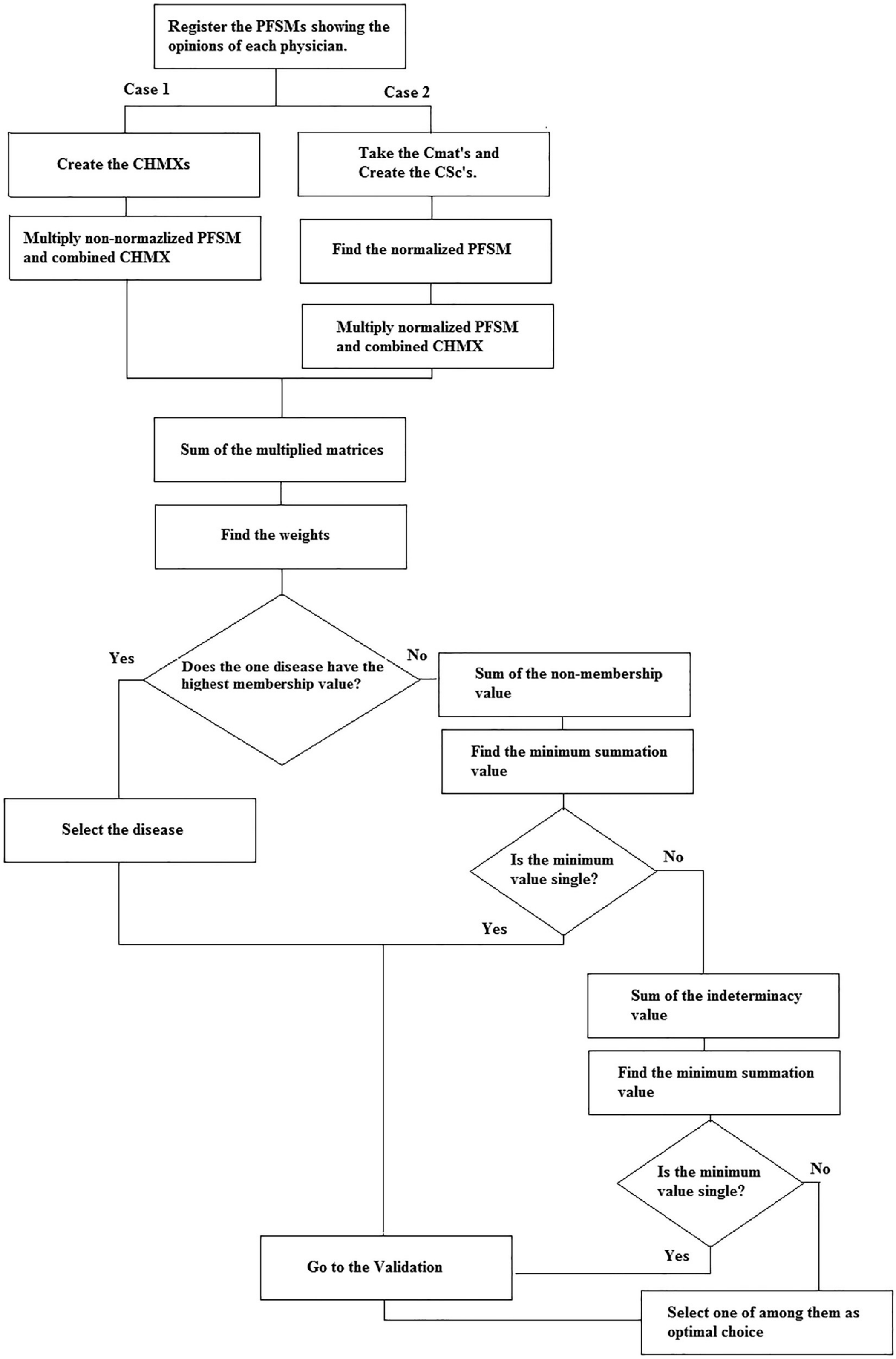

Fig. 2. Flow charts of the algorithm.

$$car(F) = \left\{ \frac{(m_{carF}(x), n_{carF}(x))}{x} : x \in \mathcal{P} \right\} \tag{5}$$

is called the CS of PFSS F, where $car(F)$ is a PFS on $\mathcal{P}$. Let $|\mathcal{U}|$ denote the cardinality of the $\mathcal{U}$. Then, $m_{carF}(x), n_{carF}(x)$ are defined by

$$m_{carF}(x) = \sum_{d \in U} \frac{m_{F(A)}(d)}{|\mathcal{U}|} \tag{6}$$

$$n_{carF}(x) = \sum_{d \in U} \frac{n_{F(A)}(d)}{|\mathcal{U}|}. \tag{7}$$

(6) and (7) are called scalar cardinalities of PFSS F, for all $x \in \mathcal{P}$. Cardinal score(CSc) of a CS $car(F)$ is defined by $SC(carF) = \sum_{x \in \mathcal{P}} [m_{carF}(x)]^2 - \sum_{x \in \mathcal{P}} [n_{carF}(x)]^2$.

3.2. Algorithms

Take for m objects, the set $G = \{d_1, d_2, \cdots, d_m\} \subseteq \mathcal{U}$ and for n parameters, the set $\mathcal{P} = \{e_1, e_2, \cdots, e_n\}$. Each parameter is described by using Pythagorean Fuzzy word and or sentences. Suppose k be the number of DMKRs/ physicians, given by $H = \{p_1, p_2, \cdots, p_k\}$ and each DM selects an object from $\mathcal{U}$ according to their set of choice parameters (CPs) are represented by $K = \{c_1, c_2, \cdots, c_n\}$ where $K \subseteq \mathcal{P}$. Now the problem is to find out the optimal object from $\mathcal{U}$ which satisfies all of these CPs as much as possible.

$H = \{p_1, p_2, \cdots, p_k\}, S = \{s_1, s_2, \cdots, s_n\}, G = \{d_1, d_2, \cdots, d_m\}$ denote the sets of physicians, symptoms and diseases, respectively.

This algorithm is twofold. Firstly, the two cases related to solving procedures will be given. In Case 1 and Case 2, the non-normalized PFSM and normalized PFSM will be used, respectively. Second, the Validation Results section of the algorithm in which the results obtained by the performing the cases are checked will be given.

Case 1: For non-normalized PFSM

Step 1. The opinions of the physicians (H) related to the diverse symptoms (S) and diseases (G) of a patient are registered in PFSMs.
Step 2. The CHMXs are created according to the physician's CPs. The combined CHMXs are obtained for all DMKRs. Step 3. Multiply PFSMs(non-normalized) with combined CHMXs.
Step 4. Sum the matrices obtained from the multiplying of PFSMs and combined CHMXs.
Step 5. The weight of each disease is computed by summing the MS values of the entries of its concerned row.
Step 6. The disease having the highest MS value becomes the optimal choice disease.
Step 7. Go to the Step 8, if more than one disease have the highest MS value.
Step 8. Here one has to consider the sum of the NMS values of those entries which are associated with the equal-weighted disease. The disease with the minimum summation value is the optimal choice disease. Now if the summation values of those diseases are same then Step 9 is followed.
Step 9. When sum of the NMS values of those diseases are same, then the sum of indeterminacy values of those diseases are checked and the disease having minimum indeterminacy margin is selected. If two or more disease have same indeterminacy summation value, then any disease among them is the optimal choice.

