Research Article

Multi-Attribute Decision-Support System Based on Aggregations of Interval-Valued Complex Neutrosophic Hypersoft Set

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Hypersoft set is an emerging field of study that is meant to address the insufficiency and the limitation of existing soft-set-like models regarding the consideration and the entitlement of multi-argument approximate function. This type of function maps the multi-subparametric tuples to the power set of the universe. It focuses on the partitioning of each attribute into its attribute-valued set that is missing in existing soft-set-like structures. This study aims to introduce novel concepts of complex intuitionistic fuzzy set and complex neutrosophic set under the hypersoft set environment with interval-valued settings. Two novel structures, that is, interval-valued complex intuitionistic hypersoft set (IV-CIFHS-set) and interval-valued complex neutrosophic hypersoft set (IV-CNHS-set), are developed via employing theoretic, axiomatic, graphical, and algorithmic approaches. After conceptual characterization of essential elementary notions of these structures, decision-support systems are presented with the proposal of algorithms to assist the decision-making process. The proposed algorithms are validated with the help of real-world applications. A comprehensive inter-cum-intra comparison of proposed structures is discussed with the existing relevant models, and their generalization is elaborated under certain evaluating features.

1. Introduction

The traditional logic (i.e., Boolean logic) is not always pertinent in real-world scenarios, where the available data is vague or imprecise. To deal with such kinds of situations, a particular class of sets known as fuzzy sets (F.Sets) that were proposed by Zadeh [1] is considered appropriate. In these sets, every member of the universe is specified by a membership grade in a unit closed interval. However, to tackle scenarios having more complexity and uncertainty, it was observed that the concept of F.Sets is not sufficient, and therefore, these concepts were expanded with few extensions. Intuitionistic fuzzy sets (IF.Sets) by Atanassov [2] was one of such major developments. Due to the consideration of nonmembership grade, IF.Sets are more effective in tackling with the vagueness of data. Moreover, IF.Sets are proficient to emulate the available information more precisely and rationally. As far as the consideration of the degree of indeterminacy was concerned, both F.Sets and IF.Set were inadequate for such kind of grade, so neutrosophic sets (N.Sets) were initiated by Smarandache [3] to cope with such shortcoming. N.Sets are more capable to maintain impreciseness in the contents of information and may facilitate approximate reasoning behavior diligently. Although the descriptive capability of N.Sets is higher than that of the traditional F.Sets and IF.Sets due to their additional presence of nonmembership and indeterminant graded functions; however, they have fairly higher computational complexity over F.Sets and IF.Sets.

The models such as F.Sets, IF.Sets, and N.Sets depicted some sort of limitation regarding the validation for some parameterization tools. To address this scarcity, Molodtsov

[4] characterized soft sets (S.Sets) as a new mathematical parameterized model. In S.Sets, every parameter in a set of parameters maps to power set of the universe of discourse while defining single-argument approximate function. The researchers [5–11] studied the basic properties, elementary set theoretic operations, relations, and functions of S.Sets with illustrative numerical examples. To hybridize the characteristics of F.Sets, IF.Sets, and N.Sets with S.Sets, fuzzy soft sets (FS.Sets) [12, 13], intuitionistic fuzzy soft sets (IFS.Sets) [14, 15], and neutrosophic soft sets (NS.Sets) [16] were conceptualized. Although there are many researchers who contributed a lot towards the expansion and extension of these hybridized structures with the interval-valued setting, the contributions of researchers [17–21] are more prominent relevant to these models. They not only discussed the fundamentals of interval-valued fuzzy soft-set-like models but also employed certain techniques for their applications in different situations.

In certain real-world scenarios, the classification of attributes into subattributive values in the form of sets is necessary. The existing concept of S.Sets is not sufficient and incompatible with such scenarios so Smarandache [22] introduced the concept of hypersoft sets (HS.Sets) to address the insufficiency of S.Sets and to cope with the situations with multi-argument approximate function. The rudiments and elementary axioms of HS.Sets have been discussed in [23] and elaborated with numerical examples. Rahman et al. [24–30] investigated the hybridized properties of HS.Sets under the environments of complex set, convexity and concavity, parameterization, and bijection. They employed decision-making algorithmic approaches to solve real-world problems. Saeed et al. [31–33] developed the theories of neutrosophic hypersoft mappings and complex multi-fuzzy hypersoft sets with applications in decision-making and clinical diagnosis.

1.1. Research Gap and Motivation.

The following points depict the need and motivation behind the proposed study:

(1) Many researchers discussed the hybridized structures of a complex set with fuzzy set, intuitionistic fuzzy set, and neutrosophic set under soft set environments. The literature review of the most relevant models [34–42] is presented in Table 1.

(2) It is vivid that these structures consider only one set of parameters and use the single-argument approximate function. They depict some kind of insufficiency to tackle the scenarios (recruitment process, product selection, medical diagnosis, etc.) where further classification of parameters into their subparametric values in the form of disjoint sets is necessary for deep learning and observation in decision-support systems.

(3) Along these lines, another construction requests its place in writing for tending to such obstacle, so the hypersoft set is conceptualized to handle such situations (Figure 1 depicts the vivid comparison of soft set model and hypersoft set model. It presents the optimal selection of a mobile with the help of suitable parameters in the case of soft set and suitable subparametric values in the case of hypersoft set). It has made the decision-making process more flexible and reliable. Also, it not only fulfills the requirements of existing soft set-like literature for multi-argument approximate functions but also supports the decision-makers to make decisions with the deep inspection.

(4) Although the models IV-CNSS [36] and IV-CFSS [37, 41] have been developed to tackle the scenarios with periodic and interval type data under soft set-environment, these are inadequate to deal sub-attribute values in the form of disjoint sets as a collective domain of multi-argument approximate function.

(5) Inspiring from the above literature in general and from [36, 37, 41] in specific, this study aims to characterize novel structures of IV-CIFHS-set and IV-CNHS-set that not only generalize the existing relevant models but also address their limitations.

1.2. Main Contributions.

The following are the possible objectives of this study:

(1) The existing relevant models, that is, [34–42], are made adequate with the entitlement of multi-argument approximate function through development of IV-CIFHS-set and IV-CNHS-set

(2) The scenarios where parameters are further partitioned into subparametric values in the form of sets are tackled by using IV-CIFHS-set and IV-CNHS-set

(3) Some essential rudiments, that is, properties, elementary laws, and set theoretic operations of IV-CIFHS-set and IV-CNHS-set, are characterized

(4) Two algorithms based on IV-CIFHS-set and IV-CNHS-set are proposed to deal with daily-life decision-making problems having periodic and interval type data/information

(5) The proposed study is compared with some existing relevant models by considering some important evaluating indicators so that the advantageous aspect of the proposed study may be depicted

(6) The generalization and particular cases of proposed models are discussed with the pictorial depiction

(7) The advantages and future directions of the proposed study are presented

1.3. Paper Organization.

The organization of the remaining paper is given in Figure 2.

1.4. Notations and Abbreviations.

Some abbreviations and notations are used throughout the paper. Their full names are given in Table 2 to facilitate the readers for proper understanding of the concept.

TABLE 1: Literature review on complex fuzzy-set-like models under soft set.

| Authors	Structure	Domain of approximate function	Range setting
Ali et al. [34]	Complex intuitionistic fuzzy soft set	Set of parameters	Complex intuitionistic fuzzy set
Al-Quran et al. [35]	Complex neutrosophic soft expert set	Soft expert set	Complex neutrosophic set
Al-Sharqi et al. [36]	Interval-valued complex neutrosophic soft set	Set of parameters	Interval-valued complex neutrosophic set
Fan et al. [37]	Interval-valued complex fuzzy soft set	Set of parameters	Interval-valued complex fuzzy set
Khan et al. [38]	Complex intuitionistic fuzzy soft set	Set of parameters	Complex intuitionistic fuzzy set
Kumar et al. [39]	Complex intuitionistic fuzzy soft set	Set of parameters	Complex intuitionistic fuzzy set
Smarandache et al. [40]	Complex neutrosophic soft set	Set of parameters	Complex neutrosophic set
Selvachandran et al. [41]	Interval-valued complex fuzzy soft set	Set of parameters	Interval-valued complex fuzzy set
Thirunavukarasu et al. [42]	Complex fuzzy soft set	Set of parameters	Complex fuzzy set
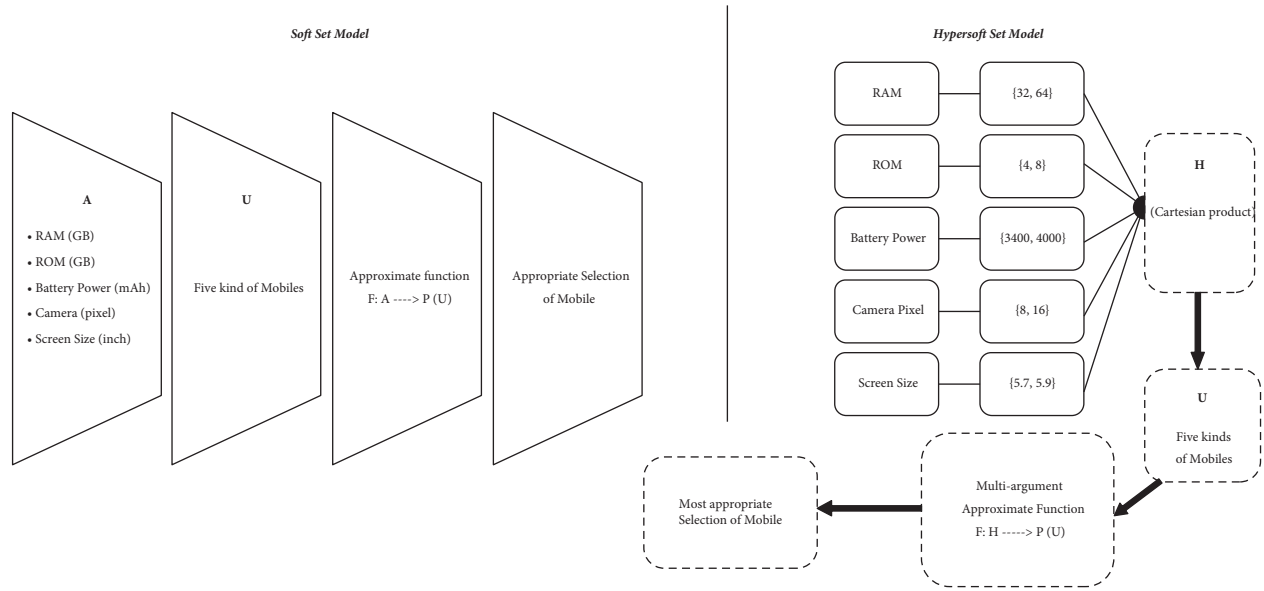
FIGURE 1: Comparison of soft set and hypersoft set.

2. Preliminaries

In this section, some fundamental definitions from literature are presented for the vivid understanding of the proposed study.

Definition 1 (see [1]). Let $\mathcal{U}$ be a fuzzy set over $\mathbb{Z}$ that can be written as $\mathcal{U} = \{(\nu, \alpha_{\mathcal{U}}(\nu)) | \nu \in \mathbb{Z}\}$ such that

$$\alpha_{\mathcal{U}}: \mathbb{Z} \longrightarrow [0, 1], \qquad (1)$$

where $\alpha_{\mathcal{U}}(\nu)$ is the membership degree of $\nu \in \mathcal{U}$.

Definition 2 (see [43]). A complex fuzzy set $\mathbb{F}_C$ can be written as follows:

$$\mathbb{F}_C = \left\{ \left(\delta, \eta_{\mathbb{F}_C}(\delta) \right): \delta \in \mathbb{Z} \right\} = \left\{ \left(\delta, k_{\mathbb{F}_C}(\delta) e^{i\lambda_{\mathbb{F}_C}(\delta)} \right): \delta \in \mathbb{Z} \right\}, \qquad (2)$$

where M-function of $\mathbb{F}_C$ is $\eta_{\mathbb{F}_C}(\delta)$ with $k_{\mathbb{F}_C}(\delta) \in [0, 1]$ as A-term and $\lambda_{\mathbb{F}_C}(\delta) \in (0, 2\pi]$ as P-term and $i = \sqrt{-1}$.

Buckley [44–46] and Zhang et al. [47] presented fuzzy complex sets and numbers in a different way. Amplitude terms and P-terms in the form of fuzzy sets are discussed in [43, 48].

Definition 3 (see [4]). Let $\mathfrak{E}$ be set of parameters; then soft set S_0 over $\mathbb{Z}$ is given by

$$S_0 = \left\{ \left(\delta, \zeta_{S_0}(\delta) \right): \delta \in \mathfrak{E}_1 \right\}, \qquad (3)$$

where $\zeta_{S_0}: \mathfrak{E}_1 \longrightarrow P(\mathbb{Z})$ and $\mathfrak{E}_1$ is subset of $\mathfrak{E}$.

Definition 4 (see [13]). The fuzzy soft set $\Omega_{\mathfrak{E}_1}$ on $\mathbb{Z}$ is given by

$$\Omega_{\mathfrak{E}_1} = \left\{ \left(\delta, \omega_{\mathfrak{E}_1}(\delta) \right): \delta \in \mathfrak{E}_1, \omega_{\mathfrak{E}_1}(\delta) \in \mathbb{F}(\mathbb{Z}) \right\}, \qquad (4)$$

where $\omega_{\mathfrak{E}_1}: \mathfrak{E}_1 \longrightarrow \mathbb{F}(\mathbb{Z})$, where $\omega_{\mathfrak{E}_1}(\delta) = \varnothing$ for $\delta \notin \mathfrak{E}_1$ and

$$\omega_{\mathfrak{E}_1}(\delta) = \left\{ \eta_{\omega_{\mathfrak{E}_1}(\delta)}(\nu)/\nu: \nu \in \mathbb{Z}, \eta_{\omega_{\mathfrak{E}_1}(\delta)}(\nu) \in I \right\} \qquad (5)$$
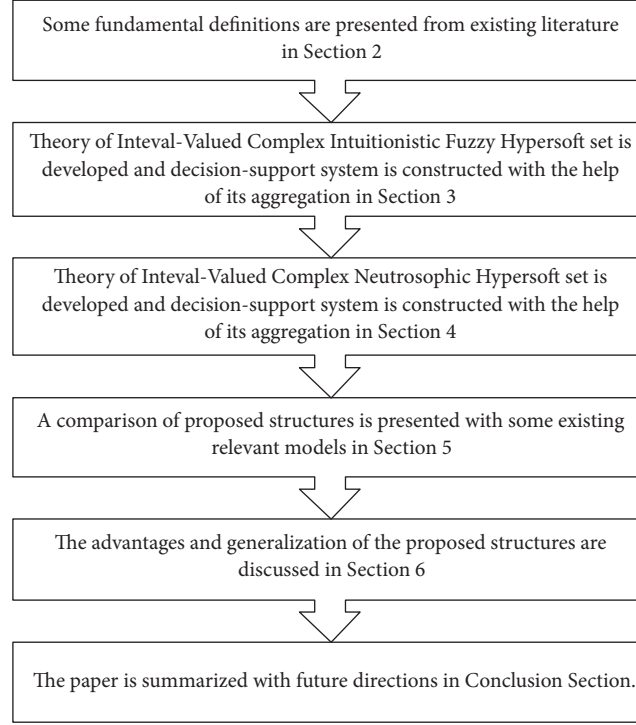
FIGURE 2: Organization of paper.

TABLE 2: Notations and abbreviations.

Notation	Used for	Notation	Used for
$\mathbb{Z}$	Universe of discourse	$P(\mathbb{Z})$	Power set of $\mathbb{Z}$
$\mathbb{F}(\mathbb{Z})$	Collection of fuzzy sets	$\mathbb{IF}(\mathbb{Z})$	Collection of intuitionistic fuzzy sets
$\mathbb{N}(\mathbb{Z})$	Collection of neutrosophic sets	$C(\mathbb{Z})$	Collection of complex fuzzy sets
$C_{\text{Int}}(\mathbb{Z})$	Collection of complex intuitionistic fuzzy sets	$C_{\text{Neu}}(\mathbb{U})$	Collection of complex neutrosophic sets
A-term	Amplitude term	M-function	Membership function
NM-function	Nonmembership function	ID-function	Indeterminacy function
P-term	Phase term	$C_{\text{IV}}(\mathbb{Z})$	Collection of interval-valued complex fuzzy sets

is a fuzzy set over $\mathbb{Z}$. Here, $\omega_{\mathfrak{G}_1}$ is the approximate function of $\Omega_{\mathfrak{G}_1}$, where $\omega_{\mathfrak{G}_1}(\delta)$ is a fuzzy set known as δ-element of $\Omega_{\mathfrak{G}_1}$. If $\omega_{\mathfrak{G}_1}(\delta) = \varnothing$, then $(\delta, \omega_{\mathfrak{G}_1}(\delta)) \notin \Omega_{\mathfrak{G}_1}$.

Definition 5 (see [42]). A complex fuzzy soft set $\xi_{\mathfrak{G}_1}$ over $\mathbb{Z}$ is given by

$$\xi_{\mathfrak{G}_1} = \left\{ \left(\delta, \chi_{\mathfrak{G}_1}(\delta) \right) : \delta \in \mathfrak{G}_1, \chi_{\mathfrak{G}_1}(\delta) \in C(\mathbb{Z}) \right\}, \quad (6)$$

where $\chi_{\mathfrak{G}_1} : \mathfrak{G}_1 \longrightarrow C(\mathbb{Z})$, where $\chi_{\mathfrak{G}_1}(\delta) = \varnothing$ for $\delta \notin \mathfrak{G}_1$ is the complex fuzzy approximate function of $\xi_{\mathfrak{G}_1}$, where $\chi_{\mathfrak{G}_1}(\delta)$ is called δ-member of $\xi_{\mathfrak{G}_1}$ for all $\delta \in \mathfrak{G}_1$. Set operations of complex fuzzy set and complex fuzzy soft set have been described in [42, 47], respectively.

Definition 6 (see [39]). Let E be a set of attributes with $A \subseteq E$. Then complex intuitionistic fuzzy soft set $\xi_A = (\Psi, A)$ over $\mathbb{Z}$ is defined as follows:

$$\xi_A = \left\{ (a, \Psi(a)) : a \in A, \Psi(a) \in C_{\text{Int}}(\mathbb{Z}) \right\}, \quad (7)$$

where

$$\Psi : A \longrightarrow C_{\text{Int}}(\mathbb{Z}),$$
$$\Psi(a) = \varnothing \text{ if } a \notin A, \quad (8)$$

is a complex intuitionistic fuzzy approximate function of ξ_A and $\Psi(a) = \langle \Psi^T(a), \Psi^F(a) \rangle$.

$\Psi^T(a) = p_T e^{i\theta_T}$ and $\Psi^F(a) = p_F e^{i\theta_F}$ are complex-valued M-function and complex-valued NM-function of ξ_A, respectively, and all are lying within unit circle in the complex plane such that $p_T, p_F \in [0, 1]$ with $0 \le p_T + p_F \le 1$ (or $0 \le |p_T + p_F| \le 1$) and $\theta_T, \theta_F \in (0, 2\pi]$. The value $\Psi(a)$ is called a-member of $\xi_A = (\Psi, A) \forall a \in A$.

Definition 7 (see [40]). Let E be a set of attributes with $A \subseteq E$. Then complex neutrosophic soft set $\xi_A = (\Psi, A)$ over $\mathbb{Z}$ is defined as follows:

$$\xi_A = \left\{ (a, \Psi(a)) : a \in A, \Psi(a) \in C_{\text{Neu}}(\mathbb{U}) \right\}, \quad (9)$$

where

$$\Psi\colon A \longrightarrow C_{\text{Neu}}(\mathbb{Z}),$$
$$\Psi(a) = \varnothing \text{ if } a \notin A, \tag{10}$$

is a complex neutrosophic approximate function of ξ_A and $\Psi(a) = \langle \Psi^T(a), \Psi^I(a), \Psi^F(a) \rangle$.

