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TWO PROPERTIES OF THE SPECIAL OCTAGON

Ion Patrascu

"Fratii Buzesti" National College, Craiova, Romania

Mario Dalcín

"Artigas" Secondary School Teachers Institute-CFE, Montevideo, Uruguay

Florentin Smarandache

University of New Mexico-Gallup, USA

This article is based on the article "From Newton's Theorem to a Theorem of Inscribable Octagon" in [2, pp. 96-99]. On this occasion we announce that Theorem 3 regarding the circumscribable octagon stated there is false.

In this article we will introduce the notions of *special octagon* and *quasi-center of a special octagon* and we will state and prove two properties related to these notions.

Definition 1. We call a *special octagon* a circumscribed octagon whose property as the four lines determined by the points of contact with the circle of the opposite sides are concurrent. The point of competition of these lines we will call the *quasi-center of the special octagon*.

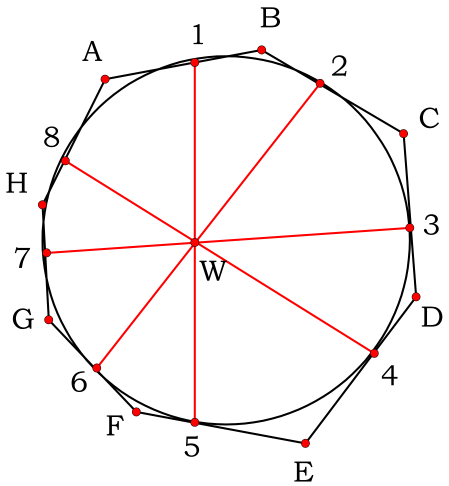

Figure 1

In Figure 1, the octagon ABCDEFGH is special. I marked with 1, 2, 3, 4, 5, 6, 7, 8 the points of tangency with the circle of the sides AB, BC, CD, DE, EF, FG, GH, HA. Lines 15, 26, 37 and 48 are concurrent in the note point W - quasi-center of the special octagon.

Property 1. *In a special octagon, the diagonals determined by the opposite vertices of the octagon are concurrent in its quasi-center.*

To demonstrate this property we will use two lemmas.

Lemma 1. (Theorem I. Newton). *In a circumscribed convex quadrilateral, the diagonals and lines determined by the points of tangent to the circle of the opposite sides are four competing lines*.

Demonstration. Let A_1, B_1, C_1, D_1 be the tangent points of the sides with the circle (see figure 2). On the extensions of the sides AB, BC, CD, DA of the circumscribed quadrilateral ABCD we construct respectively the points M, N; P, Q; R, S; U, V such that: $A_1M = A_1N = B_1P = B_1Q = C_1R = C_1S = D_1U = D_1V$.

