Numerical Simulations and Optimization under Uncertainty: Recent Trends, Challenges, and Applications in Mechanical Systems - Research Article

Advances in Mechanical Engineering

Advances in Mechanical Engineering
2022, Vol. 14(10) 1–19

DOI: 10.1177/16878132221132146
journals.sagepub.com/home/ade
SAGE

An innovative decisive framework for optimized agri-automobile evaluation and HRM pattern recognition via possibility fuzzy hypersoft setting

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Abstract

In multi-attribute decision-making system, decision-makers have to face two kinds of situations: (i) the opted parameters are likely to be classified into their respective parametric-valued sub-collections and (ii) the acceptance degree for approximate opinions of decision-makers is required to be assessed by possibility setting. The literature related to fuzzy soft sets is unable to provide any model which can tackle such situations collectively. Therefore, this study aims to address this scarcity through the development of a novel structure, that is, possibility fuzzy hypersoft set (*pfhs*-set). Firstly, the algebraic properties and set-theoretic operations of *pfhs*-set are characterized by mathematical illustration. Secondly, two algorithms based on AND and OR-operations of *pfhs*-set are proposed and authenticated through application in multi-attribute decision-making real-world problems for the evaluation of agri-automobile and then their suitability is judged through vivid comparison. Thirdly, similarity measures between *pfhs*-sets are formulated and validated with the help of application in recruitment-based pattern recognition and its significance is assessed through comparison with most relevant models. Lastly, the advantageous aspects of the proposed structure are analyzed by its comparison with possibility fuzzy soft set-like models by observing some important evaluating features.

Keywords

Fuzzy hypersoft set, decision-support system, similarity measures, recruitment pattern recognition

Date received: 4 July 2022; accepted: 16 September 2022

Handling Editor: Chenhui Liang

Introduction

Dealing with information-based vagueness and uncertainty has always been a challenging task for the researchers. Several models have already been developed to tackle such vagueness and uncertainty. Out of these models, the most prominent model is fuzzy soft set (*fs*-set)[1] which is a hybridized model of fuzzy set (f-set)[2] and soft set (*s*-set).[3] It not only equips f-set with a parameterization tool to tackle uncertainties and vagueness efficiently but also makes *s*-set adequate for membership grading of each element in the universal set corresponding to attributes. In *fs*-set, approximate function of *s*-set is employed to deal with uncertain scenarios. This function maps the set of parameters to

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the collection of fuzzy subsets. Alcantud,[4] Petchimuthuet al.,[5] and Roy and Maji[6] applied the concept of *fs*-set in real-world decision-making (*DM*) through algorithmic techniques. Basu et al.,[7] Liet al.,[8] and Xiao[9] discussed the applicability of *fs*-set in clinical diagnosis. Many researchers applied *fs*-set in different fields of study but the contributions of Akram and Nawaz,[10] Ma and Qin,[11] and Mahanta and Das[12] in graphs, matrices, parameter reduction, and topological spaces respectively are more significant. Enginoğlu et al.[13] discussed TOPSIS based grouped *DM* technique by using the characterization of *fs*-set. Khan and Zhu[14] employed an innovative approach for parameter-reduction by utilizing the concept of *fs*-set. Sadiq and Devi[15] utilized a *fs*-set operational approach for the optimum ranking of functional needs of software. Zeng et al.,[16] Aydnı and Enginoğlu,[17] and Verma and Merigó[18] investigated several applications of *fs*-set with the entitlement of non-membership grades for approximate elements. Zeng et al.,[19] Zulqarnain et al.,[20] and Hussain et al.[21] discussed *DM* approaches based on Pythagorean *fs*-set environment for the evaluation of delivery vehicles, the green supply chain management and roughness respectively.

In various real-world *DM* scenarios, attributes require further classification into their respective attribute-valued disjoint sets to have reliable and unbiased decisions. The existing literatures on *s*-set and *fs*-set are insufficient for managing with such scenarios. Smarandache[22] initiated the concept of hypersoft set (*hs*-set) as an extension of *s*-set to tackle such scenarios. Abbas et al.[23] and Saeed et al.[24] characterized various elementary axiomatic properties, set-theoretic operations, relations, functions, and matrices of *hs*-set for its applicability in different fields. Rahman et al.[25] developed a novel gluing structure of *hs*-set by considering bijective settings. Kamacı[26] combined a rough set with *hs*-set and investigated notions of this hybrid with examples. Deli[27] developed hybrid set structures of *hs*-set by considering its parameterization under uncertainty. Yolcu and Ozturk[28] and Debnath[29] initiated the concept of fuzzy hypersoft set (*fhs*-set) by combining *f*-set with *hs*-set and discussed its some rudiments with numerical illustrations. Bavia et al.[30] extended the concept of *fhs*-set to a fuzzy whole hypersoft set (*fwhs*-set) and discussed its certain properties. In order to tackle with multi-decisive opinions of experts, Kamacı and Saqlain[31] combined *fhs*-set with an expert set and discussed its various properties. Musa and Asaad[32] studied the bi-polarity of *hs*-set by developing a novel structure called bipolar *hs*-set and discussed its set-theoretic operations. Rahman et al.[33–37] applied the concept of *fhs*-set in real world *DM* scenarios like medical diagnosis for heart diseases, set-theoretic and modular inequalities, supplier selection and risk analysis for real estate projects respectively. Ihsan et al.[38,39] investigated various set-theoretic characteristics, set operations, and applications of hypersoft hybrids with an expert set environment to tackle with multi-decisive opinions of experts. Arshad et al.[40] developed an intelligent multiattribute decision making framework based on aggregation operations of vague hypersoft hybrids for dealing with informational uncertainties. Saeed et al.[41] characterized some weak structures for the development of topological spaces under a hypersoft set environment. Khan et al.[42] discussed the basic set theoretic operations of q-rung orthopair *fhs*-set and explained the results with examples. Dalkılıç[43] determines membership degrees within (0, 1) for *hs*-set independent of decision-makers. Musa and Asaad[44] developed various topology-based structures by employing the characterization of bipolar *hs*-set.

Theory of possibility (poss-theory)[45] is the replacement of probability theory for dealing with the uncertain nature of information. Zadeh[46] discussed various aspects of *f*-sets as the basis for poss-theory. Fedrizzi[47] employed a blended approach of *f*-set and poss-theory to handle with various optimization models. Dubois and Prade[48] emphasized on the clarification of several features of poss-theory and probability theory. The poss-theory uses the measures of possibility of any entity between 0 and 1 in the objective space. The poss-theory has already been applied by many authors in *fs*-set like models as possibility degrees to measure the uncertain nature of approximate components collectively. In the following lines, some relevant literature is reviewed to appraise the research-gap and necessity of proposed study. Yager[49] and Alkhazaleh et al.[50] introduced novel gluing concept (i.e., possibility fuzzy soft set (*pfs*-set)) of *f*-set and *s*-set by assigning a specific grade that is possibility grade to each *fs*-number to assess its uncertainty. Alkhazaleh et al., in addition, worked out the similarity measures between two *pfs*-sets and utilized it in medical-diagnosis based pattern recognition. Bashir and Salleh[51] characterized the possibility fuzzy soft expert set (*pfse*-set), an extension of *pfs*-set, to manage with its limitation for the deliberation of multi-specialist judgments. Zhang et al.[52,53] developed possibility multi-*fs*-set (*pmfs*-set) and possibility interval-valued *fs*-set (*pivfs*-set) as the generalization of multi-*fs*-set and interval-valued *fs*-set respectively. Kalaiselvi and Seenivasan[54] and Ponnalagu and Mounika[55] discussed the utilization of *pfse*-sets in some real-world problems. Khalil and Hassan[56,57] explored some algorithm-based approaches using possibility m-polar *fs*-sets (*pmpfs*-set) and *pmfs*-set respectively.

In order to have reliable decisions, it is observed that the entitlement of attributes is not sufficient in those situations which require mandatory partitioning of attributes into disjoint sets having their respective values (Figures 1 and 2 present the significant comparison between *s*-set and *hs*-set through the elaboration of an

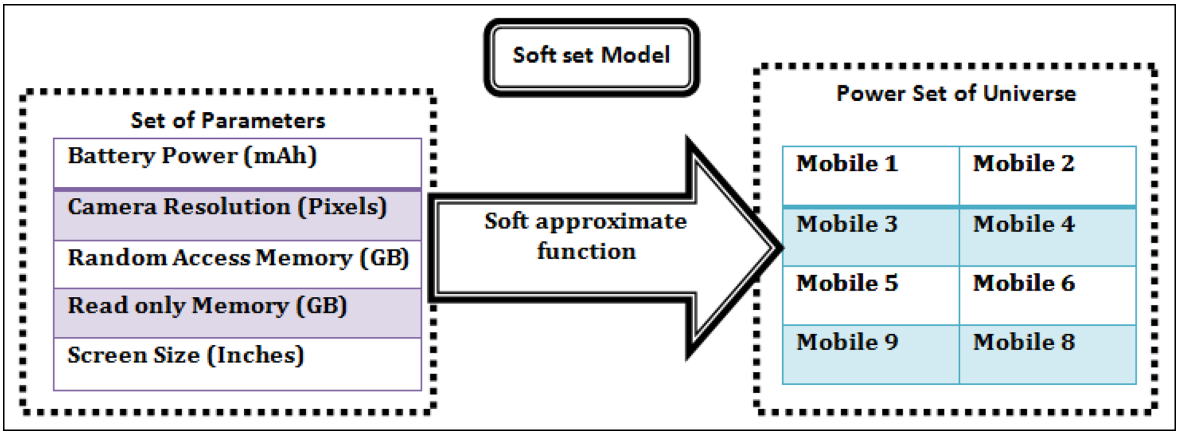

Figure 1. Pictorial representation of *s*-set model.

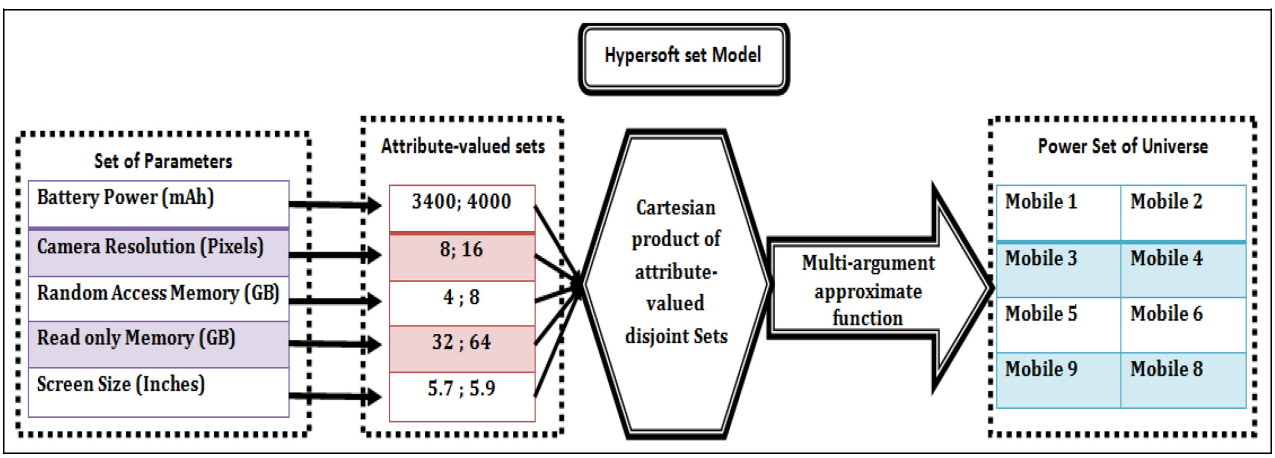

Figure 2. Pictorial representation of *hs*-set model.

illustrative example). Disregarding such partitioning may lead to biased and dubious decisions. The above described models and other *fs*-set like approaches are not capable of managing with such situations. In these models, a single set of parameters is considered as domain of approximate function which is inept for the consideration of attribute-valued classification. This scarcity of existing literature leads to the development of *hs*-set and *fhs*-set. Under above discussion, it is viewed that literature requires an innovative concept which may tackle with the situations: (i) which have mandatory partitioning of attributes, (ii) which assigns fuzzy membership grades to each alternative for managing with its uncertainty corresponding to each attribute, and (iii) which is enable to assess the uncertain nature of approximate elements with possibility degree-based setting, collectively in one model. This requisite provides the motivational basis of this study. Therefore instigating from above description in general, and from[50,54] in particular, a novel structure "possibility *fhs*-set (*pfhs*-set)" is developed along with its utilization in resolving real-world problems based on its aggregations.

The major contributions of the proposed work are outlined as

1. An innovative model *pfhs*-set is characterized which is more flexible and reliable as it is capable to handle with the real-world situations such as (i) sub-parametric classification for each parameter is required to be considered and (ii) the collective uncertain nature of approximate elements. The part (i) is managed by using the *hs*-set environment and the part (ii) is measured by assigning a possibility degree within [0, 1] to each approximate element.
2. Based on AND and OR-operations of *pfhs*-set, two algorithms are proposed having descriptive steps of input, construction and computation for the *DM* process. These algorithms are then validated with the help of a real-world based application for the optimum evaluation of

agri-automobile that is tractor. The computational results of these proposed algorithms are assessed through their comparison and AND-operation based algorithm is found more consistent as compared to OR-operation based algorithm that is OR-operation based algorithm requires some weights to be applied for its consistency.

3. As a structural requirement, similarity measures between two *pfhs*-sets are calculated and then an algorithmic approach is employed to assess its role in real-world pattern recognition for the evaluation of applicant suitability for a specific job.
4. In order to prove the reliability and flexibility of proposed model *pfhs*-set, its comprehensive structural comparison is discussed with some relevant existing structures and its generalization is discussed by describing its some particular cases.

The sectional arrangement of the rest of the paper is given as: some important essential definitions are reviewed in section "Preliminaries" to support the main results. The basic operational-properties along with aggregation operations of *pfhs*-sets are characterized with examples in section "The development of possibility fuzzy hypersoft set (*pfhs*-set)". Two novel algorithms based on aggregation operations of *pfhs*-set are proposed and then explained through their implementation in real-world *DM* problem in section "Decision support scheme based on aggregations of *pfhs*-sets". The modified similarity measures between two *pfhs*-sets are computed and then applied in pattern recognition for recruitment process in section "Similarity measures between *pfhs*-sets". The comparison, generalization, and merits are provided in section "Discussion and comparison analysis" and final section "Conclusion" presents the future directions of the proposed study.

