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Utilization of neutrosophic Kuhn-Tucker's optimality conditions for Solving Pythagorean fuzzy Two-Level Linear Programming Problems

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Abstract

This article considers a bi-level linear programming with single valued trapezoidal fuzzy neutrosophic cost coefficient matrix and Pythagorean fuzzy parameters in the set of constraints both in the right and left sides. Based on the score functions of the neutrosophic numbers and Pythagorean fuzzy numbers, the model is changed to the corresponding crisp bi-level linear programming (BLP) problem. This problem is designated as a Pythagorean fuzzy bi-level linear programming (PFBLP) problem under neutrosophic environment. Kuhn-Tucker's conditions for optimality are necessary and sufficient for the existence of the optimal solution to a BLP problem. Using the suggested methodology, the problem is formulated as a single-objective non-linear programming problem with several variables and constraints. Two typical numerical examples are examined to illustrate the proposed approach.

Keywords: Optimization; Optimization problems; Bi-level programming; Pythagorean fuzzy number; Neutrosophic set; Single valued neutrosophic numbers; Treapezoidal neutrosophic numbers; Kuhn-Tucker's optimality conditions; Decision Making; GAMS computer package.

1. Introduction

The process of decision making in many planning problems involves hierarchical administrative structure, which includes selection of several individual with independent and conflicting objectives. Bi-level programming is a mathematical technique where there are two decision makers (DMs) at each level in hierarchical optimization situation. The fundamental ideology concept in the Bi-level programming approach is that the DM at the upper level and the lower level are termed as the leader and follower respectively. As, when the leader optimizes his/her objective, the follower attempts to determine an optimal solution without affecting the leader's decisions (e.g. Bard and Falk [1], Sinha and Biswal [2]).

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Numerous methods for solving the bi-level linear programming (BLP) problems have been proposed in literature. Bialas and Karwan [3] contributed a lot to the development of BLP problems and their solution methodologies. Ye and Zhu [4] studied the linear BLP problem by determining the constraint region. Chen and Florian [5] investigated the optimal conditions for BLP problem in crisp environment. It is observed they did not consider the uncertainty in the parameters of the BLP problem. In real-life situations, the nature of model parameters is imprecise, which results uncertainty or vagueness.

The fuzzy sets were introduced by Zadeh [6] to deal the uncertainty. After that, several researchers studied the fuzzy sets, for instance, Dubois and Prade [7], Kaufmann and Gupta [8], Zimmermann [9], etc. In 1986, Atanassov [10] proposed the generalization of fuzzy sets to intuitionistic fuzzy sets on a universe. In this framework, not only the degree of membership of each element was considered, but also the degree of non-membership. In 1999, Smarandache [11], known as the founder of neutrosophic sets, first developed the idea of neutrosophic sets as generalizations of intuitionistic fuzzy sets. Neutrosophic sets consist of three independent components: the degree of membership, the degree of non-membership, and the degree of indeterminacy.

Falk and Liu [12] investigated the BLP using the stability of local optimizers. Gendreau et al. [13] argued that obtaining the BLP model is a consequence of a linear hierarchical decision model in which the constraint set at the follower level is independent of the decisions made at the leader level. They further implemented their solution methodology to study the NP-hard problem using the Tabu search method. Dempe and Schmidt [14] introduced an algorithm for solving BLP, where they studied a direction estimating model regularizing the follower constraints. On the other side, the objective function in leader problem was considered in the regularization to provide the unique solution to follower problem. Safaei and Saraj [15] studied the fully fuzzy bi-level linear programming (FFBLP) problem, where the problem involving fuzzy numbers in all the parameters as well as model variables. They partitioned the model to three deterministic linear programs considering the variables constraints as bounded.

Ren [16] introduced a solution methodology to find the fuzzy optimal solution for the FFLP model. Hossein and Edalatpanah [17] considered the Lexicography approach to study a FFLP model. Rajarajeswari and Sudha [17] presented a decision making methodology to compute the optimal solutions of the FFLP problems**.** Neutrosophic sets have been widely adopted and applied to challenging problems in different areas, including but not limited to multiple attribute decision-making. Very recently, various applications are presented using Kuhn-Tucker conditions, for instance, game theory models (Khalifa and Kumar, [18]), etc.

There are situations where the sum of degree of membership and degree of non-membership is greater than one, unlike the situations appear in intuitionistic fuzzy set. This drawback in intuitionistic fuzzy sets caused to a new concept, referred as Pythagorean fuzzy sets (Kumar et al. [19]). Pythagorean fuzzy sets are one of the latest technique to handle the uncertainty (Yager, [20]). Pythagorean fuzzy sets have attracted great attentions of several decision makes and planers, and subsequently, the idea has been adopted in modelling many real-life situations such as decision-making, and career placements.
After reviewing the above works in the literature, it is observed that the BLP problems in Pythagorean fuzzy environment are not discussed and solved using the Kuhn-Tucker's conditions. Therefore, there is a need to fill this research gap by considering the Pythagorean fuzzy bi-level linear programming (PFBLP) model. This motivates to incorporate the Pythagorean fuzzy sets into Kuhn-Tucker's conditions. In the current work, a PFBLP model is developed. The developed PFBLP model is transformed to a new BLP model in crisp environment using the score function. The main advantage of the suggested solution methodology is the occurrence of a single-objective non-linear programming problem, where the Kuhn-Tucker's conditions for optimality are implemented. Recently, many researchers studied bi- level programming in uncertain environment (for instance see [21-28])

The rest of the article is structured as below: Section 2 presents some basic terminology related to neutrosophic numbers and Pythagorean fuzzy number too. Section 3 formulates the PFBLP problem. Section 4 formulates the solution methodology for BLP model. Afterwards, Section 5 implements the solution methodology as discussed in Section 4, by conducting a number of numerical experimentations. In the end, some concluding remarks as well as further research directions are summarized in Section 6.

