Lattice Ordered Neutrosophic Soft Set and Its Usage for a Large Scale Company in Adopting Best NGO to Deliver Funds During COVID-19

J. Vimala[1,*], **S. Rajareega** [2], **D. Preethi**[3], **M. Al-Tahan** [4], **P. Mahalakshmi**[5], **K. Jeevitha** [6] and **K. Ashma Banu** [7]

[1,2,3,5,6,7] Department of Mathematics, Alagappa University, Karaikudi, Tamilnadu, India.

[4] Department of Mathematics, Lebanese International University, Bekaa, Lebanon

[1,*] vimaljey@alagappauniversity.ac.in,

[2] reega948@gmail.com, [3] preethi06061996@gmail.com,

[4] altahan.madeleine@gmail.com, [5] mahalakshmi4812@gmail.com,

[6] kjeevitha991@gmail.com, [7] ashmabanu1998@gmail.com

Abstract

The notion called lattice ordered neutrosophic soft set is initiated with some properties. Using this theory, an application is developed to assist the decision makers in choosing an NGO to utilize the Covid-19 fund of a large scale company.

Keywords: Neutrosophic soft sets, Lattice ordered neutrosophic soft sets, Anti-lattice ordered neutrosophic soft sets, decision making.

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1 Introduction

Molodtsov [17] made known the definition of soft sets, a contemporary mathematical way to vagueness. The soft sets have been exercised to varied fields with great boom. In detail Maji et al. [13] studied the idea of soft sets and conferred an application. Smarandache [7] introduces the neutrosophic set comprising inconsistent, indeterminancy and indefinite data. The properties and applications have been developed increasingly [5, 6, 3]. Maji [14] fusioned the above notions by formulating neutrosophic soft set with many application.

Birkhoff [4] initiated the notion called lattice. Lattices are dominant mathematical tool that have been used nicely to solve many essential problems in computer science, mainly in the fields of combinatorial optimizations, cryptography and decision making. Muhammad Irfan Ali et al. pro-offered soft sets with order in parameters and proved some important theorems of lattice ordered soft sets [15]. Further lattice ordered structure to various concepts and their applications in myraid of science fields were studied in detail by Vimala et al. [12, 16, 21, 18, 19].

Keeping in view the importance of neutrosophic soft sets and its generalizations, our purpose is to launch lattice ordered neutrosophic soft sets. And also the basic union, basic intersection, restricted union, restricted intersection, extended union and extended intersection on neutrosophic soft sets and lattice ordered neutrosophic soft sets are analyzed. A group decision making problem has been handled by utilizing these concept.

2 Preliminaries

Throughout this manuscript, $\mathcal{U}$ denotes the universal set, $\mathcal{E}$ represents the set of parameters and $\bar{\mathcal{A}} \subset \mathcal{E}$.

Definition 2.1. [9] Let $F\bar{P}(\mathcal{U})$ denote the set of all fuzzy sets on $\mathcal{U}$ and $\bar{\mathcal{A}} \bar{\subseteq} \mathcal{E}$. Then a pair $(\mathcal{F}, \bar{\mathcal{A}})$ is known as fuzzy soft set over $\mathcal{U}$, where $\mathcal{F}$ is a mapping given by $\mathcal{F} : \bar{\mathcal{A}} \to F\bar{P}(\mathcal{U})$.

Definition 2.2. [2] Let $(\mathcal{F}, \bar{\mathcal{A}})$ be a fuzzy soft set over $\mathcal{U}$. Then it is known to be lattice(antilattice) ordered fuzzy soft set over $\mathcal{U}$, where $\mathcal{F} : \bar{\mathcal{A}} \to F\bar{P}(\mathcal{U})$, if $\bar{\varepsilon}_1 \leq \bar{\varepsilon}_2$, then $\mathcal{F}(\bar{\varepsilon}_1) \bar{\subseteq} \mathcal{F}(\bar{\varepsilon}_2) (\mathcal{F}(\bar{\varepsilon}_2) \bar{\subseteq} \mathcal{F}(\bar{\varepsilon}_1))$, for every $\bar{\varepsilon}_1, \bar{\varepsilon}_2 \in \bar{\mathcal{A}}$.

Definition 2.3. [14] A neutrosophic set (NS) $\bar{\mathcal{A}}$ in $\mathcal{U}$ is distinguised by a truth-membership $\mathcal{T}_{\mathcal{A}}$ function, an indeterminacy membership $\mathcal{I}_{\mathcal{A}}$ function and a falsity-membership $\mathcal{F}_{\mathcal{A}}$ function, where $\mathcal{T}_{\mathcal{A}}$, $\mathcal{I}_{\mathcal{A}}$ and $\mathcal{F}_{\mathcal{A}}$ are in [0, 1]. It can be represented as
$\bar{\mathcal{A}} = \{(x, (\mathcal{T}_{\mathcal{A}}(x), \mathcal{I}_{\mathcal{A}}(x), \mathcal{F}_{\mathcal{A}}(x))) : x \in \mathcal{U} \; and \; \mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}} \in \,]^{-}0, 1^{+}[\; \}$
There is no restriction on the sum of $\mathcal{T}_{\mathcal{A}}(\bar{u})$, $\mathcal{I}_{\mathcal{A}}(\bar{u})$ and $\mathcal{F}_{\mathcal{A}}(\bar{u})$ and so
$0^{-} \leq \mathcal{T}_{\mathcal{A}}(\bar{u}) + \mathcal{I}_{\mathcal{A}}(\bar{u}) + \mathcal{F}_{\mathcal{A}}(\bar{u}) \leq 3^{+}$.

Definition 2.4. [14] Let $NS(\mathcal{U})$ denote the set of all neutrosophic sets of $\mathcal{U}$. Then a pair $(\mathcal{F}, \bar{\mathcal{A}})$ is termed to be a neutrosophic soft set (NSS) over $\mathcal{U}$ where $\mathcal{F} : \bar{\mathcal{A}} \to NS(\mathcal{U})$.
i.e. $(\mathcal{F}, \bar{\mathcal{A}}) = \{\langle u, \mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u})\rangle : \bar{\varepsilon} \in \bar{\mathcal{A}}, \bar{u} \in \mathcal{U} \; and \; \mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}) \in [0, 1]\}$
There is no restriction on the sum $\mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u})$, $\mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u})$ and $\mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u})$ and so
$0^{-} \leq \mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}) + \mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}) + \mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}) \leq 3^{+}$.

Definition 2.5. [14] Let $(\mathcal{F}, \bar{\mathcal{A}}), (\mathcal{G}, \bar{\mathcal{B}}) \in NSS(\mathcal{U})$. Then $(\mathcal{F}, \bar{\mathcal{A}})$ is a neutrosophic soft subset of $(\mathcal{G}, \bar{\mathcal{B}})$ if
(i) $\bar{\mathcal{A}} \bar{\subseteq} \bar{\mathcal{B}}$ and
(ii) $\mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}) \leq \mathcal{T}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u})$, $\mathcal{I}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u}) \leq \mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u})$ and $\mathcal{F}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u}) \leq \mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u})$, for every $\bar{\varepsilon} \in \bar{\mathcal{A}}$ and $\bar{u} \in \mathcal{U}$.
We denote it by $(\mathcal{F}, \bar{\mathcal{A}}) \bar{\subseteq} (\mathcal{G}, \bar{\mathcal{B}})$.

3 Lattice Ordered Neutrosophic Soft Set

Definition 3.1. Let $(\mathcal{F}, \bar{\mathcal{A}})$ be a neutrosophic soft set over $\mathcal{U}$. Then it is known as lattice ordered neutrosophic soft set over $\mathcal{U}$ $(\mathcal{LONSS}(\mathcal{U}))$, where $\mathcal{F}$ is a mapping defined by $\mathcal{F} : \bar{\mathcal{A}} \to NS(\mathcal{U})$, if $\bar{\varepsilon}_i, \bar{\varepsilon}_j \in \bar{\mathcal{A}}$ such that $\bar{\varepsilon}_i \leq \bar{\varepsilon}_j$, then $\mathcal{F}(\bar{\varepsilon}_i) \bar{\subseteq} \mathcal{F}(\bar{\varepsilon}_j)$.
(i.e.), $\mathcal{T}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u})$, $\mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u}) \leq \mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u})$ and $\mathcal{F}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u}) \leq \mathcal{F}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u})$, for all $\bar{u} \in \mathcal{U}$.

Example 1. Let $\mathcal{U} = \{h_1, h_2, h_3, h_4, h_5\}$ be the set of houses. Consider
$\mathcal{E} = \{wooden, costly, in\ bad\ repair, beautiful, very\ costly, green\ surroundings,$
$in\ good\ repair, moderate, cheap, expensive\}$.
Let $\bar{\mathcal{A}} = \{\bar{\varepsilon}_1(moderate), \bar{\varepsilon}_2(beautiful), \bar{\varepsilon}_3(costly)\ and\ \bar{\varepsilon}_4(very\ costly)\}\bar{\subseteq}\mathcal{E}$.
The order among the elements of $\bar{\mathcal{A}}$ is $\bar{\varepsilon}_1 \leq \bar{\varepsilon}_2 \leq \bar{\varepsilon}_4$ and $\bar{\varepsilon}_1 \leq \bar{\varepsilon}_3 \leq \bar{\varepsilon}_4$.

Table 1: Table shows the example of $\mathcal{LONSS}(\mathcal{U})$

$(\mathcal{F}, \bar{\mathcal{A}})$	h_1	h_2	h_3	h_4	h_5
$\bar{\varepsilon}_1$	(0.4, 0.7, 0.9)	(0.5, 0.9, 0.8)	(0.5, 0.6, 0.7)	(0.3, 0.8, 0.7)	(0.7, 0.5, 0.9)
$\bar{\varepsilon}_2$	(0.5, 0.4, 0.6)	(0.7, 0.5, 0.4)	(0.6, 0.2, 0.5)	(0.5, 0.2, 0.6)	(0.8, 0.3, 0.5)
$\bar{\varepsilon}_3$	(0.7, 0.6, 0.8)	(0.6, 0.4, 0.3)	(0.8, 0.2, 0.6)	(0.6, 0.3, 0.4)	(0.8, 0.3, 0.7)
$\bar{\varepsilon}_4$	(0.8, 0.3, 0.4)	(0.8, 0.3, 0.2)	(0.9, 0.1, 0.4)	(0.7, 0.1, 0.3)	(0.9, 0.2, 0.4)

Clearly $\mathcal{F}(\bar{\varepsilon}_1)\bar{\subseteq}\mathcal{F}(\bar{\varepsilon}_2)\bar{\subseteq}\mathcal{F}(\bar{\varepsilon}_4)$ and $\mathcal{F}(\bar{\varepsilon}_1)\bar{\subseteq}\mathcal{F}(\bar{\varepsilon}_3)\bar{\subseteq}\mathcal{F}(\bar{\varepsilon}_4)$.

