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INTUITIONISTIC FUZZY HYPERSOFT SETS

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ABSTRACT. In this paper, a new environment namely, intuitionistic fuzzy hypersoft set (IFHSS) is defined. We introduce some fundamental operators of intuitionistic fuzzy hypersoft sets such as subset, null set, absolute set, complement, union, intersection, equal set etc. Validity and application are presented with appropriate examples. For greater precision and accuracy, in the future, proposed operations in decision making processes such as personal selection, management issues and others will play a vital role.

1. INTRODUCTION

The fuzzy set theory identified in 1965 by Zadeh [26] is one of the most popular theories of recent times. Zadeh specified that there is a considerable amount of ambiguity in most real-life situations and physical problems that the classical set theory and its normal mathematical theories centered on such set theory did not give us the necessary knowledge and inferences. Fuzzy set theory has brought a great paradigmatic change in mathematics, but this theory also has some structural difficulties in its nature. Fuzzy set structure is defined with the help of membership function. It's difficult to create a membership function for each event, according to Molodtsov, because creating a membership function is too individual.

Molodtsov [15] introduced the soft set theory in 1999 which he felt was more practical. This theory is a relatively new approach to solving problems involving decision making and uncertainty. The major benefit of soft set theory is that in fuzzy set and other theories it is free from difficulties. Soft set theory has become popular among researchers in a short time and many scientific studies are carried

out on this theory every year [8, 16, 17, 27]. Maji et al. [12, 14] proposed an implementation of soft sets focused on parameter reduction to preserve optimum selection artifacts in decision-making problems. Chen [5] investigated a new definition and various applications of the reduction of soft sets to parameters. Pei and Miao [19] have shown that soft sets are a special class of information systems. Kong et al. [9] introduced the reduction and algorithm of soft sets to normal parameters. Zou and Xiao [28] discussed the soft data analysis approach. Aktaş and Çağman [2] defined the algebraic structure of soft set theory.

The intuitionistic fuzzy set (IFS) theory [3, 4] was created by adding a non-membership function to the fuzzy set structure. The non-member function makes IFS more functional in decision-making problems. Maji et al. [10, 13] combined soft set theory with the theory of the intuitionistic fuzzy sets and called intuitionistic fuzzy soft Set (IFSS). The parameterization and hesitancy acquired by IFSS from this mixture facilitates very accurately the description of real-world situations. IFSS is a valuable method for addressing data uncertainty and vagueness. Many scientific paper have shown the suitability of IFSS to issue decision making [7, 11].

Smarandache [21] introduced a new technique to deal with uncertainty. By converting the function into multiple decision functions, he generalized the soft to hypersoft set. Many studies have been done recently using hypersoft set structure [1, 6, 18, 20, 22, 23, 24, 25].

Multi-criteria decision-making (MCDM) is concerned with coordinating and taking care of matters of preference and preparation, including multi-criteria. Intuitionistic fuzzy soft set environments can not be used to solve certain types of problems if attributes are more than one and further bifurcated. Therefore, there was a serious need to identify a new approach to solve such problems, so a new setting, namely the intuitionistic fuzzy hypersoft Sets (IFHSS), is established for this reason. In the present paper, we introduce intuitionistic fuzzy hypersoft set theory. Intuitionistic fuzzy hypersoft set theory is a mixture of IFS theory and the hypersoft set theory. The complement, subset, equal set, "AND", "OR", intersection, union notions are defined on intuitionistic fuzzy hypersoft sets. This paper also is supported by many suitable examples.

2. Preliminary

Definition 1. *[3] An intuitionistic fuzzy set H in U is $H = \{(u, \theta_H(u), \sigma_H(u)) : u \in U\}$, where $\theta_H : U \to [0,1]$, $\sigma_H : U \to [0,1]$ with the condition $0 \leq \theta_H(u) + \sigma_H(u) \leq 1$, $\forall u \in U$. $\theta_H, \sigma_H \in [0,1]$ denote the degree of membership and non-membership of u to H, respectively. The set of all intuitionistic fuzzy sets over U will be denoted by $IFP(U)$.*

Definition 2. *[15] Let U be an initial universe and E be a set of parameters. A pair (H, E) is called a soft set over U, where H is a mapping $H : E \to \mathcal{P}(U)$. In other words, the soft set is a parameterized family of subsets of the set U.*

Definition 3. *[13] Let U be an initial universe and E be a set of parameters. A pair (H, E) is called an intuitionistic fuzzy soft set over U, where H is a mapping given by, $H : E \to IFP(U)$.*

In general, for every $e \in E$, $H(e)$ is an intuitionistic fuzzy set of U and it is called intuitionistic fuzzy value set of parameter e. Clearly, $H(e)$ can be written as a intuitionistic fuzzy set such that $H(e) = \{(u, \theta_H(u), \sigma_H(u)) : u \in U\}$.

Definition 4. *[21] Let U be the universal set and $P(U)$ be the power set of U. Consider $e_1, e_2, e_3, ..., e_n$ for $n \geq 1$, be n well-defined attributes, whose corresponding attribute values are resspectively the sets $E_1, E_2, ..., E_n$ with $E_i \cap E_j = \emptyset$, for $i \neq j$ and $i, j \in \{1, 2, ..., n\}$, then the pair $(H, E_1 \times E_2 \times ... \times E_n)$ is said to be Hypersoft set over U where $H : E_1 \times E_2 \times ... \times E_n \to P(U)$.*

3. Intuitionistic Fuzzy Hypersoft Sets

Definition 5. *Let U be the universal set and $IFP(U)$ be the intuitionistic fuzzy power set of U. Consider $e_1, e_2, e_3, ..., e_n$ for $n \geq 1$, be n well-defined attributes, whose corresponding attribute values are respectively the sets $E_1, E_2, ..., E_n$ with $E_i \cap E_j = \emptyset$, for $i \neq j$ and $i, j \in \{1, 2, ..., n\}$. Let A_i be the nonempty subset of E_i for each $i = 1, 2, ..., n$. An intuitionistic fuzzy hypersoft set defined as the pair $(H, A_1 \times A_2 \times ... \times A_n)$ where; $H : A_1 \times A_2 \times ... \times A_n \to IFP(U)$ and*

$$H(A_1 \times A_2 \times ... \times A_n) = \{< \xi, (\frac{u}{\theta_{H(\xi)}(u), \sigma_{H(\xi)}(u)}) >: u \in U, \xi \in A_1 \times A_2 \times ... \times A_n \subseteq E_1 \times E_2 \times ... \times E_n\}$$

where θ and σ are the membership and non-membership value, respectively such that $0 \leq \theta_{H(\xi)}(u) + \sigma_{H(\xi)}(u) \leq 1$ and $\theta_{H(\xi)}(u), \sigma_{H(\xi)}(u) \in [0, 1]$. For sake of simplicity, we write the symbols Δ for $E_1 \times E_2 \times ... \times E_n$, Ω for $A_1 \times A_2 \times ... \times A_n$ and ξ for an element of the set Ω.

