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ARTICLE

Decision Making Algorithmic Approaches Based on Parameterization of Neutrosophic Set under Hypersoft Set Environment with Fuzzy, Intuitionistic Fuzzy and Neutrosophic Settings

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ABSTRACT

Hypersoft set is an extension of soft set as it further partitions each attribute into its corresponding attribute-valued set. This structure is more flexible and useful as it addresses the limitation of soft set for dealing with the scenarios having disjoint attribute-valued sets corresponding to distinct attributes. The main purpose of this study is to make the existing literature regarding neutrosophic parameterized soft set in line with the need of multi-attribute approximate function. Firstly, we conceptualize the neutrosophic parameterized hypersoft sets under the settings of fuzzy set, intuitionistic fuzzy set and neutrosophic set along with some of their elementary properties and set theoretic operations. Secondly, we propose decision-making-based algorithms with the help of these theories. Moreover, illustrative examples are presented which depict the structural validity for successful application to the problems involving vagueness and uncertainties. Lastly, the generalization of the proposed structure is discussed.

KEYWORDS

Neutrosophic set; hypersoft set; neutrosophic hypersoft set; parameterized soft set; parameterized hypersoft set

1 Introduction

Fuzzy sets theory (FST) [1] and intuitionistic fuzzy set theory (IFST) [2] are considered apt mathematical modes to tackle many intricate problems involving various uncertainties, in different mathematical disciplines. The former one emphasizes on the degree of true belongingness of a certain object from the initial sample space whereas the later one accentuates on degree of true membership and degree of non-membership with condition of their dependency on each other. These theories depict some kind of inadequacy regarding the provision of due status to degree of indeterminacy. Such impediment is addressed with the introduction of neutrosophic set theory (NST) [3,4] which not only considers the due status of degree of indeterminacy but also waives off the condition of dependency. This theory is more flexible and appropriate to deal with uncertainty and vagueness. NST has attracted the keen concentration of many researchers [5–19] to further utilization in statistics, topological spaces as well as in the development of certain neutrosophic-like blended structures with other existing models for useful applications in decision making. Edalatpanah [20] studied a system of neutrosophic linear equations (SNLE) based on the embedding approach. He used (α, β, γ)-cut for transformation of SNLE into a crisp linear system. Kumar et al. [21] exhibited a novel linear programming approach for finding the neutrosophic shortest path problem (NSSPP) considering Gaussian valued neutrosophic number.

FST, IFST and NST have some kind of complexities which restrain them to solve problems involving uncertainty professionally. The reason for these hurdles is, possibly, the inadequacy of the parametrization tool. It demands a mathematical tool free of all such impediments to tackle such issues. This scantiness is resolved with the development of soft set theory (SST) [22] which is a new parameterized family of subsets of the universe of discourse. The researchers [23–34] studied and investigated some elementary properties, operations, laws and hybrids of SST with applications in decision making. The gluing concept of NST and SST, is studied in [35,36] to make the NST adequate with parameterized tool. In many real life situations, distinct attributes are further partitioned in disjoint attribute-valued sets but existing SST is insufficient for dealing with such kind of attribute-valued sets. Hypersoft set theory (HST) [37] is developed to make the SST in line with attribute-valued sets to tackle real life scenarios. HST is an extension of SST as it transforms the single argument approximate function into a multi-argument approximate function. Certain elementary properties, aggregation operations, laws, relations and functions of HST, are investigated by [38–40] for proper understanding and further utilization in different fields. The applications of HST in decision making is studied by [41–44] and the intermingling study of HST with complex sets, convex and concave sets is studied by [45,46]. Deli [47] characterized hybrid set structures under uncertainly parameterized hypersoft sets with theory and applications. Gayen et al. [48] analyzed some essential aspects of plithogenic hypersoft algebraic structures. They also investigated the notions and basic properties of plithogenic hypersoft subgroups, i.e., plithogenic fuzzy hypersoft subgroup, plithogenic intuitionistic fuzzy hypersoft subgroup, plithogenic neutrosophic hypersoft subgroup.

1.1 Motivation

In miscellany of real-life applications, the attributes are required to be further partitioned into attribute values for more vivid understanding. Hypersoft set as a generalization of soft set, accomplishes this limitation and accentuates the disjoint attribute-valued sets for distinct attributes. This generalization reveals that the hypersoft set with neutrosophic, intuitionistic, and fuzzy set theory will be very helpful to construct a connection between alternatives and attributes. It is interesting that the hypersoft theory can be applied on any decision-making problem without the limitations of the selection of the values by the decision-makers. This theory can successfully be applied to Multi-criteria decision making (MCDM), Multi-criteria group decision making (MCGDM), shortest path selection, employee selection, e-learning, graph theory, medical diagnosis, probability theory, topology, and many others. It is pertinent that the existing literature regarding soft set should be adequate with the existence and the consideration of attribute-valued sets, therefore, this study aims to develop novel theories of embedding structures of parameterized neutrosophic set and hypersoft set with the setting of fuzzy, intuitionistic fuzzy and neutrosophic sets through the extension of concept investigated in [49–54]. Moreover, decision-making based algorithms are proposed for each setting to solve a real life problem relating to the purchase of most suitable and appropriate product with the help of some essential operations of these presented theories.

1.2 Organization of Paper

The rest of the paper is systemized as:

Section 2	Some essential definitions and terminologies are recalled.
Section 3	Theory of neutrosophic parameterized fuzzy hypersoft set is developed with suitable examples.
Section 4	Theory of neutrosophic parameterized intuitionistic fuzzy hypersoft set is characterized with suitable examples.
Section 5	Theory of neutrosophic parameterized neutrosophic hypersoft set is investigated with suitable examples.
Section 6	Analysis of proposed structure is discussed.
Section 7	Paper is summarized with future directions.

2 Preliminaries

Here some basic terms are recalled from existing literature to support the proposed work. Throughout the paper, $\mathbb{X}$, $\mathbb{P}(\mathbb{X})$ and $\mathbb{I}$ will denote the universe of discourse, power set of $\mathbb{X}$ and closed unit interval respectively. In this work, algorithmic approaches are followed from decision making methods stated in [49–54].

Definition 2.1. [1]

A *fuzzy set* $\mathcal{F}$ defined as $\mathcal{F} = \{(\hat{a}, A_{\mathcal{F}}(\hat{a})) \mid \hat{a} \in \mathbb{X}\}$ such that $A_{\mathcal{F}}: \mathbb{X} \to \mathbb{I}$ where $A_{\mathcal{F}}(\hat{a})$ denotes the belonging value of $\hat{a} \in \mathcal{F}$.

Definition 2.2. [2]

An *intuitionistic fuzzy set* $\mathcal{Y}$ defined as $\mathcal{Y} = \{(\hat{b}, < A_{\mathcal{Y}}(\hat{b}), B_{\mathcal{Y}}(\hat{b}) >) \mid \hat{b} \in \mathbb{X}\}$ such that $A_{\mathcal{Y}}: \mathbb{X} \to \mathbb{I}$ and $B_{\mathcal{Y}}: \mathbb{X} \to \mathbb{I}$, where $A_{\mathcal{Y}}(\hat{b})$ and $B_{\mathcal{Y}}(\hat{b})$ denote the belonging value and not-belonging value of $\hat{b} \in \mathcal{Y}$ with condition of $0 \leq A_{\mathcal{Y}}(\hat{b}) + B_{\mathcal{Y}}(\hat{b}) \leq 1$.

Definition 2.3. [3]

A *neutrosophic set* $\mathcal{Z}$ defined as $\mathcal{Z} = \{(\hat{c}, < \mathcal{A}_{\mathcal{Z}}(\hat{c}), \mathcal{B}_{\mathcal{Z}}(\hat{c}), \mathcal{C}_{\mathcal{Z}}(\hat{c}) >) \mid \hat{c} \in \mathbb{X}\}$ such that $\mathcal{A}_{\mathcal{Z}}(\hat{c}), \mathcal{B}_{\mathcal{Z}}(\hat{c}), \mathcal{C}_{\mathcal{Z}}(\hat{c}): \mathbb{X} \to (^{-}0, 1^{+})$, where $\mathcal{A}_{\mathcal{Z}}(\hat{c}), \mathcal{B}_{\mathcal{Z}}(\hat{c})$ and $\mathcal{C}_{\mathcal{Z}}(\hat{c})$ denote the degrees of membership, indeterminacy and non-membership of $\hat{c} \in \mathcal{Z}$ with condition of $^{-}0 \leq \mathcal{A}_{\mathcal{Z}}(\hat{c}) + \mathcal{B}_{\mathcal{Z}}(\hat{c}) + \mathcal{C}_{\mathcal{Z}}(\hat{c}) \leq 3^{+}$.

Definition 2.4. [22]

A pair (F_S, Λ) is called a *soft set* over $\mathbb{X}$, where $F_S: \Lambda \to \mathbb{P}(\mathbb{X})$ and Λ be a subset of a set of attributes E.

For more detail on soft set, see [23–32].

Definition 2.5. [37]

The pair $(\mathcal{W}, \mathcal{G})$ is called a hypersoft set over $\mathbb{X}$, where $\mathcal{G}$ is the cartesian product of n disjoint sets $\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3, \ldots, \mathcal{G}_n$ having attribute values of n distinct attributes $\hat{g}_1, \hat{g}_2, \hat{g}_3, \ldots, \hat{g}_n$ respectively and $\mathcal{W}: \mathcal{G} \to \mathbb{P}(\mathbb{X})$.

For more definitions and operations of hypersoft set, see [38–40].

3 Neutrosophic Parameterized Fuzzy Hypersoft Set (*npfhs*-Set) with Application

In this section, *npfhs*-set theory is conceptualized and a decision making application is discussed.

Definition 3.1. Let $\boldsymbol{\mathcal{X}} = \{\boldsymbol{\mathcal{X}}_1, \boldsymbol{\mathcal{X}}_2, \boldsymbol{\mathcal{X}}_3, \ldots, \boldsymbol{\mathcal{X}}_n\}$ be a collection of disjoint attribute-valued sets corresponding to n distinct attributes $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n$, respectively. A *npfhs*-set $\Psi_{\boldsymbol{\mathcal{A}}}$ over $\mathbb{X}$ is defined as

$$\Psi_{\boldsymbol{\mathcal{A}}} = \{(< P_{\boldsymbol{\mathcal{A}}}(g), Q_{\boldsymbol{\mathcal{A}}}(g), R_{\boldsymbol{\mathcal{A}}}(g) > /g, \psi_{\boldsymbol{\mathcal{A}}}(g)) : g \in \mathbb{G}, \psi_{\boldsymbol{\mathcal{A}}}(g) \in F(\mathbb{X})\}$$

where

(i) $F(\mathbb{X})$ is a collection of all fuzzy sets over $\mathbb{X}$
(ii) $\mathbb{G} = \boldsymbol{\mathcal{X}}_1 \times \boldsymbol{\mathcal{X}}_2 \times \boldsymbol{\mathcal{X}}_3 \times \ldots \times \boldsymbol{\mathcal{X}}_n$
(iii) $\boldsymbol{\mathcal{A}}$ is a neutrosophic set over $\mathbb{G}$ with $P_{\boldsymbol{\mathcal{A}}}, Q_{\boldsymbol{\mathcal{A}}}, R_{\boldsymbol{\mathcal{A}}} : \mathbb{G} \to \mathbb{I}$ as membership function, indeterminacy function and nonmembership function of *npfhs*-set.
(iv) $\psi_{\boldsymbol{\mathcal{A}}}(g)$ is a fuzzy set for all $g \in \mathbb{G}$ with $\psi_{\boldsymbol{\mathcal{A}}} : \mathbb{G} \to F(\mathbb{X})$ and is called approximate function of *npfhs*-set.

Note that collection of all *npfhs*-sets is represented by $\Omega_{NPFHS}(\mathbb{X})$.

Definition 3.2. Let $\Psi_{\boldsymbol{\mathcal{A}}} \in \Omega_{NPFHS}(\mathbb{X})$. If $\psi_{\boldsymbol{\mathcal{A}}}(g) = \phi$, $P_{\boldsymbol{\mathcal{A}}}(g) = 0$, $Q_{\boldsymbol{\mathcal{A}}}(g) = 1$, $R_{\boldsymbol{\mathcal{A}}}(g) = 1$ for all $g \in \mathbb{G}$, then $\Psi_{\boldsymbol{\mathcal{A}}}$ is called $\boldsymbol{\mathcal{A}}$-empty *npfhs*-set, denoted by $\Psi_{\Phi_{\boldsymbol{\mathcal{A}}}}$. If $\boldsymbol{\mathcal{A}} = \phi$, then $\boldsymbol{\mathcal{A}}$-empty *npfhs*-set is called an empty *npfhs*-set, denoted by Ψ_{Φ}.

Definition 3.3. Let $\Psi_{\boldsymbol{\mathcal{A}}} \in \Omega_{NPFHS}(\mathbb{X})$. If $\psi_{\boldsymbol{\mathcal{A}}}(g) = \mathbb{X}$, $P_{\boldsymbol{\mathcal{A}}}(g) = 1$, $Q_{\boldsymbol{\mathcal{A}}}(g) = 0$, $R_{\boldsymbol{\mathcal{A}}}(g) = 0$ for all $g \in \mathbb{G}$, then $\Psi_{\boldsymbol{\mathcal{A}}}$ is called $\boldsymbol{\mathcal{A}}$-universal *npfhs*-set, denoted by $\Psi_{\tilde{\boldsymbol{\mathcal{A}}}}$. If $\boldsymbol{\mathcal{A}} = \mathbb{G}$, then the $\boldsymbol{\mathcal{A}}$-universal *npfhs*-set is called universal *npfhs*-set, denoted by $\Psi_{\tilde{\mathbb{G}}}$.

Example 3.1. Consider $\mathbb{X} = \{u_1, u_2, u_3, u_4, u_5\}$ and $\boldsymbol{\mathcal{X}} = \{\boldsymbol{\mathcal{X}}_1, \boldsymbol{\mathcal{X}}_2, \boldsymbol{\mathcal{X}}_3\}$ with $\boldsymbol{\mathcal{X}}_1 = \{\hat{x}_{11}, \hat{x}_{12}\}$, $\boldsymbol{\mathcal{X}}_2 = \{\hat{x}_{21}, \hat{x}_{22}\}$, $\boldsymbol{\mathcal{X}}_3 = \{\hat{x}_{31}\}$, then $\mathbb{G} = \boldsymbol{\mathcal{X}}_1 \times \boldsymbol{\mathcal{X}}_2 \times \boldsymbol{\mathcal{X}}_3$

$$\mathbb{G} = \left\{\left(\hat{x}_{11}, \hat{x}_{21}, \hat{x}_{31}\right), \left(\hat{x}_{11}, \hat{x}_{22}, \hat{x}_{31}\right), \left(\hat{x}_{12}, \hat{x}_{21}, \hat{x}_{31}\right), \left(\hat{x}_{12}, \hat{x}_{22}, \hat{x}_{31}\right)\right\} = \{g_1, g_2, g_3, g_4\}.$$

Case 1.

If $\boldsymbol{\mathcal{A}}_1 = \{< 0.2, 0.3, 0.4 > /g_2, < 0, 1, 1 > /g_3, < 1, 0, 0 > /g_4\}$ and

$\psi_{\boldsymbol{\mathcal{A}}_1}(g_2) = \{0.4/u_2, 0.6/u_4\}$, $\psi_{\boldsymbol{\mathcal{A}}_1}(g_3) = \emptyset$, and $\psi_{\boldsymbol{\mathcal{A}}_1}(g_4) = \mathbb{X}$, then

$$\Psi_{\boldsymbol{\mathcal{A}}_1} = \{(< 0.2, 0.3, 0.4 > /g_2, \{0.4/u_2, 0.6/u_4\}), (< 0, 1, 1 > /g_3, \emptyset), (< 1, 0, 0 > /g_4, \mathbb{X})\}.$$

Case 2.

If $\boldsymbol{\mathcal{A}}_2 = \{< 0, 1, 1 > /g_2, < 0, 1, 1 > /g_3\}, \psi_{\boldsymbol{\mathcal{A}}_2}(g_2) = \emptyset$ and $\psi_{\boldsymbol{\mathcal{A}}_2}(g_3) = \emptyset$, then $\Psi_{\boldsymbol{\mathcal{A}}_2} = \Psi_{\Phi_{\boldsymbol{\mathcal{A}}_2}}$.

Case 3.

If $\boldsymbol{\mathcal{A}}_3 = \emptyset$ corresponding to all elements of $\mathbb{G}$, then $\Psi_{\boldsymbol{\mathcal{A}}_3} = \Psi_{\Phi}$.

Case 4.

If $\boldsymbol{\mathcal{A}}_4 = \{< 1, 0, 0 > /g_1, < 1, 0, 0 > /g_2\}, \psi_{\boldsymbol{\mathcal{A}}_4}(g_1) = \mathbb{X}$, and $\psi_{\boldsymbol{\mathcal{A}}_4}(g_2) = \mathbb{X}$, then $\Psi_{\boldsymbol{\mathcal{A}}_4} = \Psi_{\tilde{\boldsymbol{\mathcal{A}}}_4}$.

Case 5.

If $\boldsymbol{\mathcal{A}}_5 = \mathbb{X}$ with respect to all elements of $\mathbb{G}$, then $\Psi_{\boldsymbol{\mathcal{A}}_5} = \Psi_{\tilde{\mathbb{G}}}$.

Definition 3.4. Let $\Psi_{\mathcal{A}_1}$, $\Psi_{\mathcal{A}_2} \in \Omega_{NPFHS}(\mathbb{X})$ then $\Psi_{\mathcal{A}_1}$ is an *npfhs*-subset of $\Psi_{\mathcal{A}_2}$, denoted by $\Psi_{\mathcal{A}_1} \tilde{\subseteq}_f \Psi_{\mathcal{A}_2}$ if

$P_{\mathcal{A}_1}(g) \leq P_{\mathcal{A}_2}(g), Q_{\mathcal{A}_1}(g) \geq Q_{\mathcal{A}_2}(g), R_{\mathcal{A}_1}(g) \geq R_{\mathcal{A}_2}(g)$ and $\psi_{\mathcal{A}_1}(g) \subseteq_f \psi_{\mathcal{A}_2}(g)$ for all $g \in \mathbb{G}$.

Definition 3.5. Let $\Psi_{\mathcal{A}_1}, \Psi_{\mathcal{A}_2} \in \Omega_{NPFHS}(\mathbb{X})$ then, $\Psi_{\mathcal{A}_1}$ and $\Psi_{\mathcal{A}_2}$ are *npfhs*-equal, represented as $\Psi_{\mathcal{A}_1} = \Psi_{\mathcal{A}_2}$, if and only if $P_{\mathcal{A}_1}(g) = P_{\mathcal{A}_2}(g)$, $Q_{\mathcal{A}_1}(g) = Q_{\mathcal{A}_2}(g)$, $R_{\mathcal{A}_1}(g) = R_{\mathcal{A}_2}(g)$ and $\psi_{\mathcal{A}_1}(g) =_f \psi_{\mathcal{A}_2}(g)$ for all $g \in \mathbb{G}$.

Definition 3.6. Let $\Psi_{\mathcal{A}} \in \Omega_{NPFHS}(\mathbb{X})$ then, complement of $\Psi_{\mathcal{A}}$ (i.e., $\Psi^{\tilde{c}}_{\mathcal{A}}$) is an *npfhs*-set given as $P^{\tilde{c}}_{\mathcal{A}}(g) = 1 - P_{\mathcal{A}}(g)$, $Q^{\tilde{c}}_{\mathcal{A}}(g) = 1 - Q_{\mathcal{A}}(g)$, $R^{\tilde{c}}_{\mathcal{A}}(g) = 1 - R_{\mathcal{A}}(g)$ and $\psi^{\tilde{c}}_{\mathcal{A}}(g) = \mathbb{X} \setminus_f \psi_{\mathcal{A}}(g)$.

Proposition 3.1. Let $\Psi_{\mathcal{A}} \in \Omega_{NPFHS}(\mathbb{X})$ then,

1. $(\Psi^{\tilde{c}}_{\mathcal{A}})^{\tilde{c}} = \Psi_{\mathcal{A}}$.
2. $\Psi^{\tilde{c}}_{\phi} = \Psi_{\tilde{\mathbb{G}}}$.