Case 2: For normalized PFSM

Step 1. The opinions of the physicians (H) related to the diverse symptoms (S) and diseases (G) of a patient are registered in PFSM.
Step 2. Take the CMat for PFSM and compute their CSc's.
Step 3. Find the normalized PFSMs which defined by $[a_{ij}] = [CSc * (a_{ij})]_{mxn}$
Step 4. Multiply PFSMs(normalized) with combined CHMXs.
Step 5. Sum the matrices obtained in Step 4.
Step 6. Calculate the weights as similar to Step 5 in Case 1.
Step 7. The disease having the highest MV becomes the optimal choice disease.
Step 8. Go to the Step 9, if more than one disease have the highest MV.
Step 9. Here one has to consider the sum of the NVs of those entries which are associated with the equal-weighted disease. The disease with the minimum summation value is the optimal choice disease. Now if the summation values of those diseases are same then Step 10 is followed.
Step 10. When sum of the NVs of those diseases are same, then the sum of indeterminacy values of those diseases are checked and the disease having minimum indeterminacy margin is selected. If two or more disease have same indeterminacy summation value, then any disease among them is the optimal choice.

Validation of results:

Steps A and B are used to validate the results obtained by performing the Case 1 and Case 2.

Step A. Distance measurements are investigated between the individual physician's opinion and the hospital information system where lowest distance points out the proper diagnosis.
Step B. The disease is selected where opinions of a majority of physicians are found to be similar.

4. Medical case study

The five infectious diseases Hepatitis C, Crimean-Congo Hemorrhagic Fever(CCHF), sandfly fever, influenza A(H1N1), norovirus ($D = \{d_1, d_2, d_3, d_4, d_5\}$) were chosen based on the number of people affected by infectious diseases in Turkey in recent years Ergonul, 2016. Symptoms were headache, temperature, nausea, vomiting, anorexia ($S = \{s_1, s_2, s_3, s_4, s_5\}$). Choose a group of four physicians as $P = \{p_1, p_2, p_3, p_4\}$. These physicians are examining the symptoms of the disease in the hospital information system. According to the observed symptoms, the PFSMs $p_{ij}, q_{ij}, r_{ij}, t_{ij}$ are constructed for physicians p_i.

Case 1:

In this case, we can use non-normalized PFSM.

Step 1: We establish the PFSMs as follows:

$$P = [p_{ij}] = \begin{bmatrix} (0.9, 0.2) & (0.7, 0.7) & (0.7, 0.4) & (0.8, 0.2) & (0.4, 0.7) \\ (0.6, 0.5) & (0.9, 0.3) & (0.5, 0.8) & (0.6, 0.7) & (0.5, 0.6) \\ (0.7, 0.6) & (0.8, 0.5) & (0.7, 0.6) & (0.9, 0.4) & (0.7, 0.5) \\ (0.4, 0.6) & (0.4, 0.7) & (0.5, 0.8) & (0.3, 0.8) & (0.7, 0.7) \\ (0.5, 0.6) & (0.8, 0.4) & (0.7, 0.6) & (0.5, 0.8) & (0.5, 0.6) \end{bmatrix},$$

$$Q = [q_{ij}] = \begin{bmatrix} (0.8, 0.3) & (0.7, 0.4) & (0.5, 0.6) & (0.9, 0.1) & (0.8, 0.2) \\ (0.7, 0.5) & (0.4, 0.8) & (0.6, 0.5) & (0.7, 0.4) & (0.6, 0.6) \\ (0.6, 0.4) & (0.6, 0.7) & (0.7, 0.3) & (0.9, 0.3) & (0.8, 0.5) \\ (0.9, 0.2) & (0.8, 0.3) & (0.7, 0.4) & (0.6, 0.5) & (0.5, 0.6) \\ (0.4, 0.6) & (0.6, 0.5) & (0.8, 0.4) & (0.2, 0.9) & (0.5, 0.7) \end{bmatrix}$$

$$R = [r_{ij}] = \begin{bmatrix} (0.8,0.5) & (0.7,0.7) & (0.6,0.5) & (0.7,0.4) & (0.1,0.9) \\ (0.2,0.9) & (0.8,0.2) & (0.5,0.8) & (0.8,0.4) & (0.7,0.4) \\ (0.6,0.6) & (0.7,0.4) & (0.9,0.2) & (0.7,0.6) & (0.9,0.4) \\ (0.9,0.4) & (0.4,0.7) & (0.4,0.8) & (0.8,0.4) & (0.7,0.7) \\ (0.8,0.3) & (0.2,0.9) & (0.9,0.1) & (0.6,0.5) & (0.5,0.6) \end{bmatrix},$$

$$T = [t_{ij}] = \begin{bmatrix} (0.7,0.5) & (0.6,0.5) & (0.6,0.7) & (0.9,0.3) & (0.7,0.5) \\ (0.4,0.7) & (0.5,0.7) & (0.8,0.6) & (0.4,0.7) & (0.8,0.4) \\ (0.7,0.3) & (0.6,0.5) & (0.4,0.7) & (0.7,0.6) & (0.9,0.4) \\ (0.8,0.5) & (0.7,0.4) & (0.6,0.5) & (0.7,0.6) & (0.8,0.5) \\ (0.9,0.3) & (0.7,0.6) & (0.8,0.4) & (0.7,0.4) & (0.7,0.5) \end{bmatrix}$$

Step 2: Now, we will give combined CHMX for physicians $PhysicianA = p_1, PhysicianB = p_2, PhysicianC = p_3, PhysicianD = p_4$ as follows:

In $CM_{p_1}, CM_{p_2}, CM_{p_3}, CM_{p_4}$, rows show $s_{p_1}, s_{p_2}, s_{p_3}, s_{p_4}$ and columns show $s_{p_2\wedge p_3\wedge p_4}$, $s_{p_1\wedge p_3\wedge p_4}$, $s_{p_1\wedge p_2\wedge p_4}$, $s_{p_1\wedge p_2\wedge p_3}$, respectively.

$$CM_{p_1} = \begin{bmatrix} (1,0) & (1,0) & (0,1) & (0,1) & (0,1) \\ (1,0) & (1,0) & (0,1) & (0,1) & (0,1) \\ (1,0) & (1,0) & (0,1) & (0,1) & (0,1) \\ (1,0) & (1,0) & (0,1) & (0,1) & (0,1) \\ (1,0) & (1,0) & (0,1) & (0,1) & (0,1) \end{bmatrix},$$

$$CM_{p_2} = \begin{bmatrix} (1,0) & (1,0) & (0,1) & (0,1) & (1,0) \\ (1,0) & (1,0) & (0,1) & (0,1) & (1,0) \\ (1,0) & (1,0) & (0,1) & (0,1) & (1,0) \\ (1,0) & (1,0) & (0,1) & (0,1) & (1,0) \\ (0,1) & (0,1) & (0,1) & (0,1) & (0,1) \end{bmatrix}$$

$$CM_{p_3} = \begin{bmatrix} (1,0) & (1,0) & (0,1) & (1,0) & (0,1) \\ (1,0) & (1,0) & (0,1) & (1,0) & (0,1) \\ (1,0) & (1,0) & (0,1) & (1,0) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) & (0,1) \\ (1,0) & (1,0) & (0,1) & (1,0) & (0,1) \end{bmatrix},$$

$$CM_{p_4} = \begin{bmatrix} (1,0) & (1,0) & (1,0) & (0,1) & (0,1) \\ (1,0) & (1,0) & (1,0) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) & (0,1) \\ (1,0) & (1,0) & (1,0) & (0,1) & (0,1) \\ (1,0) & (1,0) & (1,0) & (0,1) & (0,1) \end{bmatrix}$$

