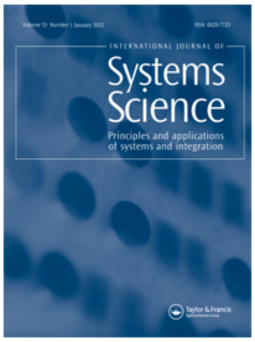

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Determining the membership degrees in the range (0, 1) for hypersoft sets independently of the decision-maker

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ABSTRACT

Today, Molodtsov's soft set has been generalised to hypersoft sets, and the use of hypersoft set theory for uncertain data has become more preferable than soft sets. However, the membership degree of an object in hypersoft sets is 0 or 1. In order to express this situation in the range (0, 1), many mathematical models have been constructed by considering hypersoft sets together with fuzzy sets and their derivatives. However, these mathematical models require the decision-maker to express the membership degrees. It is a very difficult task for a decision-maker to determine a value in (0, 1) and the probability of an error is very high. For this reason, the concepts relational hypersoft membership degree and inverse relational hypersoft membership degree, which are given less dependent on decision-makers, are proposed in this paper. Moreover, two decision-making algorithms are given to use these concepts in an environment of uncertainty. Finally, the decision-making process for the given algorithms is analysed.

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1. Introduction

Uncertain data encountered in many fields need to be addressed during data analysis to increase the robustness of the results. Since classical mathematics cannot successfully model uncertain data, researchers have made different efforts. Fuzzy set theory, one of the first results of these efforts, was proposed by Zadeh (1965). This theory, which can express the belonging (i.e. membership degree in the range $[0, 1]$) of an element to any set, is a very successful mathematical model. In particular, it is noteworthy that membership degrees, which are only expressed as 0 and 1 in classical mathematics, are generalised. However, fuzzy sets have some difficulties in modelling uncertainty problems, especially in the application phase. Molodtsov (1999), who thinks that the main reason for these difficulties is the lack of a parameterisation tool, introduced the soft set theory to the literature in 1999. Since these sets allow us to express objects associated with a particular parameter set, better mathematical models for uncertainty problems can be developed. Moreover, this theory has been successfully applied in many fields such as the theory of measurement, game theory, Riemann Integration, smoothness of functions and so on. Since this theory proposed by Molodtsov can successfully express uncertain situations, the application areas of soft sets continue to increase rapidly (Bordbar et al., 2021; Dalkılıç, 2021b, 2021d; Dalkılıç & Demirtaş, 2020, 2021a, 2021b; Demirtaş & Dalkılıç, 2020; Hayat et al., 2020; Sarwar et al., 2021).

The contribution of the parameterisation tool in soft set theory has led to the construction of different mathematical models. One of them is inverse soft sets (Cetkin et al., 2016), which offer a different approach to decision-making problems by considering the inverse of the mapping between parameter and object set in soft set theory. It can be said that the studies on inverse soft set theory have increased especially in recent years (Dalkılıç, 2021a; Demirtaş et al., 2020; Kamacı et al., 2018; Khalil & Hassan, 2019; Petchimuthu & Kamacı, 2019; Suebsan, 2019). Another important mathematical model is the hypersoft set theory introduced by Smarandache (2018). These sets are constructed by replacing the approximate function of Molodtsov's soft sets with the multi-argument approximation function. Moreover, the inverse hypersoft sets (Gilbert Rani & Muthulakshmi, 2020) are suggested for hypersoft set, inspired by inverse soft sets.

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For all these mathematical models developed for uncertainty problems based on parametric data, the membership degrees are expressed as 0 and 1. In order to overcome this situation, many theories have been proposed by considering all these theories together with fuzzy sets and their derivatives (Dalkılıç, 2021b, 2021c; Demirtaş & Dalkılıç, 2019; Gilbert Rani & Muthulakshmi, 2020; Ihsan et al., 2021; Rahman et al., 2021; Saeed et al., 2021; Saqlain, Jafar, et al., 2020; Smarandache, 2018; Zulqarnain et al., 2021). However, the task of expressing the membership degrees expressed in these theories is dependent on the decision-makers. In other words, the membership degree expressed by the decision-makers is directly accepted and the uncertainty is tried to be overcome. However, accurately expressing a membership degree in $(0, 1)$ is quite a challenge. Therefore, the motivation of this paper is to express the membership degrees more mathematically as independently as possible from the decision-makers. For this purpose, the concepts relational hypersoft membership degree and inverse relational hypersoft membership degree are proposed. Moreover, some of its properties are examined and examples are given for a better understanding of the concepts. In addition, two algorithms are proposed to better construct the decision-making process of the proposed concepts under uncertain data. How these decision-making algorithms can be applied on an uncertainty problem is also illustrated. Finally, the decision-making process under uncertainty for the proposed algorithms is discussed.

2. Preliminaries

In this section, some theories from the literature are reminded to support the concepts expressed in the rest of the proposed paper. Some supporting information is also provided. More detailed explanations can be found in Smarandache (2018), Gilbert Rani and Muthulakshmi (2020), Saeed et al. (2020), Saqlain, Jafar, et al. (2020), Saqlain, Moin, et al. (2020), especially for the hypersoft sets mentioned in this section.

Throughout this paper; $R = \{r_1, r_2, \ldots, r_m\}$ is an initial universe and 2^R is the power set of R.

Definition 2.1 (Zadeh, 1965): A fuzzy set F over R is a set defined by $\mu_F : R \to [0, 1]$. μ_F is called the membership function of F, and the value $\mu_F(r)$ is called the grade of membership of $r \in R$. The value represents the degree of r belonging to the fuzzy set F. Thus a fuzzy set F over R can be represented as follows:

$$F = \{\langle \mu_F(r)/r \rangle : r \in R\} \quad (1)$$

Example 2.2: Let $R = \{r_1 : \mathit{yellow}, r_2 : \mathit{pink}, r_3 : \mathit{orange}, r_4 : \mathit{brown}, r_5 : \mathit{purble}, r_6 : \mathit{blue}\}$ be a set of colours. If the memberships of yellow, pink, orange, brown, purble and blue are defined as 0.42, 0.36, 0.7, 0, 0.85 and 0.2, respectively; then the fuzzy set F on R can be written as

$$F = \{\langle 0.42/r_1 \rangle, \langle 0.36/r_2 \rangle, \langle 0.7/r_3 \rangle, \langle 0/r_4 \rangle, \langle 0.85/r_5 \rangle, \langle 0.2/r_6 \rangle\}$$

or

$$F = \{\langle 0.42/r_1 \rangle, \langle 0.36/r_2 \rangle, \langle 0.7/r_3 \rangle, \langle 0.85/r_5 \rangle, \langle 0.2/r_6 \rangle\}.$$

Definition 2.3 (Gilbert Rani & Muthulakshmi, 2020; Smarandache, 2018): Let $\xi_1, \xi_2, \ldots, \xi_n$, for $n \geq 1$, be the distinct attributes whose corresponding attribute values belong to the sets

$$\begin{aligned} P_1 &= \{p_1^1, p_1^2, \ldots, p_1^{l_1}, \ldots, p_1^{m_1}\} \quad \text{for } 1 \leq l_1 \leq m_1, \\ P_2 &= \{p_2^1, p_2^2, \ldots, p_2^{l_2}, \ldots, p_2^{m_2}\} \quad \text{for } 1 \leq l_2 \leq m_2, \\ &\vdots \qquad\qquad \vdots \\ P_n &= \{p_n^1, p_n^2, \ldots, p_n^{l_n}, \ldots, p_n^{m_n}\} \quad \text{for } 1 \leq l_n \leq m_n, \end{aligned}$$

respectively, where $P_i \cap P_j = \emptyset$ for $1 \leq i, j \leq n$ and $i \neq j$. Then,

(i) a pair (Φ, P) is called a hypersoft set over R, where Φ is the mapping given by $\Phi : P \to 2^R$, where $P = \prod_{i=1}^{n} P_i = P_1 \times P_2 \times \cdots \times P_n$. Thus a hypersoft set (Φ, P) over R can be represented by the set of ordered pairs,

$$(\Phi, P) = \left\{ \begin{array}{l} (\{p_1^{l_1}, p_2^{l_2}, \ldots, p_n^{l_n}\}, \\ \Phi(\{p_1^{l_1}, p_2^{l_2}, \ldots, p_n^{l_n}\})) : \\ \{p_1^{l_1}, p_2^{l_2}, \ldots, p_n^{l_n}\} \in P, \\ \Phi(\{p_1^{l_1}, p_2^{l_2}, \ldots, p_n^{l_n}\}) \in 2^R \end{array} \right\} \quad (2)$$

(ii) a pair (Ψ, R) is called an inverse hypersoft set over P, where Ψ is the mapping given by $\Psi : R \to 2^P$, where $P = \prod_{i=1}^{n} P_i = P_1 \times P_2 \times \cdots \times P_n$. Here 2^P is the power set of P. Thus an inverse hypersoft set (Ψ, R) over P can be represented by the set of ordered pairs,

$$(\Psi, R) = \{(r, \Psi(r)) : r \in R, \Psi(r) \in 2^P\} \quad (3)$$

State that the set of all the hypersoft sets [inverse hypersoft sets] over R [P] will be denoted by $HS(R)$ [$IHS(P)$].