Definition 3.2. Let $(\mathcal{F}, \bar{\mathcal{A}}), (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$. Then their restricted union is denoted and is defined by $(\mathcal{F}, \bar{\mathcal{A}})\bar{\cup}_{RES}(\mathcal{G}, \bar{\mathcal{B}}) = (\mathcal{H}, \bar{\mathcal{C}})$, where $\bar{\mathcal{C}} = \bar{\mathcal{A}}\bar{\cap}\bar{\mathcal{B}}$ and $\forall\ \bar{\varepsilon} \in \bar{\mathcal{C}}$, $\bar{u} \in \mathcal{U}$, $\mathcal{H}(\bar{\varepsilon}) = \mathcal{F}(\bar{\varepsilon})\bar{\cup}\mathcal{G}(\bar{\varepsilon})$

$$\mathcal{T}_{\mathcal{H}(\bar{\varepsilon})}(\bar{u}) = Max\{\mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{T}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u})\}$$
$$\mathcal{I}_{\mathcal{H}(\bar{\varepsilon})}(\bar{u}) = Min\{\mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{I}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u})\}$$
$$\mathcal{F}_{\mathcal{H}(\bar{\varepsilon})}(\bar{u}) = Min\{\mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{F}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u})\}$$

Proposition 3.3. *Let* $(\mathcal{F}, \bar{\mathcal{A}}), (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$. *Then* $(\mathcal{F}, \bar{\mathcal{A}})\bar{\cup}_{RES}(\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$.

Proof. Let $(\mathcal{F}, \bar{\mathcal{A}}), (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$. Then by Definition 3.2

$\mathcal{F}(\bar{\varepsilon})\bar{\cup}\mathcal{G}(\bar{\varepsilon}) = \mathcal{H}(\bar{\varepsilon})$, where $\bar{\varepsilon} \in \bar{\mathcal{C}} = \bar{\mathcal{A}}\bar{\cap}\bar{\mathcal{B}}$.

If $\bar{\mathcal{A}}\bar{\cap}\bar{\mathcal{B}} = \phi$, then the result is trivial.

Now for $\bar{\mathcal{A}}\bar{\cap}\bar{\mathcal{B}} \neq \phi$, since $\bar{\mathcal{A}}, \bar{\mathcal{B}}\bar{\subseteq}\mathcal{E}$, $\therefore$ for any $\bar{\varepsilon}_i \leq_{\bar{\mathcal{A}}} \bar{\varepsilon}_j$

$\mathcal{F}(\bar{\varepsilon}_i)\bar{\subseteq}\mathcal{F}(\bar{\varepsilon}_j)$, $\forall\ \bar{\varepsilon}_i, \bar{\varepsilon}_j \in \bar{\mathcal{A}}$

and for any $\eta_i \leq_B \eta_j$, $\mathcal{G}(\eta_i)\bar{\subseteq}\mathcal{G}(\eta_j)$, $\forall\ \eta_i, \eta_j \in \bar{\mathcal{B}}$

Now for any $\gamma_i, \gamma_j \in C$ and $\gamma_i \leq \gamma_j$

$\Rightarrow \gamma_i, \gamma_j \in \bar{\mathcal{A}}\bar{\cap}\bar{\mathcal{B}}$

$\Rightarrow \gamma_i, \gamma_j \in \bar{\mathcal{A}}\ and\ \gamma_i, \gamma_j \in \bar{\mathcal{B}}$

$\Rightarrow \mathcal{F}(\gamma_i)\bar{\subseteq}\mathcal{F}(\gamma_j)\ and\ \mathcal{G}(\gamma_i)\bar{\subseteq}\mathcal{G}(\gamma_j)\ whenever\ \gamma_i \leq_{\bar{\mathcal{A}}} \gamma_j, \gamma_i \leq_{\bar{\mathcal{B}}} \gamma_j$

$\Rightarrow$

$$\mathcal{T}_{\mathcal{F}(\gamma_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{F}(\gamma_j)}(\bar{u}), \mathcal{T}_{\mathcal{G}(\gamma_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{G}(\gamma_j)}(\bar{u})$$
$$\mathcal{I}_{\mathcal{F}(\gamma_j)}(\bar{u}) \leq \mathcal{I}_{\mathcal{F}(\gamma_i)}(\bar{u}), \mathcal{I}_{\mathcal{G}(\gamma_j)}(\bar{u}) \leq \mathcal{I}_{\mathcal{G}(\gamma_i)}(\bar{u})$$
$$\mathcal{F}_{\mathcal{F}(\gamma_j)}(\bar{u}) \leq \mathcal{F}_{\mathcal{F}(\gamma_i)}(\bar{u}), \mathcal{F}_{\mathcal{G}(\gamma_j)}(\bar{u}) \leq \mathcal{F}_{\mathcal{G}(\gamma_i)}(\bar{u})$$

$\Rightarrow$

$$Max\{\mathcal{T}_{\mathcal{F}(\gamma_i)}(\bar{u}), \mathcal{T}_{\mathcal{G}(\gamma_i)}(\bar{u})\} \leq Max\{\mathcal{T}_{\mathcal{F}(\gamma_j)}(\bar{u}), \mathcal{T}_{\mathcal{G}(\gamma_j)}(\bar{u})\}$$
$$Min\{\mathcal{I}_{\mathcal{F}(\gamma_j)}(\bar{u}), \mathcal{I}_{\mathcal{G}(\gamma_j)}(\bar{u})\} \leq Min\{\mathcal{I}_{\mathcal{F}(\gamma_i)}(\bar{u}), \mathcal{I}_{\mathcal{G}(\gamma_i)}(\bar{u})\}$$
$$Min\{\mathcal{F}_{\mathcal{F}(\gamma_j)}(\bar{u}), \mathcal{F}_{\mathcal{G}(\gamma_j)}(\bar{u})\} \leq Min\{\mathcal{F}_{\mathcal{F}(\gamma_i)}(\bar{u}), \mathcal{F}_{\mathcal{G}(\gamma_i)}(\bar{u})\}$$

$\Rightarrow$

$$\mathcal{T}_{\mathcal{F}(\gamma_i)\bar{\cup}\mathcal{G}(\gamma_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{F}(\gamma_j)\bar{\cup}\mathcal{G}(\gamma_j)}(\bar{u}),$$
$$\mathcal{I}_{\mathcal{F}(\gamma_j)\bar{\cup}\mathcal{G}(\gamma_j)}(\bar{u}) \leq \mathcal{I}_{\mathcal{F}(\gamma_i)\bar{\cup}\mathcal{G}(\gamma_i)}(\bar{u}),$$

$$\mathcal{F}_{\mathcal{F}(\gamma_j)\bar{\cup}\mathcal{G}(\gamma_j)}(\bar{u}) \leq \mathcal{F}_{\mathcal{F}(\gamma_i)\bar{\cup}\mathcal{G}(\gamma_i)}(\bar{u})$$

$\Rightarrow$

$$\mathcal{T}_{(\mathcal{F}\bar{\cup}\mathcal{G})(\gamma_i)}(\bar{u}) \leq \mathcal{T}_{(\mathcal{F}\bar{\cup}\mathcal{G})(\gamma_j)}(\bar{u}),$$
$$\mathcal{I}_{(\mathcal{F}\bar{\cup}\mathcal{G})(\gamma_j)}(\bar{u}) \leq \mathcal{I}_{(\mathcal{F}\bar{\cup}\mathcal{G})(\gamma_i)}(\bar{u}),$$
$$\mathcal{F}_{(\mathcal{F}\bar{\cup}\mathcal{G})(\gamma_j)}(\bar{u}) \leq \mathcal{F}_{(\mathcal{F}\bar{\cup}\mathcal{G})(\gamma_i)}(\bar{u})$$

$\Rightarrow$

$$\mathcal{T}_{\mathcal{H}(\gamma_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{H}(\gamma_j)}(\bar{u}),$$
$$\mathcal{I}_{\mathcal{H}(\gamma_j)}(\bar{u}) \leq \mathcal{I}_{\mathcal{H}(\gamma_i)}(\bar{u}),$$
$$\mathcal{F}_{\mathcal{H}(\gamma_j)}(\bar{u}) \leq \mathcal{F}_{\mathcal{H}(\gamma_i)}(\bar{u})$$

$\Rightarrow$

$$\mathcal{H}(\gamma_i)\bar{\subseteq}\mathcal{H}(\gamma_j) \text{ for } \gamma_i \leq \gamma_j$$

$\Rightarrow$

$$(\mathcal{F}, \bar{\mathcal{A}})\bar{\cup}_{RES}(\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U}).$$ □

EXAMPLE 2. Suppose $\mathcal{U} = \{u_1, u_2, u_3\}$ is a set of cars and $\mathcal{E} = \{\bar{\varepsilon}_1(Color), \bar{\varepsilon}_2(Price), \bar{\varepsilon}_3(Tax), \bar{\varepsilon}_4(Speed)\}$ is the parameters set and $\bar{\mathcal{A}}, \bar{\mathcal{B}}\bar{\subseteq}\mathcal{E}$, $\bar{\mathcal{A}} = \{\bar{\varepsilon}_1, \bar{\varepsilon}_2, \bar{\varepsilon}_3\}$, $\bar{\mathcal{B}} = \{\bar{\varepsilon}_2, \bar{\varepsilon}_3, \bar{\varepsilon}_4\}$ and $\bar{\mathcal{A}}\bar{\cap}\bar{\mathcal{B}} = \{\bar{\varepsilon}_2, \bar{\varepsilon}_3\}$.
The order among the elements of $\bar{\mathcal{A}}$ is $\bar{\varepsilon}_1 \leq_{\bar{\mathcal{A}}} \bar{\varepsilon}_2 \leq_{\bar{\mathcal{A}}} \bar{\varepsilon}_3$.

$(\mathcal{F}, \bar{\mathcal{A}})$	u_1	u_2	u_3
$\bar{\varepsilon}_1$	(0.2, 0.3, 0.6)	(0.4, 0.3, 0.6)	(0.5, 0.4, 0.7)
$\bar{\varepsilon}_2$	(0.4, 0.2, 0.2)	(0.8, 0.2, 0.4)	(0.6, 0.2, 0.5)
$\bar{\varepsilon}_3$	(0.6, 0.1, 0.1)	(0.9, 0.1, 0.2)	(0.7, 0.1, 0.3)

Clearly $\mathcal{F}(\bar{\varepsilon}_1)\bar{\subseteq}\mathcal{F}(\bar{\varepsilon}_2)\bar{\subseteq}\mathcal{F}(\bar{\varepsilon}_3)$.
The order among the elements of $\bar{\mathcal{B}}$ is $\bar{\varepsilon}_2 \leq_{\bar{\mathcal{B}}} \bar{\varepsilon}_3 \leq_{\bar{\mathcal{B}}} \bar{\varepsilon}_4$.