Step 3: We will multiply PFSMs (non-normalized) with combined CHMXs which obtained from Step 2. The multiplication process here will be done according to the multiplication rule of PFSMs.

$$[p_{ij}] \times CM_{p_1} = \begin{bmatrix} (0.9,0.2) & (0.9,0.2) & (0,1) & (0,1) & (0,1) \\ (0.9,0.3) & (0.9,0.3) & (0,1) & (0,1) & (0,1) \\ (0.9,0.4) & (0.9,0.4) & (0,1) & (0,1) & (0,1) \\ (0.7,0.6) & (0.7,0.6) & (0,1) & (0,1) & (0,1) \\ (0.8,0.4) & (0.8,0.4) & (0,1) & (0,1) & (0,1) \end{bmatrix},$$

$$[q_{ij}] \times CM_{p_2} = \begin{bmatrix} (0.9,0.1) & (0.9,0.1) & (0,1) & (0,1) & (0.9,0.1) \\ (0.7,0.4) & (0.7,0.4) & (0,1) & (0,1) & (0.7,0.4) \\ (0.9,0.2) & (0.9,0.2) & (0,1) & (0,1) & (0.9,0.2) \\ (0.9,0.2) & (0.9,0.2) & (0,1) & (0,1) & (0.9,0.2) \\ (0.8,0.4) & (0.8,0.4) & (0,1) & (0,1) & (0.8,0.4) \end{bmatrix}$$

$$[r_{ij}] \times CM_{p_3} = \begin{bmatrix} (0.8,0.5) & (0.8,0.5) & (0,1) & (0.8,0.5) & (0,1) \\ (0.8,0.2) & (0.8,0.2) & (0,1) & (0.8,0.2) & (0,1) \\ (0.9,0.2) & (0.9,0.2) & (0,1) & (0.9,0.2) & (0,1) \\ (0.9,0.4) & (0.9,0.4) & (0,1) & (0.9,0.4) & (0,1) \\ (0.9,0.1) & (0.9,0.1) & (0,1) & (0.9,0.1) & (0,1) \end{bmatrix},$$

$$[t_{ij}] \times CM_{p_4} = \begin{bmatrix} (0.9,0.3) & (0.9,0.3) & (0.9,0.3) & (0,1) & (0,1) \\ (0.8,0.2) & (0.8,0.2) & (0.8,0.2) & (0,1) & (0,1) \\ (0.9,0.3) & (0.9,0.3) & (0.9,0.3) & (0,1) & (0,1) \\ (0.8,0.4) & (0.8,0.4) & (0.8,0.4) & (0,1) & (0,1) \\ (0.9,0.3) & (0.9,0.3) & (0.9,0.3) & (0,1) & (0,1) \end{bmatrix}$$

Step 4: The sum of the product matrices obtained in step 3.

$$\begin{bmatrix} (0.9,0.2) & (0.9,0.2) & (0,1) & (0,1) & (0,1) \\ (0.9,0.3) & (0.9,0.3) & (0,1) & (0,1) & (0,1) \\ (0.9,0.4) & (0.9,0.4) & (0,1) & (0,1) & (0,1) \\ (0.7,0.6) & (0.7,0.6) & (0,1) & (0,1) & (0,1) \\ (0.8,0.4) & (0.8,0.4) & (0,1) & (0,1) & (0,1) \end{bmatrix}$$
$$+ \begin{bmatrix} (0.9,0.1) & (0.9,0.1) & (0,1) & (0,1) & (0.9,0.1) \\ (0.7,0.4) & (0.7,0.4) & (0,1) & (0,1) & (0.7,0.4) \\ (0.9,0.2) & (0.9,0.2) & (0,1) & (0,1) & (0.9,0.2) \\ (0.9,0.2) & (0.9,0.2) & (0,1) & (0,1) & (0.9,0.2) \\ (0.8,0.4) & (0.8,0.4) & (0,1) & (0,1) & (0.8,0.4) \end{bmatrix}$$
$$+ \begin{bmatrix} (0.8,0.5) & (0.8,0.5) & (0,1) & (0.8,0.5) & (0,1) \\ (0.8,0.2) & (0.8,0.2) & (0,1) & (0.8,0.2) & (0,1) \\ (0.9,0.2) & (0.9,0.2) & (0,1) & (0.9,0.2) & (0,1) \\ (0.9,0.4) & (0.9,0.4) & (0,1) & (0.9,0.4) & (0,1) \\ (0.9,0.1) & (0.9,0.1) & (0,1) & (0.9,0.1) & (0,1) \end{bmatrix}$$
$$+ \begin{bmatrix} (0.9,0.3) & (0.9,0.3) & (0.9,0.3) & (0,1) & (0,1) \\ (0.8,0.2) & (0.8,0.2) & (0.8,0.2) & (0,1) & (0,1) \\ (0.9,0.3) & (0.9,0.3) & (0.9,0.3) & (0,1) & (0,1) \\ (0.8,0.4) & (0.8,0.4) & (0.8,0.4) & (0,1) & (0,1) \\ (0.9,0.3) & (0.9,0.3) & (0.9,0.3) & (0,1) & (0,1) \end{bmatrix}$$
$$= \begin{bmatrix} (0.9,0.1) & (0.9,0.1) & (0.9,0.3) & (0.8,0.5) & (0.9,1) \\ (0.9,0.2) & (0.9,0.2) & (0.8,0.2) & (0.8,0.2) & (0.7,0.4) \\ (0.9,0.2) & (0.9,0.2) & (0.9,0.2) & (0.9,0.2) & (0.9,0.2) \\ (0.9,0.2) & (0.9,0.2) & (0.8,0.4) & (0.9,0.4) & (0.9,0.2) \\ (0.9,0.1) & (0.9,0.1) & (0.9,0.3) & (0.9,0.1) & (0.8,0.4) \end{bmatrix}$$

Step 5: Calculate the weights of these diseases: $w(d_1) = 0.9 + 0.9 + 0.9 + 0.8 + 0.9 = 4.4, w(d_2) = 0.9 + 0.9 + 0.8 + 0.8 + 0.7 = 4.1, w(d_3) = 0.9 + 0.9 + 0.9 + 0.9 + 0.9 = 4.5, w(d_4) = 0.9 + 0.9 + 0.8 + 0.8 + 0.9 = 4.4, w(d_5) = 0.9 + 0.9 + 0.9 + 0.9 + 0.8 = 4.4.$

Step 6: Now as the disease d_3 has the highest MV, therefore it is the optimal observation by all of the physicians. More precisely, it is stated that all the four physicians have reached a common opinion from their individual observations that the patient is suffering from H1N1.

Case 2:

The normalized PFSM s used in this process.

Step 1: Let's use the PSFMs $p_{ij}, q_{ij}, r_{ij}, t_{ij}$ in Case 1.