2. Basic Terminology

To identify our problem in a more effective manner, we recollect fundamental concepts and conclusions regarding intuitionistic trapezoidal fuzzy numbers (ITFN), trapezoidal fuzzy numbers (TFN), and the neutrosophic set (NS), and Pythagorean fuzzy number.

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2.1. Single valued trapezoidal fuzzy numbers

Definition 1. (TFNs, [30]). A fuzzy number $\tilde{F} = (r_1, r_2, r_3, r_4)$. Is a TrFN where $r_1, r_2, r_3, r_4 \in R$ and has membership function (MF) defined as:

$$\mu_{\tilde{F}}(\chi) = \begin{cases} \frac{\chi - r_1}{r_2 - r_1}, & r_1 \le \chi \le r_2, \\ 1, & r_2 \le \chi \le r_3, \\ \frac{r_4 - \chi}{r_4 - r_3}, & r_3 \le \chi \le r_4, \\ 0, & \text{otherwise}, \end{cases}$$

Definition 2. (Intuitionistic fuzzy set, [10]). A fuzzy set $\tilde{F}$ is an intuitionistic fuzzy set $\tilde{F}^{IN}$ of a non-empty set Z if $\tilde{F}^{IN} = \{\langle \chi, \mu_{\tilde{F}^{IN}}, \rho_{\tilde{F}^{IN}} \rangle : \chi \in Z\}$, where $\mu_{\tilde{F}^{IN}}$, and $\rho_{\tilde{F}^{IN}}$ are the MF and the non-MF functions where $\mu_{\tilde{F}^{IN}}, \rho_{\tilde{F}^{IN}} : Z \to [0, 1]$ and $0 \le \mu_{\tilde{F}^{IN}} + \rho_{\tilde{F}^{IN}} \le 1, \ \forall \chi \in Z$.

Definition 3. (Intuitionistic fuzzy number, [10]). An intuitionistic fuzzy set $\tilde{F}^{IN}$ on R is called an Intuitionistic fuzzy number if each of the following conditions valid:

1. $\exists u \in R: \mu_{\tilde{F}^{IN}}(u) = 1$, and $\rho_{\tilde{F}^{IN}}(u) = 0$,
2. $\mu_{\tilde{F}^{IN}}: R \to [0, 1]$ is continuous and $0 \le \mu_{\tilde{F}^{IN}} + \rho_{\tilde{F}^{IN}} \le 1, \forall \chi \in Z$,
3. $\mu_{\tilde{F}^{IN}}$, and $\rho_{\tilde{F}^{IN}}$ are

$$\mu_{\tilde{F}^{IN}}(\chi) = \begin{cases} 0, & \chi < r_1 \\ H(\chi), & r_1 \le \chi \le r_2 \\ 1, & \chi = r_2 \\ I(\chi), & r_2 \le \chi \le r_3 \\ 0, & r_4 \le \chi. \end{cases} \quad \text{and} \quad \rho_{\tilde{F}^{IN}}(\chi) = \begin{cases} 0, & \chi < r_1 \\ F(\chi), & r_1 \le \chi \le r_2 \\ 1, & \chi = r_2 \\ G(\chi), & r_2 \le \chi \le r_3 \\ 0, & r_4 \le \chi. \end{cases}$$

Where $H, I, F, G: R \to [0, 1]$, H and G are monotonic increasing functions, I and F are monotonic decreasing functions and satisfy $0 \le H(\chi) + F(\chi) \le 1$, and $0 \le I(\chi) + G(\chi) \le 1$.

Definition 4. (Trapezoidal intuitionistic fuzzy number, [30]).

A trapezoidal intuitionistic fuzzy number is denoted by $\tilde{F}^{IN} = (r_1, r_2, r_3, r_4), (r'_1, r_2, r_3, r'_4)$, where $r'_1 \le r_1 \le r_2 \le r_3 \le r_4 \le r'_4$ with defined as

$$\mu_{\tilde{F}^{INT}}(\chi) = \begin{cases} \frac{\chi - r_1}{r_2 - r_1}, & r_1 \le \chi \le r_2, \\ 1, & r_2 \le \chi \le r_3, \\ \frac{r_4 - \chi}{r_4 - r_3}, & r_3 \le \chi \le r_4, \\ 0, & \text{otherwise}, \end{cases} \quad \text{and} \quad \rho_{\tilde{F}^{INT}}(\chi) = \begin{cases} \frac{\chi - r'_1}{r_2 - r'_1}, & r'_1 \le \chi \le r_2, \\ 1, & r_2 \le \chi \le r_3, \\ \frac{r'_4 - \chi}{r'_4 - r_3}, & r_3 \le \chi \le r'_4, \\ 0, & \text{otherwise}, \end{cases}$$