Preliminaries

In this section, certain essential terminologies and terms are recalled to support main findings. The symbol $\mathcal{Z}$ will be used for initial universe in the whole paper.

Definition 0.1.[2] A f-set $\mathcal{F}$ is a set characterized as

$$\mathcal{F} = \{(\hat{z}, \hat{\psi}_{\mathcal{F}}(\hat{z}))|\hat{z} \in \mathcal{Z}\}$$

such that $\hat{\psi}_{\mathcal{F}} : \mathcal{Z} \rightarrow \mathbb{I}$ where $\hat{\psi}_{\mathcal{F}}(\hat{z})$ denotes the belonging value of $\hat{z} \in \mathcal{Z}$.

Definition 0.2.[2] Let $\mathcal{F}_1$ and $\mathcal{F}_2$ are two f-sets and $\hat{z} \in \mathcal{Z}$, then

(i) $\mathcal{F}_1 \cup \mathcal{F}_2 = \{(\hat{z}, max\{\hat{\psi}_{\mathcal{F}_1}(\hat{z}), \hat{\psi}_{\mathcal{F}_2}(\hat{z})\})\}$
(ii) $\mathcal{F}_1 \cap \mathcal{F}_2 = \{(\hat{z}, min\{\hat{\psi}_{\mathcal{F}_1}(\hat{z}), \hat{\psi}_{\mathcal{F}_2}(\hat{z})\})\}$
(iii) $\mathcal{F}_1{}^c = \{(\hat{z}, 1 - \hat{\psi}_{\mathcal{F}_1}(\hat{z}))\}$

Definition 0.3.[3] Let $\mathcal{E}$ be a set of attributes and $\mathcal{D}$ be its subset then the pair $(\Psi_{ss}, \mathcal{D})$ is called s-set on $\mathcal{Z}$ such that $\Psi_{ss} : \mathcal{D} \rightarrow \mathcal{P}(\mathcal{Z})$.

Definition 0.4.[1] Let $\mathcal{E}$ be a set of attributes and $\mathcal{D}_1$ be its subset then the pair $(\Psi_{fs}, \mathcal{D}_1)$ is called a fs-et on $\mathcal{Z}$ such that $\Psi_{fs} : \mathcal{D}_1 \rightarrow \mathcal{P}(\mathcal{FS})$ and $\mathcal{P}(\mathcal{FS})$ is a set having all f-subsets on $\mathcal{Z}$.

Definition 0.5.[22] The pair $(\xi_{hs}, \mathcal{M})$ is called a hs-set on $\mathcal{Z}$, where $\xi_{hs} : \mathcal{M} \rightarrow \mathcal{P}(\mathcal{Z})$ and $\mathcal{M} = \mathcal{M}_1 \times \mathcal{M}_2 \times \mathcal{M}_3 \times ..... \times \mathcal{M}_n$, $\mathcal{M}_i$ are disjoint parameter-valued sets corresponding to distinct parameters $m_1, m_2, ..., m_n$ respectively.

Definition 0.6.[22] The pair $(\zeta_{fhs}, \mathfrak{M})$ is called a fhs-set on $\mathcal{Z}$, where $\zeta_{fhs} : \mathcal{M} \rightarrow \mathcal{P}(\mathcal{FS})$ and $\mathcal{M} = \mathcal{M}_1 \times \mathcal{M}_2 \times \mathcal{M}_3 \times ..... \times \mathcal{M}_n$, $\mathcal{M}_i$ are disjoint parameter-valued sets corresponding to distinct parameters $m_1, m_2, ..., m_n$ respectively.

The development of possibility fuzzy hypersoft set (*pfhs*-set)

Before the characterization of *pfhs*-set, we first discuss its necessity in real-world scenarios. Consider the scenario of the recruitment process. It is a matter of common observation that any interview panel is headed by a chairperson who is empowered to issue the decision on the basis of recommendations (opinions) submitted by other members (decision-makers) of the panel. Suppose they agreed upon a set of parameters like qualification, age, and work experience with their sub-parametric valued sets $\{Bachelor Degree, Master Degree\}$, $\{25 years, 30 years\}$, and $\{5 years, 10 years\}$ respectively to evaluate the ranking of five candidates. The decision-makers interview the candidates according to their areas of expertise in the presence of chairperson and evaluate each candidate corresponding to each parametric tuple by assigning a fuzzy membership to each candidate. This fuzzy membership is for managing the uncertain nature of parametric tuples corresponding to each candidate. After the completion of the interviewing process, all decision-makers submit their recommendations (approximate elements of *fhs*-set) to the chairperson who evaluates the received recommendations with the help of poss-theory by assigning possibility grade to each recommendation. In other words, the chairperson and other members of the interviewing panel are applying the concept of *pfhs*-set to the evaluation of candidates for the specific job. This specific grade measures the degree of possibility for the acceptance of the recommendations toward the final decision. Now we present the characterization of basic concepts for *pfhs*-set with the support of illustrated examples.

Note: In all the matrix notations, the sub-parametric tuples are arranged in rows and the members of initial universe (alternatives) are arranged in columns.

Definition 0.7. The pair $(\mathfrak{F}_\mu, \mathfrak{A})$ is called a *pfhs*-set on hypersoft universe $(\mathcal{Z}, \mathfrak{A})$ if $\mathfrak{F}_\mu : \mathfrak{A} \to I_\mathcal{Z} \times I_\mathcal{Z}$ defined by $\mathfrak{F}_\mu(\alpha) = (\mathfrak{F}(\alpha)(u), \mu(\alpha)(u)), \forall u \in \mathcal{Z}$ where

1. $\mathfrak{A} = \mathfrak{A}_1 \times \mathfrak{A}_2 \times .... \times \mathfrak{A}_n$ with $\mathfrak{A}_i \cap \mathfrak{A}_j = \emptyset, i \neq j$, $\mathfrak{A}_i$ are sets having sub-parametric values for parameters $a_i, i = 1, 2, ..., n$ respectively,
2. $\mathfrak{F} : \mathfrak{A} \to I_\mathcal{Z}$ and $\mu : \mathfrak{A} \to I_\mathcal{Z}$, $I_\mathcal{Z}$ is collection of all fuzzy subsets of $\mathcal{Z}$,
3. $\mathfrak{F}(\alpha)(u)$ and $\mu(\alpha)(u)$ represent belonging and possibility grades respectively of $u \in \mathcal{Z}$ in $\mathfrak{F}(\alpha)$.

so $\mathfrak{F}_\mu(\alpha_i)$ can be written as:

$$\mathfrak{F}_\mu(\alpha_i) = \left\{ \begin{array}{l} \left(\frac{u_1}{\mathfrak{F}(\alpha_i)(u_1)}, \mu(\alpha_i)(u_1)\right), \\ \left(\frac{u_2}{\mathfrak{F}(\alpha_i)(u_2)}, \mu(\alpha_i)(u_2)\right), \\ \ldots\ldots\ldots\ldots\ldots\ldots, \\ \ldots\ldots\ldots\ldots\ldots\ldots, \\ \ldots\ldots\ldots\ldots\ldots\ldots, \\ \left(\frac{u_n}{\mathfrak{F}(\alpha_i)(u_n)}, \mu(\alpha_i)(u_n)\right) \end{array} \right\}$$

For simplicity, *pfhs*-set is symbolized as $\mathfrak{F}_\mu$ and its family as Ω_{pfhss}.

Example 0.8. Mrs. Smith is the head of a technical institution which is providing training services in different technical trades to female individuals of the locality. She wants to purchase some overlock sewing machines to start training classes for learning various techniques of sewing and stitching. She constitutes a committee of some faculty members (having relevant experience) for this purchase. There are four kinds of machines which constitute a set of alternatives $\mathcal{Z} = \{m_1, m_2, m_3, m_4\}$. The members of the committee consider the attributes, that is, a_1 = speed (stitches per minute), a_2 = stitch length (millimeter), and a_3 = oil lubrication system. The non-overlapping sets having their sub-attribute values are $\mathfrak{A}_1 = \{a_{11} = 6000, a_{12} = 6500\}$, $\mathfrak{A}_2 = \{a_{21} = 2, a_{22} = 4\}$, $\mathfrak{A}_3 = \{a_{31} = automatic\}$

then $\mathfrak{A} = \mathfrak{A}_1 \times \mathfrak{A}_2 \times \mathfrak{A}_3 = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ where every α_i is a 3-tuple entity. Now

$$\mathfrak{F}_\mu(\alpha_1) = \left\{ \left(\tfrac{m_1}{0.1}, 0.2\right), \left(\tfrac{m_2}{0.2}, 0.3\right), \left(\tfrac{m_3}{0.3}, 0.4\right), \left(\tfrac{m_4}{0.4}, 0.5\right) \right\}$$
$$\mathfrak{F}_\mu(\alpha_2) = \left\{ \left(\tfrac{m_1}{0.2}, 0.8\right), \left(\tfrac{m_2}{0.3}, 0.8\right), \left(\tfrac{m_3}{0.4}, 0.7\right), \left(\tfrac{m_4}{0.5}, 0.6\right) \right\}$$
$$\mathfrak{F}_\mu(\alpha_3) = \left\{ \left(\tfrac{m_1}{0.3}, 0.1\right), \left(\tfrac{m_2}{0.4}, 0.2\right), \left(\tfrac{m_3}{0.5}, 0.3\right), \left(\tfrac{m_4}{0.6}, 0.4\right) \right\}$$
$$\mathfrak{F}_\mu(\alpha_4) = \left\{ \left(\tfrac{m_1}{0.4}, 0.2\right), \left(\tfrac{m_2}{0.5}, 0.3\right), \left(\tfrac{m_3}{0.6}, 0.4\right), \left(\tfrac{m_4}{0.7}, 0.5\right) \right\}$$

Then $\mathfrak{F}_\mu$ is a *pfhs*-set over $(\mathcal{Z}, \mathfrak{A})$ which is represented in matrix notation as

$$\mathfrak{F}_\mu = \begin{pmatrix} (0.1, 0.2) & (0.2, 0.3) & (0.3, 0.4) & (0.4, 0.5) \\ (0.2, 0.8) & (0.3, 0.8) & (0.4, 0.7) & (0.5, 0.6) \\ (0.3, 0.1) & (0.4, 0.2) & (0.5, 0.3) & (0.6, 0.4) \\ (0.4, 0.2) & (0.5, 0.3) & (0.6, 0.4) & (0.7, 0.5) \end{pmatrix}$$

Definition 0.9. A *pfhs*-set $\mathfrak{F}_\mu$ is called a *pfhs*-subset of another *pfhs*-set $\mathfrak{G}_\eta$, represented as $\mathfrak{F}_\mu \subseteq \mathfrak{G}_\eta$, if $\mu(\alpha)$ and $\mathfrak{F}(\alpha)$ are fuzzy subsets of $\eta(\alpha)$ and $\mathfrak{G}(\alpha)$ respectively $\forall \alpha \in \mathfrak{A}$.

Example 0.10. Reassuming $\mathfrak{F}_\mu$ as stated in Example 0.8 and let

$$\mathfrak{G}_\eta(\alpha_1) = \left\{\left(\tfrac{m_1}{0.2}, 0.3\right), \left(\tfrac{m_2}{0.3}, 0.4\right), \left(\tfrac{m_3}{0.4}, 0.5\right), \left(\tfrac{m_4}{0.5}, 0.6\right)\right\}$$
$$\mathfrak{G}_\eta(\alpha_2) = \left\{\left(\tfrac{m_1}{0.3}, 0.9\right), \left(\tfrac{m_2}{0.4}, 0.9\right), \left(\tfrac{m_3}{0.5}, 0.8\right), \left(\tfrac{m_4}{0.6}, 0.7\right)\right\}$$
$$\mathfrak{G}_\eta(\alpha_3) = \left\{\left(\tfrac{m_1}{0.4}, 0.2\right), \left(\tfrac{m_2}{0.5}, 0.3\right), \left(\tfrac{m_3}{0.6}, 0.4\right), \left(\tfrac{m_4}{0.7}, 0.5\right)\right\}$$
$$\mathfrak{G}_\eta(\alpha_4) = \left\{\left(\tfrac{m_1}{0.5}, 0.3\right), \left(\tfrac{m_2}{0.6}, 0.4\right), \left(\tfrac{m_3}{0.7}, 0.5\right), \left(\tfrac{m_4}{0.8}, 0.6\right)\right\}$$

then $\mathfrak{F}_\mu \subseteq \mathfrak{G}_\eta$.

Definition 0.11. The complement of a *pfhs*-set $\mathfrak{F}_\mu$, denoted by $\mathfrak{F}^c_\mu$, is defined by $\mathfrak{F}^c_\mu = \mathfrak{G}_\eta$ such that $\eta(\alpha) = \mu^c(\alpha)$ and $\mathfrak{G}(\alpha) = \mathfrak{F}^c(\alpha), \forall \alpha \in \mathfrak{A}$, where c is a fuzzy complement.

Example 0.12. From Example 0.8, reconsidering $\mathfrak{F}_\mu$ then its complement is

$$\mathfrak{F}^c_\mu = \mathfrak{G}_\eta$$
$$= \begin{pmatrix} (0.9, 0.8) & (0.8, 0.7) & (0.7, 0.6) & (0.6, 0.5) \\ (0.8, 0.2) & (0.7, 0.2) & (0.6, 0.3) & (0.5, 0.4) \\ (0.7, 0.9) & (0.6, 0.8) & (0.5, 0.7) & (0.4, 0.6) \\ (0.6, 0.8) & (0.5, 0.7) & (0.4, 0.6) & (0.3, 0.5) \end{pmatrix}$$

Set theoretic operations of pfhs-sets

In this part, characterization of aggregation operations of *pfhs*-sets is discussed with suitable examples.

Definition 0.13. Let $\mathfrak{J}_\mu, \mathfrak{K}_\eta \in \Omega_{pfhss}$ then

1. $\mathfrak{J}_\mu \cup \mathfrak{K}_\eta$ is called their union which is a *pfhs*-set $\mathfrak{L}_\nu$ with $\mathfrak{L}(\alpha) = max\{\mathfrak{J}(\alpha), \mathfrak{K}(\alpha)\}$ and $\nu(\alpha) = max\{\mu(\alpha),\ \eta(\alpha)\}$.
2. $\mathfrak{J}_\mu \cap \mathfrak{K}_\eta$ is called their intersection which is a *pfhs*-set $\mathfrak{D}_\omega$ with $\mathfrak{D}(\alpha) = min\{\mathfrak{J}(\alpha), \mathfrak{K}(\alpha)\}$ and $\omega(\alpha) = min\ \{\mu(\alpha), \eta(\alpha)\}$.