Definition 6. *i) An intuitionistic fuzzy hypersoft set (H, Δ) over the universe U is said to be null intuitionistic fuzzy hypersoft set and denoted by $0_{(U_{IFH}, \Delta)}$ if for all $u \in U$ and $\xi \in \Delta$, $\theta_{H(\xi)}(u) = 0$ and $\sigma_{H(\xi)}(u) = 1$.*

ii) An intuitionistic fuzzy hypersoft set (H, Δ) over the universe U is said to be absolute intuitionistic fuzzy hypersoft set and denoted by $1_{(U_{IFH}, \Delta)}$ if for all $u \in U$ and $\xi \in \Delta$, $\theta_{H(\xi)}(\alpha) = 1$ and $\sigma_{H(\xi)}(u) = 0$.

Example 7. *Let U be the set of cars given as $U = \{u_1, u_2, u_3\}$ also consider the set of attributes as $E_1 = Fuel$, $E_2 = Transmission$, $E_3 = Color$ and their respective attributes are given as*

$$\begin{aligned} E_1 &= Fuels = \{Gasoline(\alpha_1), Diesel(\alpha_2), Electric(\alpha_3)\} \\ E_2 &= Transmissions = \{Automatic(\beta_1), Manual(\beta_2)\} \\ E_3 &= Colors = \{Black(\gamma_1), Blue(\gamma_2), White(\gamma_3)\} \end{aligned}$$

Suppose that

$$A_1 = \{\alpha_3\}, \ A_2 = \{\beta_1, \beta_2\}, \ A_3 = \{\gamma_1, \gamma_3\}$$

$$B_1 \quad = \quad \{\alpha_1, \alpha_3\}, \ B_2 = \{\beta_1\}, \ B_3 = \{\gamma_1, \gamma_2\}$$

are subset of E_i for each $i = 1, 2, 3$. Then the intuitionistic fuzzy hypersofts (H, Ω_1) and (G, Ω_2) defined as follows;

$$(H, \Omega_1) = \left\{ \begin{array}{l} < (\alpha_3, \beta_1, \gamma_1), \{\frac{u_1}{(0.6, 0.3)}, \frac{u_2}{(0.4, 0.2)}\} >, < (\alpha_3, \beta_1, \gamma_3), \{\frac{u_2}{(0.2, 0.6)}, \frac{u_3}{(0.3, 0.1)}\} >, \\ < (\alpha_3, \beta_2, \gamma_1), \{\frac{u_1}{(0.7, 0.3)}, \frac{u_2}{(0.3, 0.5)}, \frac{u_3}{(0.1, 0.6)}\} >, < (\alpha_3, \beta_2, \gamma_3), \{\frac{u_1}{(0.8, 0.2)}, \frac{u_2}{(0.2, 0.4)}\} > \end{array} \right\}$$

$$(G, \Omega_2) = \left\{ \begin{array}{l} < (\alpha_1, \beta_1, \gamma_1), \{\frac{u_1}{(0.3, 0.3)}, \frac{u_2}{(0.1, 0.2)}, \frac{u_3}{(0.6, 0.4)}\} >, < (\alpha_1, \beta_1, \gamma_2), \{\frac{u_1}{(0.7, 0.2)}, \frac{u_2}{(0.6, 0.1)}\} >, \\ < (\alpha_3, \beta_1, \gamma_1), \{\frac{u_1}{(0.9, 0.1)}, \frac{u_2}{(0.2, 0.7)}\} >, < (\alpha_3, \beta_1, \gamma_2), \{\frac{u_2}{(0.5, 0.3)}, \frac{u_3}{(0.3, 0.7)}\} > \end{array} \right\}$$

Tabular form of these sets are given as follows:

TABLE 1. Tabular form of IFHSS (H, Ω_1)

(H, Ω_1)	u_1	u_2	u_3
$(\alpha_3, \beta_1, \gamma_1)$	$(0.6, 0.3)$	$(0.4, 0.2)$	$(0, 1)$
$(\alpha_3, \beta_1, \gamma_3)$	$(0, 1)$	$(0.2, 0.6)$	$(0.3, 0.1)$
$(\alpha_3, \beta_2, \gamma_1)$	$(0.7, 0.3)$	$(0.3, 0.5)$	$(0.1, 0.6)$
$(\alpha_3, \beta_2, \gamma_3)$	$(0.8, 0.2)$	$(0.2, 0.4)$	$(0, 1)$

TABLE 2. Tabular form of IFHSS (G, Ω_2)

(G, Ω_2)	u_1	u_2	u_3
$(\alpha_1, \beta_1, \gamma_1)$	$(0.3, 0.3)$	$(0.1, 0.2)$	$(0.6, 0.4)$
$(\alpha_1, \beta_1, \gamma_2)$	$(0.7, 0.2)$	$(0.6, 0.1)$	$(0, 1)$
$(\alpha_3, \beta_1, \gamma_1)$	$(0.9, 0.1)$	$(0.2, 0.7)$	$(0, 1)$
$(\alpha_3, \beta_1, \gamma_2)$	$(0, 1)$	$(0.5, 0.3)$	$(0.3, 0.7)$

Corollary 8. *It is clear that each intuitionistic fuzzy hypersoft set is also intuitionistic fuzzy soft set. An example of this situation is provided below.*