Definition 5. (Neutrosophic set, [11]). A NS $\bar{F}^N$ of non-empty set Z is described as

$\bar{F}^N = \{\langle z, I_{\bar{F}^N}(z), J_{\bar{F}^N}(z), V_{\bar{F}^N}(z) \rangle : z \in Z, I_{\bar{F}^N}(z), J_{\bar{F}^N}(z), V_{\bar{F}^N}(z) \in (0_-, 1^+)\}$,where $I_{\bar{F}^N}(z), J_{\bar{F}^N}(z)$, and $V_{\bar{F}^N}(z)$ are the truth, the indeterminacy, and the falsity membership functions, $\nexists$ any restrictions on, $0^- \le I_{\bar{F}^N}(z) + J_{\bar{F}^N}(z) + V_{\bar{F}^N}(z) \le 3^+$, $(0_-, 1^+)$ is a non-standard unit interval.

Definition 6. (Single-valued neutrosophic set, [29]). A Single-valued neutrosophic set $\bar{F}^{SVN}$ of a set $Z, Z \ne \varphi$ is defined as

$\bar{F}^{SVN} = \{\langle z, I_{\bar{F}^N}(z), J_{\bar{F}^N}(z), V_{\bar{F}^N}(z) \rangle : z \in Z\}$, where $I_{\bar{F}^N}(z), J_{\bar{F}^N}(z)$, and $V_{\bar{F}^N}(z) \in [0, 1] \quad \forall z \in Z$ and $0 \le I_{\bar{F}^N}(z) + J_{\bar{F}^N}(z) + V_{\bar{F}^N}(z) \le 3$.

Definition 7. (Single-valued neutrosophic number, ([29]). Let $\tau_{\tilde{f}}, \varphi_{\tilde{f}}, \omega_{\tilde{f}} \in [0, 1]$ and $r_1, r_2, r_3, r_4 \in R$ and has (MF) defined as and. Then $\tilde{f}^N = \langle (r_1, r_2, r_3, r_4) : \tau_{\tilde{f}}, \varphi_{\tilde{f}}, \omega_{\tilde{f}} \rangle$ is a specific NS on R, whose truth, indeterminacy, and falsity MFs are

$$\tau_{\tilde{f}^N}(z) = \begin{cases} \tau_{\tilde{f}^N}\left(\frac{z - r_1}{r_2 - r_1}\right), & r_1 \le z < r_2 \\ \tau_{\tilde{f}^N}, & r_2 \le z \le r_3 \\ \tau_{\tilde{f}^N}\left(\frac{r_4 - z}{z - r_3}\right), & r_3 \le z \le r_4 \\ 0, & \text{otherwise}, \end{cases} ,$$

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$$\varphi_{\tilde{f}}{}^{N}(z) = \begin{cases} \frac{r_2 - z + \varphi_{\tilde{f}^N}(z - r_1)}{r_2 - r_1}, & r_1 \le z < r_2 \\ \varphi_{\tilde{f}^N}, & r_2 \le z \le r_3 \\ \frac{z - r_3 + \varphi_{\tilde{f}^N}(r_4 - z)}{r_4 - r_3}, & r_3 \le z \le r_4 \\ 1, & \text{otherwise,} \end{cases} \text{, and}$$

$$\omega_{\tilde{f}}{}^{N}(z) = \begin{cases} \frac{r_2 - z + \omega_{\tilde{f}}{}^{N}(z - r_1)}{r_2 - r_1}, & r_1 \le z < r_2 \\ \omega_{\tilde{f}}{}^{N}, & r_2 \le z \le r_3 \\ \frac{z - r_3 + \omega_{\tilde{f}}{}^{N}(r_4 - z)}{r_4 - r_3}, & r_3 \le z \le r_4 \\ 1, & \text{otherwise.} \end{cases}$$

A single- valued trapezoidal neutrosophic number (SVTNN) $\tilde{f}^N = \langle (r_1, r_2, r_3, r_4): \tau_{\tilde{f}^N}, \varphi_{\tilde{f}^N}, \omega_{\tilde{f}^N} \rangle$ may represented in ill-defined quantity about f, which is nearly equal to $[r_2, r_3]$.

Definition 8. [29]. Let $\tilde{f}^N = \langle (r_1, r_2, r_3, r_4): \tau_{\tilde{f}^N}, \varphi_{\tilde{f}^N}, \omega_{\tilde{f}^N} \rangle$, and $\tilde{g}^N = \langle (r'_1, r'_2, r'_3, r'_4): \tau_{\tilde{g}^N}, \varphi_{\tilde{g}^N}, \omega_{\tilde{g}^N} \rangle$ be two SVTNNs, the arithmetic operations on $\tilde{f}^N$, and $g^N$are:

1. $\tilde{f}^N \oplus \tilde{g}^N = \langle (r_1 + r'_1, r_2 + r'_2, r_3 + r'_3, r_4 + r'_4);\ \tau_{\tilde{f}^N} \wedge \tau_{\tilde{g}^N}, \varphi_{\tilde{f}^N} \vee \varphi_{\tilde{g}^N}, \omega_{\tilde{f}^N} \vee \omega_{\tilde{g}^N} \rangle$,
2. $\tilde{f}^N \ominus \tilde{g}^N = \langle (r_1 - r'_4, r_2 - r'_3, r_3 - r'_2, r_4 - r'_1);\ \tau_{\tilde{f}^N} \wedge \tau_{\tilde{g}^N}, \varphi_{\tilde{f}^N} \vee \varphi_{\tilde{g}^N}, \omega_{\tilde{f}^N} \vee \omega_{\tilde{g}^N} \rangle$,
3. $\tilde{f}^N \otimes \tilde{g}^N = \begin{cases} \langle (r_1 r'_1, r_2 r'_2, r_3 r'_3, r_4 r'_4);\ \tau_{\tilde{f}^N} \wedge \tau_{\tilde{g}^N}, \varphi_{\tilde{f}^N} \vee \varphi_{\tilde{g}^N}, \omega_{\tilde{f}^N} \vee \omega_{\tilde{g}^N} \rangle, r_4 > 0,\ r'_4 > 0 \\ \langle (r_1 r'_4, r_2 r'_3, r_3 r'_2, r_4 r'_1);\ \tau_{\tilde{f}^N} \wedge \tau_{\tilde{g}^N}, \varphi_{\tilde{f}^N} \vee \varphi_{\tilde{g}^N}, \omega_{\tilde{f}^N} \vee \omega_{\tilde{g}^N} \rangle, r_4 < 0,\ r'_4 > 0 \\ \langle (r_4 r'_4, r_3 r'_3, r_2 r'_2, r_1 r'_1{}');\ \tau_{\tilde{f}^N} \wedge \tau_{\tilde{g}^N}, \varphi_{\tilde{f}^N} \vee \varphi_{\tilde{g}^N}, \omega_{\tilde{f}^N} \vee \omega_{\tilde{g}^N} \rangle, r_4 < 0,\ r'_4 < 0, \end{cases}$
4. $\tilde{f}^N \oslash \tilde{g}^N = \begin{cases} \langle (r_1/r'_4, r_2/r'_3, r_3/r'_2, r_4/r'_1);\ \tau_{\tilde{f}^N} \wedge \tau_{\tilde{g}^N}, \varphi_{\tilde{f}^N} \vee \varphi_{\tilde{g}^N}, \omega_{\tilde{f}^N} \vee \omega_{\tilde{g}^N} \rangle, r_4 > 0,\ r'_4 > 0 \\ \langle (r_4/r'_4, r_3/r'_3, r_2/r'_2, r_1/r'_1);\ \tau_{\tilde{f}^N} \wedge \tau_{\tilde{g}^N}, \varphi_{\tilde{f}^N} \vee \varphi_{\tilde{g}^N}, \omega_{\tilde{f}^N} \vee \omega_{\tilde{g}^N} \rangle, r_4 < 0,\ r'_4 > 0 \\ \langle (r_4/r'_1, r_3/r'_2, r_2/r'_3, r_1/r'_4);\ \tau_{\tilde{f}^N} \wedge \tau_{\tilde{g}^N}, \varphi_{\tilde{f}^N} \vee \varphi_{\tilde{g}^N}, \omega_{\tilde{f}^N} \vee \omega_{\tilde{g}^N} \rangle, r_4 < 0,\ r'_4 < 0, \end{cases}$
5. $\alpha\tilde{f}^N = \begin{cases} \langle (\alpha r_1, \alpha r_2, \alpha r_3, \alpha r_4);\ \tau_{\tau_{\tilde{f}^N}},\ \varphi_{\tau_{\tilde{f}^N}}, \omega_{\tau_{\tilde{f}^N}} \rangle, \alpha > 0, \\ \langle (\alpha r_4, \alpha r_3, \alpha r_2, \alpha r_1);\ \tau_{\tau_{\tilde{f}^N}},\ \varphi_{\tau_{\tilde{f}^N}}, \omega_{\tau_{\tilde{f}^N}} \rangle, \alpha < 0, \end{cases}$
6. $\tilde{f}^{N^{-1}} = \langle (1/r_4, 1/r_3, 1/r_2, 1/r_1);\ \tau_{\tau_{\tilde{f}^N}},\ \varphi_{\tau_{\tilde{f}^N}}, \omega_{\tau_{\tilde{f}^N}} \rangle, \tilde{f}^N \neq 0.$

Definition 9. [29]. (Score and Accuracy functions of SVTNN). Any two SVTNNs $\tilde{f}$, and $\tilde{g}$ can be ordered according to their score and accuracy functions as:

1. Accuracy function $AC(\tilde{f}^N) = \left(\frac{1}{16}\right)[r_1 + r_2 + r_3 + r_4] * \left[\tau_{\tau_{\tilde{f}^N}} + \left(1 - \varphi_{\tau_{\tilde{f}^N}}(z)\right) + \left(1 + \omega_{\tau_{\tilde{f}^N}}(z)\right)\right]$,
2. Score function $SC(\tilde{f}^N) = \left(\frac{1}{16}\right)[r_1 + r_2 + r_3 + r_4] * \left[\tau_{\tau_{\tilde{f}^N}} + \left(1 - \varphi_{\tau_{\tilde{f}^N}}(z)\right) + \left(1 - \omega_{\tau_{\tilde{f}^N}}(z)\right)\right]$.