$(\mathcal{G}, \bar{\mathcal{B}})$	u_1	u_2	u_3
$\bar{\varepsilon}_2$	(0.3, 0.6, 0.7)	(0.4, 0.6, 0.9)	(0.5, 0.8, 0.3)
$\bar{\varepsilon}_3$	(0.4, 0.2, 0.4)	(0.5, 0.3, 0.7)	(0.6, 0.6, 0.2)
$\bar{\varepsilon}_4$	(0.8, 0.1, 0.3)	(0.6, 0.2, 0.5)	(0.7, 0.4, 0.1)

Clearly $\mathcal{G}(\bar{\varepsilon}_2)\bar{\subseteq}\mathcal{G}(\bar{\varepsilon}_3)\bar{\subseteq}\mathcal{G}(\bar{\varepsilon}_4)$.
Then $(\mathcal{F}, \bar{\mathcal{A}})\bar{\cup}_{RES}(\mathcal{G}, \bar{\mathcal{B}}) = (\mathcal{H}, \bar{\mathcal{C}})$ is the restricted union of two $\mathcal{LONSS}$ over $\mathcal{U}$.

$(\mathcal{H}, \bar{\mathcal{C}})$	u_1	u_2	u_3
$\bar{\varepsilon}_2$	(0.4, 0.2, 0.2)	(0.8, 0.2, 0.4)	(0.6, 0.2, 0.3)
$\bar{\varepsilon}_3$	(0.6, 0.1, 0.1)	(0.9, 0.1, 0.2)	(0.7, 0.1, 0.2)

$\Rightarrow \mathcal{H}(\bar{\varepsilon}_2)\bar{\subseteq}\mathcal{H}(\bar{\varepsilon}_3)$, for $\bar{\varepsilon}_2 \leq \bar{\varepsilon}_3$.
$\Rightarrow (\mathcal{F}, \bar{\mathcal{A}})\bar{\cup}_{RES}(\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$.

Definition 3.4. Let $(\mathcal{F}, \bar{\mathcal{A}}), (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$. Then their restricted intersection is denoted and is defined by $(\mathcal{F}, \bar{\mathcal{A}})\bar{\cap}_{RES}(\mathcal{G}, \bar{\mathcal{B}}) = (\mathcal{H}, \bar{\mathcal{C}})$, where $\bar{\mathcal{C}} = \bar{\mathcal{A}}\bar{\cap}\bar{\mathcal{B}}$ and $\forall\ \bar{\varepsilon} \in \bar{\mathcal{C}}$, $\bar{u} \in \mathcal{U}$, $\mathcal{H}(\bar{\varepsilon}) = \mathcal{F}(\bar{\varepsilon})\bar{\cap}\mathcal{G}(\bar{\varepsilon})$

$\mathcal{T}_{\mathcal{H}(\bar{\varepsilon})}(\bar{u}) = Min\{\mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{T}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u})\}$
$\mathcal{I}_{\mathcal{H}(\bar{\varepsilon})}(\bar{u}) = Max\{\mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{I}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u})\}$
$\mathcal{F}_{\mathcal{H}(\bar{\varepsilon})}(\bar{u}) = Max\{\mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{F}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u})\}$

Proposition 3.5. *Let $(\mathcal{F}, \bar{\mathcal{A}}), (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$. Then $(\mathcal{F}, \bar{\mathcal{A}})\bar{\cap}_{RES}(\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$.*

Example 3. Suppose $\mathcal{U} = \{u_1, u_2, u_3\}$ is a set of shoes and $\mathcal{E} = \{\bar{\varepsilon}_1(Price), \bar{\varepsilon}_2(Color), \bar{\varepsilon}_3(Quality), \bar{\varepsilon}_4(Comfort)\}$ is the parameters set and $\bar{\mathcal{A}}, \bar{\mathcal{B}}\bar{\subseteq}\mathcal{E}$, $\bar{\mathcal{A}} = \{\bar{\varepsilon}_1, \bar{\varepsilon}_2, \bar{\varepsilon}_3\}$, $\bar{\mathcal{B}} = \{\bar{\varepsilon}_2, \bar{\varepsilon}_3, \bar{\varepsilon}_4\}$ and $\bar{\mathcal{A}}\bar{\cap}\bar{\mathcal{B}} = \{\bar{\varepsilon}_2, \bar{\varepsilon}_3\}$.
The order among the elements of $\bar{\mathcal{A}}$ is $\bar{\varepsilon}_1 \leq_{\bar{\mathcal{A}}} \bar{\varepsilon}_2 \leq_{\bar{\mathcal{A}}} \bar{\varepsilon}_3$.

$(\mathcal{F}, \bar{\mathcal{A}})$	u_1	u_2	u_3
$\bar{\varepsilon}_1$	(0.2, 0.3, 0.7)	(0.4, 0.3, 0.6)	(0.5, 0.4, 0.8)
$\bar{\varepsilon}_2$	(0.4, 0.2, 0.4)	(0.8, 0.2, 0.5)	(0.6, 0.2, 0.5)
$\bar{\varepsilon}_3$	(0.6, 0.1, 0.2)	(0.9, 0.1, 0.4)	(0.7, 0.1, 0.2)

Clearly $\mathcal{F}(\bar{\varepsilon}_1)\bar{\subseteq}\mathcal{F}(\bar{\varepsilon}_2)\bar{\subseteq}\mathcal{F}(\bar{\varepsilon}_3)$.
The order among the elements of $\bar{\mathcal{B}}$ is $\bar{\varepsilon}_2 \leq_{\bar{\mathcal{B}}} \bar{\varepsilon}_3 \leq_{\bar{\mathcal{B}}} \bar{\varepsilon}_4$.

$(\mathcal{G}, \bar{\mathcal{B}})$	u_1	u_2	u_3
$\bar{\varepsilon}_2$	(0.3, 0.6, 0.4)	(0.4, 0.6, 0.7)	(0.5, 0.6, 0.7)
$\bar{\varepsilon}_3$	(0.4, 0.2, 0.3)	(0.5, 0.3, 0.6)	(0.6, 0.5, 0.4)
$\bar{\varepsilon}_4$	(0.8, 0.1, 0.2)	(0.6, 0.2, 0.5)	(0.7, 0.4, 0.2)

Clearly $\mathcal{G}(\bar{\varepsilon}_2)\bar{\subseteq}\mathcal{G}(\bar{\varepsilon}_3)\bar{\subseteq}\mathcal{G}(\bar{\varepsilon}_4)$.
Then $(\mathcal{F}, \bar{\mathcal{A}})\bar{\cap}_{RES}(\mathcal{G}, \bar{\mathcal{B}}) = (\mathcal{H}, \bar{\mathcal{C}})$ is the restricted intersection of two $\mathcal{LONSS}$ over $\mathcal{U}$.

$(\mathcal{H}, \bar{\mathcal{C}})$	u_1	u_2	u_3
$\bar{\varepsilon}_2$	(0.3, 0.6, 0.4)	(0.4, 0.6, 0.7)	(0.5, 0.6, 0.7)
$\bar{\varepsilon}_3$	(0.4, 0.2, 0.3)	(0.5, 0.3, 0.6)	(0.6, 0.5, 0.4)

$\Rightarrow \mathcal{H}(\bar{\varepsilon}_2)\bar{\subseteq}\mathcal{H}(\bar{\varepsilon}_3)$, for $\bar{\varepsilon}_2 \leq \bar{\varepsilon}_3$.
$\Rightarrow (\mathcal{F}, \bar{\mathcal{A}})\bar{\cap}_{RES}(\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$.

Definition 3.6. Let $(\mathcal{F}, \bar{\mathcal{A}}), (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$. Then their extended union is denoted and is defined by $(\mathcal{F}, \bar{\mathcal{A}})\bar{\cup}_{EXT}(\mathcal{G}, \bar{\mathcal{B}}) = (\mathcal{H}, \bar{\mathcal{C}})$ where $\bar{\mathcal{C}} = \bar{\mathcal{A}}\bar{\cup}\bar{\mathcal{B}}$

$$(\mathcal{H}, \bar{\mathcal{C}}) = \begin{cases} \langle \mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u})\rangle \; if \; e \in \bar{\mathcal{A}} - \bar{\mathcal{B}} \\ \langle \mathcal{T}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u}), \mathcal{I}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u}), \mathcal{F}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u})\rangle \; if \; \bar{\varepsilon} \in \bar{\mathcal{B}} - \bar{\mathcal{A}} \\ \langle Max\{\mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{T}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u})\}, Min\{\mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{I}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u})\}, Min\{\mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{F}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u})\}\rangle \; if \; \bar{\varepsilon} \in \bar{\mathcal{A}}\bar{\cap}\bar{\mathcal{B}} \end{cases}$$

Proposition 3.7. *Let $(\mathcal{F}, \bar{\mathcal{A}}), (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$. Then $(\mathcal{F}, \bar{\mathcal{A}})\bar{\cup}_{EXT}(\mathcal{G}, \bar{\mathcal{B}}) = (\mathcal{H}, \bar{\mathcal{C}}) \in \mathcal{LONSS}(\mathcal{U})$, if one of them is a lattice ordered neutrosophic soft subset of other.*

Proof. Let $(\mathcal{F}, \bar{\mathcal{A}}), (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$. Then by Definition 3.6
$\mathcal{F}(\bar{\varepsilon})\bar{\cup}\mathcal{G}(\bar{\varepsilon}) = \mathcal{H}(\bar{\varepsilon})$ where $\bar{\varepsilon} \in \bar{\mathcal{C}} = \bar{\mathcal{A}}\bar{\cup}\bar{\mathcal{B}}$
Suppose $(\mathcal{F}, \bar{\mathcal{A}})\bar{\subseteq}(\mathcal{G}, \bar{\mathcal{B}})$. Then $\bar{\mathcal{A}}\bar{\subseteq}\bar{\mathcal{B}}$ and

$\mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}) \leq \mathcal{T}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u})$, $\mathcal{I}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u}) \leq \mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u})$ and $\mathcal{F}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u}) \leq \mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u})$, for every $\bar{\varepsilon} \in \bar{\mathcal{A}}$ and $\bar{u} \in \mathcal{U}$
Since $\bar{\mathcal{A}}, \bar{\mathcal{B}} \bar{\subseteq} \mathcal{E}$, $\therefore$ for any $\bar{\varepsilon}_i \leq_{\bar{\mathcal{A}}} \bar{\varepsilon}_j$