$\Psi^T(a) = p_T e^{i\theta_T}$, $\Psi^I(a) = p_I e^{i\theta_I}$, and $\Psi^F(a) = p_F e^{i\theta_F}$ are complex-valued truth M-function, complex-valued indeterminacy M-function, and complex-valued falsity M-function of ξ_A, respectively, and all are lying within unit circle in the complex plane such that $p_T, p_I, p_F \in [0, 1]$ with $0^- \leq p_T + p_I + p_F \leq 3^+$ (or $0 \leq |p_T + p_I + p_F| \leq 3$) and $\theta_T, \theta_I, \theta_F \in (0, 2\pi]$. The value $\Psi(a)$ is called a-member of $\xi_A = (\Psi, A) \forall a \in A$.

Definition 8 (see [22]). $(H_s, \mathcal{D})$ is known as a hypersoft set over $\mathbb{Z}$, if $H_s\colon \mathcal{D} \longrightarrow P(\mathbb{Z})$ where $\mathcal{D}$ is the Cartesian product of finite number of disjoint sets $H_{s_1}, H_{s_2}, H_{s_3}, \ldots, H_{s_n}$ with n distinct attributes $h_{s_1}, h_{s_2}, h_{s_3}, \ldots, h_{s_n}$, respectively.

Definition 9 (see [22]). Fuzzy hypersoft set, intuitionistic fuzzy hypersoft set, and neutrosophic hypersoft set are hypersoft sets defined over fuzzy universe, intuitionistic fuzzy universe, and neutrosophic universe, respectively.

The fundamental properties and set theoretic operations of hypersoft set are discussed in [23].

Definition 10 (see [24]). The complex fuzzy hypersoft set CFHS– set $\xi_{\mathcal{D}}$ over $\mathbb{Z}$ is given by

$$\xi_{\mathcal{D}} = \{(\underline{\delta}, \chi(\underline{\delta})): \underline{\delta} \in \mathcal{D}, \chi(\underline{\delta}) \in C(\mathbb{Z})\}, \tag{11}$$

where $\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, \ldots, \mathcal{L}_n$ are disjoint sets having n distinct attributes $l_1, l_2, l_3, \ldots, l_n$ for $n \geq 1$, $\mathcal{D} = \mathcal{L}_1 \times \mathcal{L}_2 \times \mathcal{L}_3 \times \cdots \times \mathcal{L}_n$, and $\chi(\breve{x})$ be a CF– set over $\mathbb{Z}$ for all $\underline{\delta} = (d_1, d_2, d_3, \ldots, d_n) \in \mathcal{D}$.

$$\chi\colon \mathcal{D} \longrightarrow C(\mathbb{Z}),$$
$$\chi(\underline{\delta}) = \varnothing \text{ if } \underline{\delta} \notin \mathcal{D}, \tag{12}$$

is a complex fuzzy approximate function of ξ_G and $\chi(\underline{\delta})$ is $\underline{\delta}$– member of CFHS– set $\forall \underline{\delta} \in \mathcal{D}$.

Definition 11 (see [30]). Let $W_1, W_2, W_3, \ldots, W_n$ are disjoint sets having attribute values of n distinct attributes $w_1, w_2, w_3, \ldots, w_n$, respectively, for $n \geq 1$, $W = W_1 \times W_2 \times W_3 \times \cdots \times W_n$ and $\Psi(\underline{\omega})$ be an interval-valued complex fuzzy set over $\mathbb{Z}$ for all $\underline{\omega} = (b_1, b_2, b_3, \ldots, b_n) \in W$. Then interval-valued complex fuzzy hypersoft set (IV-CFHS-set) $\Omega_W = (\Psi, W)$ over $\mathbb{Z}$ is defined as follows:

$$\Omega_W = \{(\underline{\omega}, \Psi(\underline{\omega})): \underline{\omega} \in W, \Psi(\underline{\omega}) \in C_{IV}(\mathbb{Z})\} \tag{13}$$

where

$$\Psi\colon W \longrightarrow C_{IV}(\mathbb{Z}),$$
$$\Psi(\underline{\omega}) = \varnothing \text{ if } \underline{\omega} \notin W, \tag{14}$$

is an interval-valued complex fuzzy approximate function of Ω_W and $\Psi(\underline{\omega}) = (\overleftarrow{\Psi}(\underline{\omega}), \overrightarrow{\Psi}(\underline{\omega}))$. $\overleftarrow{\Psi}(\underline{\omega}) = \overleftarrow{r} e^{i\theta}$ and $\overrightarrow{\Psi}(\underline{\omega}) = \overrightarrow{r} e^{i\overrightarrow{\theta}}$ are lower and upper bounds of the M-function of Ω_W, respectively, and its value $\Psi(\underline{\omega})$ is called $\underline{\omega}$-member of IV-CFHS $\forall \underline{\omega} \in W$.

3. Interval-Valued Complex Intuitionistic Fuzzy Hypersoft Set (IV-CIFHS-Set)

Consider the daily-life scenario of the clinical study to diagnose heart diseases in patients, doctors (decision-makers) usually prefer chest pain type, resting blood pressure, serum cholesterol, and so on as diagnostic parameters. After keen analysis, it is vivid that these parameters are required to be further partitioning into their subparametric values, that is, chest pain type (typical angina, atypical angina, etc.), resting blood pressure (110 mmHg, 150 mmHg, 180 mmHg, etc.), and serum cholesterol (210 mg/dl, 320 mg/dl, 430 mg/dl, etc.). Patients are advised to visit medical laboratories for test reports regarding indicated parameters. As the efficiency of medical instruments in laboratories varies that leads to different observations (data) for each patient. This may be categorized in the form of the set having a range of data from the minimum value (lower bounds) to the maximum (upper bounds) that is treated as interval data. Sometimes test reports have repeated values corresponding to these prescribed parameters. This can be of either lab-to-lab basis or day-to-day basis. Such type of data is treated as periodic data. The existing fuzzy set-like literature has no suitable model to deal with (i) subattribute values in the form of disjoint sets, (ii) interval-type data, and (iii) periodic nature of data collectively. In order to meet the demand of literature, the models IV-CIFHS-set and IV-CNHS-set are being characterized. Case (i) is addressed by considering multi-argument approximate function that considers the Cartesian product of attribute-valued disjoint sets as its domain and then maps it to power set of the initial universe (collection of intuitionistic fuzzy sets or neutrosophic sets). Case (ii) is tackled by considering lower and upper limits of reported intervals, and case (iii) is dealt with the introduction of amplitude and phase terms in the Argand plane.

Now we develop the theory of complex intuitionistic fuzzy hypersoft set with interval settings in the remaining part of this section.

Definition 12. Let $\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3, \ldots, \mathcal{M}_n$ are disjoint sets having n distinct attributes $m_1, m_2, m_3, \ldots, m_n$, respectively, for $n \geq 1$, $\mathcal{M} = \mathcal{M}_1 \times \mathcal{M}_2 \times \mathcal{M}_3 \times \cdots \times \mathcal{M}_n$ and $\Lambda(\underline{\lambda})$ be an IV-CIFS over $\mathbb{Z}$ for all $\underline{\lambda} = (b_1, b_2, b_3, \ldots, b_n) \in \mathcal{M}$. Then interval-valued complex intuitionistic fuzzy hypersoft set (IV-CIFHS-set), denoted by $\Omega_{\mathcal{M}} = (\Lambda, \mathcal{M})$, over $\mathbb{Z}$ is defined as follows:

$$\Omega_{\mathcal{M}} = \{(\underline{\lambda}, \Lambda(\underline{\lambda})): \underline{\lambda} \in \mathcal{M}, \Lambda(\underline{\lambda}) \in C_{IV}(\mathbb{Z})\}, \tag{15}$$

where

$$\Lambda: \mathcal{M} \longrightarrow C_{IV}(\mathbb{Z}),$$
$$\Lambda(\underline{\lambda}) = \varnothing \text{ if } \underline{\lambda} \notin \mathcal{M}, \tag{16}$$

is a IV$-$CIF approximate function of $\Omega_{\mathcal{M}}$ and $\Lambda(\underline{\lambda}) = \langle [\overleftarrow{\Lambda}_1(\underline{\lambda}), \overrightarrow{\Lambda}_1(\underline{\lambda})], [\overleftarrow{\Lambda}_2(\underline{\lambda}), \overrightarrow{\Lambda}_2(\underline{\lambda})] \rangle$ with the following:

(i) $\overleftarrow{\Lambda}_1(\underline{\lambda}) = \overleftarrow{\gamma} e^{i\overleftarrow{\theta}}$ and $\overrightarrow{\Lambda}_1(\underline{\lambda}) = \overrightarrow{\gamma} e^{i\overrightarrow{\theta}}$ are lower and upper bounds of the M-function of $\Omega_{\mathcal{M}}$, respectively

(ii) $\overleftarrow{\Lambda}_2(\underline{\lambda}) = \overleftarrow{\gamma} e^{i\overleftarrow{\theta}}$ and $\overrightarrow{\Lambda}_2(\underline{\lambda}) = \overrightarrow{\gamma} e^{i\overrightarrow{\theta}}$ are lower and upper bounds of the NM-function of $\Omega_{\mathcal{M}}$,

respectively, and its value $\Lambda(\underline{\lambda})$ is called $\underline{\lambda}$-member of IV-CIFHS-set for all values of $\underline{\lambda} \in \mathcal{M}$

Note: The collection of all IV-CIFHS-sets is denoted by $\underset{\text{IVCIFHS}}{\uplus}$.

Example 1. Consider $\mathcal{M} = \{e_1, e_2, e_3, \ldots, e_8\}$ with interval-valued complex intuitionistic fuzzy sets $\Lambda_{\mathcal{M}}(e_1), \Lambda_{\mathcal{M}}(e_2), \ldots, \Lambda_{\mathcal{M}}(e_8)$ given by

$$\Lambda_{\mathcal{M}}(e_1) = \left\{ \begin{array}{l} \dfrac{([0.4,0.5],[0.3,0.4])e^{i([0.3,0.5],[0.3,0.4])\pi}}{t_1}, \dfrac{([0.1,0.7],[0.2,0.2])e^{i([0.3,0.5],[0.1,0.3])\pi}}{t_2} \\[3mm] \dfrac{([0.3,0.3],[0.3,0.5])e^{i([0.1,0.8],[0.1,0.1])\pi}}{t_3}, \dfrac{([0.3,0.4],[0.2,0.5])e^{i([0.2,0.6],[0.2,0.3])\pi}}{t_4} \end{array} \right\},$$

$$\Lambda_{\mathcal{M}}(e_2) = \left\{ \begin{array}{l} \dfrac{([0.3,0.6],[0.2,0.3])e^{i([0.1,0.7],[0.1,0.2])\pi}}{t_1}, \dfrac{([0.3,0.4],[0.1,0.2])e^{i([0.2,0.6],[0.1,0.3])\pi}}{t_2} \\[3mm] \dfrac{([0.2,0.5],[0.3,0.4])e^{i([0.1,0.5],[0.3,0.4])\pi}}{t_3}, \dfrac{([0.4,0.5],[0.1,0.4])e^{i([0.2,0.5],[0.2,0.3])\pi}}{t_4} \end{array} \right\},$$

$$\Lambda_{\mathcal{M}}(e_3) = \left\{ \begin{array}{l} \dfrac{([0.4,0.5],[0.2,0.3])e^{i([0.2,0.6],[0.1,0.2])\pi}}{t_1}, \dfrac{([0.1,0.8],[0.1,0.1])e^{i([0.2,0.3],[0.2,0.5])\pi}}{t_2} \\[3mm] \dfrac{([0.1,0.4],[0.1,0.2])e^{i([0.2,0.5],[0.3,0.4])\pi}}{t_3}, \dfrac{([0.2,0.6],[0.1,0.3])e^{i([0.1,0.5],[0.3,0.4])\pi}}{t_4} \end{array} \right\},$$

$$\Lambda_{\mathcal{M}}(e_4) = \left\{ \begin{array}{l} \dfrac{([0.2,0.3],[0.3,0.6])e^{i([0.1,0.7],[0.2,0.2])\pi}}{t_1}, \dfrac{([0.2,0.4],[0.2,0.3])e^{i([0.1,0.5],[0.2,0.4])\pi}}{t_2} \\[3mm] \dfrac{([0.4,0.5],[0.3,0.4])e^{i([0.1,0.4],[0.4,0.5])\pi}}{t_3}, \dfrac{([0.4,0.5],[0.2,0.4])e^{i([0.2,0.5],[0.1,0.3])\pi}}{t_4} \end{array} \right\},$$

$$\Lambda_{\mathcal{M}}(e_5) = \left\{ \begin{array}{l} \dfrac{([0.1,0.2],[0.4,0.5])e^{i([0.4,0.5],[0.1,0.3])\pi}}{t_1}, \dfrac{([0.2,0.2],[0.2,0.7])e^{i([0.1,0.4],[0.3,0.5])\pi}}{t_2} \\[3mm] \dfrac{([0.2,0.6],[0.1,0.3])e^{i([0.1,0.2],[0.2,0.3])\pi}}{t_3}, \dfrac{([0.3,0.5],[0.2,0.3])e^{i([0.3,0.4],[0.2,0.3])\pi}}{t_4} \end{array} \right\},$$

$$\Lambda_{\mathcal{M}}(e_6) = \left\{ \begin{array}{l} \dfrac{([0.4,0.5],[0.1,0.4])e^{i([0.2,0.3],[0.3,0.6])\pi}}{t_1}, \dfrac{([0.1,0.3],[0.4,0.4])e^{i([0.2,0.5],[0.2,0.3])\pi}}{t_2} \\[3mm] \dfrac{([0.2,0.7],[0.1,0.2])e^{i([0.1,0.8],[0.1,0.1])\pi}}{t_3}, \dfrac{([0.2,0.5],[0.2,0.5])e^{i([0.1,0.6],[0.2,0.3])\pi}}{t_4} \end{array} \right\},$$

$$\Lambda_{\mathcal{M}}(e_7) = \left\{ \begin{array}{l} \dfrac{([0.2,0.4],[0.1,0.2])e^{i([0.2,0.3],[0.1,0.6])\pi}}{t_1}, \dfrac{([0.1,0.3],[0.3,0.5])e^{i([0.2,0.3],[0.2,0.6])\pi}}{t_2} \\[3mm] \dfrac{([0.1,0.5],[0.3,0.4])e^{i([0.1,0.6],[0.2,0.3])\pi}}{t_3}, \dfrac{([0.4,0.5],[0.3,0.4])e^{i([0.2,0.5],[0.1,0.2])\pi}}{t_4} \end{array} \right\},$$

$$\Lambda_{\mathcal{M}}(e_8) = \left\{ \begin{array}{l} \dfrac{([0.3,0.3],[0.1,0.4])e^{i([0.4,0.5],[0.3,0.4])\pi}}{t_1}, \dfrac{([0.2,0.3],[0.2,0.6])e^{i([0.3,0.6],[0.1,0.3])\pi}}{t_2} \\[3mm] \dfrac{([0.1,0.2],[0.4,0.5])e^{i([0.3,0.5],[0.3,0.4])\pi}}{t_3}, \dfrac{([0.2,0.5],[0.1,0.4])e^{i([0.4,0.5],[0.3,0.4])\pi}}{t_4} \end{array} \right\},$$

$$\tag{17}$$

and then IV-CIFHS-set $\Omega_{\mathscr{M}}$ is written by

$$
\Omega_{\mathscr{M}} = \left\{
\begin{array}{l}
\left(e_1, \left\{
\begin{array}{l}
\dfrac{([0.4,0.5],[0.3,0.4])e^{i([0.3,0.5],[0.3,0.4])\pi}}{t_1}, \dfrac{([0.1,0.7],[0.2,0.2])e^{i([0.3,0.5],[0.1,0.3])\pi}}{t_2}, \\[4mm]
\dfrac{([0.3,0.3],[0.3,0.5])e^{i([0.1,0.8],[0.1,0.1])\pi}}{t_3}, \dfrac{([0.3,0.4],[0.2,0.5])e^{i([0.2,0.6],[0.2,0.3])\pi}}{t_4}
\end{array}
\right\} \right), \\[10mm]
\left(e_2, \left\{
\begin{array}{l}
\dfrac{([0.3,0.6],[0.2,0.3])e^{i([0.1,0.7],[0.1,0.2])\pi}}{t_1}, \dfrac{([0.3,0.4],[0.1,0.2])e^{i([0.2,0.6],[0.1,0.3])\pi}}{t_2}, \\[4mm]
\dfrac{([0.2,0.5],[0.3,0.4])e^{i([0.1,0.5],[0.3,0.4])\pi}}{t_3}, \dfrac{([0.4,0.5],[0.1,0.4])e^{i([0.2,0.5],[0.2,0.3])\pi}}{t_4}
\end{array}
\right\} \right), \\[10mm]
\left(e_3, \left\{
\begin{array}{l}
\dfrac{([0.4,0.5],[0.2,0.3])e^{i([0.2,0.6],[0.1,0.2])\pi}}{t_1}, \dfrac{([0.1,0.8],[0.1,0.1])e^{i([0.2,0.3],[0.2,0.5])\pi}}{t_2}, \\[4mm]
\dfrac{([0.1,0.4],[0.1,0.2])e^{i([0.2,0.5],[0.3,0.4])\pi}}{t_3}, \dfrac{([0.2,0.6],[0.1,0.3])e^{i([0.1,0.5],[0.3,0.4])\pi}}{t_4}
\end{array}
\right\} \right), \\[10mm]
\left(e_4, \left\{
\begin{array}{l}
\dfrac{([0.2,0.3],[0.3,0.6])e^{i([0.1,0.7],[0.2,0.2])\pi}}{t_1}, \dfrac{([0.2,0.4],[0.2,0.3])e^{i([0.1,0.5],[0.2,0.4])\pi}}{t_2}, \\[4mm]
\dfrac{([0.4,0.5],([0.3,0.4]))e^{i([0.1,0.4],[0.4,0.5])\pi}}{t_3}, \dfrac{([0.4,0.5],[0.2,0.4])e^{i([0.2,0.5],[0.1,0.3])\pi}}{t_4}
\end{array}
\right\} \right), \\[10mm]
\left(e_5, \left\{
\begin{array}{l}
\dfrac{([0.1,0.2],[0.4,0.5])e^{i([0.4,0.5],[0.1,0.3])\pi}}{t_1}, \dfrac{([0.2,0.2],[0.2,0.7])e^{i(([0.1,0.4]),[0.3,0.5])\pi}}{t_2}, \\[4mm]
\dfrac{([0.2,0.6],[0.1,0.3])e^{i([0.1,0.2],[0.2,0.3])\pi}}{t_3}, \dfrac{([0.3,0.5],[0.2,0.3])e^{i([0.3,0.4],[0.2,0.3])\pi}}{t_4}
\end{array}
\right\} \right), \\[10mm]
\left(e_6, \left\{
\begin{array}{l}
\dfrac{([0.4,0.5],t[0.1,0.4])e^{i([0.2,0.3],[0.3,0.6])\pi}}{t_1}, \dfrac{([0.1,0.3],t[0.4,0.4])e^{i([0.2,0.5],t[0.2,0.3])\pi}}{t_2}, \\[4mm]
\dfrac{([0.2,0.7],t[0.1,0.2])e^{i([0.1,0.8],[0.1,0.1])\pi}}{t_3}, \dfrac{([0.2,0.5],t[0.2,0.5])e^{i([0.1,0.6],t[0.2,0.3])\pi}}{t_4}
\end{array}
\right\} \right), \\[10mm]
\left(e_7, \left\{
\begin{array}{l}
\dfrac{([0.2,0.4],[0.1,0.2])e^{i([0.2,0.3],[0.1,0.6])\pi}}{t_1}, \dfrac{([0.1,0.3],[0.3,0.5])e^{i([0.2,0.3],[0.2,0.6])\pi}}{t_2}, \\[4mm]
\dfrac{([0.1,0.5],[0.3,0.4])e^{i([0.1,0.6],[0.2,0.3])\pi}}{t_3}, \dfrac{([0.4,0.5],[0.3,0.4])e^{i([0.2,0.5],[0.1,0.2])\pi}}{t_4}
\end{array}
\right\} \right), \\[10mm]
\left(e_8, \left\{
\begin{array}{l}
\dfrac{([0.3,0.3],[0.1,0.4])e^{i([0.4,0.5],[0.3,0.4])\pi}}{t_1}, \dfrac{([0.2,0.3],[0.2,0.6])e^{i([0.3,0.6],[0.1,0.3])\pi}}{t_2}, \\[4mm]
\dfrac{([0.1,0.2],[0.4,0.5])e^{i([0.3,0.5],[0.3,0.4])\pi}}{t_3}, \dfrac{([0.2,0.5],[0.1,0.4])e^{i([0.4,0.5],[0.3,0.4])\pi}}{t_4}
\end{array}
\right\} \right)
\end{array}
\right\}. \tag{18}
$$