Remark 2.4: Each hypersoft set can be uniquely represented as an inverse hypersoft set.

Example 2.5: Let $R = \{r_1, r_2, r_3, r_4, r_5, r_6, r_7\}$ be the set of most preferred hybrid cars. Let 'ξ_1 : *engine capacity*', 'ξ_2 : *colour*', 'ξ_3 : *fuel type*' distinct attributes whose attribute values belong to the sets P_1, P_2, P_3, respectively. Also, let $P_1 = \{p_1^1 : \textit{smallest}, p_1^2 : \textit{biggest}\}$, $P_2 = \{p_2^1 : \textit{white}, \quad p_2^2 : \textit{red}, \quad p_2^3 = \textit{blue}\}$, $P_3 = \{p_3^1 : \textit{diesel}\}$. Here, since $P = \prod_{i=1}^{3} P_i = P_1 \times P_2 \times P_3$, then

$$P = \left\{ \begin{array}{c} \{p_1^1, p_2^1, p_3^1\}, \{p_1^1, p_2^2, p_3^1\}, \{p_1^1, p_2^3, p_3^1\}, \\ \{p_1^2, p_2^1, p_3^1\}, \{p_1^2, p_2^2, p_3^1\}, \{p_1^2, p_2^3, p_3^1\} \end{array} \right\}$$

Then;

(i) by Definition 2.3(i), we can describe the hypersoft sets as $\Phi(\{p_1^1, p_2^1, p_3^1\}) = \{r_1, r_3, r_4\}$, $\Phi(\{p_1^1, p_2^2, p_3^1\}) = \{r_2, r_4, r_6, r_7\}$, $\Phi(\{p_1^1, p_2^3, p_3^1\}) = \{r_2, r_6\}$, $\Phi(\{p_1^2, p_2^1, p_3^1\}) = \{r_3, r_5, r_7\}$, $\Phi(\{p_1^2, p_2^2, p_3^1\}) = \{r_4, r_5, r_7\}$ and $\Phi(\{p_1^2, p_2^3, p_3^1\}) = \{r_2, r_3, r_5, u_7\}$. Thus we obtain a hypersoft set

$$(\Phi, P) = \left\{ \begin{array}{c} (\{p_1^1, p_2^1, p_3^1\}, \{1/r_1, 1/r_3, 1/r_4\}), \\ (\{p_1^1, p_2^2, p_3^1\}, \{1/r_2, 1/r_4, 1/r_6, 1/r_7\}), \\ (\{p_1^1, p_2^3, p_3^1\}, \{1/r_2, 1/r_6\}) \\ (\{p_1^2, p_2^1, p_3^1\}, \{1/r_3, 1/r_5, 1/r_7\}), \\ (\{p_1^2, p_2^2, p_3^1\}, \{1/r_4, 1/r_5, 1/r_7\}), \\ (\{p_1^2, p_2^3, p_3^1\}, \{1/r_2, 1/r_3, 1/r_5, 1/r_7\}) \end{array} \right\}$$

or

$$(\Phi, P) = \left\{ \begin{array}{c} (\{p_1^1, p_2^1, p_3^1\}, \{r_1, r_3, r_4\}), \\ (\{p_1^1, p_2^2, p_3^1\}, \{r_2, r_4, r_6, r_7\}), \\ (\{p_1^1, p_2^3, p_3^1\}, \{r_2, r_6\}) \\ (\{p_1^2, p_2^1, p_3^1\}, \{r_3, r_5, r_7\}), \\ (\{p_1^2, p_2^2, p_3^1\}, \{r_4, r_5, r_7\}), \\ (\{p_1^2, p_2^3, p_3^1\}, \{r_2, r_3, r_5, r_7\}) \end{array} \right\}$$

(ii) by Definition 2.3(ii), we can describe the inverse hypersoft sets as

$$\Psi(r_1) = \{\{p_1^1, p_2^1, p_3^1\}\},$$

$$\Psi(r_2) = \{\{p_1^1, p_2^2, p_3^1\}, \{p_1^1, p_2^3, p_3^1\}, \{p_1^2, p_2^3, p_3^1\}\},$$

$$\Psi(r_3) = \{\{p_1^1, p_2^1, p_3^1\}, \{p_1^2, p_2^1, p_3^1\}, \{p_1^2, p_2^3, p_3^1\}\},$$

$$\Psi(r_4) = \{\{p_1^1, p_2^1, p_3^1\}, \{p_1^1, p_2^2, p_3^1\}, \{p_1^2, p_2^2, p_3^1\}\},$$

$$\Psi(r_5) = \{\{p_1^2, p_2^1, p_3^1\}, \{p_1^2, p_2^2, p_3^1\}, \{p_1^2, p_2^3, p_3^1\}\},$$

$$\Psi(r_6) = \{\{p_1^1, p_2^2, p_3^1\}, \{p_1^1, p_2^3, p_3^1\}\},$$

$$\Psi(r_7) = \{\{p_1^1, p_2^2, p_3^1\}, \{p_1^2, p_2^1, p_3^1\}, \{p_1^2, p_2^2, p_3^1\}, \{p_1^2, p_2^3, p_3^1\}\}.$$

Thus we obtain an inverse hypersoft set

$$(\Psi, R) = \left\{ \begin{array}{c} (r_1, \{1/\{p_1^1, p_2^1, p_3^1\}\}), \\ (r_2, \{1/\{p_1^1, p_2^2, p_3^1\}, 1/\{p_1^1, p_2^3, p_3^1\}, \\ 1/\{p_1^2, p_2^3, p_3^1\}\}), \\ (r_3, \{1/\{p_1^1, p_2^1, p_3^1\}, 1/\{p_1^2, p_2^1, p_3^1\}, \\ 1/\{p_1^2, p_2^3, p_3^1\}\}), \\ (r_4, \{1/\{p_1^1, p_2^1, p_3^1\}, 1/\{p_1^1, p_2^2, p_3^1\}, \\ 1/\{p_1^2, p_2^2, p_3^1\}\}), \\ (r_5, \{1/\{p_1^2, p_2^1, p_3^1\}, 1/\{p_1^2, p_2^2, p_3^1\}, \\ 1/\{p_1^2, p_2^3, p_3^1\}\}), \\ (r_6, \{1/\{p_1^1, p_2^2, p_3^1\}, 1/\{p_1^1, p_2^3, p_3^1\}\}), \\ (r_7, \{1/\{p_1^1, p_2^2, p_3^1\}, 1/\{p_1^2, p_2^1, p_3^1\}, \\ 1/\{p_1^2, p_2^2, p_3^1\}, 1/\{p_1^2, p_2^3, p_3^1\}\}) \end{array} \right\}$$

or

$$(\Psi, R) = \left\{ \begin{array}{c} (r_1, \{\{p_1^1, p_2^1, p_3^1\}\}), \\ (r_2, \{\{p_1^1, p_2^2, p_3^1\}, \{p_1^1, p_2^3, p_3^1\}, \\ \{p_1^2, p_2^3, p_3^1\}\}), \\ (r_3, \{\{p_1^1, p_2^1, p_3^1\}, \{p_1^2, p_2^1, p_3^1\}, \\ \{p_1^2, p_2^3, p_3^1\}\}), \\ (r_4, \{\{p_1^1, p_2^1, p_3^1\}, \{p_1^1, p_2^2, p_3^1\}, \\ \{p_1^2, p_2^2, p_3^1\}\}), \\ (r_5, \{\{p_1^2, p_2^1, p_3^1\}, \{p_1^2, p_2^2, p_3^1\}, \\ \{p_1^2, p_2^3, p_3^1\}\}), \\ (r_6, \{\{p_1^1, p_2^2, p_3^1\}, \{p_1^1, p_2^3, p_3^1\}\}), \\ (r_7, \{\{p_1^1, p_2^2, p_3^1\}, \{p_1^2, p_2^1, p_3^1\}, \\ \{p_1^2, p_2^2, p_3^1\}, \{p_1^2, p_2^3, p_3^1\}\}) \end{array} \right\}$$

In the remainder of the paper, the parameters will be shown as $\{p_1^{l_1}, p_2^{l_2}, \ldots, p_n^{l_n}\} = p_{1,\ldots,n}^{l_1,\ldots,l_n}$ for convenience.