Example 9. *We consider that Example-7. If we select the parameters from a single attribute set such as E_1 while creating the intuitionistic fuzzy hypersoft set, then the resulting set becomes the intuitionistic fuzzy soft set. Therefore, it is clear that each intuitionistic fuzzy hypersoft set is also intuitionistic fuzzy soft set. That is, the intuitionistic fuzzy hypersoft set structure is the generalized version of the intuitionistic fuzzy soft sets.*

Definition 10. *Let U be an initial universe set and $(H, \Omega_1), (G, \Omega_2)$ be two intuitionistic fuzzy hypersoft sets over the universe U. We say that (H, Ω_1) is an intuitionistic fuzzy hypersoft subset of (G, Ω_2) and denote $(H, \Omega_1) \widetilde{\subseteq} (G, \Omega_2)$ if*

   i) $\Omega_1 \subseteq \Omega_2$
   ii) For any $\xi \in \Omega_1$, $H(\xi) \subseteq G(\xi)$,
   That is for all $u \in U$ and $\xi \in \Omega_1$, $\theta_{H(\xi)}(u) \leq \theta_{G(\xi)}(u)$ and $\sigma_{H(\xi)}(u) \geq \sigma_{G(\xi)}(u)$.

Example 11. *We consider that attributes in Example-7 and Let (H, Ω_1), (G, Ω_2) be two intuitionistic fuzzy hypersoft set over the same universe $U = \{u_1, u_2, u_3\}$. Tabular forms of (H, Ω_1) and (G, Ω_2) are following:*

<center>TABLE 3. Tabular form of IFHSS (H, Ω_1)</center>

(H, Ω_1)	u_1	u_2	u_3
$(\alpha_3, \beta_1, \gamma_1)$	$(0.5, 0.4)$	$(0.1, 0.8)$	$(0.3, 0.4)$
$(\alpha_3, \beta_1, \gamma_3)$	$(0.2, 0.7)$	$(0.3, 0.6)$	$(0.1, 0.6)$
$(\alpha_3, \beta_2, \gamma_1)$	$(0, 1)$	$(0, 1)$	$(0, 1)$
$(\alpha_3, \beta_2, \gamma_3)$	$(0, 1)$	$(0, 1)$	$(0, 1)$

<center>TABLE 4. Tabular form of IFHSS (G, Ω_2)</center>

(G, Ω_2)	u_1	u_2	u_3
$(\alpha_3, \beta_1, \gamma_1)$	$(0.6, 0.1)$	$(0.2, 0.5)$	$(0.7, 0.3)$
$(\alpha_3, \beta_1, \gamma_3)$	$(0.6, 0.4)$	$(0.4, 0.6)$	$(0.7, 0.4)$
$(\alpha_3, \beta_2, \gamma_1)$	$(0, 1)$	$(0, 1)$	$(0, 1)$
$(\alpha_3, \beta_2, \gamma_3)$	$(0.4, 0.3)$	$(0.8, 0.2)$	$(0.6, 0.4)$

It is clear that $(H, \Omega_1)\widetilde{\subseteq}(G, \Omega_2)$. we can also written as

$$(H, \Omega_1)\widetilde{\subseteq}(G, \Omega_2) = \left\{ \begin{array}{c} < (\alpha_3, \beta_1, \gamma_1), \{\frac{u_1}{(0.5,0.4)}, \frac{u_2}{(0.1,0.8)}, \frac{u_3}{(0.3,0.4)}\} >, \\ < (\alpha_3, \beta_1, \gamma_3), \{\frac{u_1}{(0.2,0.7)}, \frac{u_2}{(0.3,0.6)}, \frac{u_3}{(0.1,0.6)}\} >, \\ < (\alpha_3, \beta_2, \gamma_1), \{\frac{u_1}{(0,1)}, \frac{u_2}{(0,1)}, \frac{u_3}{(0,1)}\} >, \\ < (\alpha_3, \beta_2, \gamma_3), \{\frac{u_1}{(0,1)}, \frac{u_2}{(0,1)}, \frac{u_3}{(0,1)}\} > \end{array} \right\}$$

$$\widetilde{\subseteq} \left\{ \begin{array}{c} < (\alpha_3, \beta_1, \gamma_1), \{\frac{u_1}{(0.6,0.1)}, \frac{u_2}{(0.2,0.5)}, \frac{u_3}{(0.7,0.3)}\} >, \\ < (\alpha_3, \beta_1, \gamma_3), \{\frac{u_1}{(0.6,0.4)}, \frac{u_2}{(0.4,0.6)}, \frac{u_3}{(0.7,0.4)}\} >, \\ < (\alpha_3, \beta_2, \gamma_1), \{\frac{u_1}{(0,1)}, \frac{u_2}{(0,1)}, \frac{u_3}{(0,1)}\} >, \\ < (\alpha_3, \beta_2, \gamma_3), \{\frac{u_1}{(0.4,0.3)}, \frac{u_2}{(0.8,0.2)}, \frac{u_3}{(0.6,0.4)}\} > \end{array} \right\}$$

Definition 12. *Let U be an initial universe set and $(H, \Omega_1), (G, \Omega_2)$ be two intuitionistic fuzzy hypersoft sets over the universe U. We say that (H, Ω_1) is an intuitionistic fuzzy hypersoft equal (G, Ω_2) and denote $(H, \Omega_1)\widetilde{=}(G, \Omega_2)$ if for all $u \in U$ and $\xi \in \Delta$, $\theta_{H(\xi)}(u) = \theta_{G(\xi)}(u)$ and $\sigma_{H(\xi)}(u) = \sigma_{G(\xi)}(u)$.*

Theorem 13. *Let U be an initial universe set, $\Omega_1, \Omega_2, \Omega_3 \subseteq \Delta$ and $(H, \Omega_1), (G, \Omega_2), (K, \Omega_3)$ be intuitionistic fuzzy hypersoft sets over the universe U. Then,*

i) $(H, \Omega_1)\widetilde{\subseteq}1_{(U_{IFH}, \Delta)}$,

ii) $0_{(U_{IFH}, \Delta)}\widetilde{\subseteq}(H, \Omega_1)$,

iii) $(H, \Omega_1)\widetilde{\subseteq}(G, \Omega_2)$ and $(G, \Omega_2)\widetilde{\subseteq}(K, \Omega_3) \Rightarrow (H, \Omega_1)\widetilde{\subseteq}(K, \Omega_3)$.