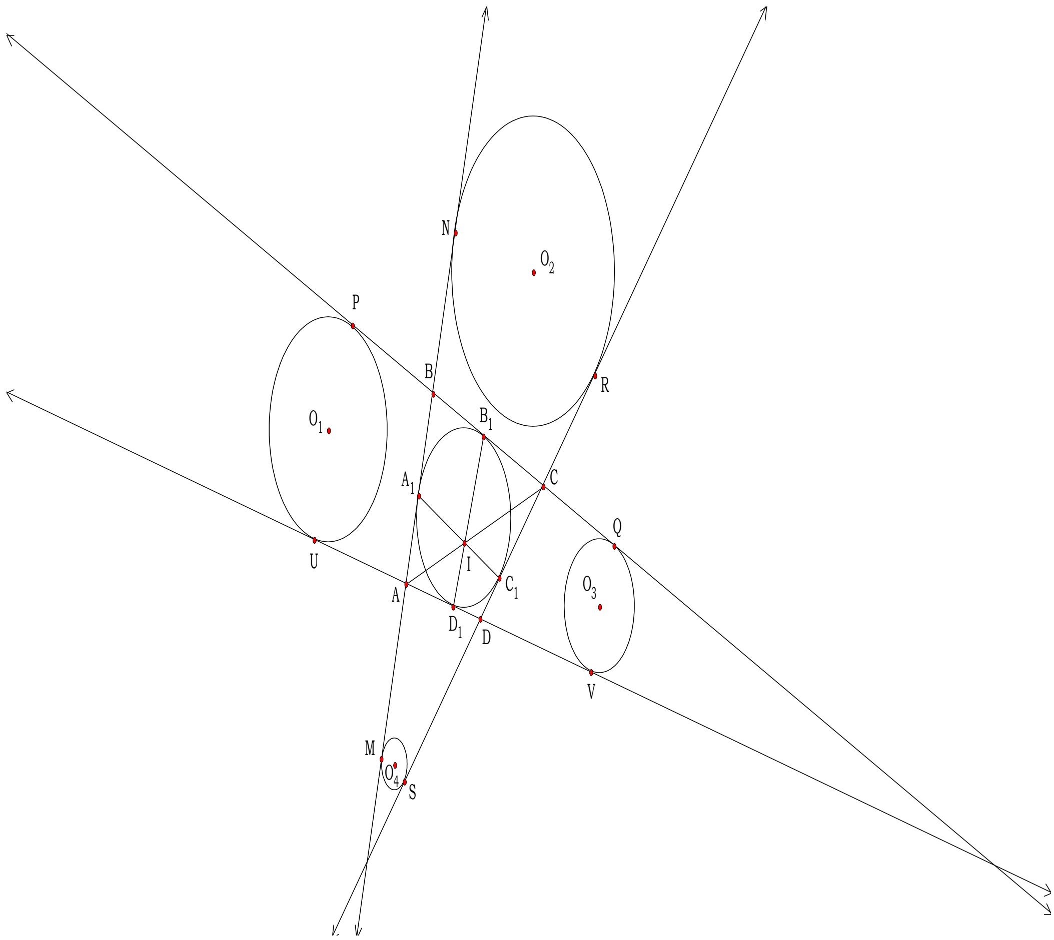

Figure 2

We denote by O_1, O_2, O_3, O_4 the centers of the tangent circles respectively in P and U of the lines BC and AD, in N and R of the lines AB and CD, in Q and V of the lines BC and AD and in M and S of the lines AB and CD .

From: $A_1M = A_1N = C_1R = C_1S$ it follows that A_1C_1 is the radical axis for the circles (O_2) and (O_4). (1)

The relations $B_1P = B_1Q = D_1U = D_1V$ lead to the conclusion that

B_1D_1 is the radical axis of the circles (O_3) and (O_1). (2)

Noting $\{I\} = A_1C_1 \cap B_1D_1$ from relations (1) and (2) we deduce that:

I has equal powers over circles (O_1), (O_2), (O_3) and (O_4). (3)

Since $BA_1 = BB_1$ (tangents taken from point B to the circle) and $B_1P = A_1N$ we obtain that $BP = BN$. (4)

Also from $D_1U = C_1R$ and $DD_1 = DC_1$ it results that $DU = DR$. (5)

Relationships (4) and (5) show that BD is the radical axis of the circles (O_1), (O_2). (6)

From relations (3) and (6) we retain that $I \in (BD)$ (7). Analogously it is shown that $I \in (AC)$ and consequently $\{I\} = A_1C_1 \cap B_1D_1 \cap AC \cap BD$. (8)

Lemma 2. *In a circumscribed concave quadrilateral the diagonals and lines determined by the points of tangent to the circle of the opposite sides are four competing lines*.

The demonstration of this lemma being very similar to that of lemma 1, we do not reproduce it here. The reader can make this demonstration using possibly figure 3. We mention that a circumscribable concave quadrilateral has two adjacent sides tangent to the circle and the extensions of the other two sides are tangent to the circle.