Definition 3.7. Let $\Psi_{\mathcal{A}_1}, \Psi_{\mathcal{A}_2} \in \Omega_{NPFHS}(\mathbb{X})$ then, union of $\Psi_{\mathcal{A}_1}$ and $\Psi_{\mathcal{A}_2}$, denoted by $\Psi_{\mathcal{A}_1} \tilde{\cup}_f \Psi_{\mathcal{A}_2}$, is an *npfhs*-set defined by

(i) $P_{\mathcal{A}_1 \tilde{\cup} \mathcal{A}_2}(g) = max\{P_{\mathcal{A}_1}(x), P_{\mathcal{A}_2}(g)\}$,
(ii) $Q_{\mathcal{A}_1 \tilde{\cup} \mathcal{A}_2}(g) = min\{Q_{\mathcal{A}_1}(x), Q_{\mathcal{A}_2}(g)\}$,
(iii) $R_{\mathcal{A}_1 \tilde{\cup} \mathcal{A}_2}(g) = min\{R_{\mathcal{A}_1}(x), R_{\mathcal{A}_2}(g)\}$,
(iv) $\psi_{\mathcal{A}_1 \tilde{\cup} \mathcal{A}_2}(g) = \psi_{\mathcal{A}_1}(g) \tilde{\cup}_f \psi_{\mathcal{A}_2}(g)$, for all $g \in \mathbb{G}$.

Definition 3.8. Let $\Psi_{\mathcal{A}_1}, \Psi_{\mathcal{A}_2} \in \Omega_{NPFHS}(\mathbb{X})$ then intersection of $\Psi_{\mathcal{A}_1}$ and $\Psi_{\mathcal{A}_2}$, denoted by $\Psi_{\mathcal{A}_1} \tilde{\cap}_f \Psi_{\mathcal{A}_2}$, is an *npfhs*-set defined by

(i) $P_{\mathcal{A}_1 \tilde{\cap} \mathcal{A}_2}(g) = min\{P_{\mathcal{A}_1}(x), P_{\mathcal{A}_2}(g)\}$,
(ii) $Q_{\mathcal{A}_1 \tilde{\cap} \mathcal{A}_2}(g) = max\{Q_{\mathcal{A}_1}(x), Q_{\mathcal{A}_2}(g)\}$,
(iii) $R_{\mathcal{A}_1 \tilde{\cap} \mathcal{A}_2}(g) = max\{R_{\mathcal{A}_1}(x), R_{\mathcal{A}_2}(g)\}$,
(iv) $\psi_{\mathcal{A}_1 \tilde{\cap} \mathcal{A}_2}(g) = \psi_{\mathcal{A}_1}(g) \tilde{\cap}_f \psi_{\mathcal{A}_2}(g)$, for all $g \in \mathbb{G}$.

Remark 3.1. Let $\Psi_{\mathcal{A}} \in \Omega_{NPFHS}(\mathbb{X})$. If $\Psi_{\mathcal{A}} \neq_f \Psi_{\tilde{\mathbb{G}}}$, then $\Psi_{\mathcal{A}} \tilde{\cup}_f \Psi^{\tilde{c}}_{\mathcal{A}} \neq_f \Psi_{\tilde{\mathbb{G}}}$ and $\Psi_{\mathcal{A}} \tilde{\cap}_f \Psi^{\tilde{c}}_{\mathcal{A}} \neq_f \Psi_{\Phi}$

Proposition 3.2. Let $\Psi_{\mathcal{A}_1}, \Psi_{\mathcal{A}_2} \in \Omega_{NPFHS}(\mathbb{X})$ D. Morgan laws are valid

1. $(\Psi_{\mathcal{A}_1} \tilde{\cup}_f \Psi_{\mathcal{A}_2})^{\tilde{c}} = \Psi^{\tilde{c}}_{\mathcal{A}_1} \tilde{\cap}_f \Psi^{\tilde{c}}_{\mathcal{A}_2}$.
2. $(\Psi_{\mathcal{A}_1} \tilde{\cap}_f \Psi_{\mathcal{A}_2})^{\tilde{c}} = \Psi^{\tilde{c}}_{\mathcal{A}_1} \tilde{\cup}_f \Psi^{\tilde{c}}_{\mathcal{A}_2}$.

Proof. For all $g \in \mathbb{G}$,

(1). Since
$$
\begin{aligned}
(P_{\mathcal{A}_1 \tilde{\cup} \mathcal{A}_2})^{\tilde{c}}(g) &= 1 - P_{\mathcal{A}_1 \tilde{\cup} \mathcal{A}_2}(g) \\
&= 1 - max\{P_{\mathcal{A}_1}(g), P_{\mathcal{A}_2}(g)\} \\
&= min\{1 - P_{\mathcal{A}_1}(g), 1 - P_{\mathcal{A}_2}(g)\} \\
&= min\{P^{\tilde{c}}_{\mathcal{A}_1}(g), P^{\tilde{c}}_{\mathcal{A}_2}(g)\} \\
&= P^{\tilde{c}}_{\mathcal{A}_1 \tilde{\cap} \mathcal{A}_2}(g)
\end{aligned}
$$

also

$$\begin{aligned}(Q_{\mathcal{A}_1\tilde{\cup}\mathcal{A}_2})^{\tilde{c}}(g) &= 1 - Q_{\mathcal{A}_1\tilde{\cup}\mathcal{A}_2}(g)\\ &= 1 - min\{Q_{\mathcal{A}_1}(g), Q_{\mathcal{A}_2}(g)\}\\ &= max\{1 - Q_{\mathcal{A}_1}(g), 1 - Q_{\mathcal{A}_2}(g)\}\\ &= max\{Q^{\tilde{c}}_{\mathcal{A}_1}(g), Q^{\tilde{c}}_{\mathcal{A}_2}(g)\}\\ &= Q^{\tilde{c}}_{\mathcal{A}_1\tilde{\cap}\mathcal{A}_2}(g)\end{aligned}$$

and

$$\begin{aligned}(R_{\mathcal{A}_1\tilde{\cup}\mathcal{A}_2})^{\tilde{c}}(g) &= 1 - R_{\mathcal{A}_1\tilde{\cup}\mathcal{A}_2}(g)\\ &= 1 - min\{R_{\mathcal{A}_1}(g), R_{\mathcal{A}_2}(g)\}\\ &= max\{1 - R_{\mathcal{A}_1}(g), 1 - R_{\mathcal{A}_2}(g)\}\\ &= max\{R^{\tilde{c}}_{\mathcal{A}_1}(g), R^{\tilde{c}}_{\mathcal{A}_2}(g)\}\\ &= R^{\tilde{c}}_{\mathcal{A}_1\tilde{\cap}\mathcal{A}_2}(g)\end{aligned}$$

and

$$\begin{aligned}(\psi_{\mathcal{A}_1\tilde{\cup}\mathcal{A}_2})^{\tilde{c}}(g) &= \mathbb{X} \setminus_f \psi_{\mathcal{A}_1\tilde{\cup}\mathcal{A}_2}(g)\\ &= \mathbb{X} \setminus_f (\psi_{\mathcal{A}_1}(g)\tilde{\cup}_f \psi_{\mathcal{A}_2}(g))\\ &= (\mathbb{X} \setminus_f \psi_{\mathcal{A}_1}(g))\tilde{\cap}_f(\mathbb{X} \setminus_f \psi_{\mathcal{A}_2}(g))\\ &= \psi^{\tilde{c}}_{\mathcal{A}_1}(g)\,\tilde{\cap}_f\, \psi^{\tilde{c}}_{\mathcal{A}_2}(g)\\ &= \psi^{\tilde{c}}_{\mathcal{A}_1\tilde{\cap}\mathcal{A}_2}(g).\end{aligned}$$

similarly (2) can be proved easily.

Proposition 3.3. Let $\Psi_{\mathcal{A}_1}, \Psi_{\mathcal{A}_2}, \Psi_{\mathcal{A}_3} \in \Omega_{NPFHS}(\mathbb{X})$ then

1. $\Psi_{\mathcal{A}_1}\,\tilde{\cup}_f\,(\Psi_{\mathcal{A}_2}\,\tilde{\cap}_f\,\Psi_{\mathcal{A}_3}) = (\Psi_{\mathcal{A}_1}\,\tilde{\cup}_f\,\Psi_{\mathcal{A}_2})\,\tilde{\cap}_f\,(\Psi_{\mathcal{A}_1}\,\tilde{\cup}_f\,\Psi_{\mathcal{A}_3})$.
2. $\Psi_{\mathcal{A}_1}\,\tilde{\cap}_f\,(\Psi_{\mathcal{A}_2}\,\tilde{\cup}_f\,\Psi_{\mathcal{A}_3}) = (\Psi_{\mathcal{A}_1}\,\tilde{\cap}_f\,\Psi_{\mathcal{A}_2})\,\tilde{\cup}_f\,(\Psi_{\mathcal{A}_1}\,\tilde{\cap}_f\,\Psi_{\mathcal{A}_3})$.

Proof. For all $g \in \mathbb{G}$,

(1). Since
$$\begin{aligned}P_{\mathcal{A}_1\tilde{\cup}(\mathcal{A}_2\tilde{\cap}\mathcal{A}_3)}(g) &= max\{P_{\mathcal{A}_1}(g), P_{\mathcal{A}_2\tilde{\cap}\mathcal{A}_3}(g)\}\\ &= max\{P_{\mathcal{A}_1}(g), min\{P_{\mathcal{A}_2}(g), P_{\mathcal{A}_3}(g)\}\}\\ &= min\{max\{P_{\mathcal{A}_1}(g), P_{\mathcal{A}_2}(g)\}, max\{P_{\mathcal{A}_1}(g), P_{\mathcal{A}_3}(g)\}\}\\ &= min\{P_{\mathcal{A}_1\tilde{\cup}\mathcal{A}_2}(g), P_{\mathcal{A}_1\tilde{\cup}\mathcal{A}_3}(g)\}\\ &= P_{(\mathcal{A}_1\tilde{\cup}\mathcal{A}_2)\tilde{\cap}(\mathcal{A}_1\tilde{\cup}\mathcal{A}_3)}(g)\end{aligned}$$

and

$$
\begin{aligned}
Q_{\mathcal{A}_1 \tilde{\cup} (\mathcal{A}_2 \tilde{\cap} \mathcal{A}_3)}(g) &= min\{Q_{\mathcal{A}_1}(g),\, Q_{\mathcal{A}_2 \tilde{\cap} \mathcal{A}_3}(g)\} \\
&= min\{Q_{\mathcal{A}_1}(g),\, max\{Q_{\mathcal{A}_2}(g),\, Q_{\mathcal{A}_3}(g)\}\} \\
&= max\{min\{Q_{\mathcal{A}_1}(g),\, Q_{\mathcal{A}_2}(g)\},\, min\{Q_{\mathcal{A}_1}(g),\, Q_{\mathcal{A}_3}(g)\}\} \\
&= max\{Q_{\mathcal{A}_1 \tilde{\cup} \mathcal{A}_2}(g),\, Q_{\mathcal{A}_1 \tilde{\cup} \mathcal{A}_3}(g)\} \\
&= Q_{(\mathcal{A}_1 \tilde{\cup} \mathcal{A}_2) \tilde{\cap} (\mathcal{A}_1 \tilde{\cup} \mathcal{A}_3)}(g)
\end{aligned}
$$

and

$$
\begin{aligned}
R_{\mathcal{A}_1 \tilde{\cup} (\mathcal{A}_2 \tilde{\cap} \mathcal{A}_3)}(g) &= min\{R_{\mathcal{A}_1}(g), R_{\mathcal{A}_2 \tilde{\cap} \mathcal{A}_3}(g)\} \\
&= min\{R_{\mathcal{A}_1}(g), max\{R_{\mathcal{A}_2}(g),\, R_{\mathcal{A}_3}(g)\}\} \\
&= max\{min\{R_{\mathcal{A}_1}(g),\, R_{\mathcal{A}_2}(g)\},\, min\{R_{\mathcal{A}_1}(g),\, R_{\mathcal{A}_3}(g)\}\} \\
&= max\{R_{\mathcal{A}_1 \tilde{\cup} \mathcal{A}_2}(g),\, R_{\mathcal{A}_1 \tilde{\cup} \mathcal{A}_3}(g)\} \\
&= R_{(\mathcal{A}_1 \tilde{\cup} \mathcal{A}_2) \tilde{\cap} (\mathcal{A}_1 \tilde{\cup} \mathcal{A}_3)}(g)
\end{aligned}
$$

and

$$
\begin{aligned}
\psi_{\mathcal{A}_1 \tilde{\cup} (\mathcal{A}_2 \tilde{\cap} \mathcal{A}_3)}(g) &= \psi_{\mathcal{A}_1}(g) \cup_f \psi_{\mathcal{A}_2 \tilde{\cap} \mathcal{A}_3}(g) \\
&= \psi_{\mathcal{A}_1}(g) \cup_f (\psi_{\mathcal{A}_2}(g) \cap_f \psi_{\mathcal{A}_3}(g)) \\
&= (\psi_{\mathcal{A}_1}(g) \cup_f \psi_{\mathcal{A}_2}(g)) \cap_f (\psi_{\mathcal{A}_1}(g) \cup_f \psi_{\mathcal{A}_3}(g)) \\
&= \psi_{\mathcal{A}_1 \tilde{\cup} \mathcal{A}_2}(g) \cap_f \psi_{\mathcal{A}_1 \tilde{U} \mathcal{A}_3}(g) \\
&= \psi_{(\mathcal{A}_1 \tilde{\cup} \mathcal{A}_2) \tilde{\cap} (\mathcal{A}_1 \tilde{\cup} \mathcal{A}_3)}(g)
\end{aligned}
$$

In the same way, (2) can be proved.

Definition 3.9. Let $\Psi_{\mathcal{A}_1}, \Psi_{\mathcal{A}_2} \in \Omega_{NPFHS}(\mathbb{X})$ then OR-operation of $\Psi_{\mathcal{A}_1}$ and $\Psi_{\mathcal{A}_2}$, denoted by $\Psi_{\mathcal{A}_1} \tilde{\Cup} \Psi_{\mathcal{A}_2}$, is an *npfhs*-set defined by

(i) $P_{\mathcal{A}_1 \tilde{\Cup} \mathcal{A}_2}(g_1, g_2) = max\{P_{\mathcal{A}_1}(g_1),\, P_{\mathcal{A}_2}(g_2)\}$,
(ii) $Q_{\mathcal{A}_1 \tilde{\Cup} \mathcal{A}_2}(g_1, g_2) = min\{Q_{\mathcal{A}_1}(g_1),\, Q_{\mathcal{A}_2}(g_2)\}$,
(iii) $R_{\mathcal{A}_1 \tilde{\Cup} \mathcal{A}_2}(g_1, g_2) = min\{R_{\mathcal{A}_1}(g_1),\, R_{\mathcal{A}_2}(g_2)\}$,
(iv) $\psi_{\mathcal{A}_1 \tilde{\Cup} \mathcal{A}_2}(g_1, g_2) = \psi_{\mathcal{A}_1}(g_1) \cup_f \psi_{\mathcal{A}_2}(g_2)$, for all $(g_1, g_2) \in \mathcal{A}_1 \times \mathcal{A}_2$.

Definition 3.10. Let $\Psi_{\mathcal{A}_1}, \Psi_{\mathcal{A}_2} \in \Omega_{NPFHS}(\mathbb{X})$ then AND-operation of $\Psi_{\mathcal{A}_1}$ and $\Psi_{\mathcal{A}_2}$, denoted by $\Psi_{\mathcal{A}_1} \tilde{\Cap} \Psi_{\mathcal{A}_2}$, is an *npfhs*-set defined by

(i) $P_{\mathcal{A}_1 \tilde{\Cap} \mathcal{A}_2}(g_1, g_2) = min\{P_{\mathcal{A}_1}(g_1),\, P_{\mathcal{A}_2}(g_2)\}$,
(ii) $Q_{\mathcal{A}_1 \tilde{\Cap} \mathcal{A}_2}(g_1, g_2) = max\{Q_{\mathcal{A}_1}(g_1),\, Q_{\mathcal{A}_2}(g_2)\}$,
(iii) $R_{\mathcal{A}_1 \tilde{\Cap} \mathcal{A}_2}(g_1, g_2) = max\{R_{\mathcal{A}_1}(g_1),\, R_{\mathcal{A}_2}(g_2)\}$,
(iv) $\psi_{\mathcal{A}_1 \tilde{\Cap} \mathcal{A}_2}(g_1, g_2) = \psi_{\mathcal{A}_1}(g_1) \cap_f \psi_{\mathcal{A}_2}(g_2)$, for all $(g_1, g_2) \in \mathcal{A}_1 \times \mathcal{A}_2$.

Proposition 3.4. Let $\Psi_{\mathcal{A}_1}, \Psi_{\mathcal{A}_2}, \Psi_{\mathcal{A}_3} \in \Omega_{NPFHS}(\mathbb{X})$ then

1. $\Psi_{\mathcal{A}_1} \tilde{\Cap} \Psi_{\Phi} = \Psi_{\Phi}$.
2. $(\Psi_{\mathcal{A}_1} \tilde{\Cap} \Psi_{\mathcal{A}_2}) \tilde{\Cap} \Psi_{\mathcal{A}_3} = \Psi_{\mathcal{A}_1} \tilde{\Cap} (\Psi_{\mathcal{A}_2} \tilde{\Cap} \Psi_{\mathcal{A}_3})$.
3. $(\Psi_{\mathcal{A}_1} \tilde{\Cup} \Psi_{\mathcal{A}_2}) \tilde{\Cup} \Psi_{\mathcal{A}_3} = \Psi_{\mathcal{A}_1} \tilde{\Cup} (\Psi_{\mathcal{A}_2} \tilde{\Cup} \Psi_{\mathcal{A}_3})$.

3.1 Neutrosophic Decision Set of npfhs-Set

An algorithm is presented with the help of characterization of neutrosophic decision set on *npfhs*-set which based on decision making technique and is explained with example.

Definition 3.11. Let $\Psi_{\mathcal{A}} \in \Omega_{NPFHS}(\mathbb{X})$ then a neutrosophic decision set of $\Psi_{\mathcal{A}}$ (i.e., $\Psi^D_{\mathcal{A}}$) is represented as

$$\Psi^D_{\mathcal{A}} = \left\{ < \mathcal{T}^D_{\mathcal{A}}(u), \mathcal{I}^D_{\mathcal{A}}(u), \mathcal{F}^D_{\mathcal{A}}(u) > /u: \ u \in \mathbb{X} \right\}$$

where $\mathcal{T}^D_{\mathcal{A}}, \mathcal{I}^D_{\mathcal{A}}, \mathcal{F}^D_{\mathcal{A}}: \ \mathbb{X} \to \mathbb{I}$ and

$$\mathcal{T}^D_{\mathcal{A}}(u) = \frac{1}{|\mathbb{X}|} \sum_{v \in S(\mathcal{A})} \mathcal{T}_{\mathcal{A}}(v) \Gamma_{\psi_{\mathcal{A}(v)}}(u)$$

$$\mathcal{I}^D_{\mathcal{A}}(u) = \frac{1}{|\mathbb{X}|} \sum_{v \in S(\mathcal{A})} \mathcal{I}_{\mathcal{A}}(v) \Gamma_{\psi_{\mathcal{A}(v)}}(u)$$

$$\mathcal{F}^D_{\mathcal{A}}(u) = \frac{1}{|\mathbb{X}|} \sum_{v \in S(\mathcal{A})} \mathcal{F}_{\mathcal{A}}(v) \Gamma_{\psi_{\mathcal{A}(v)}}(u)$$

where $|\bullet|$ denotes set cardinality with

$$\Gamma_{\psi_{\mathcal{A}(v)}}(u) = \begin{cases} \psi_{\mathcal{A}}(v); & u \in \Gamma_{\psi_{\mathcal{A}}}(v) \\ 0; & u \notin \Gamma_{\psi_{\mathcal{A}}}(v) \end{cases}$$

Definition 3.12. If $\Psi_{\mathcal{A}} \in \Omega_{NPFHS}(\mathbb{X})$ with neutrosophic decision set $\Psi^D_{\mathcal{A}}$ then reduced fuzzy set of $\Psi^D_{\mathcal{A}}$ is a fuzzy set represented as

$$\mathbb{R}(\Psi^D_{\mathcal{A}}) = \left\{ \zeta_{\Psi^D_{\mathcal{A}}}(u)/u : u \in \mathbb{X} \right\}$$

where $\zeta_{\Psi^D_{\mathcal{A}}}: \ \mathbb{X} \to \mathbb{I}$ with $\zeta_{\Psi^D_{\mathcal{A}}}(u) = \mathcal{T}^D_{\mathcal{A}}(u) + \mathcal{I}^D_{\mathcal{A}}(u) - \mathcal{F}^D_{\mathcal{A}}(u)$

Algorithm 3.1. Once $\Psi^D_{\mathcal{A}}$ has been established, it may be indispensable to select the best single substitute from the options. Therefore, decision can be set up with the help of following algorithm.

