A Novel TOPSIS Method under Fermatean Fuzzy Hypersoft Set Based on Correlation Coefficients for Selection of Hip Prothesis Materials

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Abstract: A statistical technique called correlation analysis is used to determine the link between two variables and gauge how strongly two variables are linearly related. The degree of change in one variable as a result of the other's change is determined via correlation analysis. The correlation coefficient is one of the statistical ideas that are most relevant to this kind of investigation. The correlation coefficient, which is easily recognized by its symbol r and typical value without units falling between 1 and -1, is the unit of measurement used to determine the intensity of the linear relationship between the variables included in correlation analysis. One of the best at representing completely ambiguous and uncertain information is the Fermatean fuzzy set. In this research, we present correlation coefficients and weighted correlation coefficient formulations to evaluate the relationship between two Fermatean fuzzy hypersoft sets, taking into account that the correlation coefficient plays a significant role in statistics and engineering disciplines. The Fermatean fuzzy hypersoft set is a parameterized family that deals with the subattributes of the parameters and is an appropriate extension of the Fermatean fuzzy soft set. It is also the generalization of the intuitionistic fuzzy hypersoft set and the Pythagorean fuzzy hypersoft set, which are used to accurately assess insufficiency, anxiety, and uncertainties in decision-making. The Fermatean fuzzy hypersoft set can accommodate more uncertainties compared to the intuitionistic fuzzy hypersoft set and Pythagorean fuzzy hypersoft set, and it is the most substantial methodology to describe fuzzy information in the decision-making process. One of the aims of this study is to give Fermatean fuzzy hypersoft sets and to examine their basic properties. The second objective of this study is to develop the notion and features of the correlation coefficient and the weighted correlation coefficient for the Fermatean fuzzy hypersoft set and to introduce the aggregation operators such as Fermatean fuzzy hypersoft weighted average and Fermatean fuzzy hypersoft weighted geometric operators under the Fermatean fuzzy hypersoft set scenario. A prioritization technique for order preference by similarity to the ideal solution (TOPSIS) under Fermatean fuzzy hypersoft set based on correlation coefficients and weighted correlation coefficients is presented. Through the developed methodology, a technique for solving the multi-attribute group decision-making problem is planned. In addition, examples of medical decision-making are presented for the importance and application of the developed methodology.

Keywords: Fermatean fuzzy hypersoft set, correlation coefficients, informational energy, group decision-making, TOPSIS, medical decision-making.

1 Introduction

1.1 Multi-criteria decision-making and uncertainty

Cognitive science, philosophy, artificial intelligence, and psychology are just a few of the academic disciplines that study how people think and make decisions in everyday situations. Several mathematical and statistical models are frequently used to characterize these processes. The issue of decision-making (DM) emerges during this procedure. DM is the process of choosing one or more of the available possibilities for action when a person or organization is striving to achieve a specified objective. In real life, although many everyday decisions can be made instinctively, it is a well-known fact that complex and critical decisions require more thought. For making such complex decisions, Multi-Criteria Decision Making (MCDM) is a set of analytical tools that evaluate the benefits and drawbacks of various options based on various criteria. MCDM techniques are used to select one or more alternatives from a collection of alternatives with varying characteristics according to competing criteria or to rank these alternatives to aid the DM process. In other words, decision-makers use MCDM techniques to rank options based on various characteristics by comparing them to multiple criteria. In other words, MCDM is a set of procedures that are used every day at all levels and in all aspects of life.

MCDM is based on the handle of modeling the decision procedure using criteria and analyzing the decision maker in such a way that the benefit obtained at the end of the process is maximized. Assume you are in the process of making a home purchase decision. You should be asked to evaluate the two house options available to you based on two criteria, such as price and house size, and make a decision. In such a decision problem, a simple comparison can be used to select the alternative that will provide you with the greatest benefit (the house with the greatest area at the lowest price) without resorting to any decision-making method. However, if we apply the problem of buying a house to real life, we can predict that the number of alternatives will be much greater than two, and the number of criteria you must consider during the buying process will be much greater. In this case, you will be unable to conduct logical tests in order to make an effective decision, and making decisions based on intuition will not yield very productive results. In such a case, utilizing a scientific decision support system to aid your decision-making process will allow you to obtain an effective output as a result of the DM process. The decision problem described above on a simple example becomes more complex in real life, particularly in the decision processes of businesses, hospitals, and disease diagnosis, economies, and government plans, and becomes more important in terms of costs when it is resolved. The MCDM approaches proposed for use in this type of decision problem include approaches and methods that attempt to find the "best/suitable" solution that meets more than one conflicting criterion. The best solution is the decision made as a result of the decision process that provides the greatest benefit at the lowest cost. To overcome such issues, decision-makers can use MCDM techniques to make deterministic and, thus, more effective decisions. MCDM is a collection of strategies that are widely employed in all aspects of life and at all levels.

Uncertainty is a key notion in DM problems. Unpredictable events characterize uncertainty. In uncertain circumstances, routine decisions cannot be debated. In ambiguous situations, it is critical to weigh both the benefits and drawbacks of potential outcomes. It is critical at this point to conduct a thorough examination of the environmental factors. Even when future judgments are uncontested, drawing on prior experiences and decisions is not always successful when there is uncertainty. Because of Zadeh's concept of "fuzzy sets" (FS) [51], everyday linguistic phrases have become "computable." Thanks to fuzzy logic, the grading system was able to expand the area of classical mathematics that was previously limited to certainty. As a result of its effective implementation in everyday situations, this concept prompted a paradigm change that expanded throughout the world. A distinct function element is either an element of a set in the conventional sense or it is not. According to the FS concept, the membership function (MF), which assigns each item a degree of membership in the range $[0, 1]$, determines whether or not an apple is a member of a set.

The degree to which an element belongs to a set in FS A is $\rho(A)$, while the degree to which it does not belong is $1-\rho(A)$. However, in a number of cases, this condition falls short of fully addressing the uncertainty. As a result, Atanassov [1] developed the intuitionistic fuzzy set (IFS) theory as an extension to the FS theory. In addition to the membership degree (MD), IFS theory specifies the non-membership degree (ND), whereas FS theory only reveals the membership degree (MD)($\in [0,1]$). In the theory of IFS, MD and ND are both in the $[0,1]$ range. Yager [46] proposed Pythagorean fuzzy sets (PFS) as an extension of IFSs in some cases because IFSs cannot adequately represent uncertainty. PFSs are based on the idea that $MD^2+ND^2 \leq 1$. There is a lot of study on FS and its numerous expansions in the literature([6, 7, 8, 9, 11, 13, 14, 32, 33, 50]).

1.2 Correlation Coefficients

Any statistical relationship, whether causal or not, between two random variables or bivariate data is referred to as correlation or dependence. Although the term "correlation" can apply to any sort of link, it is most commonly used in statistics to describe the degree to which two variables are linearly connected. As a result, in correlation research, the correlation coefficient (KK) is the exact measurement used to determine the power of the linear link between 2 variables. Correlations are useful because they can forecast a relationship that can be used in practice. In general, the appearance of a correlation does not imply the presence of a causal relationship (i.e., a correlation does not imply causation). If random variables do not meet a mathematical characteristic known as probabilistic independence, they are called dependent. Correlation is often used interchangeably with dependence. When used in a technical sense, correlation refers to any of many distinct types of mathematical operations conducted between the tested variables and their corresponding predicted values. A correlation is a measure of co-change. We can use this statistical method to determine the strength and direction of a relationship between two or more variables. The letter r represents the letter KK. According to mathematics, the KK can have values ranging from -1 to $+1$. The direction of the link between the two variables is indicated by the sign of this coefficient and the degree of correlation between the two variables. Consider the reciprocal connection to gain an understanding of the correlation. A correlation is a reciprocal relationship that can represent the difference or resemblance between two variables rather than a cause-and-effect relationship. Because of correlation, if the relationship between the variables is known, the other can be estimated by studying the value of one variable, or the values of the connected variables can be maximized if the influencing factors are managed.

When there are more than two variables, a KK is a multivariate statistic; when there are only two variables, it is a bivariate statistic. As a result, several fields of study have emerged, ranging from engineering to physics, and from medicine to economics. Existing probabilistic approaches have several benefits but also drawbacks. Probabilistic techniques, for example, achieve the required confidence level by collecting large amounts of random data. The complex system, on the other hand, has widespread fuzzy uncertainty, making it impossible to predict every possible outcome of the events. Because specialists can only act on numerical information, probabilistic outcomes may not always give helpful information. Furthermore, there are times when there is insufficient data to operate parameter statistics appropriately in day-to-day operations. Because of these constraints, probability theory's conclusions may not always ensure beneficial information to specialists, and the probabilistic approximation is thus insufficient to account for the underlying uncertainties in the data. There are several approaches to overcoming these obstacles. Methods based on FS theory are among the most successful outcomes of these alternatives for reducing uncertainties and imprecision in DM.

Finding a KK between any two parameters or variables is a fairly common statistical task. Pearson's KK has been used in data processing and categorization, pattern recognition, clustering, medical diagnosis, and decision-making statistics research. It has been demonstrated that classical correlation cannot handle ambiguous data. To address and quantify ambiguities in human perception and reasoning, the fuzzy logic approach is used. Scientists are accustomed to analyzing data using binary logic. Because human logic is imprecise and complicated, employing binary logic to study human cognitive processes results in some distortion. The foundation of fuzzy logic is human cognitive processes. The modeling of thinking and decision mechanisms that enables people to reach consistent and correct conclusions in the face of incomplete and inaccurate information is one definition of fuzzy logic. The fuzzy type KKs have been extended with the help of mathematical statistics, statistical KKs, and the fuzzy logic method. The KK generated for fuzzy data not only displays the degree of association between FSs, but also whether they are favorably or negatively connected.

1.3 Motivation

In decision-making situations, tools such as aggregation operators and information measures are routinely employed. The KKs assessment of the amount of dependence between two sets may also be used to choose the optimal alternative. Using KKs, one may determine how strongly two variables are related. Because the information in various settings is typically unclear, ambiguous, and partial, numerous scholars have created KKs in fuzzy environments. Chiang and Lin [2] developed a strategy for KK of FSs in addition to providing the correlation for fuzzy information in line with traditional statistics. The KK of fuzzy information has been examined in [24] using a mathematical programming approximation, according to the standard notion of KKs. According to the findings of the FS theory, Atanassov [1] claimed that IFS results were more complete and precise. The IFS theory considers both MD and ND, and it demands that their sum be one or less than one. The IFS-derived KKs have a variety of applications, including DM, cluster analysis, image processing, pattern recognition, and so on ([23], [41], [42], [43], [44]). Several DM problems utilizing Pythagorean fuzzy information have been described in the literature as a result of the PFS ([13], [12], [31], [32], [45], [46], [47], [53]), which was designed to address an IFS issue.

Many area experts and scholars have expressed interest in the PFS model since its creation. For example, Yager and Abbasov, for example, examined the relationship between Pythagorean MGs and complex numbers. Zhang and Xu developed an extended Pythagorean fuzzy TOPSIS model to address various MCDM situations utilizing Pythagorean fuzzy data. Furthermore, it offers a wide range of possible applications, including domestic airline service quality, DM, and so on. However, in other cases, the PFS strategy could not be approved. Consider an expert team that was separated into two groups. The first team of experts estimates the MD to be 0.9, whereas the second team estimates the ND to be 0.8. It is obvious that $0.92 + 0.82$ is larger than one. The IFS and PFS were unable to depict this circumstance.

Senapati and Yager [34] developed the concept of FFSs as an extension of the IFSs and PFSs to address this difficulty. In an FFS, the cubic sum of an object's MD and ND is limited by 1 ($0 \leq MD^3 + ND^3 \leq 1$). Consider $0.9 + 0.6 > 1$, $(0.9)^2 + (0.6)^2 > 1$, and $(0.9)^3 + (0.6)^3 < 1$ as examples. That is because the total of the cubes of the MD and ND of FFSs is in the $[0, 1]$, FFSs give a more comprehensive view for FSs. When dealing with unclear data, FFSs are more adaptable and efficient than IFSs and PFSs. FFS is now playing an important role in a variety of disciplines since it is a strong notion for dealing with imprecise and unclear information in a Fermatean fuzzy environment.

Some FFS characteristics, score, and accuracy functions are provided in [34]. Additionally, the TOPSIS approach, which is widely used to solve MCDM issues, has been employed to solve FFS. Additionally, Senapati and Yager [34] have used the TOPSIS method, which is often used in MCDM issues, to solve FFS difficulties. Senapati and Yager [35] continued this work by investigating a number of additional operations including arithmetic mean operations over FFSs in addition to using the FF weighted product model to address MCDM issues. New aggregation operations that are FFS-related have been described and their associated attributes have been looked at in [36].

Shahzadi and Akram [37] developed the new aggregated operators and a decision support algorithm for the FFSS. Garg et al. [10] introduced novel FFS types of aggregated operators defined by t-norm and t-conorm. In their investigation, Donghai et al. [3] propose the notion of FF linguistic word sets. These sets' operations, scoring, and accuracy functions were provided. In [4], a new similarity metric for FF linguistic word sets is developed. The new metric is a hybrid of the Euclidean distance measure and the cosine similarity measure. Kirisci [20] designed FF soft sets and provided an entropy metric based on them. In [18], a novel hesitant fuzzy set known as the "fermatean hesitant fuzzy set" is presented and its properties are studied. Kirisci and Simsek [19] offer aggregation operations to extend FFHSs to interval-valued Fermatean hesitant fuzzy sets (IVFHFS) and to improve MCGDM methods to IVFHF environments. The ELECTRE I approach is defined in [15] using Fermatean fuzzy sets and the group DM process, in which more than one individual engages at the same time. [17] defines many FF reference relations (consistent, incomplete, consistent incomplete, acceptable incomplete). A priority vector-based additive consistency has been provided. In addition, a methodology for obtaining missing judgments in incomplete FF preference relations is described. Garg et al. [10] examined decision-making analysis using Fermatean fuzzy Yager aggregation operators (for use in coronavirus disease 2019 (COVID-19) testing facilities). Yang et al. [49] addressed Fermatean fuzzy function differential calculus. Shahzadi and Akram [37] created a unique DM paradigm based on Fermatean fuzzy soft information to pick an antiviral mask. Furthermore, in a Fermatean fuzzy environment, Akram et al. [38] suggested an unique DM approach for selecting an efficient sanitizer to decrease COVID-19. Based on the linguistic scale function, Liu et al. [25] established the idea of distance measures for Fermatean fuzzy linguistic word sets. Their application in the creation of TODIM and TOPSIS approaches exemplifies this.

The originality:

Fuzzy KK, IF KK, and PF KK are examples of expansions to the classical KKs. The KKs' performance has increased as a result of these extensions. FFSs can handle ambiguity and partial information problems more effectively than IFSs and PFSs (Figure 1). In this study, first of all, the concept of the FFHS is introduced and its basic properties are examined. Secondly, new KKs based on FFHSS were defined and the theoretical basis of these coefficients was demonstrated. The reason for using FFSs to define new correlation coefficients is that since the $MD^3 + ND^3 \leq 1$ requirement for an object is met, it is likely to cover more items than IFSs and PFSs. The MCGDM algorithm was produced by combining the new KKs and the TOPSIS method, and an example for the selection of hip prosthesis materials is given to demonstrate the operability and reliability of the method. KKs given in previous studies and KKs defined in this study were compared.

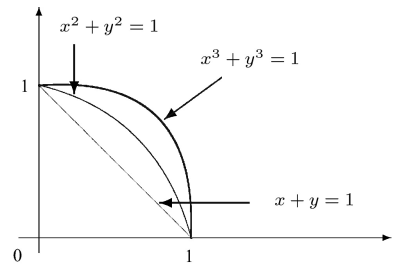

Figure 1: Comparison of FFSs, PFSs and IFSs [34]

The organization of this work's structure: Fundamental ideas and findings from the concept of HSSs are given in Section 2. The new FFHSSs, which are an extension of the FFSS and also a parameterized family that deals with the sub-attributes of the parameters are given in the third chapter, and their properties are examined in detail. The informational energies, correlations, KKs, and weighted KKs of FFHSSs are defined in Section 4. Section 5 presents a prioritizing strategy based on KKs and weighted KKs for order preference by the resemblance to the ideal solution (TOPSIS) under FFHSS. An approach for tackling MCGDM issues is envisaged using the described approach. The 6th chapter is based on an illustrative example of our new method. In this section, a study was carried out on the selection of hip prosthesis materials. A comparison of the new method with previous methods is made in Section 7 and this section is concluded with a subsection explaining the superiority of the new method.

2 Preliminaries

Throughout the article, $\mathfrak{U}$, and $\mathfrak{E}$ will be used as a universe and as a set of attributes, respectively.

Definition 2.1. *[28] $F_{\mathscr{A}}$ is called a SS over $\mathfrak{U}$, where $F : \mathscr{A} \to P(\mathfrak{U})$, $\mathscr{A} \subseteq \mathfrak{E}$, and $P(\mathfrak{U})$ is the power set of $\mathfrak{U}$.*

SS can also be written as:

$$F_{\mathscr{A}} \quad = \quad \{F(e) \in P(\mathfrak{U}) : e \in \epsilon, F(e) = \emptyset, if e \notin \mathscr{A}\}. \tag{1}$$

Maji et al. [26] investigated SS and FS and proposed a more extended form to handle uncertainty in comparison to current FS and SS, as well as its unique properties. This is sometimes referred to as a fuzzy soft set (FSS), which is a mix of FS and SS.