Proof. i) $(H,\Omega_1)\widetilde{\subseteq}1_{(U_{IFH},\Delta)}$, since $\theta_{H(\xi)}(u) \leq \theta_{H(\Delta)}(u) = 1$ and
$\sigma_{H(\xi)}(u) \geq \sigma_{H(\Delta)}(u) = 0$ for all $\xi \in \Delta$, $u \in U$,

ii)$0_{(U_{IFH},\Delta)}\widetilde{\subseteq}(H,\Omega_1)$, since $0 = \theta_{\tilde{\emptyset}}(u) \leq \theta_{H(\varepsilon)}(u)$ and $1 = \sigma_{\tilde{\emptyset}}(u) \geq \sigma_{H(\xi)}(u)$ for all $\xi \in \Delta$, $u \in U$,

iii) $(H,\Omega_1)\widetilde{\subseteq}(G,\Omega_2) \Rightarrow \theta_{H(\xi)}(u) \leq \theta_{G(\xi)}(u)$ and $\sigma_{H(\xi)}(u) \geq \sigma_{H(\xi)}(u)$ for all $\xi \in \Delta$, $u \in U$. Also $(G,\Omega_2)\widetilde{\subseteq}(K,\Omega_3) \Rightarrow \theta_{G(\xi)}(u) \leq \theta_{K(\xi)}(u)$ and $\sigma_{G(\xi)}(u) \geq \sigma_{K(\xi)}(u)$ for all $\xi \in \Delta$, $u \in U$. Therefore $\theta_{H(\xi)}(u) \leq \theta_{K(\xi)}(u)$ and $\sigma_{H(\xi)}(u) \geq \sigma_{K(\xi)}(u)$. Thus, we obtain $(H,\Omega_1)\widetilde{\widetilde{\subseteq}}(K,\Omega_3)$. $\square$

Definition 14. *The complement of intutionistic fuzzy hypersoft set* (H,Ω) *over the universe* U *is denoted by* $(H,\Omega)^c$ *and defined as* $(H,\Omega)^c = (H^c,\Omega)$, *where* $H^c : (E_1 \times E_2 \times ... \times E_n) = \Delta \rightarrow IFP(U)$ *and* $H^c(\Omega) = (H(\Omega))^c$ *for all* $\Omega \subseteq \Delta$. *Thus if* $(H,\Omega) = \{< \xi, (\frac{u}{\theta_{H(\xi)}(u),\sigma_{H(\xi)}(u)}) >: u \in U, \xi \in \Omega)\}$, *then* $(H,\Omega)^c = \{< \xi, (\frac{u}{\sigma_{H(\xi)}(u),\theta_{H(\xi)}(u)}) >: u \in U, \xi \in \Omega)\}$.

Example 15. *According to Example-7, consider the intuitionistic fuzzy hypersoft set* (H,Ω) *over the universe* $U = \{\alpha_1,\alpha_2,\alpha_3,\alpha_4\}$.

$$(H,\Omega_1) = \left\{ \begin{array}{c} < (\alpha_3,\beta_1,\gamma_1), \{\frac{u_1}{(0.6,0,3)}, \frac{u_2}{(0.4,0.2)}\} >, < (\alpha_3,\beta_1,\gamma_3), \{\frac{u_2}{(0.2,0.6)}, \frac{u_3}{(0.3,0.1)}\} >, \\ < (\alpha_3,\beta_2,\gamma_1), \{\frac{u_1}{(0.7,0.3)}, \frac{u_2}{(0.3,0.5)}, \frac{u_3}{(0.1,0.6)}\} >, < (\alpha_3,\beta_2,\gamma_3), \{\frac{u_1}{(0.8,0.2)}, \frac{u_2}{(0.2,0.4)}\} > \end{array} \right\}$$

Then the complement of (H,Ω) *is written as;*

$$(H,\Omega_1)^c = \left\{ \begin{array}{c} < (\alpha_3,\beta_1,\gamma_1), \{\frac{u_1}{(0.3,0,6)}, \frac{u_2}{(0.2,0.4)}, \frac{u_3}{(1,0)}\} >, < (\alpha_3,\beta_1,\gamma_3), \{\frac{u_1}{(1,0)}, \frac{u_2}{(0.6,0.2)}, \frac{u_3}{(0.1,0.3)}\} >, \\ < (\alpha_3,\beta_2,\gamma_1), \{\frac{u_1}{(0.3,0.7)}, \frac{u_2}{(0.5,0.3)}, \frac{u_3}{(0.6,0.1)}\} >, < (\alpha_3,\beta_2,\gamma_3), \{\frac{u_1}{(0.2,0.8)}, \frac{u_2}{(0.4,0.2)}\} > \end{array} \right\}.$$

Theorem 16. *Let* (H,Ω) *be any intuitionistic fuzzy hypersoft set over the universe* U. *Then,*

i) $((H,\Omega)^c)^c = (H,\Omega)$

ii) $0^c_{(U_{IFH},\Delta)} = 1_{(U_{IFH},\Delta)}$

iii) $1^c_{(U_{IFH},\Delta)} = 0_{(U_{IFH},\Delta)}$

Proof. Proofs are trivial. $\square$

Definition 17. *Let* U *be an initial universe set,* $\Omega_1,\Omega_2 \subseteq \Delta$ *and* $(H,\Omega_1),(G,\Omega_2)$ *be two intuitionistic fuzzy hypersoft sets over the universe* U. *The union of* (H,Ω_1) *and* (G,Ω_2) *is denoted by* $(H,\Omega_1)\breve{\cup}(G,\Omega_2) = (K,\Omega_3)$ *where* $\Omega_3 = \Omega_1 \cup \Omega_2$ *and*

$$\theta_{K(\xi)}(u) = \begin{cases} H(\xi) & if \quad \xi \in \Omega_1 - \Omega_2 \\ G(\xi) & if \quad \xi \in \Omega_2 - \Omega_1 \\ \max(H(\xi),(G(\xi)) & if \quad \xi \in \Omega_1 \cap \Omega_2 \end{cases}$$

$$\sigma_{K(\xi)}(u) = \begin{cases} H(\xi) & if \quad \xi \in \Omega_1 - \Omega_2 \\ G(\xi) & if \quad \xi \in \Omega_2 - \Omega_1 \\ \min(H(\xi),(G(\xi)) & if \quad \xi \in \Omega_1 \cap \Omega_2 \end{cases}$$