Step 2: As in Example 1, let's first calculate the cardinals for $[p_{ij}], [q_{ij}], [r_{ij}], [t_{ij}]$. CMat of the the PFSM,

$[q_{ij}]_{1\times5} = [(0.68, 0.40) \quad (0.62, 0.54) \quad (0.66, 0.44) \quad (0.66, 0.44) \quad (0.64, 0.52)]$ *and* $SC(carQ) = 2.1276 - 1.1092 \cong 1$,
$[r_{ij}]_{1\times5} = [(0.66, 0.54) \quad (0.56, 0.58) \quad (0.66, 0.48) \quad (0.72, 0.46) \quad (0.58, 0.60)]$ *and* $SC(carR) = 1.1624 - 0.682 \cong 0.6$,
$[t_{ij}]_{1\times5} = [(0.70, 0.46) \quad (0.62, 0.54) \quad (0.64, 0.58) \quad (0.68, 0.52) \quad (0.78, 0.46)]$ *and* $SC(carT) = 1.328 - 0.912 \cong 1$.

Step 3: Normalized PFSMs are $NP = N_{PFSM}[p_{ij}] = [4.6 * p_{ij}]$, $NQ = N_{PFSM}[q_{ij}] = [1.0 * q_{ij}]$, $NR = N_{PFSM}[r_{ij}] = [0.6 * r_{ij}]$, $NT = N_{PFSM}[t_{ij}] = [1 * t_{ij}]$ are as follows:

$$NP = \begin{bmatrix} (0.414, 0.092) & (0.322, 0.322) & (0.322, 0.184) & (0.368, 0.092) & (0.184, 0.322) \\ (0.276, 0.230) & (0.414, 0.138) & (0.230, 0.368) & (0.276, 0.322) & (0.230, 0.276) \\ (0.322, 0.276) & (0.368, 0.230) & (0.322, 0.276) & (0.414, 0.230) & (0.322, 0.230) \\ (0.184, 0.276) & (0.184, 0.322) & (0.230, 0.368) & (0.138, 0.368) & (0.322, 0.322) \\ (0.230, 0.276) & (0.414, 0.184) & (0.322, 0.276) & (0.230, 0.368) & (0.230, 0.276) \end{bmatrix},$$

$$NQ = \begin{bmatrix} (0.8, 0.3) & (0.7, 0.4) & (0.5, 0.6) & (0.9, 0.1) & (0.8, 0.2) \\ (0.7, 0.5) & (0.4, 0.8) & (0.6, 0.5) & (0.7, 0.4) & (0.6, 0.6) \\ (0.6, 0.4) & (0.6, 0.7) & (0.7, 0.3) & (0.9, 0.3) & (0.8, 0.5) \\ (0.9, 0.2) & (0.8, 0.3) & (0.7, 0.4) & (0.6, 0.5) & (0.5, 0.6) \\ (0.4, 0.6) & (0.6, 0.5) & (0.8, 0.4) & (0.2, 0.9) & (0.5, 0.7) \end{bmatrix}$$

$$NR = \begin{bmatrix} (0.48, 0.30) & (0.42, 0.42) & (0.36, 0.30) & (0.42, 0.24) & (0.06, 0.54) \\ (0.12, 0.54) & (0.48, 0.12) & (0.30, 0.48) & (0.48, 0.24) & (0.42, 0.24) \\ (0.36, 0.36) & (0.42, 0.24) & (0.54, 0.12) & (0.42, 0.36) & (0.54, 0.24) \\ (0.54, 0.24) & (0.24, 0.42) & (0.54, 0.06) & (0.36, 0.30) & (0.30, 0.36) \\ (0.48, 0.18) & (0.12, 0.54) & (0.54, 0.06) & (0.36, 0.30) & (0.30, 0.36) \end{bmatrix},$$

$$NT = \begin{bmatrix} (0.9, 0.3) & (0.9, 0.3) & (0.9, 0.3) & (0, 1) & (0, 1) \\ (0.8, 0.2) & (0.8, 0.2) & (0.8, 0.2) & (0, 1) & (0, 1) \\ (0.9, 0.3) & (0.9, 0.3) & (0.9, 0.3) & (0, 1) & (0, 1) \\ (0.8, 0.4) & (0.8, 0.4) & (0.8, 0.4) & (0, 1) & (0, 1) \\ (0.9, 0.3) & (0.9, 0.3) & (0.9, 0.3) & (0, 1) & (0, 1) \end{bmatrix}$$

Step 4: Take the combined CHMX as in Case 1.

Step 5: We will multiply normalized PFSMs with combined CHMXs $(NCP = N_{PFSM}[p_{ij}] \times CM_{p_1}, NCQ = N_{PFSM}[q_{ij}] \times CM_{p_2}, NCR = N_{PFSM}[r_{ij}] \times CM_{p_3}, NCT = N_{PFSM}[t_{ij}] \times CM_{p_4})$.

$$NCP = \begin{bmatrix} (0.414, 0.092) & (0.414, 0.092) & (0, 1) & (0, 1) & (0, 1) \\ (0.414, 0.138) & (0.414, 0.138) & (0, 1) & (0, 1) & (0, 1) \\ (0.414, 0.230) & (0.414, 0.230) & (0, 1) & (0, 1) & (0, 1) \\ (0.322, 0.276) & (0.322, 0.276) & (0, 1) & (0, 1) & (0, 1) \\ (0.414, 0.184) & (0.414, 0.184) & (0, 1) & (0, 1) & (0, 1) \end{bmatrix},$$

$$NCQ = \begin{bmatrix} (0.9, 0.1) & (0.9, 0.1) & (0, 1) & (0, 1) & (0.9, 0.1) \\ (0.7, 0.4) & (0.7, 0.4) & (0, 1) & (0, 1) & (0.7, 0.4) \\ (0.9, 0.3) & (0.9, 0.3) & (0, 1) & (0, 1) & (0.9, 0.3) \\ (0.9, 0.2) & (0.9, 0.2) & (0, 1) & (0, 1) & (0.9, 0.2) \\ (0.8, 0.4) & (0.8, 0.4) & (0, 1) & (0, 1) & (0.8, 0.4) \end{bmatrix}$$

$$NCR = \begin{bmatrix} (0.48, 0.24) & (0.48, 0.24) & (0, 1) & (0.48, 0.24) & (0, 1) \\ (0.48, 0.12) & (0.48, 0.12) & (0, 1) & (0.48, 0.12) & (0, 1) \\ (0.54, 0.12) & (0.54, 0.12) & (0, 1) & (0.54, 0.12) & (0, 1) \\ (0.54, 0.06) & (0.54, 0.06) & (0, 1) & (0.54, 0.06) & (0, 1) \\ (0.54, 0.06) & (0.54, 0.06) & (0, 1) & (0.54, 0.06) & (0, 1) \end{bmatrix},$$

$$NCT = \begin{bmatrix} (0.9, 0.3) & (0.9, 0.3) & (0.9, 0.3) & (0, 1) & (0, 1) \\ (0.8, 0.4) & (0.8, 0.4) & (0.8, 0.4) & (0, 1) & (0, 1) \\ (0.9, 0.3) & (0.9, 0.3) & (0.9, 0.3) & (0, 1) & (0, 1) \\ (0.8, 0.4) & (0.8, 0.42) & (0.8, 0.4) & (0, 1) & (0, 1) \\ (0.9, 0.3) & (0.92, 0.3) & (0.9, 0.3) & (0, 1) & (0, 1) \end{bmatrix}$$

Step 6: The sum of the product matrices obtained in step 5.