Definition 13. Let $\Omega_{\mathscr{M}_1} = (\Lambda_1, \mathscr{M}_1)$ and $\Omega_{\mathscr{M}_2} = (\Lambda_2, \mathscr{M}_2)$ be two IV-CIFHS-sets over the same universe. The set $\Omega_{\mathscr{M}_1} = (\Lambda_1, \mathscr{M}_1)$ is a subset of $\Omega_{\mathscr{M}_2} = (\Lambda_2, \mathscr{M}_2)$, if

(i) $\mathscr{M}_1 \subseteq \mathscr{M}_2$

(ii) $\forall \breve{x} \in \mathscr{M}_1, \Lambda_1(\breve{x}) \subseteq \Lambda_2(\breve{x})$ implies $\overleftarrow{\Lambda}_1(\breve{x}) \subseteq \overleftarrow{\Lambda}_2(\breve{x}), \overrightarrow{\Lambda}_1(\breve{x}) \subseteq \overrightarrow{\Lambda}_2(\breve{x})$, that is,

$\overleftarrow{\gamma}_{\mathscr{M}_1}(\breve{x}) \leq \overleftarrow{\gamma}_{\mathscr{M}_2}(\breve{x}), \overrightarrow{\gamma}_{\mathscr{M}_1}(\breve{x}) \leq \overrightarrow{\gamma}_{\mathscr{M}_2}(\breve{x}), \overleftarrow{\theta}_{\mathscr{M}_1}(\breve{x}) \leq \overleftarrow{\theta}_{\mathscr{M}_2}(\breve{x})$, and $\overrightarrow{\theta}_{\mathscr{M}_1}(\breve{x}) \leq \overrightarrow{\theta}_{\mathscr{M}_2}(\breve{x})$,

where amplitude and P-terms of each $\Lambda_i, i = 1, 2$ are given below:

$(\overleftarrow{\gamma}_{\mathscr{M}_1}(\breve{x}), \overleftarrow{\theta}_{\mathscr{M}_1}(\breve{x}))$ for $\overleftarrow{\Lambda}_1(\breve{x})$
$(\overrightarrow{\gamma}_{\mathscr{M}_1}(\breve{x}), \overrightarrow{\theta}_{\mathscr{M}_1}(\breve{x}))$ for $\overrightarrow{\Lambda}_1(\breve{x})$
$(\overleftarrow{\gamma}_{\mathscr{M}_2}(\breve{x}), \overleftarrow{\theta}_{\mathscr{M}_2}(\breve{x}))$ for $\overleftarrow{\Lambda}_2(\breve{x})$
$(\overrightarrow{\gamma}_{\mathscr{M}_2}(\breve{x}), \overrightarrow{\theta}_{\mathscr{M}_2}(\breve{x}))$ for $\overrightarrow{\Lambda}_2(\breve{x})$

Definition 14. Two IV-CIFHS-sets $\Omega_{\mathscr{M}_1} = (\Lambda_1, \mathscr{M}_1)$ and $\Omega_{\mathscr{M}_2} = (\Lambda_2, \mathscr{M}_2)$ over the same universe are equal if

(i) $(\Lambda_1, \mathscr{M}_1) \subseteq (\Lambda_2, \mathscr{M}_2)$

(ii) $(\Lambda_2, \mathscr{M}_2) \subseteq (\Lambda_1, \mathscr{M}_1)$

Definition 15. Let $(\Lambda, \mathscr{M})$ be an IV-CIFHS-set over $\mathbb{Z}$. Then

(i) It is a null IV-CIFHS-set, symbolized by $(\Lambda, \mathscr{M})_\Phi$ if for all values of $\breve{x} \in \mathscr{M}$, the A-term of the M-function is given by $\overleftarrow{\gamma}_{\mathscr{M}}(\breve{x}) = \overrightarrow{\gamma}_{\mathscr{M}}(\breve{x}) = 0$, whereas the P-terms of is given by $\overleftarrow{\theta}_{\mathscr{M}}(\breve{x}) = \overrightarrow{\theta}_{\mathscr{M}}(\breve{x}) = 0\pi$

(ii) It is an absolute IV-CIFHS-set, symbolized by $(\Lambda, \mathscr{M})_\Delta$; if for all values of $\breve{x} \in \mathscr{M}$, the A-terms of the M-function is given by $\overleftarrow{\gamma}_{\mathscr{M}}(\breve{x}) = \overrightarrow{\gamma}_{\mathscr{M}}(\breve{x}) = 1$, whereas the P-terms is given by $\overleftarrow{\theta}_{\mathscr{M}}(\breve{x}) = \overrightarrow{\theta}_{\mathscr{M}}(\breve{x}) = 2\pi$.

Definition 16. For two IV-CIFHS-sets $(\Lambda_1, \mathscr{M}_1)$ and $(\Lambda_2, \mathscr{M}_2)$ over the same universe $\mathbb{Z}$, the following definitions hold:

(i) An IV-CIFHS-set $(\Lambda_1, \mathscr{M}_1)$ is a homogeneous IV-CIFHS-set, symbolized by $(\Lambda_1, \mathscr{M}_1)_{\text{Hom}}$ if $\Lambda_1(\breve{x})$ is homogeneous CIF-set $\forall \breve{x} \in \mathscr{M}_1$

(ii) An IV-CIFHS-set $(\Lambda_1, \mathscr{M}_1)$ is a completely homogeneous IV-CIFHS-set, symbolized by $(\Lambda_1, \mathscr{M}_1)_{\text{CHom}}$ if $\Lambda_1(\breve{x})$ is homogeneous with $\Lambda_1(\underline{y}) \forall \breve{x}, \underline{y} \in \mathscr{M}_1$

(iii) An IV-CIFHS-set $(\Lambda_1, \mathscr{M}_1)$ is a completely homogeneous IV-CIFHS-set with $(\Lambda_2, \mathscr{M}_2)$ if $\Lambda_1(\breve{x})$ is a homogeneous with $\Lambda_2(\breve{x}) \forall (\breve{x}) \in \mathscr{M}_1 \prod \mathscr{M}_2$

3.1. Set Operations and Laws on IV-CIFHS-Set. In this section, some basic set theoretic operations and laws are discussed on IV-CIFHS-set.

Definition 17. The complement of IV-CIFHS-set $(\Lambda, \mathscr{M})$, denoted by $(\Lambda, \mathscr{M})^c$, is given by

$$(\Lambda, \mathscr{M})^c = \left\{ \left(\breve{x}, (\Lambda(\breve{x}))^c \right) : \breve{x} \in \mathscr{M}, (\Lambda(\breve{x}))^c \in C_{\text{IV}}(\mathbb{Z}) \right\}, \tag{19}$$

where the M-function $(\Lambda(\breve{x}))^c$ has the A-term given by $(\overleftarrow{\gamma}_{\mathscr{M}}(\breve{x}))^c = 1 - \overleftarrow{\gamma}_{\mathscr{M}}(\breve{x}) (\overrightarrow{\gamma}_{\mathscr{M}}(\breve{x}))^c = 1 - \overrightarrow{\gamma}_{\mathscr{M}}(\breve{x})$ and P-term given by $(\overleftarrow{\theta}_{\mathscr{M}}(\breve{x}))^c = 2\pi - \overleftarrow{\theta}_{\mathscr{M}}(\breve{x})$ and $(\overrightarrow{\theta}_{\mathscr{M}}(\breve{x}))^c = 2\pi - \overrightarrow{\theta}_{\mathscr{M}}(\breve{x})$.

Proposition 1. *Let $(\Lambda, \mathscr{M})$ be an IV-CIFHS-set over $\mathbb{Z}$. Then $((\Lambda, \mathscr{M})^c)^c = (\Lambda, \mathscr{M})$.*

Proof. As $\Lambda(\breve{x}) \in C_{\text{IV}}(\mathbb{Z})$, so in terms of its A- and P-terms, $(\Lambda, \mathscr{M})$ can be expressed as follows:

$$(\Lambda, \mathscr{M}) = \left\{ \left(\breve{x}, \left(\overleftarrow{\gamma}_{\mathscr{M}}(\breve{x}) e^{i\overleftarrow{\theta}_{\mathscr{M}}(\breve{x})}, \overrightarrow{\gamma}_{\mathscr{M}}(\breve{x}) e^{i\overrightarrow{\theta}_G(\breve{x})} \right) \right) : \breve{x} \in \mathscr{M} \right\}. \tag{20}$$

Now

$$(\Lambda, \mathcal{M})^c (\breve{x}) = \left\{ \left(\breve{x}, \left(\left(\overleftarrow{\gamma}_{\mathcal{M}}(\breve{x}) \right)^c e^{i \left(\overline{\theta}_{\mathcal{M}}(\breve{x}) \right)^c}, \left(\overrightarrow{\gamma}_{\mathcal{M}}(\breve{x}) \right)^c e^{i \left(\overrightarrow{\theta}_{\mathscr{D}}(\breve{x}) \right)^c} \right) \right) : \breve{x} \in \mathcal{M} \right\},$$

$$(\Lambda, \mathcal{M})^c (\breve{x}) = \left\{ \left(\breve{x}, \left(\left(1 - \overleftarrow{\gamma}_{\mathcal{M}}(\breve{x}) \right)^c e^{i \left(2\pi - \overline{\theta}_{\mathcal{M}}(\breve{x}) \right)^c}, \left(1 - \overrightarrow{\gamma}_{\mathcal{M}}(\breve{x}) \right)^c e^{i \left(2\pi - \overrightarrow{\theta}_{\mathscr{D}}(\breve{x}) \right)^c} \right) \right) : \breve{x} \in \mathcal{M} \right\},$$

$$((\chi, \mathscr{D})^c)^c = \left\{ \left(\breve{x}, \left(\left(1 - \overleftarrow{\gamma}_{\mathcal{M}}(\breve{x}) \right)^c e^{i \left(2\pi - \overline{\theta}_{\mathcal{M}}((\breve{x})) \right)^c}, \left(1 - \overrightarrow{\gamma}_{\mathcal{M}}((\breve{x})) \right)^c e^{i \left(2\pi - \overrightarrow{\theta}_{G}((\breve{x})) \right)^c} \right) \right) : (\breve{x}) \in \mathcal{M} \right\}, \tag{21}$$

$$((\Lambda, \mathcal{M})^c)^c = \left\{ \left(\breve{x}, \left(\left(1 - \left(1 - \overleftarrow{\gamma}_{\mathcal{M}}(\breve{x}) \right) \right) e^{i \left(2\pi - \left(2\pi - \overline{\theta}_{\mathcal{M}}(\breve{x}) \right) \right)}, \left(1 - \left(1 - \overrightarrow{\gamma}_{\mathcal{M}}(\breve{x}) \right) \right) e^{i \left(2\pi - \left(2\pi - \overrightarrow{\theta}_{G}(\breve{x}) \right) \right)} \right) \right) : \breve{x} \in \mathcal{M} \right\},$$

$$((\Lambda, \mathcal{M})^c)^c = \left\{ \left(\breve{x}, \left(\overleftarrow{\gamma}_{\mathcal{M}}(\breve{x}) e^{i \overline{\theta}_{\mathcal{M}}(\breve{x})}, \overrightarrow{\gamma}_{\mathcal{M}}(\breve{x}) e^{i \overrightarrow{\theta}_{G}(\breve{x})} \right) \right) : \breve{x} \in \mathcal{M} \right\}.$$

From equations (20) and (21), we have $((\Lambda, \mathcal{M})^c)^c = (\Lambda, \mathcal{M})$. □

Proposition 2. *Let $(\Lambda, \mathcal{M})$ be an IV-CIFHS-set over $\mathbb{Z}$. Then*

(i) $((\Lambda, \mathcal{M})_\Phi)^c = (\Lambda, \mathcal{M})_\Delta$

(ii) $((\Lambda, \mathcal{M})_\Delta)^c = (\Lambda, \mathcal{M})_\Phi$

Definition 18. For two IV-CIFHS-set $(\Lambda_1, \mathcal{M}_1)$ and $(\Lambda_2, \mathcal{M}_2)$, the intersection $(\Lambda_1, \mathcal{M}_1) \prod (\Lambda_2, \mathcal{M}_2)$ over the same universe $\mathbb{Z}$ is the IV-CIFHS-set $(\Lambda_3, \mathcal{M}_3)$, where $\mathcal{M}_3 = \mathcal{M}_1 \coprod \mathcal{M}_2$, and for all $\breve{x} \in \mathcal{M}_3$,

$$\overleftarrow{\Lambda}_3(\breve{x}) = \begin{cases} \overleftarrow{\gamma}_{\mathcal{M}_1}(\breve{x}) e^{i\overline{\theta}_{\mathcal{M}_1}(\breve{x})}, & \text{if } \breve{x} \in \mathcal{M}_1 \mathcal{M}_2, \\ \overleftarrow{\gamma}_{\mathcal{M}_2}(\breve{x}) e^{i\overline{\theta}_{\mathcal{M}_2}(\breve{x})}, & \text{if } \breve{x} \in \mathcal{M}_2 \mathcal{M}_1, \\ \min\left[\overleftarrow{\gamma}_{\mathcal{M}_1}(\breve{x}), \overleftarrow{\gamma}_{\mathcal{M}_2}(\breve{x}) \right] e^{i\min\left[\overline{\theta}_{\mathcal{M}_1}(\breve{x}), \overline{\theta}_{\mathcal{M}_2}(\breve{x}) \right]}, & \text{if } \breve{x} \in \mathcal{M}_1 \prod \mathcal{M}_2, \end{cases}$$

$$\overrightarrow{\Lambda}_3(\breve{x}) = \begin{cases} \overrightarrow{\gamma}_{\mathcal{M}_1}(\breve{x}) e^{i\overrightarrow{\theta}_{\mathcal{M}_1}(\breve{x})}, & \text{if } \breve{x} \in \mathcal{M}_1 \mathcal{M}_2, \\ \overrightarrow{\gamma}_{\mathcal{M}_2}(\breve{x}) e^{i\overrightarrow{\theta}_{\mathcal{M}_2}(\breve{x})}, & \text{if } \breve{x} \in \mathcal{M}_2 \mathcal{M}_1, \\ \min\left[\overrightarrow{\gamma}_{\mathcal{M}_1}(\breve{x}), \overrightarrow{\gamma}_{\mathcal{M}_2}(\breve{x}) \right] e^{i\min\left[\overrightarrow{\theta}_{\mathcal{M}_1}(\breve{x}), \overrightarrow{\theta}_{\mathcal{M}_2}(\breve{x}) \right]} & \text{if } \breve{x} \in \mathcal{M}_1 \prod \mathcal{M}_2. \end{cases} \tag{22}$$

Definition 19. For two IV-CIFHS-set $(\Lambda_1, \mathcal{M}_1)$ and $(\Lambda_2, \mathcal{M}_2)$, the difference is defined as follows:

$$(\Lambda_1, \mathcal{M}_1) \smallsetminus (\Lambda_2, \mathcal{M}_2) = (\Lambda_1, \mathcal{M}_1) \prod (\Lambda_2, \mathcal{M}_2)^c. \tag{23}$$

Definition 20. The union $(\Lambda_1, \mathcal{M}_1) \coprod (\Lambda_2, \mathcal{M}_2)$ of two IV-CIFHS-set $(\Lambda_1, \mathcal{M}_1)$ and $(\Lambda_2, \mathcal{M}_2)$ over the same universe $\mathbb{Z}$ is the IV-CIFHS-set $(\Lambda_3, \mathcal{M}_3)$, where $\mathcal{M}_3 = \mathcal{M}_1 \coprod \mathcal{M}_2$, and for all $\breve{x} \in \mathcal{M}_3$,

$$
\overleftarrow{\Lambda}_3(\breve{x}) = \begin{cases} \overleftarrow{\gamma}_{\mathcal{M}_1}(\breve{x})e^{i\overleftarrow{\theta}_{\mathcal{M}_1}(\breve{x})}, & \text{if } (\breve{x}) \in \mathcal{M}_1 \setminus \mathcal{M}_2, \\[2mm] \overleftarrow{\gamma}_{\mathcal{M}_2}(\breve{x})e^{i\overleftarrow{\theta}_{\mathcal{M}_2}(\breve{x})}, & \text{if } (\breve{x}) \in \mathcal{M}_2 \setminus \mathcal{M}_1, \\[2mm] \max\left[\overleftarrow{\gamma}_{\mathcal{M}_1}(\breve{x}), \overleftarrow{\gamma}_{\mathcal{M}_2}(\breve{x})\right]e^{i\max\left[\overleftarrow{\theta}_{\mathcal{M}_1}(\breve{x}), \overleftarrow{\theta}_{\mathcal{M}_2}(\breve{x})\right]}, & \text{if } (\breve{x}) \in \mathcal{M}_1 \prod \mathcal{M}_2, \end{cases}
$$

$$
\overrightarrow{\Lambda}_3(\breve{x}) = \begin{cases} \overrightarrow{\gamma}_{\mathcal{M}_1}(\breve{x})e^{i\overrightarrow{\theta}_{\mathcal{M}_1}(\breve{x})}, & \text{if } \breve{x} \in \mathcal{M}_1 \mathcal{M}_2, \\[2mm] \overrightarrow{\gamma}_{\mathcal{M}_2}(\breve{x})e^{i\overrightarrow{\theta}_{\mathcal{M}_2}(\breve{x})}, & \text{if } \breve{x} \in \mathcal{M}_2 \mathcal{M}_1, \\[2mm] \max\left[\overrightarrow{\gamma}_{\mathcal{M}_1}(\breve{x})\overrightarrow{\gamma}_{\mathcal{M}_2}(\breve{x})\right]e^{i\max\left[\overrightarrow{\theta}_{\mathcal{M}_1}(\breve{x}), \overrightarrow{\theta}_{\mathcal{M}_2}(\breve{x})\right]}, & \text{if } \breve{x} \in \mathcal{M}_1 \prod \mathcal{M}_2. \end{cases}
\tag{24}
$$

Proposition 3. *Let $(\Lambda, \mathcal{M})$ be an IV-CIFHS-set over $\mathbb{Z}$. Then the following results hold true:*

(i) $(\Lambda, \mathcal{M}) \bigsqcup (\Lambda, \mathcal{M})_\Phi = (\Lambda, \mathcal{M})$

(ii) $(\Lambda, \mathcal{M}) \bigsqcup (\Lambda, \mathcal{M})_\Delta = (\Lambda, \mathcal{M})_\Delta$

(iii) $(\Lambda, \mathcal{M}) \prod (\Lambda, \mathcal{M})_\Phi = (\Lambda, \mathcal{M})_\Phi$

(iv) $(\Lambda, \mathcal{M}) \prod (\Lambda, \mathcal{M})_\Delta = (\Lambda, \mathcal{M})$

(v) $(\Lambda, \mathcal{M})_\Phi \bigsqcup (\Lambda, \mathcal{M})_\Delta = (\Lambda, \mathcal{M})_\Delta$

(vi) $(\Lambda, \mathcal{M})_\Phi \prod (\Lambda, \mathcal{M})_\Delta = (\Lambda, \mathcal{M})_\Phi$

Proposition 4. *Let $(\Lambda_1, \mathcal{M}_1)$, $(\Lambda_2, \mathcal{M}_2)$, and $(\Lambda_3, \mathcal{M}_3)$ are three IV-CIFHS-sets over the same universe $\mathbb{Z}$. Then the following commutative and associative laws hold true:*

(i) $(\Lambda_1, \mathcal{M}_1) \prod (\Lambda_2, \mathcal{M}_2) = (\Lambda_2, \mathcal{M}_2) \prod (\Lambda_1, \mathcal{M}_1)$

(ii) $(\Lambda_1, \mathcal{M}_1) \bigsqcup (\Lambda_2, \mathcal{M}_2) = (\Lambda_2, \mathcal{M}_2) \bigsqcup (\Lambda_1, \mathcal{M}_1)$

(iii) $(\Lambda_1, \mathcal{M}_1) \prod ((\Lambda_2, \mathcal{M}_2) \prod (\Lambda_3, \mathcal{M}_3)) = ((\Lambda_1, \mathcal{M}_1) \prod (\Lambda_2, \mathcal{M}_2)) \prod (\Lambda_3, \mathcal{M}_3)$

(iv) $(\Lambda_1, \mathcal{M}_1) \bigsqcup ((\Lambda_2, \mathcal{M}_2) \bigsqcup (\Lambda_3, \mathcal{M}_3)) = ((\Lambda_1, \mathcal{M}_1) \bigsqcup (\Lambda_2, \mathcal{M}_2)) \bigsqcup (\Lambda_3, \mathcal{M}_3)$

Proposition 5. *Let $(\Lambda_1, \mathcal{M}_1)$ and $(\Lambda_2, \mathcal{M}_2)$ are two IV-CIFHS-sets over the same universe $\mathbb{Z}$. Then the following De Morgans's laws hold true:*

(i) $((\Lambda_1, \mathcal{M}_1) \prod (\Lambda_2, \mathcal{M}_2))^c = (\Lambda_1, \mathcal{M}_1)^c \bigsqcup (\Lambda_2, \mathcal{M}_2)^c$

(ii) $((\Lambda_1, \mathcal{M}_1) \bigsqcup (\Lambda_2, \mathcal{M}_2))^c = (\Lambda_1, \mathcal{M}_1)^c \prod (\Lambda_2, \mathcal{M}_2)^c$

3.2. Aggregation of Interval-Valued Complex Intuitionistic Fuzzy Hypersoft Set. In this section, we define an aggregation operator on IV-CIFHS-set that produces an aggregate fuzzy set from an IV-CIFHS-set and its cardinal set. The approximate functions of an IV-CIFHS-set are fuzzy. Here, $D, \mathfrak{E}, \xi_\mathcal{D}$ and $\underset{\text{IVCIFHS}}{\uplus}$ will be in accordance with Definition 12.