Definition 2.2. *[26] $F_{\mathscr{A}}$ is called a FSS over $\mathfrak{U}$, where $F : \mathscr{A} \to F(\mathfrak{U})$, $\mathscr{A} \subseteq \mathfrak{E}$, and $P(\mathfrak{U})$ is a collection of all fuzzy subsets of $\mathfrak{U}$.*

Definition 2.3. *[40] Let $\alpha = \{\alpha_1, \alpha_2, \cdots, \alpha_n\}$ $(n \geq 1)$ be a set of attributes, and set Σ_i be a set of corresponding sub-attributes of α_i, respectively, with $\Sigma_i \cap \Sigma_j = \varphi$ for $n \geq 1$ $\forall i, j \in \{1, 2, \cdots, n\}$, and $i \neq j$. Consider $\Sigma_1 \times \Sigma_2 \times \cdots \times \Sigma_n = \mathscr{A} = \{m_{1p} \times m_{2r} \times \cdots \times m_{nx}\}$ be a collection of multi-attributes, $(1 \leq p \leq a$, $1 \leq r \leq b$, and $1 \leq s \leq c$ and $a, b, c \in \mathbb{N})$, and $P(\mathfrak{U})$ is a power set of $\mathfrak{U}$. Therefore, $F_{\Sigma_1 \times \Sigma_2 \times \cdots \times \Sigma_n = \mathscr{A}}$ is called a HSS, where*

$$F : \Sigma_1 \times \Sigma_2 \times \cdots \times \Sigma_n = \overline{\mathrm{A}} \to P(\mathfrak{U}). \tag{2}$$

HSS can also be written as:

$$F_{\overline{\mathscr{A}}} \quad = \quad \left\{(\tilde{m}, F(\overline{\mathscr{A}})(\tilde{m})) : \tilde{m} \in \overline{\mathscr{A}}, F(\overline{\mathscr{A}})(\tilde{m}) \in P(\mathfrak{U})\right\}. \tag{3}$$

Definition 2.4. *Let* $\alpha = \{\alpha_1, \alpha_2, \cdots, \alpha_n\}$ $(n \geq 1)$ *be a set of attributes, and set* Σ_i *be a set of corresponding sub-attributes of* α_i*, respectively, with* $\Sigma_i \cap \Sigma_j = \varphi$ *for* $n \geq 1$ $\forall i, j \in \{1, 2, \cdots, n\}$*, and* $i \neq j$*. Consider* $\Sigma_1 \times \Sigma_2 \times \cdots \times \Sigma_n = \mathscr{A} = \{m_{1p} \times m_{2r} \times \cdots \times m_{nx}\}$ *be a collection of multi-attributes, (*$1 \leq p \leq a$*,* $1 \leq r \leq b$*, and* $1 \leq s \leq c$ *and* $a, b, c \in \mathbb{N}$*), and* $P^{\mathfrak{U}}$ *is a collection of all fuzzy subset of* $\mathfrak{U}$*. Therefore,* $F_{\Sigma_1 \times \Sigma_2 \times \cdots \times \Sigma_n = \mathscr{A}}$ *is said to be fuzzy HSS(FHSS), where*

$$F : \Sigma_1 \times \Sigma_2 \times \cdots \times \Sigma_n = \overline{\mathscr{A}} \rightarrow P^{\mathfrak{U}}. \tag{4}$$

.

FHSS can also be written as:

$$F_{\overline{\mathscr{A}}} \quad = \quad \left\{(\tilde{m}, F(\overline{\mathscr{A}})(\tilde{m})) : \tilde{m} \in \overline{\mathscr{A}}, F(\overline{\mathscr{A}})(\tilde{m}) \in P^{\mathfrak{U}}\right\}. \tag{5}$$

Definition 2.5. *Let* $\alpha = \{\alpha_1, \alpha_2, \cdots, \alpha_n\}$ $(n \geq 1)$ *be a set of attributes, and set* Σ_i *be a set of corresponding sub-attributes of* α_i*, respectively, with* $\Sigma_i \cap \Sigma_j = \varphi$ *for* $n \geq 1$ $\forall i, j \in \{1, 2, \cdots, n\}$*, and* $i \neq j$*. Consider* $\Sigma_1 \times \Sigma_2 \times \cdots \times \Sigma_n = \mathscr{A} = \{m_{1p} \times m_{2r} \times \cdots \times m_{nx}\}$ *be a collection of multi-attributes, (*$1 \leq p \leq a$*,* $1 \leq r \leq b$*, and* $1 \leq s \leq c$ *and* $a, b, c \in \mathbb{N}$*), and* $IFS^{\mathfrak{U}}$ *is a collection of all intuitionistic fuzzy subset of* $\mathfrak{U}$*. Therefore,* $F_{\Sigma_1 \times \Sigma_2 \times \cdots \times \Sigma_n = \mathscr{A}}$ *is said to be intuitionistic fuzzy HSS(IFHSS), where*

$$F : \Sigma_1 \times \Sigma_2 \times \cdots \times \Sigma_n = \overline{\mathscr{A}} \rightarrow IFS^{\mathfrak{U}}. \tag{6}$$

.

It can be also given as

$$F_{\overline{\mathscr{A}}} \quad = \quad \left\{(\tilde{m}, F(\overline{\mathscr{A}})(\tilde{m})) : \tilde{m} \in \overline{\mathscr{A}}, F(\overline{\mathscr{A}})(\tilde{m}) \in IFS^{\mathfrak{U}}\right\}. \tag{7}$$

where $F(\overline{\mathscr{A}})(\tilde{m})) = \left\{\lambda, \zeta_{F(\tilde{m})}(\lambda), \eta_{F(\tilde{m})}(\lambda) : \delta \in \mathscr{U}\right\}$, in which $\zeta_{F(\tilde{m})}(\lambda)$ and $\eta_{F(\tilde{m})}(\lambda)$ show the MV and NV of the attributes such as $\zeta_{F(\tilde{m})}(\lambda), \eta_{F(\tilde{m})}(\lambda) \in [0, 1]$, and $0 \leq \zeta_{F(\tilde{m})}(\lambda) + \eta_{F(\tilde{m})}(\lambda) \leq 1$.

For $\mathscr{L} = \{(x, y) : x, y \in [0, 1], x^3 + y^3 < 1\}$, let $(\mathscr{L}, \preceq_{\mathscr{L}})$ denotes a complete lattice, where $\preceq_{\mathscr{L}}$ is the corresponding partial order with $(x, y) \preceq_{\mathscr{L}} (m, n) \Leftrightarrow x \preceq mandy \preceq n$ for all $(x, y), (m, n) \in \mathscr{L}$. The ordered pair $(x, y) \in \mathscr{L}$ is said to be FF value(FFV) or FF number(PFN).

According to the definition of complete lattice, the FFS can be given as:

Let $\mathscr{D} : \mathfrak{U} \rightarrow \mathscr{L}$ be a $\mathscr{L}$-fuzzy set. Then, the FFS $\mathscr{D}(a) = \{(a, \zeta_{\mathscr{D}}(a), \eta_{\mathscr{D}}(a)) : a \in \mathfrak{U}\}$ can be identified as $\mathscr{D}(a) = (\zeta_{\mathscr{D}}(a), \eta_{\mathscr{D}}(a))$ for all $a \in \mathfrak{U}$.

3 Fermatean Fuzzy Hypersoft Sets

In this section, we present FFHSS and some fundamental operations with their characteristics under the FFHS environment.

Definition 3.1. *Let* $\alpha = \{\alpha_1, \alpha_2, \cdots, \alpha_n\}$ $(n \geq 1)$ *be a set of attributes, and set* Σ_i *be a set of corresponding sub-attributes of* α_i*, respectively, with* $\Sigma_i \cap \Sigma_j = \varphi$ *for* $n \geq 1$ $\forall i, j \in \{1, 2, \cdots, n\}$*, and* $i \neq j$*. Consider* $\Sigma_1 \times \Sigma_2 \times \cdots \times \Sigma_n = \mathscr{A} = \{m_{1p} \times m_{2r} \times \cdots \times m_{nx}\}$ *be a collection of multi-attributes, (*$1 \leq p \leq a$*,* $1 \leq r \leq b$*, and* $1 \leq s \leq c$ *and* $a, b, c \in \mathbb{N}$*),* *and $FFS^{\mathfrak{U}}$ is a collection of all fermatean fuzzy subset of $\mathfrak{U}$. Therefore, $F_{\Sigma_1 \times \Sigma_2 \times \cdots \times \Sigma_n = \mathscr{A}}$ is said to be Fermatean fuzzy HSS(FFHSS), where*

$$F : \Sigma_1 \times \Sigma_2 \times \cdots \times \Sigma_n = \overline{\mathscr{A}} \to FFS^{\mathfrak{U}}. \tag{8}$$

.

It also be defined as:

$$F_{\overline{\mathscr{A}}} = \left\{ (\tilde{m}, F(\overline{\mathscr{A}})(\tilde{m})) : \tilde{m} \in \overline{\mathscr{A}}, F(\overline{\mathscr{A}})(\tilde{m}) \in FFS^{\mathfrak{U}} \right\}. \tag{9}$$

where $F(\overline{\mathscr{A}})(\tilde{m})) = \left\{ \lambda, \zeta_{F(\tilde{m})}(\lambda), \eta_{F(\tilde{m})}(\lambda) : \delta \in \mathscr{U} \right\}$, in which $\zeta_{F(\tilde{m})}(\lambda)$ and $\eta_{F(\tilde{m})}(\lambda)$ show the MV and NV of the attributes such as $\zeta_{F(\tilde{m})}(\lambda), \eta_{F(\tilde{m})}(\lambda) \in [0,1]$, and $0 \leq \zeta_{F(\tilde{m})}(\lambda)^3 + \eta_{F(\tilde{m})}(\lambda)^3 \leq 1$.

Example 3.2. *Let $\mathfrak{U} = \{\lambda_1, \lambda_2\}$ be the universe of discourse. The collection of attributes denoted by $\mathscr{L} = \{technology\ methodolgy, subjects, classes\} = \{\mathscr{L}_1, \mathscr{L}_2, \mathscr{L}_3\}$. Their corresponding attribute values given as*

$$\mathscr{L}_1 = \{project\ base, class\ discussion\} = \{m_{11}, m_{12}\}$$
$$\mathscr{L}_2 = \{Mathematics, ComputerScience, Statistics\} = \{m_{21}, m_{22}, m_{23}\}$$
$$\mathscr{L}_3 = \{Masters, Doctoral\} = \{m_{11}, m_{12}\}$$

Let $\mathscr{A} = \mathscr{L}_1 \times \mathscr{L}_2 \times \mathscr{L}_3$ be a set of attributes:

$$\overline{\mathscr{A}} = \mathscr{L}_1 \times \mathscr{L}_2 \times \mathscr{L}_3 = \{n_{11}, n_{12}\} \times \{n_{21}, n_{22}, n_{23}\} \times \{n_{11}, n_{12}\}$$
$$= \Big\{ (n_{11}, n_{21}, n_{31}), (n_{11}, n_{21}, n_{32}), (n_{11}, n_{22}, n_{31}), (n_{11}, n_{22}, m_{32}), (m_{11}, m_{23}, m_{31}), (m_{11}, m_{23}, m_{32}),$$
$$(m_{12}, m_{21}, m_{31}), (m_{12}, m_{21}, m_{32}), (m_{12}, m_{22}, m_{31}), (m_{12}, m_{22}, m_{32}), (m_{12}, m_{23}, m_{31}), (m_{12}, m_{23}, m_{32}) \Big\}$$
$$= \{\tilde{m}_1, \tilde{m}_2, \tilde{m}_3, \tilde{m}_4, \tilde{m}_5, \tilde{m}_6, \tilde{m}_7, \tilde{m}_8, \tilde{m}_9, \tilde{m}_{10}, \tilde{m}_{11}, \tilde{m}_{12}\}.$$

Then, the FFHSS over $\mathfrak{U}$ is given as follows:

$$\begin{aligned}
F_{\overline{\mathscr{A}}} = \Big\{ & (\tilde{m}_1, (\lambda_1, (0.7, 0.4)), (\lambda_2, (0.4, 0.6))), (\tilde{m}_2, (\lambda_1, (0.3, 0.8)), (\lambda_2, (0.2, 0.6))), \\
& (\tilde{m}_3, (\lambda_1, (0.4, 0.8)), (\lambda_2, (0.4, 0.6))), (\tilde{m}_4, (\lambda_1, (0.3, 0.7)), (\lambda_2, (0.3, 0.8))), \\
& (\tilde{m}_5, (\lambda_1, (0.5, 0.5)), (\lambda_2, (0.3, 0.7))), (\tilde{m}_6, (\lambda_1, (0.3, 0.8)), (\lambda_2, (0.2, 0.8))) \\
& (\tilde{m}_7, (\lambda_1, (0.3, 0.6)), (\lambda_2, (0.5, 0.3))), (\tilde{m}_8, (\lambda_1, (0.5, 0.7)), (\lambda_2, (0.3, 0.2))), \\
& (\tilde{m}_9, (\lambda_1, (0.5, 0.7)), (\lambda_2, (0.4, 0.8))), (\tilde{m}_{10}, (\lambda_1, (0.5, 0.7)), (\lambda_2, (0.8, 0.3))), \\
& (\tilde{m}_{11}, (\lambda_1, (0.5, 0.7)), (\lambda_2, (0.4, 0.6))), (\tilde{m}_{12}, (\lambda_1, (0.3, 0.7)), (\lambda_2, (0.3, 0.7))) \Big\}
\end{aligned}$$

Remark. *(1) If both $(\zeta_{F(\tilde{m})}(\lambda))^3 + (\zeta_{F(\tilde{m})}(\lambda))^3 \leq 1$ and $(\zeta_{F(\tilde{m})}(\delta))^2 + (\zeta_{F(\tilde{m})}(\delta))^2 \leq 1$ hold, then FFHSS as reduced to PFHSS [55].*

(2) If $(\zeta_{F(\tilde{m})}(\lambda))^3 + (\zeta_{F(\tilde{m})}(\lambda))^3 \leq 1$ and each parameter of the set of attributes consists of no sub-attribute, then FFHSS was reduced to FFSS [20].

(3) If both $(\zeta_{F(\tilde{m})}(\lambda))^3 + (\zeta_{F(\tilde{m})}(\lambda))^3 \leq 1$ and $(\zeta_{F(\tilde{m})}(\lambda))^2 + (\zeta_{F(\tilde{m})}(\lambda))^2 \leq 1$ hold and a set of attributes contains only one parameter with no sub-attributes, then FFHSS was reduced PFSS [31].

Definition 3.3. *Let $F_{\overline{\mathscr{A}}}$ and $G_{\overline{\mathscr{B}}}$ be two FFHSS over $\mathfrak{U}$.*

(i.) For $\zeta_{F(\tilde{m})}(\lambda) \leq \zeta_{G(\tilde{m})}$ and $\eta_{F(\tilde{m})}(\lambda) \leq \eta_{G(\tilde{m})}$, $\lambda \in \mathfrak{U}$, if

a. $\overline{\mathscr{A}} \subseteq \overline{\mathscr{B}}$,

b. $F_{\overline{\mathscr{A}}}(\tilde{a})(\lambda) \subseteq G_{\overline{\mathscr{B}}}(\tilde{a})(\lambda)$ for all $\lambda \in \mathfrak{U}$,

then $F_{\overline{\mathscr{A}}}$ is called a FFHS subset of $G_{\overline{\mathscr{B}}}$.

(ii.) If $\zeta_{F(\tilde{m})}(\lambda) = 0$, and $\eta_{F(\tilde{m})}(\lambda) = 1$ for all $\tilde{m} \in \overline{\mathscr{A}}$ and $\lambda \in \mathfrak{U}$, ($\emptyset_{\overline{\mathscr{A}}} = \{\tilde{m}, (\lambda, (0, 1)) : \lambda \in \mathfrak{U}, \tilde{m} \in \overline{\mathscr{A}}\}$ is called empty FFHSS(denoted by $\emptyset_{F(\tilde{m})}(\lambda)$).

(iii.) If $\zeta_{F(\tilde{m})}(\lambda) = 1$, and $\eta_{F(\tilde{m})}(\lambda) = 0$ for all $\tilde{m} \in \overline{\mathscr{A}}$ and $\lambda \in \mathscr{U}$, ($E_{\overline{\mathscr{A}}} = \{\tilde{m}, (\lambda, (1, 0)) : \lambda \in \mathfrak{U}, \tilde{m} \in \overline{\mathscr{A}}\}$ is called universal FFHSS(denoted by $E_{F(\tilde{m})}(\lambda)$).

(iv.) If for all $\lambda \in \mathfrak{U}$ and $\tilde{m} \in \mathfrak{U}$, $\zeta_{F_{\overline{\mathscr{A}}}(\tilde{m})}(\lambda) = \zeta_{G_{\overline{A}}(\tilde{m})}(\lambda)$ and $\eta_{F_{\overline{\mathscr{A}}}(\tilde{m})}(\lambda) = \eta_{G_{\overline{\mathscr{A}}}(\tilde{m})}(\lambda)$, then $F_{\overline{\mathscr{A}}}$ and $G_{\overline{\mathscr{B}}}$ is called equal FFHSS.

Theorem 3.4. *Let $F_{\overline{\mathscr{A}}}$, $G_{\overline{\mathscr{B}}}$ and $H_{\overline{\mathscr{C}}}$ be three FFHSS over the universe $\mathfrak{U}$. Hence*

(i.) $F_{\overline{\mathscr{A}}} \subseteq E_{\overline{\mathscr{A}}}$,

(ii.) $\emptyset_{\overline{\mathscr{A}}} \subseteq F_{\overline{\mathscr{A}}}$,

(iii.) $F_{\overline{\mathscr{A}}} \subseteq G_{\overline{\mathscr{B}}}$ and $G_{\overline{\mathscr{B}}} \subseteq H_{\overline{\mathscr{C}}} \Rightarrow F_{\overline{\mathscr{A}}} \subseteq H_{\overline{\mathscr{C}}}$.