Definition 10. Based on the accuracy and the score functions the order relations between $\tilde{f}^N$ and $\tilde{g}^N$ are:

1. If $SC(\tilde{f}^N) < SC(\tilde{g}^N), \Rightarrow \tilde{f}^N < \tilde{g}^N$
2. If $SC(\tilde{f}^N) = SC(\tilde{g}^N), \Rightarrow \tilde{f}^N = \tilde{g}^N$,
3. If $AC(\tilde{f}^N) < AC(\tilde{g}^N), \Rightarrow \tilde{f}^N < \tilde{g}^N$,
4. If $AC(\tilde{f}^N) > AC(\tilde{g}^N), \Rightarrow \tilde{f}^N < \tilde{g}^N$,
5. If $AC(\tilde{f}^N) = AC(\tilde{g}^N),\ \Rightarrow\ \tilde{f}^N = \tilde{g}^N$.

To illustrate the basic properties, let $\tilde{f}^N = \langle (4, 8, 10, 16):\ .5,\ .3\ , .6\ \rangle$ and
$\tilde{g}^N = \langle (3, 7, 11, 14):\ .4,\ .5\ , .6\ \rangle$ be two single valued trapezoidal neutrosophic numbers, then

1. $\tilde{f}^N \oplus \tilde{g}^N = \langle (7, 15, 21, 30):\ .4,\ .5\ , .6\ \rangle$,
2. $\tilde{f}^N \ominus \tilde{g}^N = \langle (-10, -3, 3, 13):\ .4,\ .5\ , .6\ \rangle$,
3. $\tilde{f}^N \otimes \tilde{g}^N = \langle (12, 56, 110, 224):\ .4,\ .5\ , .6\ \rangle$,
4. $\tilde{f}^N \oslash \tilde{g}^N = \langle \left(\frac{4}{14}, \frac{8}{11}, \frac{10}{7}, \frac{16}{3}\right):\ .4,\ .5\ , .6\ \rangle$,
5. $4\tilde{f}^N = \langle (16, 32, 40, 64):\ .4,\ .5\ , .6\ \rangle$,

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6. $\tilde{f}^{N^{-1}} = \langle\left(\frac{1}{16}, \frac{1}{10}, \frac{1}{8}, \frac{1}{4}\right):\ .4,\ .5\ ,.6\ \rangle,$
7. $SC(\tilde{f}^N) = \left(\frac{1}{16}\right)(4+8+10+16) \times (.5 + (1-.3) + (1-.6)) = 3.8,$
8. $AC(\tilde{f}^N) = \left(\frac{1}{16}\right)(4+8+10+16) \times \left(.5 + (1-.3) + (1+.6)\right) = 6.65.$

2.2. Pythagorean fuzzy numbers

Definition 11. (Pathade and Ghadle, [30]). Let X be a fixed set, a Pythagorean fuzzy set is as defined as

$P = \{\langle x, (\alpha_P(x), \beta_P(x))\rangle : x \in X\}$. Where, $_P(x): X \to [0,1]$, and $\beta_P(x): X \to [0,1]$ are the degree of membership and non- membership functions, respectively. Also, it holds that:
$\left(_P(x)\right)^2 + \left(\beta_P(x)\right)^2 \leq 1.$

Definition 12. **(**Pathade and Ghadle, [30] Let $\tilde{a}^P = \left(\alpha_i^P,\ \beta_k^P\right)$ and $\tilde{b}^P = \left(\alpha_l^P,\ \beta_s^P\right)$ be two Pythagorean fuzzy numbers (PFN). Then, the arithmetic's operations are as follows:

(i) $\tilde{a}^P(+)\tilde{b}^P = \left(\sqrt{(\alpha_i^P)^2 + (\alpha_l^P)^2 - (\alpha_i^P)^2.(\alpha_l^P)^2}, \beta_k^P.\beta_s^P\right),$

(ii) $\tilde{a}^P(\times)\tilde{b}^P = \left(\ \beta_i^P.\beta_k^P,\ \sqrt{(\alpha_k^P)^2 + (\alpha_s^P)^2 - (\alpha_k^P)^2.(\alpha_s^P)^2}\right),$

(iii) $k.\tilde{a}^P = \left(\sqrt{1-(1-\alpha_i^P)^k}, \left(\beta_k^P\right)^k\right), k \succ 0.$

Definition 13. **(**Pathade and Ghadle, [30] Let $\tilde{a}^P = \left(\alpha_i^P,\ \beta_k^P\right)$ and $\tilde{b}^P = \left(\alpha_l^P,\ \beta_s^P\right)$ be two PFNs. Then

(i) Score function: $S(\tilde{a}^P) = \frac{1}{2}\left(1 - \left(\alpha_i^P\right)^2 - \left(\beta_k^P\right)^2\right).$

(ii) Accuracy function: $A(\tilde{a}^P) = \left(\alpha_i^P\right)^2 + \left(\beta_k^P\right)^2.$