Step 1 Determine $\mathcal{A} = \{< \mathcal{T}_{\mathcal{A}}(g), \mathcal{I}_{\mathcal{A}}(g), \mathcal{F}_{\mathcal{A}}(g) > /g: \ \mathcal{T}_{\mathcal{A}}(g), \mathcal{I}_{\mathcal{A}}(g), \mathcal{F}_{\mathcal{A}}(g) \in \mathbb{I}, g \in \mathbb{G}\}$,

Step 2 Find $\psi_{\mathcal{A}}(g)$

Step 3 Construct $\Psi_{\mathcal{A}}$ over $\mathbb{X}$,

Step 4 Compute $\Psi^D_{\mathcal{A}}$,

Step 5 Choose the maximum of $\zeta_{\Psi^D_{\mathcal{A}}}(u)$.

Example 3.2. Suppose that Mr. James Peter wants to buy a mobile tablet from a mobile market. There are eight kinds of tablets (options) which form the set of discourse $\mathbb{X} = \{\hat{T}_1, \hat{T}_2, \hat{T}_3, \hat{T}_4, \hat{T}_5, \hat{T}_6, \hat{T}_7, \hat{T}_8\}$. The best selection may be evaluated by observing the attributes, i.e., a_1 = Storage (GB), a_2 = Camera Resolution (mega pixels), a_3 = Size (inches), a_4 = RAM (GB), and a_5 = Battery power (mAh). The attribute-valued sets corresponding to these attributes are:

$A_1 = \{a_{11} = 64, a_{12} = 128\}$

$A_2 = \{a_{21} = 8, a_{22} = 16\}$

$A_3 = \{a_{31} = 10, a_{32} = 11\}$

$A_4 = \{a_{41} = 2, a_{42} = 4\}$

$A_5 = \{a_{51} = 5000\}$

then $\mathbb{R} = A_1 \times A_2 \times A_3 \times A_4 \times A_5$

$\mathbb{R} = \{r_1, r_2, r_3, r_4, \ldots, r_{16}\}$ where each r_i, $i = 1, 2, \ldots, 16$, is a 5-tuples element.

Step 1:

From Tabs. 1–3, we can construct $\boldsymbol{\mathcal{A}}$ as

$$\boldsymbol{\mathcal{A}} = \left\{ \begin{array}{l} <0.1, 0.2, 0.3> /r_1, \quad <0.2, 0.3, 0.4> /r_2, \quad <0.3, 0.4, 0.5> /r_3, \quad <0.4, 0.5, 0.6> /r_4, \\ <0.5, 0.6, 0.7> /r_5, \quad <0.6, 0.7, 0.8> /r_6, \quad <0.7, 0.8, 0.9> /r_7, \quad <0.8, 0.9, 0.1> /r_8, \\ <0.9, 0.1, 0.2> /r_9, \quad <0.16, 0.27, 0.37> /r_{10}, \quad <0.25, 0.35, 0.45> /r_{11}, \\ <0.45, 0.55, 0.65> /r_{12}, \quad <0.35, 0.45, 0.55> /r_{13}, \quad <0.75, 0.85, 0.95> /r_{14}, \\ <0.65, 0.75, 0.85> /r_{15}, \quad <0.85, 0.95, 0.96> /r_{16} \end{array} \right\}.$$

Step 2:

Tab. 4 presents $\psi_{\boldsymbol{\mathcal{A}}}(r_i)$ corresponding to each element of $\mathbb{G}$.

Table 1: Degrees of membership $\mathcal{T}_{\boldsymbol{\mathcal{A}}}(r_i)$

$\mathcal{T}_{\boldsymbol{\mathcal{A}}}(r_i)$	Degree	$\mathcal{T}_{\boldsymbol{\mathcal{A}}}(r_i)$	Degree
$\mathcal{T}_{\boldsymbol{\mathcal{A}}}(r_1)$	0.1	$\mathcal{T}_{\boldsymbol{\mathcal{A}}}(r_9)$	0.9
$\mathcal{T}_{\boldsymbol{\mathcal{A}}}(r_2)$	0.2	$\mathcal{T}_{\boldsymbol{\mathcal{A}}}(r_{10})$	0.16
$\mathcal{T}_{\boldsymbol{\mathcal{A}}}(r_3)$	0.3	$\mathcal{T}_{\boldsymbol{\mathcal{A}}}(r_{11})$	0.25
$\mathcal{T}_{\boldsymbol{\mathcal{A}}}(r_4)$	0.4	$\mathcal{T}_{\boldsymbol{\mathcal{A}}}(r_{12})$	0.45
$\mathcal{T}_{\boldsymbol{\mathcal{A}}}(r_5)$	0.5	$\mathcal{T}_{\boldsymbol{\mathcal{A}}}(r_{13})$	0.35
$\mathcal{T}_{\boldsymbol{\mathcal{A}}}(r_6)$	0.6	$\mathcal{T}_{\boldsymbol{\mathcal{A}}}(r_{14})$	0.75
$\mathcal{T}_{\boldsymbol{\mathcal{A}}}(r_7)$	0.7	$\mathcal{T}_{\boldsymbol{\mathcal{A}}}(r_{15})$	0.65
$\mathcal{T}_{\boldsymbol{\mathcal{A}}}(r_8)$	0.8	$\mathcal{T}_{\boldsymbol{\mathcal{A}}}(r_{16})$	0.85

Table 2: Degrees of indeterminacy $\mathcal{I}_{\mathcal{A}}(r_i)$

$\mathcal{I}_{\mathcal{A}}(r_i)$	Degree	$\mathcal{I}_{\mathcal{A}}(r_i)$	Degree
$\mathcal{I}_{\mathcal{A}}(r_1)$	0.2	$\mathcal{I}_{\mathcal{A}}(r_9)$	0.1
$\mathcal{I}_{\mathcal{A}}(r_2)$	0.3	$\mathcal{I}_{\mathcal{A}}(r_{10})$	0.27
$\mathcal{I}_{\mathcal{A}}(r_3)$	0.4	$\mathcal{I}_{\mathcal{A}}(r_{11})$	0.35
$\mathcal{I}_{\mathcal{A}}(r_4)$	0.5	$\mathcal{I}_{\mathcal{A}}(r_{12})$	0.55
$\mathcal{I}_{\mathcal{A}}(r_5)$	0.6	$\mathcal{I}_{\mathcal{A}}(r_{13})$	0.45
$\mathcal{I}_{\mathcal{A}}(r_6)$	0.7	$\mathcal{I}_{\mathcal{A}}(r_{14})$	0.85
$\mathcal{I}_{\mathcal{A}}(r_7)$	0.8	$\mathcal{I}_{\mathcal{A}}(r_{15})$	0.75
$\mathcal{I}_{\mathcal{A}}(r_8)$	0.9	$\mathcal{I}_{\mathcal{A}}(r_{16})$	0.95

Table 3: Degrees of non-membership $\mathcal{F}_{\mathcal{A}}(r_i)$

$\mathcal{F}_{\mathcal{A}}(r_i)$	Degree	$\mathcal{F}_{\mathcal{A}}(r_i)$	Degree
$\mathcal{F}_{\mathcal{A}}(r_1)$	0.3	$\mathcal{F}_{\mathcal{A}}(r_9)$	0.2
$\mathcal{F}_{\mathcal{A}}(r_2)$	0.4	$\mathcal{F}_{\mathcal{A}}(r_{10})$	0.37
$\mathcal{F}_{\mathcal{A}}(r_3)$	0.5	$\mathcal{F}_{\mathcal{A}}(r_{11})$	0.45
$\mathcal{F}_{\mathcal{A}}(r_4)$	0.6	$\mathcal{F}_{\mathcal{A}}(r_{12})$	0.65
$\mathcal{F}_{\mathcal{A}}(r_5)$	0.7	$\mathcal{F}_{\mathcal{A}}(r_{13})$	0.55
$\mathcal{F}_{\mathcal{A}}(r_6)$	0.8	$\mathcal{F}_{\mathcal{A}}(r_{14})$	0.95
$\mathcal{F}_{\mathcal{A}}(r_7)$	0.9	$\mathcal{F}_{\mathcal{A}}(r_{15})$	0.85
$\mathcal{F}_{\mathcal{A}}(r_8)$	0.1	$\mathcal{F}_{\mathcal{A}}(r_{16})$	0.96

Table 4: Approximate functions $\psi_{\mathcal{A}}(r_i)$

r_i	$\psi_{\mathcal{A}}(r_i)$	r_i	$\psi_{\mathcal{A}}(r_i)$
r_1	$\{0.2/\hat{T}_1, 0.3/\hat{T}_2\}$	r_9	$\{0.4/\hat{T}_2, 0.6/\hat{T}_7, 0.5/\hat{T}_8\}$
r_2	$\{0.1/\hat{T}_1, 0.5/\hat{T}_2, 0.1/\hat{T}_3\}$	r_{10}	$\{0.2/\hat{T}_6, 0.6/\hat{T}_7, 0.4/\hat{T}_8\}$
r_3	$\{0.4/\hat{T}_2, 0.5/\hat{T}_3, 0.6/\hat{T}_4\}$	r_{11}	$\{0.5/\hat{T}_2, 0.6/\hat{T}_4, 0.7/\hat{T}_6\}$
r_4	$\{0.6/\hat{T}_4, 0.7/\hat{T}_5, 0.8/\hat{T}_6\}$	r_{12}	$\{0.7/\hat{T}_2, 0.8/\hat{T}_3, 0.9/\hat{T}_6\}$
r_5	$\{0.2/\hat{T}_6, 0.1/\hat{T}_7, 0.4/\hat{T}_8\}$	r_{13}	$\{0.2/\hat{T}_3, 0.4/\hat{T}_5, 0.6/\hat{T}_7\}$
r_6	$\{0.4/\hat{T}_2, 0.3/\hat{T}_3, 0.4/\hat{T}_4\}$	r_{14}	$\{0.2/\hat{T}_1, 0.5/\hat{T}_3, 0.6/\hat{T}_5\}$
r_7	$\{0.2/\hat{T}_1, 0.3/\hat{T}_3, 0.4/\hat{T}_5\}$	r_{15}	$\{0.6/\hat{T}_5, 0.4/\hat{T}_7, 0.2/\hat{T}_8\}$
r_8	$\{0.1/\hat{T}_2, 0.3/\hat{T}_3, 0.5/\hat{T}_7\}$	r_{16}	$\{0.3/\hat{T}_4, 0.5/\hat{T}_5, 0.7/\hat{T}_6\}$

Step 3:

With the help of Step 1 and Step 2, we can construct $\Psi_{\mathcal{A}}$ as

$$\Psi_{\mathcal{A}} = \left\{ \begin{array}{l} \left(< 0.1, 0.2, 0.3 > /r_1, \{0.2/\hat{T}_1, 0.3/\hat{T}_2\}\right), \\ \left(< 0.2, 0.3, 0.4 > /r_2, \{0.1/\hat{T}_1, 0.5/\hat{T}_2, 0.1\hat{T}_3\}\right), \\ \left(< 0.3, 0.4, 0.5 > /r_3, \{0.4/\hat{T}_2, 0.5/\hat{T}_3, 0.6/\hat{T}_4\}\right), \\ \left(< 0.4, 0.5, 0.6 > /r_4, \{0.6/\hat{T}_4, 0.7/\hat{T}_5, 0.8/\hat{T}_6\}\right), \\ \left(< 0.5, 0.6, 0.7 > /r_5, \{0.2/\hat{T}_6, 0.1/\hat{T}_7, 0.4/\hat{T}_8\}\right), \\ \left(< 0.6, 0.7, 0.8 > /r_6, \{0.4/\hat{T}_2, 0.3/\hat{T}_3, 0.4/\hat{T}_4\}\right), \\ \left(< 0.7, 0.8, 0.9 > /r_7, \{0.2/\hat{T}_1, 0.3/\hat{T}_3, 0.4/\hat{T}_5\}\right), \\ \left(< 0.8, 0.9, 0.1 > /r_8, \{0.1/\hat{T}_2, 0.3/\hat{T}_3, 0.5/\hat{T}_7\}\right), \\ \left(< 0.9, 0.1, 0.2 > /r_9, \{0.4/\hat{T}_2, 0.6/\hat{T}_7, 0.5/\hat{T}_8\}\right), \\ \left(< 0.16, 0.27, 0.37 > /r_{10}, \{0.2/\hat{T}_6, 0.6/\hat{T}_7, 0.4/\hat{T}_8\}\right), \\ \left(< 0.25, 0.35, 0.45 > /r_{11}, \{0.5/\hat{T}_2, 0.6/\hat{T}_4, 0.7/\hat{T}_6\}\right), \\ \left(< 0.45, 0.55, 0.65 > /r_{12}, \{0.7/\hat{T}_2, 0.8/\hat{T}_3, 0.9/\hat{T}_6\}\right), \\ \left(< 0.35, 0.45, 0.55 > /r_{13}, \{0.2/\hat{T}_3, 0.4/\hat{T}_5, 0.6/\hat{T}_7\}\right), \\ \left(< 0.75, 0.85, 0.95 > /r_{14}, \{0.2/\hat{T}_1, 0.5/\hat{T}_3, 0.6/\hat{T}_5\}\right), \\ \left(< 0.65, 0.75, 0.85 > /r_{15}, \{0.6/\hat{T}_5, 0.4/\hat{T}_7, 0.2/\hat{T}_8\}\right), \\ \left(< 0.85, 0.95, 0.96 > /r_{16}, \{0.3/\hat{T}_4, 0.5/\hat{T}_5, 0.7/\hat{T}_6\}\right) \end{array} \right\}$$

Step 4:

From Tabs. 5–8, we can construct $\mathbb{R}(\Psi_{\mathcal{A}}^D)$ as

$$\mathbb{R}(\Psi_{\mathcal{A}}^D) = \left\{ \begin{array}{l} 0.0325/\hat{T}_1,\ 0.1412/\hat{T}_2,\ 0.1968/\hat{T}_3,\ 0.1052/\hat{T}_4, \\ 0.2112/\hat{T}_5,\ 0.1675/\hat{T}_6,\ 0.2158/\hat{T}_7,\ 0.0867/\hat{T}_8 \end{array} \right\}$$

The graphical representation of this decision system is presented in Fig. 1.

Step 5:

Since maximum of $\zeta_{\Psi^D_{\mathcal{A}}}(\hat{T}_i)$ is 0.2158 so the tablet $\hat{T}_7$ is selected.

Table 5: Membership values $\mathcal{T}^D_{\mathcal{A}}(\hat{T}_i)$

$\hat{T}_i$	$\mathcal{T}^D_{\mathcal{A}}(\hat{T}_i)$	$\hat{T}_i$	$\mathcal{T}^D_{\mathcal{A}}(\hat{T}_i)$
$\hat{T}_1$	0.0413	$\hat{T}_5$	0.2456
$\hat{T}_2$	0.1700	$\hat{T}_6$	0.2034
$\hat{T}_3$	0.2006	$\hat{T}_7$	0.1945
$\hat{T}_4$	0.1331	$\hat{T}_8$	0.1055

Table 6: Indeterminacy values $\mathcal{I}^D_{\mathcal{A}}(\hat{T}_i)$

$\hat{T}_i$	$\mathcal{I}^D_{\mathcal{A}}(\hat{T}_i)$	$\hat{T}_i$	$\mathcal{I}^D_{\mathcal{A}}(\hat{T}_i)$
$\hat{T}_1$	0.0500	$\hat{T}_5$	0.2856
$\hat{T}_2$	0.1650	$\hat{T}_6$	0.2474
$\hat{T}_3$	0.2381	$\hat{T}_7$	0.1628
$\hat{T}_4$	0.1644	$\hat{T}_8$	0.0685

Table 7: Non-membership values $\mathcal{F}^D_{\mathcal{A}}(\hat{T}_i)$

$\hat{T}_i$	$\mathcal{F}^D_{\mathcal{A}}(\hat{T}_i)$	$\hat{T}_i$	$\mathcal{F}^D_{\mathcal{A}}(\hat{T}_i)$
$\hat{T}_1$	0.0588	$\hat{T}_5$	0.3200
$\hat{T}_2$	0.1938	$\hat{T}_6$	0.2833
$\hat{T}_3$	0.2419	$\hat{T}_7$	0.1415
$\hat{T}_4$	0.1923	$\hat{T}_8$	0.0873

Table 8: Reduced fuzzy membership $\zeta_{\Psi^D_{\mathcal{A}}}(\hat{T}_i)$

$\hat{T}_i$	$\zeta_{\Psi^D_{\mathcal{A}}}(\hat{T}_i)$	$\hat{T}_i$	$\zeta_{\Psi^D_{\mathcal{A}}}(\hat{T}_i)$
$\hat{T}_1$	0.0325	$\hat{T}_5$	0.2112
$\hat{T}_2$	0.1412	$\hat{T}_6$	0.1675
$\hat{T}_3$	0.1968	$\hat{T}_7$	0.2158
$\hat{T}_4$	0.1052	$\hat{T}_8$	0.0867

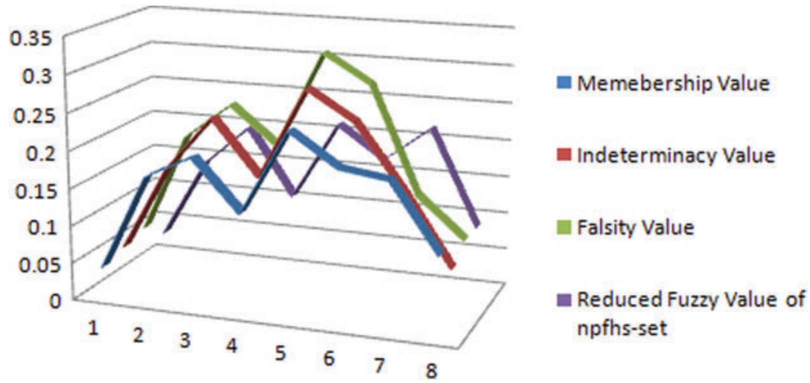

Figure 1: Neutrosophic decision system on npfhs-set

4 Neutrosophic Parameterized Intuitionistic Fuzzy Hypersoft Set (*npifhs*-set) with Application

In this section, *npifhs*-set theory is developed and decision making based application is presented.

Definition 4.1. Let $\mathcal{Y} = \{\mathcal{Y}_1, \mathcal{Y}_2, \mathcal{Y}_3, \ldots, \mathcal{Y}_n\}$ be a collection of disjoint attribute-valued sets corresponding to n distinct attributes $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n$, respectively. A *npifhs*-set $\Psi_{\mathcal{B}}$ over $\mathbb{X}$ is defined as

$$\Psi_{\mathcal{B}} = \{(< L_{\mathcal{B}}(g), M_{\mathcal{B}}(g), N_{\mathcal{B}}(g) > /g, \psi_{\mathcal{B}}(g)) : g \in \mathbb{G}, \psi_{\mathcal{A}}(g) \in IF(\mathbb{X})\}$$

where

(i) $IF(\mathcal{U})$ is a collection of all intuitionistic fuzzy sets over $\mathbb{X}$
(ii) $\mathbb{G} = \mathcal{Y}_1 \times \mathcal{Y}_2 \times \mathcal{Y}_3 \times \ldots \times \mathcal{Y}_n$
(iii) $\mathcal{B}$ is a neutrosophic set over $\mathbb{G}$ with $L_{\mathcal{B}}, M_{\mathcal{B}}, N_{\mathcal{B}} : \mathbb{G} \to \mathbb{I}$ as membership function, indeterminacy function and nonmembership function of *npifhs*-set.
(iv) $\psi_{\mathcal{B}}(g)$ is a fuzzy set for all $g \in \mathbb{G}$ with $\psi_{\mathcal{B}} : \mathbb{G} \to IF(\mathbb{X})$ and is called approximate function of *npifhs*-set.