$$\begin{bmatrix} (0.414, 0.092) & (0.414, 0.092) & (0, 1) & (0, 1) & (0, 1) \\ (0.414, 0.138) & (0.414, 0.138) & (0, 1) & (0, 1) & (0, 1) \\ (0.414, 0.230) & (0.414, 0.230) & (0, 1) & (0, 1) & (0, 1) \\ (0.322, 0.276) & (0.322, 0.276) & (0, 1) & (0, 1) & (0, 1) \\ (0.414, 0.184) & (0.414, 0.184) & (0, 1) & (0, 1) & (0, 1) \end{bmatrix} + \begin{bmatrix} (0.9, 0.1) & (0.9, 0.1) & (0, 1) & (0, 1) & (0.9, 0.1) \\ (0.7, 0.4) & (0.7, 0.4) & (0, 1) & (0, 1) & (0.7, 0.4) \\ (0.9, 0.3) & (0.9, 0.3) & (0, 1) & (0, 1) & (0.9, 0.3) \\ (0.9, 0.2) & (0.9, 0.2) & (0, 1) & (0, 1) & (0.9, 0.2) \\ (0.8, 0.4) & (0.8, 0.4) & (0, 1) & (0, 1) & (0.8, 0.4) \end{bmatrix}$$
$$+ \begin{bmatrix} (0.48, 0.24) & (0.48, 0.24) & (0, 1) & (0.48, 0.24) & (0, 1) \\ (0.48, 0.12) & (0.48, 0.12) & (0, 1) & (0.48, 0.12) & (0, 1) \\ (0.54, 0.12) & (0.54, 0.12) & (0, 1) & (0.54, 0.12) & (0, 1) \\ (0.54, 0.06) & (0.54, 0.06) & (0, 1) & (0.54, 0.06) & (0, 1) \\ (0.54, 0.06) & (0.54, 0.06) & (0, 1) & (0.54, 0.06) & (0, 1) \end{bmatrix} + \begin{bmatrix} (0.9, 0.3) & (0.9, 0.3) & (0.9, 0.3) & (0, 1) & (0, 1) \\ (0.8, 0.4) & (0.8, 0.4) & (0.8, 0.4) & (0, 1) & (0, 1) \\ (0.9, 0.3) & (0.9, 0.3) & (0.9, 0.3) & (0, 1) & (0, 1) \\ (0.8, 0.4) & (0.8, 0.42) & (0.8, 0.4) & (0, 1) & (0, 1) \\ (0.9, 0.3) & (0.92, 0.3) & (0.9, 0.3) & (0, 1) & (0, 1) \end{bmatrix}$$
$$= \begin{bmatrix} (0.9, 0.092) & (0.9, 0.092) & (0.9, 0.3) & (0.48, 0.24) & (0.9, 0.1) \\ (0.8, 0.12) & (0.8, 0.12) & (0.8, 0.4) & (0.48, 0.12) & (0.7, 0.4) \\ (0.9, 0.12) & (0.9, 0.12) & (0.9, 0.3) & (0.54, 0.12) & (0.9, 0.3) \\ (0.9, 0.06) & (0.9, 0.06) & (0.8, 0.4) & (0.54, 0.06) & (0.9, 0.2) \\ (0.9, 0.06) & (0.9, 0.06) & (0.9, 0.3) & (0.54, 0.06) & (0.8, 0.4) \end{bmatrix}$$

Step 7: Calculate the weights of these diseases: $w(d_1) = 0.9 + 0.9 + 0.9 + 0.48 + 0.9 = 4.08$, $w(d_2) = 0.8 + 0.8 + 0.8 + 0.48 + 0.7 = 3.58$, $w(d_3) = 0.9 + 0.9 + 0.9 + 0.54 + 0.9 = 4.14$, $w(d_4) = 0.9 + 0.9 + 0.8 + 0.54 + 0.9 = 4.04$, $w(d_5) = 0.9 + 0.9 + 0.9 + 0.54 + 0.8 = 4.04$.

Step 8: In this also case, the result was the same. That is, the disease d_3 has the highest MV and it is the optimal observation by all of the physicians.

The same result has been achieved in both cases of the algorithm.

5. Validation section

We will measure and compare the results obtained with both cases of the algorithm in subsection 3.2 with the distance measurement tools of PFSS.

A PFSS table was given with the opinions of physicians on symptoms and diseases(Table 2). The values given in this table according to PFSS are MVs and NVs. The degree of indeterminacy is important in distance measurements and will be used in all calculations. In this section, Hamming and Euclidean distances are given for PFSS will be used. The values from these distances will be compared with the results obtained in Case 1 and Case 2 of the algorithm. The lowest of the results obtained from the distances will represent an appropriate diagnosis. We will now give an example of how values in Hamming distance and Euclidean distance are calculated. We will use $p_{\mathcal{C}}$, Eqs. (1), (2) for the calculation. Sample calculation for Physician A is as follows:

Set $\mathcal{A} = \{(0.7, 0.4), (0.6, 0.5), (0.4, 0.8), (0.3, 0.9), (0.9, 0.3)\}$ (from Table 2) indicates the values of symptoms of Hepatitis C disease.

Set $\mathcal{B} = \{(0.9, 0.2), (0.7, 0.7), (0.7, 0.4), (0.8, 0.2), (0.4, 0.7)\}$ (from PFSM P) gives the values of opinions of Physician A about symptoms. Then,

Table 2
Hospital information system.

	Hepatitis C	CCHF	H1N1	sandfly fever	norovirus
headache	(0.7, 0.4)	(0.8, 0.3)	(0.9, 0.2)	(0.8, 0.4)	(0.4, 0.7)
temperature	(0.6, 0.5)	(0.6, 0.5)	(0.8, 0.4)	(0.7, 0.6)	(0.5, 0.6)
nausea	(0.4, 0.8)	(0.3, 0.9)	(0.8, 0.3)	(0.3, 0.8)	(0.9, 0.4)
vomiting	(0.3, 0.9)	(0.5, 0.8)	(0.7, 0.6)	(0.3, 0.7)	(0.8, 0.3)
anorexia	(0.9, 0.3)	(0.8, 0.4)	(0.7, 0.4)	(0.1, 0.9)	(0.1, 0.9)

$$D_H(\mathcal{A},\mathcal{B}) = \frac{1}{2\times 5}[|0.7-0.9| + |0.6-0.7| + |0.4-0.7| + |0.3-0.8| + |0.9-0.4| + |0.4-0.2| + |0.5-0.7| + |0.8-0.4| + |0.9-0.2| + |0.3-0.7| + |0.6-0.38| + |0.62-1| + |0.45-0.6| + |0.32-0.56| + |0.32-0.6|] = 0.49$$

$$D_E(\mathcal{A},\mathcal{B}) = \left[\frac{1}{2\times 5}\left\{(0.7-0.9)^2+(0.6-0.7)^2+(0.4-0.7)^2 + (0.3-0.8)^2+(0.9-0.4)^2+(0.4-0.2)^2+(0.5-0.7)^2+(0.8-0.4)^2+(0.9-0.2)^2 + (0.3-0.7)^2+(0.6-0.38)^2+(0.62-1)^2+(0.45-0.6)^2+(0.32-0.56)^2+(0.32-0.6)^2\right\}\right]^{1/2} = 0.44.$$

The measurements of Hamming and Euclidean distances are given in Tables 3 and 4, respectively.When Tables 4 and 5 are examined, it is seen that the opinion of Physician A is H1N1. Looking at the tables for all physicians, it is seen that the Hamming distance of Physician D is Hepatitis C. In the tables, the common opinion of all other physicians appeared as H1N1. This result coincides with the results of case 1 and case 2 in the algorithm.