Example 0.14. Let $\mathfrak{J}_\mu, \mathfrak{K}_\eta \in \Omega_{pfhss}$ with matrix notations as

$$\mathfrak{J}_{\mu} = \begin{pmatrix} (0.1, 0.2) & (0.2, 0.3) & (0.3, 0.4) & (0.4, 0.5) \\ (0.2, 0.8) & (0.3, 0.8) & (0.4, 0.7) & (0.5, 0.6) \\ (0.3, 0.1) & (0.4, 0.2) & (0.5, 0.3) & (0.6, 0.4) \\ (0.4, 0.2) & (0.5, 0.3) & (0.6, 0.4) & (0.7, 0.5) \end{pmatrix}$$

and

$$\mathfrak{K}_{\eta} = \begin{pmatrix} (0.2,0.3) & (0.3,0.4) & (0.4,0.5) & (0.5,0.6) \\ (0.3,0.9) & (0.4,0.9) & (0.5,0.8) & (0.6,0.7) \\ (0.4,0.2) & (0.5,0.3) & (0.6,0.4) & (0.7,0.5) \\ (0.5,0.3) & (0.6,0.4) & (0.7,0.5) & (0.8,0.6) \end{pmatrix}$$

then

$$\mathfrak{L}_{\nu} = \mathfrak{J}_{\mu} \cup \mathfrak{K}_{\eta} = \mathfrak{K}_{\eta} \text{ and } \mathfrak{D}_{\omega} = \mathfrak{J}_{\mu} \cap \mathfrak{K}_{\eta} = \mathfrak{J}_{\mu}.$$

Proposition 0.15. Let $\mathfrak{J}_{\mu}, \mathfrak{K}_{\eta}, \mathfrak{E}_{\psi} \in \Omega_{pfhss}$. Then

1. $\mathfrak{J}_{\mu} \cup (\mathfrak{K}_{\eta} \cap \mathfrak{E}_{\psi}) = (\mathfrak{J}_{\mu} \cup \mathfrak{K}_{\eta}) \cap (\mathfrak{J}_{\mu} \cup \mathfrak{E}_{\psi})$
2. $\mathfrak{J}_{\mu} \cap (\mathfrak{K}_{\eta} \cup \mathfrak{E}_{\psi}) = (\mathfrak{J}_{\mu} \cap \mathfrak{K}_{\eta}) \cup (\mathfrak{J}_{\mu} \cap \mathfrak{E}_{\psi})$

Definition 0.16. Let $(\mathbb{P}_{\mu}, \mathbb{C}), (\mathbb{Q}_{\eta}, \mathbb{D}) \in \Omega_{pfhss}$, then
(i) $(\mathscr{P}_{\mu}, \mathscr{C}) \wedge (\mathscr{Q}_{\eta}, \mathscr{D})$ is known as AND-operation that is a *pfhs*-set $(\mathscr{R}_{\nu}, \mathscr{G})$ characterized by $(\mathscr{R}_{\nu}, \mathscr{G}) = (\mathscr{R}_{\nu}, \mathscr{C} \times \mathscr{D})$ with $\mathscr{R}_{\nu}(\ddot{c}, \ddot{d}) = (\mathscr{R}(\ddot{c}, \ddot{d})(\ddot{z}), \nu(\ddot{c}, \ddot{d})(\ddot{z}))$,

for all $(\ddot{c}, \ddot{d}) \in \mathscr{C} \times \mathscr{D}$, such that

$$\mathscr{R}(\ddot{c}, \ddot{d}) = min\{\mathscr{P}(\ddot{c}), \mathscr{Q}(\ddot{d})\} \text{and}$$

$\nu(\ddot{c}, \ddot{d}) = min\{\mu(\ddot{c}), \eta(\ddot{d})\}$, for all $(\ddot{c}, \ddot{d}) \in \mathcal{C} \times \mathcal{D}$ and $\ddot{z} \in \mathcal{Z}$.

(ii) $(\mathscr{P}_{\mu}, \mathscr{C}) \vee (\mathscr{Q}_{\eta}, \mathscr{D})$ is known as OR-operation that is a *pfhs*-set $(\mathscr{S}_{\kappa}, \mathscr{H})$ stated as $(\mathscr{S}_{\kappa}, \mathscr{H}) = (\mathscr{S}_{\kappa}, \mathscr{C} \times \mathscr{D})$ with $\mathscr{S}_{\kappa}(\ddot{c}, \ddot{d}) = (\mathscr{S}(\ddot{c}, \ddot{d})(\ddot{z}), \nu(\ddot{c}, \ddot{d})(\ddot{z}))$,

for all $(\ddot{c}, \ddot{d}) \in \mathcal{C} \times \mathcal{D}$, such that

$$\mathscr{S}(\ddot{c}, \ddot{d}) = max\{\mathscr{P}(\ddot{c}), \mathscr{Q}(\ddot{d})\} \textit{and}$$

$\kappa(\ddot{c}, \ddot{d}) = max\{\mu(\ddot{c}), \eta(\ddot{d})\}$, for all $(\ddot{c}, \ddot{d}) \in \mathscr{C} \times \mathscr{D}$ and $\ddot{z} \in \mathcal{Z}$.

Example 0.17. Considering sets from 0.14, we have

$$(\mathbb{R}_{\nu}, \mathbb{G}) = \begin{pmatrix} (0.1, 0.2) & (0.2, 0.3) & (0.3, 0.4) & (0.4, 0.5) \\ (0.1, 0.2) & (0.2, 0.3) & (0.3, 0.4) & (0.4, 0.5) \\ (0.1, 0.2) & (0.2, 0.3) & (0.3, 0.4) & (0.4, 0.5) \\ (0.1, 0.2) & (0.2, 0.3) & (0.3, 0.4) & (0.4, 0.5) \\ (0.2, 0.3) & (0.3, 0.4) & (0.4, 0.5) & (0.5, 0.6) \\ (0.2, 0.8) & (0.3, 0.8) & (0.4, 0.7) & (0.5, 0.6) \\ (0.2, 0.2) & (0.3, 0.3) & (0.4, 0.4) & (0.5, 0.5) \\ (0.2, 0.3) & (0.3, 0.4) & (0.4, 0.5) & (0.5, 0.6) \\ (0.2, 0.1) & (0.3, 0.2) & (0.4, 0.3) & (0.5, 0.4) \\ (0.3, 0.1) & (0.4, 0.2) & (0.5, 0.3) & (0.6, 0.4) \\ (0.3, 0.1) & (0.4, 0.2) & (0.5, 0.3) & (0.6, 0.4) \\ (0.3, 0.1) & (0.4, 0.2) & (0.5, 0.3) & (0.6, 0.4) \\ (0.2, 0.2) & (0.3, 0.3) & (0.4, 0.4) & (0.5, 0.5) \\ (0.3, 0.2) & (0.4, 0.3) & (0.5, 0.4) & (0.6, 0.5) \\ (0.4, 0.2) & (0.5, 0.3) & (0.6, 0.4) & (0.7, 0.5) \\ (0.4, 0.2) & (0.5, 0.3) & (0.6, 0.4) & (0.7, 0.5) \end{pmatrix}$$

and

$$(\mathbb{S}_{\kappa}, \mathbb{H}) = \begin{pmatrix} (0.2,0.3) & (0.3,0.4) & (0.4,0.5) & (0.5,0.6) \\ (0.3,0.9) & (0.4,0.9) & (0.5,0.8) & (0.6,0.7) \\ (0.4,0.2) & (0.5,0.3) & (0.6,0.4) & (0.7,0.5) \\ (0.5,0.3) & (0.6,0.4) & (0.7,0.5) & (0.8,0.6) \\ (0.2,0.8) & (0.3,0.8) & (0.4,0.7) & (0.5,0.6) \\ (0.3,0.9) & (0.4,0.9) & (0.5,0.8) & (0.6,0.7) \\ (0.4,0.8) & (0.5,0.8) & (0.6,0.7) & (0.7,0.6) \\ (0.5,0.8) & (0.6,0.8) & (0.7,0.7) & (0.8,0.6) \\ (0.3,0.3) & (0.4,0.4) & (0.5,0.5) & (0.6,0.6) \\ (0.3,0.9) & (0.4,0.9) & (0.5,0.8) & (0.6,0.7) \\ (0.4,0.2) & (0.5,0.3) & (0.6,0.4) & (0.7,0.5) \\ (0.5,0.3) & (0.6,0.4) & (0.7,0.5) & (0.8,0.6) \\ (0.4,0.3) & (0.5,0.4) & (0.6,0.5) & (0.7,0.6) \\ (0.4,0.9) & (0.5,0.9) & (0.6,0.8) & (0.7,0.7) \\ (0.4,0.2) & (0.5,0.3) & (0.6,0.4) & (0.7,0.5) \\ (0.5,0.3) & (0.6,0.4) & (0.7,0.5) & (0.8,0.6) \end{pmatrix}.$$

Decision support scheme based on aggregations of *pfhs*-sets

The aim of this part is to design an innovative *DM* context through the proposal of two intelligent algorithms by using aggregation operations (i.e. AND and OR) of *pfhs*-set for optimized evaluation of agri-automobile i.e. tractor.

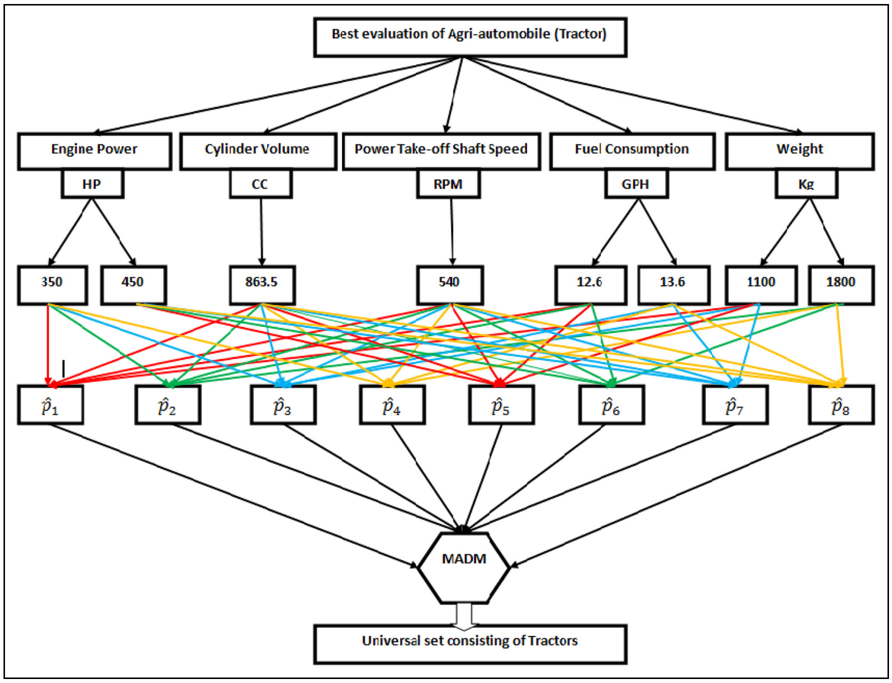

Figure 3. Hierarchy model for the optimum evaluation agri-automobile.

Recognition of problem and its brief description

Agriculture performs a decisive function in the financial system of developing countries, and supplies the most important sources of income, employment, and food to rural populations. The farmers are using both traditional and advanced machinery for cultivation to enhance their agricultural yield. Tractor is the one which is considered both traditional and advanced sources for cultivation. Usually, tractors have been utilized on farms to automate numerous farming activities. Tractors are an indispensable requirement of farming as they supply mechanical control for carrying out farming functions. They accomplish some major tasks like countryside maintenance, caring of lawn and clearance of bushes, spreading fertilizers, and pulling different farm tools for cultivating crops. In short, the finest farming can be possible only by using tractors. Not only in developing countries but all over the world the tractors are the most vital farm machines. Modern agriculture cannot be imagined without the use of tractors. To fulfill particular farm needs, the tractors with different features are being manufactured by several companies. However, many quality-based concerns are being reported due to its manufacturing diversity, therefore it is pertinent to adopt an intelligent approach to evaluate tractors so that risk factors may be avoided. The MADM approach is considered more reliable in this regard. In upcoming segments of the paper, a MADM-based algorithmic approach is employed to assist the farmers in purchasing and evaluating good quality tractors. The hierarchy model for the best selection of tractor is presented in Figure 3.

The flowing procedure of Algorithm 1 is presented in Figure 4.