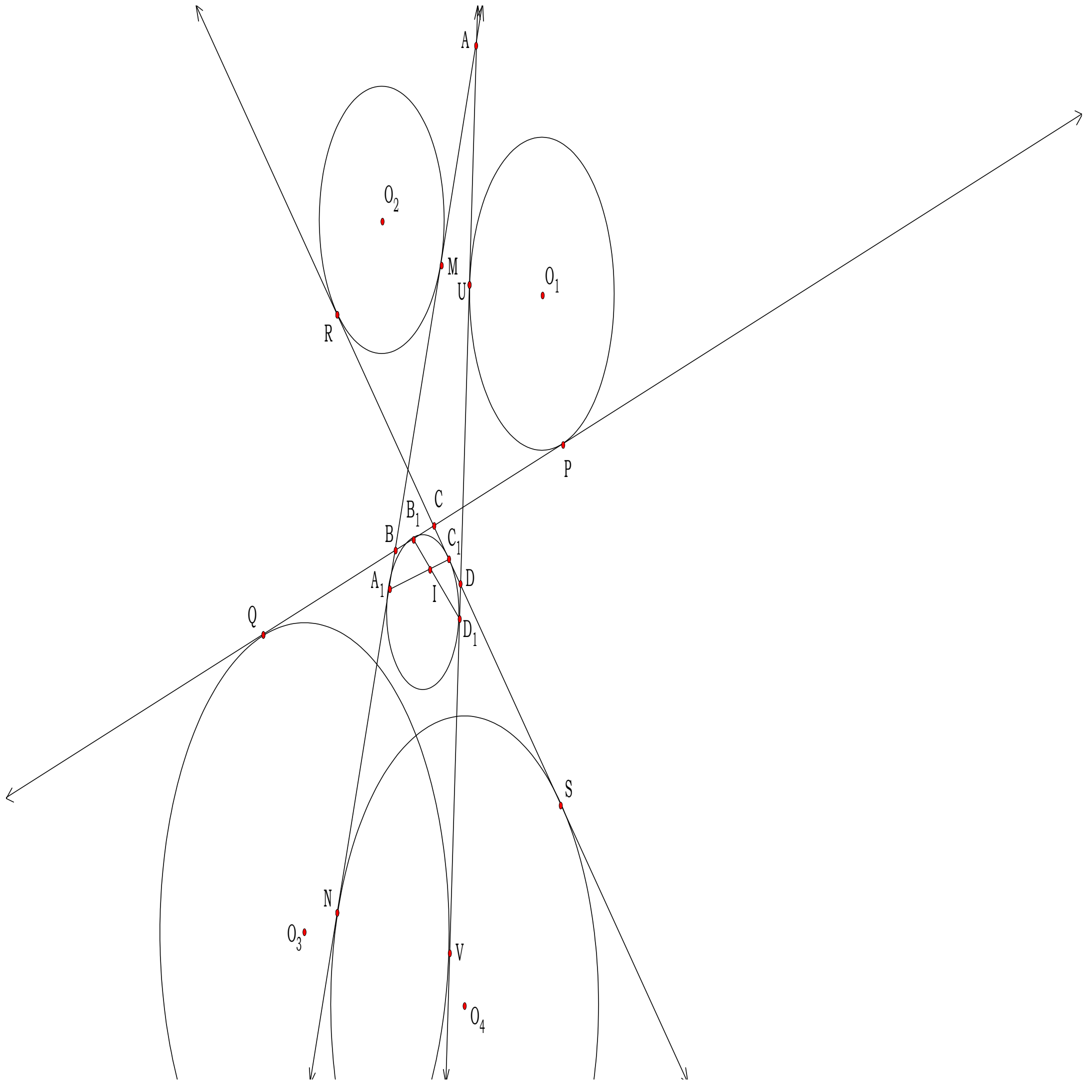

Figure 3

Proof of Ownership 1.

Note {X} = BA ∩ FG and {Y} = BC ∩ FE (see figure 4). In the circumscribed convex quadrilateral BXFY applying Lemma 1 we have as: 15 ∩ 26 ∩ BF ∩ XY = {W}. Remember that W ∈ BF. (9)