6. Discussion

The physician's opinion plays a vital role in group DM. Each physician should have a weight in group DM, since each physician's knowledge and experience will be different. How to determine these weights is a problem. In this study, each physician was given a weight according to their opinions. The weight assigned to the physicians was calculated by CSc. CSc is more when physicians notification more views related to disease symptoms, and less when they notification fewer views. This situation is suitable in real life. This situation also helps to eliminate the problem of one or more physicians' opinions being dominant in data entry. That is, when CSc is more, opinion becomes so important. Thus, the reliability of the final decision is increased.

Fig. 2 is the flowchart of the proposed algorithm. In this flowchart, the non-normalized PFSM (in Case 1) and the normalized PFSM (in Case 2) were used. Weights were calculated after these PFSMs. If there is the highest membership value according to these weights, the disease is selected and the validation section process starts. After calculating the weights, if the highest membership value is not found, the minimum of the sum of the non-membership values is checked. If the minimum value is found, the validation section is operated. If the minimum value cannot be found, one of them is selected as the most appropriate choice and the validation section is passed. It can be easily seen from this flowchart that the algorithm is an easy and practical working principle.

In this study, after the opinions of the physicians about the symptoms were collected, the nearness between the hospital information system and these opinions was examined. The hospital information system is used in the validation results section, not in the solving procedure section of our proposed DM method.

Table 3
Hamming distance measurements.

	Hepatitis C	CCHF	H1N1	sandfly fever	norovirus
Physician A	0.49	0.26	**0.22**	0.32	0.35
Physician B	0.29	0.35	**0.26**	0.32	0.29
Physician C	0.46	0.36	**0.20**	0.33	0.36
Physician D	**0.23**	0.32	0.25	0.33	0.33

Table 4
Euclidean distance measurements.

	Hepatitis C	CCHF	H1N1	sandfly fever	norovirus
Physician A	0.44	0.23	**0.22**	0.32	0.43
Physician B	0.35	0.32	**0.25**	0.33	0.38
Physician C	0.45	0.40	**0.26**	0.33	0.33
Physician D	0.30	0.32	**0.27**	0.33	0.35

Table 5
Tabular representation of $\mathcal{U}(\mathcal{P})$.

$\mathcal{U}/\mathcal{P}$	s_1	s_2	s_3	s_4	s_4	s_5
d_1	(0.46,0.71)	(0.69,0.34)	(0.76,0.28)	(0.88,0.24)	(0.61,0.54)	
d_2	(0.89,0.44)	(0.57,0.46)	(0.37,0.68)	(0.62,0.65)	(0.63,0.39)	
d_3	(0.84,0.49)	(0.72,0.39)	(0.43,0.57)	(0.55,0.44)	(0.70,0.52)	
d_4	(0.63,0.56)	(0.68,0.43)	(0.56,0.52)	(0.67,0.58)	(0.84,0.40)	
d_5	(0.77,0.57)	(0.80,0.51)	(0.69,0.37)	(0.58,0.49)	(0.76,0.47)	

The CHMX and combined CHMX are used for the physicians' parameter selection. Since a common idea is drawn from the opinions of all doctors, personalization of the results obtained is prevented. This paper includes a new algorithm to solve the group DM problems for disease diagnosis using PFSS. The first part of the algorithm is a solving procedure, which carries out the DM process. The second part is validation for the results obtained. A set of diseases with a set of common symptoms was used in this method. In this method, the opinions of physicians for a patient were examined using PFSM. A PFSM was created for each physician, consisting of diseases and symptoms. The way each physician thinks of symptoms may be different. The physician may also have ignored one or more symptoms in a patient. Confident weight is obtained in this system, which gives more weight to the physician who uses the most symptoms. We used a hospital information system in the DM process. In this way, the accuracy of the final decision on diseases will be more strong.

7. Conclusion

In medical and other applications, to solve real-world problems, easy-to-use and high-accuracy decision-making, forecasting and data analysis approaches can be investigated using different fuzzy environments. It is more effective to obtain a common result by taking the opinions of more than one physician about the diagnosis, rather than being satisfied with the opinion of a physician on an important issue such as the diagnosis of disease. Therefore, the development of methods for solving group decision-making problems will continue. Inspired by this study, interval-valued Pythagorean Fuzzy Soft Set will be defined in the future, and group decision-making methods related to medical diagnosis will be obtained according to this new set. The samples and symptoms of the most important epidemic disease of the last period, COVID-19, and the data of service and intensive care patients will be examined with these new methods.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A

Examples of subsections 2.3, 2.5, 3.1 are as follows, respectively:

Example 7.1. The five infectious diseases Hepatitis C, Crimean-Congo Hemorrhagic Fever(CCHF), sandfly fever, influenza A(H1N1), norovirus ($D = \{d_1, d_2, d_3, d_4, d_5\}$) were chosen based on the number of people affected by infectious diseases in Turkey in recent years Ergonul, 2016. Symptoms were headache, temperature, nausea, vomiting, anorexia ($S = \{s_1, s_2, s_3, s_4, s_5\}$). s_i's are common symptoms of diseases given in set $\mathcal{U}$. Then,

$$\begin{aligned}
\mathcal{U}(s_1) &= \{d_1/(0.46, 0.71), d_2/(0.69, 0.34), d_3/(0.76, 0.28), d_4/(0.88, 0.24), d_5/(0.61, 0.54)\} \\
\mathcal{U}(s_2) &= \{d_1/(0.89, 0.44), d_2/(0.57, 0.46), d_3/(0.37, 0.68), d_4/(0.62, 0.65), d_5/(0.63, 0.39)\} \\
\mathcal{U}(s_3) &= \{d_1/(0.84, 0.49), d_2/(0.72, 0.39), d_3/(0.43, 0.57), d_4/(0.55, 0.44), d_5/(0.70, 0.52)\} \\
\mathcal{U}(s_4) &= \{d_1/(0.63, 0.56), d_2/(0.68, 0.43), d_3/(0.56, 0.52), d_4/(0.67, 0.58), d_5/(0.84, 0.40)\} \\
\mathcal{U}(s_5) &= \{d_1/(0.77, 0.57), d_2/(0.80, 0.51), d_3/(0.69, 0.37), d_4/(0.58, 0.49), d_5/(0.76, 0.47)\}
\end{aligned}$$

Then, PFSS is given as follows:

$$\begin{aligned}
\mathcal{U}(\mathcal{P}) = \ & [(s_1, \{d_1/(0.46, 0.71), d_2/(0.69, 0.34), d_3/(0.76, 0.28), d_4/(0.88, 0.24), d_5/(0.61, 0.54)\}), \\
& (s_2, \{d_1/(0.89, 0.44), d_2/(0.57, 0.46), d_3/(0.37, 0.68), d_4/(0.62, 0.65), d_5/(0.63, 0.39)\}), \\
& (s_3, \{d_1/(0.84, 0.49), d_2/(0.72, 0.39), d_3/(0.43, 0.57), d_4/(0.55, 0.44), d_5/(0.70, 0.52)\}), \\
& (s_4, \{d_1/(0.63, 0.56), d_2/(0.68, 0.43), d_3/(0.56, 0.52), d_4/(0.67, 0.58), d_5/(0.84, 0.40)\}), \\
& (s_5, \{d_1/(0.77, 0.57), d_2/(0.80, 0.51), d_3/(0.69, 0.37), d_4/(0.58, 0.49), d_5/(0.76, 0.47)\})]
\end{aligned}$$