Example 0.18. (Input Stage: 1.1–1.3) Mr. Smith is a landlord who is intended to purchase a tractor for cultivation of his land. He is very much concerned about the tractors available in the market with substandard quality. During the survey, it is observed that there are two models of tractors provided by two distinct manufacturers. The tractors $\hat{T}^r{}_1, \hat{T}^r{}_2$, and $\hat{T}^r{}_3$ are provided by company I and the tractors $\hat{T}^r{}_4$ and $\hat{T}^r{}_5$ are provided by company II. All these tractors are considered as the elements of universal set $\mathcal{Z} = \left\{\hat{T}^r{}_1, \hat{T}^r{}_2, \hat{T}^r{}_3, \hat{T}^r{}_4, \hat{T}^r{}_5\right\}$. As he is not well-familiar with procurement expertise therefore he is assisted by two friends (decision makers) having wide experience of procurement. After reviewing various features of tractors, they have finalized a set of parameters

$$\ddot{\mathfrak{A}} = \left\{ \begin{array}{l} y_1 = engine\,power(hp), y_2 = cylinder \\ volume(cc : cubic\,centimetres), \\ y_3 = power\,take - off\ shaft\,speed \\ (rpm : rotations\,per\,minute), \\ y_4 = fuel\,consumption \\ (gph : gallons\,per\,hour), \\ y_5 = weight(kg : kilograms) \end{array} \right\}$$

Algorithm 1: Procedural description of an optimized tractor evaluation based on AND-operation of *pfhs*-sets

▶ **Start**
▶ **Input:**
1.1. Assume a list of available tractors as universal set $\mathcal{Z}$.
1.2. Enlist suitable criteria as a collection of n evaluating features (attributes) $\ddot{\mathfrak{A}}$.
1.3. In order to satisfy hypersoft settings, determine disjoint sub-classes $\ddot{\mathfrak{A}}^1, \ddot{\mathfrak{A}}^2, \ddot{\mathfrak{A}}^3, \ldots, \ddot{\mathfrak{A}}^n$ having parametric values.
▶ **Construction:**
1.4. Find out $\ddot{\mathfrak{A}} = \ddot{\mathfrak{A}}^1 \times \ddot{\mathfrak{A}}^2 \times \ddot{\mathfrak{A}}^3 \times \ldots \times \ddot{\mathfrak{A}}^n$.
1.5. With the assistance of experts, formulate two *pfhs*-sets: $\mathscr{A}_\zeta$ and $\mathscr{B}_\xi$.
▶ **Computation:**
1.6. Using $\mathscr{A}_\zeta$ and $\mathscr{B}_\xi$, compute AND-operation $\mathcal{V}_\delta$.
1.7. Encapsulate the numerical values of $\mathcal{V}_\delta$ in matrix version and highlight the maximum numerical grade ρ row-wise.
1.8. Using $\sum_i \rho_i.\sigma_i$, compute score values Δ_i for each alternative.
▶ **Output:**
1.9. Select the alternative with maximum score as discrete optimum selection.
▶ **End**

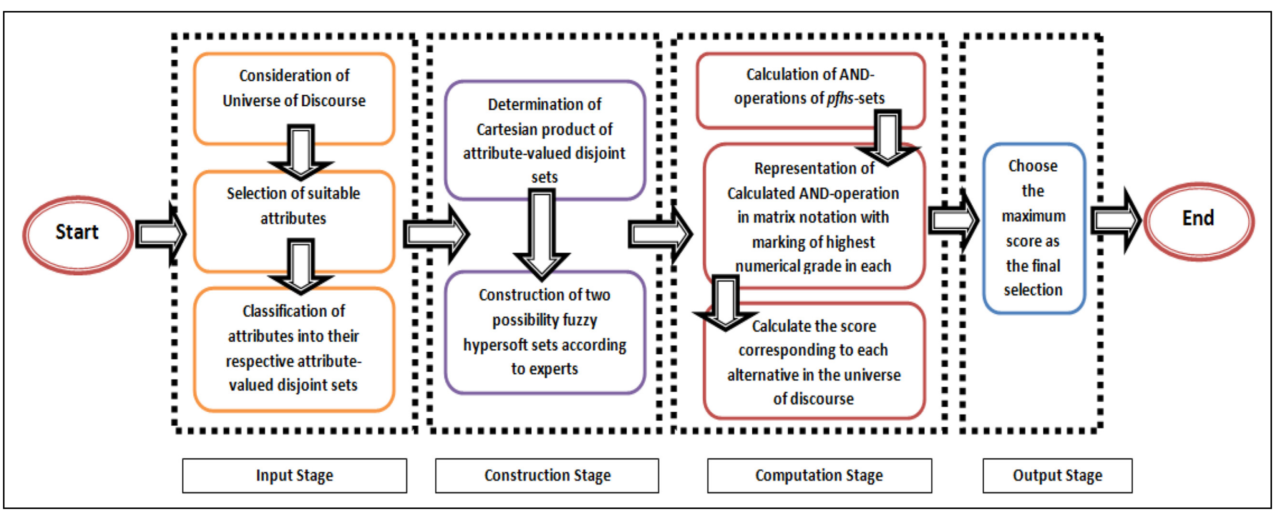

Figure 4. Flow chart of Algorithm 1.

with their mutual understanding for this evaluation. After keen observation of relevant literature on various features of tractors provided by companies, the disjoint subclasses of above parameters along with their relevant parametric values are collected which are stated as below

$$\ddot{\mathfrak{A}}_1 = \{y_{11} = 350, y_{12} = 450\},$$
$$\ddot{\mathfrak{A}}_2 = \{y_{21} = 863.5\},$$
$$\ddot{\mathfrak{A}}_3 = \{y_{31} = 540\},$$
$$\ddot{\mathfrak{A}}_4 = \{y_{41} = 12.6, y_{42} = 13.6\},$$
$$\ddot{\mathfrak{A}}_5 = \{y_{51} = 1100, y_{52} = 1800\}.$$

Construction Stage: 1.4–1.5

Now in order to fulfill the requirements of hypersoft setting, the Cartesian product of above stated parametric valued sub-classes is computed as $\ddot{\mathfrak{A}} = \ddot{\mathfrak{A}}_1 \times \ddot{\mathfrak{A}}_2 \times \ddot{\mathfrak{A}}_3 \times \ddot{\mathfrak{A}}_4 \times \ddot{\mathfrak{A}}_5 = \{\hat{p}_1, \hat{p}_2, \hat{p}_3, \hat{p}_4, \hat{p}_5, \hat{p}_6, \hat{p}_7, \hat{p}_8\}$. For computational convenience, the decision makers decide to ignore the value y_{42} in $\ddot{\mathfrak{A}}_4$ therefore the subset $\ddot{\mathfrak{B}} = \{\hat{p}_1, \hat{p}_2, \hat{p}_5, \hat{p}_6\} \subseteq \ddot{\mathfrak{A}}$ is obtained. Now the decision makers provide their expert opinions in terms of two *pfhs*-sets $\mathscr{A}_\zeta$ and $\mathscr{B}_\xi$ which are collectively stated as follows:

$$\mathscr{A}_\zeta(\hat{p}_1) = \left\{ \begin{array}{l} \left(\frac{\hat{\mathcal{T}}_1}{0.5}, 0.2\right), \left(\frac{\hat{\mathcal{T}}_2}{0.2}, 0.3\right), \left(\frac{\hat{\mathcal{T}}_3}{0.3}, 0.4\right), \\ \left(\frac{\hat{\mathcal{T}}_4}{0.4}, 0.5\right), \left(\frac{\hat{\mathcal{T}}_5}{0.4}, 0.6\right) \end{array} \right\},$$

$$\mathscr{A}_{\zeta}(\hat{p}_2) = \left\{ \begin{array}{l} \left(\frac{\hat{T}'_1}{0.2}, 0.9\right), \left(\frac{\hat{T}'_2}{0.3}, 0.8\right), \left(\frac{\hat{T}'_3}{0.4}, 0.7\right), \\ \left(\frac{\hat{T}'_4}{0.5}, 0.6\right), \left(\frac{\hat{T}'_5}{0.4}, 0.5\right) \end{array} \right\},$$

$$\mathscr{A}_{\zeta}(\hat{p}_5) = \left\{ \begin{array}{l} \left(\frac{\hat{T}'_1}{0.3}, 0.8\right), \left(\frac{\hat{T}'_2}{0.4}, 0.7\right), \left(\frac{\hat{T}'_3}{0.5}, 0.6\right), \\ \left(\frac{\hat{T}'_4}{0.6}, 0.7\right), \left(\frac{\hat{T}'_5}{0.4}, 0.8\right) \end{array} \right\}.$$

$$\mathscr{A}_{\zeta}(\hat{p}_6) = \left\{ \begin{array}{l} \left(\frac{\hat{T}'_1}{0.4}, 0.4\right), \left(\frac{\hat{T}'_2}{0.5}, 0.5\right), \left(\frac{\hat{T}'_3}{0.6}, 0.6\right), \\ \left(\frac{\hat{T}'_4}{0.7}, 0.7\right), \left(\frac{\hat{T}'_5}{0.4}, 0.8\right) \end{array} \right\}$$

$$\mathscr{B}_{\xi}(\hat{p}_1) = \left\{ \begin{array}{l} \left(\frac{\hat{T}'_1}{0.9}, 0.1\right), \left(\frac{\hat{T}'_2}{0.8}, 0.2\right), \left(\frac{\hat{T}'_3}{0.7}, 0.3\right), \\ \left(\frac{\hat{T}'_4}{0.6}, 0.4\right), \left(\frac{\hat{T}'_5}{0.5}, 0.5\right) \end{array} \right\},$$

$$\mathscr{B}_{\xi}(\hat{p}_2) = \left\{ \begin{array}{l} \left(\frac{\hat{T}'_1}{0.8}, 0.3\right), \left(\frac{\hat{T}'_2}{0.7}, 0.4\right), \left(\frac{\hat{T}'_3}{0.5}, 0.5\right), \\ \left(\frac{\hat{T}'_4}{0.5}, 0.6\right), \left(\frac{\hat{T}'_5}{0.6}, 0.7\right) \end{array} \right\},$$

$$\mathscr{B}_{\xi}(\hat{p}_5) = \left\{ \begin{array}{l} \left(\frac{\hat{T}'_1}{0.4}, 0.9\right), \left(\frac{\hat{T}'_2}{0.5}, 0.8\right), \left(\frac{\hat{T}'_3}{0.6}, 0.7\right), \\ \left(\frac{\hat{T}'_4}{0.7}, 0.6\right), \left(\frac{\hat{T}'_5}{0.8}, 0.5\right) \end{array} \right\}.$$

$$\mathscr{B}_{\xi}(\hat{p}_6) = \left\{ \begin{array}{l} \left(\frac{\hat{T}'_1}{0.4}, 0.1\right), \left(\frac{\hat{T}'_2}{0.5}, 0.2\right), \left(\frac{\hat{T}'_3}{0.6}, 0.3\right), \\ \left(\frac{\hat{T}'_4}{0.7}, 0.4\right), \left(\frac{\hat{T}'_5}{0.8}, 0.5\right) \end{array} \right\}.$$

Computation Stage: 1.6–1.8

Now $\mathscr{A}_{\zeta} \wedge \mathscr{B}_{\xi} = \mathcal{V}_{\delta}$ where

$$\mathcal{V}_{\delta}(\hat{p}_1, \hat{p}_1) = \left\{ \begin{array}{l} \left(\frac{\hat{T}'_1}{0.5}, 0.1\right), \left(\frac{\hat{T}'_2}{0.2}, 0.2\right), \left(\frac{\hat{T}'_3}{0.3}, 0.3\right), \\ \left(\frac{\hat{T}'_4}{0.4}, 0.4\right), \left(\frac{\hat{T}'_5}{0.4}, 0.5\right) \end{array} \right\},$$

$$\mathcal{V}_{\delta}(\hat{p}_1, \hat{p}_2) = \left\{ \begin{array}{l} \left(\frac{\hat{T}'_1}{0.5}, 0.2\right), \left(\frac{\hat{T}'_2}{0.2}, 0.3\right), \left(\frac{\hat{T}'_3}{0.3}, 0.4\right), \\ \left(\frac{\hat{T}'_4}{0.4}, 0.5\right), \left(\frac{\hat{T}'_5}{0.4}, 0.6\right) \end{array} \right\},$$

$$\mathcal{V}_{\delta}(\hat{p}_1, \hat{p}_5) = \left\{ \begin{array}{l} \left(\frac{\hat{T}'_1}{0.4}, 0.2\right), \left(\frac{\hat{T}'_2}{0.2}, 0.3\right), \left(\frac{\hat{T}'_3}{0.3}, 0.4\right), \\ \left(\frac{\hat{T}'_4}{0.4}, 0.5\right), \left(\frac{\hat{T}'_5}{0.4}, 0.5\right) \end{array} \right\},$$

$$\mathcal{V}_{\delta}(\hat{p}_1, \hat{p}_6) = \left\{ \begin{array}{l} \left(\frac{\hat{T}'_1}{0.4}, 0.1\right), \left(\frac{\hat{T}'_2}{0.2}, 0.2\right), \left(\frac{\hat{T}'_3}{0.3}, 0.3\right), \\ \left(\frac{\hat{T}'_4}{0.4}, 0.4\right), \left(\frac{\hat{T}'_5}{0.4}, 0.5\right) \end{array} \right\},$$

$$\mathcal{V}_{\delta}(\hat{p}_2, \hat{p}_1) = \left\{ \begin{array}{l} \left(\frac{\hat{T}'_1}{0.2}, 0.1\right), \left(\frac{\hat{T}'_2}{0.3}, 0.2\right), \left(\frac{\hat{T}'_3}{0.4}, 0.3\right), \\ \left(\frac{\hat{T}'_4}{0.5}, 0.4\right), \left(\frac{\hat{T}'_5}{0.4}, 0.5\right) \end{array} \right\},$$

$$\mathcal{V}_{\delta}(\hat{p}_2, \hat{p}_2) = \left\{ \begin{array}{l} \left(\frac{\hat{T}'_1}{0.2}, 0.3\right), \left(\frac{\hat{T}'_2}{0.3}, 0.4\right), \left(\frac{\hat{T}'_3}{0.4}, 0.5\right), \\ \left(\frac{\hat{T}'_4}{0.5}, 0.6\right), \left(\frac{\hat{T}'_5}{0.4}, 0.5\right) \end{array} \right\},$$

$$\mathcal{V}_{\delta}(\hat{p}_2, \hat{p}_5) = \left\{ \begin{array}{l} \left(\frac{\hat{T}'_1}{0.2}, 0.9\right), \left(\frac{\hat{T}'_2}{0.3}, 0.8\right), \left(\frac{\hat{T}'_3}{0.4}, 0.7\right), \\ \left(\frac{\hat{T}'_4}{0.5}, 0.6\right), \left(\frac{\hat{T}'_5}{0.4}, 0.5\right) \end{array} \right\},$$

$$\mathcal{V}_{\delta}(\hat{p}_2, \hat{p}_6) = \left\{ \begin{array}{l} \left(\frac{\hat{T}'_1}{0.2}, 0.1\right), \left(\frac{\hat{T}'_2}{0.3}, 0.2\right), \left(\frac{\hat{T}'_3}{0.4}, 0.3\right), \\ \left(\frac{\hat{T}'_4}{0.5}, 0.4\right), \left(\frac{\hat{T}'_5}{0.4}, 0.5\right) \end{array} \right\}$$

$$\mathcal{V}_{\delta}(\hat{p}_5, \hat{p}_1) = \left\{ \begin{array}{l} \left(\frac{\hat{T}'_1}{0.3}, 0.1\right), \left(\frac{\hat{T}'_2}{0.4}, 0.2\right), \left(\frac{\hat{T}'_3}{0.5}, 0.3\right), \\ \left(\frac{\hat{T}'_4}{0.6}, 0.4\right), \left(\frac{\hat{T}'_5}{0.4}, 0.5\right) \end{array} \right\},$$

$$\mathcal{V}_{\delta}(\hat{p}_5, \hat{p}_2) = \left\{ \begin{array}{l} \left(\frac{\hat{T}'_1}{0.3}, 0.3\right), \left(\frac{\hat{T}'_2}{0.4}, 0.4\right), \left(\frac{\hat{T}'_3}{0.5}, 0.5\right), \\ \left(\frac{\hat{T}'_4}{0.5}, 0.6\right), \left(\frac{\hat{T}'_5}{0.4}, 0.7\right) \end{array} \right\},$$