Theorem 18. *Let U be an initial universe set, $\Omega_1, \Omega_2, \Omega_3 \subseteq \Delta$ and $(H, \Omega_1), (G, \Omega_2),$ (S, Ω_3) be intuitionistic fuzzy hypersoft sets over the universe U. Then;*

i) $(H, \Omega_1) \tilde{\cup} (H, \Omega_1) = (H, \Omega_1)$

ii) $0_{(U_{IFH}, \Delta)} \tilde{\cup} (H, \Omega_1) = (H, \Omega_1)$

iii) $(H, \Omega_1) \tilde{\cup} 1_{(U_{IFH}, \Delta)} = 1_{(U_{IFH}, \Delta)}$

iv) $(H, \Omega_1) \tilde{\cup} (G, \Omega_2) = (G, \Omega_2) \tilde{\cup} (H, \Omega_1)$

v) $((H, \Omega_1) \tilde{\cup} (G, \Omega_2)) \tilde{\cup} (S, \Omega_3) = (H, \Omega_1) \tilde{\cup} ((G, \Omega_2) \tilde{\cup} (S, \Omega_3))$

Proof. Proofs are trivial. $\qquad \square$

Definition 19. *Let U be an initial universe set, $\Omega_1, \Omega_2 \subseteq \Delta$ and $(H, \Omega_1), (G, \Omega_2)$ be two intuitionistic fuzzy hypersoft sets over the universe U. The intersection of (H, Ω_1) and (G, Ω_2) is denoted by $(H, \Omega_1) \tilde{\cap} (G, \Omega_2) = (K, \Omega_3)$ where $\Omega_3 = \Omega_1 \cap \Omega_2$,*

$$(K, \Omega_3) = \{< \xi, \left(\frac{u}{(\min\{\theta_{H(\xi)}(u), \theta_{G(\xi)}(u)\}, \max\{\theta_{H(\xi)}(u), \theta_{G(\xi)}(u)\})} \right) >: u \in U, \xi \in \Omega)\}$$

Theorem 20. *Let U be an initial universe set, $\Omega_1, \Omega_2, \Omega_3 \subseteq \Delta$ and $(H, \Omega_1), (G, \Omega_2),$ (S, Ω_3) be intuitionistic fuzzy hypersoft sets over the universe U. Then;*

i) $(H, \Omega_1) \tilde{\cap} (H, \Omega_1) = (H, \Omega_1)$

ii) $0_{(U_{IFH}, \Delta)} \tilde{\cap} (H, \Omega_1) = 0_{(U_{IFH}, \Delta)}$

iii) $(H, \Omega_1) \tilde{\cap} 1_{(U_{IFH}, \Delta)} = (H, \Omega_1)$

iv) $(H, \Omega_1) \tilde{\cap} (G, \Omega_2) = (G, \Omega_2) \tilde{\cap} (H, \Omega_1)$

v) $((H, \Omega_1) \tilde{\cap} (G, \Omega_2)) \tilde{\cap} (S, \Omega_3) = (H, \Omega_1) \tilde{\cap} ((G, \Omega_2) \tilde{\cap} (S, \Omega_3))$

Proof. Proofs are trivial. $\qquad \square$

Definition 21. *Let U be an initial universe set, $\Omega_1, \Omega_2 \subseteq \Delta$ and $(H, \Omega_1), (G, \Omega_2)$ be two intuitionistic fuzzy hypersoft sets over the universe U. The difference of (H, Ω_1) and (G, Ω_2) is denoted by $(H, \Omega_1) \widetilde{\setminus} (G, \Omega_2) = (K, \Omega_3)$ where $(H, \Omega_1) \tilde{\cap} (G, \Omega_2)^c = (K, \Omega_3)$.*

Example 22. *We consider that attributes in Example-7 and Let $(H, \Omega_1), (G, \Omega_2)$ be any two intuitionistic fuzzy hypersoft set over the same universe $U = \{u_1, u_2, u_3\}$. The IFHSS (H, Ω_1) and (G, Ω_2) defined as follows;*

$$(H, \Omega_1) = \left\{ \begin{array}{c} < (\alpha_3, \beta_1, \gamma_1), \{\frac{u_1}{(0.6, 0.3)}, \frac{u_2}{(0.4, 0.2)}\} >, < (\alpha_3, \beta_1, \gamma_3), \{\frac{u_2}{(0.2, 0.6)}, \frac{u_3}{(0.3, 0.1)}\} >, \\ < (\alpha_3, \beta_2, \gamma_1), \{\frac{u_1}{(0.7, 0.3)}, \frac{u_2}{(0.3, 0.5)}, \frac{u_3}{(0.1, 0.6)}\} >, < (\alpha_3, \beta_2, \gamma_3), \{\frac{u_1}{(0.8, 0.2)}, \frac{u_2}{(0.2, 0.4)}\} > \end{array} \right\}$$

$$(G, \Omega_2) = \left\{ \begin{array}{c} < (\alpha_1, \beta_1, \gamma_1), \{\frac{u_1}{(0.3, 0.3)}, \frac{u_2}{(0.1, 0.2)}, \frac{u_3}{(0.6, 0.4)}\} >, < (\alpha_1, \beta_1, \gamma_2), \{\frac{u_1}{(0.7, 0.2)}, \frac{u_2}{(0.6, 0.1)}\} >, \\ < (\alpha_3, \beta_1, \gamma_1), \{\frac{u_1}{(0.9, 0.1)}, \frac{u_2}{(0.2, 0.7)}\} >, < (\alpha_3, \beta_1, \gamma_2), \{\frac{u_2}{(0.5, 0.3)}, \frac{u_3}{(0.3, 0.7)}\} > \end{array} \right\}$$