Definition 21. Let $\xi_\mathcal{D} \in \uplus_{\text{IVCIFHS}}$. Assume that $\mathbb{Z} = \{v_1, v_2, \ldots, v_m\}$ and $\mathfrak{E} = \{\mathcal{L}_1, \mathcal{L}_2, \ldots, \mathcal{L}_n\}$ with $\mathcal{L}_1 = \{e_{11}, e_{12}, \ldots, e_{1n}\}, \mathcal{L}_2 = \{e_{21}, e_{22}, \ldots, e_{2n}\}, \ldots, \mathcal{L}_n = \{e_{n1}, e_{n2}, \ldots, e_{nn}\}$ and $\mathcal{D} = \mathcal{L}_1 \times \mathcal{L}_2 \times \cdots \times \mathcal{L}_n = \{\breve{x}_1, \breve{x}_2, \ldots, \breve{x}_n, \ldots, \breve{x}_{n^n} = \breve{x}_r\}$, each $\breve{x}_i$ is n-tuple element of $\mathcal{D}$ and $|\mathcal{D}| = r = n^n$; the representation of $\xi_\mathcal{D}$ can be seen in Table 3.

In Table 3, $\eta^1_{\chi_\mathcal{D}(x)}$ and $\eta^2_{\chi_\mathcal{D}(x)}$ are M-function and NM-function of $\chi_\mathcal{D}$, respectively, with interval-valued intuitionistic fuzzy values. If $\alpha_{ij} = (\eta^1_{\chi_\mathcal{D}(\breve{x}_j)}(v_i), \eta^2_{\chi_\mathcal{D}(\breve{x}_j)}(v_i))$, for $i = \mathbb{N}_1^m$ and $j = \mathbb{N}_1^r$, then IV-CIFHS-set $\xi_\mathcal{D}$ is uniquely characterized by the following matrix:

$$
[\alpha_{ij}] = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1r} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{m1} & \alpha_{m2} & \cdots & \alpha_{mr} \end{bmatrix},
\tag{25}
$$

which is called an $m \times r$ IV-CIFHS-set matrix.

Definition 22. If $\xi_\mathcal{D} \in \underset{\text{IVCIFHS}}{\uplus}$, then cardinal set of $\xi_\mathcal{D}$ is defined as follows:

$$
\|\xi_\mathcal{D}\| = \left\{ \frac{\left(\eta^1_{\|\xi_\mathcal{D}\|}(\breve{x}), \eta^2_{\|\xi_\mathcal{D}\|}(\breve{x})\right)}{\breve{x}} : \breve{x} \in \mathcal{D} \right\},
\tag{26}
$$

where $\eta^1_{\|\xi_\mathcal{D}\|} : \mathcal{D} \longrightarrow [0,1]$ is an M-function of $\|\xi_\mathcal{D}\|$ with $\eta^1_{\|\xi_\mathcal{D}\|}(\breve{x}) = |\chi_\mathcal{D}(\breve{x})|/|\mathbb{Z}|$ and $\eta^2_{\|\xi_\mathcal{D}\|} : \mathcal{D} \longrightarrow [0,1]$ is an NM-function of $\|\xi_\mathcal{D}\|$ with $\eta^2_{\|\xi_\mathcal{D}\|}(\breve{x}) = |\chi_\mathcal{D}(\breve{x})|/|\mathbb{Z}|$. Both have interval-valued intuitionistic fuzzy values. Note that $\|C_{\text{ivcifhss}}(\mathbb{Z})\|$ is the collection of all cardinal sets of IV-CIFHS-sets and $\|C_{\text{ivcifhss}}(\mathbb{Z})\| \subseteq \mathbb{IVIF}(\mathcal{D})$.

Definition 23. Let $\xi_\mathcal{D} \in C_{\text{ivcifhss}}(\mathbb{Z})$ and $\|\xi_\mathcal{D}\| \in \|C_{\text{ivcifhss}}(\mathbb{Z})\|$. Consider $\mathfrak{E}$ as in Definition 12, the representation of $\|\xi_\mathcal{D}\|$ can be seen in Table 4.

If $\alpha_{1j} = (\eta^1_{\|\xi_\mathcal{D}\|}(\breve{x}_j), \eta^2_{\|\xi_\mathcal{D}\|}(\breve{x}_j))$, for $j = \mathbb{N}_1^r$, then the cardinal set $\|\xi_\mathcal{D}\|$ is represented by the following matrix:

$$
[\alpha_{ij}]_{1 \times r} = [\alpha_{11} \quad \alpha_{12} \quad \cdots \quad \alpha_{1r}],
\tag{27}
$$

which is called the cardinal matrix of $\|\xi_\mathcal{D}\|$.

Definition 24. Let $\xi_\mathcal{D} \in C_{\text{ivcifhss}}(\mathbb{Z})$ and $\|\xi_\mathcal{D}\| \in \|C_{\text{ivcifhss}}(\mathbb{Z})\|$. Then IV-CIFHS-aggregation operator is defined as follows:

$$
\widetilde{\xi_\mathcal{D}} = A_{\text{ivcifhss}}(\|\xi_\mathcal{D}\|, \xi_\mathcal{D}),
\tag{28}
$$

where

TABLE 3: Tabular representation of $\xi_{\mathscr{D}}$.

$\xi_{\mathscr{D}}$	$\breve{x}_1$	$\breve{x}_2$	$\cdots$	$\breve{x}_r$
ν_1	$(\eta^1_{\chi_{\mathscr{D}}(\breve{x}_1)}(\nu_1), \eta^2_{\chi_{\mathscr{D}}(\breve{x}_1)}(\nu_1))$	$(\eta^1_{\chi_{\mathscr{D}}(\breve{x}_2)}(\nu_1), \eta^2_{\chi_{\mathscr{D}}(\breve{x}_2)}(\nu_1))$	$\cdots$	$(\eta^1_{\chi_{\mathscr{D}}(\breve{x}_r)}(\nu_1), \eta^2_{\chi_{\mathscr{D}}(\breve{x}_r)}(\nu_1))$
ν_1	$(\eta^1_{\chi_{\mathscr{D}}(\breve{x}_1)}(\nu_2), \eta^2_{\chi_{\mathscr{D}}(\breve{x}_1)}(\nu_2))$	$(\eta^1_{\chi_{\mathscr{D}}(\breve{x}_2)}(\nu_2), \eta^2_{\chi_{\mathscr{D}}(\breve{x}_2)}(\nu_2))$	$\cdots$	$(\eta^1_{\chi_{\mathscr{D}}(\breve{x}_r)}(\nu_2), \eta^2_{\chi_{\mathscr{D}}(\breve{x}_r)}(\nu_2))$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
ν_1	$(\eta^1_{\chi_{\mathscr{D}}(\breve{x}_1)}(\nu_m), \eta^2_{\chi_{\mathscr{D}}(\breve{x}_1)}(\nu_m))$	$(\eta^1_{\chi_{\mathscr{D}}(\breve{x}_2)}(\nu_m), \eta^2_{\chi_{\mathscr{D}}(\breve{x}_2)}(\nu_m))$	$\cdots$	$(\eta^1_{\chi_{\mathscr{D}}(\breve{x}_r)}(\nu_m), \eta^2_{\chi_{\mathscr{D}}(\breve{x}_r)}(\nu_m))$

TABLE 4: Tabular representation of $\|\xi_{\mathscr{D}}\|$.

$\mathscr{D}$	$\breve{x}_1$	$\breve{x}_2$	$\cdots$	$\breve{x}_r$
$\eta_{\|\xi_{\mathscr{D}}\|}$	$(\eta^1_{\|\xi_{\mathscr{D}}\|}(\breve{x}_1), \eta^2_{\|\xi_{\mathscr{D}}\|}(\breve{x}_1))$	$(\eta^1_{\|\xi_{\mathscr{D}}\|}(\breve{x}_2), \eta^2_{\|\xi_{\mathscr{D}}\|}(\breve{x}_2))$	$\cdots$	$(\eta^1_{\|\xi_{\mathscr{D}}\|}(\breve{x}_r), \eta^2_{\|\xi_{\mathscr{D}}\|}(\breve{x}_r))$

$$A_{\text{ivcifhss}}: \left\| C_{\text{ivcifhss}}(\mathbb{Z}) \right\| \times C_{\text{ivcifhss}}(\mathbb{Z}) \longrightarrow \mathbb{F}(\mathbb{Z}), \qquad (29)$$

where $\widetilde{\xi_{\mathscr{D}}}$ is called the aggregate fuzzy set of IV-CIFHS-set $\xi_{\mathscr{D}}$.

Its M-function is given as follows:

$$\eta_{\widetilde{\xi_{\mathscr{D}}}}: \mathbb{Z} \longrightarrow [0, 1], \qquad (30)$$

with

$$\eta_{\widetilde{\xi_{\mathscr{D}}}}(\nu) = \frac{1}{|\mathscr{D}|} \sum_{\breve{x} \in \mathscr{D}} \eta_{\text{Card}(\xi_{\mathscr{D}})}(\breve{x}) \eta_{\text{Card}(\chi_{\mathscr{D}})}(\nu). \qquad (31)$$

Definition 25. Let $\xi_{\mathscr{D}} \in C_{\text{ivcifhss}}(\mathbb{Z})$ and $\widetilde{\xi_{\mathscr{D}}}$ be its aggregate fuzzy set. Assume that $\mathbb{Z} = \{\nu_1, \nu_2, \ldots, \nu_m\}$, then $\widetilde{\xi_{\mathscr{D}}}$ can be presented as follows:

$$\begin{bmatrix} \xi_{\mathscr{D}} & \vdots & \eta_{\widetilde{\xi_{\mathscr{D}}}} \\ \cdots\cdots & \vdots & \cdots\cdots \\ \nu_1 & \vdots & \eta_{\widetilde{\xi_{\mathscr{D}}}}(\nu_1) \\ \nu_2 & \vdots & \eta_{\widetilde{\xi_{\mathscr{D}}}}(\nu_2) \\ \vdots & \vdots & \vdots \\ \nu_m & \vdots & \eta_{\widetilde{\xi_{\mathscr{D}}}}(\nu_m) \end{bmatrix}. \qquad (32)$$

If $\alpha_{i1} = \eta_{\widetilde{\xi_{\mathscr{D}}}}(\nu_i)$ for $i = \mathbb{N}_1^m$, then $\widetilde{\xi_{\mathscr{D}}}$ is represented by the following matrix:

$$[\alpha_{i1}]_{m \times 1} = \begin{bmatrix} \alpha_{11} \\ \alpha_{21} \\ \vdots \\ \alpha_{m1} \end{bmatrix}, \qquad (33)$$

which is called the aggregate matrix of $\widetilde{\xi_{\mathscr{D}}}$ over $\mathbb{Z}$.

3.3. Applications of Interval-Valued Complex Intuitionistic Fuzzy Hypersoft Set. In this section, an algorithm is presented to solve the problems in decision-making by having under consideration the concept of aggregations defined in the previous section. An example is demonstrated to explain the proposed algorithm. It is necessary to determine an aggregate fuzzy set of IV-CIFHS-set for choosing the best option (parameter) from the given set (set of choices/alternatives). The following algorithm is proposed based on the definitions given in Subsection 3.2 that may help make optimal decision.

Now, Algorithm 1 is explained with the help of the following example.

Example 2. Suppose a businessman wants to buy a share from the share market. There are four same kinds of share that form the set, $\mathbb{Z} = \{s_1, s_2, s_3, s_4\}$. The expert committee considers a set of attributes, $\mathfrak{E} = \{e_1, e_2, e_3\}$. For $i = 1, 2, 3$, the attributes e_i stand for "current trend of company performance," "particular company's stock price for last one year," and "Home country inflation rate," respectively. Corresponding to each attribute, the sets of attribute values are $\mathscr{L}_1 = \{e_{11}, e_{12}\}$, $\mathscr{L}_2 = \{e_{21}\}$, and $\mathscr{L}_3 = \{e_{31}, e_{32}\}$. Then the set $\mathscr{D} = \mathscr{L}_1 \times \mathscr{L}_2 \times \mathscr{L}_3 = \{\delta_1, \delta_2, \delta_3, \delta_4\}$, where each δ_i is a three-tuple. IV-CIFHS-sets $\chi_{\mathscr{D}}(\delta_1), \chi_{\mathscr{D}}(\delta_2), \chi_{\mathscr{D}}(\delta_3), \chi_{\mathscr{D}}(\delta_4)$ are defined as follows:

▷ Start
▷ Input Stage:
————1. Consider $\mathbb{Z}$ as the universe of discourse
————2. Consider $\mathfrak{E}$ as a set of n parameters
————3. Classify n parameters into disjoint parametric-valued sets $\mathscr{L}_1, \mathscr{L}_2, \mathscr{L}_3, \ldots, \mathscr{L}_n$
▷ Construction Stage:
————4. $\mathscr{D} = \mathscr{L}_1 \times \mathscr{L}_2 \times \mathscr{L}_3 \times \cdots \times \mathscr{L}_n$
————5. Construct IV-CIFHS-set $\xi_{\mathscr{D}}$ over $\mathbb{Z}$ in accordance with Definition 12
▷ Computation Stage:
————6. Compute $\|\xi_{\mathscr{D}}\|$ for A-term and P-term separately by using Definition 22
————7. Compute $\widetilde{\xi_{\mathscr{D}}}$ for A-term and P-term separately with the help of Definition 24
————8. Compute $\eta_{\widetilde{\xi_{\mathscr{D}}}}(\nu)$ by using Definition 24
▷ Output Stage:
————9. Find the best option by the max modulus of $\eta_{\widetilde{\xi_{\mathscr{D}}}}(\nu)$ with the help of Definition 25
▷ End

ALGORITHM 1: Decision-support algorithm based on aggregations of an IV-CIFHS-set

$$
\begin{aligned}
\chi_{\mathscr{D}}(\delta_1) &= \left\{ \begin{array}{l} \dfrac{([0.4,0.5],[0.3,0.4])e^{i([0.1,0.4],[0.3,0.5])\pi}}{s_1}, \dfrac{([0.2,0.3],[0.2,0.6])e^{i([0.2,0.3],[0.3,0.5])\pi}}{s_2} \\[3mm] \dfrac{([0.1,0.4],[0.4,0.5])e^{i([0.3,0.4],[0.1,0.3])\pi}}{s_3}, \dfrac{([0.4,0.5],[0.2,0.4])e^{i([0.1,0.4],[0.1,0.2])\pi}}{s_4} \end{array} \right\}, \\[5mm]
\chi_{\mathscr{D}}(\delta_2) &= \left\{ \begin{array}{l} \dfrac{([0.1,0.2],[0.2,0.7])e^{i([0.2,0.4],[0.3,0.5])\pi}}{s_1}, \dfrac{([0.1,0.6],[0.2,0.3])e^{i([0.4,0.5],[0.2,0.3])\pi}}{s_2} \\[3mm] \dfrac{([0.2,0.3],[0.1,0.6])e^{i([0.2,0.2],[0.1,0.5])\pi}}{s_3}, \dfrac{([0.2,0.4],[0.3,0.4])e^{i([0.1,0.3],[0.3,0.6])\pi}}{s_4} \end{array} \right\}. \\[5mm]
\chi_{\mathscr{D}}(\delta_3) &= \left\{ \begin{array}{l} \dfrac{([0.2,0.3],[0.2,0.5])e^{i([0.1,0.4],[0.4,0.5])\pi}}{s_1}, \dfrac{([0.2,0.4],[0.1,0.3])e^{i([0.3,0.6],[0.2,0.3])\pi}}{s_2} \\[3mm] \dfrac{([0.2,0.4],[0.1,0.3])e^{i([0.2,0.5],[0.1,0.2])\pi}}{s_3}, \dfrac{([0.3,0.3],[0.2,0.6])e^{i([0.2,0.5],[0.2,0.4])\pi}}{s_4} \end{array} \right\}, \\[5mm]
\chi_{\mathscr{D}}(\delta_4) &= \left\{ \begin{array}{l} \dfrac{([0.3,0.5],[0.3,0.4])e^{i([0.4,0.5],[0.3,0.4])\pi}}{s_1}, \dfrac{([0.1,0.3],[0.3,0.5])e^{i([0.4,0.4],[0.3,0.5])\pi}}{s_2} \\[3mm] \dfrac{([0.1,0.4],[0.3,0.5])e^{i([0.3,0.5],[0.1,0.2])\pi}}{s_3}, \dfrac{([0.4,0.5],[0.2,0.4])e^{i([0.3,0.5],[0.2,0.3])\pi}}{s_4} \end{array} \right\},
\end{aligned}
\tag{34}
$$