$$\mathcal{V}_{\delta}(\hat{p}_5, \hat{p}_5) = \left\{ \begin{array}{l} \left(\frac{\hat{T}'_1}{0.3}, 0.8\right), \left(\frac{\hat{T}'_2}{0.4}, 0.7\right), \left(\frac{\hat{T}'_3}{0.5}, 0.6\right), \\ \left(\frac{\hat{T}'_4}{0.6}, 0.6\right), \left(\frac{\hat{T}'_5}{0.4}, 0.5\right) \end{array} \right\},$$

$$\mathcal{V}_{\delta}(\hat{p}_5, \hat{p}_6) = \left\{ \begin{array}{l} \left(\frac{\hat{T}'_1}{0.7}, 0.1\right), \left(\frac{\hat{T}'_2}{0.4}, 0.2\right), \left(\frac{\hat{T}'_3}{0.5}, 0.3\right), \\ \left(\frac{\hat{T}'_4}{0.6}, 0.4\right), \left(\frac{\hat{T}'_5}{0.4}, 0.5\right) \end{array} \right\},$$

$$\mathcal{V}_{\delta}(\hat{p}_6, \hat{p}_1) = \left\{ \begin{array}{l} \left(\frac{\hat{T}'_1}{0.4}, 0.1\right), \left(\frac{\hat{T}'_2}{0.5}, 0.2\right), \left(\frac{\hat{T}'_3}{0.6}, 0.3\right), \\ \left(\frac{\hat{T}'_4}{0.6}, 0.4\right), \left(\frac{\hat{T}'_5}{0.4}, 0.5\right) \end{array} \right\},$$

$$\mathcal{V}_{\delta}(\hat{p}_6, \hat{p}_2) = \left\{ \begin{array}{l} \left(\frac{\hat{T}'_1}{0.4}, 0.3\right), \left(\frac{\hat{T}'_2}{0.5}, 0.4\right), \left(\frac{\hat{T}'_3}{0.5}, 0.5\right), \\ \left(\frac{\hat{T}'_4}{0.5}, 0.6\right), \left(\frac{\hat{T}'_5}{0.4}, 0.7\right) \end{array} \right\},$$

$$\mathcal{V}_{\delta}(\hat{p}_6, \hat{p}_5) = \left\{ \begin{array}{l} \left(\frac{\hat{T}'_1}{0.4}, 0.4\right), \left(\frac{\hat{T}'_2}{0.5}, 0.5\right), \left(\frac{\hat{T}'_3}{0.6}, 0.6\right), \\ \left(\frac{\hat{T}'_4}{0.7}, 0.6\right), \left(\frac{\hat{T}'_5}{0.4}, 0.5\right) \end{array} \right\},$$

$$\mathcal{V}_{\delta}(\hat{p}_6, \hat{p}_6) = \left\{ \begin{array}{l} \left(\frac{\hat{T}'_1}{0.4}, 0.1\right), \left(\frac{\hat{T}'_2}{0.5}, 0.2\right), \left(\frac{\hat{T}'_3}{0.6}, 0.3\right), \\ \left(\frac{\hat{T}'_4}{0.7}, 0.4\right), \left(\frac{\hat{T}'_5}{0.4}, 0.5\right) \end{array} \right\},$$

$$\mathcal{V}_{\delta}(\hat{p}_6, \hat{p}_1) = \left\{ \begin{array}{l} \left(\frac{\hat{T}'_1}{0.4}, 0.1\right), \left(\frac{\hat{T}'_2}{0.5}, 0.2\right), \left(\frac{\hat{T}'_3}{0.6}, 0.3\right), \\ \left(\frac{\hat{T}'_4}{0.6}, 0.4\right), \left(\frac{\hat{T}'_5}{0.4}, 0.5\right) \end{array} \right\},$$

$$\mathcal{V}_{\delta}(\hat{p}_6, \hat{p}_2) = \left\{ \begin{array}{l} \left(\frac{\hat{T}'_1}{0.4}, 0.3\right), \left(\frac{\hat{T}'_2}{0.5}, 0.4\right), \left(\frac{\hat{T}'_3}{0.5}, 0.5\right), \\ \left(\frac{\hat{T}'_4}{0.5}, 0.6\right), \left(\frac{\hat{T}'_5}{0.4}, 0.7\right) \end{array} \right\},$$

$$\mathcal{V}_{\delta}(\hat{p}_6, \hat{p}_5) = \left\{ \begin{array}{l} \left(\frac{\hat{T}'_1}{0.4}, 0.4\right), \left(\frac{\hat{T}'_2}{0.5}, 0.5\right), \left(\frac{\hat{T}'_3}{0.6}, 0.6\right), \\ \left(\frac{\hat{T}'_4}{0.7}, 0.6\right), \left(\frac{\hat{T}'_5}{0.4}, 0.5\right) \end{array} \right\},$$

Table 1. AND-operation based grade table.

$\mathcal{V}_\delta$	$\hat{T}^r{}_i$	Highest numerical grade	σ_i
$(\hat{p}_1, \hat{p}_1)$	$\hat{T}^r{}_1$	0	0
$(\hat{p}_1, \hat{p}_2)$	$\hat{T}^r{}_1$	0.5	0.2
$(\hat{p}_1, \hat{p}_5)$	$\hat{T}^r{}_1, \hat{T}^r{}_4, \hat{T}^r{}_5$	0.4, 0.4, 0.4	0.2, 0.5, 0.5
$(\hat{p}_1, \hat{p}_6)$	$\hat{T}^r{}_1, \hat{T}^r{}_4, \hat{T}^r{}_5$	0.4, 0.4, 0.4	0.1, 0.4, 0.5
$(\hat{p}_2, \hat{p}_1)$	$\hat{T}^r{}_4$	0.5	0.4
$(\hat{p}_2, \hat{p}_2)$	$\hat{T}^r{}_4$	0	0
$(\hat{p}_2, \hat{p}_5)$	$\hat{T}^r{}_4$	0.5	0.6
$(\hat{p}_2, \hat{p}_6)$	$\hat{T}^r{}_4$	0.5	0.4
$(\hat{p}_5, \hat{p}_1)$	$\hat{T}^r{}_4$	0.6	0.4
$(\hat{p}_5, \hat{p}_2)$	$\hat{T}^r{}_3, \hat{T}^r{}_4$	0.5, 0.5	0.5, 0.6
$(\hat{p}_5, \hat{p}_5)$	$\hat{T}^r{}_4$	0	0
$(\hat{p}_5, \hat{p}_6)$	$\hat{T}^r{}_1$	0.7	0.1
$(\hat{p}_6, \hat{p}_1)$	$\hat{T}^r{}_3, \hat{T}^r{}_4$	0.6, 0.6	0.3, 0.4
$(\hat{p}_6, \hat{p}_2)$	$\hat{T}^r{}_2, \hat{T}^r{}_3, \hat{T}^r{}_4$	0.5, 0.5, 0.5	0.4, 0.5, 0.6
$(\hat{p}_6, \hat{p}_5)$	$\hat{T}^r{}_4$	0.7	0.6
$(\hat{p}_6, \hat{p}_6)$	$\hat{T}^r{}_4$	0	0

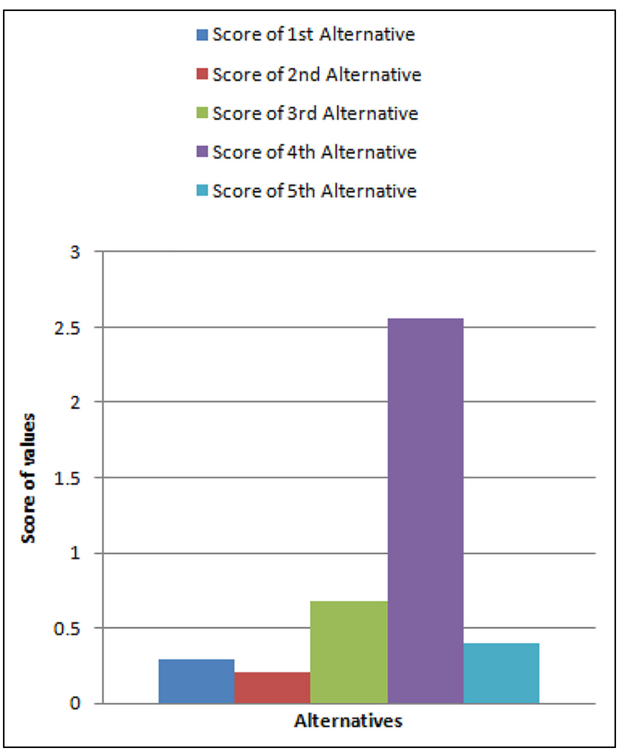

Figure 5. Score values for AND-operation.

$$\mathcal{V}_\delta(\hat{p}_6, \hat{p}_6) = \left\{ \begin{array}{l} \left(\frac{\hat{T}^r{}_1}{0.4}, 0.1\right), \left(\frac{\hat{T}^r{}_2}{0.5}, 0.2\right), \left(\frac{\hat{T}^r{}_3}{0.6}, 0.3\right), \\ \left(\frac{\hat{T}^r{}_4}{0.7}, 0.4\right), \left(\frac{\hat{T}^r{}_5}{0.4}, 0.5\right) \end{array} \right\},$$

Matrix notation of $\mathcal{V}_\delta$ is given as

$$\mathcal{V}_\delta = \begin{pmatrix} (\mathbf{0.5},0.1) & (0.2,0.2) & (0.3,0.3) & (0.4,0.4) & (\mathbf{0.4},0.5) \\ (\mathbf{0.5},0.2) & (0.2,0.3) & (0.3,0.4) & (0.4,0.5) & (\mathbf{0.4},0.6) \\ (\mathbf{0.4},0.2) & (0.2,0.3) & (0.3,0.4) & (\mathbf{0.4},0.5) & (0.4,0.5) \\ (\mathbf{0.4},0.1) & (0.2,0.2) & (0.3,0.3) & (\mathbf{0.4},0.4) & (0.4,0.5) \\ (0.2,0.1) & (0.3,0.2) & (0.4,0.3) & (\mathbf{0.5},0.4) & (0.4,0.5) \\ (0.2,0.3) & (0.3,0.4) & (0.4,0.5) & (\mathbf{0.5},0.6) & (0.4,0.5) \\ (0.2,0.9) & (0.3,0.8) & (0.4,0.7) & (\mathbf{0.5},0.6) & (0.4,0.5) \\ (0.2,0.1) & (0.3,0.2) & (0.4,0.3) & (\mathbf{0.5},0.4) & (0.4,0.5) \\ (0.3,0.1) & (0.4,0.2) & (0.5,0.3) & (\mathbf{0.6},0.4) & (0.4,0.5) \\ (0.3,0.3) & (0.4,0.4) & (\mathbf{0.5},0.5) & (\mathbf{0.5},0.6) & (0.4,0.7) \\ (0.3,0.8) & (0.4,0.7) & (0.5,0.6) & (\mathbf{0.6},0.6) & (0.4,0.5) \\ (\mathbf{0.7},0.1) & (0.4,0.2) & (0.5,0.3) & (0.6,0.4) & (0.4,0.5) \\ (0.4,0.1) & (0.5,0.2) & (\mathbf{0.6},0.3) & (\mathbf{0.6},0.4) & (0.4,0.5) \\ (0.4,0.3) & (\mathbf{0.5},0.4) & (\mathbf{0.5},0.5) & (\mathbf{0.5},0.6) & (0.4,0.7) \\ (0.4,0.4) & (0.5,0.5) & (0.6,0.6) & (\mathbf{0.7},0.6) & (0.4,0.5) \\ (0.4,0.1) & (0.5,0.2) & (0.6,0.3) & (\mathbf{0.7},0.4) & (0.4,0.5) \end{pmatrix}$$

Table 1 provides values of ρ and σ. Both ρ and σ are taken 0 for pairs $(\hat{p}_i, \hat{p}_i)$, $i = 1, 2, 3$.

$$\Delta(\hat{T}^r{}_1) = (0 \times 0) + (0.5 \times 0.2) + (0.4 \times 0.2) + (0.4 \times 0.1) + (0.7 \times 0.1) = 0.29.$$

$$\Delta(\hat{T}^r{}_2) = (0.5 \times 0.4) = 0.20.$$

$$\Delta(\hat{T}^r{}_3) = (0.5 \times 0.5) + (0.6 \times 0.3) + (0.5 \times 0.5) = 0.68.$$

$$\Delta(\hat{T}^r{}_4) = (0.4 \times 0.5) + (0.4 \times 0.4) + (0.5 \times 0.4) + (0 \times 0) + (0.5 \times 0.6) + (0.5 \times 0.4) + (0.6 \times 0.4) + (0.5 \times 0.6) + (0 \times 0) + (0.6 \times 0.4) + (0.5 \times 0.6) + (0.7 \times 0.6) + (0 \times 0) = 2.56. \Delta(\hat{T}^r{}_5) = (0.4 \times 0.5) + (0.4 \times 0.5) = 0.4.$$

Output Stage: 1.9

Score of $\hat{T}^r{}_4$ is maximum see Figure 5, therefore it is selected.

The flowing procedure of Algorithm 2 is presented in Figure 6.

Example 0.19. (Input Stage: 1.1–1.3) Reconsidering the statement of Example 0.18, these three steps are repeated as they are discussed in Algorithm 1.

Construction Stage: 1.4–1.5

Similarly these two steps are also followed as they are presented in Algorithm 1.

Computation Stage: 1.6–1.8

Now $\mathscr{A}_\zeta \vee \mathscr{B}_\xi = \mathcal{X}_\rho$ where

Algorithm 2: Procedural description of an optimized tractor evaluation based on OR-operation of *pfhs*-sets

▶ **Start**

▶ **Input:**

1.1. As in Algorithm 1.

1.2. As in Algorithm 1.

1.3. As in Algorithm 1s.

▶ **Construction:**

1.4. As in Algorithm 1.

1.5. As in Algorithm 1.

▶ **Computation:**

1.6. Using $\mathscr{A}_{\zeta}$ and $\mathscr{B}_{\xi}$, compute OR-operation $\mathcal{X}_{\rho}$.

1.7. Tabulate the computational values of $\mathcal{X}_{\rho}$ in matrix form and mention the maximum computational grade ρ row-wise.