$\mathcal{F}(\bar{\varepsilon}_i) \bar{\subseteq} \mathcal{F}(\bar{\varepsilon}_j)$, for every $\bar{\varepsilon}_i, \bar{\varepsilon}_j \in \bar{\mathcal{A}}$
for any $\eta_i \leq_{\bar{\mathcal{B}}} \eta_j$, $\mathcal{G}(\eta_i) \bar{\subseteq} \mathcal{G}(\eta_j)$, for every $\eta_i, \eta_j \in \bar{\mathcal{B}}$
$\therefore$ for any $\gamma_i, \gamma_j \in \bar{\mathcal{C}}$ and $\gamma_i \leq \gamma_j$
$\Rightarrow \gamma_i, \gamma_j \in \bar{\mathcal{A}} \bar{\cup} \bar{\mathcal{B}}$
$\Rightarrow \gamma_i, \gamma_j \in \bar{\mathcal{A}} \bar{\cap} \bar{\mathcal{B}}$ or $\gamma_i, \gamma_j \in \bar{\mathcal{B}}$ and $\gamma_i, \gamma_j \notin \bar{\mathcal{A}}$ because $\bar{\mathcal{A}} \bar{\subseteq} \bar{\mathcal{B}}$
Now take $\gamma_i, \gamma_j \in \bar{\mathcal{A}} \bar{\cap} \bar{\mathcal{B}}$
$\Rightarrow \gamma_i, \gamma_j \in \bar{\mathcal{A}}$ and $\gamma_i, \gamma_j \in \bar{\mathcal{B}}$
$\Rightarrow \mathcal{F}(\gamma_i) \bar{\subseteq} \mathcal{F}(\gamma_j)$ and $\mathcal{G}(\gamma_i) \bar{\subseteq} \mathcal{G}(\gamma_j)$ whenever $\gamma_i \leq_{\bar{\mathcal{A}}} \gamma_j$ and $\gamma_i \leq_{\bar{\mathcal{B}}} \gamma_j$
$\Rightarrow$

$$\mathcal{T}_{\mathcal{F}(\gamma_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{F}(\gamma_j)}(\bar{u}), \mathcal{T}_{\mathcal{G}(\gamma_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{G}(\gamma_j)}(\bar{u})$$
$$\mathcal{I}_{\mathcal{F}(\gamma_j)}(\bar{u}) \leq \mathcal{I}_{\mathcal{F}(\gamma_i)}(\bar{u}), \mathcal{I}_{\mathcal{G}(\gamma_j)}(\bar{u}) \leq \mathcal{I}_{\mathcal{G}(\gamma_i)}(\bar{u})$$
$$\mathcal{F}_{\mathcal{F}(\gamma_j)}(\bar{u}) \leq \mathcal{F}_{\mathcal{F}(\gamma_i)}(\bar{u}), \mathcal{F}_{\mathcal{G}(\gamma_j)}(\bar{u}) \leq \mathcal{F}_{\mathcal{G}(\gamma_i)}(\bar{u})$$

$\Rightarrow$

$$Max\{\mathcal{T}_{\mathcal{F}(\gamma_i)}(\bar{u}), \mathcal{T}_{\mathcal{G}(\gamma_i)}(\bar{u})\} \leq Max\{\mathcal{T}_{\mathcal{F}(\gamma_j)}(\bar{u}), \mathcal{T}_{\mathcal{G}(\gamma_j)}(\bar{u})\}$$
$$Min\{\mathcal{I}_{\mathcal{F}(\gamma_j)}(\bar{u}), \mathcal{I}_{\mathcal{G}(\gamma_j)}(\bar{u})\} \leq Min\{\mathcal{I}_{\mathcal{F}(\gamma_i)}(\bar{u}), \mathcal{I}_{\mathcal{G}(\gamma_i)}(\bar{u})\}$$
$$Min\{\mathcal{F}_{\mathcal{F}(\gamma_j)}(\bar{u}), \mathcal{F}_{\mathcal{G}(\gamma_j)}(\bar{u})\} \leq Min\{\mathcal{F}_{\mathcal{F}(\gamma_i)}(\bar{u}), \mathcal{F}_{\mathcal{G}(\gamma_i)}(\bar{u})\}$$

$\Rightarrow$

$$\mathcal{T}_{\mathcal{F}(\gamma_i) \bar{\cup} \mathcal{G}(\gamma_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{F}(\gamma_j) \bar{\cup} \mathcal{G}(\gamma_j)}(\bar{u}),$$
$$\mathcal{I}_{\mathcal{F}(\gamma_j) \bar{\cup} \mathcal{G}(\gamma_j)}(\bar{u}) \leq \mathcal{I}_{\mathcal{F}(\gamma_i) \bar{\cup} \mathcal{G}(\gamma_i)}(\bar{u}),$$
$$\mathcal{F}_{\mathcal{F}(\gamma_j) \bar{\cup} \mathcal{G}(\gamma_j)}(\bar{u}) \leq \mathcal{F}_{\mathcal{F}(\gamma_i) \bar{\cup} \mathcal{G}(\gamma_i)}(\bar{u})$$

$\Rightarrow$

$$\mathcal{T}_{(\mathcal{F} \bar{\cup} \mathcal{G})(\gamma_i)}(\bar{u}) \leq \mathcal{T}_{(\mathcal{F} \bar{\cup} \mathcal{G})(\gamma_j)}(\bar{u}),$$
$$\mathcal{I}_{(\mathcal{F} \bar{\cup} \mathcal{G})(\gamma_j)}(\bar{u}) \leq \mathcal{I}_{(\mathcal{F} \bar{\cup} \mathcal{G})(\gamma_i)}(\bar{u}),$$
$$\mathcal{F}_{(\mathcal{F} \bar{\cup} \mathcal{G})(\gamma_j)}(\bar{u}) \leq \mathcal{F}_{(\mathcal{F} \bar{\cup} \mathcal{G})(\gamma_i)}(\bar{u})$$

$\Rightarrow$

$$\mathcal{T}_{\mathcal{H}(\gamma_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{H}(\gamma_j)}(\bar{u}),$$
$$\mathcal{I}_{\mathcal{H}(\gamma_j)}(\bar{u}) \leq \mathcal{I}_{\mathcal{H}(\gamma_i)}(\bar{u}),$$
$$\mathcal{F}_{\mathcal{H}(\gamma_j)}(\bar{u}) \leq \mathcal{F}_{\mathcal{H}(\gamma_i)}(\bar{u})$$

$\Rightarrow$

$\mathcal{H}(\gamma_i) \bar{\subseteq} \mathcal{H}(\gamma_j)$ for $\gamma_i \leq \gamma_j$
Thus $(\mathcal{F}, \bar{\mathcal{A}}) \bar{\cup}_{EXT} (\mathcal{G}, \bar{\mathcal{B}}) = (\mathcal{H}, \bar{\mathcal{C}}) \in \mathcal{LONSS}(\mathcal{U})$ if $\gamma_i, \gamma_j \in \bar{\mathcal{A}} \bar{\cap} \bar{\mathcal{B}}$
Now suppose for any $\gamma_i, \gamma_j \in B$ and $\gamma_i, \gamma_j \notin \bar{\mathcal{A}}$ and $\gamma_i \leq_{\bar{\mathcal{B}}} \gamma_j$
$\Rightarrow$ $\mathcal{G}(\gamma_i) \bar{\subseteq} \mathcal{G}(\gamma_j)$ whenever $\gamma_i \leq_{\bar{\mathcal{B}}} \gamma_j$
implies this is also a $\mathcal{LONSS}$.
Hence $(\mathcal{F}, \bar{\mathcal{A}}) \bar{\cup}_{EXT} (\mathcal{G}, \bar{\mathcal{B}}) = (\mathcal{H}, \bar{\mathcal{C}}) \in \mathcal{LONSS}(\mathcal{U})$ for both cases.
$\Rightarrow (\mathcal{F}, \bar{\mathcal{A}}) \bar{\cup}_{EXT} (\mathcal{G}, \bar{\mathcal{B}}) = (\mathcal{H}, \bar{\mathcal{C}}) \in \mathcal{LONSS}(\mathcal{U})$, if one of them is a lattice ordered neutrosophic soft subset of other. $\square$

Example 4. Let $\mathcal{U} = \{u_1, u_2, u_3\}$ be the set of men and
$\mathcal{E} = \{\bar{\varepsilon}_1(educated), \bar{\varepsilon}_2(businessman), \bar{\varepsilon}_3(smart), \bar{\varepsilon}_4(government\ employee), \bar{\varepsilon}_5(bank\ balance)\}$
is the parameters set $\bar{\mathcal{A}}, \bar{\mathcal{B}} \bar{\subseteq} E$ and $\bar{\mathcal{A}} \bar{\subseteq} \bar{\mathcal{B}}$, $\bar{\mathcal{A}} = \{\bar{\varepsilon}_2, \bar{\varepsilon}_3, \bar{\varepsilon}_4\}$, $\bar{\mathcal{B}} = \{\bar{\varepsilon}_1, \bar{\varepsilon}_2, \bar{\varepsilon}_3, \bar{\varepsilon}_4, \bar{\varepsilon}_5\}$.

The order among the elements of $\bar{\mathcal{A}}$ is $\bar{\varepsilon}_2 \leq_{\bar{\mathcal{A}}} \bar{\varepsilon}_3 \leq_{\bar{\mathcal{A}}} \bar{\varepsilon}_4$.

$(\mathcal{F}, \bar{\mathcal{A}})$	u_1	u_2	u_3
$\bar{\varepsilon}_2$	(0.1, 0.9, 0.7)	(0.1, 0.7, 0.6)	(0.3, 0.7, 0.9)
$\bar{\varepsilon}_3$	(0.2, 0.7, 0.6)	(0.3, 0.6, 0.5)	(0.4, 0.6, 0.8)
$\bar{\varepsilon}_4$	(0.6, 0.3, 0.5)	(0.6, 0.4, 0.4)	(0.5, 0.4, 0.7)

Clearly $\mathcal{F}(\bar{\varepsilon}_2)\bar{\subseteq}\mathcal{F}(\bar{\varepsilon}_3)\bar{\subseteq}\mathcal{F}(\bar{\varepsilon}_4)$.
The order among the elements of $\bar{\mathcal{B}}$ is $\bar{\varepsilon}_1 \leq_{\bar{\mathcal{B}}} \bar{\varepsilon}_2 \leq_{\bar{\mathcal{B}}} \bar{\varepsilon}_3 \leq_{\bar{\mathcal{B}}} \bar{\varepsilon}_4 \leq_{\bar{\mathcal{B}}} \bar{\varepsilon}_5$.