Step 1. IV-CIFHS-set $\xi_{\mathscr{D}}$ is written as follows:

$$
\xi_{\mathcal{D}} = \left\{ \begin{array}{l}
\left(\delta_1, \left\{ \begin{array}{l}
\dfrac{([0.4,0.5],[0.3,0.4])e^{i([0.1,0.4],[0.3,0.5])\pi}}{s_1}, \dfrac{([0.2,0.3],[0.2,0.6])e^{i([0.2,0.3],[0.3,0.5])\pi}}{s_2}, \\[2em]
\dfrac{([0.1,0.4],[0.4,0.5])e^{i([0.3,0.4],[0.1,0.3])\pi}}{s_3}, \dfrac{([0.4,0.5],[0.2,0.4])e^{i([0.1,0.4],[0.1,0.2])\pi}}{s_4}
\end{array} \right\} \right) \\[4em]
\left(\delta_2, \left\{ \begin{array}{l}
\dfrac{([0.1,0.2],[0.2,0.7])e^{i([0.2,0.4],[0.3,0.5])\pi}}{s_1}, \dfrac{([0.1,0.6],[0.2,0.3])e^{i([0.4,0.5],[0.2,0.3])\pi}}{s_2}, \\[2em]
\dfrac{([0.2,0.3],[0.1,0.6])e^{i([0.2,0.2],[0.1,0.5])\pi}}{s_3}, \dfrac{([0.2,0.4],[0.3,0.4])e^{i([0.1,0.3],[0.3,0.6])\pi}}{s_4}
\end{array} \right\} \right) \\[4em]
\left(\delta_3, \left\{ \begin{array}{l}
\dfrac{([0.2,0.3],[0.2,0.5])e^{i([0.1,0.4],[0.4,0.5])\pi}}{s_1}, \dfrac{([0.2,0.4],[0.1,0.3])e^{i([0.3,0.6],[0.2,0.3])\pi}}{s_2}, \\[2em]
\dfrac{([0.2,0.4],[0.1,0.3])e^{i([0.2,0.5],[0.1,0.2])\pi}}{s_3}, \dfrac{([0.3,0.3],[0.2,0.6])e^{i([0.2,0.5],[0.2,0.4])\pi}}{s_4}
\end{array} \right\} \right) \\[4em]
\left(\delta_4, \left\{ \begin{array}{l}
\dfrac{([0.3,0.5],[0.3,0.4])e^{i([0.4,0.5],[0.3,0.4])\pi}}{s_1}, \dfrac{([0.1,0.3],[0.3,0.5])e^{i([0.4,0.4],[0.3,0.5])\pi}}{s_2}, \\[2em]
\dfrac{([0.1,0.4],[0.3,0.5])e^{i([0.3,0.5],[0.1,0.2])\pi}}{s_3}, \dfrac{([0.4,0.5],[0.2,0.4])e^{i([0.3,0.5],[0.2,0.3])\pi}}{s_4}
\end{array} \right\} \right)
\end{array} \right\}. \tag{35}
$$

Step 2. The cardinal is computed as follows:

$$
\|\xi_{\mathcal{D}}\| (\text{Amplitude Term}) = \left\{ \begin{array}{l}
\dfrac{([0.275,0.425],[0.275,0.475])}{\delta_1}, \dfrac{([0.150,0.375],[0.200,0.500])}{\delta_2}, \\[2em]
\dfrac{([0.225,0.350],[0.150,0.425])}{\delta_3}, \dfrac{([0.225,0.425],[0.275,0.450])}{\delta_4}
\end{array} \right\},
$$

$$
\|\xi_{\mathcal{D}}\| (\text{Phase Term}) = \left\{ \begin{array}{l}
\dfrac{([0.175,0.375],[0.200,0.375])}{\delta_1}, \dfrac{([0.225,0.350],[0.225,0.475])}{\delta_2}, \\[2em]
\dfrac{([0.200,0.500],[0.225,0.350])}{\delta_3}, \dfrac{([0.350,0.475],[0.225,0.350])}{\delta_4}
\end{array} \right\}. \tag{36}
$$

Step 3. The set $\widetilde{\xi_{\mathcal{D}}}$ can be determined as follows:

$$\widetilde{\xi_{\mathcal{D}}} \, (\text{Apmlitude Term}) = \frac{1}{4} \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \begin{bmatrix} [0.275, 0.425], [0.275, 0.475] \\ [0.150, 0.375], [0.200, 0.500] \\ [0.225, 0.350], [0.150, 0.425] \\ [0.225, 0.425], [0.275, 0.450] \end{bmatrix}, \tag{37}$$

where

$$m_{11} = [0.4, 0.5], [0.3, 0.4], m_{12} = [0.1, 0.2], [0.2, 0.7], m_{13} = [0.2, 0.3], [0.2, 0.5], m_{14} = [0.3, 0.5], [0.3, 0.4],$$

$$m_{21} = [0.2, 0.3], [0.2, 0.6], m_{22} = [0.1, 0.6], [0.2, 0.3], m_{23} = [0.2, 0.4], [0.1, 0.3], m_{24} = [0.1, 0.3], [0.3, 0.5],$$

$$m_{31} = [0.1, 0.4], [0.4, 0.5], m_{32} = [0.2, 0.3], [0.1, 0.6], m_{33} = [0.2, 0.4], [0.1, 0.3], m_{34} = [0.1, 0.4], [0.3, 0.5],$$

$$m_{41} = [0.4, 0.5], [0.2, 0.4], m_{42} = [0.2, 0.4], [0.3, 0.4], m_{43} = [0.3, 0.3], [0.2, 0.6], m_{44} = [0.4, 0.5], [0.2, 0.4]$$

$$\tag{38}$$

$$= \frac{1}{4} \begin{bmatrix} 0.2 & 0.0 & 0.1 & 0.2 \\ 0.1 & 0.4 & 0.3 & 0.0 \\ 0.0 & 0.2 & 0.3 & 0.1 \\ 0.3 & 0.1 & 0.1 & 0.3 \end{bmatrix} \begin{bmatrix} 0.150 \\ 0.175 \\ 0.200 \\ 0.150 \end{bmatrix} = \begin{bmatrix} 0.020000 \\ 0.036250 \\ 0.027500 \\ 0.031875 \end{bmatrix}.$$

$$\widetilde{\xi_{\mathcal{D}}} \, (\text{PhaseTerm}) =$$

$$\frac{1}{4} \begin{bmatrix} [0.1, 0.4], [0.3, 0.5] & [0.2, 0.4], [0.3, 0.5] & [0.1, 0.4], [0.4, 0.5] & [0.4, 0.5], [0.3, 0.4] \\ [0.2, 0.3], [0.3, 0.5] & [0.4, 0.5], [0.2, 0.3] & [0.3, 0.6], [0.2, 0.3] & [0.4, 0.4], [0.3, 0.5] \\ [0.3, 0.4], [0.1, 0.3] & [0.2, 0.2], [0.1, 0.5] & [0.2, 0.5], [0.1, 0.2] & [0.3, 0.5], [0.1, 0.2] \\ [0.1, 0.4], [0.1, 0.2] & [0.1, 0.3], [0.3, 0.6] & [0.2, 0.5], [0.2, 0.4] & [0.3, 0.5], [0.2, 0.3] \end{bmatrix} \begin{bmatrix} [0.175, 0.375], [0.200, 0.375] \\ [0.225, 0.350], [0.225, 0.475] \\ [0.200, 0.500], [0.225, 0.350] \\ [0.350, 0.475], [0.225, 0.350] \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 0.1 & 0.1 & 0.0 & 0.2 \\ 0.0 & 0.3 & 0.4 & 0.1 \\ 0.3 & 0.1 & 0.4 & 0.4 \\ 0.3 & 0.0 & 0.3 & 0.3 \end{bmatrix} \begin{bmatrix} 0.175 \\ 0.125 \\ 0.275 \\ 0.250 \end{bmatrix} = \begin{bmatrix} 0.020000 \\ 0.043125 \\ 0.068750 \\ 0.052500 \end{bmatrix},$$

$$\widetilde{\xi_{\mathcal{D}}} = \left\{ \frac{0.020000 e^{i0.020000\pi}}{s_1}, \frac{0.036250 e^{i0.043125\pi}}{s_2}, \frac{0.027500 e^{i0.068750\pi}}{s_3}, \frac{0.031875 e^{i0.052500\pi}}{s_4} \right\}.$$

$$\tag{39}$$

Consider the modulus value of $\max(\eta_{\widetilde{\xi_{\mathcal{D}}}}) = \max\{0.02124999361/s_1, \quad 0.01624997922/s_2, 0.0131496449/$ $s_3, 0.00749999154/s_4\} = 0.02124999361/s_1$. This means that s_1 may be recommended for suitable investment.

4. Interval-Valued Complex Neutrosophic Hypersoft Set (IV-CNHS-Set)

The basic theory of the IV-CNHS-set is developed in this section.

Definition 27. Let $\mathscr{M}_1, \mathscr{M}_2, \mathscr{M}_3, \ldots, \mathscr{M}_n$ are disjoint sets of n distinct attributes $m_1, m_2, m_3, \ldots, m_n$, respectively, for $n \geq 1$, $\mathscr{M} = \mathscr{M}_1 \times \mathscr{M}_2 \times \mathscr{M}_3 \times \ldots \times \mathscr{M}_n$ and $\Lambda(\underline{\lambda})$ be an IV-CNFS over $\mathbb{Z}$ for all $\underline{\lambda} = (b_1, b_2, b_3, \ldots, b_n) \in \mathscr{M}$. Then, interval-valued complex neutrosophic hypersoft set (IV-CNHS-set), denoted by $\Omega_{\mathscr{M}} = (\Lambda, \mathscr{M})$, over $\mathbb{Z}$ is given as follows:

$$\Omega_{\mathscr{M}} = \{(\underline{\lambda}, \Lambda(\underline{\lambda})): \underline{\lambda} \in \mathscr{M}, \Lambda(\underline{\lambda}) \in C_{IV}(\mathbb{Z})\}, \tag{40}$$

where

$$\begin{aligned} \Lambda: \mathscr{M} &\longrightarrow C_{IV}(\mathbb{Z}), \\ \Lambda(\underline{\lambda}) &= \varnothing \text{ if } \underline{\lambda} \notin \mathscr{M}, \end{aligned} \tag{41}$$

which is a $IV - CN$ approximate function of $\Omega_{\mathscr{M}}$ and $\Lambda(\underline{\lambda}) = \langle [\overleftarrow{\Lambda_1}(\underline{\lambda}), \overrightarrow{\Lambda_1}(\underline{\lambda})], [\overleftarrow{\Lambda_2}(\underline{\lambda}), \overrightarrow{\Lambda_2}(\underline{\lambda})], [\overleftarrow{\Lambda_3}(\underline{\lambda}), \overrightarrow{\Lambda_3}(\underline{\lambda})] \rangle$ with lower and upper bounds of membership, nonmembership, and indeterminacy function are given below, respectively:

(i) $(\overleftarrow{\Lambda_1}(\underline{\lambda}) = \overleftarrow{\gamma} e^{i\overleftarrow{\theta}}, \overrightarrow{\Lambda_1}(\underline{\lambda}) = \overrightarrow{\gamma} e^{i\overrightarrow{\theta}})$ for the M-function of $\Omega_{\mathscr{M}}$

(ii) $(\overleftarrow{\Lambda_2}(\underline{\lambda}) = \overleftarrow{\gamma} e^{i\overleftarrow{\theta}}, \overrightarrow{\Lambda_2}(\underline{\lambda}) = \overrightarrow{\gamma} e^{i\overrightarrow{\theta}})$ for the NM-function of $\Omega_{\mathscr{M}}$

(iii) $(\overleftarrow{\Lambda_3}(\underline{\lambda}) = \overleftarrow{\gamma} e^{i\overleftarrow{\theta}}, \overrightarrow{\Lambda_3}(\underline{\lambda}) = \overrightarrow{\gamma} e^{i\overrightarrow{\theta}})$ for the ID-function of $\Omega_{\mathscr{M}}$ and its value $\Lambda(\underline{\lambda})$ is called $\underline{\lambda}$-member of IV-CNHS-set $\forall \underline{\lambda} \in \mathscr{M}$

Note: The collection of all IV-CNHS-sets is denoted by $\underset{IVCNHS}{\uplus}$.

Example 3. Consider $\mathscr{M} = \{e_1, e_2, e_3, \ldots, e_8\}$ with IV-CNHS-sets $\Lambda_{\mathscr{M}}(e_1), \Lambda_{\mathscr{M}}(e_2), \ldots, \Lambda_{\mathscr{M}}(e_8)$ that are defined as follows:

$$\Lambda_{\mathscr{M}}(e_1) = \left\{ \begin{array}{c} \dfrac{([0.1, 0.2], [0.1, 0.4], [0.2, 0.3])e^{i([0.1,0.2],[0.3,0.4],[0.3,0.3])\pi}}{t_1}, \quad \dfrac{([0.1, 0.3], [0.2, 0.4], [0.2, 0.2])e^{i([0.1,0.2][0.3,0.4],[0.1,0.3])\pi}}{t_2} \\[4mm] \dfrac{([0.1, 0.2], [0.3, 0.3], [0.3, 0.4])e^{i([0.1,0.1],[0.4,0.5],[0.3,0.3])\pi}}{t_3}, \quad \dfrac{([0.1, 0.2], [0.3, 0.4], [0.2, 0.3])e^{i([0.2,0.2],[0.2,0.4],[0.2,0.3])\pi}}{t_4} \end{array} \right\},$$

$$\Lambda_{\mathscr{M}}(e_2) = \left\{ \begin{array}{c} \dfrac{([0.2, 0.3], [0.3, 0.3], [0.1, 0.3])e^{i([0.1,0.2],[0.1,0.5],[0.1,0.2])\pi}}{t_1}, \quad \dfrac{([0.3, 0.4], [0.3, 0.4], [0.1, 0.1])e^{i([0.1,0.2],[0.2,0.4],[0.2,0.3])\pi}}{t_2} \\[4mm] \dfrac{([0.2, 0.3], [0.2, 0.4], [0.2, 0.3])e^{i([0.1,0.2],[0.1,0.4],[0.3,0.3])\pi}}{t_3}, \quad \dfrac{([0.1, 0.1], [0.4, 0.4], [0.3, 0.4])e^{i([0.1,0.3],[0.2,0.2],[0.2,0.3])\pi}}{t_4} \end{array} \right\},$$

$$\Lambda_{\mathscr{M}}(e_3) = \left\{ \begin{array}{c} \dfrac{([0.1, 0.3], [0.2, 0.4], [0.2, 0.2])e^{i([0.1,0.2],[0.2,0.3],[0.3,0.4])\pi}}{t_1}, \quad \dfrac{([0.1, 0.3], [0.1, 0.2], [0.3, 0.4])e^{i([0.1,0.2],[0.2,0.3],[0.2,0.4])\pi}}{t_2} \\[4mm] \dfrac{([0.1, 0.1], [0.1, 0.4], [0.1, 0.2])e^{i([0.1,0.2],[0.2,0.4],[0.3,0.3])\pi}}{t_3}, \quad \dfrac{([0.1, 0.3], [0.2, 0.3], [0.1, 0.3])e^{i([0.2,0.3],[0.1,0.2],[0.3,0.4])\pi}}{t_4} \end{array} \right\},$$

$$\Lambda_{\mathscr{M}}(e_4) = \left\{ \begin{array}{c} \dfrac{([0.1, 0.2], [0.2, 0.3], [0.1, 0.4])e^{i([0.1,0.2],[0.1,0.3],[0.2,0.2])\pi}}{t_1}, \quad \dfrac{([0.1, 0.3], [0.2, 0.4], [0.1, 0.2])e^{i([0.1,0.2],[0.1,0.3],[0.2,0.4])\pi}}{t_2} \\[4mm] \dfrac{([0.4, 0.4], [0.1, 0.2], [0.1, 0.1])e^{i([0.1,0.2],[0.1,0.4],[0.2,0.3])\pi}}{t_3}, \quad \dfrac{([0.1, 0.1], [0.2, 0.3], [0.2, 0.4])e^{i([0.1,0.3],[0.2,0.2],[0.1,0.3])\pi}}{t_4} \end{array} \right\},$$

$$\Lambda_{\mathscr{M}}(e_5) = \left\{ \begin{array}{c} \dfrac{([0.1, 0.2], [0.4, 0.5], [0.2, 0.2])e^{i([0.1,0.3],[0.2,0.3],[0.1,0.3])\pi}}{t_1}, \quad \dfrac{([0.1, 0.2], [0.2, 0.2], [0.2, 0.5])e^{i([0.1,0.1],[0.1,0.4],[0.3,0.4])\pi}}{t_2} \\[4mm] \dfrac{([0.1, 0.2], [0.2, 0.6], [0.1, 0.1])e^{i([0.1,0.1],[0.1,0.2],[0.2,0.3])\pi}}{t_3}, \quad \dfrac{([0.1, 0.3], [0.3, 0.4], [0.1, 0.2])e^{i([0.1,0.2],[0.3,0.4],[0.1,0.3])\pi}}{t_4} \end{array} \right\},$$

TABLE 5: Tabular representation of $\xi_{\mathscr{D}}$.

$\xi_{\mathscr{D}}$	$\breve{x}_1$	$\breve{x}_2$	$\cdots$	$\breve{x}_r$
ν_1	$\begin{pmatrix} \eta^1_{\chi_{\mathscr{D}}(\breve{x}_1)}(\nu_1), \\ \eta^2_{\chi_{\mathscr{D}}(\breve{x}_1)}(\nu_1), \\ \eta^3_{\chi_{\mathscr{D}}(\breve{x}_1)}(\nu_1) \end{pmatrix}$	$\begin{pmatrix} \eta^1_{\chi_{\mathscr{D}}(\breve{x}_2)}(\nu_1), \\ \eta^2_{\chi_{\mathscr{D}}(\breve{x}_2)}(\nu_1), \\ \eta^3_{\chi_{\mathscr{D}}(\breve{x}_2)}(\nu_1) \end{pmatrix}$	$\cdots$	$\begin{pmatrix} \eta^1_{\chi_{\mathscr{D}}(\breve{x}_r)}(\nu_1), \\ \eta^2_{\chi_{\mathscr{D}}(\breve{x}_r)}(\nu_1), \\ \eta^3_{\chi_{\mathscr{D}}(\breve{x}_r)}(\nu_1) \end{pmatrix}$
ν_2	$\begin{pmatrix} \eta^1_{\chi_{\mathscr{D}}(\breve{x}_1)}(\nu_2), \\ \eta^2_{\chi_{\mathscr{D}}(\breve{x}_1)}(\nu_2), \\ \eta^3_{\chi_{\mathscr{D}}(\breve{x}_1)}(\nu_2) \end{pmatrix}$	$\begin{pmatrix} \eta^1_{\chi_{\mathscr{D}}(\breve{x}_2)}(\nu_2), \\ \eta^2_{\chi_{\mathscr{D}}(\breve{x}_2)}(\nu_2), \\ \eta^3_{\chi_{\mathscr{D}}(\breve{x}_2)}(\nu_2) \end{pmatrix}$	$\cdots$	$\begin{pmatrix} \eta^1_{\chi_{\mathscr{D}}(\breve{x}_r)}(\nu_2), \\ \eta^2_{\chi_{\mathscr{D}}(\breve{x}_r)}(\nu_2), \\ \eta^3_{\chi_{\mathscr{D}}(\breve{x}_r)}(\nu_2) \end{pmatrix}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
ν_m	$\begin{pmatrix} \eta^1_{\chi_{\mathscr{D}}(\breve{x}_1)}(\nu_m), \\ \eta^2_{\chi_{\mathscr{D}}(\breve{x}_1)}(\nu_m), \\ \eta^3_{\chi_{\mathscr{D}}(\breve{x}_1)}(\nu_m) \end{pmatrix}$	$\begin{pmatrix} \eta^1_{\chi_{\mathscr{D}}(\breve{x}_2)}(\nu_m), \\ \eta^2_{\chi_{\mathscr{D}}(\breve{x}_2)}(\nu_m), \\ \eta^3_{\chi_{\mathscr{D}}(\breve{x}_2)}(\nu_m) \end{pmatrix}$	$\cdots$	$\begin{pmatrix} \eta^1_{\chi_{\mathscr{D}}(\breve{x}_r)}(\nu_m), \\ \eta^2_{\chi_{\mathscr{D}}(\breve{x}_r)}(\nu_m), \\ \eta^3_{\chi_{\mathscr{D}}(\breve{x}_r)}(\nu_m) \end{pmatrix}$

$$\Lambda_{\mathscr{M}}(e_6) = \left\{ \begin{array}{ll} \dfrac{([0.2,0.2],[0.3,0.4],[0.1,0.2])e^{i([0.1,0.2],[0.2,0.3],[0.3,0.4])\pi}}{t_1}, & \dfrac{([0.1,0.1],[0.1,0.3],[0.4,0.4])e^{i([0.1,0.1],[0.2,0.5],[0.2,0.3])\pi}}{t_2}, \\[4mm] \dfrac{([0.1,0.2],[0.2,0.4],[0.1,0.3])e^{i([0.1,0.2],[0.1,0.6],[0.1,0.1])\pi}}{t_3}, & \dfrac{([0.1,0.1],[0.2,0.5],[0.2,0.3])e^{i([0.1,0.2],[0.1,0.6],[0.1,0.1])\pi}}{t_4} \end{array} \right\},$$

$$\Lambda_{\mathscr{M}}(e_7) = \left\{ \begin{array}{ll} \dfrac{([0.1,0.2],[0.2,0.4],[0.1,0.3])e^{i([0.1,0.1],[0.2,0.3],[0.1,0.5])\pi}}{t_1}, & \dfrac{([0.1,0.2],[0.1,0.3],[0.3,0.4])e^{i([0.1,0.2],[0.2,0.2],[0.2,0.5])\pi}}{t_2}, \\[4mm] \dfrac{([0.1,0.2],[0.1,0.3],[0.3,0.4])e^{i([0.1,0.2],[0.1,0.3],[0.2,0.4])\pi}}{t_3}, & \dfrac{([0.1,0.2],[0.4,0.5],[0.1,0.2])e^{i([0.1,0.1],[0.2,0.5],[0.1,0.2])\pi}}{t_4} \end{array} \right\},$$

$$(42)$$

$$\Lambda_{\mathscr{M}}(e_8) = \left\{ \begin{array}{ll} \dfrac{([0.1,0.2],[0.3,0.3],[0.1,0.4])e^{i([0.1,0.2],[0.4,0.5],[0.1,0.2])\pi}}{t_1}, & \dfrac{([0.1,0.2],[0.2,0.3],[0.2,0.4])e^{i([0.1,0.2],[0.1,0.2],[0.1,0.3])\pi}}{t_2}, \\[4mm] \dfrac{([0.1,0.1][0.1,0.2],[0.4,0.5])e^{i([0.1,0.2],[0.1,0.3],[0.3,0.4])\pi}}{t_3}, & \dfrac{([0.1,0.2],[0.2,0.3],[0.1,0.4])e^{i([0.1,0.3],[0.4,0.5],[0.1,0.1])\pi}}{t_4} \end{array} \right\}.$$

$$(43)$$

Then IV-CNHS-set $\Omega_{\mathscr{M}}$ is written by

$\Omega_W =$

$$\left\{ \begin{array}{l} \left(e_1, \left\{ \begin{array}{l} \dfrac{([0.1,0.2],[0.1,0.4],[0.2,0.3])e^{i([0.1,0.2],[0.3,0.4],[0.3,0.3])\pi}}{t_1}, \dfrac{([0.1,0.3],[0.2,0.4],[0.2,0.2])e^{i([0.1,0.2][0.3,0.4],[0.1,0.3])\pi}}{t_2} \\[4mm] \dfrac{([0.1,0.2],[0.3,0.3],[0.3,0.4])e^{i([0.1,0.1],[0.4,0.5],[0.3,0.3])\pi}}{t_3}, \dfrac{([0.1,0.2],[0.3,0.4],[0.2,0.3])e^{i([0.2,0.2],[0.2,0.4],[0.2,0.3])\pi}}{t_4} \end{array} \right\} \right) \\[10mm]