1.8. Using $\sum_i \rho_i.\sigma_i$, compute score values Δ_i for each alternative.

▶ **Output:**

1.9. Select the alternative with maximum score as discrete optimum selection.

▶ **End**

$$\mathcal{X}_{\varrho}(\hat{p}_1,\hat{p}_1)=\left\{\begin{array}{l}\left(\frac{\hat{\mathcal{T}}'_1}{0.9},0.2\right),\left(\frac{\hat{\mathcal{T}}'_2}{0.8},0.3\right),\left(\frac{\hat{\mathcal{T}}'_3}{0.7},0.4\right),\\ \left(\frac{\hat{\mathcal{T}}'_4}{0.6},0.5\right),\left(\frac{\hat{\mathcal{T}}'_5}{0.5},0.6\right)\end{array}\right\},$$

$$\mathcal{X}_{\varrho}(\hat{p}_1,\hat{p}_2)=\left\{\begin{array}{l}\left(\frac{\hat{\mathcal{T}}'_1}{0.8},0.3\right),\left(\frac{\hat{\mathcal{T}}'_2}{0.7},0.4\right),\left(\frac{\hat{\mathcal{T}}'_3}{0.5},0.5\right),\\ \left(\frac{\hat{\mathcal{T}}'_4}{0.5},0.6\right),\left(\frac{\hat{\mathcal{T}}'_5}{0.6},0.7\right)\end{array}\right\},$$

$$\mathcal{X}_{\varrho}(\hat{p}_1,\hat{p}_5)=\left\{\begin{array}{l}\left(\frac{\hat{\mathcal{T}}'_1}{0.5},0.9\right),\left(\frac{\hat{\mathcal{T}}'_2}{0.5},0.8\right),\left(\frac{\hat{\mathcal{T}}'_3}{0.6},0.7\right),\\ \left(\frac{\hat{\mathcal{T}}'_4}{0.7},0.6\right),\left(\frac{\hat{\mathcal{T}}'_5}{0.8},0.6\right)\end{array}\right\},$$

$$\mathcal{X}_{\varrho}(\hat{p}_1,\hat{p}_6)=\left\{\begin{array}{l}\left(\frac{\hat{\mathcal{T}}'_1}{0.5},0.2\right),\left(\frac{\hat{\mathcal{T}}'_2}{0.5},0.3\right),\left(\frac{\hat{\mathcal{T}}'_3}{0.6},0.4\right),\\ \left(\frac{\hat{\mathcal{T}}'_4}{0.7},0.5\right),\left(\frac{\hat{\mathcal{T}}'_5}{0.8},0.6\right)\end{array}\right\},$$

$$\mathcal{X}_{\varrho}(\hat{p}_2,\hat{p}_1)=\left\{\begin{array}{l}\left(\frac{\hat{\mathcal{T}}'_1}{0.9},0.9\right),\left(\frac{\hat{\mathcal{T}}'_2}{0.8},0.8\right),\left(\frac{\hat{\mathcal{T}}'_3}{0.7},0.7\right),\\ \left(\frac{\hat{\mathcal{T}}'_4}{0.6},0.6\right),\left(\frac{\hat{\mathcal{T}}'_5}{0.5},0.5\right)\end{array}\right\},$$

$$\mathcal{X}_{\varrho}(\hat{p}_2,\hat{p}_2)=\left\{\begin{array}{l}\left(\frac{\hat{\mathcal{T}}'_1}{0.8},0.9\right),\left(\frac{\hat{\mathcal{T}}'_2}{0.7},0.8\right),\left(\frac{\hat{\mathcal{T}}'_3}{0.5},0.7\right),\\ \left(\frac{\hat{\mathcal{T}}'_4}{0.5},0.6\right),\left(\frac{\hat{\mathcal{T}}'_5}{0.6},0.7\right)\end{array}\right\},$$

$$\mathcal{X}_{\varrho}(\hat{p}_2,\hat{p}_5)=\left\{\begin{array}{l}\left(\frac{\hat{\mathcal{T}}'_1}{0.4},0.9\right),\left(\frac{\hat{\mathcal{T}}'_2}{0.5},0.8\right),\left(\frac{\hat{\mathcal{T}}'_3}{0.6},0.7\right),\\ \left(\frac{\hat{\mathcal{T}}'_4}{0.7},0.6\right),\left(\frac{\hat{\mathcal{T}}'_5}{0.8},0.5\right)\end{array}\right\},$$

$$\mathcal{X}_{\varrho}(\hat{p}_2,\hat{p}_6)=\left\{\begin{array}{l}\left(\frac{\hat{\mathcal{T}}'_1}{0.4},0.9\right),\left(\frac{\hat{\mathcal{T}}'_2}{0.5},0.8\right),\left(\frac{\hat{\mathcal{T}}'_3}{0.6},0.7\right),\\ \left(\frac{\hat{\mathcal{T}}'_4}{0.7},0.6\right),\left(\frac{\hat{\mathcal{T}}'_5}{0.8},0.5\right)\end{array}\right\},$$

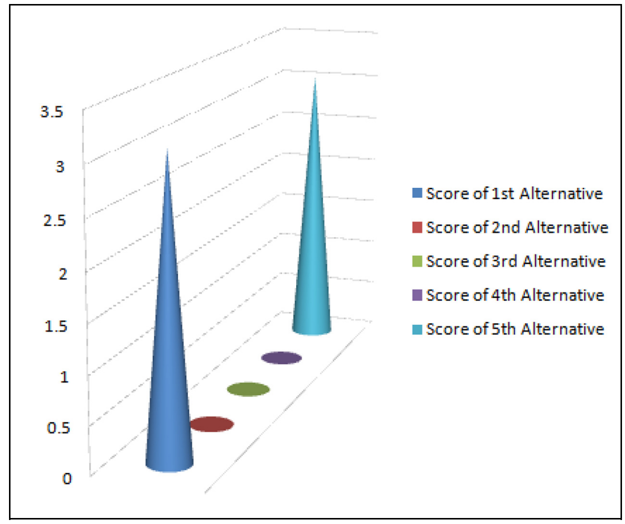

Figure 6. Flow chart of Algorithm 2.

$$\mathcal{X}_{\varrho}(\hat{p}_5,\hat{p}_1)=\left\{\begin{array}{l}\left(\frac{\hat{\mathcal{T}}'_1}{0.9},0.8\right),\left(\frac{\hat{\mathcal{T}}'_2}{0.8},0.7\right),\left(\frac{\hat{\mathcal{T}}'_3}{0.7},0.6\right),\\ \left(\frac{\hat{\mathcal{T}}'_4}{0.6},0.7\right),\left(\frac{\hat{\mathcal{T}}'_5}{0.5},0.8\right)\end{array}\right\},$$

$$\mathcal{X}_{\varrho}(\hat{p}_5,\hat{p}_2)=\left\{\begin{array}{l}\left(\frac{\hat{\mathcal{T}}'_1}{0.8},0.8\right),\left(\frac{\hat{\mathcal{T}}'_2}{0.7},0.7\right),\left(\frac{\hat{\mathcal{T}}'_3}{0.5},0.6\right),\\ \left(\frac{\hat{\mathcal{T}}'_4}{0.6},0.7\right),\left(\frac{\hat{\mathcal{T}}'_5}{0.6},0.8\right)\end{array}\right\},$$

$$\mathcal{X}_{\varrho}(\hat{p}_5,\hat{p}_5)=\left\{\begin{array}{l}\left(\frac{\hat{\mathcal{T}}'_1}{0.4},0.9\right),\left(\frac{\hat{\mathcal{T}}'_2}{0.5},0.8\right),\left(\frac{\hat{\mathcal{T}}'_3}{0.6},0.7\right),\\ \left(\frac{\hat{\mathcal{T}}'_4}{0.7},0.7\right),\left(\frac{\hat{\mathcal{T}}'_5}{0.8},0.8\right)\end{array}\right\},$$

$$\mathcal{X}_{\varrho}(\hat{p}_5,\hat{p}_6)=\left\{\begin{array}{l}\left(\frac{\hat{\mathcal{T}}'_1}{0.4},0.8\right),\left(\frac{\hat{\mathcal{T}}'_2}{0.5},0.7\right),\left(\frac{\hat{\mathcal{T}}'_3}{0.6},0.6\right),\\ \left(\frac{\hat{\mathcal{T}}'_4}{0.7},0.7\right),\left(\frac{\hat{\mathcal{T}}'_5}{0.8},0.8\right)\end{array}\right\},$$

$$\mathcal{X}_{\varrho}(\hat{p}_6,\hat{p}_1)=\left\{\begin{array}{l}\left(\frac{\hat{\mathcal{T}}'_1}{0.9},0.4\right),\left(\frac{\hat{\mathcal{T}}'_2}{0.8},0.5\right),\left(\frac{\hat{\mathcal{T}}'_3}{0.7},0.6\right),\\ \left(\frac{\hat{\mathcal{T}}'_4}{0.6},0.7\right),\left(\frac{\hat{\mathcal{T}}'_5}{0.5},0.8\right)\end{array}\right\},$$

$$\mathcal{X}_{\varrho}(\hat{p}_6,\hat{p}_2)=\left\{\begin{array}{l}\left(\frac{\hat{\mathcal{T}}'_1}{0.8},0.4\right),\left(\frac{\hat{\mathcal{T}}'_2}{0.7},0.5\right),\left(\frac{\hat{\mathcal{T}}'_3}{0.6},0.6\right),\\ \left(\frac{\hat{\mathcal{T}}'_4}{0.7},0.7\right),\left(\frac{\hat{\mathcal{T}}'_5}{0.6},0.8\right)\end{array}\right\},$$

$$\mathcal{X}_{\varrho}(\hat{p}_6,\hat{p}_5)=\left\{\begin{array}{l}\left(\frac{\hat{\mathcal{T}}'_1}{0.4},0.9\right),\left(\frac{\hat{\mathcal{T}}'_2}{0.5},0.8\right),\left(\frac{\hat{\mathcal{T}}'_3}{0.6},0.7\right),\\ \left(\frac{\hat{\mathcal{T}}'_4}{0.7},0.7\right),\left(\frac{\hat{\mathcal{T}}'_5}{0.8},0.8\right)\end{array}\right\},$$

$$\mathcal{X}_{\varrho}(\hat{p}_6,\hat{p}_6)=\left\{\begin{array}{l}\left(\frac{\hat{\mathcal{T}}'_1}{0.4},0.4\right),\left(\frac{\hat{\mathcal{T}}'_2}{0.5},0.5\right),\left(\frac{\hat{\mathcal{T}}'_3}{0.6},0.6\right),\\ \left(\frac{\hat{\mathcal{T}}'_4}{0.7},0.7\right),\left(\frac{\hat{\mathcal{T}}'_5}{0.8},0.8\right)\end{array}\right\},$$

Matrix notation of $\mathcal{X}_{\rho}$ is given as

$$\mathcal{X}_\varrho = \begin{pmatrix} (0.9,0.2) & (0.8,0.3) & (0.7,0.4) & (0.6,0.5) & (0.5,0.6) \\ (0.8,0.3) & (0.7,0.4) & (0.5,0.5) & (0.5,0.6) & (0.6,0.7) \\ (0.5,0.9) & (0.5,0.8) & (0.6,0.7) & (0.7,0.6) & (0.8,0.6) \\ (0.5,0.2) & (0.5,0.3) & (0.6,0.4) & (0.7,0.5) & (0.8,0.6) \\ (0.9,0.9) & (0.8,0.8) & (0.7,0.7) & (0.6,0.6) & (0.5,0.5) \\ (0.8,0.9) & (0.7,0.8) & (0.5,0.7) & (0.5,0.6) & (0.6,0.7) \\ (0.4,0.9) & (0.5,0.8) & (0.6,0.7) & (0.7,0.6) & (0.8,0.5) \\ (0.4,0.9) & (0.5,0.8) & (0.6,0.7) & (0.7,0.6) & (0.8,0.5) \\ (0.9,0.8) & (0.8,0.7) & (0.7,0.6) & (0.6,0.7) & (0.6,0.8) \\ (0.8,0.8) & (0.7,0.7) & (0.5,0.6) & (0.6,0.7) & (0.6,0.8) \\ (0.4,0.9) & (0.5,0.8) & (0.6,0.7) & (0.7,0.7) & (0.8,0.8) \\ (0.4,0.8) & (0.5,0.7) & (0.6,0.6) & (0.7,0.7) & (0.8,0.8) \\ (0.9,0.4) & (0.8,0.5) & (0.7,0.6) & (0.6,0.7) & (0.5,0.8) \\ (0.8,0.4) & (0.7,0.5) & (0.6,0.6) & (0.7,0.7) & (0.6,0.8) \\ (0.4,0.9) & (0.5,0.8) & (0.6,0.7) & (0.7,0.7) & (0.8,0.8) \\ (0.4,0.4) & (0.5,0.5) & (0.6,0.6) & (0.7,0.7) & (0.8,0.8) \end{pmatrix}$$

Table 2 presents the values of ρ and σ. Both ρ and σ are considered 0 for pairs $(\hat{p}_i, \hat{p}_i), i = 1, 2, 3$.

$$Score(\hat{T}^{r'}_1) = \Delta(\hat{T}^{r'}_1) = (0 \times 0) + (0.8 \times 0.3) + (0.9 \times 0.9) + (0 \times 0) + (0.9 \times 0.8) + (0.8 \times 0.8) + (0.9 \times 0.4) + (0.8 \times 0.4) = 3.09$$

$$\Delta(\hat{T}^{r}_2) = 0$$

$$\Delta(\hat{T}^{r}_3) = 0$$

$$\Delta(\hat{T}^{r}_4) = 0$$

$$\Delta(\hat{T}^{r'}_5) = (0.8 \times 0.6) + (0.8 \times 0.6) + (0.8 \times 0.5) + (0.8 \times 0.5) + (0 \times 0) + (0.8 \times 0.8) + (0.8 \times 0.8) + (0 \times 0) = 3.04$$

Output Stage: 1.9

As score of $\hat{T}^r_1$ is maximum (see Figure 7), therefore it is selected.

The computed ranking comparison of both algorithms is depicted in Table 3 and Figure 8. After calculating mean score of both methods that is 0.826 for AND-based algorithm and 1.226 for OR-based algorithm. As $0.826 \in [0,1]$ but $1.226 \notin [0,1]$ implies that certain weights are required to be applied for OR-operation to make it consistent.

Similarity measures between *pfhs*-sets

Now this segment of the article presents a formulation criterion to compute the similarity between two *pfhs*-sets.