Note that collection of all *npifhs*-sets is represented by $\Omega_{NPIFHS}(\mathbb{X})$.

Definition 4.2. Let $\Psi_{\mathcal{B}} \in \Omega_{NPIFHS}(\mathbb{X})$. If $\psi_{\mathcal{B}}(g) = \emptyset$, $L_{\mathcal{B}}(g) = 0$, $M_{\mathcal{B}}(g) = 1$, $N_{\mathcal{B}}(g) = 1$ for all $g \in \mathbb{G}$, then $\Psi_{\mathcal{B}}$ is called $\mathcal{B}$-empty *npifhs*-set, denoted by $\Psi_{\Phi_{\mathcal{B}}}$. If $\mathcal{B} = \emptyset$, then $\mathcal{B}$-empty *npifhs*-set is called an empty *npifhs*-set, denoted by Ψ_{Φ}.

Definition 4.3. Let $\Psi_{\mathcal{B}} \in \Omega_{NPIFHS}(\mathbb{X})$. If $\psi_{\mathcal{B}}(g) = \mathbb{X}$, $L_{\mathcal{B}}(g) = 1$, $M_{\mathcal{B}}(g) = 0$, $N_{\mathcal{B}}(g) = 0$ for all $g \in \mathbb{G}$, then $\Psi_{\mathcal{B}}$ is called $\mathcal{B}$-universal *npifhs*-set, denoted by $\Psi_{\tilde{\mathcal{B}}}$. If $\mathcal{B} = \mathbb{G}$, then the $\mathcal{B}$-universal *npifhs*-set is called universal *npifhs*-set, denoted by $\Psi_{\tilde{\mathbb{G}}}$.

Example 4.1. Consider $\mathbb{X} = \{u_1, u_2, u_3, u_4, u_5\}$ and $\mathcal{Y} = \{\mathcal{Y}_1, \mathcal{Y}_2, \mathcal{Y}_3\}$ with

$\mathcal{Y}_1 = \{\hat{y}_{11}, \hat{y}_{12}\}$, $\mathcal{Y}_2 = \{\hat{y}_{21}, \hat{y}_{22}\}$, $\mathcal{Y}_3 = \{\hat{y}_{31}\}$, then

$$\mathbb{G} = \mathcal{Y}_1 \times \mathcal{Y}_2 \times \mathcal{Y}_3$$

$$\mathbb{G} = \{(\hat{y}_{11}, \hat{y}_{21}, \hat{y}_{31}), (\hat{y}_{11}, \hat{y}_{22}, \hat{y}_{31}), (\hat{y}_{12}, \hat{y}_{21}, \hat{y}_{31}), (\hat{y}_{12}, \hat{y}_{22}, \hat{y}_{31})\} = \{g_1, g_2, g_3, g_4\}.$$

Case 1.

If $\mathcal{B}_1 = \{<0.2, 0.3, 0.4>/g_2, <0, 1, 1>/g_3, <1, 0, 0>/g_4\}$ and $\psi_{\mathcal{B}_1}(g_2) = \{<0.2, 0.4>/u_2, <0.3, 0.5>/u_4\}$, $\psi_{\mathcal{B}_1}(g_3) = \phi$, and $\psi_{\mathcal{B}_1}(g_4) = \mathbb{X}$, then

$$\Psi_{\mathcal{B}_1} = \left\{\begin{matrix} (<0.2, 0.3, 0.4>/g_2, \{<0.2, 0.4>/u_2, <0.3, 0.5>/u_4\}), \\ (<0, 1, 1>/g_3, \phi), (<1, 0, 0>/g_4, \mathbb{X}) \end{matrix}\right\}.$$

Case 2.

If $\mathcal{B}_2 = \{<0, 1, 1>/g_2, <0, 1, 1>/g_3\}, \psi_{\mathcal{B}_2}(g_2) = \phi$ and $\psi_{\mathcal{B}_2}(g_3) = \phi$, then $\Psi_{\mathcal{B}_2} = \Psi_{\Phi_{\mathcal{B}_2}}$.

Case 3.

If $\mathcal{B}_3 = \phi$ corresponding to all elements of $\mathbb{G}$, then $\Psi_{\mathcal{B}_3} = \Psi_\Phi$.

Case 4.

If $\mathcal{B}_4 = \{<1, 0, 0>/g_1, <1, 0, 0>/g_2\}, \psi_{\mathcal{B}_4}(g_1) = \mathbb{X}$, and $\psi_{\mathcal{B}_4}(g_2) = \mathbb{X}$, then $\Psi_{\mathcal{B}_4} = \Psi_{\tilde{\mathcal{B}}_4}$.

Case 5.

If $\mathcal{B}_5 = \mathbb{X}$ with respect to all elements of $\mathbb{G}$, then $\Psi_{\mathcal{B}_5} = \Psi_{\tilde{\mathbb{G}}}$.

Definition 4.4. Let $\Psi_{\mathcal{B}_1}$, $\Psi_{\mathcal{B}_2} \in \Omega_{NPIFHS}(\mathbb{X})$ then $\Psi_{\mathcal{B}_1}$ is an *npifhs*-subset of $\Psi_{\mathcal{B}_2}$, denoted by $\Psi_{\mathcal{B}_1} \tilde{\subseteq}_{if} \Psi_{\mathcal{B}_2}$ if

$L_{\mathcal{B}_1}(g) \leq L_{\mathcal{B}_2}(g), M_{\mathcal{B}_1}(g) \geq M_{\mathcal{B}_2}(g), N_{\mathcal{B}_1}(g) \geq N_{\mathcal{B}_2}(g)$ and $\psi_{\mathcal{B}_1}(g) \subseteq_{if} \psi_{\mathcal{B}_2}(g)$ for all $g \in \mathbb{G}$.

Definition 4.5. Let $\Psi_{\mathcal{B}_1}, \Psi_{\mathcal{B}_2} \in \Omega_{NPIFHS}(\mathbb{X})$ then, $\Psi_{\mathcal{B}_1}$ and $\Psi_{\mathcal{B}_2}$ are *npifhs*-equal, represented as $\Psi_{\mathcal{B}_1} = \Psi_{\mathcal{B}_2}$, if and only if $L_{\mathcal{B}_1}(g) = L_{\mathcal{B}_2}(g), M_{\mathcal{B}_1}(g) = M_{\mathcal{B}_2}(g), N_{\mathcal{B}_1}(g) = N_{\mathcal{B}_2}(g)$ and $\psi_{\mathcal{B}_1}(g) = \psi_{\mathcal{B}_2}(g)$ for all $g \in \mathbb{G}$.

Definition 4.6. Let $\Psi_{\mathcal{B}} \in \Omega_{NPIFHS}(\mathbb{X})$ then, complement of $\Psi_{\mathcal{B}}$ (i.e., $\Psi^{\tilde{c}}_{\mathcal{B}}$) is an *npifhs*-set given as $P^{\tilde{c}}_{\mathcal{B}}(g) = 1 - L_{\mathcal{B}}(g), Q^{\tilde{c}}_{\mathcal{B}}(g) = 1 - M_{\mathcal{B}}(g), R^{\tilde{c}}_{\mathcal{B}}(g) = 1 - N_{\mathcal{B}}(g)$ and $\psi^{\tilde{c}}_{\mathcal{B}}(g) = \mathbb{X} \setminus \psi_{\mathcal{B}}(g)$

Proposition 4.1. Let $\Psi_{\mathcal{B}} \in \Omega_{NPIFHS}(\mathbb{X})$ then,

1. $(\Psi^{\tilde{c}}_{\mathcal{B}})^{\tilde{c}} = \Psi_{\mathcal{B}}$.
2. $\Psi^{\tilde{c}}_{\phi} = \Psi_{\tilde{\mathbb{G}}}$.

Definition 4.7. Let $\Psi_{\mathcal{B}_1}, \Psi_{\mathcal{B}_2} \in \Omega_{NPIFHS}(\mathbb{X})$ then, union of $\Psi_{\mathcal{B}_1}$ and $\Psi_{\mathcal{B}_2}$, denoted by $\Psi_{\mathcal{B}_1} \tilde{\cup}_{if} \Psi_{\mathcal{B}_2}$, is an *npifhs*-set defined by

(i) $L_{\mathcal{B}_1 \tilde{\cup} \mathcal{B}_2}(g) = max\{L_{\mathcal{B}_1}(x), L_{\mathcal{B}_2}(g)\}$,
(ii) $M_{\mathcal{B}_1 \tilde{\cup} \mathcal{B}_2}(g) = min\{M_{\mathcal{B}_1}(x), M_{\mathcal{B}_2}(g)\}$,
(iii) $N_{\mathcal{B}_1 \tilde{\cup} \mathcal{B}_2}(g) = min\{N_{\mathcal{B}_1}(x), N_{\mathcal{B}_2}(g)\}$,
(iv) $\psi_{\mathcal{B}_1 \tilde{\cup} \mathcal{B}_2}(g) = \psi_{\mathcal{B}_1}(g) \tilde{\cup}_{if} \psi_{\mathcal{B}_2}(g)$, for all $g \in \mathbb{G}$.

Definition 4.8. Let $\Psi_{\mathcal{B}_1}, \Psi_{\mathcal{B}_2} \in \Omega_{NPIFHS}(\mathbb{X})$ then intersection of $\Psi_{\mathcal{B}_1}$ and $\Psi_{\mathcal{B}_2}$, denoted by $\Psi_{\mathcal{B}_1} \tilde{\cap}_{if} \Psi_{\mathcal{B}_2}$, is an *npifhs*-set defined by

(i) $L_{\mathcal{B}_1 \tilde{\cap} \mathcal{B}_2}(g) = min\{L_{\mathcal{B}_1}(x), L_{\mathcal{B}_2}(g)\}$,
(ii) $M_{\mathcal{B}_1 \tilde{\cap} \mathcal{B}_2}(g) = max\{M_{\mathcal{B}_1}(x), M_{\mathcal{B}_2}(g)\}$,
(iii) $N_{\mathcal{B}_1 \tilde{\cap} \mathcal{B}_2}(g) = max\{N_{\mathcal{B}_1}(x), N_{\mathcal{B}_2}(g)\}$,
(iv) $\psi_{\mathcal{B}_1 \tilde{\cap} \mathcal{B}_2}(g) = \psi_{\mathcal{B}_1}(g) \tilde{\cap}_{if} \psi_{\mathcal{B}_2}(g)$, for all $g \in \mathbb{G}$.

Remark 4.1. Let $\Psi_{\mathcal{B}} \in \Omega_{NPIFHS}(\mathbb{X})$. If $\Psi_{\mathcal{B}} \neq_{if} \Psi_{\tilde{\mathbb{G}}}$, then $\Psi_{\mathcal{B}} \tilde{\cup}_{if} \Psi^{\tilde{c}}_{\mathcal{B}} \neq_{if} \Psi_{\tilde{\mathbb{G}}}$ and $\Psi_{\mathcal{B}} \tilde{\cap}_{if} \Psi^{\tilde{c}}_{\mathcal{B}} \neq_{if} \Psi_{\Phi}$

Proposition 4.2. Let $\Psi_{\mathcal{B}_1}, \Psi_{\mathcal{B}_2} \in \Omega_{NPIFHS}(\mathbb{X})$ then following D. Morgan laws are valid:

1. $(\Psi_{\mathcal{B}_1} \tilde{\cup}_{if} \Psi_{\mathcal{B}_2})^{\tilde{c}} = \Psi^{\tilde{c}}_{\mathcal{B}_1} \tilde{\cap}_{if} \Psi^{\tilde{c}}_{\mathcal{B}_2}$.
2. $(\Psi_{\mathcal{B}_1} \tilde{\cap}_{if} \Psi_{\mathcal{B}_2})^{\tilde{c}} = \Psi^{\tilde{c}}_{\mathcal{B}_1} \tilde{\cup}_{if} \Psi^{\tilde{c}}_{\mathcal{B}_2}$.

Proof. For all $g \in \mathbb{G}$,

(1). Since
$$\begin{aligned}(L_{\mathcal{B}_1 \tilde{\cup} \mathcal{B}_2})^{\tilde{c}}(g) &= 1 - L_{\mathcal{B}_1 \tilde{\cup} \mathcal{B}_2}(g) \\ &= 1 - max\{L_{\mathcal{B}_1}(g), L_{\mathcal{B}_2}(g)\} \\ &= min\{1 - L_{\mathcal{B}_1}(g), 1 - L_{\mathcal{B}_2}(g)\} \\ &= min\{P^{\tilde{c}}_{\mathcal{B}_1}(g), P^{\tilde{c}}_{\mathcal{B}_2}(g)\} \\ &= P^{\tilde{c}}_{\mathcal{B}_1 \tilde{\cap} \mathcal{B}_2}(g)\end{aligned}$$

also

$$\begin{aligned}(M_{\mathcal{B}_1 \tilde{\cup} \mathcal{B}_2})^{\tilde{c}}(g) &= 1 - M_{\mathcal{B}_1 \tilde{\cup} \mathcal{B}_2}(g) \\ &= 1 - min\{M_{\mathcal{B}_1}(g), M_{\mathcal{B}_2}(g)\} \\ &= max\{1 - M_{\mathcal{B}_1}(g), 1 - M_{\mathcal{B}_2}(g)\} \\ &= max\{Q^{\tilde{c}}_{\mathcal{B}_1}(g), Q^{\tilde{c}}_{\mathcal{B}_2}(g)\} \\ &= Q^{\tilde{c}}_{\mathcal{B}_1 \tilde{\cap} \mathcal{B}_2}(g)\end{aligned}$$

and

$$\begin{aligned}(N_{\mathcal{B}_1 \tilde{\cup} \mathcal{B}_2})^{\tilde{c}}(g) &= 1 - N_{\mathcal{B}_1 \tilde{\cup} \mathcal{B}_2}(g) \\ &= 1 - min\{N_{\mathcal{B}_1}(g), N_{\mathcal{B}_2}(g)\} \\ &= max\{1 - N_{\mathcal{B}_1}(g), 1 - N_{\mathcal{B}_2}(g)\} \\ &= max\{R^{\tilde{c}}_{\mathcal{B}_1}(g), R^{\tilde{c}}_{\mathcal{B}_2}(g)\} \\ &= R^{\tilde{c}}_{\mathcal{B}_1 \tilde{\cap} \mathcal{B}_2}(g)\end{aligned}$$

and

$$
\begin{aligned}
(\psi_{\mathcal{B}_1\tilde{\cup}\mathcal{B}_2})^{\tilde{c}}(g) &= \mathbb{X}\setminus_{if}\ \psi_{\mathcal{B}_1\tilde{\cup}\mathcal{B}_2}(g)\\
&= \mathbb{X}\setminus_{if}\ (\psi_{\mathcal{B}_1}(g)\tilde{\cup}_{if}\psi_{\mathcal{B}_2}(g))\\
&= (\mathbb{X}\setminus_{if}\ \psi_{\mathcal{B}_1}(g))\tilde{\cap}_{if}\ (\mathbb{X}\setminus_{if}\ \psi_{\mathcal{B}_2}(g))\\
&= \psi^{\tilde{c}}_{\mathcal{B}_1}(g)\,\tilde{\cap}_{if}\ \psi^{\tilde{c}}_{\mathcal{B}_2}(g)\\
&= \psi^{\tilde{c}}_{\mathcal{B}_1\tilde{\cap}\mathcal{B}_2}(g).
\end{aligned}
$$

similarly (2) can be proved easily.

Proposition 4.3. Let $\Psi_{\mathcal{B}_1}, \Psi_{\mathcal{B}_2}, \Psi_{\mathcal{B}_3} \in \Omega_{NPIFHS}(\mathbb{X})$ then

1. $\Psi_{\mathcal{B}_1}\,\tilde{\cup}_{if}\,(\Psi_{\mathcal{B}_2}\,\tilde{\cap}_{if}\,\Psi_{\mathcal{B}_3}) = (\Psi_{\mathcal{B}_1}\,\tilde{\cup}_{if}\,\Psi_{\mathcal{B}_2})\,\tilde{\cap}_{if}\,(\Psi_{\mathcal{B}_1}\,\tilde{\cup}_{if}\,\Psi_{\mathcal{B}_3})$.
2. $\Psi_{\mathcal{B}_1}\,\tilde{\cap}_{if}\,(\Psi_{\mathcal{B}_2}\,\tilde{\cup}_{if}\,\Psi_{\mathcal{B}_3}) = (\Psi_{\mathcal{B}_1}\,\tilde{\cap}_{if}\,\Psi_{\mathcal{B}_2})\,\tilde{\cup}_{if}\,(\Psi_{\mathcal{B}_1}\,\tilde{\cap}_{if}\,\Psi_{\mathcal{B}_3})$.

Proof. For all $g \in \mathbb{G}$,

(1). Since
$$
\begin{aligned}
L_{\mathcal{B}_1\tilde{\cup}(\mathcal{B}_2\tilde{\cap}\mathcal{B}_3)}(g) &= max\{L_{\mathcal{B}_1}(g), L_{\mathcal{B}_2\tilde{\cap}\mathcal{B}_3}(g)\}\\
&= max\{L_{\mathcal{B}_1}(g), min\{L_{\mathcal{B}_2}(g), L_{\mathcal{B}_3}(g)\}\}\\
&= min\{max\{L_{\mathcal{B}_1}(g), L_{\mathcal{B}_2}(g)\}, max\{L_{\mathcal{B}_1}(g), L_{\mathcal{B}_3}(g)\}\}\\
&= min\{L_{\mathcal{B}_1\tilde{\cup}\mathcal{B}_2}(g), L_{\mathcal{B}_1\tilde{\cup}\mathcal{B}_3}(g)\}\\
&= L_{(\mathcal{B}_1\tilde{\cup}\mathcal{B}_2)\tilde{\cap}(\mathcal{B}_1\tilde{\cup}\mathcal{B}_3)}(g)
\end{aligned}
$$

and

$$
\begin{aligned}
M_{\mathcal{B}_1\tilde{\cup}(\mathcal{B}_2\tilde{\cap}\mathcal{B}_3)}(g) &= min\{M_{\mathcal{B}_1}(g), M_{\mathcal{B}_2\tilde{\cap}\mathcal{B}_3}(g)\}\\
&= min\{M_{\mathcal{B}_1}(g), max\{M_{\mathcal{B}_2}(g), M_{\mathcal{B}_3}(g)\}\}\\
&= max\{min\{M_{\mathcal{B}_1}(g), M_{\mathcal{B}_2}(g)\}, min\{M_{\mathcal{B}_1}(g), M_{\mathcal{B}_3}(g)\}\}\\
&= max\{M_{\mathcal{B}_1\tilde{\cup}\mathcal{B}_2}(g), M_{\mathcal{B}_1\tilde{\cup}\mathcal{B}_3}(g)\}\\
&= M_{(\mathcal{B}_1\tilde{\cup}\mathcal{B}_2)\tilde{\cap}(\mathcal{B}_1\tilde{\cup}\mathcal{B}_3)}(g)
\end{aligned}
$$

and

$$
\begin{aligned}
N_{\mathcal{B}_1\tilde{\cup}(\mathcal{B}_2\tilde{\cap}\mathcal{B}_3)}(g) &= min\{N_{\mathcal{B}_1}(g), N_{\mathcal{B}_2\tilde{\cap}\mathcal{B}_3}(g)\}\\
&= min\{N_{\mathcal{B}_1}(g), max\{N_{\mathcal{B}_2}(g), N_{\mathcal{B}_3}(g)\}\}\\
&= max\{min\{N_{\mathcal{B}_1}(g), N_{\mathcal{B}_2}(g)\}, min\{N_{\mathcal{B}_1}(g), N_{\mathcal{B}_3}(g)\}\}\\
&= max\{N_{\mathcal{B}_1\tilde{\cup}\mathcal{B}_2}(g), N_{\mathcal{B}_1\tilde{\cup}\mathcal{B}_3}(g)\}\\
&= N_{(\mathcal{B}_1\tilde{\cup}\mathcal{B}_2)\tilde{\cap}(\mathcal{B}_1\tilde{\cup}\mathcal{B}_3)}(g)
\end{aligned}
$$

and

$$\begin{aligned}\psi_{\mathcal{B}_1\tilde{\cup}(\mathcal{B}_2\tilde{\cap}\mathcal{B}_3)}(g) &= \psi_{\mathcal{B}_1}(g)\tilde{\cup}_{if}\psi_{\mathcal{B}_2\tilde{\cap}\mathcal{B}_3}(g)\\ &= \psi_{\mathcal{B}_1}(g)\tilde{\cup}_{if}(\psi_{\mathcal{B}_2}(g)\tilde{\cap}_{if}\psi_{\mathcal{B}_3}(g))\\ &= (\psi_{\mathcal{B}_1}(g)\tilde{\cup}_{if}\psi_{\mathcal{B}_2}(g))\tilde{\cap}_{if}(\psi_{\mathcal{B}_1}(g)\tilde{\cup}_{if}\psi_{\mathcal{B}_3}(g))\\ &= \psi_{\mathcal{B}_1\tilde{\cup}\mathcal{B}_2}(g)\tilde{\cap}_{if}\psi_{\mathcal{B}_1\tilde{U}\mathcal{B}_3}(g)\\ &= \psi_{(\mathcal{B}_1\tilde{\cup}\mathcal{B}_2)\tilde{\cap}(\mathcal{B}_1\tilde{\cup}\mathcal{B}_3)}(g)\end{aligned}$$

In the same way, (2) can be proved.