Proof. (i.) $F_{\overline{\mathscr{A}}} \subseteq E_{\overline{\mathscr{A}}}$, since $\zeta_{F_{\overline{A}}(\tilde{m})}(\lambda) \leq \zeta_{G_{\overline{A}}(\tilde{m})}(\lambda) = 1$ and $\eta_{F_{\overline{A}}(\tilde{m})}(\lambda) \geq \eta_{G_{\overline{A}}(\tilde{m})}(\lambda) = 0$ $\forall \tilde{m} \in \overline{\mathscr{A}}$, $\lambda \in \mathfrak{U}$.
(ii.) $\emptyset_{\overline{\mathscr{A}}} \subseteq F_{\overline{\mathscr{A}}}$, since $0 = \zeta_{F_{\overline{A}}(\tilde{m})}(\lambda) \leq \zeta_{G_{\overline{A}}(\tilde{m})}(\lambda)$ and $1 = \eta_{F_{\overline{A}}(\tilde{m})}(\lambda) \geq \eta_{G_{\overline{A}}(\tilde{m})}(\lambda)$ $\forall \tilde{m} \in \overline{\mathscr{A}}$, $\lambda \in \mathfrak{U}$.
(iii.) $F_{\overline{\mathscr{A}}} \subseteq g_{.} \Rightarrow \zeta_{F_{\overline{A}}(\tilde{m})}(\lambda) \leq \zeta_{G_{\overline{B}}(\tilde{m})}(\lambda)$ and $\eta_{G_{\overline{B}}(\tilde{m})}(\lambda) \geq \eta_{F_{\overline{A}}(\tilde{m})}(\lambda)$ $\forall \tilde{m} \in \overline{\mathscr{A}}$, $\lambda \in \mathfrak{U}$. Therefore, $\zeta_{G_{\overline{B}}(\tilde{m})}(\lambda) \leq \zeta_{H_{\overline{C}}(\tilde{m})}(\lambda)$ and $\eta_{H_{\overline{C}}(\tilde{m})}(\lambda) \geq \eta_{G_{\overline{B}}(\tilde{m})}(\lambda)$ $\forall \tilde{m} \in \overline{A}$, $\lambda \in \mathfrak{U}$. Hence, we have $F_{\overline{\mathscr{A}}} \subseteq H_{\overline{\mathscr{C}}}$. □

Definition 3.5. *Let $F_{\overline{\mathscr{A}}} = \{(\tilde{m}, [\lambda, \zeta_{F(\tilde{m})}(\lambda), \eta_{F(\tilde{m})}(\lambda)] : \lambda \in \mathfrak{U}) : \tilde{m} \in \overline{\mathscr{A}}\}$ be a FFHSS over $\mathfrak{U}$. $F^{c}_{\overline{\mathscr{A}}} = \{(\tilde{m}, [\lambda, \eta_{F(\tilde{n})}(\lambda), \zeta_{F(\tilde{m})}(\lambda)] : \lambda \in \mathfrak{U}) : \tilde{m} \in \overline{\mathscr{A}}\}$ is called complement of $F_{\overline{\mathscr{A}}}$.*

Proposition 3.6. *If $F_{\overline{\mathscr{A}}}$ be a FFHSS, then*

i. $(F^{c}_{\overline{\mathscr{A}}})^{c} = F_{\overline{\mathscr{A}}}$

ii. $\emptyset^{c}_{\overline{\mathscr{A}}} = E_{\overline{\mathscr{A}}}$

iii. $E^{c}_{\overline{\mathscr{A}}} = \emptyset_{\overline{\mathscr{A}}}$

Proof. We will only prove the item (i.). The items (ii.) and (iii.) can be easily proved as similar to (i.)

Let $F_{\overline{\mathscr{A}}} = \{(\tilde{m}, [\lambda, \sigma_{F(\tilde{m})}(\lambda), \tau_{F(\tilde{m})}(\lambda)] : \lambda \in \mathfrak{U}) : \tilde{m} \in \overline{\mathscr{A}}\}$ be a FFHSS over $\mathfrak{U}$. Then, the using Definition 3.5, we have $F^{c}_{\overline{\mathscr{A}}} = \{(\tilde{m}, [\delta, \tau_{F(\tilde{m})}(\lambda), \sigma_{F(\tilde{m})}(\lambda)] : \lambda \in \mathfrak{U}) : \tilde{m} \in \overline{\mathscr{A}}\}$. Again using the Definition 3.5, $(F^{c}_{\overline{\mathscr{A}}})^{c} = \{(\tilde{m}, [\lambda, \sigma_{F(\tilde{m})}(\lambda), \tau_{F(\tilde{m})}(\lambda)] : \lambda \in \mathfrak{U}) : \tilde{m} \in \overline{\mathscr{A}}\}$. Hence $(F^{c}_{\overline{\mathscr{A}}})^{c} = F_{\overline{\mathscr{A}}}$. □

Definition 3.7. *Let $F_{\overline{\mathscr{A}}}$ and $G_{\overline{\mathscr{B}}}$ be two FFHSS over $\mathfrak{U}$. The union of $F_{\overline{\mathscr{A}}}$ and $G_{\overline{\mathscr{B}}}$ is given as:*

$$F_{\overline{\mathscr{A}}}\tilde{\cup}G_{\overline{\mathscr{B}}} = H_{\overline{\mathscr{C}}} \quad and \quad \overline{\mathscr{C}} = \overline{\mathscr{A}} \cup \overline{\mathscr{B}}, \tag{10}$$

$$\zeta_{H(\tilde{m})}(\lambda) = \begin{cases} F(\tilde{m}) & , \text{ if } \tilde{m}a \in \overline{\mathscr{A}} - \overline{\mathscr{B}} \\ G(\tilde{m}) & , \text{ if } \tilde{m} \in \overline{\mathscr{B}} - \overline{\mathscr{A}} \\ \max\{F(\tilde{m}), G(\tilde{m})\} & , \text{ if } \tilde{m} \in \overline{\mathscr{A}} \cap \overline{\mathscr{B}} \end{cases} \tag{11}$$

$$\eta_{H(\tilde{m})}(\lambda) = \begin{cases} F(\tilde{m}) & , \text{ if } \tilde{m} \in \overline{\mathscr{A}} - \overline{\mathscr{B}} \\ G(\tilde{m}) & , \text{ if } \tilde{m} \in \overline{\mathscr{B}} - \overline{\mathscr{A}} \\ \min\{F(\tilde{m}), G(\tilde{m})\} & , \text{ if } \tilde{m} \in \overline{\mathscr{A}} \cap \overline{\mathscr{B}} \end{cases} \tag{12}$$

Definition 3.8. *Let $F_{\overline{\mathscr{A}}}$ and $G_{\overline{\mathscr{B}}}$ be two FFHSS over $\mathfrak{U}$. The intersection of $F_{\overline{\mathscr{A}}}$ and $G_{\overline{\mathscr{B}}}$ is given as:*

$$F_{\overline{\mathscr{A}}}\tilde{\cap}G_{\overline{\mathscr{B}}} = H_{\overline{\mathscr{C}}}) \quad and \quad \overline{\mathscr{C}} = \overline{\mathscr{A}} \cap \overline{\mathscr{B}}, \tag{13}$$

$$\zeta_{H(\tilde{m})}(\lambda) = \begin{cases} F(\tilde{m}) & , \text{ if } \tilde{m} \in \overline{\mathscr{A}} - \overline{\mathscr{B}} \\ G(\tilde{m}) & , \text{ if } \tilde{m} \in \overline{\mathscr{B}} - \overline{\mathscr{A}} \\ \min\{F(\tilde{m}), G(\tilde{m})\} & , \text{ if } \tilde{m} \in \overline{\mathscr{A}} \cap \overline{\mathscr{B}} \end{cases} \tag{14}$$

$$\eta_{H(\tilde{a})}(\delta) = \begin{cases} F(\tilde{a}) & , \text{ if } \tilde{a} \in \overline{\mathscr{A}} - \overline{\mathscr{B}} \\ G(\tilde{a}) & , \text{ if } \tilde{a} \in \overline{\mathscr{B}} - \overline{\mathscr{A}} \\ \max\{F(\tilde{a}), G(\tilde{a})\} & , \text{ if } \tilde{a} \in \overline{\mathscr{A}} \cap \overline{\mathscr{B}} \end{cases} \tag{15}$$

Definition 3.9. *Let $\mathscr{A}, \mathscr{B} \subseteq \mathfrak{E}$, $\mathscr{C} = \mathscr{A} \cup \mathscr{H} \neq \emptyset$, $\mathscr{F} \subseteq \mathscr{G}$, and $F_{\overline{\mathscr{A}}} \subseteq G_{\overline{\mathscr{B}}}$. The restricted union of $F_{\overline{\mathscr{A}}}$ and $G_{\overline{\mathscr{B}}}$ is characterized by $(F_{\overline{\mathscr{A}}}\tilde{\cup}_r G_{\overline{\mathscr{B}}} = H_{\overline{\mathscr{C}}}$ such that*

$$\begin{aligned} &H_{\cdot} = F_{\overline{\mathscr{A}}} \cup_r G_{\overline{\mathscr{B}}} \\ &\zeta_{H(\tilde{m})}(\lambda) = \max\{\zeta_{F(\tilde{m})}(\lambda), \zeta_{G(\tilde{m})}(\lambda)\}; \\ &\eta_{H(\tilde{m})}(\lambda) = \min\{\eta_{F(\tilde{m})}(\lambda), \eta_{G(\tilde{m})}(\lambda)\}. \end{aligned}$$

Definition 3.10. *Let $\mathscr{A}, \mathscr{B} \subseteq \mathfrak{E}$, $\mathscr{C} = \mathscr{A} \cup \mathscr{H} \neq \emptyset$, $\mathscr{F} \subseteq \mathscr{G}$, $F_{\overline{\mathscr{A}}} \subseteq G_{\overline{\mathscr{B}}}$. The restricted union of $F_{\overline{\mathscr{A}}}$ and $G_{\overline{\mathscr{B}}}$ is defined as $F_{\overline{\mathscr{A}}}\tilde{\cap}_r G_{\overline{\mathscr{B}}} = M_{\overline{\mathscr{D}}}$ such that*

$$\begin{aligned} &M_{\overline{\mathscr{D}}} = F_{\overline{\mathscr{A}}} \cap_r G_{\overline{\mathscr{B}}} \\ &\zeta_{M(\tilde{m})}(\lambda) = \min\{\zeta_{F(\tilde{m})}(\lambda), \zeta_{G(\tilde{m})}(\lambda)\}; \\ &\eta_{M(\tilde{m})}(\lambda) = \max\{\eta_{F(\tilde{m})}(\lambda), \eta_{G(\tilde{m})}(\lambda)\}. \end{aligned}$$

Definition 3.11. *For two FFHSS $F_{\overline{\mathscr{A}}}$ and $G_{\overline{\mathscr{B}}}$, the "OR" operation on $F_{\overline{\mathscr{A}}}$ and $G_{\overline{\mathscr{B}}}$ is described as*

$$F_{\overline{\mathscr{A}}} \vee G_{\overline{\mathscr{B}}}) \quad = \quad \mathscr{H}_{overline\mathscr{A} \times \overline{\mathscr{B}}} = (K, \tilde{a} \times \tilde{b})$$

where

$$\begin{aligned} (\mathscr{K}, \tilde{m} \times \tilde{n}) \quad &= \quad F_{\overline{\mathscr{A}}_{(\tilde{m})}} \cup G_{\overline{\mathscr{B}}_{(\tilde{n})}}, \quad \forall (\tilde{m} \times \tilde{n}) \in \overline{\mathscr{A}} \times \overline{\mathscr{B}} \\ &= \quad \big\{\{\max(\zeta_{F(\tilde{m})}), \zeta_{G(\tilde{m})}\}, \{\min(\eta_{F(\tilde{m})}), \eta_{G(\tilde{m})}\} : \lambda \in \mathscr{U}, \tilde{m}, \tilde{n} \in \overline{\mathscr{A}}, \overline{\mathscr{B}}\big\} \end{aligned}$$

Definition 3.12. *For two FFHSS $F_{\overline{\mathscr{A}}}$ and $G_{\overline{\mathscr{B}}}$, the "AND" operation on $F_{\overline{\mathscr{A}}}$ and $G_{\overline{\mathscr{B}}}$ is denoted as*

$$F_{\overline{\mathscr{A}}} \wedge G_{\overline{\mathscr{B}}} \quad = \quad \mathscr{H}_{overline\mathscr{A} \times \overline{\mathscr{B}}} = (K, \tilde{a} \times \tilde{b})$$

where

$$\begin{aligned} (K, \tilde{m} \times \tilde{n}) &= F_{\overline{\mathscr{A}}_{(\tilde{m})}} \cup G_{\overline{\mathscr{B}}_{(\tilde{n})}}, \quad \forall (\tilde{m} \times \tilde{n}) \in \overline{\mathscr{A}} \times \overline{\mathscr{B}} \\ &= \big\{\{\min(\zeta_{F(\tilde{m})}), \zeta_{G(\tilde{m})}\}, \{\max(\eta_{F(\tilde{m})}), \eta_{G(\tilde{m})}\} : \lambda \in \mathscr{U}, \tilde{m}, \tilde{n} \in \overline{\mathscr{A}}, \overline{\mathscr{B}}\big\} \end{aligned}$$

Proposition 3.13. *For three FFHSS $F_{\overline{\mathscr{A}}}$, $G_{\overline{\mathscr{B}}}$, and $H_{\overline{\mathscr{C}}}$,*

i. $F_{\overline{\mathscr{A}}} \vee G_{\overline{\mathscr{B}}} = G_{\overline{\mathscr{B}}} \vee F_{\overline{\mathscr{A}}}$,

ii. $F_{\overline{\mathscr{A}}} \wedge G_{\overline{\mathscr{B}}} = G_{\overline{\mathscr{B}}} \wedge F_{\overline{\mathscr{A}}}$,

iii. $F_{\overline{\mathscr{A}}} \vee [G_{\overline{\mathscr{B}}} \vee H_{\overline{\mathscr{C}}}] = [F_{\overline{\mathscr{A}}} \vee G_{\overline{\mathscr{B}}}] \vee H_{\overline{\mathscr{C}}}$,

iv. $F_{\overline{\mathscr{A}}} \wedge [G_{\overline{\mathscr{B}}} \wedge H_{\overline{\mathscr{C}}}] = [F_{\overline{\mathscr{A}}} \wedge G_{\overline{\mathscr{B}}}] \wedge H_{\overline{\mathscr{C}}}$,

v. $[F_{\overline{\mathscr{A}}} \vee G_{\overline{\mathscr{B}}}]^c = F^c_{\overline{\mathscr{A}}} \wedge G^c_{\overline{\mathscr{B}}}$,

vi. $(F_{\overline{\mathscr{A}}} \wedge G_{\overline{\mathscr{B}}})^c = F^c_{\overline{\mathscr{A}}} \vee G^c_{\overline{\mathscr{B}}}$.

Using the Definitions "OR" and "AND", this proposition is easily proved.

Let $F_{\overline{\mathscr{A}}}$ be a FFHSS and $u, v \in [0, 1]$, such that $u^3 + v^3 \leq 1$. Hence, $F_{\overline{\mathscr{A}}}$ is called $(u, v)-$constant FFHSS(shown by $C^{(u,v)}_{\overline{\mathscr{A}}}$), if $\zeta_{F(\tilde{m})} = \tilde{u}$ and $\eta_{F(\tilde{m})} = \tilde{v}$ $\forall \tilde{m}$.

Definition 3.14. *For three FFHSS $F_{\overline{\mathscr{A}}}$, $G_{\overline{\mathscr{B}}}$, and $H_{\overline{\mathscr{C}}}$,*
(i.) If $F_{\overline{\mathscr{A}}} = C^{(0,1)}_{A}$ and $\zeta(\tilde{m}) = 0$, $\eta(\tilde{m}) = 1$ for all $\tilde{m} \in \overline{\mathscr{A}}$, $F_{\overline{\mathscr{A}}}$ is said to be a relative null FFHSS with respect to $\overline{\mathscr{A}}$ (shown by $F_\emptyset$).
(ii.) If $F_{\overline{\mathscr{A}}} = C^{(1,0)}_{A}$ and $\zeta(\tilde{m}) = 1$, $\eta(\tilde{m}) = 0$ for all $\tilde{m} \in \overline{\mathscr{A}}$, ($F_{\overline{\mathscr{A}}}$ is called a relative whole FFHSS with respect to $\overline{\mathscr{A}}$ (shown by F_E).

Obviously, if $F = G$ is taken, $F_\emptyset$ and F_E will be null FFHSS and the whole FFHSS.

Proposition 3.15. *For the FFHSSs $F_{\overline{\mathscr{A}}}$, and $G_{\overline{\mathscr{B}}}$*

i. $F_{\overline{\mathscr{A}}} \tilde{\cup} G_{\overline{\mathscr{B}}} = F_{\overline{\mathscr{A}}} \cup_r G_{\overline{\mathscr{B}}} = F_{\overline{\mathscr{A}}}$,

ii. $F_{\overline{\mathscr{A}}} \tilde{\cap} G_{\overline{\mathscr{B}}} = F_{\overline{\mathscr{A}}} \cap_r G_{\overline{\mathscr{B}}} = F_{\overline{\mathscr{A}}}$,

iii. $(F_{\overline{\mathscr{A}}} \tilde{\cup} F_\emptyset = F_{\overline{\mathscr{A}}} \cup_r F_\emptyset = F_{\overline{\mathscr{A}}}$,

iv. $F_{\overline{\mathscr{A}}} \tilde{\cap} F_\emptyset = F_{\overline{\mathscr{A}}} \cap_r F_\emptyset = F_\emptyset$.

v. $F_{\overline{\mathscr{A}}} \tilde{\cup} F_E = F_{\overline{\mathscr{A}}} \cup_r F_E = F_E$,

vi. $F_{\overline{\mathscr{A}}} \tilde{\cap} F_E = F_{\overline{\mathscr{A}}} \cap_r F_E = F_E$.

Using Definitions 3.1, 3.7, 3.8, 3.9, 3.10, 3.14, this proposition can be easily proved.

4 New Coefficients for Fermatean Fuzzy Hypersoft Set

This section is devoted to the notion of KKs and weighted KKs and some aggregation operators on FFHSS. Further, some fundamental characteristics of new KKs have been investigated.

Definition 4.1. *Choose two FFHSSs* $F_{\overline{\mathscr{A}}} = \{(\lambda_i, \zeta_{F(\tilde{m}_k)}(\lambda_i), \eta_{F(\tilde{m}_k)}(\lambda_i)) : \lambda_i \in \mathfrak{U}\}$ *and* $G_{\overline{\mathscr{B}}} = \{(\lambda_i, \zeta_{G(\tilde{m}_k)}(\lambda_i), \eta_{G(\tilde{m}_k)}(\lambda_i)) : \lambda_i \in \mathfrak{U}\}$. *Then,*

$$IE(F_{\overline{\mathscr{A}}}) = \sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\zeta_{F(\tilde{m}_k)}(\lambda_i)\right)^6 + \left(\eta_{F(\tilde{m}_k)}(\lambda_i)\right)^6\right), \tag{16}$$

$$IE(G_{\overline{\mathscr{B}}}) = \sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\zeta_{G(\tilde{m}_k)}(\lambda_i)\right)^6 + \left(\eta_{G(\tilde{m}_k)}(\lambda_i)\right)^6\right). \tag{17}$$

are called the informational energies of $F_{\overline{\mathscr{A}}}$, *and* $G_{\overline{\mathscr{B}}}$.