The union, intersection and difference operator of above IFHSS is written as;

$$(H, \Omega_1) \tilde{\cup} (G, \Omega_2) =$$

$$\left\{ \begin{array}{c} < (\alpha_3, \beta_1, \gamma_1), \{\frac{u_1}{(0.9, 0.1)}, \frac{u_2}{(0.4, 0.2)}\} >, < (\alpha_3, \beta_1, \gamma_3), \{\frac{u_2}{(0.2, 0.6)}, \frac{u_3}{(0.3, 0.1)}\} >, \\ < (\alpha_3, \beta_2, \gamma_1), \{\frac{u_1}{(0.7, 0.3)}, \frac{u_2}{(0.3, 0.5)}, \frac{u_3}{(0.1, 0.6)}\} >, < (\alpha_3, \beta_2, \gamma_3), \{\frac{u_1}{(0.8, 0.2)}, \frac{u_2}{(0.2, 0.4)}\} >, \\ < (\alpha_1, \beta_1, \gamma_1), \{\frac{u_1}{(0.3, 0.3)}, \frac{u_2}{(0.1, 0.2)}, \frac{u_3}{(0.6, 0.4)}\} >, < (\alpha_1, \beta_1, \gamma_2), \{\frac{u_1}{(0.7, 0.2)}, \frac{u_2}{(0.6, 0.1)}\} > \\ < (\alpha_3, \beta_1, \gamma_2), \{\frac{u_2}{(0.5, 0.3)}, \frac{u_3}{(0.3, 0.7)}\} > \end{array} \right\}$$

$$(H,\Omega_1)\tilde{\cap}(G,\Omega_2) = \left\{ < (\alpha_3,\beta_1,\gamma_1), \{\frac{u_1}{(0.6,0,3)}, \frac{u_2}{(0.2,0.7)}\} > \right\}$$

and

$$(H,\Omega_1)\widetilde{\backslash}(G,\Omega_2) = \left\{ < (\alpha_3,\beta_1,\gamma_1), \{\frac{u_1}{(0.1,0,9)}, \frac{u_2}{(0.4,0.2)}\} > \right\}$$

Theorem 23. *Let U be an initial universe set, $\Omega_1, \Omega_2 \subseteq \Delta$ and $(H,\Omega_1),(G,\Omega_2)$ be two intuitionistic fuzzy hypersoft sets over the universe U. Then De-Morgan Laws are hold.*
  i) $((H,\Omega_1)\tilde{\cup}(G,\Omega_2))^c = (H,\Omega_1)^c\tilde{\cap}(G,\Omega_2)^c$
  ii)$((H,\Omega_1)\tilde{\cap}(G,\Omega_2))^c = (H,\Omega_1)^c\tilde{\cup}(G,\Omega_2)^c$

Proof. We only prove $((H,\Omega_1)\tilde{\cup}(G,\Omega_2))^c = (H,\Omega_1)^c\tilde{\cap}(G,\Omega_2)^c$. The other properties can be similarly proved. Suppose that $((H,\Omega_1)\tilde{\cup}(G,\Omega_2))^c = (K,\Omega_1 \cup \Omega_2)$ and $(H,\Omega_1)^c\tilde{\cap}(G,\Omega_2)^c = (I,\Omega_1 \cup \Omega_2)$. For any $\xi \in \Omega_1 \cup \Omega_2$, we consider the following cases.

  Case 1: $\xi \in \Omega_1 - \Omega_2$. Then $K(\xi) = H^c(\xi) = I(\xi)$.
  Case 2: $\xi \in \Omega_2 - \Omega_1$. Then $K(\xi) = G^c(\xi) = I(\xi)$.
  Case 3: $\xi \in \Omega_1 \cap \Omega_2$. Then $K(\xi) = (\sigma_{H(\xi)}(u) \cap \sigma_{G(\xi)}(u), \theta_{H(\xi)}(u) \cup \theta_{G(\xi)}(u)) = H^c(\xi) \cap G^c(\xi) = I(\xi)$.

  Therefore, K and I are same operators, and so
$((H,\Omega_1)\tilde{\cup}(G,\Omega_2))^c = (H,\Omega_1)^c\tilde{\cap}(G,\Omega_2)^c$. $\qquad\qquad\qquad\square$

Definition 24. *Let U be an initial universe set, $\Omega_1, \Omega_2 \subseteq \Delta$ and $(H,\Omega_1),(G,\Omega_2)$ be two intuitionistic fuzzy hypersoft sets over the universe U. The "AND" operation on them is denoted by $(H,\Omega_1) \wedge (G,\Omega_2) = (K,\Omega_1 \times \Omega_2)$ is given as;*

$$(K,\Omega_1\times\Omega_2) = \left\{ < (\xi_1,\xi_2), (\frac{u}{\theta_{K(\xi_1,\xi_2)}(u), \sigma_{K(\xi_1,\xi_2)}(u)}) >: u \in U, (\xi_1,\xi_2) \in \Omega_1 \times \Omega_2 \right\}$$

where

$$\begin{aligned} \theta_{K(\xi_1,\xi_2)}(u) &= \min\left\{\theta_{H(\xi_1)}(u),\theta_{G(\xi_2)}(u)\right\} \\ \sigma_{K(\xi_1,\xi_2)}(u) &= \max\left\{\sigma_{H(\xi_1)}(u),\sigma_{G(\xi_2)}(u)\right\} \end{aligned}$$

Definition 25. *Let U be an initial universe set, $\Omega_1, \Omega_2 \subseteq \Delta$ and $(H,\Omega_1),(G,\Omega_2)$ be two intuitionistic fuzzy hypersoft sets over the universe U. The "OR" operation on them is denoted by $(H,\Omega_1) \vee (G,\Omega_2) = (K,\Omega_1 \times \Omega_2)$ is given as;*

$$(K,\Omega_1\times\Omega_2) = \left\{ < (\xi_1,\xi_2), (\frac{u}{\theta_{K(\xi_1,\xi_2)}(u), \sigma_{K(\xi_1,\xi_2)}(u)}) >: u \in U, (\xi_1,\xi_2) \in \Omega_1 \times \Omega_2 \right\}$$

where

$$\begin{aligned} \theta_{K(\xi_1,\xi_2)}(u) &= \max\left\{\theta_{H(\xi_1)}(u),\theta_{G(\xi_2)}(u)\right\} \\ \sigma_{K(\xi_1,\xi_2)}(u) &= \min\left\{\sigma_{H(\xi_1)}(u),\sigma_{G(\xi_2)}(u)\right\} \end{aligned}$$