$(\mathcal{G}, \bar{\mathcal{B}})$	u_1	u_2	u_3
$\bar{\varepsilon}_1$	(0.1, 0.9, 0.7)	(0.1, 0.9, 0.6)	(0.3, 0.7, 0.8)
$\bar{\varepsilon}_2$	(0.2, 0.6, 0.5)	(0.3, 0.4, 0.5)	(0.5, 0.5, 0.7)
$\bar{\varepsilon}_3$	(0.5, 0.4, 0.4)	(0.5, 0.3, 0.4)	(0.6, 0.3, 0.5)
$\bar{\varepsilon}_4$	(0.7, 0.2, 0.3)	(0.7, 0.2, 0.3)	(0.7, 0.2, 0.4)
$\bar{\varepsilon}_5$	(0.8, 0.1, 0.2)	(0.9, 0.1, 0.2)	(0.9, 0.1, 0.3)

Clearly $\mathcal{G}(\bar{\varepsilon}_1)\bar{\subseteq}\mathcal{G}(\bar{\varepsilon}_2)\bar{\subseteq}\mathcal{G}(\bar{\varepsilon}_3)\bar{\subseteq}\mathcal{G}(\bar{\varepsilon}_4)\bar{\subseteq}\mathcal{G}(\bar{\varepsilon}_5)$ and $(\mathcal{F}, \bar{\mathcal{A}})\bar{\subseteq}(\mathcal{G}, \bar{\mathcal{B}})$ and extended union is defined as

$(\mathcal{H}, \bar{\mathcal{C}})$	u_1	u_2	u_3
$\bar{\varepsilon}_1$	(0.1, 0.9, 0.7)	(0.1, 0.9, 0.6)	(0.3, 0.7, 0.8)
$\bar{\varepsilon}_2$	(0.2, 0.6, 0.5)	(0.3, 0.4, 0.5)	(0.5, 0.5, 0.7)
$\bar{\varepsilon}_3$	(0.5, 0.4, 0.4)	(0.5, 0.3, 0.4)	(0.6, 0.3, 0.5)
$\bar{\varepsilon}_4$	(0.7, 0.2, 0.3)	(0.7, 0.2, 0.3)	(0.7, 0.2, 0.4)
$\bar{\varepsilon}_5$	(0.8, 0.1, 0.2)	(0.9, 0.1, 0.2)	(0.9, 0.1, 0.3)

$\Rightarrow H(\bar{\varepsilon}_1)\bar{\subseteq}H(\bar{\varepsilon}_2)\bar{\subseteq}H(\bar{\varepsilon}_3)\bar{\subseteq}H(\bar{\varepsilon}_4)\bar{\subseteq}H(\bar{\varepsilon}_5)$, for $\bar{\varepsilon}_1 \leq \bar{\varepsilon}_2 \leq \bar{\varepsilon}_3 \leq \bar{\varepsilon}_4 \leq \bar{\varepsilon}_5$
$\Rightarrow (\mathcal{F}, \bar{\mathcal{A}})\bar{\cup}_{EXT}(\mathcal{G}, \bar{\mathcal{B}}) = (\mathcal{H}, \bar{\mathcal{C}}) \in \mathcal{LONSS}(\mathcal{U})$.

Definition 3.8. Let $(\mathcal{F}, \bar{\mathcal{A}}) \in \mathcal{LONSS}(\mathcal{U})$. Then complement of $(\mathcal{F}, \bar{\mathcal{A}})$ is denoted by $(\mathcal{F}, \bar{\mathcal{A}})^c$ and is

$$(\mathcal{F}, \bar{\mathcal{A}})^c = \{\langle u, \mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), 1 - \mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u})\rangle : \ \bar{\varepsilon} \in \bar{\mathcal{A}} \ and \ \bar{u} \in \mathcal{U}\}$$

Definition 3.9. Let $(\mathcal{F}, \bar{\mathcal{A}}) \in \mathcal{LONSS}(\mathcal{U})$.
If $\mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}) = 0$ and $\mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}) = \mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}) = 1$, $\forall\ \bar{\varepsilon} \in \bar{\mathcal{A}}$ and $\bar{u} \in \mathcal{U}$, then $(\mathcal{F}, \bar{\mathcal{A}})$ is called a relative null $\mathcal{LONSS}$ and denoted by $\Phi_{\bar{\mathcal{A}}}$

Similarly, the relativenull $\mathcal{LONSS}$is the null $\mathcal{LONSS}$with respect to $\mathcal{E}$and is indicated by Φ.

Definition 3.10. Let $(\mathcal{F}, \bar{\mathcal{A}}) \in \mathcal{LONSS}(\mathcal{U})$.
If $\mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}) = 1$ and $\mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}) = \mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}) = 0\ \ \forall\ \ \bar{\varepsilon} \in \bar{\mathcal{A}}$ and for all $\bar{u} \in \mathcal{U}$, then $(\mathcal{F}, \bar{\mathcal{A}})$ is called a relative universal $\mathcal{LONSS}$ and denoted by $\bar{\mathcal{U}}_{\bar{\mathcal{A}}}$.

Similarly, the relative universal neutrosophic soft set with respect to the set of parameters E is called universal $\mathcal{LONSS}$ and denoted by $\bar{\mathcal{U}}$.

Proposition 3.11. *Let* $(\mathcal{F}, \bar{\mathcal{A}}) \in \mathcal{LONSS}(\mathcal{U})$*. Then*
1. $(\mathcal{F}, \bar{\mathcal{A}}) \bar{\cap}_{RES} (\mathcal{F}, \bar{\mathcal{A}}) = (\mathcal{F}, \bar{\mathcal{A}})$
2. $(\mathcal{F}, \bar{\mathcal{A}}) \bar{\cup}_{RES} (\mathcal{F}, \bar{\mathcal{A}}) = (\mathcal{F}, \bar{\mathcal{A}})$
3. $(\mathcal{F}, \bar{\mathcal{A}}) \bar{\cap}_{RES} \phi_{\bar{\mathcal{A}}} = \phi_{\bar{\mathcal{A}}}$
4. $(\mathcal{F}, \bar{\mathcal{A}}) \bar{\cup}_{RES} \phi_{\bar{\mathcal{A}}} = (\mathcal{F}, \bar{\mathcal{A}})$*.*

Proof. Straightforward. □

Definition 3.12. Let $(\mathcal{F}, \bar{\mathcal{A}}) \in NSS(\mathcal{U})$. Then it is known to be an anti-lattice ordered neutrosophic soft set ($\mathcal{ALONSS}$) over $\mathcal{U}$, where $\mathcal{F}$ is a mapping defined by $\mathcal{F} : \bar{\mathcal{A}} \to NS(\mathcal{U})$, if $\bar{\varepsilon}_i, \bar{\varepsilon}_j \in \bar{\mathcal{A}}$ such that $\bar{\varepsilon}_i \leq \bar{\varepsilon}_j$, then
$\mathcal{F}(\bar{\varepsilon}_j) \bar{\subseteq} \mathcal{F}(\bar{\varepsilon}_i)$
i.e. $\mathcal{T}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u}) \leq \mathcal{T}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u})$, $\mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}) \leq \mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u})$ and $\mathcal{F}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}) \leq \mathcal{F}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u})$

Proposition 3.13. *Let* $(\mathcal{F}, \bar{\mathcal{A}}) \in \mathcal{LONSS}(\mathcal{U})$*. Then complement of* $(\mathcal{F}, \bar{\mathcal{A}})$ *is an* $\mathcal{ALONSS}$ *over* $\mathcal{U}$*.*

Proof. Given that $(\mathcal{F}, \bar{\mathcal{A}}) \in \mathcal{LONSS}(\mathcal{U})$.
For $\bar{\varepsilon}_i \leq_{\bar{\mathcal{A}}} \bar{\varepsilon}_j$
$\Rightarrow \mathcal{F}(\bar{\varepsilon}_i) \bar{\subseteq} \mathcal{F}(\bar{\varepsilon}_j)$
$\Rightarrow$
$\mathcal{T}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u})$,
$\mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u}) \leq \mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u})$ and
$\mathcal{F}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u}) \leq \mathcal{F}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u})$
$\Rightarrow$
$\mathcal{T}_{\mathcal{F}^c(\bar{\varepsilon}_j)}(\bar{u}) \leq \mathcal{T}_{\mathcal{F}^c(\bar{\varepsilon}_i)}(\bar{u})$,
$1 - \mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}) \leq 1 - \mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u})$ and
$\mathcal{F}_{\mathcal{F}^c(\bar{\varepsilon}_i)}(\bar{u}) \leq \mathcal{F}_{\mathcal{F}^c(\bar{\varepsilon}_j)}(\bar{u})$
$\Rightarrow$
$\mathcal{T}_{\mathcal{F}^c(\bar{\varepsilon}_j)}(\bar{u}) \leq \mathcal{T}_{\mathcal{F}^c(\bar{\varepsilon}_i)}(\bar{u})$,
$\mathcal{I}_{\mathcal{F}^c(\bar{\varepsilon}_i)}(\bar{u}) \leq \mathcal{I}_{\mathcal{F}^c(\bar{\varepsilon}_j)}(\bar{u})$ and
$\mathcal{F}_{\mathcal{F}^c(\bar{\varepsilon}_i)}(\bar{u}) \leq \mathcal{F}_{\mathcal{F}^c(\bar{\varepsilon}_j)}(\bar{u})$
$\Rightarrow$
$\mathcal{F}^c(\bar{\varepsilon}_j) \bar{\subseteq} \mathcal{F}^c(\bar{\varepsilon}_i)$ whenever $\bar{\varepsilon}_i \leq_{\bar{\mathcal{A}}} \bar{\varepsilon}_j$
$\Rightarrow (\mathcal{F}, \bar{\mathcal{A}})^c$ is an $\mathcal{ALONSS}$. □

Proposition 3.14. *Let* $(\mathcal{F}, \bar{\mathcal{A}}) \in \mathcal{LONSS}(\mathcal{U})$*. Then* $((\mathcal{F}, \bar{\mathcal{A}})^c)^c = (\mathcal{F}, \bar{\mathcal{A}})$*.*

Proof. Let $(\mathcal{F}, \bar{\mathcal{A}}) \in \mathcal{LONSS}(\mathcal{U})$.
Then the complement of $(\mathcal{F}, \bar{\mathcal{A}})$ is
$\mathcal{T}_{\mathcal{F}^c(\bar{\varepsilon})}(\bar{u}) = \mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u})$,
$\mathcal{I}_{\mathcal{F}^c(\bar{\varepsilon})}(\bar{u}) = 1 - \mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u})$ and
$\mathcal{F}_{\mathcal{F}^c(\bar{\varepsilon})}(\bar{u}) = \mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u})$, where $\bar{\varepsilon} \in \bar{\mathcal{A}}$
Now the complement of $(\mathcal{F}, \bar{\mathcal{A}})^c$ is
$\mathcal{T}_{(\mathcal{F}^c)^c(\bar{\varepsilon})}(\bar{u}) = \mathcal{F}_{\mathcal{F}^c(\bar{\varepsilon})}(\bar{u}) = \mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u})$,
$\mathcal{I}_{(\mathcal{F}^c)^c(\bar{\varepsilon})}(\bar{u}) = 1 - \mathcal{I}_{\mathcal{F}^c(\bar{\varepsilon})}(\bar{u}) = 1 - \{1 - \mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u})\} = \mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u})$, and

$\mathcal{F}_{(\mathcal{F}^c)^c(\bar{\varepsilon})}(\bar{u}) = \mathcal{T}_{\mathcal{F}^c(\bar{\varepsilon})}(\bar{u}) = \mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u})$, where $\bar{\varepsilon} \in \bar{\mathcal{A}}$
$\Rightarrow ((\mathcal{F}, \bar{\mathcal{A}})^c)^c = (\mathcal{F}, \bar{\mathcal{A}})$. □

Definition 3.15. Let $(\mathcal{F}, \bar{\mathcal{A}}), (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$. Then $(\mathcal{F}, \bar{\mathcal{A}}) \vee (\mathcal{G}, \bar{\mathcal{B}}) = (\mathcal{H}, \bar{\mathcal{C}})$ is known to be basic union of two $\mathcal{LONSS}$s over $\mathcal{U}$, where $\bar{\mathcal{C}} = \bar{\mathcal{A}} \times \bar{\mathcal{B}}$ and define $\mathcal{H}(\bar{\varepsilon}_i, \bar{\varepsilon}_j) = \mathcal{F}(\bar{\varepsilon}_i)\bar{\cup}_{RES}\mathcal{G}(\bar{\varepsilon}_j)$ and

$\mathcal{T}_{H(\bar{\varepsilon}_i,\bar{\varepsilon}_j)}(\bar{u}) = Max\{\mathcal{T}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}), \mathcal{T}_{\mathcal{G}(\bar{\varepsilon}_j)}(\bar{u})\}$
$\mathcal{I}_{H(\bar{\varepsilon}_i,\bar{\varepsilon}_j)}(\bar{u}) = Min\{\mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}), \mathcal{I}_{\mathcal{G}(\bar{\varepsilon}_j)}(\bar{u})\}$
$\mathcal{F}_{H(\bar{\varepsilon}_i,\bar{\varepsilon}_j)}(\bar{u}) = Min\{\mathcal{F}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}), \mathcal{F}_{\mathcal{G}(\bar{\varepsilon}_j)}(\bar{u})\}$ *for all* $(\bar{\varepsilon}_i, \bar{\varepsilon}_j) \in \bar{\mathcal{C}}, \bar{u} \in \mathcal{U}$

Proposition 3.16. *Let* $(\mathcal{F}, \bar{\mathcal{A}}), (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$. *Then* $(\mathcal{F}, \bar{\mathcal{A}}) \vee (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$.