Definition 4.2. *Choose two FFHSSs* $F_{\overline{\mathscr{A}}} = \{(\lambda_i, \zeta_{F(\tilde{m}_k)}(\lambda_i), \eta_{F(\tilde{m}_k)}(\lambda_i)) : \lambda_i \in \mathfrak{U}\}$ *and* $G_{\overline{\mathscr{B}}} = \{(\lambda_i, \zeta_{G(\tilde{m}_k)}(\lambda_i), \eta_{G(\tilde{m}_k)}(\lambda_i)) : \lambda_i \in \mathfrak{U}\}$. *Then,*

$$C(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}) = \sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\zeta_{F(\tilde{m}_k)}(\lambda_i)\right)^3 * \left(\zeta_{G(\tilde{m}_k)}(\lambda_i)\right)^3 + \left(\eta_{F(\tilde{m}_k)}(\lambda_i)\right)^3 * \left(\eta_{G(\tilde{m}_k)}(\lambda_i)\right)^3\right) \tag{18}$$

is called the correlation measure between $F_{\overline{\mathscr{A}}}$, $G_{\overline{\mathscr{B}}}$.

Theorem 4.3. *Let* $F_{\overline{\mathscr{A}}} = \{(\lambda_i, \zeta_{F(\tilde{d}_k)}(\lambda_i), \eta_{F(\tilde{m}_k)}(\lambda_i)) : \lambda_i \in \mathfrak{U}\}$, *and* $G_{\overline{\mathscr{B}}} = \{(\lambda_i, \zeta_{G(\tilde{m}_k)}(\lambda_i), \eta_{G(\tilde{m}_k)}(\lambda_i)) : \lambda_i \in \mathfrak{U}\}$ *be two FFHSSs. Hence,*
(1) $C(F_{\overline{\mathscr{A}}}, F_{\overline{\mathscr{A}}}) = IE(F_{\overline{\mathscr{A}}})$,
(2) $C(G_{\overline{\mathscr{B}}}, G_{\overline{\mathscr{B}}}) = IE(G_{\overline{\mathscr{B}}})$.

Definition 4.4. *Choose two FFHSSs* $F_{\overline{\mathscr{A}}} = \{(\lambda_i, \zeta_{F(\tilde{m}_k)}(\lambda_i), \eta_{F(\tilde{m}_k)}(\lambda_i)) : \lambda_i \in \mathfrak{U}\}$ *and* $G_{\overline{\mathscr{B}}} = \{(\lambda_i, \zeta_{G(\tilde{m}_k)}(\lambda_i), \eta_{G(\tilde{m}_k)}(\lambda_i)) : \lambda_i \in \mathfrak{U}\}$. *Then,*

$$CC(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}) = \frac{C(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}})}{\sqrt{IE(F_{\overline{\mathscr{A}}})} * \sqrt{IE(G_{\overline{\mathscr{B}}})}} \tag{19}$$

$$= \frac{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\zeta_{F(\tilde{m}_k)}(\lambda_i)\right)^3 * \left(\zeta_{G(\tilde{m}_k)}(\lambda_i)\right)^3 + \left(\eta_{F(\tilde{m}_k)}(\lambda_i)\right)^3 * \left(\eta_{G(\tilde{m}_k)}(\lambda_i)\right)^3\right)}{\sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\zeta_{F(\tilde{m}_k)}(\lambda_i)\right)^6 + \left(\eta_{F(\tilde{m}_k)}(\lambda_i)\right)^6\right)} * \sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\zeta_{G(\tilde{m}_k)}(\lambda_i)\right)^6 + \left(\eta_{G(\tilde{m}_k)}(\lambda_i)\right)^6\right)}}$$

is called the KK *between* $F_{\overline{\mathscr{A}}}$, $G_{\overline{\mathscr{B}}}$.

Theorem 4.5. *Let* $F_{\overline{\mathscr{A}}} = \{(\lambda_i, \zeta_{F(\tilde{m}_k)}(\lambda_i), \eta_{F(\tilde{m}_k)}(\lambda_i)) : \lambda_i \in \mathfrak{U}\}$ *and* $G_{\overline{\mathscr{B}}}) = \{(\lambda_i, \zeta_{G(\tilde{m}_k)}(\lambda_i), \eta_{G(\tilde{m}_k)}(\lambda_i)) : \lambda_i \in \mathfrak{U}\}$ *be two FFHSSs. For the* KK *between them,*
(1) $0 \leq CC(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}) \leq 1$.
(2) $CC((F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}) = CC(G_{\overline{\mathscr{B}}}, G_{\overline{\mathscr{B}}})$,
(3) If $F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}$, *that is, for each* i, k, $\zeta_{F(\tilde{m}_k)}(\lambda_i) = \zeta_{G(\tilde{m}_k)}(\lambda_i))$ *and* $\eta_{F(\tilde{m}_k)}(\lambda_i) = \eta_{G(\tilde{m}_k)}(\lambda_i))$, *then* $CC(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}) = 1$.

Proof. It is clear that it is $CC(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}) \geq 0$. So we need to prove the $CC(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}) \leq 1$

part. Using the Equation 18,

$$
\begin{aligned}
C(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}) &= \sum_{k=1}^{m}\sum_{i=1}^{n}\Big(\big(\zeta_{F(\tilde{m}_k)}(\lambda_i)\big)^3 * \big(\zeta_{G(\tilde{m}_k)}(\lambda_i)\big)^3 + \big(\eta_{F(\tilde{m}_k)}(\lambda_i)\big)^3 * \big(\eta_{G(\tilde{m}_k)}(\lambda_i)\big)^3\Big) \\
&= \sum_{k=1}^{m}\Big(\big(\zeta_{F(\tilde{m}_k)}(\lambda_1)\big)^3 * \big(\zeta_{G(\tilde{m}_k)}(\lambda_1)\big)^3 + \big(\eta_{F(\tilde{m}_k)}(\lambda_1)\big)^3 * \big(\eta_{G(\tilde{m}_k)}(\lambda_1)\big)^3\Big) \\
&+ \Big(\big(\zeta_{F(\tilde{m}_k)}(\lambda_2)\big)^3 * \big(\zeta_{G(\tilde{m}_k)}(\lambda_2)\big)^3 + \big(\eta_{F(\tilde{m}_k)}(\lambda_2)\big)^3 * \big(\eta_{G(\tilde{m}_k)}(\lambda_2)\big)^3\Big) \\
&+ \cdots \\
&+ \Big(\big(\zeta_{F(\tilde{m}_k)}(\lambda_n)\big)^3 * \big(\zeta_{G(\tilde{m}_k)}(\lambda_n)\big)^3 + \big(\eta_{F(\tilde{m}_k)}(\lambda_n)\big)^3 * \big(\eta_{G(\tilde{m}_k)}(\lambda_n)\big)^3\Big) \\
C(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}) &= \Bigg(\Big(\big(\zeta_{F(\tilde{m}_1)}(\lambda_1)\big)^3 * \big(\zeta_{G(\tilde{m}_1)}(\lambda_1)\big)^3 + \big(\eta_{F(\tilde{m}_1)}(\lambda_1)\big)^3 * \big(\eta_{G(\tilde{m}_1)}(\lambda_1)\big)^3\Big) \\
&+ \Big(\big(\zeta_{F(\tilde{m}_2)}(\lambda_1)\big)^3 * \big(\zeta_{G(\tilde{m}_2)}(\lambda_1)\big)^3 + \big(\eta_{F(\tilde{m}_2)}(\lambda_1)\big)^3 * \big(\eta_{G(\tilde{m}_2)}(\lambda_1)\big)^3\Big) \\
&+ \cdots \\
&+ \Big(\big(\zeta_{F(\tilde{m}_m)}(\lambda_1)\big)^3 * \big(\zeta_{G(\tilde{m}_m)}(\lambda_1)\big)^3 + \big(\eta_{F(\tilde{m}_m)}(\lambda_1)\big)^3 * \big(\eta_{G(\tilde{m}_m)}(\lambda_1)\big)^3\Big)\Bigg) \\
&+ \Bigg(\Big(\big(\zeta_{F(\tilde{m}_1)}(\lambda_2)\big)^3 * \big(\zeta_{G(\tilde{m}_1)}(\lambda_2)\big)^3 + \big(\eta_{F(\tilde{m}_1)}(\lambda_2)\big)^3 * \big(\eta_{G(\tilde{m}_1)}(\lambda_2)\big)^3\Big) \\
&+ \Big(\big(\zeta_{F(\tilde{m}_2)}(\lambda_1)\big)^3 * \big(\zeta_{G(\tilde{m}_2)}(\lambda_2)\big)^3 + \big(\eta_{F(\tilde{m}_2)}(\lambda_2)\big)^3 * \big(\eta_{G(\tilde{m}_2)}(\lambda_2)\big)^3\Big) \\
&+ \cdots \\
&+ \Big(\big(\zeta_{F(\tilde{m}_m)}(\lambda_2)\big)^3 * \big(\zeta_{G(\tilde{m}_m)}(\lambda_2)\big)^3 + \big(\eta_{F(\tilde{m}_m)}(\lambda_2)\big)^3 * \big(\eta_{G(\tilde{m}_m)}(\lambda_2)\big)^3\Big)\Bigg) \\
&+ \cdots \\
&+ \Bigg(\Big(\big(\zeta_{F(\tilde{m}_1)}(\lambda_n)\big)^3 * \big(\zeta_{G(\tilde{m}_1)}(\lambda_n)\big)^3 + \big(\eta_{F(\tilde{m}_1)}(\lambda_n)\big)^3 * \big(\eta_{G(\tilde{m}_1)}(\lambda_n)\big)^3\Big) \\
&+ \Big(\big(\zeta_{F(\tilde{m}_2)}(\lambda_n)\big)^3 * \big(\zeta_{G(\tilde{m}_2)}(\lambda_n)\big)^3 + \big(\eta_{F(\tilde{m}_2)}(\lambda_n)\big)^3 * \big(\eta_{G(\tilde{m}_n)}(\lambda_2)\big)^3\Big) \\
&+ \cdots \\
&+ \Big(\big(\zeta_{F(\tilde{m}_m)}(\lambda_n)\big)^3 * \big(\zeta_{G(\tilde{m}_m)}(\lambda_n)\big)^3 + \big(\eta_{F(\tilde{m}_m)}(\lambda_n)\big)^3 * \big(\eta_{G(\tilde{m}_m)}(\lambda_n)\big)^3\Big)\Bigg) \\
&= \sum_{k=1}^{m}\Bigg(\Bigg(\big(\zeta_{F(\tilde{m}_k)}(\lambda_1)\big)^3 * \big(\zeta_{G(\tilde{m}_k)}(\lambda_1)\big)^3 + \big(\zeta_{F(\tilde{m}_k)}(\lambda_2)\big)^3 * \big(\zeta_{G(\tilde{m}_k)}(\lambda_2)\big)^3 + \cdots \\
&+ \big(\zeta_{F(\tilde{m}_k)}(\lambda_n)\big)^3 * \big(\zeta_{G(\tilde{m}_k)}(\lambda_n)\big)^3\Bigg)\Bigg) \\
&+ \sum_{k=1}^{m}\Bigg(\Bigg(\big(\eta_{F(\tilde{m}_k)}(\lambda_1)\big)^3 * \big(\eta_{G(\tilde{m}_k)}(\lambda_1)\big)^3 + \big(\eta_{F(\tilde{m}_k)}(\lambda_2)\big)^3 * \big(\eta_{G(\tilde{m}_k)}(\lambda_2)\big)^3 + \cdots \\
&+ \big(\eta_{F(\tilde{m}_k)}(\lambda_n)\big)^3 * \big(\eta_{G(\tilde{m}_k)}(\lambda_n)\big)^3\Bigg)\Bigg)
\end{aligned}
$$

Now we will use the Cauchy-Schwarz inequality,

$$\begin{aligned}
[C(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}})]^2 &\leq \sum_{k=1}^{m}\Bigg(\Big(\big(\zeta_{F(\tilde{m}_k)}(\lambda_1)\big)^6 + \big(\zeta_{F(\tilde{m}_k)}(\lambda_2)\big)^6 + \cdots + \big(\zeta_{F(\tilde{m}_k)}(\lambda_n)\big)^6\Big) \\
&+ \Big(\big(\eta_{F(\tilde{m}_k)}(\lambda_1)\big)^6 + \big(\eta_{F(\tilde{m}_k)}(\lambda_2)\big)^6 + \cdots + \big(\eta_{F(\tilde{m}_k)}(\lambda_n)\big)^6\Big) \\
&\times \Big(\big(\zeta_{G(\tilde{m}_k)}(\lambda_1)\big)^6 + \big(\zeta_{G(\tilde{m}_k)}(\lambda_2)\big)^6 + \cdots + \big(\zeta_{G(\tilde{m}_k)}(\lambda_n)\big)^6\Big) \\
&+ \Big(\big(\eta_{FG(\tilde{m}_k)}(\lambda_1)\big)^6 + \big(\eta_{G(\tilde{m}_k)}(\lambda_2)\big)^6 + \cdots + \big(\eta_{G(\tilde{m}_k)}(\lambda_n)\big)^6\Big)\Bigg) \\
C(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}})^2 &\leq \sum_{k=1}^{m}\sum_{i=1}^{n}\Big(\big(\zeta_{F(\tilde{m}_k)}(\lambda_i)\big)^6 + \big(\eta_{F(\tilde{m}_k)}(\lambda_i)\big)^6\Big) \\
&\times \sum_{k=1}^{m}\sum_{i=1}^{n}\Big(\big(\zeta_{G(\tilde{m}_k)}(\lambda_i)\big)^6 + \big(\eta_{G(\tilde{m}_k)}(\lambda_i)\big)^6\Big) \\
C(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}})^2 &\leq IE(F_{\overline{\mathscr{A}}}) \times IE(G_{\overline{\mathscr{A}}}).
\end{aligned}$$

Hence, $CC(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}) \leq 1$. So $0 \leq CC(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}) \leq 1$.
(2) The proof of this item is obvious.
(3) We will use the Equation 19. As we know that $\zeta_{F(\tilde{m}_k)}(\lambda_i) = \zeta_{G(\tilde{m}_k)}(\lambda_i)$ and $\eta_{F(\tilde{m}_k)}(\lambda_i) = \eta_{G(\tilde{m}_k)}(\lambda_i)$, for i, k, we get

$$\begin{aligned}
&CC(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}) = \frac{C(F_{\overline{\mathscr{A}}}).C(G_{\overline{\mathscr{G}}})}{\sqrt{IE(F_{\overline{\mathscr{A}}})}.\sqrt{G_{\overline{\mathscr{B}}}}} \qquad (20) \\
&= \frac{\sum_{k=1}^{m}\sum_{i=1}^{n}\Big(\big(\zeta_{F(\tilde{m}_k)}(\lambda_i)\big)^6 + \big(\eta_{F(\tilde{m}_k)}(\lambda_i)\big)^6\Big)}{\sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\Big(\big(\zeta_{F(\tilde{m}_k)}(\lambda_i)\big)^6 + \Big(\eta_{F(\tilde{md}_k)}(\lambda_i)\Big)^6\Big)} * \sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\Big(\big(\zeta_{G(\tilde{m}_k)}(\lambda_i)\big)^6 + \big(\eta_{G(\tilde{m}_k)}(\lambda_i)\big)^6\Big)}} \\
&= 1.
\end{aligned}$$

□

Definition 4.6. *Let* $F_{\overline{\mathscr{A}}} = \{(\lambda_i, \zeta_{F(\tilde{m}_k)}(\lambda_i), \eta_{F(\tilde{m}_k)}(\lambda_i)) : \lambda_i \in \mathfrak{U}\}$ *and* $G_{\overline{\mathscr{B}}} = \{(\lambda_i, \zeta_{G(\tilde{m}_k)}(\lambda_i), \eta_{G(\tilde{m}_k)}(lambda_i)) : \lambda_i \in \mathfrak{U}\}$ *be two FFHSSs. Then, their KK is defined as*

$$\begin{aligned}
&CC_M(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}) = \frac{C(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}})}{\max\{IE(F_{\overline{\mathscr{A}}}), IE(G_{\overline{\mathscr{B}}})\}} \qquad (21) \\
&= \frac{\sum_{k=1}^{m}\sum_{i=1}^{n}\Big(\big(\zeta_{F(\tilde{m}_k)}(\lambda_i)\big)^3 * \big(\zeta_{G(\tilde{m}_k)}(\lambda_i)\big)^3 + \big(\eta_{F(\tilde{m}_k)}(\lambda_i)\big)^3 * \big(\eta_{G(\tilde{m}_k)}(\lambda_i)\big)^3\Big)}{\max\Big\{\sum_{k=1}^{m}\sum_{i=1}^{n}\Big(\big(\zeta_{F(\tilde{m}_k)}(\lambda_i)\big)^6 + \big(\eta_{F(\tilde{m}_k)}(\lambda_i)\big)^6\Big), \sum_{k=1}^{m}\sum_{i=1}^{n}\Big(\big(\zeta_{G(\tilde{m}_k)}(\lambda_i)\big)^6 + \big(\eta_{G(\tilde{m}_k)}(\lambda_i)\big)^6\Big)\Big\}}
\end{aligned}$$

Theorem 4.7. *Let* $F_{\overline{\mathscr{A}}} = \{(\lambda_i, \zeta_{F(\tilde{m}_k)}(\lambda_i), \eta_{F(\tilde{m}_k)}(\lambda_i)) : \lambda_i \in \mathfrak{U}\}$ *and* $G_{\overline{\mathscr{B}}} = \{(\lambda_i, \zeta_{G(\tilde{m}_k)}(\lambda_i), \eta_{G(\tilde{m}_k)}(\lambda_i)) : \lambda_i \in \mathfrak{U}\}$ *be two FFHSSs. For the KK in Equation 21,*
(1) $0 \leq CC_M(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}) \leq 1$.
(2) $CC_M(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}) = CC_M(G_{\overline{\mathscr{B}}}, F_{\overline{\mathscr{A}}})$,
(3) If $F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}$, *that is, for each* i, k, $\zeta_{F(\tilde{m}_k)}(\lambda_i) = \zeta_{G(\tilde{m}_k)}(\lambda_i))$ *and* $\eta_{F(\tilde{m}_k)}(\lambda_i) = \eta_{G(\tilde{m}_k)}(\lambda_i))$, *then* $CC_M(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}) = 1$.