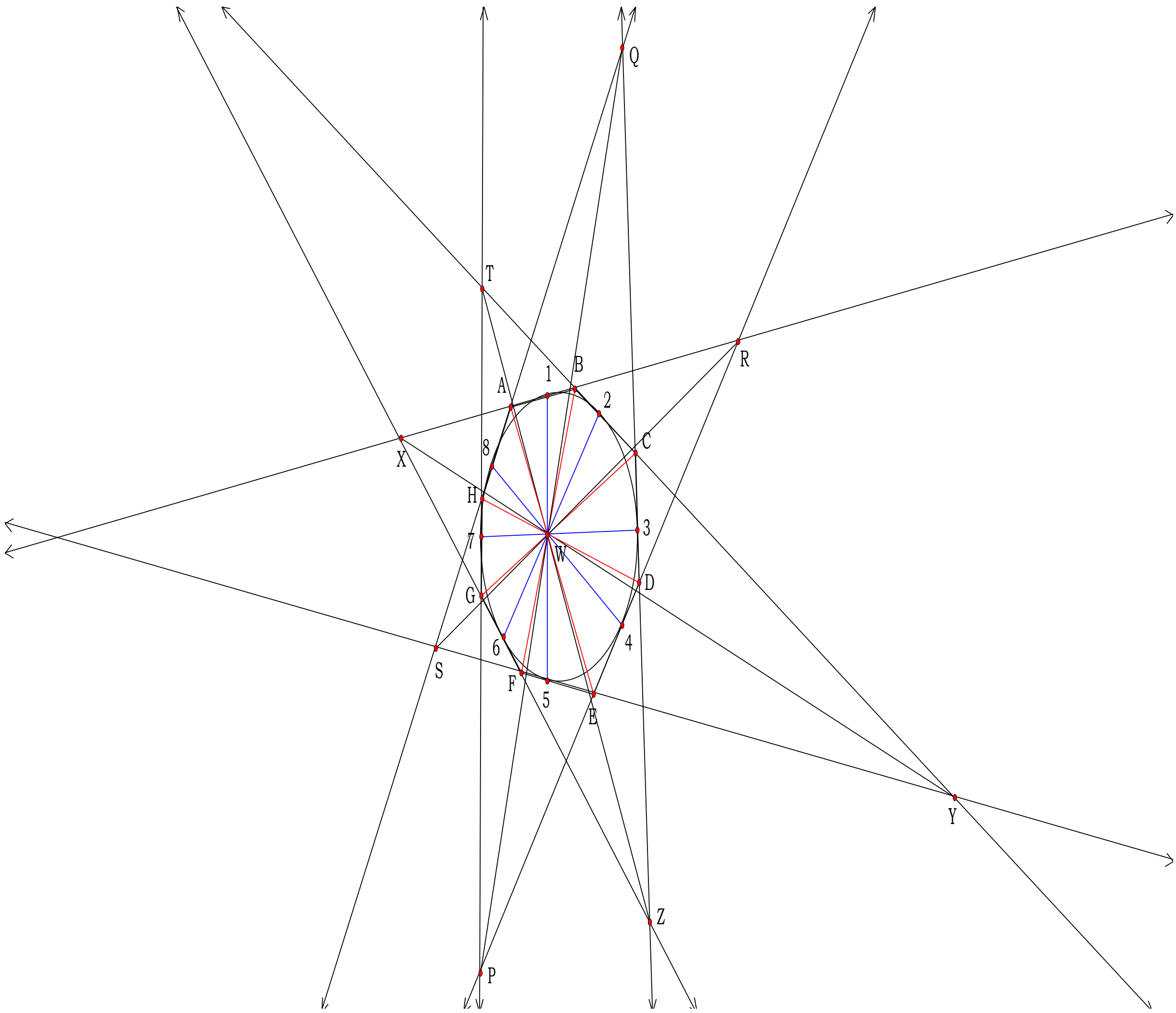

Figure 4

We denote {Z} = CD ∩ FG and {T} = CB ∩ GH, we apply the lemma in the circumscribed convex quadrilateral CZGT we obtain that; 26 ∩ 37 ∩ CG ∩ ZT = {W}. We note here that W ∈ CG. (10)

Note {R} = AB ∩ ED and {S} = AH ∩ EF. In the circumscribed convex quadrilateral ARES applying Lemma 1 we obtain that: 15 ∩ 48 ∩ AE ∩ RS = {W}, consequently W ∈ AE. (11)

Note {P} = DE ∩ HG and {Q} = CD ∩ AH. The circumscribed concave quadrilateral QDPH and Lemma 2 lead to 37 ∩ 48 ∩ DH ∩ QP = {W}. We note from here that W ∈ AE. (12)

Relationships (9), (10), (11) and (12) show that the diagonals BF, CG, AE and DH of the special octagon ABCDEFGH are concurrent in its quasi-center W.

Property 2. *The opposite sides of a special octagon and the opposite sides of the octagon determined by the tangent points of the sides of the given special octagon with the circle intersect two by two in 8 collinear points*.

Demonstration. Let ABCDEFGH be the special octagon given and die 12345678 the octagon formed by the points of tangent to the circle of the sides AB, BC, CD, DE, EF, FG, GH, HA (see figure 5).