\left(e_2, \left\{ \begin{array}{l} \dfrac{([0.2,0.3],[0.3,0.3],[0.1,0.3])e^{i([0.1,0.2],[0.1,0.5],[0.1,0.2])\pi}}{t_1}, \dfrac{([0.3,0.4],[0.3,0.4],[0.1,0.1])e^{i([0.1,0.2],[0.2,0.4],[0.2,0.3])\pi}}{t_2} \\[4mm] \dfrac{([0.2,0.3],[0.2,0.4],[0.2,0.3])e^{i([0.1,0.2],[0.1,0.4],[0.3,0.3])\pi}}{t_3}, \dfrac{([0.1,0.1],[0.4,0.4],[0.3,0.4])e^{i([0.1,0.3],[0.2,0.2],[0.2,0.3])\pi}}{t_4} \end{array} \right\} \right) \\[10mm]

\left(e_3, \left\{ \begin{array}{l} \dfrac{([0.1,0.3],[0.2,0.4],[0.2,0.2])e^{i([0.1,0.2],[0.2,0.3],[0.3,0.4])\pi}}{t_1}, \dfrac{([0.1,0.3],[0.1,0.2],[0.3,0.4])e^{i([0.1,0.2],[0.2,0.3],[0.2,0.4])\pi}}{t_2} \\[4mm] \dfrac{([0.1,0.1],[0.1,0.4],[0.1,0.2])e^{i([0.1,0.2],[0.2,0.4],[0.3,0.3])\pi}}{t_3}, \dfrac{([0.1,0.3],[0.2,0.3],[0.1,0.3])e^{i([0.2,0.3],[0.1,0.2],[0.3,0.4])\pi}}{t_4} \end{array} \right\} \right) \\[10mm]

\left(e_4, \left\{ \begin{array}{l} \dfrac{([0.1,0.2],[0.2,0.3],[0.1,0.4])e^{i([0.1,0.2],[0.1,0.3],[0.2,0.2])\pi}}{t_1}, \dfrac{([0.1,0.3],[0.2,0.4],[0.1,0.2])e^{i([0.1,0.2],[0.1,0.3],[0.2,0.4])\pi}}{t_2} \\[4mm] \dfrac{([0.4,0.4],[0.1,0.2],[0.1,0.1])e^{i([0.1,0.2],[0.1,0.4],[0.2,0.3])\pi}}{t_3}, \dfrac{([0.1,0.1],[0.2,0.3],[0.2,0.4])e^{i([0.1,0.3],[0.2,0.2],[0.1,0.3])\pi}}{t_4} \end{array} \right\} \right) \\[10mm]

\left(e_5, \left\{ \begin{array}{l} \dfrac{([0.1,0.2],[0.4,0.5],[0.2,0.2])e^{i([0.1,0.3],[0.2,0.3],[0.1,0.3])\pi}}{t_1}, \dfrac{([0.1,0.2],[0.2,0.2],[0.2,0.5])e^{i([0.1,0.1],[0.1,0.4],[0.3,0.4])\pi}}{t_2} \\[4mm] \dfrac{([0.1,0.2],[0.2,0.6],[0.1,0.1])e^{i([0.1,0.1],[0.1,0.2],[0.2,0.3])\pi}}{t_3}, \dfrac{([0.1,0.3],[0.3,0.4],[0.1,0.2])e^{i([0.1,0.2],[0.3,0.4],[0.1,0.3])\pi}}{t_4} \end{array} \right\} \right) \\[10mm]

\left(e_6, \left\{ \begin{array}{l} \dfrac{([0.2,0.2],[0.3,0.4],[0.1,0.2])e^{i([0.1,0.2],[0.2,0.3],[0.3,0.4])\pi}}{t_1}, \dfrac{([0.1,0.1],[0.1,0.3],[0.4,0.4])e^{i([0.1,0.1],[0.2,0.5],[0.2,0.3])\pi}}{t_2}, \\[4mm] \dfrac{([0.1,0.2],[0.2,0.4],[0.1,0.3])e^{i([0.1,0.2],[0.1,0.6],[0.1,0.1])\pi}}{t_3}, \dfrac{([0.1,0.1],[0.2,0.5],[0.2,0.3])e^{i([0.1,0.2],[0.1,0.6],[0.1,0.1])\pi}}{t_4} \end{array} \right\} \right) \\[10mm]

\left(e_7, \left\{ \begin{array}{l} \dfrac{([0.1,0.2],[0.2,0.4],[0.1,0.3])e^{i([0.1,0.1],[0.2,0.3],[0.1,0.5])\pi}}{t_1}, \dfrac{([0.1,0.2],[0.1,0.3],[0.3,0.4])e^{i([0.1,0.2],[0.2,0.2],[0.2,0.5])\pi}}{t_2} \\[4mm] \dfrac{([0.1,0.2],[0.1,0.3],[0.3,0.4])e^{i([0.1,0.2],[0.1,0.3],[0.2,0.4])\pi}}{t_3}, \dfrac{([0.1,0.2],[0.4,0.5],[0.1,0.2])e^{i([0.1,0.1],[0.2,0.5],[0.1,0.2])\pi}}{t_4} \end{array} \right\} \right) \\[10mm]

\left(e_8, \left\{ \begin{array}{l} \dfrac{([0.1,0.2],[0.2,0.4],[0.1,0.3])e^{i([0.1,0.1],[0.2,0.3],[0.1,0.5])\pi}}{t_1}, \dfrac{([0.1,0.2],[0.1,0.3],[0.3,0.4])e^{i([0.1,0.2],[0.2,0.2],[0.2,0.5])\pi}}{t_2} \\[4mm] \dfrac{([0.1,0.2],[0.1,0.3],[0.3,0.4])e^{i([0.1,0.2],[0.1,0.3],[0.2,0.4])\pi}}{t_3}, \dfrac{([0.1,0.2],[0.4,0.5],[0.1,0.2])e^{i([0.1,0.1],[0.2,0.5],[0.1,0.2])\pi}}{t_4} \end{array} \right\} \right) \end{array} \right\}. \qquad (44)$$

Definition 28. Let $\Omega_{\mathcal{M}_1} = (\Lambda_1, \mathcal{M}_1)$ and $\Omega_{\mathcal{M}_2} = (\Lambda_2, \mathcal{M}_2)$ be two IV-CNHS-set over the same $\mathbb{Z}$. The set $\Omega_{\mathcal{M}_1} = (\Lambda_1, \mathcal{M}_1)$ is said to be the subset of $\Omega_{\mathcal{M}_2} = (\Lambda_2, \mathcal{M}_2)$, if

(i) $\mathcal{M}_1 \subseteq \mathcal{M}_2$

(ii) $\forall \breve{x} \in \mathcal{M}_1, \Lambda_1(\breve{x}) \subseteq \Lambda_2(\breve{x})$ implies $\overleftarrow{\Lambda}_1(\breve{x}) \subseteq \overleftarrow{\Lambda}_2(\breve{x})$, $\overrightarrow{\Lambda}_1(\breve{x}) \subseteq \overrightarrow{\Lambda}_2(\breve{x})$, that is,

$\overleftarrow{\gamma}_{\mathcal{M}_1}(\breve{x}) \le \overleftarrow{\gamma}_{\mathcal{M}_2}(\breve{x}), \overrightarrow{\gamma}_{\mathcal{M}_1}(\breve{x}) \le \overrightarrow{\gamma}_{\mathcal{M}_2}(\breve{x}), \overleftarrow{\theta}_{\mathcal{M}_1}(\breve{x}) \le \overleftarrow{\theta}_{\mathcal{M}_2}(\breve{x})$ and $\overrightarrow{\theta}_{\mathcal{M}_1}(\breve{x}) \le \overrightarrow{\theta}_{\mathcal{M}_2}(\breve{x})$,

where A- and P-terms of each $\Lambda_i, i = 1, 2$ are given below:

$(\overleftarrow{\gamma}_{\mathcal{M}_1}(\breve{x}), \overleftarrow{\theta}_{\mathcal{M}_1}(\breve{x}))$ for $\overleftarrow{\Lambda}_1(\breve{x})$
$(\overrightarrow{\gamma}_{\mathcal{M}_1}(\breve{x}), \overrightarrow{\theta}_{\mathcal{M}_1}(\breve{x}))$ for $\overrightarrow{\Lambda}_1(\breve{x})$
$(\overrightarrow{\gamma}_{\mathcal{M}_2}(\breve{x}), \overrightarrow{\theta}_{\mathcal{M}_2}(\breve{x}))$ for $\overrightarrow{\Lambda}_2(\breve{x})$
$(\overrightarrow{\gamma}_{\mathcal{M}_2}(\breve{x}), \overrightarrow{\theta}_{\mathcal{M}_2}(\breve{x}))$ for $\Lambda_2(\breve{x})$

Definition 29. Two IV-CNHS-set $\Omega_{\mathcal{M}_1} = (\Lambda_1, \mathcal{M}_1)$ and $\Omega_{\mathcal{M}_2} = (\Lambda_2, \mathcal{M}_2)$ over the same $\mathbb{Z}$, are said to be equal if

(i) $(\Lambda_1, \mathcal{M}_1) \subseteq (\Lambda_2, \mathcal{M}_2)$

(ii) $(\Lambda_2, \mathcal{M}_2) \subseteq (\Lambda_1, \mathcal{M}_1)$

Definition 30. Let $(\Lambda, \mathcal{M})$ be a IV-CNHS-set over $\mathbb{Z}$. Then

(i) $(\Lambda, \mathcal{M})$ is called a null IV-CNHS-set, presented by $(\Lambda, \mathcal{M})_\Phi$; if for all values of $\breve{x} \in \mathcal{M}$, the A-term of the M-function is given by $\overleftarrow{\gamma}_{\mathcal{M}}(\breve{x}) = \overrightarrow{\gamma}_{\mathcal{M}}(\breve{x}) = 0$ and P-term by $\overleftarrow{\theta}_{\mathcal{M}}(\breve{x}) = \overrightarrow{\theta}_{\mathcal{M}}(\breve{x}) = 0\pi$

(ii) $(\Lambda, \mathcal{M})$ is called an absolute IV-CNHS-set, represented by $(\Lambda, \mathcal{M})_\Delta$; if $\forall \breve{x} \in \mathcal{M}$, the A-term of the M-function is given by $\overleftarrow{\gamma}_{\mathcal{M}}(\breve{x}) = \overrightarrow{\gamma}_{\mathcal{M}}(\breve{x}) = 1$ whereas P-term by $\overleftarrow{\theta}_{\mathcal{M}}(\breve{x}) = \overrightarrow{\theta}_{\mathcal{M}}(\breve{x}) = 2\pi$

Definition 31. For two IV-CNHS-sets $(\Lambda_1, \mathcal{M}_1)$ and $(\Lambda_2, \mathcal{M}_2)$ over the same universe $\mathbb{Z}$, the following definitions hold:

(i) A IV-CNHS-set $(\Lambda_1, \mathcal{M}_1)$ is a homogeneous IV-CNHS-set, represented by $(\Lambda_1, \mathcal{M}_1)_{\text{Hom}}$ if and only if $\Lambda_1(\breve{x})$ is a homogeneous IV-CNHS-set $\forall \breve{x} \in \mathcal{M}_1$

(ii) A IV-CNHS-set $(\Lambda_1, \mathcal{M}_1)$ is a completely homogeneous IV-CNHS-set, represented by $(\Lambda_1, \mathcal{M}_1)_{\text{CHom}}$ if and only if $\Lambda_1(\breve{x})$ is a homogeneous with $\Lambda_1(y) \forall \breve{x}, y \in \mathcal{M}_1$

(iii) A IV-CNHS-set $(\Lambda_1, \mathcal{M}_1)$ is a completely homogeneous IV-CNHS-set with $(\Lambda_2, \mathcal{M}_2)$ if and only if $\Lambda_1(\breve{x})$ is a homogeneous with $\Lambda_2(\breve{x}) \forall \breve{x} \in \mathcal{M}_1 \prod \mathcal{M}_2$

4.1. Set Theoretic Operations and Laws on IV-CNHS-Set. Some basic set operations and laws are discussed on IV-CNHS-set in this section.

Definition 32. The complement of IV-CNHS-set $(\Lambda, \mathcal{M})$, denoted by $(\Lambda, \mathcal{M})^c$, is stated as follows:

$$(\Lambda, \mathcal{M})^c = \{(\breve{x}, (\Lambda(\breve{x}))^c): \breve{x} \in \mathcal{M}, (\Lambda(\breve{x}))^c \in C_{\text{IV}}(\mathbb{Z})\}, \tag{45}$$

where the A- and P-terms of the M-function $(\Lambda(\breve{x}))^c$ are given by $(\overleftarrow{\gamma}_{\mathcal{M}}(\breve{x}))^c = 1 - \overleftarrow{\gamma}_{\mathcal{M}}(\breve{x})$ and $(\overrightarrow{\gamma}_{\mathcal{M}}((\breve{x})))^c = 1 - \overrightarrow{\gamma}_{\mathcal{M}}(\breve{x})$ and $(\overleftarrow{\theta}_{\mathcal{M}}(\breve{x}))^c = 2\pi - \overleftarrow{\theta}_{\mathcal{M}}(\breve{x})$ and $(\overrightarrow{\theta}_{\mathcal{M}}(\breve{x}))^c = 2\pi - \overrightarrow{\theta}_{\mathcal{M}}(\breve{x})$, respectively.

Proposition 6. *Let $(\Lambda, \mathcal{M})$ be a IV-CNHS-set over $\mathbb{Z}$. Then $((\Lambda, \mathcal{M})^c)^c = (\Lambda, \mathcal{M})$.*

Proof. Since $\Lambda(\breve{x}) \in C_{\text{IV}}(\mathbb{Z})$, therefore, in terms of its A- and P-terms, $(\Lambda, \mathcal{M})$ can be written as follows:

$$(\Lambda, \mathcal{M}) = \left\{ \left(\breve{x}, \left(\overleftarrow{\gamma}_{\mathcal{M}}(\breve{x}) e^{i\overleftarrow{\theta}_{\mathcal{M}}(\breve{x})}, \overrightarrow{\gamma}_{\mathcal{M}}(\breve{x}) e^{i\overrightarrow{\theta}_{\mathcal{D}}(\breve{x})} \right) \right): \breve{x} \in \mathcal{M} \right\}. \tag{46}$$

Now,

$$(\Lambda, \mathcal{M})^c\,(\breve{x}) = \left\{\left(\breve{x}, \left(\left(\overleftarrow{\gamma}_{\mathcal{M}}(\breve{x})\right)^c e^{i\left(\overleftarrow{\theta}_{\mathcal{M}}(\breve{x})\right)^c}, \left(\overrightarrow{\gamma}_{\mathcal{M}}(\breve{x})\right)^c e^{i\left(\overrightarrow{\theta}_{\mathcal{D}}(\breve{x})\right)^c}\right)\right): \breve{x}\in \mathcal{M}\right\},$$

$$(\Lambda, \mathcal{M})^c\,(\breve{x}) = \left\{\left(\breve{x}, \left(\left(1 - \overleftarrow{\gamma}_{\mathcal{M}}(\breve{x})\right)^c e^{i\left(2\pi - \overleftarrow{\theta}_{\mathcal{M}}(\breve{x})\right)^c}, \left(1 - \overrightarrow{\gamma}_{\mathcal{M}}(\breve{x})\right)^c e^{i\left(2\pi - \overrightarrow{\theta}_{\mathcal{D}}(\breve{x})\right)^c}\right)\right): \breve{x}\in \mathcal{M}\right\},$$

$$((\psi, G)^c)^c = \left\{\left(\breve{x}, \left(\left(1 - \overleftarrow{\gamma}_{\mathcal{M}}(\breve{x})\right)^c e^{i\left(2\pi - \overleftarrow{\theta}_{\mathcal{M}}(\breve{x})\right)^c}, \left(1 - \overrightarrow{\gamma}_{\mathcal{M}}(\breve{x})\right)^c e^{i\left(2\pi - \overrightarrow{\theta}_{\mathcal{D}}(\breve{x})\right)^c}\right)\right): \breve{x}\in \mathcal{M}\right\}, \tag{47}$$

$$((\Lambda, \mathcal{M})^c)^c = \left\{\left(\breve{x}, \left(\left(1 - \left(1 - \overleftarrow{\gamma}_{\mathcal{M}}(\breve{x})\right)\right)e^{i\left(2\pi - \left(2\pi - \overleftarrow{\theta}_{\mathcal{M}}(\breve{x})\right)\right)}, \left(1 - \left(1 - \overrightarrow{\gamma}_{\mathcal{M}}(\breve{x})\right)\right)e^{i\left(2\pi - \left(2\pi - \overrightarrow{\theta}_{\mathcal{D}}(\breve{x})\right)\right)}\right)\right): \breve{x}\in \mathcal{M}\right\},$$

$$((\Lambda, \mathcal{M})^c)^c = \left\{\left(\breve{x}, \left(\overleftarrow{\gamma}_{\mathcal{M}}(\breve{x})e^{i\overleftarrow{\theta}_{\mathcal{M}}(\breve{x})}, \overrightarrow{\gamma}_{\mathcal{M}}(\breve{x})e^{i\overrightarrow{\theta}_{\mathcal{D}}(\breve{x})}\right)\right): \breve{x}\in \mathcal{M}\right\}.$$

From equations (46) and (47), we have $((\Lambda, \mathcal{M})^c)^c = (\Lambda, \mathcal{M})$. □

Proposition 7. *Let $(\Lambda, \mathcal{M})$ be a IV-CNHS-set over $\mathbb{Z}$. Then*