The proof of this theorem is similar to Theorem 4.5.

In this day and age, it is critical to examine the weights of FFHSS in practical applications. The choice may change when the decision-maker assigns different weights for each alternative in the universe of discourse. As a result, it is critical to plan the weight before making a selection. Let $\Omega = \{\Omega_1, \Omega_2, \cdots, \Omega_m\}^T$ be a weight vector for experts such as $\Omega_k > 0$, $\sum_{k=1}^{m} \Omega_k = 1$, and $\gamma = \{\gamma_1, \gamma_2, \cdots, \gamma_n\}^T$ be a weight vector for parameters such as $\gamma_i > 0$, $\sum_{k=1}^{m} \gamma_k = 1$. By extending Definitions 4.4 and 4.6, we create the weighted correlation coefficient between FFHSSs in the sections that follow.

Definition 4.8. *Let* $F_{\overline{\mathscr{A}}} = \{(\lambda_i, \zeta_{F(\tilde{m}_k)}(\lambda_i), \eta_{F(\tilde{m}_k)}(\lambda_i)) : \lambda_i \in \mathfrak{U}\}$ *and* $G_{\overline{\mathscr{B}}} = \{(\lambda_i, \zeta_{G(\tilde{m}_k)}(\lambda_i), \eta_{G(\tilde{m}_k)}(\lambda_i)) : \lambda_i \in \mathfrak{U}\}$ *be two FFHSSs.*

$$CC_W(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}) = \frac{C_W(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}})}{\sqrt{IE_W(F_{\overline{\mathscr{A}}})}.\sqrt{IE_W G_{\overline{\mathscr{B}}}}} \quad ($$

$$= \frac{\sum_{k=1}^{m} \Omega_k \left(\sum_{i=1}^{n} \gamma_i \left(\left(\zeta_{F(\tilde{m}_k)}(\lambda_i)\right)^3 * \left(\zeta_{G(\tilde{m}_k)}(\lambda_i)\right)^3 + \left(\eta_{F(\tilde{m}_k)}(\lambda_i)\right)^3 * \left(\eta_{G(\tilde{m}_k)}(\lambda_i)\right)^3\right)\right)}{\sqrt{\sum_{k=1}^{m} \Omega_k \left(\sum_{i=1}^{n} \gamma_i \left(\left(\zeta_{F(\tilde{m}_k)}(\lambda_i)\right)^6 + \left(\eta_{F(\tilde{m}_k)}(\lambda_i)\right)^6\right)\right)} * \sqrt{\sum_{k=1}^{m} \Omega_k \left(\sum_{i=1}^{n} \gamma_i \left(\left(\zeta_{G(\tilde{m}_k)}(\lambda_i)\right)^6 + \left(\eta_{G(\tilde{m}_k)}(\lambda_i)\right)^6 \right.\right.}}$$

is called KK *between* $F_{\overline{\mathscr{A}}}$, $G_{\overline{\mathscr{B}}}$.

Definition 4.9. *Let* $F_{\overline{\mathscr{A}}} = \{(\lambda_i, \zeta_{F(\tilde{m}_k)}(\lambda_i), \eta_{F(\tilde{m}_k)}(\lambda_i)) : \lambda_i \in \mathfrak{U}\}$ *and* $G_{\overline{\mathscr{B}}} = \{(\lambda_i, \zeta_{G(\tilde{m}_k)}(\lambda_i), \eta_{G(\tilde{m}_k)}(\lambda_i)) : \lambda_i \in \mathfrak{U}\}$ *be two FFHSSs.*

$$CC_{MW}(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}) = \frac{C_W(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}})}{\max\left(IE_W F_{\overline{\mathscr{A}}} * IE_W(G_{\overline{\mathscr{B}}}\right)} \quad (2$$

$$= \frac{\sum_{k=1}^{m} \Omega_k \left(\sum_{i=1}^{n} \gamma_i \left(\left(\zeta_{F(\tilde{m}_k)}(\lambda_i)\right)^3 * \left(\zeta_{G(\tilde{m}_k)}(\lambda_i)\right)^3 + \left(\eta_{F(\tilde{m}_k)}(\lambda_i)\right)^3 * \left(\eta_{G(\tilde{m}_k)}(\lambda_i)\right)^3\right)\right)}{\max \sum_{k=1}^{m} \Omega_k \left(\sum_{i=1}^{n} \gamma_i \left(\left(\zeta_{F(\tilde{m}_k)}(\lambda_i)\right)^6 + \left(\eta_{F(\tilde{m}_k)}(\lambda_i)\right)^6\right)\right), \sum_{k=1}^{m} \Omega_k \left(\sum_{i=1}^{n} \gamma_i \left(\left(\zeta_{G(\tilde{m}_k)}(\lambda_i)\right)^6 + \left(\eta_{G(\tilde{m}_k)}(\lambda_i)\right)^6 \right.\right.}$$

is called KK *between* $F_{\overline{\mathscr{A}}}$, $G_{\overline{\mathscr{B}}}$.

Theorem 4.10. *Let* $F_{\overline{\mathscr{A}}} = \{(\delta_i, \zeta_{F(\tilde{m}_k)}(\lambda_i), \eta_{F(\tilde{dm_k})}(\lambda_i)) : \lambda_i \in \mathfrak{U}\}$ *and* $G_{\overline{\mathscr{B}}} = \{(\lambda_i, \zeta_{G(\tilde{m}_k)}(\lambda_i), \eta_{G(\tilde{m}_k)}(\lambda_i)) : \lambda_i \in \mathfrak{U}\}$ *be two FFHSSs. Then,*
(1) $0 \leq CC_{MW}(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}) \leq 1$.
(2) $CC_{MW}(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}) = CC_{MW}(G_{\overline{\mathscr{B}}}, F_{\overline{\mathscr{A}}})$,
(3) If $F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}$, *that is, for each* i, k, $\zeta_{F(\tilde{m}_k)}(\lambda_i) = \zeta_{G(\tilde{m}_k)}(\lambda_i))$ *and* $\eta_{F(\tilde{m}_k)}(\lambda_i) = \eta_{G(\tilde{m}_k)}(\lambda_i))$,
then $CC_{MW}(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}) = 1$.

Proof. (1) The inequality $CC_{MW}(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}) \geq 0$ is trivial, and here we only need to prove that $CC_{MW}(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}) \leq 1$. Then,

$$\begin{aligned}
C_{MW}(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}) &= \sum_{k=1}^{n} \Omega_k \left(\sum_{i=1}^{n} \gamma_i \left(\left(\zeta_{F(\tilde{m}_k)(\lambda_i)}\right)^3 * \left(\zeta_{G(\tilde{m}_k)(\lambda_i)}\right)^3 + \left(\eta_{F(\tilde{m}_k)(\lambda_i)}\right)^3 * \left(\eta_{G(\tilde{m}_k)(\lambda_i)}\right)^3 \right) \right) \\
&= \sum_{k=1}^{n} \Omega_k \left(\gamma_1 \left(\left(\zeta_{F(\tilde{m}_k)(\lambda_1)}\right)^3 * \left(\zeta_{G(\tilde{m}_k)(\lambda_1)}\right)^3 + \left(\eta_{F(\tilde{m}_k)(\lambda_1)}\right)^3 * \left(\eta_{G(\tilde{m}_k)(\lambda_1)}\right)^3 \right) \right) \\
&+ \sum_{k=1}^{m} \left(\gamma_2 \left(\left(\zeta_{F(\tilde{m}_k)(\lambda_2)}\right)^3 * \left(\zeta_{G(\tilde{m}_k)(\lambda_2)}\right)^3 + \left(\eta_{F(\tilde{m}_k)(\lambda_2)}\right)^3 * \left(\eta_{G(\tilde{m}_k)(\lambda_2)}\right)^3 \right) \right) \\
&+ \cdots + \sum_{k=1}^{n} \left(\gamma_n \left(\left(\zeta_{F(\tilde{m}_k)(\lambda_n)}\right)^3 * \left(\zeta_{G(\tilde{m}_k)(\lambda_n)}\right)^3 + \left(\eta_{F(\tilde{m}_k)(\lambda_n)}\right)^3 * \left(\eta_{G(\tilde{m}_k)(\lambda_n)}\right)^3 \right) \right) \\
&= \Bigg\{ \Omega_1 \left(\gamma_1 \left(\left(\zeta_{F(\tilde{m}_1)(\lambda_1)}\right)^3 * \left(\zeta_{G(\tilde{m}_1)(\lambda_1)}\right)^3 + \left(\eta_{F(\tilde{m}_1)(\lambda_1)}\right)^3 * \left(\eta_{G(\tilde{m}_1)(\lambda_1)}\right)^3 \right) \right) \\
&+ \Omega_2 \left(\gamma_1 \left(\left(\zeta_{F(\tilde{m}_2)(\lambda_1)}\right)^3 * \left(\zeta_{G(\tilde{m}_2)(\lambda_1)}\right)^3 + \left(\eta_{F(\tilde{m}_2)(\lambda_1)}\right)^3 * \left(\eta_{G(\tilde{m}_2)(\lambda_1)}\right)^3 \right) \right) \\
&+ \cdots + \Omega_m \left(\gamma_1 \left(\left(\zeta_{F(\tilde{m}_m)(\lambda_1)}\right)^3 * \left(\zeta_{G(\tilde{m}_m)(\lambda_1)}\right)^3 + \left(\eta_{F(\tilde{m}_m)(\lambda_1)}\right)^3 * \left(\eta_{G(\tilde{m}_m)(\lambda_1)}\right)^3 \right) \right) \Bigg\} \\
&+ \Bigg\{ \Omega_1 \left(\gamma_n \left(\left(\zeta_{F(\tilde{m}_1)(\lambda_2)}\right)^3 * \left(\zeta_{G(\tilde{m}_1)(\lambda_2)}\right)^3 + \left(\eta_{F(\tilde{m}_1)(\lambda_2)}\right)^3 * \left(\eta_{G(\tilde{m}_1)(\lambda_2)}\right)^3 \right) \right) \\
&+ \Omega_2 \left(\gamma_1 \left(\left(\zeta_{F(\tilde{m}_2)(\lambda_2)}\right)^3 * \left(\zeta_{G(\tilde{m}_2)(\lambda_2)}\right)^3 + \left(\eta_{F(\tilde{m}_2)(\lambda_2)}\right)^3 * \left(\eta_{G(\tilde{m}_2)(\lambda_2)}\right)^3 \right) \right) \\
&+ \cdots + \Omega_m \left(\gamma_1 \left(\left(\zeta_{F(\tilde{m}_m)(\lambda_2)}\right)^3 * \left(\zeta_{G(\tilde{m}_m)(\lambda_2)}\right)^3 + \left(\eta_{F(\tilde{m}_m)(\lambda_2)}\right)^3 * \left(\eta_{G(\tilde{m}_m)(\lambda_2)}\right)^3 \right) \right) \Bigg\} \\
&+ \cdots + \Bigg\{ \Omega_1 \left(\gamma_n \left(\left(\zeta_{F(\tilde{m}_1)(\lambda_n)}\right)^3 * \left(\zeta_{G(\tilde{m}_1)(\lambda_n)}\right)^3 + \left(\eta_{F(\tilde{a}_1)(\delta_n)}\right)^3 * \left(\eta_{G(\tilde{a}_1)(\delta_n)}\right)^3 \right) \right) \\
&+ \Omega_2 \left(\gamma_1 \left(\left(\zeta_{F(\tilde{m}_2)(\lambda_n)}\right)^3 * \left(\zeta_{G(\tilde{m}_2)(\lambda_n)}\right)^3 + \left(\eta_{F(\tilde{m}_2)(\lambda_n)}\right)^3 * \left(\eta_{G(\tilde{m}_2)(\lambda_n)}\right)^3 \right) \right) + \cdots \\
&+ \Omega_m \left(\gamma_1 \left(\left(\zeta_{F(\tilde{m}_m)(\lambda_n)}\right)^3 * \left(\zeta_{G(\tilde{m}_m)(\lambda_n)}\right)^3 + \left(\eta_{F(\tilde{m}_m)(\lambda_n)}\right)^3 * \left(\eta_{G(\tilde{m}_m)(\lambda_n)}\right)^3 \right) \right) \Bigg\} \\
&= \Bigg\{ \Omega_1 \left(\sqrt{\gamma_1} \left(\zeta_{F(\tilde{m}_1)(\lambda_1)}\right)^3 * \sqrt{\gamma_1} \left(\zeta_{G(\tilde{m}_1)(\lambda_1)}\right)^3 + \sqrt{\gamma_1} \left(\eta_{F(\tilde{m}_1)(\lambda_1)}\right)^3 * \sqrt{\gamma_1} \left(\eta_{G(\tilde{m}_1)(\lambda_1)}\right)^3 \right) \\
&+ \Omega_2 \left(\sqrt{\gamma_1} \left(\zeta_{F(a\tilde{n}_2)(\lambda_1)}\right)^3 * \sqrt{\gamma_1} \left(\zeta_{G(\tilde{m}_2)(\lambda_1)}\right)^3 + \sqrt{\gamma_1} \left(\eta_{F(a\tilde{m}_2)(\lambda_1)}\right)^3 * \sqrt{\gamma_1} \left(\eta_{G(\tilde{m}_2)(\lambda_1)}\right)^3 \right) \\
&+ \cdots + \Omega_m \left(\sqrt{\gamma_1} \left(\zeta_{F(\tilde{m}_m)(\lambda_1)}\right)^3 * \sqrt{\gamma_1} \left(\zeta_{G(\tilde{m}_m)(\lambda_1)}\right)^3 + \sqrt{\gamma_1} \left(\eta_{F(\tilde{m}_m)(\lambda_1)}\right)^3 * \sqrt{\gamma_1} \left(\eta_{G(\tilde{m}_m)(\lambda_1)}\right)^3 \right) \\
&+ \Omega_1 \left(\sqrt{\gamma_2} \left(\zeta_{F(\tilde{m}_1)(\lambda_2)}\right)^3 * \sqrt{\gamma_2} \left(\zeta_{G(\tilde{m}_1)(\lambda_2)}\right)^3 + \sqrt{\gamma_2} \left(\eta_{F(\tilde{m}_1)(\lambda_2)}\right)^3 * \sqrt{\gamma_2} \left(\eta_{G(\tilde{m}_1)(\lambda_2)}\right)^3 \right) \\
&+ \Omega_2 \left(\sqrt{\gamma_2} \left(\zeta_{F(\tilde{m}_2)(\lambda_2)}\right)^3 * \sqrt{\gamma_2} \left(\zeta_{G(\tilde{m}_2)(\lambda_2)}\right)^3 + \sqrt{\gamma_2} \left(\eta_{F(a\tilde{m}_2)(\lambda_2)}\right)^3 * \sqrt{\gamma_2} \left(\eta_{G(\tilde{m}_2)(\lambda_2)}\right)^3 \right) \\
&+ \cdots + \Omega_m \left(\sqrt{\gamma_2} \left(\zeta_{F(\tilde{m}_m)(\lambda_2)}\right)^3 * \sqrt{\gamma_2} \left(\zeta_{G(\tilde{m}_m)(\lambda_2)}\right)^3 + \sqrt{\gamma_2} \left(\eta_{F(\tilde{m}_m)(\lambda_2)}\right)^3 * \sqrt{\gamma_2} \left(\eta_{G(\tilde{m}_m)(\lambda_2)}\right)^3 \right) \\
&+ \cdots + \Omega_1 \left(\sqrt{\gamma_n} \left(\zeta_{F(\tilde{m}_1)(\lambda_n)}\right)^3 * \sqrt{\gamma_n} \left(\zeta_{G(\tilde{m}_1)(\lambda_1)}\right)^3 + \sqrt{\gamma_n} \left(\eta_{F(\tilde{m}_1)(\lambda_n)}\right)^3 * \sqrt{\gamma_n} \left(\eta_{G(\tilde{m}_1)(\lambda_n)}\right)^3 \right) \\
&+ \Omega_2 \left(\sqrt{\gamma_n} \left(\zeta_{F(\tilde{m}_2)(\lambda_n)}\right)^3 * \sqrt{\gamma_n} \left(\zeta_{G(\tilde{m}_2)(\lambda_n)}\right)^3 + \sqrt{\gamma_n} \left(\eta_{F(\tilde{m}_2)(\lambda_n)}\right)^3 * \sqrt{\gamma_n} \left(\eta_{G(\tilde{m}_2)(\lambda_n)}\right)^3 \right) \\
&+ \cdots + \Omega_m \left(\sqrt{\gamma_n} \left(\zeta_{F(\tilde{m}_m)(\lambda_n)}\right)^3 * \sqrt{\gamma_n} \left(\zeta_{G(\tilde{m}_m)(\lambda_n)}\right)^3 + \sqrt{\gamma_n} \left(\eta_{F(\tilde{m}_m)(\lambda_n)}\right)^3 * \sqrt{\gamma_n} \left(\eta_{G(\tilde{m}_m)(\lambda_n)}\right)^3 \right) \Bigg\}
\end{aligned}$$