3. Membership degrees for (inverse) hypersoft set theory

In this section, the membership degrees expressed by decision-makers for hypersoft sets and inverse hypersoft sets are discussed. The concepts relational hypersoft membership degree and inverse relational hypersoft membership degree are proposed to accurately determine the membership degrees expressed in the range $(0, 1)$. In addition, some properties related to these concepts were analysed.

First, let's consider the membership degrees desired to be expressed in the range $(0,1)$ by the decision-makers for hypersoft sets. For this, fuzzy hypersoft sets (Smarandache, 2018), which are actually a combination of fuzzy sets and hypersoft sets, are proposed. Moreover, many proposed mathematical models are available (Rahman et al., 2020; Saeed et al., 2021; Saqlain, Jafar, et al., 2020; Zulqarnain et al., 2021). However, since it is a very difficult task to express a value in $(0,1)$, we can talk about a possible margin of error in the membership degrees expressed by decision-makers. In addition to these, it is clear that only values 0 and 1 are expressed by the decision-makers will cause less errors. For this reason, it is aimed to determine the membership degrees in $(0,1)$ by using only values 0 and 1 for hypersoft sets. In this way, we propose the concept of relational hypersoft membership degree, which we built with this motivation for hypersoft sets. Thus we can express the membership degrees in $(0,1)$ of all objects corresponding to a parameter by making use of hypersoft sets that express an uncertainty environment.

Definition 3.1: Let $(\Phi,P)\in HS(R)$. For $r_k\in R\setminus\Phi(p_{1,\ldots,n}^{t_1,\ldots,t_n})$ and $r_j\in\Phi(p_{1,\ldots,n}^{t_1,\ldots,t_n})$, the relational hypersoft membership degree of r_k to $\Phi(p_{1,\ldots,n}^{t_1,\ldots,t_n})$ is expressed with the help of mapping given by $\overset{(\Phi,\mathcal{P})}{\Theta}$: $P^{[R\setminus\Phi(p_{1,\ldots,n}^{t_1,\ldots,t_n})]}\to[0,1)$. The mapping (Relational Hypersoft Membership Function) is given as follows: $(1\le k,j\le m, 1\le t_k\le m_k, 2\le m_k$ for $k=1,2,\ldots,n$ and $m,n\ge 2)$

$$\overset{(\Phi,\mathcal{P})}{\Theta}(\{p_{1,\ldots,n}^{t_1,\ldots,t_n}\}^{r_k}) = \frac{1}{(m-1)(-1+\prod_{i=1}^{n} m_k)} \times \sum_{r_j\in\Phi(p_{1,\ldots,n}^{t_1,\ldots,t_n})}\left[\sum_{p_{1,\ldots,n}^{l_1,\ldots,l_n}\in P}\left(\underset{p_{1,\ldots,n}^{l_1,\ldots,l_n}}{\overset{(\Phi,\mathcal{P})}{\Upsilon}}(r_k,r_j)\right)\right] \quad (4)$$

where

$$\underset{p_{1,\ldots,n}^{l_1,\ldots,l_n}}{\overset{(\Phi,\mathcal{P})}{\Upsilon}}(r_k,r_j) = \begin{cases} 1, & \mu_{\Phi\left(p_{1,\ldots,n}^{l_1,\ldots,l_n}\right)}(r_k)+\mu_{\Phi\left(p_{1,\ldots,n}^{l_1,\ldots,l_n}\right)}(r_j)=2 \\ 0, & \text{otherwise} \end{cases},$$

$$\forall p_{1,\ldots,n}^{l_1,\ldots,l_n}\in P$$

is a mapping given by $\underset{p_{1,\ldots,n}^{l_1,\ldots,l_n}}{\overset{(\Phi,\mathcal{P})}{\Upsilon}}(r_k,r_j):[R\setminus\Phi(p_{1,\ldots,n}^{t_1,\ldots,t_n})]\times\Phi(p_{1,\ldots,n}^{t_1,\ldots,t_n})\to\{0,1\}$. Here $\mu_{\Phi(p_{1,\ldots,n}^{l_1,\ldots,l_n})}$ is the membership function of Φ.

Example 3.2: Let $R=\{r_1,r_2,r_3,r_4,r_{5=m}\}$ and $P_1=\{p_1^{1=m_1}\}$, $P_2=\{p_2^1,p_2^2,p_2^{3=m_2}\}$, $P_3=\{p_3^1,p_3^{2=m_3}\}$. If

$$\Phi(\{p_1^1,p_2^1,p_3^1\})=\{r_1,r_3,r_4,r_5\},$$
$$\Phi(\{p_1^1,p_2^2,p_3^1\})=\{r_2,r_3,r_5\},$$
$$\Phi(\{p_1^1,p_2^3,p_3^1\})=\{r_1,r_2,r_3,r_4\},$$
$$\Phi(\{p_1^1,p_2^1,p_3^2\})=\{r_1,r_2,r_4,r_5\},$$
$$\Phi(\{p_1^1,p_2^2,p_3^2\})=\{r_1,r_3,r_4\},$$
$$\Phi(\{p_1^1,p_2^3,p_3^2\})=\{r_2,r_3,r_4,r_5\}$$

for $P=P_1\times P_2\times P_3$; then the hypersoft set (Φ,P) is written by

$$(\Phi,P)=\left\{\begin{array}{c}(\{p_1^1,p_2^1,p_3^1\},\{r_1,r_3,r_4,r_5\}),\\(\{p_1^1,p_2^2,p_3^1\},\{r_2,r_3,r_5\}),\\(\{p_1^1,p_2^3,p_3^1\},\{r_1,r_2,r_3,r_4\}),\\(\{p_1^1,p_2^1,p_3^2\},\{r_1,r_2,r_4,r_5\}),\\(\{p_1^1,p_2^2,p_3^2\},\{r_1,r_3,r_4\}),\\(\{p_1^1,p_2^3,p_3^2\},\{r_2,r_3,r_4,r_5\})\end{array}\right\}.$$

Now let's calculate all relational hypersoft membership degrees for (Φ,P).

For $t_1=2$, $t_2=2$, $t_3=1$:

Since $r_{2=k}\in[R\setminus\Phi(p_{1,2,3}^{1,1,1})]$, $\underset{p_{1,2,3}^{1,1,1}}{\overset{(\Phi,P)}{\Upsilon}}(r_2,r_j)=0$ for $r_j\in\Phi(p_{1,2,3}^{1,1,1})$. Thus we have

$$\overset{(\Phi,\mathcal{P})}{\Theta}(\{p_{1,2,3}^{1,1,1}\}^{r_2}) = \frac{1}{4.5}\left[\begin{array}{c}\underset{p_{1,2,3}^{1,2,1}}{\overset{(\Phi,\mathcal{P})}{\Upsilon}}(r_2,r_j)+\underset{p_{1,2,3}^{1,3,1}}{\overset{(\Phi,\mathcal{P})}{\Upsilon}}(r_2,r_j)+\underset{p_{1,2,3}^{1,1,2}}{\overset{(\Phi,\mathcal{P})}{\Upsilon}}(r_2,r_j)\\+\underset{p_{1,2,3}^{1,2,2}}{\overset{(\Phi,\mathcal{P})}{\Upsilon}}(r_2,r_j)+\underset{p_{1,2,3}^{1,3,2}}{\overset{(\Phi,\mathcal{P})}{\Upsilon}}(r_2,r_j)\end{array}\right],$$

$$\forall r_j\in\Phi(p_{1,2,3}^{1,1,1})$$

$$=\frac{\left[\begin{array}{c}(0+1+0+1)+(1+1+1+0)\\+(1+0+1+1)+(0+0+0+0)\\+(0+1+1+1)\end{array}\right]}{20}$$

$$=11/20$$

Similarly,

For $t_1 = 1$, $t_2 = 2$, $t_3 = 1$:

Since $r_1, r_4 \in [R \setminus \Phi(p_{1,2,3}^{1,2,1})]$, then $\overset{(\Phi,P)}{\Theta}(\{p_{1,2,3}^{1,2,1}\}^{r_1}) = 7/20$ and $\overset{(\Phi,P)}{\Theta}(\{p_{1,2,3}^{1,2,1}\}^{r_4}) = 10/20$.