Definition 0.20. Let $\mathscr{A}_\zeta$ and $\mathscr{B}_\xi$ are *pfhs*-sets then their similarity (symbolized as $\mathfrak{S}(\mathscr{A}_\zeta, \mathscr{B}_\xi)$) can be computed by the following method:

Table 2. OR-operation based grade table.

$\mathcal{V}_\delta$	$\hat{T}^r_i$	Highest numerical grade	σ_i
$(\hat{p}_1, \hat{p}_1)$	$\hat{T}^r_1$	0	0
$(\hat{p}_1, \hat{p}_2)$	$\hat{T}^r_1$	0.8	0.3
$(\hat{p}_1, \hat{p}_5)$	$\hat{T}^r_5$	0.8	0.6
$(\hat{p}_1, \hat{p}_6)$	$\hat{T}^r_5$	0.8	0.6
$(\hat{p}_2, \hat{p}_1)$	$\hat{T}^r_1$	0.9	0.9
$(\hat{p}_2, \hat{p}_2)$	$\hat{T}^r_1$	0	0
$(\hat{p}_2, \hat{p}_5)$	$\hat{T}^r_5$	0.8	0.5
$(\hat{p}_2, \hat{p}_6)$	$\hat{T}^r_5$	0.8	0.5
$(\hat{p}_5, \hat{p}_1)$	$\hat{T}^r_1$	0.9	0.8
$(\hat{p}_5, \hat{p}_2)$	$\hat{T}^r_1$	0.8	0.8
$(\hat{p}_5, \hat{p}_5)$	$\hat{T}^r_5$	0	0
$(\hat{p}_5, \hat{p}_6)$	$\hat{T}^r_5$	0.8	0.8
$(\hat{p}_6, \hat{p}_1)$	$\hat{T}^r_1$	0.9	0.4
$(\hat{p}_6, \hat{p}_2)$	$\hat{T}^r_1$	0.8	0.4
$(\hat{p}_6, \hat{p}_5)$	$\hat{T}^r_5$	0.8	0.8
$(\hat{p}_6, \hat{p}_6)$	$\hat{T}^r_5$	0	0

$\mathfrak{S}(\mathscr{A}_\zeta, \mathscr{B}_\xi) = \mathfrak{M}(\mathscr{A}(\varepsilon), \mathscr{B}(\varepsilon)) \times \mathfrak{M}(\zeta(\varepsilon), \xi(\varepsilon))$, such that

$$\mathfrak{M}(\mathscr{A}(\varepsilon), \mathscr{B}(\varepsilon)) = max\, \mathfrak{M}_i(\mathscr{A}(\varepsilon), \mathscr{B}(\varepsilon)),$$

$$\mathfrak{M}(\zeta(\varepsilon), \xi(\varepsilon)) = \max\, \mathfrak{M}_i(\zeta(\varepsilon), \xi(\varepsilon)),$$

with

$$\mathfrak{M}_i(\mathscr{A}(\varepsilon), \mathscr{B}(\varepsilon)) = 1 - \frac{\sum_{j=1}^{n} \left|\mathscr{A}_{ij}(\varepsilon) - \mathscr{B}_{ij}(\varepsilon)\right|}{\sum_{j=1}^{n} \left|\mathscr{A}_{ij}(\varepsilon) + \mathscr{B}_{ij}(\varepsilon)\right|}$$

and

$$\mathfrak{M}_i(\zeta(\varepsilon), \xi(\varepsilon)) = 1 - \frac{\sum_{j=1}^{n} \left|\zeta_{ij}(\varepsilon) - \xi_{ij}(\varepsilon)\right|}{\sum_{j=1}^{n} \left|\zeta_{ij}(\varepsilon) + \xi_{ij}(\varepsilon)\right|}.$$

Definition 0.21. Let $\mathscr{A}_\zeta, \mathscr{B}_\xi \in \Omega_{pfhss}$ then they are claimed as radically similar

$$if\, \mathfrak{S}(\mathscr{A}_\zeta, \mathscr{B}_\xi) \geqslant \tfrac{1}{2}.$$

Example 0.22. Consider *pfhs*-sets from Example 0.18, we have

$$\mathfrak{M}_1(\zeta(\hat{p}_1), \xi(\hat{p}_1)) = 1 - \frac{\sum_{j=1}^{5} \left|\zeta_{1j}(\hat{p}_1) - \xi_{1j}(\hat{p}_1)\right|}{\sum_{j=1}^{5} \left|\zeta_{1j}(\hat{p}_1) + \xi_{1j}(\hat{p}_1)\right|}$$

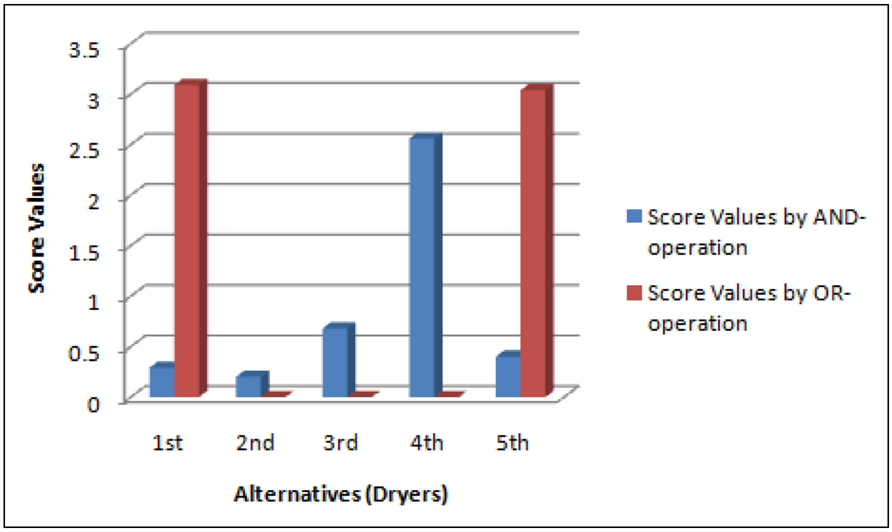

Figure 7. Score values for OR-operation.

Table 3. Tabulation of score values for AND and OR-operations.

Aggregation operation	$\hat{T}'_1$	$\hat{T}'_2$	$\hat{T}'_3$	$\hat{T}'_4$	$\hat{T}'_5$	Mean score	Ranking
AND-operation	0.29	0.20	0.68	2.56	0.40	0.826	$\hat{T}'_4 \succ \hat{T}'_3 \succ \hat{T}'_5 \succ \hat{T}'_1 \succ \hat{T}'_2$
OR-operation	3.09	0.00	0.00	0.00	3.04	1.226	$\hat{T}'_1 \succ \hat{T}'_5 \succ \hat{T}'_2 = \hat{T}'_3 = \hat{T}'_4$

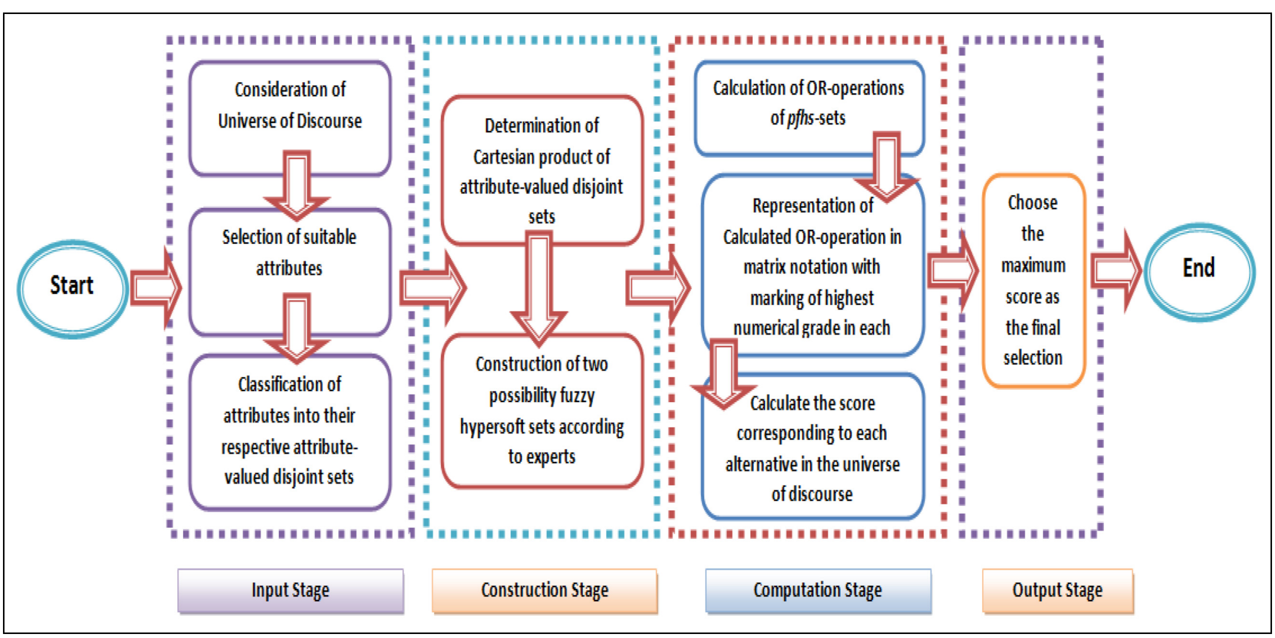

Figure 8. The relationship of scores for AND and OR-operations.

$$= 1 - \frac{|(0.2 - 0.1)| + |(0.3 - 0.2)| + |(0.4 - 0.3)| + |(0.5 - 0.4)| + |(0.6 - 0.5)|}{|(0.2 + 0.1)| + |(0.3 + 0.2)| + |(0.4 + 0.3)| + |(0.5 + 0.4)| + |(0.6 + 0.5)|} = 0.86.$$

Algorithm 3. Recruitment pattern recognition by using similarity measures of *pfhs*-sets

▷ **Start**

▷ **Input:**

1. Assume a universal set $\mathcal{Z}$ with only two possible elements identifying agree and disagree based nature.
2. Collect suitable attributes $\hat{p}_i, i = 1, 2, 3, ..., n$ for recruitment process.
3. Classify n attributes into attribute-valued non-overlapping sets $\ddot{\mathfrak{A}}_1, \ddot{\mathfrak{A}}_2, \ddot{\mathfrak{A}}_3, ..., \ddot{\mathfrak{A}}_n$.
4. Determine $\mathcal{Q} = \ddot{\mathfrak{A}}_1 \times \ddot{\mathfrak{A}}_2 \times \ddot{\mathfrak{A}}_3 \times ... \times \ddot{\mathfrak{A}}_n$.
5. Choose any suitable subset $\mathcal{Q}_1 \subseteq \mathcal{Q}$.

▷ **Construction:**

6. Construct a model *pfhs*-set $\mathscr{A}_\zeta$ for standard recruitment.
7. Construct a *pfhs*-set $\mathscr{B}_\xi$ for candidate.
8. Represent *pfhs*-sets $\mathscr{A}_\zeta$ and $\mathscr{B}_\xi$ in matrix notations.

▷ **Computation:**

9. Compute similarity measures between *pfhs*-sets $\mathscr{A}_\zeta$ and $\mathscr{B}_\xi$ that is $\mathfrak{S}(\mathscr{A}_\zeta, \mathscr{B}_\xi)$ by using Definition 0.20.

▷ **Output:**

10. If $\mathfrak{S}(\mathscr{A}_\zeta, \mathscr{B}_\xi) \geqslant \frac{1}{2}$ then similarity is radical thus recommended is opted as final decision.

▷ **End**

Similarly

$$\mathfrak{M}_2(\zeta(\hat{p}_2), \xi(\hat{p}_2)) = 0.77,$$

$$\mathfrak{M}_3(\zeta(\hat{p}_5), \xi(\hat{p}_5)) = 0.90,$$

$$\mathfrak{M}_4(\zeta(\hat{p}_6), \xi(\hat{p}_6)) = 0.67, \text{ therefore}$$

$$\mathfrak{M}(\zeta(p), \xi(p)) = 0.90.$$

Now

$$\mathfrak{M}_1(\mathscr{A}(\hat{p}_1), \mathscr{B}(\hat{p}_1)) = 1 - \frac{\sum_{j=1}^{4} \left| \mathscr{A}_{1j}(\hat{p}_1) - \mathscr{B}_{1j}(\hat{p}_1) \right|}{\sum_{j=1}^{4} \left| \mathscr{A}_{1j}(\hat{p}_1) + \mathscr{B}_{1j}(\hat{p}_1) \right|}$$

$$= 1 - \frac{|(0.5-0.9)| + |(0.2-0.8)| + |(0.3-0.7)| + |(0.4-0.6)| + |(0.4-0.5)|}{|(0.5+0.9)| + |(0.2+0.8)| + |(0.3+0.7)| + |(0.4+0.6)| + |(0.4+0.5)|} = 0.68.$$

Similarly

$$\mathfrak{M}_2(\mathscr{A}(\hat{p}_2), \mathscr{B}(\hat{p}_2)) = 0.73,$$

$$\mathfrak{M}_3(\mathscr{A}(\hat{p}_5), \mathscr{B}(\hat{p}_5)) = 0.85,$$

$$\mathfrak{M}_4(\mathscr{A}(\hat{p}_6), \mathscr{B}(\hat{p}_6)) = 0.93, \text{ therefore}$$

$$\mathfrak{M}(\mathscr{A}(p), \mathscr{B}(p)) = 0.93.$$

Thus $\mathfrak{S}(\mathscr{A}_\zeta, \mathscr{B}_\xi) = 0.90 \times 0.93 \cong 0.84$ which leads to the conclusion that the similarity of $\mathscr{A}_\zeta$ and $\mathscr{B}_\xi$ is radical.

Application of similarity between pfhs-sets in recruitment pattern recognition

In this part, it is tried to assess the possibility whether an applicant with prescribed qualification and skill, is appropriate for a post in a corporation or not. In this regards a model *pfhs*-set is designed for standard recruitment and another *pfhs*-set is designed for applicant. After computation of their mutual similarity measures, the applicant will be recommended if the computed similarity is found radical.

The flowing procedure of Algorithm 3 is presented in Figure 9.