$$
\begin{aligned}
= \quad & \Big\{ \sqrt{\Omega_1}\sqrt{\gamma_1}\left(\zeta_{F(\tilde{n}_1)(\lambda_1)}\right)^3 * \sqrt{\Omega_1}\sqrt{\gamma_1}\left(\zeta_{G(\tilde{m}_1)(\lambda_1)}\right)^3 + \sqrt{\Omega_1}\sqrt{\gamma_1}\left(\eta_{F(\tilde{m}_1)(\lambda_1)}\right)^3 * \sqrt{\Omega_1}\sqrt{\gamma_1}\left(\eta_{G(\tilde{m}_1)(\lambda_1)}\right)^3 \\
+ \quad & \sqrt{\Omega_2}\sqrt{\gamma_1}\left(\zeta_{F(\tilde{m}_2)(\lambda_1)}\right)^3 * \sqrt{\gamma_1}\left(\zeta_{G(\tilde{m}_2)(\lambda_1)}\right)^3 + \sqrt{\Omega_2}\sqrt{\gamma_1}\left(\eta_{F(\tilde{m}_2)(\lambda_1)}\right)^3 * \sqrt{\Omega_2}\sqrt{\gamma_1}\left(\eta_{G(a\tilde{m}_2)(\lambda_1)}\right)^3 \\
+ \quad & \cdots + \sqrt{\Omega_m}\sqrt{\gamma_1}\left(\zeta_{F(\tilde{m}_m)(\lambda_1)}\right)^3 * \sqrt{\Omega_m}\sqrt{\gamma_1}\left(\zeta_{G(\tilde{m}_m)(\lambda_1)}\right)^3 + \sqrt{\Omega_m}\sqrt{\gamma_1}\left(\eta_{F(\tilde{m}_m)(\lambda_1)}\right)^3 * \sqrt{\Omega_m}\sqrt{\gamma_1}\left(\eta_{G(\tilde{m}_m)(\lambda_1)}\right)^3 \\
+ \quad & \sqrt{\Omega_1}\left(\sqrt{\gamma_2}\left(\zeta_{F(\tilde{m}_1)(\lambda_2)}\right)^3 * \sqrt{\Omega_1}\sqrt{\gamma_2}\left(\zeta_{G(\tilde{m}_1)(\lambda_2)}\right)^3 + \sqrt{\Omega_1}\sqrt{\gamma_2}\left(\eta_{F(\tilde{m}_1)(\lambda_2)}\right)^3 * \sqrt{\Omega_1}\sqrt{\gamma_2}\left(\eta_{G(\tilde{m}_1)(\lambda_2)}\right)^3\right) \\
+ \quad & \sqrt{\Omega_2}\left(\sqrt{\gamma_2}\left(\zeta_{F(\tilde{m}_2)(\lambda_2)}\right)^3 * \sqrt{\Omega_2}\sqrt{\gamma_2}\left(\zeta_{G(\tilde{m}_2)(\lambda_2)}\right)^3 + \sqrt{\Omega_2}\sqrt{\gamma_2}\left(\eta_{F(\tilde{m}_2)(\lambda_2)}\right)^3 * \sqrt{\Omega_2}\sqrt{\gamma_2}\left(\eta_{G(\tilde{m}_2)(\lambda_2)}\right)^3\right) \\
+ \quad & \cdots + \sqrt{\Omega_m}\left(\sqrt{\gamma_2}\left(\zeta_{F(\tilde{m}_m)(\lambda_2)}\right)^3 * \sqrt{\Omega_m}\sqrt{\gamma_2}\left(\zeta_{G(\tilde{m}_m)(\lambda_2)}\right)^3 + \sqrt{\Omega_m}\sqrt{\gamma_2}\left(\eta_{F(\tilde{m}_m)(\lambda_2)}\right)^3 * \sqrt{\Omega_m}\sqrt{\gamma_2}\left(\eta_{G(\tilde{m}_m)(\lambda_2)}\right)^3\right) \\
+ \quad & \cdots + \sqrt{\Omega_1}\left(\sqrt{\gamma_n}\left(\zeta_{F(\tilde{m}_1)(\lambda_n)}\right)^3 * \sqrt{\Omega_1}\sqrt{\gamma_n}\left(\zeta_{G(\tilde{m}_1)(\lambda_1)}\right)^3 + \sqrt{\Omega_1}\sqrt{\gamma_n}\left(\eta_{F(\tilde{m}_1)(\lambda_n)}\right)^3 * \sqrt{\Omega_1}\sqrt{\gamma_n}\left(\eta_{G(\tilde{m}_1)(\lambda_n)}\right)^3\right) \\
+ \quad & \sqrt{\Omega_2}\left(\sqrt{\gamma_n}\left(\zeta_{F(\tilde{m}_2)(\lambda_n)}\right)^3 * \sqrt{\Omega_2}\sqrt{\gamma_n}\left(\zeta_{G(\tilde{m}_2)(\lambda_n)}\right)^3 + \sqrt{\Omega_2}\sqrt{\gamma_n}\left(\eta_{F(\tilde{m}_2)(\lambda_n)}\right)^3 * \sqrt{\Omega_2}\sqrt{\gamma_n}\left(\eta_{G(\tilde{m}_2)(\lambda_n)}\right)^3\right) \\
+ \quad & \cdots + \sqrt{\Omega_m}\left(\sqrt{\gamma_n}\left(\zeta_{F(\tilde{m}_m)(\lambda_n)}\right)^3 * \sqrt{\Omega_m}\sqrt{\gamma_n}\left(\zeta_{G(\tilde{m}_m)(\lambda_n)}\right)^3 + \sqrt{\Omega_m}\sqrt{\gamma_n}\left(\eta_{F(\tilde{m}_m)(\lambda_n)}\right)^3 * \sqrt{\Omega_m}\sqrt{\gamma_n}\left(\eta_{G(\tilde{m}_m)(\lambda_n)}\right)^3
\end{aligned}
$$

Using the Cauchy-Schwarz inequality, we get

$$
\begin{aligned}
C_{MW}(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}})^2 \leq & \sum_{k=1}^{m} \Omega_k \left(\sum_{i=1}^{m} \gamma_i \left(\left(\zeta_{F(\tilde{m}_k)}(\lambda_i)\right)^6 + \left(\eta_{F(\tilde{m}_k)}(\lambda_i)\right)^6 \right) \right) \\
& \times \sum_{k=1}^{m} \Omega_k \left(\sum_{i=1}^{m} \gamma_i \left(\left(\zeta_{G(\tilde{m}_k)}(\lambda_i)\right)^6 + \left(\eta_{G(\tilde{m}_k)}(\lambda_i)\right)^6 \right) \right)
\end{aligned}
$$

$$
C_{MW}(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}})^2 \leq IE_{MW}(F_{\overline{\mathscr{A}}}).IE_{MW}(G_{\overline{\mathscr{B}}})
$$

Therefore,

$$
C_{MW}(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}) \leq \sqrt{IE_{MW}(F_{\overline{\mathscr{A}}}) * IE_{MW}(G_{\overline{\mathscr{B}}}}
$$

and so $0 \leq C_{MW}(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}) \leq 1$.
The proof of item (2) is obvious.
(3) From Equation 22, we have

$$
\begin{aligned}
CC_W(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}) &= \frac{C_W(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}})}{\sqrt{IE_W(F_{\overline{\mathscr{A}}})} * \sqrt{IE_W(G_{\overline{\mathscr{B}}}}} \qquad (\\
&= \frac{\sum_{k=1}^{m} \Omega_k \left(\sum_{i=1}^{n} \gamma_i \left(\left(\zeta_{F(\tilde{m}_k)}(\lambda_i)\right)^3 * \left(\zeta_{G(\tilde{m}_k)}(\lambda_i)\right)^3 + \left(\eta_{F(\tilde{m}_k)}(\lambda_i)\right)^3 * \left(\eta_{G(\tilde{m}_k)}(\lambda_i)\right)^3 \right) \right)}{\sqrt{\sum_{k=1}^{m} \Omega_k \left(\sum_{i=1}^{n} \gamma_i \left(\left(\zeta_{F(\tilde{m}_k)}(\lambda_i)\right)^6 + \left(\eta_{F(\tilde{m}_k)}(\lambda_i)\right)^6 \right) \right)} * \sqrt{\sum_{k=1}^{m} \Omega_k \left(\sum_{i=1}^{n} \gamma_i \left(\left(\zeta_{G(\tilde{m}_k)}(\lambda_i)\right)^6 + \left(\eta_{G(\tilde{m}_k)}(\lambda_i)\right)^6 \right.\right.}}
\end{aligned}
$$

Take $\zeta_{F(\tilde{m}_k)}(\lambda_i) = \zeta_{G(\tilde{m}_k)}(\lambda_i)$ and $\eta_{F(\tilde{m}_k)}(\lambda_i) = \eta_{G(\tilde{m}_k)}(\lambda_i)$ for all i, k. Hence,

$$
\begin{aligned}
CC_W(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}}) &= \frac{C_W(F_{\overline{\mathscr{A}}}, G_{\overline{\mathscr{B}}})}{\sqrt{IE_W(F_{\overline{\mathscr{A}}})} * \sqrt{IE_W(G_{\overline{\mathscr{B}}}}} \qquad (\\
&= \frac{\sum_{k=1}^{m} \Omega_k \left(\sum_{i=1}^{n} \gamma_i \left(\left(\zeta_{F(\tilde{m}_k)}(\lambda_i)\right)^6 + \left(\eta_{F(\tilde{m}_k)}(\lambda_i)\right)^6 \right) \right)}{\sqrt{\sum_{k=1}^{m} \Omega_k \left(\sum_{i=1}^{n} \gamma_i \left(\left(\zeta_{F(\tilde{m}_k)}(\lambda_i)\right)^6 + \left(\eta_{F(\tilde{m}_k)}(\lambda_i)\right)^6 \right) \right)} * \sqrt{\sum_{k=1}^{m} \Omega_k \left(\sum_{i=1}^{n} \gamma_i \left(\left(\zeta_{G(\tilde{m}_k)}(\lambda_i)\right)^6 + \left(\eta_{G(\tilde{m}_k)}(\lambda_i)\right)^6 \right.\right.}} \\
&= 1. \qquad (
\end{aligned}
$$

□

Definition 4.11. *Let $T_{\tilde{d}_k} = (\zeta_{\tilde{m}_k}, \eta_{\tilde{m}_k})$, $T_{\tilde{m}_{11}} = (\zeta_{\tilde{m}_{11}}, \eta_{\tilde{m}_{11}})$, and $T_{\tilde{m}_{12}} = (\zeta_{\tilde{m}_{12}}, \eta_{\tilde{m}_{12}})$ be three FFHSNs and α be a positive real number; by algeabric norms,we have*

(1) $T_{\tilde{m}_{11}} \boxplus T_{\tilde{m}_{12}} = \left(\sqrt[3]{\zeta^3_{\tilde{m}_{11}} + \zeta^3_{\tilde{m}_{12}} - \zeta^3_{\tilde{m}_{11}}\zeta^3_{\tilde{m}_{12}}}, \eta_{\tilde{m}_{11}}\eta_{\tilde{m}_{12}}\right),$

(2) $T_{\tilde{m}_{11}} \boxtimes T_{\tilde{m}_{12}} = \left(\zeta_{\tilde{m}_{11}}\zeta_{\tilde{m}_{12}}\sqrt[3]{\eta^3_{\tilde{m}_{11}} + \eta^3_{\tilde{m}_{12}} - \eta^3_{\tilde{m}_{11}}\eta^3_{\tilde{m}_{12}}},\right)$

(3) $\alpha T_{\tilde{m}_k} = \left(\sqrt[3]{1 - \left(1 - \zeta^3_{\tilde{m}_k}\right)^{\alpha}}, \eta^{\alpha}_{\tilde{m}_k}\right)$

(4) $T^{\alpha}_{\tilde{m}_k} = \left(\zeta^{\alpha}_{\tilde{m}_k}, \sqrt[3]{1 - \left(1 - \eta^3_{\tilde{m}_k}\right)^{\alpha}},\right)$

Based on the foregoing rules for the collection of FFHSNs, certain averaging and geometric aggregation operations for FFHSSs have been established.

Definition 4.12. *Let $T_{\tilde{d}_{ij}} = (\zeta_{\tilde{m}_{ij}}, \eta_{\tilde{m}_{ij}})$ be a FFHSN, Ω_i and γ_j be the weighted vectors with $\Omega_i > 0$, $\sum_{i=1}^n \Omega_i = 1$, $\gamma_j > 0$, and $\sum_{i=1}^n \gamma_i = 1$.Hence, FFHS weighted averaging operator is characterized by $FFHSWA : \Delta^n \to \Delta$, which is given as follows:*

$$\begin{aligned} FFHSWA(T_{\tilde{m}_{11}}, T_{\tilde{m}_{12}}, \cdots, T_{\tilde{m}_{nm}}) &= \boxplus_{j=1}^m \gamma_j \left(\boxplus_{i=1}^n \Omega_i T_{\tilde{m}_{ij}}\right) \\ &= \left(\sqrt[3]{1 - \prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - \zeta^3_{\tilde{m}_{ij}}\right)^{\Omega_j}\right)^{\gamma_j}}, \prod_{j=1}^m \left(\prod_{i=1}^n \left(\eta_{\tilde{m}_{ij}}\right)^{\Omega_j}\right)^{\gamma_j}\right). \end{aligned}$$

Definition 4.13. *Let $T_{\tilde{d}_{ij}} = (\zeta_{\tilde{d}_{ij}}, \eta_{\tilde{d}_{ij}})$ be a FFHSN, Ω_i and γ_j be the weighted vectors with $\Omega_i > 0$, $\sum_{i=1}^n \Omega_i = 1$, $\gamma_j > 0$, and $\sum_{i=1}^n \gamma_i = 1$. Then, FFHS weighted geometric operator is defined as $FFHSWG : \Delta^n \to \Delta$, which is defined as follows:*

$$\begin{aligned} FFHSWG(T_{\tilde{m}_{11}}, T_{\tilde{m}_{12}}, \cdots, T_{\tilde{m}_{nm}}) &= \boxtimes_{j=1}^m \gamma_j \left(\boxtimes_{i=1}^n \Omega_i T_{\tilde{m}_{ij}}\right) \\ &= \left(\prod_{j=1}^m \left(\prod_{i=1}^n \left(\zeta_{\tilde{m}_{ij}}\right)^{\Omega_j}\right)^{\gamma_j}, \sqrt[3]{1 - \prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - \eta^3_{\tilde{m}_{ij}}\right)^{\Omega_j}\right)^{\gamma_j}}\right) \end{aligned}$$

5 New Approach

In this part, we will continue the TOPSIS approach for FFHSS information based on KKs to construct a framework for solving DM concerns. The TOPSIS approach was created and applied by Yoon Hwang [48] to promote the order of assessment components of positive(PIS) and negative ideal solutions(NIS) for DM challenges. Using the TOPSIS technique, we will be able to find the best potential options with the shortest and biggest distances to the PIS and NIS, respectively. By using rankings, the TOPSIS approach assures that the correlation measure can discriminate between positive and negative ideals. In general, researchers use the TOPSIS approach to determine proximity coefficients, unique distance forms, and similar measurements. By using rankings, the TOPSIS approach assures that the correlation measure may be utilized to discriminate between positive and negative ideals. In general, researchers use the TOPSIS approach to determine proximity coefficients, unique distance forms, and similar measurements. Because the correlation measure preserves the linear relationship between the elements investigated, the TOPSIS approach using KKs is preferable for identifying closeness coefficients rather than distance and similarity measurements. A

TOPSIS approach for selecting the best choice is shown by leveraging the created KK.

Scenario:
Let's consider a set of "s" alternatives such as $A = \{A^1, A^2, \cdots, A^s\}$ for the evaluation of the $E = \{E_1, E_2, \cdots, E_n\}$ expert team with $\Omega = (\Omega_1, \Omega_2, \cdots, \Omega_n)^T$ weights providing $\Omega_i > 0$, $\sum_{i=1}^{n} \Omega_i = 1$. Let $D = \{d_1, d_2, \cdots, d_m\}$ specified as a set of attributes. Let $T = \{(t_{1p} \times t_{2p} \times \cdots \times d_{mp})$, for all $p \in \{1, 2, \cdots, t\}\}$ be a collection of sub-attirbutes with $\gamma = (\gamma_{1p}, \gamma_{2p}, \cdots, \gamma_{mp})^T$ weights satisfying the conditions $\sum = 1\ \gamma_p > 0$, $\sum_{p=1}^{t} \gamma_p = 1$. The elements in the collection of sub-attributes are multi-valued; for the sake of convenience, the elements of T can be expressed as $T = \{\tilde{d}_\partial : \partial \in \{1, 2, cdots, k\}\}$. The team of experts $\{E_i : i, 1, 2, \cdots, n\}$ evaluate the alternatives $\{A^{(z)} : z = 1, 2, \cdots, s\}$ based on the desired sub-attributes of the considered parameters $\{\tilde{d}_\partial : \partial = 1, 2, \cdots, k\}$ given in the form of FFHSNs such as $(T^{(z)}_{\tilde{d}_\partial})_{n\times\partial} = (\zeta^{(z)}_{\tilde{d}\partial}, \eta^{(z)}_{\tilde{d}\partial})_{n\times\partial}$, where $0 \le \zeta^{(z)}_{\tilde{d}_{ij}}, \eta^{(z)}_{\tilde{d}_{ij}} \le 1$ and $\zeta^{(z)}_{\tilde{d}^3_{ij}} + \eta^{(z)}_{\tilde{d}^3_{ij}} \le 1$ for all i, j.