For $t_1 = 1$, $t_2 = 3$, $t_3 = 1$:

Since $r_5 \in [R \setminus \Phi(p_{1,2,3}^{1,3,1})]$, then $\overset{(\Phi,P)}{\Theta}(\{p_{1,2,3}^{1,3,1}\}^{r_5}) = 11/20$.

For $t_1 = 1$, $t_2 = 1$, $t_3 = 2$:

Since $r_3 \in [R \setminus \Phi(p_{1,2,3}^{1,1,2})]$, then $\overset{(\Phi,P)}{\Theta}(\{p_{1,2,3}^{1,1,2}\}^{r_3}) = 13/20$.

For $t_1 = 1$, $t_2 = 2$, $t_3 = 2$:

Since $r_2, r_5 \in [R \setminus \Phi(p_{1,2,3}^{1,2,2})]$, then $\overset{(\Phi,P)}{\Theta}(\{p_{1,2,3}^{1,2,2}\}^{r_2}) = 8/20$ and $\overset{(\Phi,P)}{\Theta}(\{p_{1,2,3}^{1,2,2}\}^{r_5}) = 8/20$.

For $t_1 = 1$, $t_2 = 3$, $t_3 = 2$:

Since $r_1 \in [R \setminus \Phi(p_{1,2,3}^{1,3,2})]$, then $\overset{(\Phi,P)}{\Theta}(\{p_{1,2,3}^{1,3,2}\}^{r_1}) = 11/20$.

In addition, we can write by matrix form as

$(\Phi, P) =$

	r_1	r_2	r_3	r_4	r_5
$p_{1,2,3}^{1,1,1}$	1	11/20	1	1	1
$p_{1,2,3}^{1,2,1}$	7/20	1	1	1/2	1
$p_{1,2,3}^{1,3,1}$	1	1	1	1	11/20
$p_{1,2,3}^{1,1,2}$	1	1	13/20	1	1
$p_{1,2,3}^{1,2,2}$	1	2/5	1	1	2/5
$p_{1,2,3}^{1,3,2}$	11/20	1	1	1	1

Now, let's focus on inverse hypersoft sets, a theory based on the inverse application of the mapping between parameter and object set. For these sets, we ask the decision-makers to specify only membership degrees 0 and 1, as well. Similarly, we aim to determine the membership degrees in (0, 1) by making use of these values. For this, the concept of inverse relational hypersoft membership degree is proposed. Thus we can express the membership degrees in (0, 1) of all parameters corresponding to an object by making use of inverse hypersoft sets that express an uncertainty environment.

Definition 3.3: Let $(\Psi, R) \in IHS(P)$. For $p_{1,\ldots,n}^{s_1,\ldots,s_n} \in P \setminus \Psi(r_j)$ and $p_{1,\ldots,n}^{t_1,\ldots,t_n} \in \Psi(r_j)$, the inverse relational hypersoft membership degree of $p_{1,\ldots,n}^{s_1,\ldots,s_n}$ to $\Psi(r_j)$ is expressed with the help of mapping given by $\overset{(\Psi,R)}{\Xi} : R^{P\setminus\psi(r_j)} \to [0,1)$. The mapping (Inverse Relational Hypersoft Membership Function) is given as follows: $(1 \le j \le m,\ 1 \le s_k, t_k \le m_k,\ 2 \le m_k$ for $k = 1, 2, \ldots, n$ and $m, n \ge 2)$

$$\overset{(\Psi,R)}{\Xi}\left(r_j^{[p_{1,\ldots,n}^{s_1,\ldots,s_n}]}\right) = \frac{1}{(m-1)(-1+\prod_{i=1}^{n} m_k)} \sum_{p_{1,\ldots,n}^{t_1,\ldots,t_n} \in \Psi(r_j)} \times \left[\sum_{r\in R} \overset{(\Psi,R)}{\Upsilon_r}(p_{1,\ldots,n}^{s_1,\ldots,s_n}, p_{1,\ldots,n}^{t_1,\ldots,t_n})\right] \quad (5)$$

where

$$\overset{(\Psi,R)}{\Upsilon_r}(p_{1,\ldots,n}^{s_1,\ldots,s_n}, p_{1,\ldots,n}^{t_1,\ldots,t_n}) = \begin{cases} 1, & \mu_{\Psi(r)}(p_{1,\ldots,n}^{s_1,\ldots,s_n}) + \mu_{\Psi(r)}(p_{1,\ldots,n}^{t_1,\ldots,t_n}) = 2 \\ 0, & \text{otherwise} \end{cases}, \quad \forall r \in R$$

is a mapping given by $\overset{(\Psi,R)}{\Upsilon_r} : [P \setminus \Psi(r_j)] \times \Psi(r_j) \to \{0,1\}$. Here $\mu_{\Psi(r)}$ is the membership function of Ψ.

Proposition 3.4: *Let* $(\Phi, P) \in HS(R)$ *and* $(\Psi, R) \in IHS(P)$. *Then,*

$$\overset{(\Phi,\mathcal{P})}{\underset{p_{1,\ldots,n}^{l_1,\ldots,l_n}}{\Upsilon}}(r_k, r_j) = \overset{(\Phi,\mathcal{P})}{\underset{p_{1,\ldots,n}^{l_1,\ldots,l_n}}{\Upsilon}}(r_j, r_k)$$

and

$$\overset{(\Psi,R)}{\Upsilon_r}(p_{1,\ldots,n}^{s_1,\ldots,s_n}, p_{1,\ldots,n}^{t_1,\ldots,t_n}) = \overset{(\Psi,R)}{\Upsilon_r}(p_{1,\ldots,n}^{t_1,\ldots,t_n}, p_{1,\ldots,n}^{s_1,\ldots,s_n}).$$

Proof: Straightforward. ■

Example 3.5: Consider Example 3.2, we have

$$\Psi(r_1) = \{\{p_1^1, p_2^1, p_3^1\}, \{p_1^1, p_2^3, p_3^1\}, \{p_1^1, p_2^1, p_3^2\}, \{p_1^1, p_2^2, p_3^2\}\},$$

$$\Psi(r_2) = \{\{p_1^1, p_2^2, p_3^1\}, \{p_1^1, p_2^3, p_3^1\}, \{p_1^1, p_2^1, p_3^2\}, \{p_1^1, p_2^3, p_3^2\}\},$$

$$\Psi(r_3) = \{\{p_1^1, p_2^1, p_3^1\}, \{p_1^1, p_2^2, p_3^1\}, \{p_1^1, p_2^3, p_3^1\}, \{p_1^1, p_2^2, p_3^2\}, \{p_1^1, p_2^3, p_3^2\}\},$$

$$\Psi(r_4) = \{\{p_1^1, p_2^1, p_3^1\}, \{p_1^1, p_2^3, p_3^1\}, \{p_1^1, p_2^1, p_3^2\}, \{p_1^1, p_2^2, p_3^2\}, \{p_1^1, p_2^3, p_3^2\}\},$$

$$\Psi(r_5) = \{\{p_1^1, p_2^1, p_3^1\}, \{p_1^1, p_2^2, p_3^1\}, \{p_1^1, p_2^1, p_3^2\}, \{p_1^1, p_2^3, p_3^2\}\}.$$

Thus the inverse hypersoft set (Ψ,R) is written by

$$(\Psi,R)=\left\{\begin{array}{c}(r_1,\{\{p_1^1,p_2^1,p_3^1\},\{p_1^1,p_2^3,p_3^1\},\\ \{p_1^1,p_2^1,p_3^2\},\{p_1^1,p_2^2,p_3^2\}\}),\\ (r_2,\{\{p_1^1,p_2^2,p_3^1\},\{p_1^1,p_2^3,p_3^1\},\\ \{p_1^1,p_2^1,p_3^2\},\{p_1^1,p_2^3,p_3^2\}\}),\\ (r_3,\{\{p_1^1,p_2^1,p_3^1\},\{p_1^1,p_2^2,p_3^1\},\\ \{p_1^1,p_2^3,p_3^1\},\{p_1^1,p_2^2,p_3^2\},\{p_1^1,p_2^3,p_3^2\}\}),\\ (r_4,\{\{p_1^1,p_2^1,p_3^1\},\{p_1^1,p_2^3,p_3^1\},\{p_1^1,p_2^1,p_3^2\},\\ \{p_1^1,p_2^2,p_3^2\},\{p_1^1,p_2^3,p_3^2\}\}),\\ (r_5,\{\{p_1^1,p_2^1,p_3^1\},\{p_1^1,p_2^2,p_3^1\},\\ \{p_1^1,p_2^1,p_3^2\},\{p_1^1,p_2^3,p_3^2\}\})\end{array}\right\}.$$