Definition 4.9. Let $\Psi_{\mathcal{B}_1}, \Psi_{\mathcal{B}_2} \in \Omega_{NPIFHS}(\mathbb{X})$ then OR-operation of $\Psi_{\mathcal{B}_1}$ and $\Psi_{\mathcal{B}_2}$, denoted by $\Psi_{\mathcal{B}_1}\tilde{\veebar}\Psi_{\mathcal{B}_2}$, is an *npifhs*-set defined by

(i) $L_{\mathcal{B}_1\tilde{\veebar}\mathcal{B}_2}(g_1,g_2) = max\{L_{\mathcal{B}_1}(g_1), L_{\mathcal{B}_2}(g_2)\}$,
(ii) $M_{\mathcal{B}_1\tilde{\veebar}\mathcal{B}_2}(g_1,g_2) = min\{M_{\mathcal{B}_1}(g_1), M_{\mathcal{B}_2}(g_2)\}$,
(iii) $N_{\mathcal{B}_1\tilde{\veebar}\mathcal{B}_2}(g_1,g_2) = min\{N_{\mathcal{B}_1}(g_1), N_{\mathcal{B}_2}(g_2)\}$,
(iv) $\psi_{\mathcal{B}_1\tilde{\veebar}\mathcal{B}_2}(g_1,g_2) = \psi_{\mathcal{B}_1}(g_1)\cup\psi_{\mathcal{B}_2}(g_2)$, for all $(g_1,g_2)\in\mathcal{B}_1\times\mathcal{B}_2$.

Definition 4.10. Let $\Psi_{\mathcal{B}_1}, \Psi_{\mathcal{B}_2} \in \Omega_{NPIFHS}(\mathbb{X})$ then AND-operation of $\Psi_{\mathcal{B}_1}$ and $\Psi_{\mathcal{B}_2}$, denoted by $\Psi_{\mathcal{B}_1}\tilde{\tilde{\wedge}}\Psi_{\mathcal{B}_2}$, is an *npifhs*-set defined by

(i) $L_{\mathcal{B}_1\tilde{\tilde{\wedge}}\mathcal{B}_2}(g_1,g_2) = min\{L_{\mathcal{B}_1}(g_1), L_{\mathcal{B}_2}(g_2)\}$,
(ii) $M_{\mathcal{B}_1\tilde{\tilde{\wedge}}\mathcal{B}_2}(g_1,g_2) = max\{M_{\mathcal{B}_1}(g_1), M_{\mathcal{B}_2}(g_2)\}$,
(iii) $N_{\mathcal{B}_1\tilde{\tilde{\wedge}}\mathcal{B}_2}(g_1,g_2) = max\{N_{\mathcal{B}_1}(g_1), N_{\mathcal{B}_2}(g_2)\}$,
(iv) $\psi_{\mathcal{B}_1\tilde{\tilde{\wedge}}\mathcal{B}_2}(g_1,g_2) = \psi_{\mathcal{B}_1}(g_1)\cap\psi_{\mathcal{B}_2}(g_2)$, for all $(g_1,g_2)\in\mathcal{B}_1\times\mathcal{B}_2$.

Proposition 4.4. Let $\Psi_{\mathcal{B}_1}, \Psi_{\mathcal{B}_2}, \Psi_{\mathcal{B}_3} \in \Omega_{NPIFHS}(\mathbb{X})$ then

1. $\Psi_{\mathcal{B}_1}\tilde{\tilde{\wedge}}\Psi_{\Phi} = \Psi_{\Phi}$.
2. $(\Psi_{\mathcal{B}_1}\tilde{\tilde{\wedge}}\Psi_{\mathcal{B}_2})\tilde{\tilde{\wedge}}\Psi_{\mathcal{B}_3} = \Psi_{\mathcal{B}_1}\tilde{\tilde{\wedge}}(\Psi_{\mathcal{B}_2}\tilde{\tilde{\wedge}}\Psi_{\mathcal{B}_3})$.
3. $(\Psi_{\mathcal{B}_1}\tilde{\veebar}\Psi_{\mathcal{B}_2})\tilde{\veebar}\Psi_{\mathcal{B}_3} = \Psi_{\mathcal{B}_1}\tilde{\veebar}(\Psi_{\mathcal{B}_2}\tilde{\veebar}\Psi_{\mathcal{B}_3})$.

4.1 Neutrosophic Decision Set of npifhs-Set

Here an algorithm is presented with the help of characterization of neutrosophic decision set on *npifhs*-set which based on decision making technique and is explained with example.

Definition 4.11. Let $\Psi_{\mathcal{B}} \in \Omega_{NPIFHS}(\mathbb{X})$ then a neutrosophic decision set of $\Psi_{\mathcal{B}}$ (i.e., $\Psi^D_{\mathcal{B}}$) is represented as

$$\Psi^D_{\mathcal{B}} = \left\{ < \mathcal{T}^D_{\mathcal{B}}(u), \mathcal{I}^D_{\mathcal{B}}(u), \mathcal{F}^D_{\mathcal{B}}(u) > /u :\ u\in\mathbb{X} \right\}$$

where $\mathcal{T}^D_{\mathcal{B}}, \mathcal{I}^D_{\mathcal{B}}, \mathcal{F}^D_{\mathcal{B}}: \ \mathbb{X} \to \mathbb{I}$ and

$$\mathcal{T}^D_{\mathcal{B}}(u) = \frac{1}{|\mathbb{X}|} \sum_{v \in S(\mathcal{B})} \mathcal{T}_{\mathcal{B}}(v) \Gamma_{\psi_{\mathcal{B}}(v)}(u)$$

$$\mathcal{I}^D_{\mathcal{B}}(u) = \frac{1}{|\mathbb{X}|} \sum_{v \in S(\mathcal{B})} \mathcal{I}_{\mathcal{B}}(v) \Gamma_{\psi_{\mathcal{B}}(v)}(u)$$

$$\mathcal{F}^D_{\mathcal{B}}(u) = \frac{1}{|\mathbb{X}|} \sum_{v \in S(\mathcal{B})} \mathcal{F}_{\mathcal{B}}(v) \Gamma_{\psi_{\mathcal{B}}(v)}(u)$$

where $|\bullet|$ denotes set cardinality with

$$\Gamma_{\psi_{\mathcal{B}}(v)}(u) = \begin{cases} |T_{\psi_{\mathcal{B}}}(u) - F_{\psi_{\mathcal{B}}}(u)|; & u \in \Gamma_{\psi_{\mathcal{B}}}(v) \\ 0; & u \notin \Gamma_{\psi_{\mathcal{B}}}(v) \end{cases}$$

Definition 4.12. If $\Psi_{\mathcal{B}} \in \Omega_{NPIFHS}(\mathbb{X})$ with neutrosophic decision set $\Psi^D_{\mathcal{B}}$ then reduced fuzzy set of $\Psi^D_{\mathcal{B}}$ is a fuzzy set represented as

$$\mathbb{R}(\Psi^D_{\mathcal{B}}) = \left\{ \zeta_{\Psi^D_{\mathcal{B}}}(u)/u : \ u \in \mathbb{X} \right\}$$

where $\zeta_{\Psi^D_{\mathcal{B}}}: \ \mathbb{X} \to \mathbb{I}$ with $\zeta_{\Psi^D_{\mathcal{B}}}(u) = \mathcal{T}^D_{\mathcal{B}}(u) + \mathcal{I}^D_{\mathcal{B}}(u) - \mathcal{F}^D_{\mathcal{B}}(u)$

4.2 Proposed Algorithm

Once $\Psi^D_{\mathcal{B}}$ has been established, it may be indispensable to select the best single substitute from the options. Therefore, decision can be set up with the help of following algorithm:

Step 1 Determine $\mathcal{B} = \{< \mathcal{T}_{\mathcal{B}}(g), \mathcal{I}_{\mathcal{B}}(g), \mathcal{F}_{\mathcal{B}}(g) > /g : \ \mathcal{T}_{\mathcal{B}}(g), \mathcal{I}_{\mathcal{B}}(g), \mathcal{F}_{\mathcal{B}}(g) \in \mathbb{I}, g \in \mathbb{G}\}$,

Step 2 Find $\psi_{\mathcal{B}}(g)$

Step 3 Construct $\Psi_{\mathcal{B}}$ over $\mathbb{X}$,

Step 4 Compute $\Psi^D_{\mathcal{B}}$,

Step 5 Choose the maximum of $\zeta_{\Psi^D_{\mathcal{B}}}(u)$.

Example 4.2. Suppose that Mrs. Andrew wants to buy a washing machine from market. There are eight kinds of washing machines (options) which form the set of discourse $\mathbb{X} = \{\hat{W}_1, \hat{W}_2, \hat{W}_3, \hat{W}_4, \hat{W}_5, \hat{W}_6, \hat{W}_7, \hat{W}_8\}$. The best selection may be evaluated by observing the attributes i.e., b_1 = Company, b_2 = Power in Watts, b_3 = Voltage, b_4 = Capacity in kg, and b_5 = Color. The attribute-valued sets corresponding to these attributes are:

$B_1 = \{b_{11} = National, b_{12} = Hier\}$

$B_2 = \{b_{21} = 400, b_{22} = 500\}$

$B_3 = \{b_{31} = 220, b_{32} = 240\}$

$B_4 = \{b_{41} = 7, b_{42} = 10\}$

$B_5 = \{b_{51} = White\}$

then $\mathbb{Q} = B_1 \times B_2 \times B_3 \times B_4 \times B_5$

$\mathbb{Q} = \{q_1, q_2, q_3, q_4, \ldots, q_{16}\}$ where each $q_i, i = 1, 2, \ldots, 16$, is a 5-tuples element.

Step 1:

From Tabs. 9–11, we can construct $\mathcal{B}$ as

$$\mathcal{B} = \left\{ \begin{array}{l} <0.1,0.2,0.3>/q_1, <0.2,0.3,0.4>/q_2, \\ <0.3,0.4,0.5>/q_3, <0.4,0.5,0.6>/q_4, \\ <0.5,0.6,0.7>/q_5, <0.6,0.7,0.8>/q_6, \\ <0.7,0.8,0.9>/q_7, <0.8,0.9,0.1>/q_8, \\ <0.9,0.1,0.2>/q_9, <0.16,0.27,0.37>/q_{10}, \\ <0.25,0.35,0.45>/q_{11}, <0.45,0.55,0.65>/q_{12}, \\ <0.35,0.45,0.55>/q_{13}, <0.75,0.85,0.95>/q_{14}, \\ <0.65,0.75,0.85>/q_{15}, <0.85,0.95,0.96>/q_{16} \end{array} \right\}$$

Table 9: Degrees of membership $\mathcal{T}_{\mathcal{B}}(q_i)$

$\mathcal{T}_{\mathcal{B}}(q_i)$	Degree	$\mathcal{T}_{\mathcal{B}}(q_i)$	Degree
$\mathcal{T}_{\mathcal{B}}(q_1)$	0.1	$\mathcal{T}_{\mathcal{B}}(q_9)$	0.9
$\mathcal{T}_{\mathcal{B}}(q_2)$	0.2	$\mathcal{T}_{\mathcal{B}}(q_{10})$	0.16
$\mathcal{T}_{\mathcal{B}}(q_3)$	0.3	$\mathcal{T}_{\mathcal{B}}(q_{11})$	0.25
$\mathcal{T}_{\mathcal{B}}(q_4)$	0.4	$\mathcal{T}_{\mathcal{B}}(q_{12})$	0.45
$\mathcal{T}_{\mathcal{B}}(q_5)$	0.5	$\mathcal{T}_{\mathcal{B}}(q_{13})$	0.35
$\mathcal{T}_{\mathcal{B}}(q_6)$	0.6	$\mathcal{T}_{\mathcal{B}}(q_{14})$	0.75
$\mathcal{T}_{\mathcal{B}}(q_7)$	0.7	$\mathcal{T}_{\mathcal{B}}(q_{15})$	0.65
$\mathcal{T}_{\mathcal{B}}(q_8)$	0.8	$\mathcal{T}_{\mathcal{B}}(q_{16})$	0.85

Table 10: Degrees of indeterminacy $\mathcal{I}_{\mathcal{B}}(q_i)$

$\mathcal{I}_{\mathcal{B}}(q_i)$	Degree	$\mathcal{I}_{\mathcal{B}}(q_i)$	Degree
$\mathcal{I}_{\mathcal{B}}(q_1)$	0.2	$\mathcal{I}_{\mathcal{B}}(q_9)$	0.1
$\mathcal{I}_{\mathcal{B}}(q_2)$	0.3	$\mathcal{I}_{\mathcal{B}}(q_{10})$	0.27
$\mathcal{I}_{\mathcal{B}}(q_3)$	0.4	$\mathcal{I}_{\mathcal{B}}(q_{11})$	0.35
$\mathcal{I}_{\mathcal{B}}(q_4)$	0.5	$\mathcal{I}_{\mathcal{B}}(q_{12})$	0.55
$\mathcal{I}_{\mathcal{B}}(q_5)$	0.6	$\mathcal{I}_{\mathcal{B}}(q_{13})$	0.45
$\mathcal{I}_{\mathcal{B}}(q_6)$	0.7	$\mathcal{I}_{\mathcal{B}}(q_{14})$	0.85
$\mathcal{I}_{\mathcal{B}}(q_7)$	0.8	$\mathcal{I}_{\mathcal{B}}(q_{15})$	0.75
$\mathcal{I}_{\mathcal{B}}(q_8)$	0.9	$\mathcal{I}_{\mathcal{B}}(q_{16})$	0.95

Table 11: Degrees of non-membership $\mathcal{F}_{\mathcal{B}}(q_i)$

$\mathcal{F}_{\mathcal{B}}(q_i)$	Degree	$\mathcal{F}_{\mathcal{B}}(q_i)$	Degree
$\mathcal{F}_{\mathcal{B}}(q_1)$	0.3	$\mathcal{F}_{\mathcal{B}}(q_9)$	0.2
$\mathcal{F}_{\mathcal{B}}(q_2)$	0.4	$\mathcal{F}_{\mathcal{B}}(q_{10})$	0.37
$\mathcal{F}_{\mathcal{B}}(q_3)$	0.5	$\mathcal{F}_{\mathcal{B}}(q_{11})$	0.45
$\mathcal{F}_{\mathcal{B}}(q_4)$	0.6	$\mathcal{F}_{\mathcal{B}}(q_{12})$	0.65
$\mathcal{F}_{\mathcal{B}}(q_5)$	0.7	$\mathcal{F}_{\mathcal{B}}(q_{13})$	0.55
$\mathcal{F}_{\mathcal{B}}(q_6)$	0.8	$\mathcal{F}_{\mathcal{B}}(q_{14})$	0.95
$\mathcal{F}_{\mathcal{B}}(q_7)$	0.9	$\mathcal{F}_{\mathcal{B}}(q_{15})$	0.85
$\mathcal{F}_{\mathcal{B}}(q_8)$	0.1	$\mathcal{F}_{\mathcal{B}}(q_{16})$	0.96

Step 2:

Tab. 12 presents $\psi_{\mathcal{B}}(q_i)$ corresponding to each element of $\mathbb{G}$.

Table 12: Approximate functions $\psi_{\mathcal{B}}(q_i)$

q_i	$\psi_{\mathcal{B}}(q_i)$	q_i	$\psi_{\mathcal{B}}(q_i)$
q_1	$\{<0.2, 0.1>/\hat{W}_1, <0.3, 0.2>/\hat{W}_2\}$	q_9	$\{<0.4, 0.3>/\hat{W}_2, <0.6, 0.4>/\hat{W}_7, <0.5, 0.4>/\hat{W}_8\}$
q_2	$\{<0.1, 0.2>/\hat{W}_1, <0.5, 0.4>/\hat{W}_2, <0.1, 0.4>/\hat{W}_3\}$	q_{10}	$\{<0.2, 0.1>/\hat{W}_6, <0.6, 0.4>/\hat{W}_7, <0.4, 0.3>/\hat{W}_8\}$
q_3	$\{<0.4, 0.3>/\hat{W}_2, <0.5, 0.4>/\hat{W}_3, <0.6, 0.3>/\hat{W}_4\}$	q_{11}	$\{<0.5, 0.4>/\hat{W}_2, <0.6, 0.3>/\hat{W}_4, <0.7, 0.2>/\hat{W}_6\}$
q_4	$\{<0.6, 0.2>/\hat{W}_4, <0.7, 0.3>/\hat{W}_5, <0.8, 0.1>/\hat{W}_6\}$	q_{12}	$\{<0.7, 0.2>/\hat{W}_2, <0.8, 0.1>/\hat{W}_3, <0.9, 0.1>/\hat{W}_6\}$
q_5	$\{<0.2, 0.1>/\hat{W}_6, <0.1, 0.2>/\hat{W}_7, <0.4, 0.3>/\hat{W}_8\}$	q_{13}	$\{<0.2, 0.1>/\hat{W}_3, <0.4, 0.3>/\hat{W}_5, <0.6, 0.1>/\hat{W}_7\}$
q_6	$\{<0.4, 0.2>/\hat{W}_2, <0.3, 0.4>/\hat{W}_3, <0.4, 0.5>/\hat{W}_4\}$	q_{14}	$\{<0.2, 0.5>/\hat{W}_1, <0.5, 0.4>/\hat{W}_3, <0.6, 0.2>/\hat{W}_5\}$
q_7	$\{<0.2, 0.3>/\hat{W}_1, <0.3, 0.4>/\hat{W}_3, <0.4, 0.3>/\hat{W}_5\}$	q_{15}	$\{<0.6, 0.3>/\hat{W}_5, <0.4, 0.3>/\hat{W}_7, <0.2, 0.4>/\hat{W}_8\}$
q_8	$\{<0.1, 0.4>/\hat{W}_2, <0.3, 0.5>/\hat{W}_3, <0.5, 0.4>/\hat{W}_7\}$	q_{16}	$\{<0.3, 0.6>/\hat{W}_4, <0.5, 0.4>/\hat{W}_5, <0.7, 0.1>/\hat{W}_6\}$

Step 3: With the help of Step 1 and Step 2, we can construct $\Psi_{\mathcal{B}}$ as performed in step of Section 3.