(i) $((\Lambda, \mathcal{M})_\Phi)^c = (\Lambda, \mathcal{M})_\Delta$

(ii) $((\Lambda, \mathcal{M})_\Delta)^c = (\Lambda, \mathcal{M})_\Phi$

Definition 33. For two IV-CNHS-set $(\Lambda_1, \mathcal{M}_1)$ and $(\Lambda_2, \mathcal{M}_2)$, the intersection $(\Lambda_1, \mathcal{M}_1)\prod(\Lambda_2, \mathcal{M}_2)$ over the same universe $\mathbb{Z}$ is the IV-CNHS-set $(\Lambda_3, \mathcal{M}_3)$, where $\mathcal{M}_3 = \mathcal{M}_1\coprod\mathcal{M}_2$, and for all $\breve{x} \in \mathcal{M}_3$,

$$\overleftarrow{\Lambda}_3(\breve{x}) = \begin{cases} \overleftarrow{\gamma}_{\mathcal{M}_1}(\breve{x})e^{i\overleftarrow{\theta}_{\mathcal{M}_1}(\breve{x})}, & \text{if } \breve{x}\in \mathcal{M}_1\backslash\mathcal{M}_2, \\ \overleftarrow{\gamma}_{\mathcal{M}_2}(\breve{x})e^{i\overleftarrow{\theta}_{\mathcal{M}_2}(\breve{x})}, & \text{if } \breve{x}\in \mathcal{M}_2\mathcal{M}_1, \\ \min\left[\overleftarrow{\gamma}_{\mathcal{M}_1}(\breve{x}), \overleftarrow{\gamma}_{\mathcal{M}_2}(\breve{x})\right]e^{i\min\left[\overleftarrow{\theta}_{\mathcal{M}_1}(\breve{x}), \overleftarrow{\theta}_{\mathcal{M}_2}(\breve{x})\right]}, & \text{if } \breve{x}\in \mathcal{M}_1\prod\mathcal{M}_2, \end{cases}$$

$$\overrightarrow{\Lambda}_3(\breve{x}) = \begin{cases} \overrightarrow{\gamma}_{\mathcal{M}_1}(\breve{x})e^{i\overrightarrow{\theta}_{\mathcal{M}_1}(\breve{x})}, & \text{if } \breve{x}\in \mathcal{M}_1\backslash\mathcal{M}_2, \\ \overrightarrow{\gamma}_{\mathcal{M}_2}(\breve{x})e^{i\overrightarrow{\theta}_{\mathcal{M}_2}(\breve{x})}, & \text{if } \breve{x}\in \mathcal{M}_2\mathcal{M}_1, \\ \min\left[\overrightarrow{\gamma}_{\mathcal{M}_1}(\breve{x}), \overrightarrow{\gamma}_{\mathcal{M}_2}(\breve{x})\right]e^{i\min\left[\overrightarrow{\theta}_{\mathcal{M}_1}(\breve{x}), \overrightarrow{\theta}_{\mathcal{M}_2}(\breve{x})\right]}, & \text{if } \breve{x}\in \mathcal{M}_1\prod\mathcal{M}_2. \end{cases} \tag{48}$$

Definition 34. The difference between two IV-CNHS-set $(\Lambda_1, \mathcal{M}_1)$ and $(\Lambda_2, \mathcal{M}_2)$ is defined as follows:

$$(\Lambda_1, \mathcal{M}_1)\backslash(\Lambda_2, \mathcal{M}_2) = (\Lambda_1, \mathcal{M}_1)\prod(\Lambda_2, \mathcal{M}_2)^c. \tag{49}$$

Definition 35. For two IV-CNHS-set $(\Lambda_1, \mathcal{M}_1)$ and $(\Lambda_2, \mathcal{M}_2)$, the union $(\Lambda_1, \mathcal{M}_1)\coprod(\Lambda_2, \mathcal{M}_2)$ over the same universe $\mathbb{Z}$ is the IV-CNHS-set $(\Lambda_3, \mathcal{M}_3)$, where $\mathcal{M}_3 = \mathcal{M}_1\coprod\mathcal{M}_2$, and for all $\breve{x} \in \mathcal{M}_3$,

$$\overleftarrow{\Lambda}_3(\breve{x}) = \begin{cases} \overleftarrow{\gamma}_{\mathscr{M}_1}(\breve{x})e^{i\overleftarrow{\theta}_{\mathscr{M}_1}(\breve{x})}, & \text{if } \breve{x} \in \mathscr{M}_1\mathscr{M}_2, \\ \overleftarrow{\gamma}_{\mathscr{M}_2}(\breve{x})e^{i\overleftarrow{\theta}_{\mathscr{M}_2}(\breve{x})}, & \text{if } \breve{x} \in \mathscr{M}_2\mathscr{M}_1, \\ \max\left[\overleftarrow{\gamma}_{\mathscr{M}_1}(\breve{x}), \overleftarrow{\gamma}_{\mathscr{M}_2}(\breve{x})\right]e^{i\max\left[\overleftarrow{\theta}_{\mathscr{M}_1}(\breve{x}),\overleftarrow{\theta}_{\mathscr{M}_2}(\breve{x})\right]}, & \text{if } \breve{x} \in \mathscr{M}_1\prod\mathscr{M}_2, \end{cases}$$

$$\overrightarrow{\Lambda}_3(\breve{x}) = \begin{cases} \overrightarrow{\gamma}_{\mathscr{M}_1}(\breve{x})e^{i\overrightarrow{\theta}_{\mathscr{M}_1}(\breve{x})}, & \text{if } \breve{x} \in \mathscr{M}_1\mathscr{M}_2, \\ \overrightarrow{\gamma}_{\mathscr{M}_2}(\breve{x})e^{i\overrightarrow{\theta}_{\mathscr{M}_2}(\breve{x})}, & \text{if } \breve{x} \in \mathscr{M}_2\mathscr{M}_1, \\ \max\left[\overrightarrow{\gamma}_{\mathscr{M}_1}(\breve{x}), \overrightarrow{\gamma}_{\mathscr{M}_2}(\breve{x})\right]e^{i\max\left[\overrightarrow{\theta}_{\mathscr{M}_1}(\breve{x}),\overrightarrow{\theta}_{\mathscr{M}_2}(\breve{x})\right]}, & \text{if } \breve{x} \in \mathscr{M}_1\prod\mathscr{M}_2. \end{cases} \tag{50}$$

Note: Propositions 3–5 are also valid for SV-NHS-sets.

4.2. Aggregation of Interval-Valued Complex Neutrosophic Hypersoft Set.

In this part of the paper, aggregation operations and their decisive system for IV-CNHS-set are established that results an aggregate fuzzy set from a CNHS-set and its cardinal set and has fuzzy-like nature. The terms $\mathscr{D}, \mathfrak{E}, \xi_{\mathscr{D}}$ and $\uplus_{\text{IVCNHS}}$ are in accordance with Definition 27.

Definition 36. Let $\xi_{\mathscr{D}} \in \uplus_{\text{IVCNHS}}$. Assume that $\mathbb{Z} = \{v_1, v_2, \ldots, v_m\}$ and $\mathfrak{E} = \{\mathscr{L}_1, \mathscr{L}_2, \ldots, \mathscr{L}_n\}$ with $\mathscr{L}_1 = \{e_{11}, e_{12}, \ldots, e_{1n}\}$, $\mathscr{L}_2 = \{e_{21}, e_{22}, \ldots, e_{2n}\}$, $\ldots, \mathscr{L}_n = \{e_{n1}, e_{n2}, \ldots, e_{nn}\}$ and $\mathscr{D} = \mathscr{L}_1 \times \mathscr{L}_2 \times \cdots \times \mathscr{L}_n = \{\breve{x}_1, \breve{x}_2, \ldots, \breve{x}_n, \ldots, \breve{x}_{n^n} = \breve{x}_r\}$, each $\breve{x}_i$ is n-tuple element of $\mathscr{D}$ and $|\mathscr{D}| = r = n^n$; the representation of $\xi_{\mathscr{D}}$ can be seen in Table 5.

In Table 5, $\eta^1_{\chi_{\mathscr{D}}(x)}, \eta^2_{\chi_{\mathscr{D}}(x)}$, and $\eta^3_{\chi_{\mathscr{D}}(x)}$ are M-function, ID-function, and NM-function of $\chi_{\mathscr{D}}$, respectively, with interval-valued neutrosophic values. If $\alpha_{ij} = (\eta^1_{\chi_{\mathscr{D}}(\breve{x}_j)}(v_i), \eta^2_{\chi_{\mathscr{D}}(\breve{x}_j)}(v_i), \eta^3_{\chi_{\mathscr{D}}(\breve{x}_j)}(v_i))$, for $i = \mathbb{N}_1^m$ and $j = \mathbb{N}_1^r$, then IV-CNHS-set $\xi_{\mathscr{D}}$ is uniquely characterized by the following matrix:

$$\left[\alpha_{ij}\right] = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1r} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{m1} & \alpha_{m2} & \cdots & \alpha_{mr} \end{bmatrix}, \tag{51}$$

Which is called an $m \times r$ IV-CNHS-set matrix.

Definition 37. If $\xi_{\mathscr{D}} \in \uplus_{\text{IVCNHS}}$, then cardinal set of $\xi_{\mathscr{D}}$ is defined as follows:

$$\|\xi_{\mathscr{D}}\| = \left\{ \frac{\left(\eta^1_{\|\xi_{\mathscr{D}}\|}(\breve{x}), \eta^2_{\|\xi_{\mathscr{D}}\|}(\breve{x}), \eta^3_{\|\xi_{\mathscr{D}}\|}(\breve{x})\right)}{\breve{x}} : \breve{x} \in \mathscr{D} \right\}, \tag{52}$$

where $\eta^1_{\|\xi_{\mathscr{D}}\|}, \eta^2_{\|\xi_{\mathscr{D}}\|}, \eta^3_{\|\xi_{\mathscr{D}}\|}: \mathscr{D} \longrightarrow [0,1]$ are M-function, ID-function, and NM-function of $\|\xi_{\mathscr{D}}\|$ with

$$\eta^1_{\|\xi_{\mathscr{D}}\|}(\breve{x}), \eta^2_{\|\xi_{\mathscr{D}}\|}(\breve{x}), \eta^3_{\|\xi_{\mathscr{D}}\|}(\breve{x}) = \frac{|\chi_{\mathscr{D}}(\breve{x})|}{|\mathbb{Z}|}, \tag{53}$$

respectively. These have interval-valued neutrosophic values. Note that $\|C_{\text{ivcnhss}}(\mathbb{Z})\|$ is the collection of all cardinal sets of IV-CNHS-sets and $\|C_{\text{ivcnhss}}(\mathbb{Z})\| \subseteq \mathbb{IVN}(\mathscr{D})$.

Definition 38. Let $\xi_{\mathscr{D}} \in C_{\text{ivcnhss}}(\mathbb{Z})$ and $\|\xi_{\mathscr{D}}\| \in \|C_{\text{ivcnhss}}(\mathbb{Z})\|$. Consider $\mathfrak{E}$ as in Definition 27; the tabular representation of $\|\xi_{\mathscr{D}}\|$ can be seen in Table 6.

If $\alpha_{1j} = (\eta^1_{\|\xi_{\mathscr{D}}\|}(\breve{x}_j), \eta^2_{\|\xi_{\mathscr{D}}\|}(\breve{x}_j), \eta^3_{\|\xi_{\mathscr{D}}\|}(\breve{x}_j))$, for $j = \mathbb{N}_1^r$. then the cardinal set $\|\xi_{\mathscr{D}}\|$ is represented by the following matrix:

$$\left[\alpha_{ij}\right]_{1 \times r} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1r} \end{bmatrix}, \tag{54}$$

which is called the cardinal matrix of $\|\xi_{\mathscr{D}}\|$.

Definition 39. Let $\xi_{\mathscr{D}} \in C_{\text{ivcnhss}}(\mathbb{Z})$ and $\|\xi_{\mathscr{D}}\| \in \|C_{\text{ivcnhss}}(\mathbb{Z})\|$. Then IV-CNHS-aggregation operator is defined as follows:

$$\widetilde{\xi_{\mathscr{D}}} = A_{\text{ivcifhss}}\left(\|\xi_{\mathscr{D}}\|, \xi_{\mathscr{D}}\right), \tag{55}$$

where

$$A_{\text{ivcnhss}}: \|C_{\text{ivcnhss}}(\mathbb{Z})\| \times C_{\text{ivcifhss}}(\mathbb{Z}) \longrightarrow \mathbb{F}(\mathbb{Z}), \tag{56}$$

where $\widetilde{\xi_{\mathscr{D}}}$ is called the aggregate fuzzy set of IV-CNHS-set $\xi_{\mathscr{D}}$.

Its M-function is given as follows:

$$\eta_{\widetilde{\xi_{\mathscr{D}}}}: \mathbb{Z} \longrightarrow [0,1], \tag{57}$$

with

$$\eta_{\widetilde{\xi_{\mathscr{D}}}}(v) = \frac{1}{|\mathscr{D}|} \sum_{\breve{x} \in \mathscr{D}} \eta_{\text{Card}(\xi_{\mathscr{D}})}(\breve{x}) \eta_{\text{Card}(\chi_{\mathscr{D}})}(v). \tag{58}$$

Definition 40. Let $\xi_{\mathscr{D}} \in C_{\text{ivcnhss}}(\mathbb{Z})$ and $\widetilde{\xi_{\mathscr{D}}}$ be its aggregate fuzzy set. Assume that $\mathbb{Z} = \{v_1, v_2, \ldots, v_m\}$, then $\widetilde{\xi_{\mathscr{D}}}$ can be presented as follows:

Table 6: Tabular representation of $\|\xi_{\mathscr{D}}\|$.

$\mathscr{D}$	$\breve{x}_1$	$\breve{x}_2$	$\cdots$	$\breve{x}_r$
$\eta_{\|\xi_{\mathscr{D}}\|}$	$\begin{pmatrix} \eta^1_{\|\xi_{\mathscr{D}}\|}(\breve{x}_1), \\ \eta^2_{\|\xi_{\mathscr{D}}\|}(\breve{x}_1), \\ \eta^3_{\|\xi_{\mathscr{D}}\|}(\breve{x}_1), \end{pmatrix}$	$\begin{pmatrix} \eta^1_{\|\xi_{\mathscr{D}}\|}(\breve{x}_2), \\ \eta^2_{\|\xi_{\mathscr{D}}\|}(\breve{x}_2), \\ \eta^3_{\|\xi_{\mathscr{D}}\|}(\breve{x}_2), \end{pmatrix}$	$\cdots$	$\begin{pmatrix} \eta^1_{\|\xi_{\mathscr{D}}\|}(\breve{x}_r), \\ \eta^2_{\|\xi_{\mathscr{D}}\|}(\breve{x}_r), \\ \eta^3_{\|\xi_{\mathscr{D}}\|}(\breve{x}_r), \end{pmatrix}$

$$
\begin{bmatrix}
\xi_{\mathscr{D}} & \vdots & \eta_{\widetilde{\xi_{\mathscr{D}}}} \\
\cdots\cdots & \vdots & \cdots\cdots \\
\nu_1 & \vdots & \eta_{\widetilde{\xi_{\mathscr{D}}}}(\nu_1) \\
\nu_2 & \vdots & \eta_{\widetilde{\xi_{\mathscr{D}}}}(\nu_2) \\
\vdots & \vdots & \vdots \\
\nu_m & \vdots & \eta_{\widetilde{\xi_{\mathscr{D}}}}(\nu_m)
\end{bmatrix}. \tag{59}
$$

If $\alpha_{i1} = \eta_{\widetilde{\xi_{\mathscr{D}}}}(\nu_i)$ for $i = \mathbb{N}_1^m$, then $\widetilde{\xi_{\mathscr{D}}}$ is represented by the following matrix:

$$
[\alpha_{i1}]_{m \times 1} = \begin{bmatrix} \alpha_{11} \\ \alpha_{21} \\ \vdots \\ \alpha_{m1} \end{bmatrix}, \tag{60}
$$

which is called the aggregate matrix of $\widetilde{\xi_{\mathscr{D}}}$ over $\mathbb{Z}$.

4.3. Applications of Interval-Valued Complex Neutrosophic Hypersoft Set.

Now considering the definitions stated in Subsection 4.2, an algorithm is presented in this segment of the paper to assist the decision-support system, and the presented algorithm will be validated with the help of an example from a daily-life scenario.

Now, Algorithm 2 is explained with the help of the following example.

Example 4. Considering the problem statement stated in Example 2, we construct IV-CNHS-sets $\psi_{\mathscr{D}}(\delta_1)$, $\psi_{\mathscr{D}}(\delta_2)$ and $\psi_{\mathscr{D}}(\delta_3)$, $\psi_{\mathscr{D}}(\delta_4)$ that are defined as follows:

$$
\psi_{\mathscr{D}}(\delta_1) = \left\{ \begin{array}{ll} \dfrac{([0.4,0.5],[0.1,0.2],[0.1,0.2])e^{i([0.1,0.2],[0.1,0.4],[0.3,0.3])\pi}}{s_1}, & \dfrac{([0.1,0.1],[0.2,0.3],[0.3,0.4])e^{i([0.1,0.3],[0.2,0.3],[0.3,0.3])\pi}}{s_2}, \\[4mm] \dfrac{([0.1,0.2],[0.1,0.4],[0.2,0.2])e^{i([0.3,0.4],[0.3,0.4],[0.1,0.2])\pi}}{s_3}, & \dfrac{([0.1,0.2],[0.4,0.5],[0.2,0.2])e^{i([0.1,0.2],[0.1,0.4],[0.1,0.2])\pi}}{s_4} \end{array} \right\},
$$

$$
\psi_{\mathscr{D}}(\delta_2) = \left\{ \begin{array}{ll} \dfrac{([0.1,0.2],[0.3,0.5],[0.2,0.2])e^{i([0.1,0.2],[0.2,0.4][0.2,0.2])\pi}}{s_1}, & \dfrac{([0.1,0.2],[0.1,0.5],[0.1,0.2])e^{i([0.1,0.2],[0.4,0.5],[0.2,0.2])\pi}}{s_2}, \\[4mm] \dfrac{([0.1,0.1],[0.2,0.3],[0.1,0.5])e^{i([0.3,0.4],[0.2,0.3],[0.1,0.2])\pi}}{s_3}, & \dfrac{([0.1,0.2],[0.2,0.4],[0.3,0.3])e^{i([0.2,0.3],[0.1,0.3],[0.3,0.3])\pi}}{s_4} \end{array} \right\},
$$

$$
\psi_{\mathscr{D}}(\delta_3) = \left\{ \begin{array}{ll} \dfrac{([0.3,0.4],[0.2,0.3],[0.1,0.2])e^{i((0.1,0.2),[0.1,0.4],[0.2,0.3])\pi}}{s_1}, & \dfrac{([0.1,0.2],[0.2,0.3],[0.1,0.3])e^{i([0.2,0.3],[0.3,0.4],[0.2,0.2])\pi}}{s_2}, \\[4mm] \dfrac{([0.1,0.2],[0.2,0.4],[0.1,0.3])e^{i([0.3,0.3],[0.2,0.3],[0.1,0.2])\pi}}{s_3}, & \dfrac{([0.1,0.2],[0.3,0.3],[0.2,0.3])e^{i([0.1,0.2],[0.2,0.5],[0.2,0.2])\pi}}{s_4} \end{array} \right\},
$$

$$
\psi_{\mathscr{D}}(\delta_4) = \left\{ \begin{array}{ll} \dfrac{([0.1,0.3],[0.3,0.4],[0.1,0.2])e^{i([0.1,0.2],[0.4,0.5],[0.2,0.2])\pi}}{s_1}, & \dfrac{([0.4,0.5],[0.1,0.3],[0.1,0.1])e^{i([0.1,0.2],[0.1,0.4],[0.2,0.3])\pi}}{s_2}, \\[4mm] \dfrac{([0.1,0.1],[0.1,0.4],[0.1,0.2])e^{i([0.2,0.3],[0.3,0.3],[0.3,0.3])\pi}}{s_3}, & \dfrac{([0.1,0.2],[0.4,0.5],[0.2,0.2])e^{i([0.1,0.2],[0.2,0.4],[0.2,0.3])\pi}}{s_4} \end{array} \right\}. \tag{61}
$$

```
▷ Start
▷ Input Stage:
————1. Consider ℤ as the universe of discourse
————2. Consider 𝔈 as a set of n parameters
————3. Classify n parameters into disjoint parametric-valued sets ℒ₁, ℒ₂, ℒ₃, . . . , ℒₙ
▷ Construction Stage:
————4. 𝒟 = ℒ₁ × ℒ₂ × ℒ₃ × · · · × ℒₙ
————5. Construct IV-CNHS-set χ_𝒟 over ℤ, in accordance with Definition 27
▷ Computation Stage:
————6. Compute ‖ξ_𝒟‖ for A-term and P-term separately by using Definition 37
————7. Compute ξ̃_𝒟 for A-term and P-term separately with the help of Definition 39
————8. Compute η_ξ̃_𝒟(ν) by using Definition 39
▷ Output Stage:
————9. Find the best option by the max modulus of η_ξ̃_𝒟(ν) with the help of Definition 40
▷ End
```

ALGORITHM 2: Decision-support algorithm based on aggregations of IV-CNHS-set.