Now let's calculate all inverse relational hypersoft membership degrees for (Ψ,R),

For $r_{1=j}$:

Since $p_{1,2,3}^{1,2,1}\in[P\setminus\Psi(r_{1=j})]$ for $s_1=1$, $s_2=2$ and $s_3=1$; then $\overset{(\Psi,R)}{\Upsilon}_{r_1}(p_{1,2,3}^{1,2,1},p_{1,2,3}^{t_1,t_2,t_3})=0$ for $p_{1,2,3}^{t_1,t_2,t_3}\in\Psi(r_1)$. Thus we have

$$\overset{(\Psi,R)}{\Xi}\left(r_1^{[p_{1,2,3}^{1,2,1}]}\right)=\frac{1}{4.5}\left[\begin{array}{c}\overset{(\Psi,R)}{\Upsilon}_{r_2}(p_{1,2,3}^{1,2,1},p_{1,2,3}^{t_1,t_2,t_3})\\ +\overset{(\Psi,R)}{\Upsilon}_{r_3}(p_{1,2,3}^{1,2,1},p_{1,2,3}^{t_1,t_2,t_3})\\ +\overset{(\Psi,R)}{\Upsilon}_{r_4}(p_{1,2,3}^{1,2,1},p_{1,2,3}^{t_1,t_2,t_3})\\ +\overset{(\Psi,R)}{\Upsilon}_{r_5}(p_{1,2,3}^{1,2,1},p_{1,2,3}^{t_1,t_2,t_3})\end{array}\right],$$
$$\forall p_{1,2,3}^{t_1,t_2,t_3}\in\Psi(r_1)$$
$$=\frac{\left[\begin{array}{c}(0+1+1+0+0)+(1+1+0+1+0)\\+(0+0+0+0+0)+(1+0+1+0+0)\end{array}\right]}{20}$$
$$=7/20$$

Similarly,

For r_1:

Since $p_{1,2,3}^{1,3,2}\in[P\setminus\Psi(r_1)]$, then $\overset{(\Psi,R)}{\Xi}(r_1^{[p_{1,2,3}^{1,3,2}]})=11/20$.

For r_2:

Since $p_{1,2,3}^{1,1,1},p_{1,2,3}^{1,2,2}\in[P\setminus\Psi(r_2)]$, then $\overset{(\Psi,R)}{\Xi}(r_2^{[p_{1,2,3}^{1,1,1}]})=11/20$ and $\overset{(\Psi,R)}{\Xi}(r_2^{[p_{1,2,3}^{1,2,2}]})=8/20$.

For r_3:

Since $p_{1,2,3}^{1,1,2}\in[P\setminus\Psi(r_3)]$, then $\overset{(\Psi,R)}{\Xi}(r_3^{[p_{1,2,3}^{1,1,2}]})=13/20$.

For r_4:

Since $p_{1,2,3}^{1,2,1}\in[P\setminus\Psi(r_4)]$, then $\overset{(\Psi,R)}{\Xi}(r_4^{[p_{1,2,3}^{1,2,1}]})=10/20$.

For r_5:

Since $p_{1,2,3}^{1,3,1},p_{1,2,3}^{1,2,2}\in[P\setminus\Psi(r_5)]$, then $\overset{(\Psi,R)}{\Xi}(r_5^{[p_{1,2,3}^{1,3,1}]})=11/20$ and $\overset{(\Psi,R)}{\Xi}(r_5^{[p_{1,2,3}^{1,2,2}]})=8/20$.

In addition, we can write by matrix form as

$$(\Psi,R)=\left\{\begin{array}{c|cccccc} & \{p_1^1,p_2^1,p_3^1\} & \{p_1^1,p_2^2,p_3^1\} & \{p_1^1,p_2^3,p_3^1\} & \{p_1^1,p_2^1,p_3^2\} & \{p_1^1,p_2^2,p_3^2\} & \{p_1^1,p_2^3,p_3^2\}\\ \hline r_1 & 1 & 7/20 & 1 & 1 & 1 & 11/20\\ r_2 & 11/20 & 1 & 1 & 1 & 2/5 & 1\\ r_3 & 1 & 1 & 1 & 13/20 & 1 & 1\\ r_4 & 1 & 1/2 & 1 & 1 & 1 & 1\\ r_5 & 1 & 1 & 11/20 & 1 & 2/5 & 1\end{array}\right\}.$$

Remark 3.6: Consider Examples 3.2 and 3.5, the comparison of the results obtained is as follows:

$$\overset{(\Phi,\mathcal{P})}{\Theta}(\{p_{1,2,3}^{1,1,1}\}^{r_2})=11/20=11/20=\overset{(\Psi,R)}{\Xi}(r_2^{[p_{1,2,3}^{1,1,1}]}),$$
$$\overset{(\Phi,\mathcal{P})}{\Theta}(\{p_{1,2,1}^{1,1,1}\}^{r_1})=7/20=7/20=\overset{(\Psi,R)}{\Xi}(r_1^{[p_{1,2,3}^{1,2,1}]}),$$
$$\overset{(\Phi,\mathcal{P})}{\Theta}(\{p_{1,2,1}^{1,1,1}\}^{r_4})=1/2=1/2=\overset{(\Psi,R)}{\Xi}(r_4^{[p_{1,2,3}^{1,2,1}]}),$$
$$\overset{(\Phi,\mathcal{P})}{\Theta}(\{p_{1,3,1}^{1,1,1}\}^{r_5})=11/20=11/20=\overset{(\Psi,R)}{\Xi}(r_5^{[p_{1,2,3}^{1,3,1}]}),$$
$$\overset{(\Phi,\mathcal{P})}{\Theta}(\{p_{1,1,2}^{1,1,1}\}^{r_3})=13/20=13/20=\overset{(\Psi,R)}{\Xi}(r_3^{[p_{1,2,3}^{1,1,2}]}),$$
$$\overset{(\Phi,\mathcal{P})}{\Theta}(\{p_{1,2,2}^{1,1,1}\}^{r_2})=2/5=2/5=\overset{(\Psi,R)}{\Xi}(r_2^{[p_{1,2,3}^{1,2,2}]}),$$
$$\overset{(\Phi,\mathcal{P})}{\Theta}(\{p_{1,2,2}^{1,1,1}\}^{r_5})=2/5=2/5=\overset{(\Psi,R)}{\Xi}(r_5^{[p_{1,2,3}^{1,2,2}]}),$$
$$\overset{(\Phi,\mathcal{P})}{\Theta}(\{p_{1,3,2}^{1,1,1}\}^{r_1})=11/20=11/20=\overset{(\Psi,R)}{\Xi}(r_1^{[p_{1,2,3}^{1,3,2}]}).$$

Moreover, the matrix representations of the results obtained are transposes of each other.

Proposition 3.7: *Let $(\Phi,P)\in HS(R)$ and $(\Psi,R)\in IHS(P)$. Then,*

$$\overset{(\Phi,\mathcal{P})}{\Theta}(\{p_{1,\ldots,n}^{a_1,\ldots,a_n}\}^{r_b})=\overset{(\Psi,R)}{\Xi}(r_b^{[p_{1,\ldots,n}^{a_1,\ldots,a_n}]})$$

for $1\le b\le m$, $1\le a_k\le m_k$ for $k=1,2,\ldots,n$ and $m,n\ge 2$.

Proof: This proof is easily obtained from Definitions 3.1 and 3.3. ■

4. The proposed decision-making approaches

In this section, two novel decision-making algorithms for uncertainty problems are proposed based on the concepts proposed in the previous section.

First, a decision-making algorithm based on the concept of relational hypersoft membership degree, which we propose for hypersoft sets, is constructed as follows:

Algorithm 1 Determine the best object based on a hypersoft sets.