Example 0.23. Consider an organization intends to recruit some employees against some vacant posts. Its human resource management (HRM) department has been assigned the task to assess the suitability of the employees one by one against the particular posts for recruitment. In this regards, only two members "agree $= z_1$" and "disagree $= z_2$" are considered in universal set that is $\mathcal{Z} = \{z_1, z_2\}$. Before the initiating the evaluation process, a committee is formulated which consists of some staff members of HRM department having expertise in recruitment process. With mutual understanding, the members of the committee (decision makers) are agreed upon some evaluating attributes for this recruitment. Let these attributes are $\hat{p}_1 =$ qualification, $\hat{p}_2 =$ age, and $\hat{p}_5 =$ experience. These parameters are further partitioned into disjoint attribute-valued sets that are given as

$$\ddot{\mathfrak{A}}_1 = \{\hat{p}_{11} = BachelorDegree, \hat{p}_{12} = MasterDegree\}$$

$$\ddot{\mathfrak{A}}_2 = \{\hat{p}_{21} = 25years, \hat{p}_{22} = 30years\}$$

$$\ddot{\mathfrak{A}}_2 = \{\hat{p}_{21} = 5years, \hat{p}_{22} = 10years\}$$

then

$$\mathcal{Q} = \ddot{\mathfrak{A}}_1 \times \ddot{\mathfrak{A}}_2 \times \ddot{\mathfrak{A}}_3 = \{q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8\}.$$

Let $\mathcal{Q}_1 = \{q_3, q_4, q_7, q_8\}$ is a subset of $\mathcal{Q}$. Consider a model pfhs-set for standard recruitment is $\mathscr{A}_\zeta$.

$$\mathscr{A}_\zeta(q_3) = \left\{ \left(\frac{z_1}{<1>}, 1\right), \left(\frac{z_2}{<1>}, 1\right) \right\},$$
$$\mathscr{A}_\zeta(q_4) = \left\{ \left(\frac{z_1}{<1>}, 1\right), \left(\frac{z_2}{<1>}, 1\right) \right\},$$
$$\mathscr{A}_\zeta(q_7) = \left\{ \left(\frac{z_1}{<1>}, 1\right), \left(\frac{z_2}{<1>}, 1\right) \right\},$$
$$\mathscr{A}_\zeta(q_8) = \left\{ \left(\frac{z_1}{<1>}, 1\right), \left(\frac{z_2}{<1>}, 1\right) \right\},$$

and its matrix representation is given as

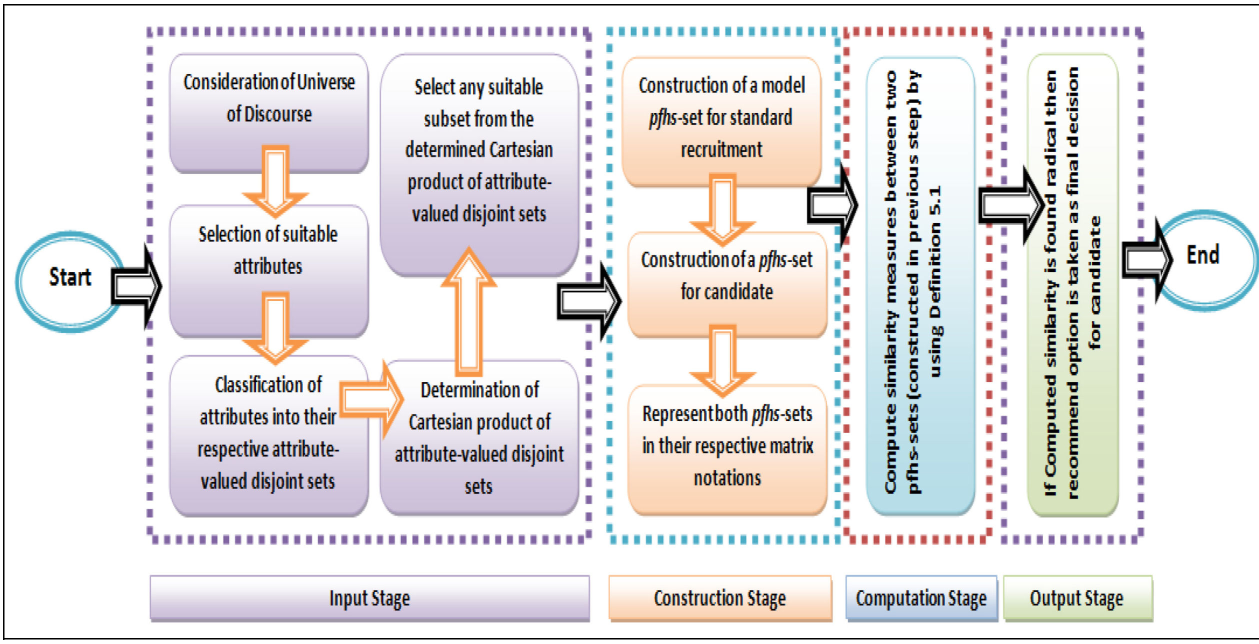

Figure 9. Algorithm for recruitment pattern recognition.

$$\mathscr{A}_{\zeta} = \begin{pmatrix} (1,1) & (1,1) \\ (1,1) & (1,1) \\ (1,1) & (1,1) \\ (1,1) & (1,1) \end{pmatrix}$$

and $\mathscr{B}_{\xi}$ is a pfhs-set for the candidate which is constructed by an expert outside the experts team of the company.

$$\begin{aligned}
\mathscr{B}_{\xi}(q_3) &= \left\{ \left(\tfrac{z_1}{<0.2>}, 0.3\right), \left(\tfrac{z_2}{<0.7>}, 0.4\right) \right\}, \\
\mathscr{B}_{\xi}(q_4) &= \left\{ \left(\tfrac{z_1}{<0.4>}, 0.4\right), \left(\tfrac{z_2}{<0.3>}, 0.5\right) \right\}, \\
\mathscr{B}_{\xi}(q_7) &= \left\{ \left(\tfrac{z_1}{<0.3>}, 0.7\right), \left(\tfrac{z_2}{<0.3>}, 0.8\right) \right\}, \\
\mathscr{B}_{\xi}(q_8) &= \left\{ \left(\tfrac{z_1}{<0.2>}, 0.8\right), \left(\tfrac{z_2}{<0.3>}, 0.9\right) \right\},
\end{aligned}$$

and its matrix representation is given as

$$\mathscr{B}_{\xi} = \begin{pmatrix} (0.2, 0.3) & (0.7, 0.4) \\ (0.4, 0.4) & (0.3, 0.5) \\ (0.3, 0.7) & (0.3, 0.8) \\ (0.2, 0.8) & (0.3, 0.9) \end{pmatrix}$$

Now we calculate similarity between $\mathcal{A}_{\zeta}$ and $\mathcal{B}_{\xi}$ according to Definition 0.20

$$\mathfrak{M}_1(\zeta(\hat{p}_5), \xi(\hat{p}_5)) = 1 - \frac{|(1-0.3)| + |(1-0.4)|}{|(1+0.3)| + |(1+0.4)|} = 0.5185.$$

Similarly

$$\mathfrak{M}_2(\zeta(\hat{p}_6), \xi(\hat{p}_6)) = 0.6207,$$
$$\mathfrak{M}_3(\zeta(\hat{p}_7), \xi(\hat{p}_7)) = 0.8571,$$
$$\mathfrak{M}_4(\zeta(\hat{p}_8), \xi(\hat{p}_8)) = 0.9189, \text{ therefore}$$
$$\mathfrak{M}(\zeta(p), \xi(p)) = 0.9189. Now$$

$$\mathfrak{M}_1(\mathscr{A}(\hat{p}_5), \mathscr{B}(\hat{p}_5)) = 1 - \frac{|(1-0.2)| + |(1-0.7)|}{|(1+0.2)| + |(1+0.7)|} = 0.6207.$$

Similarly

$$\mathfrak{M}_2(\mathscr{A}(\hat{p}_2), \mathscr{B}(\hat{p}_2)) = 0.5185,$$
$$\mathfrak{M}_3(\mathscr{A}(\hat{p}_5), \mathscr{B}(\hat{p}_5)) = 0.4615,$$
$$\mathfrak{M}_4(\mathscr{A}(\hat{p}_6), \mathscr{B}(\hat{p}_6)) = 0.4000, \text{ therefore}$$
$$\mathfrak{M}(\mathscr{A}(p), \mathscr{B}(p)) = 0.6207.$$

Hence $\mathfrak{S}(\mathscr{A}_{\zeta}, \mathscr{B}_{\xi}) = 0.9189 \times 0.6207 \cong 0.5704 > \frac{1}{2}$ that is $\mathscr{A}_{\zeta}$ and $\mathscr{B}_{\xi}$ are radically similar. Thus the option recommended is finalized.

Discussion and comparison analysis

Many researchers have already employed various algorithm based techniques to investigate the applicability of *fs*-set like models. There are various traits which have a significant function in decision making and their exclusion may lead to biased decisions. For instance, the parameters qualification, experience, and age play key roles in recruitment scenarios. It is an apt way to further consider their respective sub-attributive values as performed in Example 0.23. The proposed model is preferable in the sense that it considers such kind

Table 4. Comparison analysis.

References	Models	Computed similarity and its nature	Limitations
Alkhazaleh et al.[50]	*pfs*-set	0.74, radical	Fuzzy soft setting is considered but fuzzy hypersoft setting is ignored.
Majumdar and Samanta[58]	*fs*-set	0.617, radical	Fuzzy soft expert setting is considered but fuzzy hypersoft setting is ignored.
Presented model	*pfhs*-set	0.5704, radical	Both fuzz soft and fuzzy hypersoft settings are considered.

classification of parameters with their due status that is ignored in existing models. The decision making process becomes more reliable and trust-worthy with the entitlement of such classification. Now the proposed model is compared with the most relevant existing structures and presented in Table 4.

Flexibility of pfhs-set

In this section, we discuss the generalization of *pfhs*-set which proves its flexibility. The followings are some particular cases of *pfhs*-set:

1. If fuzzy membership grade is omitted and only the *hs*-set environment is considered with possibility degree, then it transforms to a possibility hypersoft set (*phs*-set).
2. It converts to *fhs*-set when possibility degree is ignored.
3. It matches with *hs*-set after eliminating both possibility and fuzzy membership grades.
4. It converts to *pfs*-set after ignoring *hs*-setting.
5. Possibility soft set (*ps*-set) is obtained after the removal of fuzzy membership grade from *pfs*-set.
6. It takes the form of *fs*-set when both possibility grade and *hs*-setting are omitted.
7. Finally *s*-set is obtained by ignoring fuzzy membership grade, possibility grade and *hs*-setting.

The Figure 10 presents the pictorial version of this generalization of proposed structure.

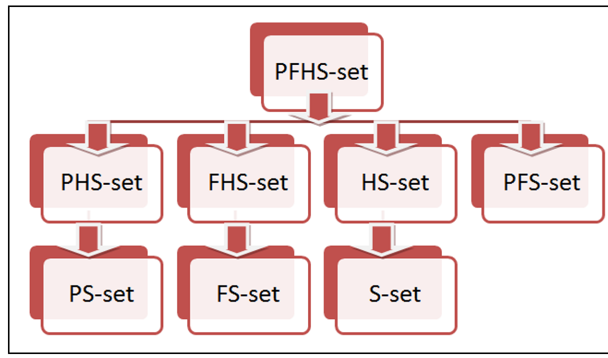

Figure 10. Generalized status of proposed structure.

Merits of proposed structure

Followings are some advantages of proposed model (i.e., *pfhs*-set):

1. The presented technique took the importance of the proposal of possibility in conjunction with the *fhs*-set to treat with contemporary *DM* concerns. The considered possibility-grade replicates the possibility of the partaking of the level of appreciation and dispensation which lead to an incredible perspective in the legitimate interpretation inside the space of computations.
2. In view of the fact that significant inspection of an attribute via consideration of its respective sub-attribute valued set is the major focus of this study therefore it may assist the decision-makers to have consistent and unbiased results.
3. It controls all the aspects of the relevant models like *fhs*-set, *hs*-set, *pfs*-set, *fs*-set, and *s*-set therefore it is not irrational to entitle it as their generalized version.

The Table 5 presents the meritorious aspects of the proposed model. The comparison is evaluated on the basis of two different aspects:

1. Main features discussed in the study.
2. Features like M.G (Membership Grade), P.G (Possibility Grade), SA-AF (Single Argument Approximate function), MA-AF (Multi Argument Approximate function), and *hs*-setting.

In Table 5, the symbols ✓ and × are meant for “Yes” and “No” respectively. Similarly “radical similarity” means that the computed values of similarity are more or equal than 0.5.

Conclusion

The following points provide the summary of the paper with future works:

Table 5. The vivid comparison of proposed structure.

Authors	Structure	M.G	SA-AF	P.G	MA-AF	*hs*-Setting
Alkhazaleh et al.[50]	*pfs*-set	✓	✓	✓	×	×
Bashir and Salleh[51]	*pfse*-set	✓	✓	✓	×	×
Zhang and Shu[52]	*pmfs*-set	✓	✓	✓	×	×
Zhang et al.[53]	*pivfs*-set	✓	✓	✓	×	×
Kalaiselvi and Seenivasan[54]	*pfs*-set	✓	✓	✓	×	×
Ponnalagu and Mounika[55]	*pfse*-set	✓	✓	✓	×	×
Khalil et al.[56]	*pmpfs*-set	✓	✓	✓	×	×
Khalil and Hassan[57]	*pmfs*-set	✓	✓	✓	×	×
Proposed model	*pfhs*-set	✓	✓	✓	✓	✓

1. An innovative concept of *pfhs*-set is investigated along with the characterization of some of its basic axiomatic properties.
2. The aggregation-operations of *pfhs*-sets are developed and explained with the support of computational examples.
3. While employing the concept of AND- and OR-operations of *pfhs*-set, two algorithms are presented and then verified through application of real-world agri-automobile evaluation based scenarios.
4. In order to tackle various pattern recognition-based problems, similarity measures between *pfhs*-sets are formulated and then validated by using real-world recruitment-based pattern recognition problems. The preference of proposed formulation is judged through vivid comparison with existing works.

In some *DM* situations, when decision-makers provide their opinions by considering falsity and indeterminacy grades for each member of the universal set corresponding to each parametric tuple then the proposed study has limitations for such kinds of situations. Therefore the future work may include the development of other fuzzy soft set-like models with possibility grade for each of their approximate members and their use in *DM* problems. The proposed study may also be applied to resolve several other real-world problems by using different *DM* techniques and approaches. The scope (managerial implications) of this study may cover the areas of soft computing (fuzzy logic), pattern recognition, image processing, human resource management, multi-criteria decision-making processes, medical diagnosis and analysis, investment management, and many other fields of mathematics and theoretical computer science.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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