Tabular representation of the PFSS $\mathcal{U}(\mathcal{P})$ is shown in Table 5. In addition, distance measures are calculated as follows:

$$\begin{aligned}
D_H(\mathcal{A}, \mathcal{B}) = \frac{1}{2 \times 5} & [|0.7 - 0.9| + |0.6 - 0.7| + |0.4 - 0.7| + |0.3 - 0.8| + |0.9 - 0.4| \\
& + |0.4 - 0.2| + |0.5 - 0.7| + |0.8 - 0.4| + |0.9 - 0.2| + |0.3 - 0.7| \\
& + |0.6 - 0.38| + |0.62 - 1| + |0.45 - 0.6| + |0.32 - 0.56| + |0.32 - 0.6|] \\
= 0.49 &
\end{aligned}$$

$$\begin{aligned}
D_E(\mathcal{A}, \mathcal{B}) = \Big[\frac{1}{2 \times 5} & \{(0.7 - 0.9)^2 + (0.6 - 0.7)^2 + (0.4 - 0.7)^2 \\
& + (0.3 - 0.8)^2 + (0.9 - 0.4)^2 + (0.4 - 0.2)^2 + (0.5 - 0.7)^2 \\
& + (0.8 - 0.4)^2 + (0.9 - 0.2)^2 + (0.3 - 0.7)^2 + (0.6 - 0.38)^2 \\
& + (0.62 - 1)^2 + (0.45 - 0.6)^2 + (0.32 - 0.56)^2 + (0.32 - 0.6)^2\}\Big]^{1/2} \\
= 0.44. &
\end{aligned}$$

For u_i, other distance measures can be calculated similarly.

Example 7.2. Choose $\mathcal{U}$ and $\mathcal{P}$ as in Example 7.1. Assume that Physician A chooses the symptoms in the set $G = \{temprature, nausea, vomiting\} = \{s_2, s_3, s_4\} \subset \mathcal{P}$. In this case, the CHMX of the Physician A will be as follows:

$$CM_A = \begin{array}{c c} & e_H \\ e_G & \begin{bmatrix} (0,1) & (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (1,0) & (1,0) & (1,0) & (0,1) \\ (0,1) & (1,0) & (1,0) & (1,0) & (0,1) \\ (0,1) & (1,0) & (1,0) & (1,0) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) & (0,1) \end{bmatrix} \end{array}$$

If there are two physicians(Physician A, Physician B) who want to predict the disease by using of these symptoms, by taking the set G for Physician A and the set $H = \{s_1, s_2, s_3, s_4\} \subset \mathcal{P}$ for Physician B, we can obtain the CHMX as follows:

$$CM_{(A,B)} = \begin{array}{c c} & e_H \\ e_G & \begin{bmatrix} (0,1) & (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (1,0) & (1,0) & (1,0) & (0,1) \\ (0,1) & (1,0) & (1,0) & (1,0) & (0,1) \\ (0,1) & (1,0) & (1,0) & (1,0) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) & (0,1) \end{bmatrix} \end{array}$$

Example 7.3. Take the PFSM $P = [p_{ij}]_{5\times5}$ as follows:

$$P = [p_{ij}]_{m\times n} = \begin{bmatrix} (0.9, 0.2) & (0.7, 0.7) & (0.7, 0.4) & (0.8, 0.2) & (0.4, 0.7) \\ (0.6, 0.5) & (0.9, 0.3) & (0.5, 0.8) & (0.6, 0.7) & (0.5, 0.6) \\ (0.7, 0.6) & (0.8, 0.5) & (0.7, 0.6) & (0.9, 0.4) & (0.7, 0.5) \\ (0.4, 0.6) & (0.4, 0.7) & (0.5, 0.8) & (0.3, 0.8) & (0.7, 0.7) \\ (0.5, 0.6) & (0.8, 0.4) & (0.7, 0.6) & (0.5, 0.8) & (0.5, 0.6) \end{bmatrix}$$

Now, we compute cardinals:

$$\begin{aligned}
m_{carP}(x_1) &= \tfrac{0.9+0.6+0.7+0.4+0.5}{5} = 1.55, \quad & n_{carP}(x_1) &= \tfrac{0.2+0.5+0.6+0.6+0.6}{5} = 0.5, \\
m_{carP}(x_2) &= \tfrac{0.7+0.9+0.8+0.4+0.8}{5} = 0.72, \quad & n_{carP}(x_2) &= \tfrac{0.7+0.3+0.5+0.7+0.4}{5} = 0.52, \\
m_{carP}(x_3) &= \tfrac{0.7+0.5+0.7+0.5+0.7}{5} = 0.62, \quad & n_{carP}(x_3) &= \tfrac{0.4+0.8+0.6+0.8+0.6}{5} = 0.64, \\
m_{carP}(x_4) &= \tfrac{0.8+0.6+0.9+0.3+0.5}{5} = 0.62, \quad & n_{carP}(x_4) &= \tfrac{0.2+0.7+0.4+0.8+0.8}{5} = 0.48, \\
m_{carP}(x_5) &= \tfrac{0.4+0.5+0.7+0.7+0.5}{5} = 1.6, \quad & n_{carP}(x_5) &= \tfrac{0.7+0.6+0.5+0.7+0.6}{5} = 0.62.
\end{aligned}$$

Cardinal matrix (CMat) of the PFSM $P = [p_{ij}]_{1\times5}$ is $[p_{ij}]_{1\times5} = [a_{11} \quad a_{12} \quad a_{13} \quad a_{14} \quad a_{15}]$. Then,

$$[p_{ij}]_{1\times5} = [(1.55, 0.5) \quad (0.72, 0.52) \quad (0.62, 0.64) \quad (0.62, 0.48) \quad (1.6, 0.62)].$$

From here, we can calculate the CSc as follows:

$$\begin{aligned} SC(carP) &= \sum_{x\in G}[m_{carP}(x)]^2 - \sum_{x\in G}[n_{carP}(x)]^2 \\ &= \left[(1.55)^2 + (0.72)^2 + (0.62)^2 + (0.62)^2 + (1.6)^2\right] \\ &- \left[(0.5)^2 + (0.52)^2 + (0.64)^2 + (0.48)^2 + (0.62)^2\right] \\ &= 6.2497 - 1.5448 \cong 4.6. \end{aligned}$$

References

Ali, M.A., Feng, F., Liu, X., Min, W.K., Shabir, M., 2009. On some new operations in soft set theory. Comput. Math. Appl. 57, 1547–1553.

Atanassov, K., 1986. Intuitionistic fuzzy sets. Fuzzy Sets Syst. 20, 87–96.

Chen, S.M., Chang, C.H., 2016. Fuzzy multi-attribute decision making based on transformation techniques of intuitionistic fuzzy values and intuitionistic fuzzy geometric averaging operators. Inf. Sci. 352–353, 133–149. https://doi.org/10.1016/j.ins.2016.02.049.

Ejegwa, P.A., 2018. Distance and similarity measures for Pythagorean fuzzy sets. Granul. Comput. 5, 225–238. https://doi.org/10.1007/s41066-018-00149-z.