Proof. Suppose $(\mathcal{F}, \bar{\mathcal{A}}), (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$. Then by Definition 3.15
$(\mathcal{F}, \bar{\mathcal{A}}) \vee (\mathcal{G}, \bar{\mathcal{B}}) = (\mathcal{H}, \bar{\mathcal{C}})$, where $\bar{\mathcal{C}} = \bar{\mathcal{A}} \times \bar{\mathcal{B}}$
Since $\bar{\mathcal{A}}, \bar{\mathcal{B}} \bar{\subseteq} \mathcal{E}$, so both $\bar{\mathcal{A}}$ and $\bar{\mathcal{B}}$ inherit the partial order from $\mathcal{E}$ also
$\mathcal{H}(\epsilon, \eta) = \mathcal{F}(\epsilon) \vee \mathcal{G}(\eta) = \mathcal{F}(\epsilon)\bar{\cup}_{RES}\mathcal{G}(\eta)$
Now, $\bar{\varepsilon}_i \leq_{\bar{\mathcal{A}}} \bar{\varepsilon}_j$ we have $\mathcal{F}(\bar{\varepsilon}_i)\bar{\subseteq}\mathcal{F}(\bar{\varepsilon}_j)$, for every $\bar{\varepsilon}_i, \bar{\varepsilon}_j \in \bar{\mathcal{A}}$
and also for $\eta_i \leq_B \eta_j$ we have $\mathcal{G}(\eta_i)\bar{\subseteq}\mathcal{G}(\eta_j)$, for every $\eta_i, \eta_j \in \bar{\mathcal{B}}$
Now for any $(\bar{\varepsilon}_i, \eta_i), (\bar{\varepsilon}_j, \eta_j) \in \bar{\mathcal{C}}$ and $\leq$ is partial order on $\bar{\mathcal{C}}$ which is induced by partial orders on $\bar{\mathcal{A}}$ and $\bar{\mathcal{B}}$
The order on $\bar{\mathcal{A}} \times \bar{\mathcal{B}}$ is $(\bar{\varepsilon}_i, \eta_i) \leq (\bar{\varepsilon}_j, \eta_j)$, whenever $\bar{\varepsilon}_i \leq_{\bar{\mathcal{A}}} \bar{\varepsilon}_j$ and $\eta_i \leq_{\bar{\mathcal{B}}} \eta_j$
$\Rightarrow \quad \mathcal{F}(\bar{\varepsilon}_i)\bar{\subseteq}\mathcal{F}(\bar{\varepsilon}_j)$ and $\mathcal{G}(\eta_i)\bar{\subseteq}\mathcal{G}(\eta_j)$
$\Rightarrow$
$\mathcal{T}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u}), \mathcal{T}_{\mathcal{G}(\eta_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{G}(\eta_j)}(\bar{u})$
$\mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u}) \leq \mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}), \mathcal{I}_{\mathcal{G}(\eta_j)}(\bar{u}) \leq \mathcal{I}_{\mathcal{G}(\eta_i)}(\bar{u})$
$\mathcal{F}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u}) \leq \mathcal{F}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}), \mathcal{F}_{\mathcal{G}(\eta_j)}(\bar{u}) \leq \mathcal{F}_{\mathcal{G}(\eta_i)}(\bar{u})$
$\Rightarrow$
$Max\{\mathcal{T}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}), \mathcal{T}_{\mathcal{G}(\eta_i)}(\bar{u})\} \leq Max\{\mathcal{T}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u}), \mathcal{T}_{\mathcal{G}(\eta_j)}(\bar{u})\}$
$Min\{\mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u}), \mathcal{I}_{\mathcal{G}(\eta_j)}(\bar{u})\} \leq Min\{\mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}), \mathcal{I}_{\mathcal{G}(\eta_i)}(\bar{u})\}$
$Min\{\mathcal{F}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u}), \mathcal{F}_{\mathcal{G}(\eta_j)}(\bar{u})\} \leq Min\{\mathcal{F}_{\mathcal{F}(\epsilon_i)}(\bar{u}), \mathcal{F}_{\mathcal{G}(\eta_i)}(\bar{u})\}$
$\Rightarrow$
$\mathcal{T}_{\mathcal{F}(\epsilon_i)\vee\mathcal{G}(\eta_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{F}(\epsilon_j)\vee\mathcal{G}(\eta_j)}(\bar{u})$,
$\mathcal{I}_{\mathcal{F}(\epsilon_j)\vee\mathcal{G}(\eta_j)}(\bar{u}) \leq \mathcal{I}_{\mathcal{F}(\epsilon_i)\vee\mathcal{G}(\eta_i)}(\bar{u})$,
$\mathcal{F}_{\mathcal{F}(\epsilon_j)\vee\mathcal{G}(\eta_j)}(\bar{u}) \leq \mathcal{F}_{\mathcal{F}(\epsilon_i)\vee\mathcal{G}(\eta_i)}(\bar{u})$
$\Rightarrow$
$\mathcal{T}_{\mathcal{H}(\epsilon_i,\eta_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{H}(\epsilon_j,\eta_j)}(\bar{u})$,
$\mathcal{I}_{\mathcal{H}(\epsilon_j,\eta_j)}(\bar{u}) \leq \mathcal{I}_{\mathcal{H}(\epsilon_i,\eta_i)}(\bar{u})$,
$\mathcal{F}_{\mathcal{H}(\epsilon_j,\eta_j)}(\bar{u}) \leq \mathcal{F}_{\mathcal{H}(\epsilon_i,\eta_i)}(\bar{u})$
$\Rightarrow \quad \mathcal{H}(\epsilon_i, \eta_i)\bar{\subseteq}\mathcal{H}(\epsilon_j, \eta_j)$, for every $(\epsilon_i, \eta_i) \leq (\epsilon_j, \eta_j)$
Therefore, $(\mathcal{F}, \bar{\mathcal{A}}) \vee (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$. □

Definition 3.17. Let $(\mathcal{F}, \bar{\mathcal{A}}), (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$. Then $(\mathcal{F}, \bar{\mathcal{A}}) \wedge (\mathcal{G}, \bar{\mathcal{B}}) = (\mathcal{H}, \bar{\mathcal{C}})$ is known to be basic intersection of two $\mathcal{LONSS}$s over $\mathcal{U}$, where $\bar{\mathcal{C}} = \bar{\mathcal{A}} \times \bar{\mathcal{B}}$ and define $\mathcal{H}(\bar{\varepsilon}_i, \bar{\varepsilon}_j) = \mathcal{F}(\bar{\varepsilon}_i)\bar{\cap}_{RES}\mathcal{G}(\bar{\varepsilon}_j)$ and

$\mathcal{T}_{\mathcal{H}(\bar{\varepsilon}_i,\bar{\varepsilon}_j)}(\bar{u}) = Min\{\mathcal{T}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}), \mathcal{T}_{\mathcal{G}(\bar{\varepsilon}_j)}(\bar{u})\}$

$\mathcal{I}_{\mathcal{H}(\bar{\varepsilon}_i,\bar{\varepsilon}_j)}(\bar{u}) = Max\{\mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}), \mathcal{I}_{\mathcal{G}(\bar{\varepsilon}_j)}(\bar{u})\}$
$\mathcal{F}_{\mathcal{H}(\bar{\varepsilon}_i,\bar{\varepsilon}_j)}(\bar{u}) = Max\{\mathcal{F}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}), \mathcal{F}_{\mathcal{G}(\bar{\varepsilon}_j)}(\bar{u})\}\ forall\ (\bar{\varepsilon}_i, \bar{\varepsilon}_j) \in \mathcal{C}, \bar{u} \in \mathcal{U}$

Proposition 3.18. *Suppose* $(\mathcal{F}, \bar{\mathcal{A}}), (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$*. Then* $(\mathcal{F}, \bar{\mathcal{A}}) \wedge (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$*.*

Proof. The proof follows from Definition 3.17 and Proposition 3.16. □

4 Application

A large scale company intends to contribute funds. The objective of the fund is to recover the people's family whose livelihood is affected in Covid-19 pandemic. In order to carry out this project, the company tends to seek suitable NGO(Non-Governmental Organization).

The parameters are considered as
(i) Positive parameter $\Rightarrow$ Value $\propto$ Preference,
(ii) Negative parameter $\Rightarrow$ Value $\frac{1}{\propto}$ Preference.
The priority value lies in [-1,1].

If the priority value,
(i) Doesn't affect the expert decision $\Rightarrow$ Priority is 0,
(ii) Affects positively the expert decision $\Rightarrow$ Priority is (0,1],
(iii) Affects negatively the expert decision $\Rightarrow$ Priority is [-1,0),
(iv) Does not given $\Rightarrow$ Priority is 0 (This can be eliminated).

If there is more than one object, we keep only one object (same values for all parameters).

$$score = \mathcal{T}_{\mathcal{F}}(1 + \mathcal{I}_F) \tag{1}$$

In fuzzy soft set, equation (1) reduces to membership score only . The algorithm formulated by Tripathy et al. [20] is considered to compute the following decision making.
Let U be a set of NGOs to to carry out the project given by $U = \{n_1, n_2, n_3, n_4, n_5, n_6\}$ and the parameter set $E = \{\bar{\varepsilon}_1, \bar{\varepsilon}_2, \bar{\varepsilon}_3, \bar{\varepsilon}_4, \bar{\varepsilon}_5, \bar{\varepsilon}_6\}$ for the parameters "good track of record, number of volunteers, service experience, transparency, office network, familiarity", respectively.
There are three experts (E_1, E_2, E_3) to analyze the skills and features of the NGOs. For each parameter, the experts analyse the priority values. As per priority values, the parameters are ranked. The highest absolute value parameter has more priority and it holds the highest rank and so on.
If the priority value is same for more than one parameter then the expert can choose the rank among the parameters.
Clearly the order among the parameters

$$\bar{\varepsilon}_1 \bar{\succeq} \bar{\varepsilon}_2 \bar{\succeq} \bar{\varepsilon}_3 \bar{\succeq} \bar{\varepsilon}_5 \bar{\succeq} \bar{\varepsilon}_4 \bar{\succeq} \bar{\varepsilon}_6$$

The NGO rankings based on each expert are represented as $\mathcal{LONSS}$ in the tabular form.
Table 3, 4, 5 represent the ratings as per experts E_1, E_2, E_3 respectively. Each NGO will get a rank from every expert. Because of null priority there is no column for $\bar{\varepsilon}_6$.