Theorem 26. *Let U be an initial universe set, $\Omega_1, \Omega_2, \Omega_3 \subseteq \Delta$ and $(H, \Omega_1), (G, \Omega_2),$ (S, Ω_3) be intuitionistic fuzzy hypersoft sets over the universe U. Then;*

i) $(H, \Omega_1) \vee [(G, \Omega_2) \vee (S, \Omega_3)] = [(H, \Omega_1) \vee (G, \Omega_2)] \vee (S, \Omega_3)$

ii) $(H, \Omega_1) \wedge [(G, \Omega_2) \wedge (S, \Omega_3)] = [(H, \Omega_1) \wedge (G, \Omega_2)] \wedge (S, \Omega_3)$

Proof. Straightforward. $\qquad\square$

Theorem 27. *Let U be an initial universe set, $\Omega_1, \Omega_2 \subseteq \Delta$ and $(H, \Omega_1), (G, \Omega_2)$ be two intuitionistic fuzzy hypersoft sets over the universe U. Then;*

i) $[(H, \Omega_1) \vee (G, \Omega_2)]^c = (H, \Omega_1)^c \wedge (G, \Omega_2)^c$

ii) $[(H, \Omega_1) \wedge (G, \Omega_2)]^c = (H, \Omega_1)^c \vee (G, \Omega_2)^c.$

Proof. We only prove (i). The other properties can be similarly proved.

For all $(\xi_1, \xi_2) \in \Omega_1 \times \Omega_2$ and $u \in U$,

$$
\begin{aligned}
(H, \Omega_1) \vee (G, \Omega_2) &= \left\{ < u, \max\left\{ \theta_{H(\xi_1)}(u), \theta_{G(\xi_2)}(u) \right\}, \min\left\{ \sigma_{H(\xi_1)}(u), \sigma_{G(\xi_2)}(u) \right\} > \right\}, \\
[(H, \Omega_1) \vee (G, \Omega_2)]^c &= \left\{ < u, \min\left\{ \sigma_{H(\xi_1)}(u), \sigma_{G(\xi_2)}(u) \right\}, \max\left\{ \theta_{H(\xi_1)}(u), \theta_{G(\xi_2)}(u) \right\} > \right\}
\end{aligned}
$$

On the other hand,

$$
\begin{aligned}
(H, \Omega_1)^c &= \left\{ < u, \sigma_{H(\xi_1)}(u), \theta_{H(\xi_1)}(u) >: \xi_1 \in \Omega_1 \right\} \\
(G, \Omega_2)^c &= \left\{ < u, \sigma_{G(\xi_2)}(u), \theta_{G(\xi_2)}(u) >: \xi_2 \in \Omega_2 \right\}
\end{aligned}
$$

Then,

$$
\begin{aligned}
(H, \Omega_1)^c \wedge (G, \Omega_2)^c &= \left\{ \left\{ < u, \min\left\{ \sigma_{H(\xi_1)}(u), \sigma_{G(\xi_2)}(u) \right\}, \max\left\{ \theta_{H(\xi_1)}(u), \theta_{G(\xi_2)}(u) \right\} > \right\} \right\} \\
&= [(H, \Omega_1) \vee (G, \Omega_2)]^c
\end{aligned}
$$

Hence, $[(H, \Omega_1) \vee (G, \Omega_2)]^c = (H, \Omega_1)^c \wedge (G, \Omega_2)^c$ is obtained. $\qquad\square$

Example 28. *We consider that attributes in Example-7. Then the fuzzy hypersoft sets (H, Ω_1) and (G, Ω_2) defined as follows;*

$$
(H, \Omega_1) = \left\{ \begin{aligned} &< (\alpha_3, \beta_1, \gamma_1), \{ \tfrac{u_1}{(0.6, 0.3)}, \tfrac{u_2}{(0.4, 0.2)} \} >, < (\alpha_3, \beta_1, \gamma_3), \{ \tfrac{u_2}{(0.2, 0.6)}, \tfrac{u_3}{(0.3, 0.1)} \} >, \\ &< (\alpha_3, \beta_2, \gamma_1), \{ \tfrac{u_1}{(0.7, 0.3)}, \tfrac{u_2}{(0.3, 0.5)}, \tfrac{u_3}{(0.1, 0.6)} \} >, < (\alpha_3, \beta_2, \gamma_3), \{ \tfrac{u_1}{(0.8, 0.2)}, \tfrac{u_2}{(0.2, 0.4)} \} > \end{aligned} \right\}
$$

$$
(G, \Omega_2) = \left\{ \begin{aligned} &< (\alpha_1, \beta_1, \gamma_1), \{ \tfrac{u_1}{(0.3, 0.3)}, \tfrac{u_2}{(0.1, 0.2)}, \tfrac{u_3}{(0.6, 0.4)} \} >, < (\alpha_1, \beta_1, \gamma_2), \{ \tfrac{u_1}{(0.7, 0.2)}, \tfrac{u_2}{(0.6, 0.1)} \} >, \\ &< (\alpha_3, \beta_1, \gamma_1), \{ \tfrac{u_1}{(0.9, 0.1)}, \tfrac{u_2}{(0.2, 0.7)} \} >, < (\alpha_3, \beta_1, \gamma_2), \{ \tfrac{u_2}{(0.5, 0.3)}, \tfrac{u_3}{(0.3, 0.7)} \} > \end{aligned} \right\}
$$

Let's assume $(\alpha_3, \beta_1, \gamma_1) = m_1$, $(\alpha_3, \beta_1, \gamma_3) = m_2$, $(\alpha_3, \beta_2, \gamma_1) = m_3$, $(\alpha_3, \beta_2, \gamma_3) = m_4$ in (H, Ω_1) and $(\alpha_1, \beta_1, \gamma_1) = n_1$, $(\alpha_1, \beta_1, \gamma_2) = n_2$, $(\alpha_3, \beta_1, \gamma_1) = n_3$, $(\alpha_3, \beta_1, \gamma_2) = n_4$ in (G, Ω_2) for easier operation. The tabular forms of these sets are as follows.