Step 4:

From Tabs. 13–16, we can construct $\mathbb{R}(\Psi^D_{\mathcal{B}})$ as

$$\mathbb{R}(\Psi^D_{\mathcal{B}}) = \begin{Bmatrix} 0.0331/\hat{W}_1, 0.1100/\hat{W}_2, 0.1019/\hat{W}_3, 0.0659/\hat{W}_4, \\ 0.0855/\hat{W}_5, 0.1394/\hat{W}_6, 0.0690/\hat{W}_7, 0.0296/\hat{W}_8 \end{Bmatrix}$$

Table 13: Membership values $\mathcal{T}^D_{\mathcal{B}}(\hat{W}_i)$

$\hat{W}_i$	$\mathcal{T}^D_{\mathcal{B}}(\hat{W}_i)$	$\hat{W}_i$	$\mathcal{T}^D_{\mathcal{B}}(\hat{W}_i)$
$\hat{W}_1$	0.0406	$\hat{W}_5$	0.1006
$\hat{W}_2$	0.0950	$\hat{W}_6$	0.1676
$\hat{W}_3$	0.1006	$\hat{W}_7$	0.0728
$\hat{W}_4$	0.0800	$\hat{W}_8$	0.0358

Table 14: Indeterminacy values $\mathcal{I}^D_{\mathcal{B}}(\hat{W}_i)$

$\hat{W}_i$	$\mathcal{I}^D_{\mathcal{B}}(\hat{W}_i)$	$\hat{W}_i$	$\mathcal{I}^D_{\mathcal{B}}(\hat{W}_i)$
$\hat{W}_1$	0.0481	$\hat{W}_5$	0.1169
$\hat{W}_2$	0.1025	$\hat{W}_6$	0.2028
$\hat{W}_3$	0.1219	$\hat{W}_7$	0.0655
$\hat{W}_4$	0.0975	$\hat{W}_8$	0.0309

Table 15: Non-membership values $\mathcal{F}^D_{\mathcal{B}}(\hat{W}_i)$

$\hat{W}_i$	$\mathcal{F}^D_{\mathcal{B}}(\hat{W}_i)$	$\hat{W}_i$	$\mathcal{F}^D_{\mathcal{B}}(\hat{W}_i)$
$\hat{W}_1$	0.0556	$\hat{W}_5$	0.1320
$\hat{W}_2$	0.0875	$\hat{W}_6$	0.2310
$\hat{W}_3$	0.1206	$\hat{W}_7$	0.0693
$\hat{W}_4$	0.1116	$\hat{W}_8$	0.0371

Table 16: Reduced fuzzy membership $\zeta_{\Psi^D_{\mathcal{B}}}(\hat{W}_i)$

$\hat{W}_i$	$\zeta_{\Psi^D_{\mathcal{B}}}(\hat{W}_i)$	$\hat{W}_i$	$\zeta_{\Psi^D_{\mathcal{B}}}(\hat{W}_i)$
$\hat{W}_1$	0.0331	$\hat{W}_5$	0.0855
$\hat{W}_2$	0.1100	$\hat{W}_6$	0.1394
$\hat{W}_3$	0.1019	$\hat{W}_7$	0.0690
$\hat{W}_4$	0.0659	$\hat{W}_8$	0.0296

The graphical representation of this decision system is presented in Fig. 2.

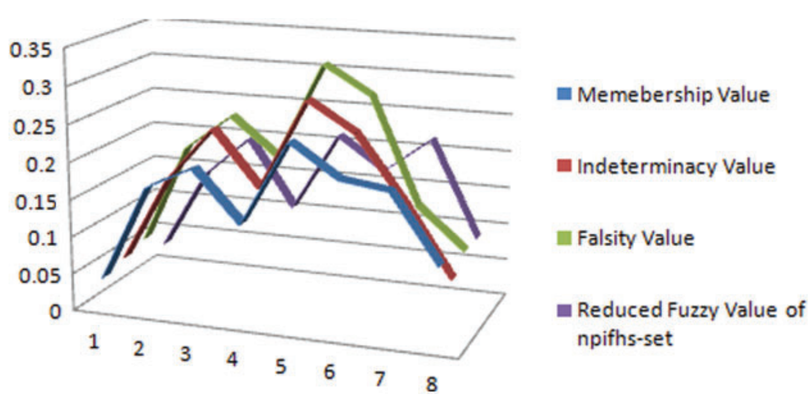

Figure 2: Neutrosophic decision system on npifhs-set

Step 5:

Since maximum of $\zeta_{\Psi^{D}_{\mathcal{B}}}(\hat{W}_i)$ is 0.5313 so the washing machine $\hat{W}_3$ is selected.

5 Neutrosophic Parameterized Neutrosophic Hypersoft Set (*npnhs*-Set) with Application

In this section, neutrosophic parameterized hypersoft set is conceptualized and some of its fundamentals are discussed.

Definition 5.1. Let $\mathcal{Z} = \{\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3, \ldots, \mathcal{Z}_n\}$ be a collection of disjoint attribute-valued sets corresponding to n distinct attributes $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n$, respectively. A *npnhs*-set $\Psi_{\mathcal{D}}$ over $\mathbb{X}$ is defined as

$$\Psi_{\mathcal{D}} = \{(< A_{\mathcal{D}}(g), B_{\mathcal{D}}(g), C_{\mathcal{D}}(g) > /g, \psi_{\mathcal{D}}(g)) : g \in \mathbb{G}, \psi_{\mathcal{D}}(g) \in N(\mathbb{X})\}$$

where

(i) $N(\mathbb{X})$ is a collection of all neutrosophic sets over $\mathbb{X}$
(i) $\mathbb{G} = \mathcal{Z}_1 \times \mathcal{Z}_2 \times \mathcal{Z}_3 \times \ldots \times \mathcal{Z}_n$
(ii) $\mathcal{D}$ is a neutrosophic set over $\mathbb{G}$ with $A_{\mathcal{D}}, B_{\mathcal{D}}, C_{\mathcal{D}} : \mathbb{G} \to \mathbb{I}$ as membership function, indeterminacy function and nonmembership function of *npnhs*-set.
(iii) $\psi_{\mathcal{D}}(g)$ is a neutrosophic set for all $g \in \mathbb{G}$ with $\psi_{\mathcal{D}} : \mathbb{G} \to N(\mathbb{X})$ and is called approximate function of *npnhs*-set.

Note that collection of all *npnhs*-sets is represented by $\Omega_{NPNHS}(\mathbb{X})$.

Definition 5.2. Let $\Psi_{\mathcal{D}} \in \Omega_{NPNHS}(\mathbb{X})$. If $\psi_{\mathcal{D}}(g) = \emptyset, A_{\mathcal{D}}(g) = 0, B_{\mathcal{D}}(g) = 1, C_{\mathcal{D}}(g) = 1$ for all $g \in \mathbb{G}$, then $\Psi_{\mathcal{D}}$ is called $\mathcal{D}$-empty *npnhs*-set, denoted by $\Psi_{\Phi_{\mathcal{D}}}$. If $\mathcal{D} = \emptyset$, then $\mathcal{D}$-empty *npnhs*-set is called an empty *npnhs*-set, denoted by Ψ_{Φ}.

Definition 5.3. Let $\Psi_{\mathcal{D}} \in \Omega_{NPNHS}(\mathbb{X})$. If $\psi_{\mathcal{D}}(g) = \mathbb{X}, A_{\mathcal{D}}(g) = 1, B_{\mathcal{D}}(g) = 0, C_{\mathcal{D}}(g) = 0$ for all $g \in \mathbb{G}$, then $\Psi_{\mathcal{D}}$ is called $\mathcal{D}$-universal *npnhs*-set, denoted by $\Psi_{\tilde{\mathcal{D}}}$. If $\mathcal{D} = \mathbb{G}$, then the $\mathcal{D}$-universal *npnhs*-set is called universal *npnhs*-set, denoted by $\Psi_{\tilde{\mathbb{G}}}$.

Example 5.1. Consider $\mathbb{X} = \{u_1, u_2, u_3, u_4, u_5\}$ and $\mathcal{Z} = \{\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3\}$ with $\mathcal{Z}_1 = \{\hat{z}_{11}, \hat{z}_{12}\}$, $\mathcal{Z}_2 = \{\hat{z}_{21}, \hat{z}_{22}\}, \mathcal{Z}_3 = \{\hat{z}_{31}\}$, then

$$\mathbb{G} = \mathcal{Z}_1 \times \mathcal{Z}_2 \times \mathcal{Z}_3$$

$$\mathbb{G} = \left\{\left(\hat{z}_{11}, \hat{z}_{21}, \hat{z}_{31}\right), \left(\hat{z}_{11}, \hat{z}_{22}, \hat{z}_{31}\right), \left(\hat{z}_{12}, \hat{z}_{21}, \hat{z}_{31}\right), \left(\hat{z}_{12}, \hat{z}_{22}, \hat{z}_{31}\right)\right\} = \{g_1, g_2, g_3, g_4\}.$$

Case 1.

If $\mathcal{D}_1 = \{< 0.2, 0.3, 0.4 > /g_2, < 0, 1, 1 > /g_3, < 1, 0, 0 > /g_4\}$ and

$\psi_{\mathcal{D}_1}(g_2) = \{< 0.2, 0.4, 0.6 > /u_2, < 0.3, 0.5, 0.7 > /u_4\}$, $\psi_{\mathcal{D}_1}(g_3) = \emptyset$, and $\psi_{\mathcal{D}_1}(g_4) = \mathbb{X}$, then

$$\Psi_{\mathcal{D}_1} = \left\{\begin{matrix} (< 0.2, 0.3, 0.4 > /g_2, \{< 0.2, 0.4, 0.6 > /u_2, 0.3, 0.5, 0.7 > /u_4\}), \\ (< 0, 1, 1 > /g_3, \emptyset), (< 1, 0, 0 > /g_4, \mathbb{X}) \end{matrix}\right\}.$$

Case 2.

If $\mathcal{D}_2 = \{< 0, 1, 1 > /g_2, < 0, 1, 1 > /g_3\}, \psi_{\mathcal{D}_2}(g_2) = \emptyset$ and $\psi_{\mathcal{D}_2}(g_3) = \emptyset$, then $\Psi_{\mathcal{D}_2} = \Psi_{\Phi_{\mathcal{D}_2}}$.

Case 3.

If $\mathcal{D}_3 = \emptyset$ corresponding to all elements of $\mathbb{G}$, then $\Psi_{\mathcal{D}_3} = \Psi_{\Phi}$.

Case 4.

If $\mathcal{D}_4 = \{< 1, 0, 0 > /g_1, < 1, 0, 0 > /g_2\}, \psi_{\mathcal{D}_4}(g_1) = \mathbb{X}$, and $\psi_{\mathcal{D}_4}(g_2) = \mathbb{X}$, then $\Psi_{\mathcal{D}_4} = \Psi_{\tilde{\mathcal{D}}_4}$.

Case 5.

If $\mathcal{D}_5 = \mathbb{X}$ with respect to all elements of $\mathbb{G}$, then $\Psi_{\mathcal{D}_5} = \Psi_{\tilde{\mathbb{G}}}$.

Definition 5.4. Let $\Psi_{\mathcal{D}_1}$, $\Psi_{\mathcal{D}_2} \in \Omega_{NPNHS}(\mathbb{X})$ then $\Psi_{\mathcal{D}_1}$ is an *npnhs*-subset of $\Psi_{\mathcal{D}_2}$, denoted by $\Psi_{\mathcal{D}_1} \tilde{\subseteq} \Psi_{\mathcal{D}_2}$ if $A_{\mathcal{D}_1}(g) \leq A_{\mathcal{D}_2}(g), B_{\mathcal{D}_1}(g) \geq B_{\mathcal{D}_2}(g), C_{\mathcal{D}_1}(g) \geq C_{\mathcal{D}_2}(g)$ and $\psi_{\mathcal{D}_1}(g) \subseteq_n \psi_{\mathcal{D}_2}(g)$ for all $g \in \mathbb{G}$.

Proposition 5.1. Let $\Psi_{\mathcal{D}_1}$, $\Psi_{\mathcal{D}_2}$, $\Psi_{\mathcal{D}_3} \in \Omega_{NPNHS}(\mathbb{X})$ then

1. $\Psi_{\mathcal{D}_1} \tilde{\subseteq} \Psi_{\tilde{\mathbb{G}}}$.
2. $\Psi_{\Phi} \tilde{\subseteq} \Psi_{\mathcal{D}_1}$.
3. $\Psi_{\mathcal{D}_1} \tilde{\subseteq} \Psi_{\mathcal{D}_1}$.
4. if $\Psi_{\mathcal{D}_1} \tilde{\subseteq} \Psi_{\mathcal{D}_2}$ and $\Psi_{\mathcal{D}_2} \tilde{\subseteq} \Psi_{\mathcal{D}_3}$ then $\Psi_{\mathcal{D}_1} \tilde{\subseteq} \Psi_{\mathcal{D}_3}$.

Definition 5.5. Let $\Psi_{\mathcal{D}_1}, \Psi_{\mathcal{D}_2} \in \Omega_{NPNHS}(\mathbb{X})$ then, $\Psi_{\mathcal{D}_1}$ and $\Psi_{\mathcal{D}_2}$ are *npnhs*-equal, represented as $\Psi_{\mathcal{D}_1} = \Psi_{\mathcal{D}_2}$, if and only if $A_{\mathcal{D}_1}(g) = A_{\mathcal{D}_2}(g), B_{\mathcal{D}_1}(g) = B_{\mathcal{D}_2}(g), C_{\mathcal{D}_1}(g) = C_{\mathcal{D}_2}(g)$ and $\psi_{\mathcal{D}_1}(g) =_n \psi_{\mathcal{D}_2}(g)$ for all $g \in \mathbb{G}$.

Proposition 5.2. Let $\Psi_{\mathcal{D}_1}, \Psi_{\mathcal{D}_2}, \Psi_{\mathcal{D}_3} \in \Omega_{NPNHS}(\mathbb{X})$ then,

1. if $\Psi_{\mathcal{D}_1} = \Psi_{\mathcal{D}_2}$ and $\Psi_{\mathcal{D}_2} = \Psi_{\mathcal{D}_3}$ then $\Psi_{\mathcal{D}_1} = \Psi_{\mathcal{D}_3}$.
2. if $\Psi_{\mathcal{D}_1} \tilde{\subseteq} \Psi_{\mathcal{D}_2}$ and $\Psi_{\mathcal{D}_2} \tilde{\subseteq} \Psi_{\mathcal{D}_1} \Leftrightarrow \Psi_{\mathcal{D}_1} = \Psi_{\mathcal{D}_2}$.

Definition 5.6. Let $\Psi_{\mathcal{D}} \in \Omega_{NPNHS}(\mathbb{X})$ then, complement of $\Psi_{\mathcal{D}}$ (i.e., $\Psi^{\tilde{c}}_{\mathcal{D}}$) is a *npnhs*-set given as $P^{\tilde{c}}_{\mathcal{D}}(g) = 1 - A_{\mathcal{D}}(g), Q^{\tilde{c}}_{\mathcal{D}}(g) = 1 - B_{\mathcal{D}}(g), R^{\tilde{c}}_{\mathcal{D}}(g) = 1 - C_{\mathcal{D}}(g)$ and $\psi^{\tilde{c}}_{\mathcal{D}}(g) = \mathbb{X} \setminus_n \psi_{\mathcal{D}}(g)$.

Proposition 5.3. Let $\Psi_{\mathcal{D}} \in \Omega_{NPNHS}(\mathbb{X})$ then,

1. $(\Psi^{\tilde{c}}_{\mathcal{D}})^{\tilde{c}} = \Psi_{\mathcal{D}}$.
2. $\Psi^{\tilde{c}}_{\phi} = \Psi_{\tilde{\mathbb{G}}}$.

Definition 5.7. Let $\Psi_{\mathcal{D}_1}, \Psi_{\mathcal{D}_2} \in \Omega_{NPNHS}(\mathbb{X})$ then, union of $\Psi_{\mathcal{D}_1}$ and $\Psi_{\mathcal{D}_2}$, denoted by $\Psi_{\mathcal{D}_1} \tilde{\cup} \Psi_{\mathcal{D}_2}$, is an *npnhs*-set defined by

(i) $A_{\mathcal{D}_1 \tilde{\cup} \mathcal{D}_2}(g) = max\{A_{\mathcal{D}_1}(x), A_{\mathcal{D}_2}(g)\}$,
(ii) $B_{\mathcal{D}_1 \tilde{\cup} \mathcal{D}_2}(g) = min\{B_{\mathcal{D}_1}(x), B_{\mathcal{D}_2}(g)\}$,
(iii) $C_{\mathcal{D}_1 \tilde{\cup} \mathcal{D}_2}(g) = min\{C_{\mathcal{D}_1}(x), C_{\mathcal{D}_2}(g)\}$,
(iv) $\psi_{\mathcal{D}_1 \tilde{\cup} \mathcal{D}_2}(g) = \psi_{\mathcal{D}_1}(g) \cup_n \psi_{\mathcal{D}_2}(g)$, for all $g \in \mathbb{G}$.

Proposition 5.4. Let $\Psi_{\mathcal{D}_1}, \Psi_{\mathcal{D}_2}, \Psi_{\mathcal{D}_3} \in \Omega_{NPNHS}(\mathbb{X})$ then,

1. $\Psi_{\mathcal{D}_1} \tilde{\cup} \Psi_{\mathcal{D}_1} = \Psi_{\mathcal{D}_1}$,
2. $\Psi_{\mathcal{D}_1} \tilde{\cup} \Psi_{\Phi} = \Psi_{\mathcal{D}_1}$,
3. $\Psi_{\mathcal{D}_1} \tilde{\cup} \Psi_{\tilde{\mathbb{G}}} = \Psi_{\tilde{\mathbb{G}}}$,
4. $\Psi_{\mathcal{D}_1} \tilde{\cup} \Psi_{\mathcal{D}_2} = \Psi_{\mathcal{D}_2} \tilde{\cup} \Psi_{\mathcal{D}_1}$,
5. $(\Psi_{\mathcal{D}_1} \tilde{\cup} \Psi_{\mathcal{D}_2}) \tilde{\cup} \Psi_{\mathcal{D}_3} = \Psi_{\mathcal{D}_1} \tilde{\cup} (\Psi_{\mathcal{D}_2} \tilde{\cup} \Psi_{\mathcal{D}_3})$.

Definition 5.8. Let $\Psi_{\mathcal{D}_1}, \Psi_{\mathcal{D}_2} \in \Omega_{NPNHS}(\mathbb{X})$ then intersection of $\Psi_{\mathcal{D}_1}$ and $\Psi_{\mathcal{D}_2}$, denoted by $\Psi_{\mathcal{D}_1} \tilde{\cap} \Psi_{\mathcal{D}_2}$, is an *npnhs*-set defined by

(i) $A_{\mathcal{D}_1 \tilde{\cap} \mathcal{D}_2}(g) = min\{A_{\mathcal{D}_1}(x), A_{\mathcal{D}_2}(g)\}$,
(ii) $B_{\mathcal{D}_1 \tilde{\cap} \mathcal{D}_2}(g) = max\{B_{\mathcal{D}_1}(x), B_{\mathcal{D}_2}(g)\}$,
(iii) $C_{\mathcal{D}_1 \tilde{\cap} \mathcal{D}_2}(g) = max\{C_{\mathcal{D}_1}(x), C_{\mathcal{D}_2}(g)\}$,
(iv) $\psi_{\mathcal{D}_1 \tilde{\cap} \mathcal{D}_2}(g) = \psi_{\mathcal{D}_1}(g) \cap_n \psi_{\mathcal{D}_2}(g)$, for all $g \in \mathbb{G}$.

Proposition 5.5. Let $\Psi_{\mathcal{D}_1}, \Psi_{\mathcal{D}_2}, \Psi_{\mathcal{D}_3} \in \Omega_{NPNHS}(\mathbb{X})$ then

1. $\Psi_{\mathcal{D}_1} \tilde{\cap} \Psi_{\mathcal{D}_1} = \Psi_{\mathcal{D}_1}$.
2. $\Psi_{\mathcal{D}_1} \tilde{\cap} \Psi_{\Phi} = \Psi_{\Phi}$.
3. $\Psi_{\mathcal{D}_1} \tilde{\cap} \Psi_{\tilde{\mathbb{G}}} = \Psi_{\tilde{\mathcal{D}}_1}$.
4. $\Psi_{\mathcal{D}_1} \tilde{\cap} \Psi_{\mathcal{D}_2} = \Psi_{\mathcal{D}_2} \tilde{\cap} \Psi_{\mathcal{D}_1}$.
5. $(\Psi_{\mathcal{D}_1} \tilde{\cap} \Psi_{\mathcal{D}_2}) \tilde{\cap} \Psi_{\Psi_{\mathcal{D}_3}} = \Psi_{\mathcal{D}_1} \tilde{\cap} (\Psi_{\mathcal{D}_2} \tilde{\cap} \Psi_{\Psi_{\mathcal{D}_3}})$.