Table 2: Parameter Data Table

U	$\bar{\varepsilon}_1$	$\bar{\varepsilon}_2$	$\bar{\varepsilon}_3$	$\bar{\varepsilon}_4$	$\bar{\varepsilon}_5$	$\bar{\varepsilon}_6$
priority	0.4	0.3	-0.15	0.05	0.1	0
Parameter Rank	1	2	3	5	4	6

Table 3: $\mathcal{LONSS}$ for E_1

U	$\bar{\varepsilon}_1$			$\bar{\varepsilon}_2$			$\bar{\varepsilon}_3$			$\bar{\varepsilon}_4$			$\bar{\varepsilon}_5$		
n_1	0.8	0.1	0.3	0.7	0.2	0.4	0.6	0.3	0.5	0.2	0.7	0.9	0.4	0.6	0.8
n_2	0.6	0.1	0.0	0.5	0.3	0.1	0.3	0.5	0.2	0.0	0.8	0.6	0.2	0.7	0.4
n_3	0.7	0.0	0.1	0.6	0.2	0.1	0.3	0.4	0.2	0.1	0.9	0.7	0.2	0.7	0.5
n_4	0.5	0.2	0.4	0.4	0.3	0.5	0.3	0.4	0.6	0.1	0.7	0.8	0.2	0.6	0.7
n_5	0.9	0.2	0.0	0.7	0.4	0.1	0.6	0.5	0.2	0.3	0.8	0.6	0.4	0.7	0.5
n_6	0.9	0.4	0.3	0.8	0.6	0.5	0.7	0.6	0.8	0.4	0.9	0.8	0.5	0.8	0.7

Table 4: $\mathcal{LONSS}$ for E_2

U	$\bar{\varepsilon}_1$			$\bar{\varepsilon}_2$			$\bar{\varepsilon}_3$			$\bar{\varepsilon}_4$			$\bar{\varepsilon}_5$		
n_1	0.6	0.1	0.2	0.5	0.2	0.3	0.3	0.4	0.5	0.0	0.6	0.8	0.2	0.5	0.7
n_2	0.9	0.4	0.3	0.8	0.6	0.5	0.7	0.6	0.8	0.4	0.9	0.8	0.5	0.8	0.7
n_3	0.7	0.1	0.2	0.5	0.1	0.4	0.4	0.2	0.6	0.2	0.5	0.8	0.3	0.4	0.7
n_4	0.6	0.5	0.2	0.4	0.6	0.5	0.3	0.7	0.6	0.0	0.9	0.8	0.1	0.8	0.7
n_5	0.9	0.1	0.2	0.7	0.3	0.4	0.4	0.5	0.6	0.0	0.9	0.8	0.2	0.7	0.6
n_6	0.8	0.1	0.3	0.5	0.2	0.4	0.4	0.3	0.5	0.1	0.5	0.8	0.3	0.4	0.7

Table 5: $\mathcal{LONSS}$ for E_3

U	$\bar{\varepsilon}_1$			$\bar{\varepsilon}_2$			$\bar{\varepsilon}_3$			$\bar{\varepsilon}_4$			$\bar{\varepsilon}_5$		
n_1	0.7	0.2	0.1	0.6	0.5	0.2	0.5	0.7	0.4	0.1	0.9	0.8	0.4	0.8	0.5
n_2	0.9	0.4	0.3	0.8	0.6	0.5	0.7	0.6	0.8	0.4	0.9	0.8	0.5	0.8	0.7
n_3	0.8	0.4	0.3	0.5	0.6	0.4	0.4	0.7	0.6	0.2	0.9	0.8	0.3	0.8	0.7
n_4	0.9	0.3	0.1	0.7	0.4	0.3	0.6	0.7	0.4	0.4	0.9	0.7	0.5	0.8	0.6
n_5	0.5	0.1	0.6	0.4	0.3	0.7	0.3	0.5	0.8	0.0	0.7	0.9	0.1	0.6	0.8
n_6	0.7	0.0	0.2	0.5	0.2	0.4	0.3	0.4	0.5	0.1	0.6	0.8	0.2	0.5	0.7

The priority tables 6, 7, 8 for each expert can be formulated by multiplying the values in the tables 3, 4, 5 with respective values fixed by the expert. Parameter 'Transfer fee' is having negative priority means negative parameter.

Calculate the entries as differences of each row sum in priority tables with those of all other rows and compute row sum in each table to create the corresponding comparison tables.

Tables 9, 10, 11 are the comparison tables for the experts E_1, E_2, E_3 respectively.

By using the (1), the decision can be developed and rank is given. If there is same score for more than one NGO then the NGO having top score in top ranked priority and so on. Similarly all the decision tables are obtained.

As illustrated in table 15, the rank table can be produced by adding the ranks assigned

Table 6: Priority table for the expert E_1

U	$\bar{\varepsilon}_1$			$\bar{\varepsilon}_2$			$\bar{\varepsilon}_3$			$\bar{\varepsilon}_4$			$\bar{\varepsilon}_5$			$\sum \mathcal{T}_F$	$\sum \mathcal{I}_F$	$\sum \mathcal{F}_F$
	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$			
n_1	0.32	0.04	0.12	0.21	0.06	0.12	-0.09	-0.045	-0.075	0.01	0.035	0.045	0.04	0.06	0.08	0.49	0.15	0.29
n_2	0.24	0.04	0.0	0.15	0.09	0.03	-0.045	-0.075	-0.03	0.0	0.04	0.03	0.02	0.07	0.04	0.365	0.165	0.07
n_3	0.28	0.0	0.04	0.18	0.06	0.03	-0.045	-0.06	-0.03	0.005	0.045	0.035	0.02	0.07	0.05	0.44	0.115	0.125
n_4	0.2	0.08	0.16	0.12	0.09	0.15	-0.045	-0.06	-0.09	0.005	0.035	0.04	0.02	0.06	0.07	0.3	0.205	0.33
n_5	0.36	0.08	0.0	0.21	0.12	0.03	-0.09	-0.075	-0.03	0.015	0.04	0.03	0.04	0.07	0.05	0.535	0.235	0.08
n_6	0.36	0.16	0.12	0.24	0.18	0.15	-0.105	-0.09	-0.12	0.02	0.045	0.04	0.05	0.08	0.07	0.565	0.375	0.26

Table 7: Priority table for the expert E_2

U	$\bar{\varepsilon}_1$			$\bar{\varepsilon}_2$			$\bar{\varepsilon}_3$			$\bar{\varepsilon}_4$			$\bar{\varepsilon}_5$			$\sum \mathcal{T}_F$	$\sum \mathcal{I}_F$	$\sum \mathcal{F}_F$
	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$			
n_1	0.24	0.04	0.08	0.15	0.06	0.09	-0.045	-0.06	-0.075	0.0	0.03	0.04	0.02	0.05	0.07	0.365	0.12	0.205
n_2	0.36	0.16	0.12	0.24	0.18	0.15	-0.105	-0.09	-0.12	0.02	0.045	0.04	0.05	0.08	0.07	0.565	0.375	0.26
n_3	0.28	0.04	0.08	0.15	0.03	0.12	-0.06	-0.03	-0.09	0.01	0.025	0.04	0.03	0.04	0.07	0.41	0.105	0.22
n_4	0.24	0.20	0.08	0.12	0.18	0.15	-0.045	-0.105	-0.09	0.0	0.045	0.04	0.01	0.08	0.07	0.325	0.4	0.25
n_5	0.36	0.04	0.08	0.21	0.09	0.12	-0.06	-0.075	-0.09	0.0	0.045	0.04	0.02	0.07	0.06	0.53	0.17	0.21
n_6	0.32	0.04	0.12	0.15	0.06	0.12	-0.06	-0.045	-0.075	0.005	0.025	0.04	0.03	0.04	0.07	0.445	0.12	0.275

Table 8: Priority table for the expert E_3

U	$\bar{\varepsilon}_1$			$\bar{\varepsilon}_2$			$\bar{\varepsilon}_3$			$\bar{\varepsilon}_4$			$\bar{\varepsilon}_5$			$\sum \mathcal{T}_F$	$\sum \mathcal{I}_F$	$\sum \mathcal{F}_F$
	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$			
n_1	0.28	0.08	0.04	0.18	0.15	0.06	-0.075	-0.105	-0.06	0.005	0.045	0.04	0.04	0.08	0.05	0.43	0.25	0.13
n_2	0.36	0.16	0.12	0.24	0.18	0.15	-0.105	-0.09	-0.12	0.02	0.045	0.04	0.05	0.08	0.07	0.565	0.375	0.26
n_3	0.32	0.16	0.12	0.15	0.18	0.12	-0.06	-0.105	-0.09	0.01	0.045	0.04	0.03	0.08	0.07	0.45	0.36	0.26
n_4	0.36	0.12	0.04	0.21	0.12	0.09	-0.09	-0.105	-0.06	0.02	0.045	0.035	0.05	0.08	0.06	0.55	0.26	0.165
n_5	0.20	0.04	0.24	0.12	0.09	0.21	-0.045	-0.075	-0.12	0.0	0.035	0.045	0.01	0.06	0.08	0.285	0.15	0.455
n_6	0.28	0.0	0.08	0.15	0.06	0.12	-0.045	-0.06	-0.075	0.005	0.03	0.04	0.02	0.05	0.07	0.41	0.08	0.235

Table 9: Comparison table for the expert E_1

U	n_1			n_2			n_3			n_4			n_5			n_6			$\sum \mathcal{T}_F$	$\sum \mathcal{I}_F$	$\sum \mathcal{F}_F$
	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$			
n_1	0	0	0	0.125	-0.015	0.22	0.05	0.035	0.165	0.19	-0.055	-0.04	-0.045	-0.085	0.21	-0.075	-0.225	0.03	0.245	-0.345	0.585
n_2	-0.125	0.015	-0.22	0	0	0	-0.075	0.05	-0.055	0.065	-0.04	-0.26	-0.17	-0.07	-0.01	-0.2	-0.21	-0.19	-0.505	-0.255	-0.735
n_3	-0.05	-0.035	-0.165	0.075	-0.05	0.055	0	0	0	0.14	-0.09	-0.205	-0.095	-0.12	0.045	-0.125	-0.26	-0.135	-0.055	-0.555	-0.405
n_4	-0.19	0.055	0.04	-0.065	0.04	0.26	-0.14	0.09	0.205	0	0	0	-0.235	-0.03	0.25	-0.265	-0.17	0.07	-0.895	-0.015	0.825
n_5	0.045	0.085	-0.21	0.17	0.07	0.01	0.095	0.12	-0.045	0.235	0.03	-0.25	0	0	0	-0.03	-0.14	-0.18	0.515	0.165	-0.675
n_6	0.075	0.225	-0.03	0.2	0.21	0.19	0.125	0.26	0.135	0.265	0.17	-0.07	0.03	0.14	0.18	0	0	0	0.695	1.005	0.405

to NGOs by each expert. If same rank sum obtained by more than one NGO, then the dispute can be solved by the similar way as in decision table formation.