Definition 5. Let $\tilde{a}^P$, and $\tilde{b}^P$ be any two PFN, then

(i) $\tilde{a}^P \succ \tilde{b}^P$ if and only if $S(\tilde{a}^P) \succ S(\tilde{b}^P)$,
(ii) $\tilde{a}^P \prec \tilde{b}^P$ if and only if $S(\tilde{a}^P) \prec S(\tilde{b}^P)$,
(iii) $S(\tilde{a}^P) = S(\tilde{b}^P)$, and $A(\tilde{a}^P) \prec A(\tilde{b}^P)$ then $\tilde{a}^P \prec \tilde{b}^P$,
(iv) $S(\tilde{a}^P) = S(\tilde{b}^P)$, and $A(\tilde{a}^P) \succ A(\tilde{b}^P)$ then $\tilde{a}^P \succ \tilde{b}^P$,
(v) $S(\tilde{a}^P) = S(\tilde{b}^P)$, and $A(\tilde{a}^P) = A(\tilde{b}^P)$ then $\tilde{a}^P = \tilde{b}^P$.

3. PROBLEM FORMULATION AND SOLUTION CONCEPTS

A neutrosophic Pythagorean fuzzy number two level programming problem for the leader and follower is formulated as follows

$$\max_{x_1} \tilde{Z}_1^N(x) = \tilde{c}_{11}^N x_1 + \tilde{c}_{12}^N x_2\ , \tag{1}$$

Where x_2 solves

$$\max_{x_2} \tilde{Z}_2^N(x) = \tilde{c}_{21}^N x_1 + \tilde{c}_{22}^N x_2\ , \tag{2}$$

Subject to

$$\tilde{A}_{i1}^P x_1 + \tilde{A}_{i2}^P x_2 \leq \tilde{b}_i^P,\ \text{for all } i = 1, 2, \dots, m;\ \text{and } x_1, x_2 \geq 0\ , \tag{3}$$

where $\tilde{Z}^N = \left(\tilde{Z}_1^N, \tilde{Z}_2^N\right)$, $\tilde{C}^N$ is the neutrosophic cost coefficient matrix, x is the decision vector, $\tilde{A}^P$ is the Pythagorean fuzzy coefficient matrix, and $\tilde{b}^P$ is the Pythagorean fuzzy right side vector.

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Denote the total number of decision variables by n, and the total number of constraints by m. Then, we designate the following variables:

$x_1 = \{x_1^1, x_1^2, \dots, x_1^{n_1}\}$: Decision variables controlled by the center,

$x_2 = \{x_2^1, x_2^2, \dots, x_2^{n_2}\}$: Decision variables controlled by the division.

Let the vector $(x_1, x_2) = x$, and $n_1 + n_2 = n$. Then, using Definitions 9 and 13 to the problem (1)−(3), we obtain

$$\max_{x_1} Z_1(x) = c_{11}x_1 + c_{12}x_2 \tag{4}$$

Where x_2 solves

$$\max_{x_2} Z_2(x) = c_{21}x_1 + c_{22}x_2 , \tag{5}$$

Subject to

$$A_{i1}x_1 + A_{i2}x_2 \le b_i, \text{ for all } i = 1, 2, \dots, m; \text{and } x_1, x_2 \ge 0 \tag{6}$$

Now that we have formulated the problem, the next section presents the solution procedure.

4. Solution Procedure

The Lagrange multipliers permit the transformation of constrained optimization problems (4)−(6) to an un-constrained optimization problem. This is performed by formulating the Lagrange function $L(x, \delta, S)$ for the fellower-level objective function as presented in problem (7) below:

$$L(x, \delta, S) = c_{21}x_1 + c_{22}x_2 - \sum_{i=1}^{m} \delta_i(A_{i1}x_1 + A_{i2}x_2 + S_i^2 - b_i) \tag{7}$$

Let us develop the Kuhn-Tucker's necessary conditions. Let $\delta = (\delta_1, \delta_2, \dots, \delta_m)$, where $\delta_1, \delta_2, \dots, \delta_m$ are the Lagrange multipliers for the constraints in problem (6). In addition, $S = (S_1^2, S_2^2, \dots, S_m^2)$, where S_i^2 is the slack variable for i^{st}constraint in (6). The necessary conditions for follower-level maximization problem (7) are stated below:

$$\begin{aligned} &\frac{\delta L}{\delta x_2} = 0; \\ &\frac{\delta L}{\delta \delta_i} = 0, \text{ for all } i = 1, 2, \dots, m; \\ &\frac{\delta L}{\delta S_i} = 0, \text{ for all } i = 1, 2, \dots, m. \end{aligned} \tag{8}$$

Equations (8) can be further simplified as follows

$$\begin{aligned} &\sum_{i=1}^{m} \delta_i A_{i2} - c_{22} = 0; \\ &A_{i1}x_1 + A_{i2}x_2 + S_i^2 - b_i = 0, \quad \text{for all } i = 1, 2, \dots, m; \\ &\delta_i(A_{i1}x_1 + A_{i2}x_2 - b_i) = 0, \quad \text{for all } i = 1, 2, \dots, m; \\ &x_2, \delta_1, \delta_2, \dots, \delta_m \ge 0. \end{aligned} \tag{9}$$

Afterwards, the center's problem is stated below

$$\max_{x_1} Z_1(x) = c_{11}x_1 + c_{12}x_2$$

Subject to (10)

$$\begin{aligned} &\sum_{i=1}^{m} \delta_i A_{i2} - c_{22} = 0; \\ &A_{i1}x_1 + A_{i2}x_2 + S_i^2 - b_i = 0, \quad \text{for all } i = 1, 2, \dots, m; \\ &\delta_i(A_{i1}x_1 + A_{i2}x_2 - b_i) = 0, \text{ for all } i = 1, 2, \dots, m; \\ &x_2, \delta_1, \delta_2, \dots, \delta_m \ge 0. \end{aligned}$$

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Problem (10) can be treated as non-linear programming and is solved using the Kuhn-Tucker's optimality conditions (Mokhtar et al. [31]). Now that we have presented the solution procedure, the next section illustrates a number of numerical examples to demonstrate the solution procedure.