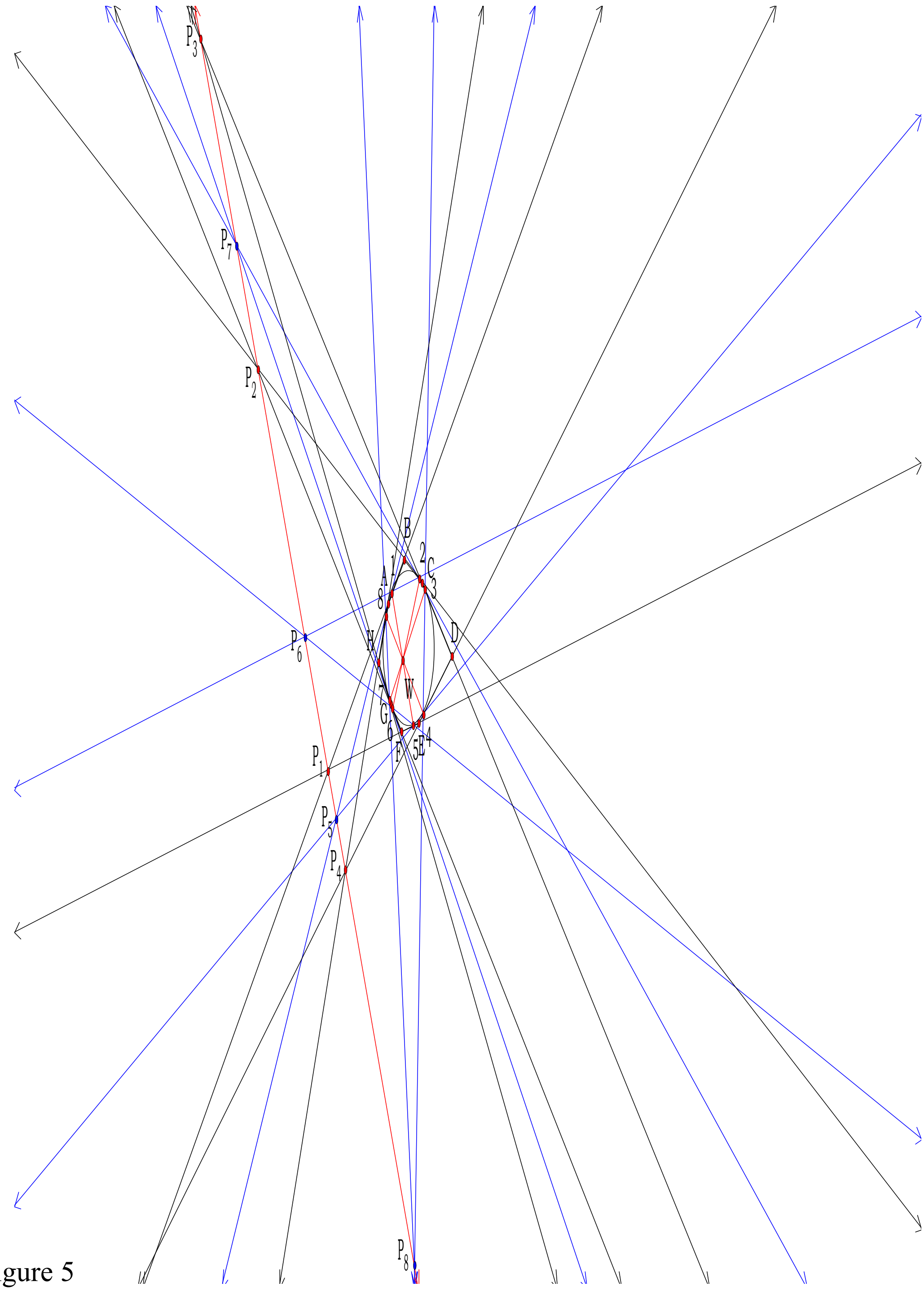

Figure 5

Transforming by duality in relation to the circle inscribed in the special octagon the configuration from figure 1 we have that the lines AB, BC, CD, DE, EF, FG, GH, HA correspond to their poles, that is the points 1, 2, 3, 4, 5, 6, 7, 8. The pole of the line 15 will be the intersection of the opposite sides AB and EF of the special octagon, we will denote this point with P_1.

The pole of line 26 will be the point P_2 - the intersection of the opposite sides BC and FG of the special octagon. The pole of line 37 will be the point P_3 - the intersection of the opposite sides CD and GH of the special octagon. The pole of line 48 is the point P_4 -the intersection of the opposite sides DE and HA of the special octagon.

Since the lines 15, 26, 37 and 48 are concurrent in the point W -quasi-center of the octagon- then the poles of these lines, ie the points P_1, P_2, P_3, P_4 will be collinear points belonging to the polar of W in relation to the circle. The polar of point A is line 18, the polar of point E is line 45, it results that the pole of the diagonal AE will be the intersection of lines 18 and 45, opposite sides in octagon 12345678, ie a point that we denote P_5, because AE passes through W we obtain that P_5 it will be on the polar of W in relation to the circle, so it will be collinear with the points P_1, P_2, P_3, P_4.

Analogously point P_6, the pole of the diagonal BF of the octagon will belong to the polar of W, P_7 the pole of the diagonal CG of the octagon will be a point on the polar of W and finally point P_8 the pole of the diagonal DH will belong to the polar of W. In conclusion the points P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8 are collinear points located on the polar quasi-center W in relation to the circle.

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