Note: It is pertinent to mention here that Propositions 5.1, 5.2, 5.4 and 5.5 are also valid for elements of $\Omega_{NPFHS}(\mathbb{X})$ and $\Omega_{NPIFHS}(\mathbb{X})$.

Remark 5.1. Let $\Psi_{\mathcal{D}} \in \Omega_{NPNHS}(\mathbb{X})$. If $\Psi_{\mathcal{D}} \neq \Psi_{\tilde{\mathbb{G}}}$, then $\Psi_{\mathcal{D}} \tilde{\cup} \Psi_{\mathcal{D}}^{\tilde{c}} \neq \Psi_{\tilde{\mathbb{G}}}$ and $\Psi_{\mathcal{D}} \tilde{\cap} \Psi_{\mathcal{D}}^{\tilde{c}} \neq \Psi_{\Phi}$

Proposition 5.6. Let $\Psi_{\mathcal{D}_1}, \Psi_{\mathcal{D}_2} \in \Omega_{NPNHS}(\mathbb{X})$ then following D. Morgan laws are valid:

1. $(\Psi_{\mathcal{D}_1} \tilde{\cup} \Psi_{\mathcal{D}_2})^{\tilde{c}} = \Psi_{\mathcal{D}_1}^{\tilde{c}} \tilde{\cap} \Psi_{\mathcal{D}_2}^{\tilde{c}}$.
2. $(\Psi_{\mathcal{D}_1} \tilde{\cap} \Psi_{\mathcal{D}_2})^{\tilde{c}} = \Psi_{\mathcal{D}_1}^{\tilde{c}} \tilde{\cup} \Psi_{\mathcal{D}_2}^{\tilde{c}}$.

Proof. For all $g \in \mathbb{G}$,

(1). Since
$$\begin{aligned}(A_{\mathcal{D}_1 \tilde{\cup} \mathcal{D}_2})^{\tilde{c}}(g) &= 1 - A_{\mathcal{D}_1 \tilde{\cup} \mathcal{D}_2}(g) \\ &= 1 - max\{A_{\mathcal{D}_1}(g), A_{\mathcal{D}_2}(g)\} \\ &= min\{1 - A_{\mathcal{D}_1}(g), 1 - A_{\mathcal{D}_2}(g)\} \\ &= min\{P_{\mathcal{D}_1}^{\tilde{c}}(g), P_{\mathcal{D}_2}^{\tilde{c}}(g)\} \\ &= P_{\mathcal{D}_1 \tilde{\cap} \mathcal{D}_2}^{\tilde{c}}(g)\end{aligned}$$

also

$$\begin{aligned}(B_{\mathcal{D}_1 \tilde{\cup} \mathcal{D}_2})^{\tilde{c}}(g) &= 1 - B_{\mathcal{D}_1 \tilde{\cup} \mathcal{D}_2}(g) \\ &= 1 - min\{B_{\mathcal{D}_1}(g), B_{\mathcal{D}_2}(g)\} \\ &= max\{1 - B_{\mathcal{D}_1}(g), 1 - B_{\mathcal{D}_2}(g)\} \\ &= max\{Q_{\mathcal{D}_1}^{\tilde{c}}(g), Q_{\mathcal{D}_2}^{\tilde{c}}(g)\} \\ &= Q_{\mathcal{D}_1 \tilde{\cap} \mathcal{D}_2}^{\tilde{c}}(g)\end{aligned}$$

and

$$
\begin{aligned}
(C_{\mathcal{D}_1\tilde{\cup}\mathcal{D}_2})^{\tilde{c}}(g) &= 1 - C_{\mathcal{D}_1\tilde{\cup}\mathcal{D}_2}(g)\\
&= 1 - min\{C_{\mathcal{D}_1}(g), C_{\mathcal{D}_2}(g)\}\\
&= max\{1 - C_{\mathcal{D}_1}(g), 1 - C_{\mathcal{D}_2}(g)\}\\
&= max\{R^{\tilde{c}}_{\mathcal{D}_1}(g), R^{\tilde{c}}_{\mathcal{D}_2}(g)\}\\
&= R^{\tilde{c}}_{\mathcal{D}_1\tilde{\cap}\mathcal{D}_2}(g)
\end{aligned}
$$

and

$$
\begin{aligned}
(\psi_{\mathcal{D}_1\tilde{\cup}\mathcal{D}_2})^{\tilde{c}}(g) &= \mathbb{X} \setminus_n \psi_{\mathcal{D}_1\tilde{\cup}\mathcal{D}_2}(g)\\
&= \mathbb{X} \setminus_n (\psi_{\mathcal{D}_1}(g) \cup_n \psi_{\mathcal{D}_2}(g))\\
&= (\mathbb{X} \setminus_n \psi_{\mathcal{D}_1}(g)) \cap_n (\mathbb{X} \setminus_n \psi_{\mathcal{D}_2}(g))\\
&= \psi^{\tilde{c}}_{\mathcal{D}_1}(g) \tilde{\cap}_n \psi^{\tilde{c}}_{\mathcal{D}_2}(g)\\
&= \psi^{\tilde{c}}_{\mathcal{D}_1\tilde{\cap}\mathcal{D}_2}(g).
\end{aligned}
$$

similarly (2) can be proved easily.

Proposition 5.7. Let $\Psi_{\mathcal{D}_1}, \Psi_{\mathcal{D}_2}, \Psi_{\mathcal{D}_3} \in \Omega_{NPNHS}(\mathbb{X})$ then

1. $\Psi_{\mathcal{D}_1} \tilde{\cup} (\Psi_{\mathcal{D}_2} \tilde{\cap} \Psi_{\mathcal{D}_3}) = (\Psi_{\mathcal{D}_1} \tilde{\cup} \Psi_{\mathcal{D}_2}) \tilde{\cap} (\Psi_{\mathcal{D}_1} \tilde{\cup} \Psi_{\mathcal{D}_3})$.
2. $\Psi_{\mathcal{D}_1} \tilde{\cap} (\Psi_{\mathcal{D}_2} \tilde{\cup} \Psi_{\mathcal{D}_3}) = (\Psi_{\mathcal{D}_1} \tilde{\cap} \Psi_{\mathcal{D}_2}) \tilde{\cup} (\Psi_{\mathcal{D}_1} \tilde{\cap} \Psi_{\mathcal{D}_3})$.

Proof. For all $g \in \mathbb{G}$,

(1). Since
$$
\begin{aligned}
A_{\mathcal{D}_1\tilde{\cup}(\mathcal{D}_2\tilde{\cap}\mathcal{D}_3)}(g) &= max\{A_{\mathcal{D}_1}(g), A_{\mathcal{D}_2\tilde{\cap}\mathcal{D}_3}(g)\}\\
&= max\{A_{\mathcal{D}_1}(g), min\{A_{\mathcal{D}_2}(g), A_{\mathcal{D}_3}(g)\}\}\\
&= min\{max\{A_{\mathcal{D}_1}(g), A_{\mathcal{D}_2}(g)\}, max\{A_{\mathcal{D}_1}(g), A_{\mathcal{D}_3}(g)\}\}\\
&= min\{A_{\mathcal{D}_1\tilde{\cup}\mathcal{D}_2}(g), A_{\mathcal{D}_1\tilde{\cup}\mathcal{D}_3}(g)\}\\
&= A_{(\mathcal{D}_1\tilde{\cup}\mathcal{D}_2)\tilde{\cap}(\mathcal{D}_1\tilde{\cup}\mathcal{D}_3)}(g)
\end{aligned}
$$

and

$$
\begin{aligned}
B_{\mathcal{D}_1\tilde{\cup}(\mathcal{D}_2\tilde{\cap}\mathcal{D}_3)}(g) &= min\{B_{\mathcal{D}_1}(g), B_{\mathcal{D}_2\tilde{\cap}\mathcal{D}_3}(g)\}\\
&= min\{B_{\mathcal{D}_1}(g), max\{B_{\mathcal{D}_2}(g), B_{\mathcal{D}_3}(g)\}\}\\
&= max\{min\{B_{\mathcal{D}_1}(g), B_{\mathcal{D}_2}(g)\}, min\{B_{\mathcal{D}_1}(g), B_{\mathcal{D}_3}(g)\}\}\\
&= max\{B_{\mathcal{D}_1\tilde{\cup}\mathcal{D}_2}(g), B_{\mathcal{D}_1\tilde{\cup}\mathcal{D}_3}(g)\}\\
&= B_{(\mathcal{D}_1\tilde{\cup}\mathcal{D}_2)\tilde{\cap}(\mathcal{D}_1\tilde{\cup}\mathcal{D}_3)}(g)
\end{aligned}
$$

and

$$C_{\mathcal{D}_1\tilde{\cup}(\mathcal{D}_2\tilde{\cap}\mathcal{D}_3)}(g) = min\{C_{\mathcal{D}_1}(g), C_{\mathcal{D}_2\tilde{\cap}\mathcal{D}_3}(g)\}$$
$$= min\{C_{\mathcal{D}_1}(g), max\{C_{\mathcal{D}_2}(g), C_{\mathcal{D}_3}(g)\}\}$$
$$= max\{min\{C_{\mathcal{D}_1}(g), C_{\mathcal{D}_2}(g)\}, min\{C_{\mathcal{D}_1}(g), C_{\mathcal{D}_3}(g)\}\}$$
$$= max\{C_{\mathcal{D}_1\tilde{\cup}\mathcal{D}_2}(g), C_{\mathcal{D}_1\tilde{\cup}\mathcal{D}_3}(g)\}$$
$$= C_{(\mathcal{D}_1\tilde{\cup}\mathcal{D}_2)\tilde{\cap}(\mathcal{D}_1\tilde{\cup}\mathcal{D}_3)}(g)$$

and

$$\psi_{\mathcal{D}_1\tilde{\cup}_n(\mathcal{D}_2\tilde{\cup}_n\mathcal{D}_3)}(g) = \psi_{\mathcal{D}_1}(g) \cup_n \psi_{\mathcal{D}_2\tilde{\cap}_n\mathcal{D}_3}(g)$$
$$= \psi_{\mathcal{D}_1}(g) \cup_n (\psi_{\mathcal{D}_2}(g) \cap_n \psi_{\mathcal{D}_3}(g))$$
$$= (\psi_{\mathcal{D}_1}(g) \cup_n \psi_{\mathcal{D}_2}(g)) \cap_n (\psi_{\mathcal{D}_1}(g) \cup_n \psi_{\mathcal{D}_3}(g))$$
$$= \psi_{\mathcal{D}_1\tilde{\cup}\mathcal{D}_2}(g) \cap_n \psi_{\mathcal{D}_1\tilde{\cup}\mathcal{D}_3}(g)$$
$$= \psi_{(\mathcal{D}_1\tilde{\cup}\mathcal{D}_2)\tilde{\cap}(\mathcal{D}_1\tilde{\cup}\mathcal{D}_3)}(g)$$

In the same way, (2) can be proved.

Definition 5.9. Let $\Psi_{\mathcal{D}_1}, \Psi_{\mathcal{D}_2} \in \Omega_{NPNHS}(\mathbb{X})$ then OR-operation of $\Psi_{\mathcal{D}_1}$ and $\Psi_{\mathcal{D}_2}$, denoted by $\Psi_{\mathcal{D}_1} \tilde{\oplus} \Psi_{\mathcal{D}_2}$, is an *npnhs*-set defined by

(i) $A_{\mathcal{D}_1\tilde{\oplus}\mathcal{D}_2}(g_1, g_2) = max\{A_{\mathcal{D}_1}(g_1), A_{\mathcal{D}_2}(g_2)\}$,
(ii) $B_{\mathcal{D}_1\tilde{\oplus}\mathcal{D}_2}(g_1, g_2) = min\{B_{\mathcal{D}_1}(g_1), B_{\mathcal{D}_2}(g_2)\}$,
(iii) $C_{\mathcal{D}_1\tilde{\oplus}\mathcal{D}_2}(g_1, g_2) = min\{C_{\mathcal{D}_1}(g_1), C_{\mathcal{D}_2}(g_2)\}$,
(iv) $\psi_{\mathcal{D}_1\tilde{\oplus}\mathcal{D}_2}(g_1, g_2) = \psi_{\mathcal{D}_1}(g_1) \cup \psi_{\mathcal{D}_2}(g_2)$, for all $(g_1, g_2) \in \mathcal{D}_1 \times \mathcal{D}_2$.

Definition 5.10. Let $\Psi_{\mathcal{D}_1}, \Psi_{\mathcal{D}_2} \in \Omega_{NPNHS}(\mathbb{X})$ then AND-operation of $\Psi_{\mathcal{D}_1}$ and $\Psi_{\mathcal{D}_2}$, denoted by $\Psi_{\mathcal{D}_1} \tilde{\otimes} \Psi_{\mathcal{D}_2}$, is an *npnhs*-set defined by

(i) $A_{\mathcal{D}_1\tilde{\otimes}\mathcal{D}_2}(g_1, g_2) = min\{A_{\mathcal{D}_1}(g_1), A_{\mathcal{D}_2}(g_2)\}$,
(ii) $B_{\mathcal{D}_1\tilde{\otimes}\mathcal{D}_2}(g_1, g_2) = max\{B_{\mathcal{D}_1}(g_1), B_{\mathcal{D}_2}(g_2)\}$,
(iii) $C_{\mathcal{D}_1\tilde{\otimes}\mathcal{D}_2}(g_1, g_2) = max\{C_{\mathcal{D}_1}(g_1), C_{\mathcal{D}_2}(g_2)\}$,
(iv) $\psi_{\mathcal{D}_1\tilde{\otimes}\mathcal{D}_2}(g_1, g_2) = \psi_{\mathcal{D}_1}(g_1) \cap \psi_{\mathcal{D}_2}(g_2)$, for all $(g_1, g_2) \in \mathcal{D}_1 \times \mathcal{D}_2$.

Proposition 5.8. Let $\Psi_{\mathcal{D}_1}, \Psi_{\mathcal{D}_2}, \Psi_{\mathcal{D}_3} \in \Omega_{NPNHS}(\mathbb{X})$ then

1. $\Psi_{\mathcal{D}_1} \tilde{\otimes} \Psi_{\Phi} = \Psi_{\Phi}$.
2. $(\Psi_{\mathcal{D}_1} \tilde{\otimes} \Psi_{\mathcal{D}_2}) \tilde{\otimes} \Psi_{\mathcal{D}_3} = \Psi_{\mathcal{D}_1} \tilde{\otimes} (\Psi_{\mathcal{D}_2} \tilde{\otimes} \Psi_{\mathcal{D}_3})$.
3. $(\Psi_{\mathcal{D}_1} \tilde{\oplus} \Psi_{\mathcal{D}_2}) \tilde{\oplus} \Psi_{\mathcal{D}_3} = \Psi_{\mathcal{D}_1} \tilde{\oplus} (\Psi_{\mathcal{D}_2} \tilde{\oplus} \Psi_{\mathcal{D}_3})$.

5.1 Neutrosophic Decision Set of npnhs-Set

Here an algorithm is presented with the help of characterization of neutrosophic decision set on *npnhs*-set which based on decision making technique and is explained with example.

Definition 5.11. Let $\Psi_{\mathcal{D}} \in \Omega_{NPNHS}(\mathbb{X})$ then a neutrosophic decision set of $\Psi_{\mathcal{D}}$ (i.e., $\Psi^D_{\mathcal{D}}$) is represented as

$$\Psi^D_{\mathcal{D}} = \left\{ < \mathcal{T}^D_{\mathcal{D}}(u), \mathcal{I}^D_{\mathcal{D}}(u), \mathcal{F}^D_{\mathcal{D}}(u) > /u : \ u \in \mathbb{X} \right\}$$

where $\mathcal{T}_{\mathcal{D}}^{D}, \mathcal{I}_{\mathcal{D}}^{D}, \mathcal{F}_{\mathcal{D}}^{D}: \mathbb{X} \to \mathbb{I}$ and

$$\mathcal{T}_{\mathcal{D}}^{D}(u) = \frac{1}{|\mathbb{X}|} \sum_{v \in S(\mathcal{D})} \mathcal{T}_{\mathcal{D}}(v) \Gamma_{\psi_{\mathcal{D}}(v)}(u)$$

$$\mathcal{I}_{\mathcal{D}}^{D}(u) = \frac{1}{|\mathbb{X}|} \sum_{v \in S(\mathcal{D})} \mathcal{I}_{\mathcal{D}}(v) \Gamma_{\psi_{\mathcal{D}}(v)}(u)$$

$$\mathcal{F}_{\mathcal{D}}^{D}(u) = \frac{1}{|\mathbb{X}|} \sum_{v \in S(\mathcal{D})} \mathcal{F}_{\mathcal{D}}(v) \Gamma_{\psi_{\mathcal{D}}(v)}(u)$$

where $|\bullet|$ denotes set cardinality with

$$\Gamma_{\psi_{\mathcal{D}}(v)}(u) = \begin{cases} |T_{\psi_{\mathcal{D}}}(u) + I_{\psi_{\mathcal{D}}}(u) - F_{\psi_{\mathcal{D}}}(u)|; & u \in \Gamma_{\psi_{\mathcal{D}}}(v) \\ 0; & u \notin \Gamma_{\psi_{\mathcal{D}}}(v) \end{cases}$$

Definition 5.12. If $\Psi_{\mathcal{D}} \in \Omega_{NPNHS}(\mathbb{X})$ with neutrosophic decision set $\Psi_{\mathcal{D}}^{D}$ then reduced fuzzy set of $\Psi_{\mathcal{D}}^{D}$ is a fuzzy set represented as

$$\mathbb{R}(\Psi_{\mathcal{D}}^{D}) = \left\{ \zeta_{\Psi_{\mathcal{D}}^{D}}(u)/u : u \in \mathbb{X} \right\}$$

where $\zeta_{\Psi_{\mathcal{D}}^{D}} : \mathbb{X} \to \mathbb{I}$ with $\zeta_{\Psi_{\mathcal{D}}^{D}}(u) = \mathcal{T}_{\mathcal{D}}^{D}(u) + \mathcal{I}_{\mathcal{D}}^{D}(u) - \mathcal{F}_{\mathcal{D}}^{D}(u)$.

5.2 Proposed Algorithm

Once $\Psi_{\mathcal{D}}^{D}$ has been established, it may be indispensable to select the best single substitute from the options. Therefore, decision can be set up with the help of following algorithm:

Step 1 Determine $\mathcal{D} = \{< \mathcal{T}_{\mathcal{D}}(g), \mathcal{I}_{\mathcal{D}}(g), \mathcal{F}_{\mathcal{D}}(g) > /g : \mathcal{T}_{\mathcal{D}}(g), \mathcal{I}_{\mathcal{D}}(g), \mathcal{F}_{\mathcal{D}}(g) \in \mathbb{I}, g \in \mathbb{G}\}$,

Step 2 Find $\psi_{\mathcal{D}}(g)$

Step 3 Construct $\Psi_{\mathcal{D}}$ over $\mathbb{X}$,

Step 4 Compute $\Psi_{\mathcal{D}}^{D}$,

Step 5 Choose the maximum of $\zeta_{\Psi_{\mathcal{D}}^{D}}(u)$.

Hand sanitizer is a liquid or gel mostly used to diminish infectious agents on the hands. According to the World Health Organization (WHO), in current epidemic circumstances of COVID-19, high-quality sanitation and physical distancing are the best ways to protect ourselves and everyone around us from this virus. This virus spreads by touching an ailing person. We cannot detach ourselves totally being cautious from this virus. So, high-quality sanitation can be the ultimate blockade between us and the virus. Alcohol-based hand sanitizers are recommended by WHO to remove the novel corona virus. Alcohol-based hand sanitizers avert the proteins of germs including bacteria and some viruses from functioning normally. Demand of a hand sanitizer has been increased terrifically in such serious condition of COVID-19. Therefore, it is tricky to have good and effectual hand sanitizers in local markets. Low quality hand sanitizers have also been introduced due to its increasing demand. The core motivation of this application is to select an effectual sanitizer to alleviate the spread of corona virus by applying the NPNHS-set theory.