5. Numerical Examples

Let us illustrate the following two numerical examples to describe the suggested methodology.

Example 1: Consider the PFBLP problem:

$$\max_{x_1} \tilde{Z}_1^N(x) = \langle(3,5,6,8); 0.6, 0.5, 0.4\rangle x_1 + \langle(0,1,3,6); 0.7, 0.5, 0.3\rangle x_2 ,$$

Where x_2 solves

$$\max_{x_2} \tilde{Z}_2^N(x) = \langle(1,3,4,6); 0.6, 0.3, 0.5\rangle x_1 + \langle(0,1,3,6); 0.7, 0.5, 0.3\rangle x_2$$

Subject to (11)

$$\langle 0.5, 0.4\rangle x_1 - \langle 0.7, 0.3\rangle x_2 \le \langle 0.2, 0.3\rangle;$$
$$\langle 0.5, 0.4\rangle x_1 - \langle 0.4, 0.8\rangle x_2 \le \langle 0.1, 0.3\rangle;$$
$$\langle 0.5, 0.4\rangle x_1 + \langle 0.4, 0.8\rangle x_2 \le \langle 0.2, 0.2\rangle;$$
$$\langle 0.5, 0.4\rangle x_1 + \langle 0.4, 0.2\rangle x_2 \le \langle 0.1, 0.1\rangle;$$
$$\langle 0.4, 0.8\rangle x_1 + \langle 0.5, 0.4\rangle x_2 \le \langle 0.1, 0.2\rangle;$$
$$x_1, x_2 \ge 0.$$

Using score function, we transform the problem (11) to problem (12) as below:

$$\max_{x_1} Z_1(x) = 2x_1 - x_2$$

Where x_2 solves

$$\max_{x_2} Z_2(x) = x_1 + 2x_2$$

Subject to (12)

$$0.545x_1 - 0.7x_2 \le 0.87;$$
$$0.545x_1 - 0.26x_2 \le 0.90;$$
$$0.545x_1 + 0.26x_2 \le 0.92;$$
$$0.545x_1 + 0.56x_2 \le 0.98;$$
$$0.26\, x_1 + 0.545x_2 \le 0.95;$$
$$x_1, x_2 \ge 0.$$

Using the equations (7)−(10, the center's problem can be stated as follows:

$$\max_{x_1, x_2, \delta} Z = 2x_1 - x_2$$

Subject to (13)

$$0.7\delta_1 + 0.26\delta_2 - 0.26\delta_3 - 0.56\delta_4 - 0.545\delta_5 = -2;$$

$$\delta_1(0.545x_1 - 0.7x_2 - 0.87) = 0;$$

$$\delta_2(0.545x_1 - 0.26x_2 - 0.9) = 0;$$

$$\delta_3(0.545x_1 + 0.26x_2 - 0.92) = 0;$$

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$\delta_4(0.545x_1 + 0.56x_2 - 0.98) = 0;$

$\delta_5(0.26\, x_1 + 0.545x_2 \leq 0.95) = 0;$

$0.545x_1 - 0.7x_2 \leq 0.87;$

$0.545x_1 - 0.26x_2 \leq 0.9;$

$0.545x_1 + 0.26x_2 \leq 0.92;$

$0.545x_1 + 0.56x_2 \leq 0.98;$

$0.26\, x_1 + 0.545x_2 \leq 0.95;$

$x_1, x_2, \delta_1, \delta_2, \delta_3, \delta_4, \delta_5 \geq 0.$

The solution of the above problem is
$Z_1 = 0.5436,$ $x_1 = 1.6632,$ $x_2 = 0.0521,$ $\delta_1 = \delta_2 = \delta_4 = \delta_5 = 0,$
$\delta_3 = 1.2885$ and $Z_2 = 0.4499$. Then, $\tilde{Z}_1^N(x) = \langle(1.8636,\ 8.1598,\ 9.7399, 13.3056);\ 0.6, 0.5, 0.4\rangle$,and $\tilde{Z}_2^N(x) = \langle(1.6632, 5.5106,\ 6.809, 13.1052);\ 0.6, 0.5, 0.5\rangle$

Example 2: Consider the below BLP problem:

$$\underset{x_1, x_2}{\text{Max}}\, \tilde{Z}_1^P(x) = \langle(4, 8, 11, 15); 0.6, 0.3, 0.2\rangle x_1 + \langle(15, 17, 19, 22); 0.4, 0.8, 0.4\rangle x_2 + \langle(9, 11, 14, 16); 0.5, 0.4, 0.7\rangle x_3$$
$$+\langle(3, 5, 6, 8); 0.6, 0.5, 0.4\rangle x_4$$