Then the "AND" and "OR" operations of these sets are given as below.

TABLE 5. Tabular form of FHSS (H, Ω_1)

(H, Ω_1)	u_1	u_2	u_3
m_1	$(0.6, 0, 3)$	$(0.4, 0.2)$	$(0, 1)$
m_2	$(0, 1)$	$(0.2, 0.6)$	$(0.3, 0.1)$
m_3	$(0.7, 0.3)$	$(0.3, 0.5)$	$(0.1, 0.6)$
m_4	$(0.8, 0.2)$	$(0.2, 0.4)$	$(0, 1)$

TABLE 6. Tabular form of FHSS (G, Ω_2)

(G, Ω_2)	u_1	u_2	u_3
n_1	$(0.3, 0, 3)$	$(0.1, 0.2)$	$(0.6, 0.4)$
n_2	$(0.7, 0.2)$	$(0.6, 0.1)$	$(0, 1)$
n_3	$(0.9, 0.1)$	$(0.2, 0.7)$	$(0, 1)$
n_4	$(0, 1)$	$(0.5, 0.3)$	$(0.3, 0.7)$

TABLE 7. Tabular form of FHSS $(H, \Omega_1) \wedge (G, \Omega_2)$

$(H, \Omega_1) \wedge (G, \Omega_2)$	u_1	u_2	u_3
$m_1 \times n_1$	$(0.3, 0.3)$	$(0.1, 0.2)$	$(0, 1)$
$m_1 \times n_2$	$(0.6, 0.3)$	$(0.4, 0.2)$	$(0, 1)$
$m_1 \times n_3$	$(0.6, 0.3)$	$(0.2, 0.7)$	$(0, 1)$
$m_1 \times n_4$	$(0, 1)$	$(0.4, 0.3)$	$(0, 1)$
$m_2 \times n_1$	$(0, 1)$	$(0.1, 0.6)$	$(0.3, 0.4)$
$m_2 \times n_2$	$(0, 1)$	$(0.2, 0.6)$	$(0, 1)$
$m_2 \times n_3$	$(0, 1)$	$(0.2, 0.7)$	$(0, 1)$
$m_2 \times n_4$	$(0, 1)$	$(0.2, 0.6)$	$(0.3, 0.7)$
$m_3 \times n_1$	$(0.3, 0.3)$	$(0.1, 0.5)$	$(0.1, 0.6)$
$m_3 \times n_2$	$(0.7, 0.3)$	$(0.3, 0.5)$	$(0, 1)$
$m_3 \times n_3$	$(0.7, 0.3)$	$(0.2, 0.7)$	$(0, 1)$
$m_3 \times n_4$	$(0, 1)$	$(0.3, 0.5)$	$(0.1, 0.7)$
$m_4 \times n_1$	$(0.3, 0.3)$	$(0.1, 0.4)$	$(0, 1)$
$m_4 \times n_2$	$(0.7, 0.2)$	$(0.2, 0.4)$	$(0, 1)$
$m_4 \times n_3$	$(0.8, 0.2)$	$(0.2, 0.7)$	$(0, 1)$
$m_4 \times n_4$	$(0, 1)$	$(0.2, 0.4)$	$(0, 1)$

4. CONCLUSION

The aim of this paper is to overcome the uncertainty trouble in more particular way by way of combing Intuitionistic fuzzy set with Hypersoft set. Some operations of Intuitionistic Fuzzy Hypersoft set such as subset, equal set, union, intersection, complement, AND, OR operations are presented. By defining these notions, the foundation of the intuitionistic fuzzy hypersoft set structure was built.

TABLE 8. Tabular form of FHSS $(H, \Omega_1) \vee (G, \Omega_2)$

$(H, \Omega_1) \vee (G, \Omega_2)$	u_1	u_2	u_3
$m_1 \times n_1$	$(0.6, 0.3)$	$(0.4, 0.2)$	$(0.6, 0.4)$
$m_1 \times n_2$	$(0.7, 0.2)$	$(0.6, 0.1)$	$(0, 1)$
$m_1 \times n_3$	$(0.9, 0.1)$	$(0.4, 0.2)$	$(0, 1)$
$m_1 \times n_4$	$(0.6, 0.3)$	$(0.5, 0.2)$	$(0.3, 0.7)$
$m_2 \times n_1$	$(0.3, 0.3)$	$(0.2, 0.2)$	$(0.6, 0.1)$
$m_2 \times n_2$	$(0.7, 0.2)$	$(0.6, 0.1)$	$(0.3, 0.1)$
$m_2 \times n_3$	$(0.9, 0.1)$	$(0.2, 0.6)$	$(0.3, 0.1)$
$m_2 \times n_4$	$(0, 1)$	$(0.5, 0.3)$	$(0.3, 0.1)$
$m_3 \times n_1$	$(0.7, 0.3)$	$(0.3, 0.2)$	$(0.6, 0.4)$
$m_3 \times n_2$	$(0.7, 0.2)$	$(0.6, 0.1)$	$(0.1, 0.6)$
$m_3 \times n_3$	$(0.9, 0.1)$	$(0.3, 0.5)$	$(0.1, 0.6)$
$m_3 \times n_4$	$(0.7, 0.3)$	$(0.5, 0.3)$	$(0.3, 0.6)$
$m_4 \times n_1$	$(0.8, 0.2)$	$(0.2, 0.2)$	$(0.6, 0.4)$
$m_4 \times n_2$	$(0.8, 0.2)$	$(0.6, 0.1)$	$(0, 1)$
$m_4 \times n_3$	$(0.9, 0.1)$	$(0.2, 0.4)$	$(0, 1)$
$m_4 \times n_4$	$(0.8, 0.2)$	$(0.5, 0.3)$	$(0.3, 0.7)$

The validity and implementation of the proposed operations and definitions are validated through presenting suitable instance. Matrices, similarity measure, single and multi-valued, interval valued, functions, distance measures, algorithms: score function, VIKOR, TOPSIS, AHP of Intutionistic Fuzzy Hypersoft sets will be future work. We hope that, this study will play a critical function in future decision-making research.

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Declaration of Competing Interest The authors declare that there is no conflict of interest regarding the publication of this article.

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