Ejegwa, P.A., Onasanya, B.O., 2018. Improved intuitionistic fuzzy composite relation and its application to medical diagnostic process. Notes Intuition. Fuzzy Sets 25, 43–58. https://doi.org/10.7546/nifs.2019.25.1.43-58.

Ergonul, O., 2016. Epidemiology is a field that searchs diseases and their indicators. Okmeydaní Med. J. 32, 1–7. https://doi.org/10.5222/otd.2016.001 (Turkish).

Feng, F., Fujita, H., Ali, M.A., Yager, R.R., Liu, X., 2017. Another view on generalized intuitionistic fuzzy soft sets and related multi- attribute decision making methods. IEEE Trans. Fuzzy Syst. 27 (3), 474–488. https://doi.org/10.1109/TFUZZ.2018.2860967.

Garg, H., 2020. Neutrality operations-based Pythagorean fuzzy aggregation operators and its applications to multiple attribute group decision-making process. J. Amb. Intell. Human. Comput. 11, 3021–3041. https://doi.org/10.1007/s12652-019-01448-2.

Garg, H., Arora, R., 2018. Generalized and group-based generalized intuitionistic fuzzy soft sets with applications in decision-making. Appl. Intell. 48, 343–356. https://doi.org/10.1007/s10489-017-0981-5.

Guleria, A., Bajaj, R.K., 2018. On Pythagorean fuzzy matrices, operations and their applications in decision making and medical diagnosis. Soft Comput. 23, 7889. https://doi.org/10.1007/s00500-018-3419-z.

Hashim, R.M., Gulistan, M., Rehman, I., Hassan, N., Nasruddin, A.M., 2020. Neutrosophic bipolar fuzzy set and its application in medicines preparations. Neutrosophic Sets Syst. 31, 86–100. https://doi.org/10.5281/zenodo.3639217.

Junhua, H., Pan, L., Yeng, Y., Chen, H., 2019. A group medical diagnosis model based on intuitionistic fuzzy soft sets. Appl. Soft Comput. 77, 453–466. https://doi.org/10.1016/j.asoc.2019.01.041.

Khan, M.S.A., Abdullah, S., Ali, A., 2019. Multiattribute group decision-making based on Pythagorean fuzzy Einstein prioritized aggregation operators. Int. J. Intell. Syst. 34, 1001–1033. https://doi.org/10.1002/int.22084.

Kirişci, M., 2019. Comparison the medical decision-making with Intuitionistic fuzzy parameterized fuzzy soft set and Riesz Summability. New Math. Nat. Comput. 15, 351–359. https://doi.org/10.1142/S1793005719500194.

Kirişci, M., 2020a. Medical decision making with respect to the fuzzy soft sets. Journal of interdisciplinary mathematics 23 (4), 767–776. https://doi.org/10.1080/09720502.2020.1715577.

Kirişci, M., 2020b. A case study for medical decision making with the fuzzy soft sets. Afrika Matematika 31, 557–564. https://doi.org/10.1007/s13370-019-00741-9.

Kirişci, M., 2021. Ω-soft sets and decision making application. Int. J. Comput. Math. 98, 690–704. https://doi.org/10.1080/00207160.2020.1777404.

Kirisci, M., Yilmaz, H., Saka, M.U., 2019. An ANFIS perspective for the diagnosis of type II diabetes. Ann. Fuzzy Math. Inf. 17, 101–113.

Maji, P.K., Biswas, R., Roy, A.R., 2001a. Fuzzy soft sets. J. Fuzzy Math. 9, 589–602.

Maji, P.K., Biswas, R., Roy, A.R., 2001b. Intuitionistic fuzzy soft sets. Comput. Math. Appl. 9, 677–692.

Maji, P.K., Biswas, R., Roy, A.R., 2002. An application of soft sets in a decision making problem. Comput. Math. Appl. 44, 1077–1083.

Maji, P.K., Biswas, R., Roy, A.R., 2003. Soft set theory. Comput. Math. Appl. 45, 555–562.

Molodtsov, D., 1999. Soft set theory-first results. Comput. Math. Appl. 20, 239–244.

Peng, X., Yang, Y., 2015. Some results for Pythagorean fuzzy sets. Int. J. Intell. Syst. 30, 1133–1160.

Peng, X., Yang, Y., Song, J., Jiang, Y., 2015. Pythagorean fuzzy soft set and its application. Comput. Eng. 41, 224–229.

Pratib, R., Mishra, A.R., Mardani, A., 2020. An extended Pythagorean fuzzy complex proportional assessment approach with new entropy and score function: Application in pharmacological therapy selection for type 2 diabetes. Appl. Soft Comput. 94. https://doi.org/10.1016/j.asoc.2020.106441.

Saeed, M., Saqlain, M., Mehmood, A., Naseer, K., Yaqoob, S., 2020. Multi-polar neutrosophic soft sets with application in medical diagnosis and decision-making. Neutrosophic Sets Syst. 33, 183–207. https://doi.org/10.5281/zenodo.3782895.

Smarandache, F., 2003. A unifying field in logics: Neutrosophic logic. Neutrosophy, neutrosophic set, neutrosophic probability and statistics. Phoenix, Xiquan.

Smarandache, F., 2018. Extension of Soft Set to Hypersoft Set, and then to Plithogenic Hypersoft Set. Neutrosophic Sets Syst. 22, 168–170. https://doi.org/10.5281/zenodo.2159754.

Srivasta, A., 2019. Medical diagnosis using intuitionistic fuzzy sets. AIP Conf. Proc. 2061,. https://doi.org/10.1063/1.5086651 020029.

Thao, N.X., Ali, M., Smarandache, F., 2019. An intuitionistic fuzzy clustering algorithm based on a new correlation coefficient with application in medical diagnosis. J. Intell. Fuzzy Syst. 36, 189–198. https://doi.org/10.3233/JIFS-181084.

Wei, G., Zhao, X., Wang, H., Lin, R., 2013. Fuzzy power aggregation operators and their application to multiple attribute group decision making. Technol. Econ. Dev. Econ. 19 (3), 377–396. https://doi.org/10.3846/20294913.2013.821684.

Yager, R.R., 2013. Pythagorean fuzzy subsets. Proc Joint IFSA World Congress and NAFIPS Annual Meeting, Edmonton, Canada.

Yager, R.R., 2014. Pythagorean membership grade in multicriteria decision making. IEEE Fuzzy Syst. 22, 958–965.

Yager, R.R., Abbasov, A.M., 2013. Pythagorean membership grades, complex numbers and decision making. Int. J. Intell. Syst. 28, 436–452.

Yasser, I., Twakol, A., Abd el-Khalek, A.A., Samrah, A., Salama, A.A., 2020. COVID-X: Novel Health-Fog Framework Based on Neutrosophic Classifier for Confrontation COVID-19. Neutrosophic Sets and Systems 35, 1–21, doi: 10.5281/zenodo.3951625.

Zadeh, L.A., 1965. Fuzzy sets. Inf. Comput. 8, 338–353.

Zeng, S., Peng, X., Balezentis, T., Streimikiene, D., 2019. Prioritization of low-carbon suppliers based on Pythagorean fuzzy group decision making with self-confidence level. Economic Research-Ekonomíka Istrazivanja 32.

Zhang, X.L., Xu, Z.S., 2014. Extension of TOPSIS to multi-criteria decision making with Pythagorean fuzzy sets. Int. J. Intell. Syst. 29, 1061–1078.