Decision Making:

Table 10: Comparison table for the expert E_2

U	n_1			n_2			n_3			n_4			n_5			n_6			$\sum \mathcal{T}_F$	$\sum \mathcal{I}_F$	$\sum \mathcal{F}_F$
	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$			
n_1	0	0	0	-0.2	-0.255	-0.055	-0.045	0.015	-0.015	0.04	-0.28	-0.045	-0.165	-0.05	-0.005	-0.08	0	-0.07	-0.45	-0.57	-0.19
n_2	0.2	0.255	0.055	0	0	0	0.155	0.27	0.04	0.24	-0.025	0.01	0.035	0.205	0.05	0.12	0.255	-0.015	0.75	0.96	0.14
n_3	0.045	-0.015	0.015	-0.155	-0.27	-0.04	0	0	0	0.085	-0.295	-0.03	-0.12	-0.065	0.01	-0.035	-0.015	-0.055	-0.18	-0.66	-0.1
n_4	-0.04	0.28	0.045	-0.24	0.025	-0.01	-0.085	0.295	0.03	0	0	0	-0.205	0.23	0.04	-0.12	0.28	-0.025	-0.69	1.11	0.08
n_5	0.165	0.05	0.005	-0.035	-0.205	-0.05	0.12	0.065	-0.01	0.205	-0.23	-0.04	0	0	0	0.085	0.05	-0.065	0.54	-0.27	-0.16
n_6	0.08	0.0	0.07	-0.12	-0.255	0.015	0.035	0.015	0.055	0.12	-0.28	0.025	-0.085	-0.05	0.065	0	0	0	0.03	-0.57	0.23

Table 11: Comparison table for the expert E_3

U	n_1			n_2			n_3			n_4			n_5			n_6			$\sum \mathcal{T}_F$	$\sum \mathcal{I}_F$	$\sum \mathcal{F}_F$
	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$			
n_1	0	0	0	-0.135	-0.125	-0.13	-0.02	-0.11	-0.13	-0.12	-0.01	-0.035	0.145	0.1	-0.325	0.02	0.17	-0.105	-0.11	0.025	-0.725
n_2	0.135	0.125	0.13	0	0	0	0.115	0.015	0	0.015	0.115	0.095	0.28	0.225	-0.195	0.155	0.295	0.025	0.7	0.775	0.055
n_3	0.02	0.11	0.13	-0.115	-0.015	0	0	0	0	-0.1	0.1	0.095	0.165	0.21	-0.195	0.04	0.28	0.025	0.01	0.685	0.055
n_4	0.12	0.01	0.035	-0.015	-0.115	-0.095	0.1	-0.1	-0.095	0	0	0	0.265	0.11	-0.29	0.14	0.18	-0.07	0.61	0.085	-0.515
n_5	-0.145	-0.1	0.325	-0.28	-0.225	0.195	-0.165	-0.21	0.195	-0.265	-0.11	0.29	0	0	0	-0.125	0.07	0.22	-0.98	-0.575	1.225
n_6	-0.02	-0.17	0.105	-0.155	-0.295	-0.025	-0.04	-0.28	-0.025	-0.14	-0.18	0.07	0.125	-0.07	-0.22	0	0	0	-0.23	-0.995	-0.095

Table 12: Decision table for expert E_1

	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	Score	RankR_1
n_1	0.245	-0.345	0.585	0.160475	3
n_2	-0.505	-0.255	-0.735	-0.376225	5
n_3	-0.055	-0.555	-0.405	-0.024475	4
n_4	-0.895	-0.015	0.825	-0.881575	6
n_5	0.515	0.165	-0.675	0.599975	2
n_6	0.695	1.005	0.405	1.393475	1

Table 13: Decision table for expert E_2

	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	Score	RankR_2
n_1	-0.45	-0.57	-0.19	-0.1935	5
n_2	0.75	0.96	0.14	1.47	1
n_3	-0.18	-0.66	-0.1	-0.0612	4
n_4	-0.69	1.11	0.08	-1.4559	6
n_5	0.54	-0.27	-0.16	0.3942	2
n_6	0.03	-0.57	0.23	0.0129	3

The top ranked NGO is the good one to choose. If more than one NGO is needed, then the next subsequent rank holders can be chosen.

Comparative Study:

In day-to-day life, we stumble upon with linguistic terms having particular ranking among them. Here, the decision makers given an order of importance to the elements of parameters. Hence the lattice ordered neutrosophic soft sets are more helpful to deal with decision-making problems involving linguistic phrases. Thus the results obtained by using $\mathcal{LONSS}$ taken into account.

Table 14: Decision table for expert E_3

	$\mathcal{T}_F$	$\mathcal{I}_F$	$\mathcal{F}_F$	Score	RankR_3
n_1	-0.11	0.025	-0.725	-0.11275	5
n_2	0.7	0.775	0.055	1.2425	1
n_3	0.01	0.685	0.055	0.01685	3
n_4	0.61	0.085	-0.515	0.66185	2
n_5	-0.98	-0.575	1.225	-0.4165	6
n_6	-0.23	-0.995	-0.095	-0.00115	4

Table 15: Rank Table

	E_1	E_2	E_3	Normalized Score	Final-Rank
n_1	3	5	5	0.111111111	5
n_2	5	1	1	0.244444444	1
n_3	4	4	3	0.155555556	4
n_4	6	6	2	0.0888888889	6
n_5	2	2	6	0.177777778	3
n_6	1	3	4	0.222222222	2

5 Conclusion

The idea of lattice ordered neutrosophic soft sets is proposed. Also the effects of lattice ordered neutrosophic soft sets and anti-lattice ordered neutrosophic soft sets on restricted union, restricted intersection, extended union, extended intersection, basic union and basic intersection are familirised.A group decision-making problem is solved using these notions to demonstrate the importance of the proposed theory. We can build the lattice ordered neutrosophic hypersoft set theory in the future by generalising the soft set to the hypersoft set and finding applications in various areas of medicine.

References

[1] Ahmad, B. and A. Kharal, On fuzzy soft sets, Adv. Fuzzy Syst. 2009 (2009), Article ID 586507.

[2] Aslam, M. F., Study of fuzzy soft sets with some order on set of parameters, MS Thesis IIUI, 2014.

[3] Ajay, D., Chellamani, P., Pythagorean Neutrosophic Soft Sets and Their Application to Decision-Making Scenario. In: Kahraman, C., Cebi, S., Cevik Onar, S., Oztaysi, B., Tolga, A.C., Sari, I.U. (eds) Intelligent and Fuzzy Techniques for Emerging Conditions and Digital Transformation. INFUS 2021. Lecture Notes in Networks and Systems, vol 308. Springer, Cham,(2022). DOI: 10.1007/978-3-030-85577-2_65.

[4] Birkhoff, G., Lattice Theory, American Mathematical Society (1967).

[5] Broumi, S. and F. Smarandache, Intuitionistic neutrosophic soft set, J. Inf. Comput. Sci. England, UK 8(2), pp. 130-140 (2013).

[6] Broumi, S., Generalized neutrosophic soft set , Int. J. Comput. Sci. Eng. Inf. Technol. 3(2), pp. 17-30 (2013).

[7] Florentin Smarandache, Neutrosophic set-A generalization of the intuitionistic fuzzy set, International Journal of Pure and Applied Mathematics, 24(3) pp. 287-297 (2005).

[8] Florentin Smarandache, Extension of Soft Set to Hypersoft Set, and then to Plithogenic Hypersoft Set, Neutrosophic Sets and Systems, 22, (2018), pp. 168-170. DOI: 10.5281/zenodo.2159754

[9] Halis Aygun and Abdulkadir Aygunoglu, Introduction to fuzzy soft groups, Comput. Math. Appl. Vol. 58, pp. 1279-1286.

[10] Harish Garg, J. Vimala, S. Rajareega, D. Preethi, Luis Perez-Dominguez, Complex intuitionistic fuzzy soft SWARA - COPRAS approach: An application of ERP software selection[J], AIMS Mathematics, 7(4),(2022), 5895-5909. DOI: 10.3934/math.2022327

[11] Karaaslan, Faruk, Neutrosophic Soft Sets with Applications in Decision Making, International Journal of Information Science and Intelligent System, 4, 1-20, (2015). DOI: 10.5281/zenodo.23151.

[12] Muhammad Bilal Khan, Tahir Mahmood , Muhammad Iftikhar, Some Results on Lattice (Anti-Lattice) Ordered Double Framed Soft Sets, Journal of New Theory, 29 (2019) pp.74-86.

[13] Maji, P. K., R. Biswas and A. R. Roy, Soft set theory, Comput. Math. Appl. 45 (2003) pp. 555-562.

[14] Maji, P.K., Neutrosophic soft set, Annals of fuzzy mathematics and informatics, 5(1), (2013) pp. 157-168.

[15] Muhammad Irfan Ali, Tahir Mahmood et al., On lattice ordered soft set, Applied soft computing, 36, (2015) pp. 499-505.

[16] Mahmood, Tahir & Ali, Muhammad & Malik, Muhammad & Ahmed, Waseem, On lattice ordered intuitionistic fuzzy soft sets, International Journal of Algebra and Statistics, 7, 46-61,(2018).DOI:10.20454/ijas.2018.1434.

[17] Molodtsov, D., Soft set theory-first result, Comput. Math. Appl. 37 (1999) pp. 19-31.

[18] Preethi, D., Vimala, J. and Rajareega, S., A systematic study in the applications of fuzzy hyperlattice, AIMS Mathematics, 6(2) (2020) pp. 1695-1705.

[19] Rajareega, S., Vimala, J., Preethi, D., Complex Intuitionistic Fuzzy Soft Lattice Ordered Group and Its Weighted Distance Measures. Mathematics 8, 705,(2020), DOI: 10.3390/math8050705.

[20] Tripathy, B.K., Mohanty, R.K., Sooraj, T.R., On Intuitionistic Fuzzy Soft Set and its Application in Group Decision Making, in Proc. Int. Conf. Signal, Netw., Comput., Syst., India, (2016), pp. 6773.

[21] Zhaowen Li, Shijie Li, Lattice structures of intuitionistic fuzzy soft sets, Annals of Fuzzy Mathematics and Informatics,(2013).

[22] Zadeh, L. A., Fuzzy sets, Information and Control 8 (1965) 338-353.