Require: $R = \{r_1, r_2, \dots r_t, \dots, r_m\}$, $P_k = \{p_k^1, p_k^2, \dots p_k^{t_k}, \dots, p_k^{m_k}\}$, $P = \prod_{k=1}^{n} P_k$, for $1 \leq t_k \leq m_k$, $1 \leq t \leq m$, $k = 1, 2, \dots, n$ and $m, n \geq 2$

Step 1: Input the hypersoft set $(\Phi, P) \in HS(R)$ as follows:

$$(\Phi, P) = \{(p_{1,\dots,n}^{t_1,\dots,t_n}, \Phi(p_{1,\dots,n}^{t_1,\dots,t_n})) : p_{1,\dots,n}^{t_1,\dots,t_n} \in P, \Phi(p_{1,\dots,n}^{t_1,\dots,t_n}) \in 2^R\}$$

Step 2: Calculate all relational hypersoft membership degrees using the hypersoft set.

Step 3: Calculate the $\overset{(\Phi,P)}{\bigoplus}(r_t)$ of objects r_t:

$$\overset{(\Phi,P)}{\bigoplus}(r_t) = \sum_{\forall p_{1,\dots,n}^{t_1,\dots,t_n} \in P} \mu_{\Phi(p_{1,\dots,n}^{t_1,\dots,t_n})}(r_t)$$

where

$$\mu_{\Phi(p_{1,\dots,n}^{t_1,\dots,t_n})}(r_t) = \begin{cases} 1, & r_t \in \Phi(p_{1,\dots,n}^{t_1,\dots,t_n}) \\ \overset{(\Phi,P)}{\Theta}(\{p_{1,\dots,n}^{t_1,\dots,t_n}\}^{r_t}), & r_t \in R \setminus \Phi(p_{1,\dots,n}^{t_1,\dots,t_n}) \end{cases}$$

Step 4: Find x, for which $\overset{(\Phi,P)}{\bigoplus}(r_x) = \max\{\overset{(\Phi,P)}{\bigoplus}(r_t) : 1 \leq t \leq m\}$.

Now, let's give an example to show the principles and steps of Algorithm 1 in an environment of uncertainty:

Example 4.1: Suppose a person wants to buy a house. For this purpose, he/she has determined some houses that he/she thinks are most suitable for him/her, and the set of these houses is $R = \{r_1, r_2, r_3, r_4, r_5, r_6\}$. In order to determine the most suitable house for this person, we must analyse the parametric values of the houses. Let the distinct parametric values chosen for the determined houses be expressed as 'ξ_1 : *sizes*', 'ξ_2 : *costs*', 'ξ_3 : *warming type*', 'ξ_4 : *house expenses per month*' and these values are given as $P_1 = \{p_1^1 : small, p_1^2 : large\}$, $P_2 = \{p_2^1 : cheap, p_2^2 : economic, p_2^3 = expensive\}$, $P_3 = \{p_3^1 : air \ \ conditioning\}$, $P_4 = \{p_4^1 : much, p_4^2 : little\}$; respectively. Then, for $P = \prod_{i=1}^{4} P_i$,

$$P = \left\{ \begin{array}{l} p_{1,2,3,4}^{1,1,1,1}, p_{1,2,3,4}^{1,1,1,2}, p_{1,2,3,4}^{1,2,1,1}, p_{1,2,3,4}^{1,2,1,2}, p_{1,2,3,4}^{1,3,1,1}, p_{1,2,3,4}^{1,3,1,2}, \\ p_{1,2,3,4}^{2,1,1,1}, p_{1,2,3,4}^{2,1,1,2}, p_{1,2,3,4}^{2,2,1,1}, p_{1,2,3,4}^{2,2,1,2}, p_{1,2,3,4}^{2,3,1,1}, p_{1,2,3,4}^{2,3,1,2} \end{array} \right\}$$

Step 1: We can express the evaluation result of these values for the houses with the help of a hypersoft set as follows:

$$(\Phi, P) = \left\{ \begin{array}{c} (p_{1,2,3,4}^{1,1,1,1}, \{r_1, r_2, r_3, r_5, r_6\}), \\ (p_{1,2,3,4}^{1,1,1,2}, \{r_1, r_3, r_4, r_6\}), \\ (p_{1,2,3,4}^{1,2,1,1}, \{r_2, r_3, r_5, r_6\}) \\ (p_{1,2,3,4}^{1,2,1,2}, \{r_1, r_2, r_3, r_4\}), \\ (p_{1,2,3,4}^{1,3,1,1}, \{r_3, r_4, r_5, r_6\}), \\ (p_{1,2,3,4}^{1,3,1,2}, \{r_1, r_3, r_5, r_6\}) \\ (p_{1,2,3,4}^{2,1,1,1}, \{r_1, r_2, r_4, r_5, r_6\}), \\ (p_{1,2,3,4}^{2,1,1,2}, \{r_1, r_3, r_4, r_5\}), \\ (p_{1,2,3,4}^{2,2,1,1}, \{r_2, r_3, r_4, r_5\}) \\ (p_{1,2,3,4}^{2,2,1,2}, \{r_1, r_2, r_3, r_5\}), \\ (p_{1,2,3,4}^{2,3,1,1}, \{r_1, r_3, r_4, r_5, r_6\}), \\ (p_{1,2,3,4}^{2,3,1,2}, \{r_2, r_3, r_4, r_5, r_6\}) \end{array} \right\}$$

Step 2: For (Φ, P), since

$$r_4 \in [R \setminus \Phi(p_{1,2,3,4}^{1,1,1,1})], \quad r_2, r_5 \in [R \setminus \Phi(p_{1,2,3,4}^{1,1,1,2})],$$
$$r_1, r_4 \in [R \setminus \Phi(p_{1,2,3,4}^{1,2,1,1})], \quad r_5, r_6 \in [R \setminus \Phi(p_{1,2,3,4}^{1,2,1,2})],$$
$$r_1, r_2 \in [R \setminus \Phi(p_{1,2,3,4}^{1,3,1,1})], \quad r_2, r_4 \in [R \setminus \Phi(p_{1,2,3,4}^{1,3,1,2})],$$
$$r_3 \in [R \setminus \Phi(p_{1,2,3,4}^{2,1,1,1})], \quad r_2, r_6 \in [R \setminus \Phi(p_{1,2,3,4}^{2,1,1,2})],$$
$$r_1, r_6 \in [R \setminus \Phi(p_{1,2,3,4}^{2,2,1,1})], \quad r_4, r_6 \in [R \setminus \Phi(p_{1,2,3,4}^{2,2,1,2})],$$
$$r_2 \in [R \setminus \Phi(p_{1,2,3,4}^{2,3,1,1})], \quad r_1 \in [R \setminus \Phi(p_{1,2,3,4}^{2,3,1,2})];$$

then

$$\overset{(\Phi,P)}{\Theta}(\{p_{1,2,3,4}^{1,1,1,1}\}^{r_4}) = 27/55,$$

$$\overset{(\Phi,P)}{\Theta}(\{p_{1,2,3,4}^{1,1,1,2}\}^{r_2}) = 18/55,$$

$$\overset{(\Phi,P)}{\Theta}(\{p_{1,2,3,4}^{1,1,1,2}\}^{r_5}) = 28/55,$$

$$\overset{(\Phi,P)}{\Theta}(\{p_{1,2,3,4}^{1,2,1,1}\}^{r_1}) = 24/55,$$

$$\overset{(\Phi,P)}{\Theta}(\{p^{1,2,1,1}_{1,2,3,4}\}^{r_4}) = 2/5,$$

$$\overset{(\Phi,P)}{\Theta}(\{p^{1,2,1,2}_{1,2,3,4}\}^{r_5}) = 27/55,$$

$$\overset{(\Phi,P)}{\Theta}(\{p^{1,2,1,2}_{1,2,3,4}\}^{r_6}) = 21/55,$$

$$\overset{(\Phi,P)}{\Theta}(\{p^{1,3,1,1}_{1,2,3,4}\}^{r_1}) = 23/55,$$

$$\overset{(\Phi,P)}{\Theta}(\{p^{1,3,1,1}_{1,2,3,4}\}^{r_2}) = 4/11,$$

$$\overset{(\Phi,P)}{\Theta}(\{p^{1,3,1,2}_{1,2,3,4}\}^{r_2}) = 4/11,$$

$$\overset{(\Phi,P)}{\Theta}(\{p^{1,3,1,2}_{1,2,3,4}\}^{r_4}) = 23/55,$$

$$\overset{(\Phi,P)}{\Theta}(\{p^{2,1,1,1}_{1,2,3,4}\}^{r_3}) = 36/55,$$

$$\overset{(\Phi,P)}{\Theta}(\{p^{2,1,1,2}_{1,2,3,4}\}^{r_2}) = 4/11,$$

$$\overset{(\Phi,P)}{\Theta}(\{p^{2,1,1,2}_{1,2,3,4}\}^{r_6}) = 24/55,$$

$$\overset{(\Phi,P)}{\Theta}(\{p^{2,2,1,1}_{1,2,3,4}\}^{r_1}) = 2/5,$$

$$\overset{(\Phi,P)}{\Theta}(\{p^{2,2,1,1}_{1,2,3,4}\}^{r_6}) = 23/55,$$

$$\overset{(\Phi,P)}{\Theta}(\{p^{2,2,1,2}_{1,2,3,4}\}^{r_4}) = 2/5,$$

$$\overset{(\Phi,P)}{\Theta}(\{p^{2,2,1,2}_{1,2,3,4}\}^{r_6}) = 23/55,$$

$$\overset{(\Phi,P)}{\Theta}(\{p^{2,3,1,1}_{1,2,3,4}\}^{r_2}) = 24/55,$$

$$\overset{(\Phi,P)}{\Theta}(\{p^{2,3,1,2}_{1,2,3,4}\}^{r_1}) = 27/55.$$

Step 3: The value $\overset{(\Phi,P)}{\bigoplus}(r_1)$ for $r_{1=t}$ is calculated as follows:

$$\begin{aligned}\overset{(\Phi,P)}{\bigoplus}(r_1) &= \sum_{\forall p^{t_1,\dots,t_n}_{1,\dots,n} \in P} \mu_{\Phi(p^{t_1,\dots,t_n}_{1,\dots,n})}(r_1) \\ &= \begin{bmatrix} 1+1+24/55+1+23/55+1 \\ +1+1+2/5+1+1+27/55 \end{bmatrix} \\ &= 10.836.\end{aligned}$$

Similarly,

$$\overset{(\Phi,P)}{\bigoplus}(r_2) = 8.854, \quad \overset{(\Phi,P)}{\bigoplus}(r_3) = 11.654,$$
$$\overset{(\Phi,P)}{\bigoplus}(r_4) = 9.709, \quad \overset{(\Phi,P)}{\bigoplus}(r_5) = 11,$$
$$\overset{(\Phi,P)}{\bigoplus}(r_6) = 9.654.$$

Step 4: Since $\overset{(\Phi,P)}{\bigoplus}(r_3) = \max\{\overset{(\Phi,P)}{\bigoplus}(r_t) : 1 \le t \le m\} = 11.654$, we find that r_3 is the best house choice.

Second, a decision-making algorithm based on the concept of inverse relational hypersoft membership degree, which we propose for inverse hypersoft sets, is constructed as follows:

Algorithm 2 Determine the best object based on an inverse hypersoft sets.

Require: $R = \{r_1, r_2, \dots r_j, \dots, r_m\}$, $P_k = \{p^1_k, p^2_k, \dots p^{l_k}_k, \dots, p^{m_k}_k\}$, $P = \prod_{k=1}^{n} P_k$, for $1 \le l_k \le m_k$, $1 \le j \le m$, $k = 1, 2, \dots, n$ and $m, n \ge 2$

Step 1: Input the inverse hypersoft set $(\Psi, R) \in IHS(P)$ as follows:

$$(\Psi, R) = \{(r_j, \Psi(r_j)) : r_j \in R, \Psi(r_j) \in 2^P\}.$$

Step 2: Calculate all inverse relational hypersoft membership degrees using the inverse hypersoft set.

Step 3: Calculate the $\overset{(\Psi,R)}{\bigoplus}(r_j)$ of objects r_j:

$$\overset{(\Psi,R)}{\bigoplus}(r_j) = \sum_{\forall p^{l_1,\dots,l_n}_{1,\dots,n} \in P} \mu_{\Psi(r_j)}(p^{l_1,\dots,l_n}_{1,\dots,n})$$

where

$$\mu_{\Psi(r_j)}(p^{l_1,\dots,l_n}_{1,\dots,n}) = \begin{cases} 1, & p^{l_1,\dots,l_n}_{1,\dots,n} \in \Psi(r_j) \\ \overset{(\Psi,R)}{\Xi}\left(r_j^{[p^{l_1,\dots,l_n}_{1,\dots,n}]}\right), & p^{l_1,\dots,l_n}_{1,\dots,n} \in P \setminus \Psi(r_j) \end{cases}$$

Step 4: Find x, for which $\overset{(\Psi,R)}{\bigoplus}(r_x) = \max\{\overset{(\Psi,R)}{\bigoplus}(r_j) : 1 \le j \le m\}$.

Proposition 4.2: *Let $(\Phi, P) \in HS(R)$ and $(\Psi, R) \in IHS(P)$. Then,*

(i) $\overset{(\Phi,P)}{\bigoplus}(r) = \overset{(\Psi,R)}{\bigoplus}(r); \forall r \in R.$

(ii) $\mu_{\Phi(p^{l_1,\dots,l_n}_{1,\dots,n})}(r) = \mu_{\Psi(r)}(p^{l_1,\dots,l_n}_{1,\dots,n})$ *for* $1 \le l_k \le m_k$, $k = 1, 2, \dots, n$ *and* $n \ge 2$, $\forall r \in R$.

Proof: It can be proved simply. ■

Now, let's give an example to show the principles and steps of Algorithm 2 in an environment of uncertainty:

Example 4.3: Consider Example 4.1. Then,

Step 1: The inverse hypersoft set (Ψ, R) that expresses the current uncertainty situation is given by

$$(\Psi, R) = \left\{ \begin{array}{c} (r_1, \{p^{1,1,1,1}_{1,2,3,4}, p^{1,1,1,2}_{1,2,3,4}, p^{1,2,1,2}_{1,2,3,4}, p^{1,3,1,2}_{1,2,3,4}, \\ p^{2,1,1,1}_{1,2,3,4}, p^{2,1,1,2}_{1,2,3,4}, p^{2,2,1,1}_{1,2,3,4}, p^{2,3,1,1}_{1,2,3,4}\}), \\ (r_2, \{p^{1,1,1,1}_{1,2,3,4}, p^{1,2,1,1}_{1,2,3,4}, p^{1,2,1,2}_{1,2,3,4}, p^{2,1,1,1}_{1,2,3,4}, \\ p^{2,2,1,1}_{1,2,3,4}, p^{2,2,1,2}_{1,2,3,4}, p^{2,3,1,2}_{1,2,3,4}\}), \\ \left(r_3, \left\{ \begin{array}{c} p^{1,1,1,1}_{1,2,3,4}, p^{1,1,1,2}_{1,2,3,4}, p^{1,2,1,1}_{1,2,3,4}, \\ p^{1,2,1,2}_{1,2,3,4}, p^{1,3,1,1}_{1,2,3,4}, p^{1,3,1,2}_{1,2,3,4}, p^{2,1,1,2}_{1,2,3,4}, \\ p^{2,2,1,1}_{1,2,3,4}, p^{2,2,1,2}_{1,2,3,4}, p^{2,3,1,1}_{1,2,3,4}, p^{2,3,1,2}_{1,2,3,4} \end{array} \right\} \right), \\ (r_4, \{p^{1,1,1,2}_{1,2,3,4}, p^{1,2,1,2}_{1,2,3,4}, p^{1,3,1,1}_{1,2,3,4}, p^{2,1,1,1}_{1,2,3,4}, \\ p^{2,1,1,2}_{1,2,3,4}, p^{2,2,1,1}_{1,2,3,4}, p^{2,3,1,1}_{1,2,3,4}, p^{2,3,1,2}_{1,2,3,4}\}), \\ \left(r_5, \left\{ \begin{array}{c} p^{1,1,1,1}_{1,2,3,4}, p^{1,2,1,1}_{1,2,3,4}, p^{1,3,1,1}_{1,2,3,4}, p^{1,3,1,2}_{1,2,3,4}, \\ p^{2,1,1,1}_{1,2,3,4}, p^{2,1,1,2}_{1,2,3,4}, p^{2,2,1,1}_{1,2,3,4}, \\ p^{2,2,1,2}_{1,2,3,4}, p^{2,3,1,1}_{1,2,3,4}, p^{2,3,1,2}_{1,2,3,4} \end{array} \right\} \right), \\ (r_6, \{p^{1,1,1,1}_{1,2,3,4}, p^{1,1,1,2}_{1,2,3,4}, p^{1,2,1,1}_{1,2,3,4}, p^{1,3,1,1}_{1,2,3,4}, \\ p^{1,3,1,2}_{1,2,3,4}, p^{2,1,1,1}_{1,2,3,4}, p^{2,3,1,1}_{1,2,3,4}, p^{2,3,1,2}_{1,2,3,4}\}), \end{array} \right\}.$$