Step 1: IV-CNH-set $\chi_{\mathscr{D}}$ is written as follows:

$$
\chi_{\mathscr{D}} = \left[\begin{array}{l}
\left(\delta_1, \left\{ \begin{array}{ll}
\dfrac{([0.4,0.5],[0.1,0.2],[0.1,0.2])e^{i([0.1,0.2],[0.1,0.4],[0.3,0.3])\pi}}{s_1}, & \dfrac{([0.1,0.1],[0.2,0.3],[0.3,0.4])e^{i([0.1,0.3],[0.2,0.3],[0.3,0.3])\pi}}{s_2} \\[4mm]
\dfrac{([0.1,0.2],[0.1,0.4],[0.2,0.2])e^{i([0.3,0.4],[0.3,0.4],[0.1,0.2])\pi}}{s_3}, & \dfrac{([0.1,0.2],[0.4,0.5],[0.2,0.2])e^{i([0.1,0.2],[0.1,0.4],[0.1,0.2])\pi}}{s_4}
\end{array} \right\} \right) \\[10mm]
\left(\delta_2, \left\{ \begin{array}{ll}
\dfrac{([0.1,0.2],[0.3,0.5],[0.2,0.2])e^{i([0.1,0.2],[0.2,0.4],[0.2,0.2])\pi}}{s_1}, & \dfrac{([0.1,0.2],[0.1,0.5],[0.1,0.2])e^{i([0.1,0.2],[0.4,0.5],[0.2,0.2])\pi}}{s_2} \\[4mm]
\dfrac{([0.1,0.1],[0.2,0.3],[0.1,0.5])e^{i([0.3,0.4],[0.2,0.3],[0.1,0.2])\pi}}{s_3}, & \dfrac{([0.1,0.2],[0.2,0.4],[0.3,0.3])e^{i([0.2,0.3],[0.1,0.3],[0.3,0.3])\pi}}{s_4}
\end{array} \right\} \right) \\[10mm]
\left(\delta_3, \left\{ \begin{array}{ll}
\dfrac{([0.3,0.4],[0.2,0.3],[0.1,0.2])e^{i([0.1,0.2],[0.1,0.4],[0.2,0.3])\pi}}{s_1}, & \dfrac{([0.1,0.2],[0.2,0.3],[0.1,0.3])e^{i([0.2,0.3],[0.3,0.4],[0.2,0.2])\pi}}{s_2} \\[4mm]
\dfrac{([0.1,0.2],[0.2,0.4],[0.1,0.3])e^{i([0.3,0.3],[0.2,0.3],[0.1,0.2])\pi}}{s_3}, & \dfrac{([0.1,0.2],[0.3,0.3],[0.2,0.3])e^{i([0.1,0.2],[0.2,0.5],[0.2,0.2])\pi}}{s_4}
\end{array} \right\} \right) \\[10mm]
\left(\delta_4, \left\{ \begin{array}{ll}
\dfrac{([0.1,0.3],[0.3,0.4],[0.1,0.2])e^{i([0.1,0.2],[0.4,0.5],[0.2,0.2])\pi}}{s_1}, & \dfrac{([0.4,0.5],[0.1,0.3],[0.1,0.1])e^{i([0.1,0.2],[0.1,0.4],[0.2,0.3])\pi}}{s_2} \\[4mm]
\dfrac{([0.1,0.1],[0.1,0.4],[0.1,0.2])e^{i([0.2,0.3],[0.3,0.3],[0.3,0.3])\pi}}{s_3}, & \dfrac{([0.1,0.2],[0.4,0.5],[0.2,0.2])e^{i([0.1,0.2],[0.2,0.4],[0.2,0.3])\pi}}{s_4}
\end{array} \right\} \right)
\end{array} \right].
$$

$$(62)$$

Step 2: the cardinal is computed as follows:

$\|\chi_{\mathcal{D}}\|$ (Amplitude Term) =

$$\left\{ \frac{([0.175, 0.250], [0.200, 0.350], [0.200, 0.250])}{\delta_1}, \frac{([0.100, 0.175], [0.200, 0.425], [0.175, 0.300])}{\delta_2}, \frac{([0.150, 0.250], [0.225, 0.325], [0.125, 0.275])}{\delta_3}, \frac{([0.175, 0.275], [0.225, 0.400], [0.125, 0.175])}{\delta_4} \right\}. \tag{63}$$

$\|\chi_{\mathcal{D}}\|$ (Phase Term) =

$$\left\{ \frac{([0.150, 0.275], [0.175, 0.375], [0.200, 0.250])}{\delta_1}, \frac{([0.175, 0.275], [0.225, 0.375], [0.200, 0.225])}{\delta_2}, \frac{([0.175, 0.250], [0.200, 0.400], [0.175, 0.225])}{\delta_3}, \frac{([0.125, 0.225], [0.250, 0.425], [0.225, 0.275])}{\delta_4} \right\}. \tag{64}$$

Step 3: the set $\widetilde{\chi_{\mathcal{D}}}$ can be determined as follows: $\widetilde{\chi_{\mathcal{D}}}$ (Apmlitude Term) =

$$\frac{1}{4} \begin{bmatrix} [0.4, 0.5], [0.1, 0.2], [0.1, 0.2] & [0.1, 0.2], [0.3, 0.5], [0.2, 0.2] & [0.3, 0.4], [0.2, 0.3], [0.1, 0.2] & [0.1, 0.3], [0.3, 0.4], [0.1, 0.2] \\ [0.1, 0.1], [0.2, 0.3], [0.3, 0.4] & [0.1, 0.2], [0.1, 0.5], [0.1, 0.2] & [0.1, 0.2], [0.2, 0.3], [0.1, 0.3] & [0.4, 0.5], [0.1, 0.3], [0.1, 0.1] \\ [0.1, 0.2], [0.1, 0.4], [0.2, 0.2] & [0.1, 0.1], [0.2, 0.3], [0.1, 0.5] & [0.1, 0.2], [0.2, 0.4], [0.1, 0.3] & [0.1, 0.1], [0.1, 0.4], [0.1, 0.2] \\ [0.1, 0.2], [0.4, 0.5], [0.2, 0.2] & [0.1, 0.2], [0.2, 0.4], [0.3, 0.3] & [0.1, 0.2], [0.3, 0.3], [0.2, 0.3] & [0.1, 0.2], [0.4, 0.5], [0.2, 0.2] \end{bmatrix}$$

$$\times \begin{bmatrix} [0.175, 0.250], [0.200, 0.350], [0.200, 0.250] \\ [0.100, 0.175], [0.200, 0.425], [0.175, 0.300] \\ [0.150, 0.250], [0.225, 0.325], [0.125, 0.275] \\ [0.175, 0.275], [0.225, 0.400], [0.125, 0.175] \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 0.3 & 0.3 & 0.1 & 0.1 \\ 0.4 & 0.0 & 0.1 & 0.3 \\ 0.1 & 0.2 & 0.1 & 0.1 \\ 0.4 & 0.3 & 0.3 & 0.4 \end{bmatrix} \begin{bmatrix} 0.150 \\ 0.200 \\ 0.100 \\ 0.075 \end{bmatrix} = \begin{bmatrix} 0.030625 \\ 0.023125 \\ 0.018125 \\ 0.045000 \end{bmatrix}. \tag{65}$$

$\widetilde{\chi_{\mathcal{D}}}$ (Phase Term) =

$$\frac{1}{4}\begin{bmatrix} [0.1,0.2],\,[0.1,0.4],\,[0.3,0.3] & [0.1,0.2],\,[0.2,0.4],\,[0.2,0.2] & [0.1,0.2],\,[0.1,0.4],\,[0.2,0.3] & [0.1,0.2],\,[0.4,0.5],\,[0.2,0.2] \\ [0.1,0.3],\,[0.2,0.3],\,[0.3,0.3] & [0.1,0.2],\,[0.4,0.5],\,[0.2,0.2] & [0.2,0.3],\,[0.3,0.4],\,[0.2,0.2] & [0.1,0.2],\,[0.1,0.4],\,[0.2,0.3] \\ [0.3,0.4],\,[0.3,0.4],\,[0.1,0.2] & [0.3,0.4],\,[0.2,0.3],\,[0.1,0.2] & [0.3,0.3],\,[0.2,0.3],\,[0.1,0.2] & [0.2,0.3],\,[0.3,0.3],\,[0.3,0.3] \\ [0.1,0.2],\,[0.1,0.4],\,[0.1,0.2] & [0.2,0.3],\,[0.1,0.3],\,[0.3,0.3] & [0.1,0.2],\,[0.2,0.5],\,[0.2,0.2] & [0.1,0.2],\,[0.2,0.4],\,[0.2,0.3] \end{bmatrix}$$

$$\times \begin{bmatrix} [0.150,0.275],\,[0.175,0.375],\,[0.200,0.250] \\ [0.175,0.275],\,[0.225,0.375],\,[0.200,0.225] \\ [0.175,0.250],\,[0.200,0.400],\,[0.175,0.225] \\ [0.125,0.225],\,[0.250,0.425],\,[0.225,0.275] \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 0.2 & 0.2 & 0.1 & 0.4 \\ 0.2 & 0.4 & 0.2 & 0.1 \\ 0.0 & 0.1 & 0.0 & 0.3 \\ 0.0 & 0.1 & 0.2 & 0.2 \end{bmatrix}\begin{bmatrix} 0.100 \\ 0.300 \\ 0.125 \\ 0.250 \end{bmatrix} = \begin{bmatrix} 0.048125 \\ 0.047500 \\ 0.026250 \\ 0.026250 \end{bmatrix}$$

$$\widetilde{\chi_{\mathscr{D}}} = \left\{ \frac{0.030625e^{i0.048135\pi}}{s_1}, \frac{0.023125e^{i0.047500\pi}}{s_2}, \frac{0.018125e^{i0.026250\pi}}{s_3}, \frac{0.045000e^{i0.026250\pi}}{s_4} \right\}.$$

(66)

Consider the modulus value of $\max(\eta_{\widetilde{\xi_{\mathscr{D}}}}) = \max$ $\{0.08562297237/s_1, \quad 0.08312368641/s_2, \quad 0.0812493106/s_3,$ $0.0793734965/s_4\} = 0.08562297237/s_1$. This means that s_1 may be recommended for suitable investment.

5. Comparison Analysis

In literature, various decision-making algorithmic approaches have already been discussed by [24, 34, 36–42] that are based on hybridized structures of complex set with fuzzy set, intuitionistic fuzzy set, and neutrosophic set under soft set environments. Decision-making is badly affected due to the omission of some features with a key role. For example, in stock-exchange share market-based scenario, it is insufficient to consider "current trend of company performance," "particular company's stock price for last one year," and "home country inflation rate" as only attributes because these indicators may have different values, so it is much appropriate to further classify these parameters into their disjoint attributive sets as we have done in Example 3.22. The above-mentioned existing decision-making models are insufficient either for interval-valued data or for multi-argument approximate function, but in the proposed model, the inadequacies of these models have been addressed. The consideration of multi-argument approximate function will make the decision-making process more reliable and trustworthy. We present a comparison analysis of our

proposed structure with the above relevant existing structures in Table 7 and 8.

6. Discussion

In this section, we show that our proposed structure IV-CNHS-set is a more generalized and flexible structure as compared to existing relevant models in the sense that the existing relevant models [24, 34, 36–42] are its particular cases by omitting one or more features among MD (membership degree), NMG (nonmembership degree), ID (indeterminacy degree), SAAF (single argument approximate function), MAAF (multiargument approximate function), PND (periodic nature of data), and IVD (interval-valued data). Figure 3 presents the pictorial version of this generalization of our proposed structure.

6.1. Merits of Proposed Study. In this subsection, some merits of the proposed study are highlighted, which are given below:

(i) The introduced approach took the significance of the idea of IV-CIFHS-set and IV-CNHS-set to deal with current decision-making issues. The presented idea enables the researchers to deal with the real-world scenario where the periodicity of data in the form of intervals is involved; along these lines, this

TABLE 7: Comparison of proposed structures with some existing relevant structures.

Authors	Structure	Approx. Func.	Remarks
Ali et al. [34]	CIFSS	SAAF	Insufficient for interval-valued data, degree of indeterminacy, and further partitioning of attributes into attribute-valued disjoint sets
Al-Sharqi et al. [36]	IV-CNSS	SAAF	Inadequacy for further partitioning of attributes into attribute-valued disjoint sets
Fan et al. [37]	IV-CFSS	SAAF	Inadequacy for the degree of nonmembership, degree of indeterminacy, and further partitioning of attributes into attribute-valued disjoint sets
Khan et al. [38]	CIFSS	SAAF	Insufficient for interval-valued data, degree of indeterminacy, and further partitioning of attributes into attribute-valued disjoint sets
Kumar et al. [39]	CIFSS	SAAF	Insufficient for interval-valued data, degree of indeterminacy, and further partitioning of attributes into attribute-valued disjoint sets
Smarandache et al. [40]	CNSS	SAAF	Insufficient for interval-valued data and further partitioning of attributes into attribute-valued disjoint sets

TABLE 8: Comparison of proposed structures with some existing relevant structures.

| Authors	Structure	Approx. Func.	Remarks
Selvachandran et al. [41]	IV-CFSS	SAAF	Insufficient for the degree of nonmembership, degree of indeterminacy, and further partitioning of attributes into attribute-valued disjoint sets
Thirunavukarasu et al. [42]	CFSS	SAAF	Insufficient for interval-valued data, degree of nonmembership, degree of indeterminacy, and further partitioning of attributes into attribute-valued disjoint sets
Rahman et al. [24]	CFHSS	MAAF	Insufficient for interval-valued data, degree of nonmembership, and degree of indeterminacy
Rahman et al. [24]	CIFHSS	MAAF	Insufficient for interval-valued data and degree of indeterminacy
Rahman et al. [24]	CNHSS	MAAF	Insufficient for interval-valued data
Rahman et al. [30]	IV-CFHSS	MAAF	Insufficient for the degree of nonmembership and degree of indeterminacy
Proposed structure 1	IV-CIFHSS	MAAF	It has limitations regarding the consideration of degree of indeterminacy
Proposed structure 2	IV-CNHSS	MAAF	Addresses the limitations and shortcomings of the above structures
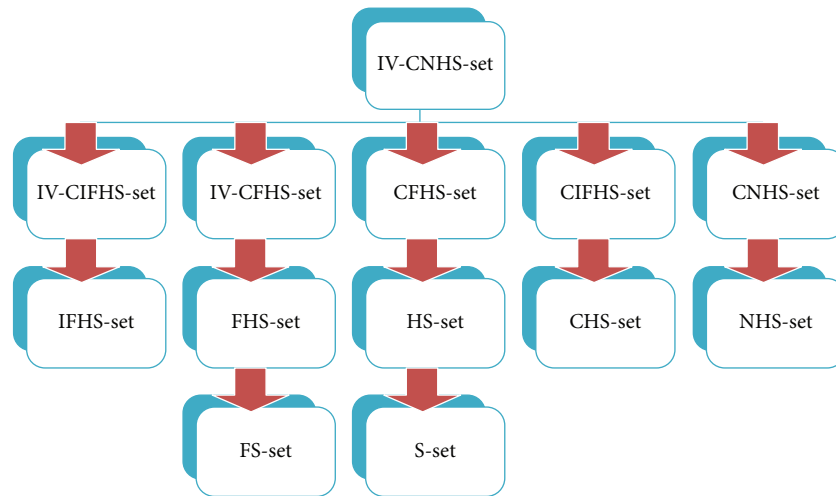
FIGURE 3: Generalization of the proposed structure.

TABLE 9: Comparison with existing models under appropriate features.

Authors	Structure	MD	NMD	ID	SAAF	MAAF	PND	IVD
Ali et al. [34]	CIFSS	√	√	×	√	×	√	×
Al-Sharqi et al. [36]	IV-CNSS	√	√	√	√	×	√	√
Fan et al. [37]	IV-CFSS	√	×	×	√	×	√	√
Khan et al. [38]	CIFSS	√	√	×	√	×	√	×
Kumar et al. [39]	CIFSS	√	√	×	√	×	√	×
Smarandache et al. [40]	CNSS	√	√	√	√	×	√	×
Selvachandran et al. [41]	IV-CFSS	√	×	×	√	×	√	√
Thirunavukarasu et al. [42]	CFSS	√	×	×	√	×	√	×
Rahman et al. [24]	CFHSS	√	×	×	√	√	√	×
Rahman et al. [24]	CIFHSS	√	√	×	√	√	√	×
Rahman et al. [24]	CNHSS	√	√	√	√	√	√	×
Rahman et al. [30]	IV-CFHSS	√	×	×	√	√	√	√
Proposed structure 1	IV-CIFHSS	√	√	×	√	√	√	√
Proposed structure 2	IV-CNHSS	√	√	√	√	√	√	√

association has tremendous potential in the genuine depiction inside the space of computational incursions.

(ii) As the proposed structure emphasizes on an in-depth study of attributes (i.e., further partitioning of attributes) rather than focusing on attributes merely, therefore, it makes the decision-making process better, flexible, and more reliable.

(iii) It covers the characteristics and properties of the existing relevant structures, that is, IV-CFHS-set, CFHS-set, CIFHS-set, CNHS-set, IV-CFSS-set, IV-CIFSS-set, IV-CNSS-set, CFSS-set, CIFSS-set, CNSS-set, and so on, so it is not unreasonable to call it the generalized form of all these structures.

The advantage of the proposed study can easily be judged from Tables 7–9. The comparison is evaluated on the basis of two different aspects as follows:

(1) Main features discussed in the study (see Tables 7 and 8)

(2) Features such as MD, NMG, ID, SAAF, MAAF, PND, and IVD (see Table 9)

7. Conclusion

The key features of this work can be summarized as follows:

(1) The novel notion of IV-CIFHS-set and IV-CNHS-set are characterized, and some of their elementary properties, that is, subset, null set, equal set, absolute set, homogeneous set, and complete homogeneous set are discussed with illustrated numerical examples.

(2) The set theoretic operations of IV-CIFHS-set and IV-CNHS-set, that is, complement, difference, union, and intersection, are characterized with the help of elaborated examples. Their fundamental laws and properties are also discussed.

(3) Aggregation operators, that is, aggregate matrix, cardinal set, cardinal matrix, and aggregate fuzzy set, of IV-CIFHS-set and IV-CNHS-set are developed.

(4) Two decision-making algorithms are proposed that are based on aggregation operators of IV-CIFHS-set and IV-CNHS-set. These are explained with the help of daily-life problems.

(5) The proposed structures along with their decision-support systems are compared with some existing relevant literature.

(6) Authors have carved out a conceptual framework for a generalized model, that is, IV-CNHS-set to deal with decision-making real-life problems by considering hypothetical data. The authors are committed to discussing some case studies based on IV-CNHS-set by using real data.

(7) Furthermore, it may also be extended to develop hybridized structures with expert sets, possibility fuzzy-set-like models, and fuzzy-set-like parameterized family and introduce algebraic structures.

Data Availability

No data were used to support the findings of the study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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