Where x_3, x_4 solve

$$\underset{x_3, x_4}{\max}\, \tilde{Z}_2^P(x) = \langle(12, 14, 16, 22; 0.8, 0.3, 0.5)\rangle x_1 + \langle(13, 15, 20, 24); 0.7, 0.6, 0.1\rangle x_2 + \langle(1, 3, 4, 6); 0.6, 0.3, 0.5\rangle x_3 +$$
$$\langle(9, 11, 14, 16); 0.5, 0.4, 0.7\rangle x_4$$

Subject to (14)

$$\langle 0.4, 0.5\rangle x_1 + \langle 0.5, 0.5\rangle x_2 + \langle 0.5, 0.7\rangle x_3 + \langle 0.4, 0.5\rangle x_4 \leq 0.98,$$
$$\langle 0.4, 0.5\rangle x_1 + \langle 0.3, 0.4\rangle x_2 + \langle 0.5, 0.7\rangle x_3 + \langle 0.5, 0.5\rangle x_4 \leq 0.95,$$
$$\langle 0.5, 0.7\rangle x_1 + \langle 0.5, 0.5\rangle x_2 + \langle 0.5, 0.7\rangle x_3 + \langle 0.5, 0.5\rangle x_4 \leq 0.92,$$
$$x_1, x_2, x_3, x_4 \geq 0.$$

Problem (14) is transformed to a new crisp BLP problem, represented by problem (15), using the score functions defined in Definitions 9 and 13, as follows:

$$\underset{x_1, x_2}{\max}\, Z_1(x) = 5x_1 + 6x_2 + 4x_3 + 2x_4\ ,$$

where x_3, x_4 solve

$$\underset{x_3, x_4}{\max}\, Z_2(x) = 8x_1 + 9x_2 + 2x_3 + 4x_4$$

Subject to (15)

$$0.295x_1 + 0.25x_2 + 0.1x_3 + 0.295x_4 \leq 0.98,$$
$$025x_1 + 0.375x_2 + 0.1x_3 + 0.25x_4 \leq 0.95,$$
$$0.1x_1 + 0.25x_2 + 0.1x_3 + 0.25x_4 \leq 0.92,$$
$$x_1, x_2, x_3, x_4 \geq 0.$$

The center's problem using the equations (7)−(10), can be stated as follows:

$$\underset{x_1, x_2, x_3, x_4}{\max}\quad Z_1(x) = 5x_1 + 6x_2 + 4x_3 + 2x_4$$

Subject to (16)

$$\delta_1 + \delta_2 + \delta_3 = 2,$$

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$$0.295\delta_1 + 0.25\delta_2 + 0.25\delta_3 = 4,$$

$$\delta_1(0.295x_1 + 0.25x_2 + 0.1x_3 + 0.295x_4 - 0.98) = 0,$$

$$\delta_2(025x_1 + 0.375x_2 + 0.1x_3 + 0.25x_4 - 0.95) = 0,$$

$$\delta_3(0.1x_1 + 0.25x_2 + 0.1x_3 + 0.25x_4 - 0.92) = 0,$$

$$0.295x_1 + 0.25x_2 + 0.1x_3 + 0.295x_4 \leq 0.98,$$

$$025x_1 + 0.375x_2 + 0.1x_3 + 0.25x_4 \leq 0.95,$$

$$0.1x_1 + 0.25x_2 + 0.1x_3 + 0.25x_4 \leq 0.92,$$

$$x_1, x_2, x_3, x_4, \delta_1, \delta_2, \delta_3 \ \geq 0.$$

The solution of problem (16) is given by
$Z_1 = 41.5$, $x_1 = 0.1538$, $x_2 = \ 0.0554$, $x_3 = 8.9077$, $x_4 = 0$, $\delta_1 = \delta_3 = 0, \delta_2 = 0.7$ and $Z_2 = 12.0359$. Then, $\tilde{Z}_1^N(x) = \langle(82.2392,108.6331, 136.9256, 146.049);\ 0.4, 0.8,0.2\rangle$,and $\tilde{Z}_2^N(x) = \langle(11.4735\ ,29.7073,49.1716, 70.1258);\ 0.5, 06,0.7\rangle$.

6. Conclusions and Future Works

In the present work, interval valued neutrosophic numbers bi-level linear programming (PFBLP) model under Pythagorean fuzzy environment has presented. With the help of score functions of neutrosophic numbers and Pythagorean fuzzy numbers, the suggested model was converted into the corresponding crisp BLP model. The BLP is further transformed to a single objective nonlinear programming problem, where the Kuhn-Tucker's optimality conditions for optimality have incorporated for obtaining the neutrosophic solution. The suggested solution methodology was demonstrated by solving two numerical examples. In the Future work might contain the additional extension of this study to other fuzzy-like structure (i. e., Neutrosophic set, interval-valued fuzzy set, Spherical fuzzy set, Pythagorean fuzzy set etc. In addition, one can consider new fuzzy systems such as interval type-2, interval type-3, Possibility Interval-valued Intuitionistic fuzzy set, Possibility Neutrosophic set, Possibility Interval-valued Neutrosophic set, Possibility Interval-valued fuzzy set, Possibility fuzzy expert set etc., with applications in decision-making.

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