Algorithm:

Step 1. Create a matrix in the form of FFHSNs for each alternative $\{A^{(z)} : z = 1, 2, \cdots, s\}$ by utilizing sub-attributes of the supplied attributes such as:

$$(A^{(z)}, T)_{n\times\delta} = \begin{pmatrix} (\zeta^{(z)}_{\tilde{d}_{11}}, \eta^{(z)}_{\tilde{d}_{11}}) & (\zeta^{(z)}_{\tilde{d}_{12}}, \eta^{(z)}_{\tilde{d}_{12}}) & \cdots & \zeta^{(z)}_{\tilde{d}_{1\partial}}, \eta^{(z)}_{\tilde{d}_{1\partial}} \\ (\zeta^{(z)}_{\tilde{d}_{21}}, \eta^{(z)}_{\tilde{d}_{21}}) & (\zeta^{(z)}_{\tilde{d}_{22}}, \eta^{(z)}_{\tilde{d}_{22}}) & \cdots & \zeta^{(z)}_{\tilde{d}_{2\partial}}, \eta^{(z)}_{\tilde{d}_{2\partial}} \\ \vdots & \vdots & \vdots & \vdots \\ (\zeta^{(z)}_{\tilde{d}_{11}}, \eta^{(z)}_{\tilde{d}_{11}}) & (\zeta^{(z)}_{\tilde{d}_{n1}}, \eta^{(z)}_{\tilde{d}_{n1}}) & \cdots & \zeta^{(z)}_{\tilde{d}_{n\partial}}, \eta^{(z)}_{\tilde{d}_{n\partial}} \end{pmatrix} \tag{27}$$

Step 2: Normalize the collective information decision matrix by using the normalization procedure to turn the rating value of the cost-type parameters into benefit-type parameters:

$$h_{ij} = \begin{cases} T^{c}_{\tilde{d}_{ij}} = \left(\eta^{(z)}_{\tilde{d}_{ij}}, \zeta^{(z)}_{\tilde{d}_{ij}}\right), & \textit{for cost-type parameter} \\ T_{\tilde{d}_{ij}} = \left(\zeta^{(z)}_{\tilde{d}_{ij}}, \eta^{(z)}_{\tilde{d}_{ij}}\right), & \textit{for benefit-type parameter} \end{cases}$$

Step 3: Construct the weighted decision matirx for each alternative $(\overline{A}^{(z)}) = (\overline{T}^{(z)}_{\tilde{d}_{ij}})_{n\times\partial}$, where

$$\overline{T}^{(z)}_{\tilde{d}_{ij}} \;=\; \gamma_j \Omega_i T^{(z)}_{\tilde{d}_{ij}} = \left(\sqrt[3]{1 - \left(\left(1 - \zeta^{3}_{\tilde{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j}}, \left(\left(\eta_{\tilde{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j} \right) = \left(\overline{\zeta}^{(z)}_{\tilde{d}_{ij}}, \overline{\eta}^{(z)}_{\tilde{d}_{ij}}\right) \tag{28}$$

Step 4: Find the indices $h_{ij} = argmax_z\{\theta^{(z)}_{ij}\}$ and $g_{ij} = argmin_z\{\theta^{(z)}_{ij}\}$ for each expert E_i and sub-attribute $\tilde{d}_j$ from correlation coefficient matrices and determine the PIA and NIA based on indices as follows:

$$L^+ = (\zeta^+_{\tilde{d}_{ij}}, \eta^+_{\tilde{d}_{ij}})_{n\times\partial} = (\overline{\zeta}^{h_{ij}}_{\tilde{d}_{ij}}, \overline{\eta}^{h_{ij}}_{\tilde{d}_{ij}}) \tag{29}$$

$$L^- = (\zeta^-_{\tilde{d}_{ij}}, \eta^-_{\tilde{d}_{ij}})_{n\times\partial} = (\overline{\zeta}^{g_{ij}}_{\tilde{d}_{ij}}, \overline{\eta}^{g_{ij}}_{\tilde{d}_{ij}}) \tag{30}$$

Step 5: Calculate the KK between each alternative of weighted decision matrices $\overline{A}^{(z)}$ and PIA L^+:

$$p^{(z)} = CC(\overline{A}^{(z)}, L^+) \tag{31}$$
$$= \frac{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\zeta_{\tilde{d}_{ij}}^{(z)} * \zeta_{\tilde{d}_{ij}}^{+} + \eta_{\tilde{d}_{ij}}^{(z)} * \eta_{\tilde{d}_{ij}}^{+}\right)}{\sqrt[3]{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\left(\zeta_{\tilde{d}_{ij}}^{(z)}\right)^3 + \left(\eta_{\tilde{d}_{ij}}^{(z)}\right)^3\right)} \cdot \sqrt[3]{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\left(\zeta_{\tilde{d}_{ij}}^{+}\right)^3 + \left(\eta_{\tilde{d}_{ij}}^{+}\right)^3\right)}}$$

Step 6: Compute the correlation coefficient between each alternative of weighted decision matrices $\overline{A}^{(z)}$ and NIA L^-:

$$q^{(z)} = CC(\overline{A}^{(z)}, L^-) \tag{32}$$
$$= \frac{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\zeta_{\tilde{d}_{ij}}^{(z)} * \zeta_{\tilde{d}_{ij}}^{-} + \eta_{\tilde{d}_{ij}}^{(z)} * \eta_{\tilde{d}_{ij}}^{-}\right)}{\sqrt[3]{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\left(\zeta_{\tilde{d}_{ij}}^{(z)}\right)^3 + \left(\eta_{\tilde{d}_{ij}}^{(z)}\right)^3\right)} \cdot \sqrt[3]{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\left(\zeta_{\tilde{d}_{ij}}^{-}\right)^3 + \left(\eta_{\tilde{d}_{ij}}^{-}\right)^3\right)}}$$

Step 7: Calculate the closeness coefficient for each alternative:

$$R^{(z)} = \frac{H(\overline{A}^{(z)}, L^-)}{H(\overline{A}^{(z)}, L^+) + H(\overline{A}^{(z)}, L^-)} \tag{33}$$

where

$$H(\overline{A}^{(z)}, L^-) = 1 - q^{(z)},$$
$$H(\overline{A}^{(z)}, L^+) = 1 - p^{(z)}.$$

Step 8: Select the alternative with a maximum value of the closeness coefficient.

Step 9: Analyze the ranking of the alternatives.

6 Numerical Examples

This section has been written to verify the suggested approach. The material for the femoral component of the hip joint prosthesis will be chosen following the procedure described in the preceding section. There are several approaches in the literature for selecting this biomedical material.

The femoral head and the acetabulum are the two main components of the hip joint, which is a crucial load-bearing joint in the human body. The femoral head is what attaches to the acetabulum, a socket in the pelvic bone, to create the hip joint. The femoral head can move inside and outward, anterior and backward, and in circles within the acetabulum (Fig. 4). The hip joint's primary function is load carrying, therefore it has to have a suitable range of motion and stability. Hip arthroplasty is a surgical treatment used to replace or rejuvenate a hip joint that has been badly damaged or calcified (osteoarthritis).

The best course of treatment for significant discomfort, mobility restrictions, and shortness that interfere with everyday activities is hip replacement. The success of applying hip

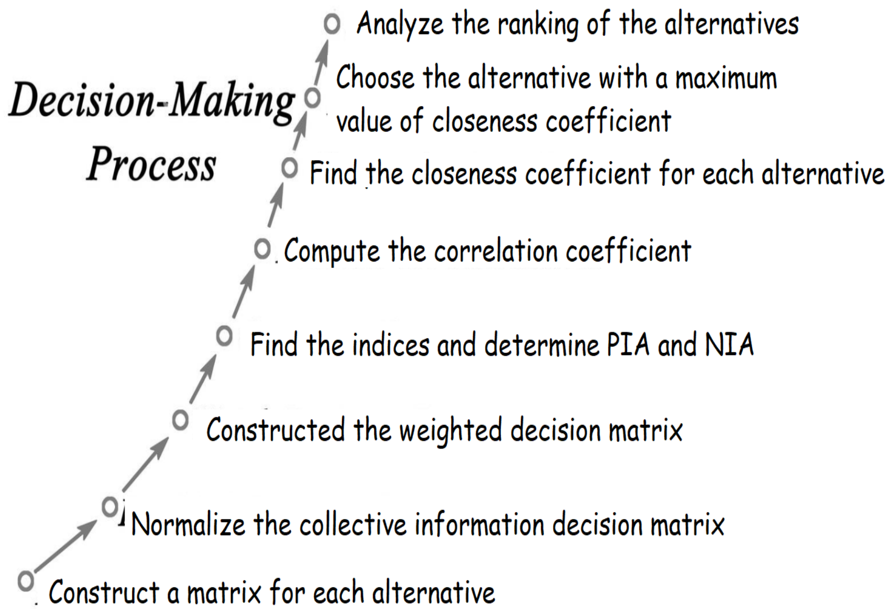

Figure 2: Decision-making process

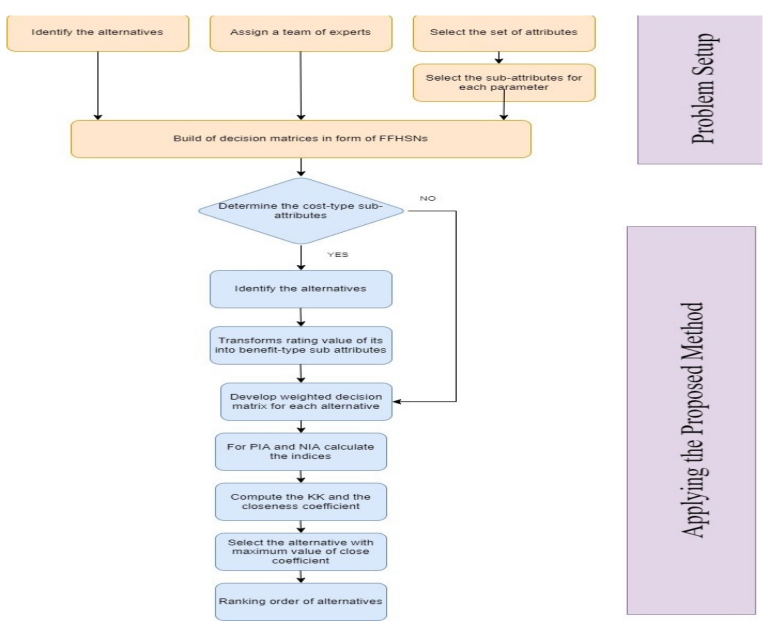

Figure 3: Flow chart

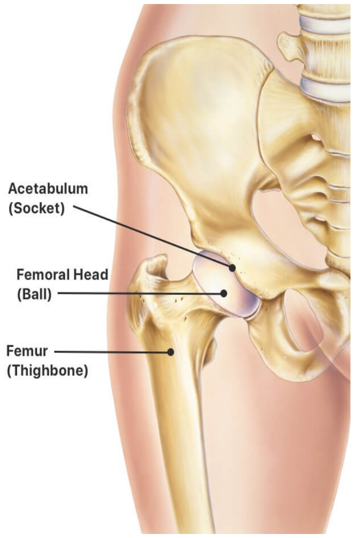

Figure 4: Healthy Hip [29]

prostheses is increased by using implants with the right materials and design elements. Numerous prostheses have been created in the present day with various construction materials and characteristics. The prosthesis should be easy to manufacture, affordable, dependable, and long-lasting thanks to the design and material characteristics of the implant that has been chosen. The choice of materials for hip replacement prostheses is problematic since the design calls for a number of distinctive key qualities that are challenging to develop in a single material.

An ideal biomaterial is expected to have extremely high biocompatibility, meaning there won't be any negative tissue reactions when it's implanted within the human body. In addition, it must have bone-like density, high mechanical power, low elastic modulus, and good corrosion and fatigue resistance.

The femoral component, the acetabular cup, and the acetabular interface are the three primary components of a hip prosthesis. The femoral component, the acetabular cup, and the acetabular interface are the three primary components of a hip prosthesis. To replace the native femoral head, a robust metal pin is placed into the hollowed-out shaft of the femur. To replace the native femoral head, a robust metal pin is placed into the hollowed-out shaft of the femur. The acetabulum is filled by an acetabular cup, a soft polymer molding linked to the ilium. The acetabular interface is positioned between the femoral component and the acetabular cup and can be made of a variety of materials to reduce frictional wear debris. The hip joint prosthesis is shown in Fig. 5 in its normal shape and location.

Various stresses and effects are applied to biomaterials in various human body components. For instance, daily activities exert strains of 4 MPa on bones and 40–80 MPa on tendons. When doing actions like jumping, the strain on a hip joint can increase to up to ten times its weight from the typical three times. Throughout the course of the day, actions including standing, jogging and sitting cause these stressors in the body repeatedly. Biomaterials may become worn down, break, or deform plastically due to these repeated motions.

It has been experimentally measured that the femoral head is loaded up to 3.5 times the

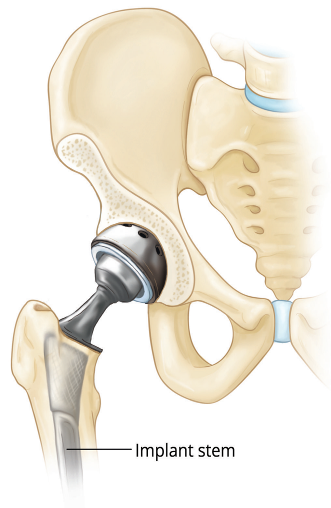

Figure 5: Hip joint prosthesis [30]

body weight (80 kg body weight) during walking, showing that total hip prostheses must be sufficiently resistant to these loads. At the same time, these prostheses must be resistant to abrasion caused by friction in the joint. Today, in a hip prosthesis; Vitallium (Co-Cr-Mo alloy), stainless steel, high-density polyethylene, polymethylmethacrylate, and titanium and titanium alloys are used.

Let's assume that there are four orthopedists in the group that will decide on the selection of biomaterials($O = \{O_1, O_2, O_3, O_4\}$).

Orthopedists with weights $(0.2, 0.3, 0.4, 0.1)^T$, including $A = \{A_1, A_2, A_3, A_4\}$, (vitallium(Co-Cr-Mo), stainless steel, high-density polyethylene, polymethylmethacrylate, titanium and titanium alloys) the set of biomedical materials, evaluate the grades of these material types. The group of orthopedists decides the criteria(attirbute) for the choice of biomedical materials as: $L = \{\ell_1, \ell_2, \ell_3\}$ (strength, resistance, tolerance). The sub-attributes corresponding to these attributes are: $\ell_1 = \{d_{11}, d_{12}\}$ (Tensile strength, Fatigue strength), $\ell_2 = \{d_{21}, d_{22}\}$ (corrosion resistance, relative wear resistance), $\ell_3 = \{d_{31}, d_{32}\}$ (Tissue tolerance, elasticity).

Let $T^{'} = \ell_1 \times \ell_2 \times \ell_3$ be a set of sub-attributes:

$$
\begin{aligned}
T^{'} &= \ell_1 \times \ell_2 \times \ell_3 = \{d_{11}, d_{12}\} \times \{d_{21}, d_{22}\} \times \{d_{31}, d_{32}\} \\
&= \Big\{ (d_{11}, d_{21}, d_{31}), (d_{11}, d_{21}, d_{32}), (d_{11}, d_{22}, d_{31}), (d_{11}, d_{22}, d_{32}), \\
&(d_{12}, d_{21}, d_{31}), (d_{12}, d_{21}, d_{32}), (d_{12}, d_{22}, d_{31}), (d_{12}, d_{22}, d_{32}) \Big\}
\end{aligned}
$$

Let $T^{'} = \{\tilde{d}_1, \tilde{d}_2, \tilde{d}_3, \tilde{d}_4, \tilde{d}_5, \tilde{d}_6, \tilde{d}_7, \tilde{d}_8\}$ be a set of all multi-sub-attributes with weights $(0.12, 0.18, 0.1, 0.15, 0.05, 0.22, 0.08)^T$. Each orthopedist will evaluate the ratings of biomedical materials in the form of FFHSNs for each sub-attribute of the considered parameters(Tables 1-4). The developed method to find the best alternative is as follows:

Step 1: Create decision matrices for each alternative under defined multi-sub-attributes based on each decision-FFHSN maker's rating.

Step 2: Because all of the criteria are of beneficial kinds, they must be normalized.

Step 3: Using Equation 28 from Tables 5-8, create a weighted decision matrix for each alternative $\overline{A}^{(z)} = (\overline{L}_{ij}^{(z)})_{n\times\partial}$.

Table 1: Decision Matrix for $A^{(1)}$

$A^{(1)}$	$\tilde{d}_1$	$\tilde{d}_2$	$\tilde{d}_3$	$\tilde{d}_4$	$\tilde{d}_5$	$\tilde{d}_6$	$\tilde{d}_7$	$\tilde{d}_8$
O_1	(0.33, 0.75)	(0.72, 0.29)	(0.64, 0.75)	(0.59, 0.32)	(0.25, 0.35)	(0.46, 0.54)	(0.52, 0.76)	(0.88, 0.32)
O_2	(0.59, 0.73)	(0.37, 0.64)	(0.25, 0.49)	(0.92, 0.27)	(0.34, 0.82)	(0.27, 0.46)	(0.73, 0.55)	(0.44, 0.65)
O_3	(0.75, 0.35)	(0.28, 0.61)	(0.17, 0.66)	(0.34, 0.48)	(0.46, 0.69)	(0.87, 0.47)	(0.61, 0.78)	(0.28, 0.53)
O_4	(0.83, 0.45)	(0.29, 0.84)	(0.25, 0.47)	(0.46, 0.68)	(0.63, 0.58)	(0.55, 0.69)	(0.47, 0.56)	(0.83, 0.37)

Table 2: Decision Matrix for $A^{(2)}$

$A^{(2)}$	$\tilde{d}_1$	$\tilde{d}_2$	$\tilde{d}_3$	$\tilde{d}_4$	$\tilde{d}_5$	$\tilde{d}_6$	$\tilde{d}_7$	$\tilde{d}_8$
O_1	(0.69, 0.64)	(0.32, 0.46)	(0.67, 0.55)	(0.33, 0.82)	(0.58, 0.52)	(0.46, 0.62)	(0.73, 0.61)	(0.47, 0.81)
O_2	(0.83, 0.55)	(0.73, 0.46)	(0.93, 0.19)	(0.75, 0.33)	(0.43, 0.65)	(0.95, 0.16)	(0.28, 0.67)	(0.31, 0.82)
O_3	(0.36, 0.72)	(0.48, 0.57)	(0.42, 0.78)	(0.35, 0.65)	(0.62, 0.65)	(0.37, 0.57)	(0.91, 0.18)	(0.68, 0.23)
O_4	(0.53, 0.54)	(0.69, 0.58)	(0.87, 0.25)	(0.86, 0.48)	(0.93, 0.25)	(0.27, 0.44)	(0.53, 0.66)	(0.67, 0.35)

Table 3: Decision Matrix for $A^{(3)}$

$A^{(3)}$	$\tilde{d}_1$	$\tilde{d}_2$	$\tilde{d}_3$	$\tilde{d}_4$	$\tilde{d}_5$	$\tilde{d}_6$	$\tilde{d}_7$	$\tilde{d}_8$
O_1	(0.51, 0.71)	(0.83, 0.46)	(0.68, 0.45)	(0.46, 0.36)	(0.48, 0.82)	(0.28, 0.38)	(0.83, 0.44)	(0.75, 0.54)
O_2	(0.79, 0.52)	(0.77, 0.34)	(0.86, 0.22)	(0.55, 0.24)	(0.53, 0.68)	(0.72, 0.46)	(0.66, 0.59)	(0.61, 0.42)
O_3	(0.63, 0.78)	(0.45, 0.56)	(0.61, 0.63)	(0.65, 0.48)	(0.69, 0.54)	(0.77, 0.46)	(0.54, 0.77)	(0.46, 0.52)
O_4	(0.48, 0.67)	(0.89, 0.31)	(0.35, 0.54)	(0.56, 0.68)	(0.36, 0.55)	(0.77, 0.51)	(0.77, 0.39)	(0.34, 0.38)