Example 5.2. Suppose that Mr. William wants to purchase an effective hand sanitizer from the local market. There are eight kinds of Hand Sanitizer (options) which form the set of discourse $\mathbb{X} = \{H_1, H_2, H_3, H_4, H_5, H_6, H_7, H_8\}$.

The best selection may be evaluated by observing the attributes i.e., k_1 = Manufacturer, k_2 = Quantity of Ethanol (percentage), k_3 = Quantity of Distilled Water (percentage), k_4 = Quantity of Glycerol (percentage), and k_5 = Quantity of Hydrogen peroxide (percentage). The attribute-valued sets corresponding to these attributes are:

$K_1 = \{k_{11} = Procter\ and\ Gamble,\ k_{12} = Unilever\}$

$K_2 = \{k_{21} = 75.15, k_{22} = 80\}$

$K_3 = \{k_{31} = 23.425, k_{32} = 18.425\}$

$K_4 = \{k_{41} = 1.30, k_{42} = 1.45\}$

$K_5 = \{k_{51} = 0.125\}$

then $\mathbb{P} = K_1 \times K_2 \times K_3 \times K_4 \times K_5$

$\mathbb{P} = \{p_1, p_2, p_3, p_4, \ldots, p_{16}\}$ where each $p_i, i = 1, 2, \ldots, 16$, is a 5-tuples element.

Step 1:

From Tabs. 17–19, we can construct $\mathcal{D}$ as

$$\mathcal{D} = \left\{ \begin{array}{l} < 0.1, 0.2, 0.3 > /p_1, < 0.2, 0.3, 0.4 > /p_2, \\ < 0.3, 0.4, 0.5 > /p_3, < 0.4, 0.5, 0.6 > /p_4, \\ < 0.5, 0.6, 0.7 > /p_5, < 0.6, 0.7, 0.8 > /p_6, \\ < 0.7, 0.8, 0.9 > /p_7, < 0.8, 0.9, 0.1 > /p_8, \\ < 0.9, 0.1, 0.2 > /p_9, < 0.16, 0.27, 0.37 > /p_{10}, \\ < 0.25, 0.35, 0.45 > /p_{11}, < 0.45, 0.55, 0.65 > /p_{12}, \\ < 0.35, 0.45, 0.55 > /p_{13}, < 0.75, 0.85, 0.95 > /p_{14}, \\ < 0.65, 0.75, 0.85 > /p_{15}, < 0.85, 0.95, 0.96 > /p_{16} \end{array} \right\}.$$

Table 17: Degrees of membership $\mathcal{T}_{\mathcal{D}}(p_i)$

$\mathcal{T}_{\mathcal{D}}(p_i)$	Degree	$\mathcal{T}_{\mathcal{D}}(p_i)$	Degree
$\mathcal{T}_{\mathcal{D}}(p_1)$	0.1	$\mathcal{T}_{\mathcal{D}}(p_9)$	0.9
$\mathcal{T}_{\mathcal{D}}(p_2)$	0.2	$\mathcal{T}_{\mathcal{D}}(p_{10})$	0.16
$\mathcal{T}_{\mathcal{D}}(p_3)$	0.3	$\mathcal{T}_{\mathcal{D}}(p_{11})$	0.25
$\mathcal{T}_{\mathcal{D}}(p_4)$	0.4	$\mathcal{T}_{\mathcal{D}}(p_{12})$	0.45
$\mathcal{T}_{\mathcal{D}}(p_5)$	0.5	$\mathcal{T}_{\mathcal{D}}(p_{13})$	0.35
$\mathcal{T}_{\mathcal{D}}(p_6)$	0.6	$\mathcal{T}_{\mathcal{D}}(p_{14})$	0.75
$\mathcal{T}_{\mathcal{D}}(p_7)$	0.7	$\mathcal{T}_{\mathcal{D}}(p_{15})$	0.65
$\mathcal{T}_{\mathcal{D}}(p_8)$	0.8	$\mathcal{T}_{\mathcal{D}}(p_{16})$	0.85

Table 18: Degrees of indeterminacy $\mathcal{I}_{\mathcal{D}}(p_i)$

$\mathcal{I}_{\mathcal{D}}(p_i)$	Degree	$\mathcal{I}_{\mathcal{D}}(p_i)$	Degree
$\mathcal{I}_{\mathcal{D}}(p_1)$	0.2	$\mathcal{I}_{\mathcal{D}}(p_9)$	0.1
$\mathcal{I}_{\mathcal{D}}(p_2)$	0.3	$\mathcal{I}_{\mathcal{D}}(p_{10})$	0.27
$\mathcal{I}_{\mathcal{D}}(p_3)$	0.4	$\mathcal{I}_{\mathcal{D}}(p_{11})$	0.35
$\mathcal{I}_{\mathcal{D}}(p_4)$	0.5	$\mathcal{I}_{\mathcal{D}}(p_{12})$	0.55
$\mathcal{I}_{\mathcal{D}}(p_5)$	0.6	$\mathcal{I}_{\mathcal{D}}(p_{13})$	0.45
$\mathcal{I}_{\mathcal{D}}(p_6)$	0.7	$\mathcal{I}_{\mathcal{D}}(p_{14})$	0.85
$\mathcal{I}_{\mathcal{D}}(p_7)$	0.8	$\mathcal{I}_{\mathcal{D}}(p_{15})$	0.75
$\mathcal{I}_{\mathcal{D}}(p_8)$	0.9	$\mathcal{I}_{\mathcal{D}}(p_{16})$	0.95

Table 19: Degrees of non-membership $\mathcal{F}_{\mathcal{D}}(p_i)$

$\mathcal{F}_{\mathcal{D}}(p_i)$	Degree	$\mathcal{F}_{\mathcal{D}}(p_i)$	Degree
$\mathcal{F}_{\mathcal{D}}(p_1)$	0.3	$\mathcal{F}_{\mathcal{D}}(p_9)$	0.2
$\mathcal{F}_{\mathcal{D}}(p_2)$	0.4	$\mathcal{F}_{\mathcal{D}}(p_{10})$	0.37
$\mathcal{F}_{\mathcal{D}}(p_3)$	0.5	$\mathcal{F}_{\mathcal{D}}(p_{11})$	0.45
$\mathcal{F}_{\mathcal{D}}(p_4)$	0.6	$\mathcal{F}_{\mathcal{D}}(p_{12})$	0.65
$\mathcal{F}_{\mathcal{D}}(p_5)$	0.7	$\mathcal{F}_{\mathcal{D}}(p_{13})$	0.55
$\mathcal{F}_{\mathcal{D}}(p_6)$	0.8	$\mathcal{F}_{\mathcal{D}}(p_{14})$	0.95
$\mathcal{F}_{\mathcal{D}}(p_7)$	0.9	$\mathcal{F}_{\mathcal{D}}(p_{15})$	0.85
$\mathcal{F}_{\mathcal{D}}(p_8)$	0.1	$\mathcal{F}_{\mathcal{D}}(p_{16})$	0.96

Step 2:

Tab. 20 presents $\psi_{\mathcal{D}}(p_i)$ corresponding to each element of $\mathbb{G}$.

Step 3:

$\Psi_{\mathcal{D}}$ can be constructed with the help of Step 1 and Step 2 same as done in Step 3 of Section 3.

Step 4:

From Tabs. 21–24, we can construct $\mathbb{R}(\Psi^{D}_{\mathcal{D}})$ as

$$\mathbb{R}(\Psi^{D}_{\mathcal{D}}) = \begin{Bmatrix} 0.0344/H_1, 0.1600/H_2, 0.1500/H_3, 0.1289/H_4, \\ 0.1367/H_5, 0.0749/H_6, 0.1538/H_7, 0.1006/H_8 \end{Bmatrix}.$$

The graphical representation of this decision system is presented in Fig. 3.

Step 5:

Since maximum of $\zeta_{\Psi^{D}_{\mathcal{D}}}(H_i)$ is 0.1600 so the Hand Sanitizer H_2 is selected.

Table 20: Approximate functions $\psi_{\mathcal{D}}(p_i)$

p_i	$\psi_{\mathcal{D}}(p_i)$	p_i	$\psi_{\mathcal{D}}(p_i)$
p_1	$\{<0.2, 0.1, 0.2>/\hat{H}_1, <0.3, 0.2, 0.1>/\hat{H}_2\}$	p_9	$\{<0.4, 0.3, 0.2>/\hat{H}_2, <0.6, 0.4, 0.3>/\hat{H}_7, <0.5, 0.4, 0.3>/\hat{H}_8\}$
p_2	$\{<0.1, 0.2, 0.1>/\hat{H}_1, <0.5, 0.4, 0.3>/\hat{H}_2, <0.1, 0.4, 0.3>/\hat{H}_3\}$	p_{10}	$\{<0.2, 0.1, 0.2>/\hat{H}_6, <0.6, 0.4, 0.5>/\hat{H}_7, <0.4, 0.3, 0.2>/\hat{H}_8\}$
p_3	$\{<0.4, 0.3, 0.1>/\hat{H}_2, <0.5, 0.4, 0.3>/\hat{H}_3, <0.6, 0.3, 0.2>/\hat{H}_4\}$	p_{11}	$\{<0.5, 0.4, 0.3>/\hat{H}_2, <0.6, 0.3, 0.2>/\hat{H}_4, <0.7, 0.2, 0.3>/\hat{H}_6\}$
p_4	$\{<0.6, 0.2, 0.3>/\hat{H}_4, <0.7, 0.3, 0.4>/\hat{H}_5, <0.8, 0.1, 0.4>/\hat{H}_6\}$	p_{12}	$\{<0.7, 0.2, 0.5>/\hat{H}_2, <0.8, 0.1, 0.5>/\hat{H}_3, <0.9, 0.1, 0.7>/\hat{H}_6\}$
p_5	$\{<0.2, 0.1, 0.1>/\hat{H}_6, <0.1, 0.2, 0.1>/\hat{H}_7, <0.4, 0.3, 0.1>/\hat{H}_8\}$	p_{13}	$\{<0.2, 0.1, 0.2>/\hat{H}_3, <0.4, 0.3, 0.2>/\hat{H}_5, <0.6, 0.1, 0.4>/\hat{H}_7\}$
p_6	$\{<0.4, 0.2, 0.3>/\hat{H}_2, <0.3, 0.4, 0.3>/\hat{H}_3, <0.4, 0.5, 0.3>/\hat{H}_4\}$	p_{14}	$\{<0.2, 0.5, 0.4>/\hat{H}_1, <0.5, 0.4, 0.6>/\hat{H}_3, <0.6, 0.2, 0.5>/\hat{H}_5\}$
p_7	$\{<0.2, 0.3, 0.4>/\hat{H}_1, <0.3, 0.4, 0.4>/\hat{H}_3, <0.4, 0.3, 0.4>/\hat{H}_5\}$	p_{15}	$\{<0.6, 0.3, 0.3>/\hat{H}_5, <0.4, 0.3, 0.4>/\hat{H}_7, <0.2, 0.4, 0.5>/\hat{H}_8\}$
p_8	$\{<0.1, 0.4, 0.3>/\hat{H}_2, <0.3, 0.5, 0.6>/\hat{H}_3, <0.5, 0.4, 0.7>/\hat{H}_7\}$	p_{16}	$\{<0.3, 0.6, 0.5>/\hat{H}_4, <0.5, 0.4, 0.8>/\hat{H}_5, <0.7, 0.1, 0.6>/\hat{H}_6\}$

Table 21: Membership values $\mathcal{T}^D_{\mathcal{D}}(H_i)$

H_i	$\mathcal{T}^D_{\mathcal{D}}(H_i)$	H_i	$\mathcal{T}^D_{\mathcal{D}}(H_i)$
H_1	0.0431	H_5	0.1656
H_2	0.1825	H_6	0.0964
H_3	0.1588	H_7	0.1588
H_4	0.1606	H_8	0.1231

Table 22: Indeterminacy values $\mathcal{I}^D_{\mathcal{D}}(H_i)$

H_i	$\mathcal{I}^D_{\mathcal{D}}(H_i)$	H_i	$\mathcal{I}^D_{\mathcal{D}}(H_i)$
H_1	0.0519	H_5	0.1956
H_2	0.1713	H_6	0.1203
H_3	0.1900	H_7	0.1081
H_4	0.1969	H_8	0.0788

Table 23: Non-membership values $\mathcal{I}^D_{\mathcal{D}}(H_i)$

H_i	$\mathcal{F}^D_{\mathcal{D}}(H_i)$	H_i	$\mathcal{F}^D_{\mathcal{D}}(H_i)$
H_1	0.0606	H_5	0.2245
H_2	0.1938	H_6	0.1418
H_3	0.1988	H_7	0.1131
H_4	0.2286	H_8	0.1013

Table 24: Reduced fuzzy membership $\zeta_{\Psi^{D}_{\mathcal{D}}}(H_i)$

H_i	$\zeta_{\Psi^{D}_{\mathcal{D}}}(H_i)$	H_i	$\zeta_{\Psi^{D}_{\mathcal{D}}}(H_i)$
H_1	0.0344	H_5	0.1367
H_2	0.1600	H_6	0.0749
H_3	0.1500	H_7	0.1538
H_4	0.1289	H_8	0.1006

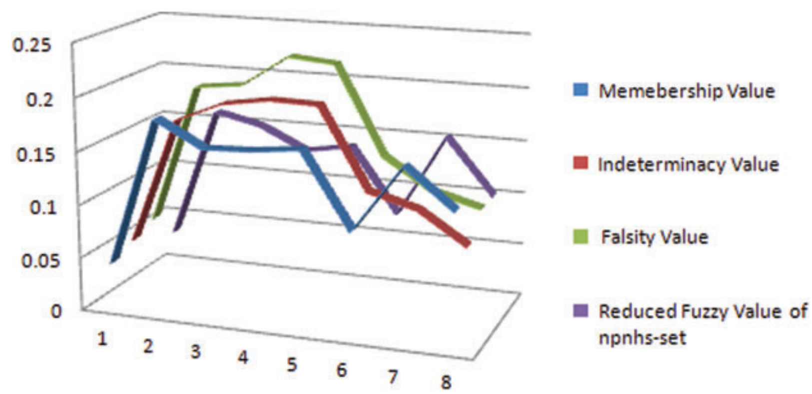

Figure 3: Neutrosophic decision system on npnhs-set

6 Discussion

The development and stability of any society depends on its justice system and the judges, lawyers and plaintiffs play a key role in its basic components. The lawyer prepares the writ petition at the request of the plaintiff but when filing the case in the Court of Justice, he/she is in a state of uncertainty for its success. This uncertain condition can be of fuzzy, intuitionistic fuzzy or even neutrosophic. And after the case is submitted, the judge concerned writes his/her decision in the light of the facts, but usually all facts have some kind of uncertainty. Such factual vagueness again may be of fuzzy, intuitionistic fuzzy or neutrosophic nature. So when initial stage (submission stage) and final stage (decisive stage) are neutrosophic valued and the process is executed with the help of parameterized data (collections of parametric values) then we say that we are tackling such problem with the help of neutrosophic parameterized neutrosophic hypersoft set (npnhs-set). Since decision makers always face some sort of uncertainties and any decision taken by ignoring uncertainty may have some extent of inclination. Indeterminacy and uncertainty are both interconnected. In this study, it has been shown (i.e., see Fig. 4) that how results are affected when indeterminacy is ignored or considered. Our proposed structure npnhs-set is very useful in dealing with many decisive systems and it is the generalization of:

(i) Neutrosophic Parameterized Intuitionistic Fuzzy Hypersoft Set (npifhs-set) if indeterminacy is ignored and remaining two are made interdependent within closed unit interval in approximate function of npnhs-set,
(ii) Neutrosophic Parameterized Fuzzy Hypersoft Set (npfhs-set) if indeterminacy and falsity are ignored and remaining be restricted within closed unit interval in approximate function of npnhs-set,

(iii) Neutrosophic Parameterized Hypersoft Set (nphs-set) if all uncertain components are ignored and approximate function of npnhs-set is a subset of universe of discourse,
(iv) Neutrosophic Parameterized Neutrosophic Soft Set (npns-set) if attribute-valued sets are replaced with only attributes in npnhs-set,
(v) Neutrosophic Parameterized Intuitionistic Fuzzy Soft Set (npifs-set) if attribute-valued sets are replaced with only attributes and indeterminacy is ignored and remaining two are made interdependent within closed unit interval in approximate function of npnhs-set,
(vi) Neutrosophic Parameterized Fuzzy Soft Set (npfs-set) if attribute-valued sets are replaced with only attributes and indeterminacy, falsity are ignored and remaining be restricted within closed unit interval in approximate function of npnhs-set,
(vii) Neutrosophic Parameterized Soft Set (nps-set) if attribute-valued sets are replaced with only attributes and all uncertain components are ignored with approximate function of npnhs-set as a subset of universe of discourse.

Fig. 5 presents the pictorial view of the generalization of the proposed structure.

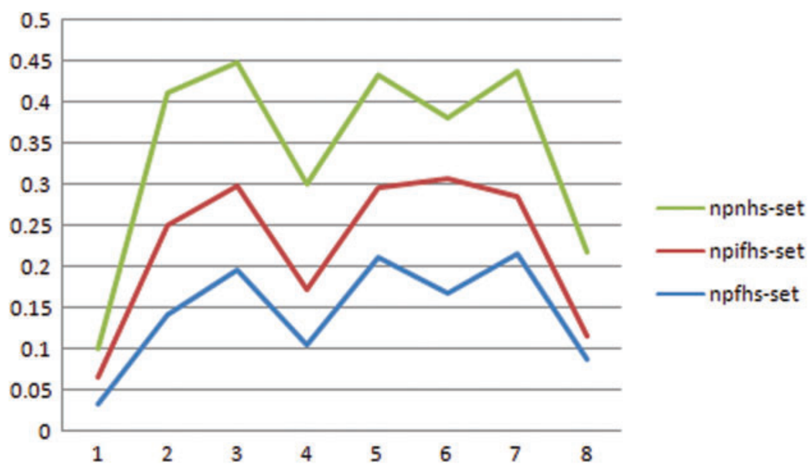

Figure 4: Comparison of neutrosophic decision system on npfhs-set, npifhs-set and npnhs-set

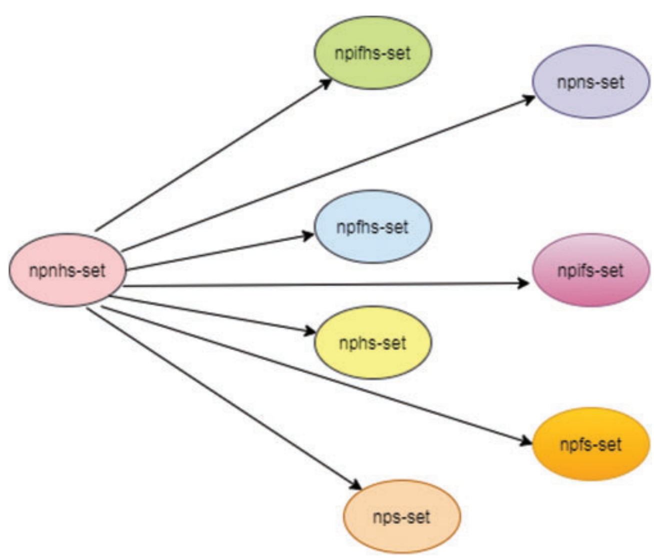

Figure 5: Generalization of npnhs-set

7 Conclusion

In this study, neutrosophic parameterized hypersoft set is conceptualized for the environments of fuzzy set, intuitionistic fuzzy set and neutrosophic set along with some of their elementary properties and theoretic operations. Novel algorithms are proposed for decision making and are validated with the help of illustrative examples for appropriate purchasing of suitable products i.e., Mobile Tablet, Washing Machines and Hand Sanitizers, from the local market. Future work may include the extension of this work for:

- The development of algebraic structures i.e., topological spaces, vector spaces, etc.,
- The development of hybrid structures with fuzzy-like environments,
- Dealing with decision making problems with multi-criteria decision making techniques,
- Applying in medical diagnosis and optimization for agricultural yield,
- Investigating and determining similarity, distance, dissimilarity measures and entropies between the proposed structures.

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