Step 2: For (Ψ, R), since

$$p^{1,2,1,1}_{1,2,3,4}, p^{1,3,1,1}_{1,2,3,4}, p^{2,2,1,1}_{1,2,3,4}, p^{2,3,1,2}_{1,2,3,4} \in [P \setminus \Psi(r_1)],$$

$$p^{1,1,1,2}_{1,2,3,4}, p^{1,3,1,1}_{1,2,3,4}, p^{1,3,1,2}_{1,2,3,4}, p^{2,1,1,2}_{1,2,3,4}, p^{2,3,1,1}_{1,2,3,4} \in [P \setminus \Psi(r_2)]$$

$$p^{2,1,1,1}_{1,2,3,4} \in [P \setminus \Psi(r_3)],$$

$$p^{1,1,1,1}_{1,2,3,4}, p^{1,2,1,1}_{1,2,3,4}, p^{1,3,1,2}_{1,2,3,4}, p^{2,2,1,2}_{1,2,3,4} \in [P \setminus \Psi(r_4)],$$

$$p^{1,1,1,2}_{1,2,3,4}, p^{1,2,1,2}_{1,2,3,4} \in [P \setminus \Psi(r_5)],$$

$$p^{1,2,1,2}_{1,2,3,4}, p^{2,1,1,2}_{1,2,3,4}, p^{2,2,1,1}_{1,2,3,4}, p^{2,2,1,2}_{1,2,3,4} \in [P \setminus \Psi(r_6)];$$

then

$$\overset{(\Psi,R)}{\Xi}(r_1^{[p^{1,2,1,1}_{1,2,3,4}]}) = 24/55, \quad \overset{(\Psi,R)}{\Xi}(r_1^{[p^{1,3,1,1}_{1,2,3,4}]}) = 23/55,$$

$$\overset{(\Psi,R)}{\Xi}(r_1^{[p^{2,2,1,1}_{1,2,3,4}]}) = 2/5, \quad \overset{(\Psi,R)}{\Xi}(r_1^{[p^{2,3,1,2}_{1,2,3,4}]}) = 27/55,$$

$$\overset{(\Psi,R)}{\Xi}(r_2^{[p^{1,1,1,2}_{1,2,3,4}]}) = 18/55, \quad \overset{(\Psi,R)}{\Xi}(r_2^{[p^{1,3,1,1}_{1,2,3,4}]}) = 4/11,$$

$$\overset{(\Psi,R)}{\Xi}(r_2^{[p^{1,3,1,2}_{1,2,3,4}]}) = 4/11, \quad \overset{(\Psi,R)}{\Xi}(r_2^{[p^{2,1,1,2}_{1,2,3,4}]}) = 4/11,$$

$$\overset{(\Psi,R)}{\Xi}(r_2^{[p^{2,3,1,1}_{1,2,3,4}]}) = 24/55, \quad \overset{(\Psi,R)}{\Xi}(r_3^{[p^{2,1,1,1}_{1,2,3,4}]}) = 36/55,$$

$$\overset{(\Psi,R)}{\Xi}(r_4^{[p^{1,1,1,1}_{1,2,3,4}]}) = 27/55, \quad \overset{(\Psi,R)}{\Xi}(r_4^{[p^{1,2,1,1}_{1,2,3,4}]}) = 2/5,$$

$$\overset{(\Psi,R)}{\Xi}(r_4^{[p^{1,3,1,2}_{1,2,3,4}]}) = 23/55, \quad \overset{(\Psi,R)}{\Xi}(r_4^{[p^{2,2,1,2}_{1,2,3,4}]}) = 2/5,$$

$$\overset{(\Psi,R)}{\Xi}(r_5^{[p^{1,1,1,2}_{1,2,3,4}]}) = 28/55, \quad \overset{(\Psi,R)}{\Xi}(r_5^{[p^{1,2,1,2}_{1,2,3,4}]}) = 27/55,$$

$$\overset{(\Psi,R)}{\Xi}(r_6^{[p^{1,2,1,2}_{1,2,3,4}]}) = 21/55, \quad \overset{(\Psi,R)}{\Xi}(r_6^{[p^{2,1,1,2}_{1,2,3,4}]}) = 24/55,$$

$$\overset{(\Psi,R)}{\Xi}(r_6^{[p^{2,2,1,1}_{1,2,3,4}]}) = 23/55, \quad \overset{(\Psi,R)}{\Xi}(r_6^{[p^{2,2,1,2}_{1,2,3,4}]}) = 23/55.$$

Step 3: The value $\overset{(\Psi,R)}{\bigoplus}(r_1)$ for $r_{1=j}$ is calculated as follows:

$$\overset{(\Psi,R)}{\bigoplus}(r_1) = \sum_{\forall p^{l_1,\ldots,l_n}_{1,\ldots,n} \in P} \mu_{\Psi(r_1)}(p^{l_1,\ldots,l_n}_{1,\ldots,n})$$
$$= \left[\begin{array}{c} 1 + 1 + 24/55 + 1 + 23/55 + 1 \\ +1 + 1 + 2/5 + 1 + 1 + 27/55 \end{array} \right]$$
$$= 10.836.$$

Similarly,

$$\overset{(\Psi,R)}{\bigoplus}(r_2) = 8.854, \quad \overset{(\Psi,R)}{\bigoplus}(r_3) = 11.654,$$
$$\overset{(\Psi,R)}{\bigoplus}(r_4) = 9.709, \quad \overset{(\Psi,R)}{\bigoplus}(r_5) = 11,$$
$$\overset{(\Psi,R)}{\bigoplus}(r_6) = 9.654.$$

Step 4: Since $\overset{(\Psi,R)}{\bigoplus}(r_3) = \max\{\overset{(\Psi,R)}{\bigoplus}(r_j) : 1 \leq j \leq m\} = 11.654$, we find that r_3 is the best house choice. As a result, we determine that r_3 is the most suitable house for the assumed problem.

Remark 4.4: In (inverse) hypersoft sets, the membership degree of an object (a parameter) is 0 or 1. Thanks to this paper, we can express this membership degree in the range $(0, 1)$. Therefore, the proposed decision-making algorithms are better for achieving near-ideal results than all existing algorithms for (inverse) hypersoft sets. Some results for the algorithms given in this paper can be given as follows:

(i) The decision-making process for the current uncertainty problem usually focuses on objects providing parameters or parameters providing objects. In both cases, it focuses on one correct result. In this case, Algorithms 1 and 2 are equivalent. In other words, no matter which algorithm is chosen, the same result is achieved.

(ii) The basis of the proposed decision-making approaches is the hypersoft set and the inverse hypersoft set. Therefore, Algorithms 1 and 2 can be applied to many mathematical models such as (plithogenic) fuzzy hypersoft sets (Smarandache, 2018), (plithogenic) intuitionistic fuzzy hypersoft sets (Smarandache, 2018), (plithogenic) neutrosophic hypersoft sets (Smarandache, 2018), plithogenic (crisp) hypersoft sets (Smarandache, 2018), single and multi-valued neutrosophic hypersoft sets (Saqlain, Jafar, et al., 2020), complex multi-fuzzy hypersoft sets (Saeed et al., 2021), pythagorean fuzzy hypersoft sets (Zulqarnain et al., 2021), neutrosophic parameterised hypersoft sets (Rahman et al., 2021) and hypersoft expert sets (Ihsan et al., 2021).

5. Conclusion

Soft sets, which were put forward to deal with uncertainty problems based on parametric data, have led to the construction of many mathematical models. Hypersoft sets, which is one of these mathematical models, is a generalisation of soft sets and has become more preferred due to the increasing number of parameters in uncertainty problems. However, in this set model, the membership degree of an object is expressed as 0 or 1. In order to express the membership degrees in the range $(0, 1)$, various fuzzy hybrid models of hypersoft sets were constructed. However, the determination of membership degrees in the range $(0, 1)$ is focused on the decision-maker. Therefore, in this paper, the concepts relational hypersoft membership degree and inverse relational hypersoft membership degree are proposed to express these values independently of the decision-maker. Some examples are given for a better understanding of these concepts. Moreover, two algorithms are proposed so that the proposed concepts can be used in the decision-making process. Finally, an analysis based on the proposed algorithms is given. We hope that the concepts given in this paper can be useful in expressing future mathematical models more independently of the decision-maker.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Data availability statement

Data sharing is not applicable to this article as no new data were created or analysed in this study.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Notes on contributor

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