Table 4: Decision Matrix for $A^{(4)}$

$A^{(4)}$	$\tilde{d}_1$	$\tilde{d}_2$	$\tilde{d}_3$	$\tilde{d}_4$	$\tilde{d}_5$	$\tilde{d}_6$	$\tilde{d}_7$	$\tilde{d}_8$
O_1	(0.50, 0.69)	(0.79, 0.54)	(0.76, 0.45)	(0.43, 0.36)	(0.48, 0.92)	(0.26, 0.46)	(0.81, 0.44)	(0.72, 0.52)
O_2	(0.77, 0.45)	(0.63, 0.48)	(0.83, 0.55)	(0.57, 0.26)	(0.53, 0.69)	(0.70, 0.46)	(0.69, 0.63)	(0.61, 0.45)
O_3	(0.56, 0.45)	(0.44, 0.77)	(0.52, 0.67)	(0.35, 0.49)	(0.72, 0.61)	(0.68, 0.55)	(0.41, 0.88)	(0.58, 0.18)
O_4	(0.43, 0.67)	(0.18, 0.34)	(0.74, 0.45)	(0.56, 0.78)	(0.33, 0.54)	(0.77, 0.34)	(0.33, 0.58)	(0.27, 0.55)

Step 4: Determine the PIA and NIA based on indices by using Equations 29 and 30:

$$L^+ = \begin{Bmatrix} (0.5987, 0.8799), & (0.5295, 0.9167), & (0.6019, 0.9426), & (0.6249, 0.9784), & (0.8194, 0.9704), \\ (0.4874, 0.9678), & (0.7787, 0.9489), & (0.6219, 0.9572) & (0.7152, 0.9726), & (0.5909, 0.9468), \\ (0.6875, 0.9436), & (0.7567, 0.9289), & (0.8977, 0.9710), & (0.6459, 0.9533), & (0.7965, 0.9899), \\ (0.6948, 0.9903), & (0.7345, 0.9428), & (0.6122, 0.9313), & (0.7170, 0.9698), & (0.8427, 0.9583), \\ (0.4957, 0.9115), & (0.7639, 0.9388), & (0.7709, 0.9617), & (0.4783, 0.8874) & (0.6523, 0.9905), \\ (0.4520, 0.9867), & (0.6987, 0.9861), & (0.7468, 0.9556), & (0.8575, 0.9914), & (0.4431, 0.9876), \\ (0.7784, 0.9961), & (0.6447, 0.9808) & & & \end{Bmatrix}$$

$$L^- = \begin{Bmatrix} (0.7465, 0.9753), & (0.6427, 0.9645), & (0.7802, 0.9726), & (0.6937, 0.9476), & (0.8895, 0.9786), \\ (0.5778, 0.9629), & (0.8219, 0.9648), & (0.7725, 0.7762) & (0.7359, 0.9576), & (0.6968, 0.9681), \\ (0.8964, 0.9123), & (0.8946, 0.9133), & (0.8895, 0.9269), & (0.8827, 0.9687), & (0.4462, 0.9036), \\ (0.8471, 0.9445), & (0.8578, 0.9075), & (0.7197, 0.9313), & (0.8892, 0.9426), & (0.8167, 0.9365), \\ (0.8026, 0.9743), & (0.6812, 0.9416), & (0.8547, 0.9702), & (0.8967, 0.9874) & (0.9053, 0.9315), \\ (0.7285, 0.9862), & (0.5028, 0.9748), & (0.7438, 0.9859), & (0.6618, 0.9772), & (0.7614, 0.9839), \\ (0.6575, 0.9576), & (0.7855, 0.9822) & & & \end{Bmatrix}$$

Step 5: Compute the KK between $\overline{A}^{(z)}$ and PIA L^+ by using Equation 31, given $p^{(1)} = 0.97724$, $p^{(2)} = 0.96823$, $p^{(3)} = 0.96182$, $p^{(4)} = 0.96221$.

Step 6: Compute the KK between $\overline{A}^{(z)}$ and NIA L^- by using Equation 31, given $q^{(1)} = 0.95651$, $q^{(2)} = 0.96107$, $q^{(3)} = 0.96463$, $q^{(4)} = 0.96501$.

Table 5: Weighted Decision Matrix for $\overline{A}^{(1)}$

$\overline{A}^{(1)}$	$\tilde{d}_1$	$\tilde{d}_2$	$\tilde{d}_3$	$\tilde{d}_4$	$\tilde{d}_5$	$\tilde{d}_6$	$\tilde{d}_7$	$\tilde{d}_8$
O_1	(0.716, 0.983)	(0.548, 0.957)	(0.6681, 0.9852)	(0.664, 0.948)	(0.815, 0.909)	(0.502, 0.987)	(0.8381, 0.9743)	(0.6719, 0.9726)
O_2	(0.7137, 0.9782)	(0.7308, 0.9827)	(0.7895, 0.9927)	(0.9190, 0.9311)	(0.8704, 0.9617)	(0.5674, 0.9314)	(0.8312, 0.9735)	(0.8272, 0.9849)
O_3	(0.7534, 0.9384)	(0.6322, 0.9412)	(0.7362, 0.9648)	(0.7469, 0.9345)	(0.9019, 0.9768)	(0.4868, 0.9258)	(0.8132, 0.9592)	(0.7791, 0.9647)
O_4	(0.6382, 0.9817)	(0.5114, 0.9696)	(0.6835, 0.9901)	(0.6355, 0.9793)	(0.8547, 0.9695)	(0.4874, 0.9873)	(0.7686, 0.9864)	(0.6434, 0.9879)

Table 6: Weighted Decision Matrix for $\overline{A}^{(2)}$

$\overline{A}^{(2)}$	$\tilde{d}_1$	$\tilde{d}_2$	$\tilde{d}_3$	$\tilde{d}_4$	$\tilde{d}_5$	$\tilde{d}_6$	$\tilde{d}_7$	$\tilde{d}_8$
O_1	(0.6786, 0.9789)	(0.6278, 0.9786)	(0.7891, 0.9794)	(0.6749, 0.9746)	(0.9011, 0.9885)	(0.5924, 0.9678)	(0.8137, 0.9768)	(0.7892, 0.9825)
O_2	(0.7609, 0.9742)	(0.6069, 0.9493)	(0.7108, 0.9504)	(0.9111, 0.9294)	(0.4599, 0.9206)	(0.8029, 0.9879)	(0.7046, 0.9879)	(0.6902, 0.9731)
O_3	(0.7912, 0.9495)	(0.7408, 0.9518)	(0.7785, 0.9902)	(0.7315, 0.9414)	(0.6377, 0.9189)	(0.8824, 0.9726)	(0.7686, 0.9392)	(0.9104, 0.9388)
O_4	(0.7316, 0.9819)	(0.4389, 0.9839)	(0.5588, 0.9812)	(0.6792, 0.9789)	(0.9899, 0.9891)	(0.4408, 0.9769)	(0.7613, 0.9913)	(0.6706, 0.4335)

Table 7: Weighted Decision Matrix for $\overline{A}^{(3)}$

$\overline{A}^{(3)}$	$\tilde{d}_1$	$\tilde{d}_2$	$\tilde{d}_3$	$\tilde{d}_4$	$\tilde{d}_5$	$\tilde{d}_6$	$\tilde{d}_7$	$\tilde{d}_8$
O_1	(0.7499, 0.9814)	(0.6099, 0.9674)	(0.7902, 0.9826)	(0.6708, 0.9730)	(0.8515, 0.9901)	(0.5978, 0.9708)	(0.8112, 0.9845)	(0.7625, 0.9826)
O_2	(0.7613, 0.9687)	(0.6095, 0.9491)	(0.7106, 0.9469)	(0.8984, 0.9903)	(0.4678, 0.9263)	(0.6102, 0.9671)	(0.8107, 0.9905)	(0.7106, 0.9866)
O_3	(0.8687, 0.9175)	(0.7005, 0.9326)	(0.9116, 0.9592)	(0.7407, 0.9510)	(0.8606, 0.9718)	(0.6909, 0.9418)	(0.8489, 0.9718)	(0.8868, 0.9488)
O_4	(0.7316, 0.9901)	(0.5143, 0.9699)	(0.7481, 0.9904)	(0.6105, 0.9830)	(0.8519, 0.9924)	(0.6102, 0.9805)	(0.7843, 0.9827)	(0.7612, 0.9804)

Step 7: Compute the closeness coefficient by using Equation 33, $R^{(1)} = 0.77498$, $R^{(2)} = 0.55478$, $R^{(3)} = 0.43854$, $R^{(4)} = 0.47487$.

Step 8: Choose the alternative with maximum closeness coefficient $R^{(1)} = 0.77498$, so $A^{(1)}$ is the best alternative.

Table 8: Weighted Decision Matrix for $\overline{A}^{(4)}$

$\overline{A}^{(4)}$	$\tilde{d}_1$	$\tilde{d}_2$	$\tilde{d}_3$	$\tilde{d}_4$	$\tilde{d}_5$	$\tilde{d}_6$	$\tilde{d}_7$	$\tilde{d}_8$
O_1	(0.6905, 0.9905)	(0.6702, 0.9547)	(0.7131, 0.9804)	(0.7104, 0.9624)	(0.8124, 0.9790)	(0.5108, 0.9615)	(0.8107, 0.9784)	(0.7291, 0.9795)
O_2	(0.7608, 0.9699)	(0.6127, 0.9526)	(0.9103, 0.9343)	(0.7801, 0.9291)	(0.9114, 0.9786)	(0.6494, 0.9535)	(0.9116, 0.9596)	(0.8617, 0.9545)
O_3	(0.8716, 0.9398)	(0.7409, 0.9467)	(0.8905, 0.9299)	(0.8497, 0.9638)	(0.6501, 0.9465)	(0.8620, 0.9704)	(0.9007, 0.9612)	(0.8514, 0.9772)
O_4	(0.7313, 0.9904)	(0.4424, 0.9718)	(0.7216, 0.9913)	(0.6721, 0.9812)	(0.8520, 0.9956)	(0.6605, 0.9560)	(0.7684, 0.9954)	(0.6909, 0.9912)

Step 9: Analyzing the ranking of the alternatives, we can see $R^{(1)} > R^{(2)} > R^{(4)} > R^{(3)}$, so the ranking of the alternatives is $\overline{A}^{(1)} > \overline{A}^{(2)} > \overline{A}^{(4)} > \overline{A}^{(3)}$.

7 Discussion

7.1 Comparison

We may use Zadeh's [51] approach to process the MD of the attributes, however, this method cannot handle the NMD and sub-attributes of the considered parameters. Zhang et al. [52] employed MD and NMD to deal with uncertainty, although these theories have limitations, such as when the sum of MD and NMD exceeds one, these theories are unable to handle the situation. Yager [46] developed the PFS to solve these concerns, but it is incapable of dealing with the parametric values of the alternatives. The FSS was created by Maji et al. [26] to handle various parameterizations.

The FSS is unaware of the NMD of the alternative's qualities and is only concerned with the MD of the attributes. In contrast, the MD and NMD are used in our proposed PFHSS to control uncertainty. Maji et al. [27] proposed using MD and NMD of features with parameterization to account for uncertainty in the IFSS. The IFSS cannot deal with problems when the sum of MD and NMD is more than one. In contrast to the IFSS, Peng et al. [31] developed the PFSS to effectively manage ambiguity. All of the aforementioned theories fail to manage situations where qualities contain sub-attributes.

The FSS does not know the NMD of the characteristics of the alternative, and it solely deals with the MD of the attributes. In contrast, our proposed PFHSS manages uncertainty by employing the MD and NMD. Maji et al. [27] suggested the IFSS account for uncertainty by employing MD and NMD of characteristics with parameterization. The IFSS is unable to tackle issues where the total of MD and NMD is more than one. In comparison to IFSS, Peng et al. [31] created the PFSS to handle ambiguity properly. When qualities contain sub-attributes, all of the aforementioned theories fail to manage the circumstance. Zulqarnain et al.[54] used the IFHSS environment to propose the TOPSIS technique for dealing with uncertain scenarios using MD and NMD in which the total of the researched parameters' sub-attributes cannot exceed one. When the sum of MD and NMD of sub-attributes exceeds one, as in $\zeta_{F(\tilde{m})}(\lambda) + \eta_{F(\tilde{m})}(\lambda) \geq 1$ IFHSS cannot handle the situation. To get over the above-mentioned limits, we transformed the IFHSS to PFHSS by modifying the condition $\zeta_{F(\tilde{m})}(\lambda) + \eta_{F(\tilde{m})}(\lambda) \leq 1$ to $\zeta_{F(\tilde{m})}(\lambda)^2 + \eta_{F(\tilde{m})}(\lambda)^2 \leq 1$. Instead, as demonstrated in Table 51, the technique we proposed is a complex strategy that can cope with alternatives with varying sub-attribute information. On the contrary, the developed method in this work addresses uncertainty by employing the MD and NMD of various sub-attributes. As a result, the suggested approach outperforms existing methodologies and, without a doubt, generates superior results for decision-makers during the decision-making process.

According to the information given in Table 9, we can briefly explain the advantages of the methods:

FS [51] handles uncertainty using fuzzy intervals. IFS[52] handles uncertainty using MD and NMD. PFS([46]) deals with more uncertainty using MD and NMD than IFS. FSS[26]

Table 9: Comparative Analysis

Set	Truthiness	Falsity	Attributes	Sub-attributes
FS [51]	YES	NO	YES	NO
IFS[52]	YES	YES	YES	NO
PFS([46])	YES	YES	YES	NO
FSS[26]	YES	NO	YES	NO
IFSS[27]	YES	YES	YES	NO
PFSS[31]	YES	YES	YES	NO
IFHSS[54]	YES	YES	YES	YES
PFHSS[55]	YES	YES	YES	YES
FFHSS(Our method)	YES	YES	YES	YES

handle uncertainty using fuzzy range with their parameterization. IFSS[27] deals with more uncertainty using MD and NMD with their parameterizations. PFSS [31] deals with more uncertainty than IFSS. IFHSS[54] handles the uncertainty of multiple sub-attributes using MD and NMD. PFHSS[55] deals with more uncertainty than IFHSS. FFHSS, by definition, deals with more uncertainty than PFHSS and IFHSS.

7.2 Superiority of the new approach

The FFS is a generalization of the classical set, which consists of the FS, IFS, and PFS. PFS, as one of the most common extensions, is differentiated by the degrees of membership and nonmembership satisfaction with regard to the criteria, where their total squares are equal to or less than 1. In some cases, the decision-maker may offer the degree of membership and nonmembership of a specific feature in such a way that the sum of the square is greater than 1. As a result, the PFS does not handle this instance effectively. To address this limitation, FFS theory is one of the more extensive ones that can handle not just partial information but also uncertain information and inconsistent information, both of which are commonly present in practical settings. As a result, actual scientific and technical applications are better suited for decision-making using Fermatean fuzzy information.

Making judgments regarding real-world situations, or finding solutions to them, is challenging and complex. Therefore, it is crucial to reduce uncertainty while making a decision in order to select the best option or alternatives. The connections between the inputs must also be handled effectively for decision-making to be most beneficial. We put out a fresh approach that combines the benefits of FFS and KK to demonstrate the presence of the association between the variables.

A combination of interval-valued fermatean fuzzy sets with Fermatean hesitant fuzzy components in the form of interval values is known as an interval-valued fermatean hesitant set. Since Fermatean hesitant fuzzy sets are effective instruments for representing more complex, ambiguous, and hazy information, interval-valued Fermatean hesitant fuzzy sets are expansions of these sets. IVFHFS is explained and used in the process of group decision-making. Since membership and non-membership are taken into account concurrently, and IVFHFS offers a greater feasible zone between 0 and 1 when decision-makers express their agreement and disagreement, it can provide decision-makers more freedom in analyzing unclear and fuzzy information.

A number of academics have developed an approach that uses the KK for PFSs, as seen from the previous studies. As was already established, some scenarios cannot be captured by PFSs, hence the related algorithm may not produce the desired outcomes.

A specific case of the KK for FFSs is the KK for PFSs. The suggested KK is therefore more applicable to a wider range of situations than the current ones, making it better suited to real-world problem solving.

We concluded from this research and comparative analysis that the outcomes of our method are more universal than those of existing procedures. However, when compared to previous decision-making methodologies, the decision-making process incorporates more information to deal with data ambiguity. Furthermore, many FS hybrid structures have evolved into FFHSS special instances, and certain acceptable criteria have been added. Among these, object-related information may be conveyed more correctly and experimentally, making it a useful tool for combining erroneous and ambiguous information in the decision-making process. As a result, our suggested solution is more effective, versatile, simple, and superior to previous hybrid fuzzy set architectures.

8 Conclusion

The concept of FFHSS is used to handle problems with insufficient information, ambiguity, and inconsistency by taking the MD and ND of the subattributes of the examined attributes into account. We created the KK and WKK for FFHSS and exhibited their favorable properties. Similarly, an enhanced TOPSIS technique is proposed based on the created correlation by taking into account the attribute set with its matching sub-attributes and decision-makers. We created correlation indices to discover PIA and NIA. The proximity coefficients were constructed based on the well-established TOPSIS approach for rating alternatives. A numerical demonstration of how to solve the MAGDM issue using the suggested TOPSIS approach has been provided. Furthermore, a comparison study was performed to validate the efficacy and presentation of the provided strategy. Finally, based on the findings, it is possible to infer that the suggested approach provides more stability and practicability for decision-makers during the decision-making process. Future studies will focus on providing ideas to various operators in the FFHSS environment about decision-making problems. In a given setting, several different structures, such as topological structures, algebraic structures, and ordered structures, can be built and examined. This research paper has pragmatic bounds and may be quite useful in real-world aspects such as sickness diagnosis, pattern detection, and economics. We are confident that this publication will open new doors for scholars in this subject.

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