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Abstract

Indeterminacy is common in practical decision-making circumstances. So the mathematical models of decision making represent the situations in a better way if the parameters are considered as neutrosophic. This article studies a general framework of multi-objective neutrosophic linear fractional programming problem (MONLFPP) and proposes unique approach. The issue's parameters are thought of as triangular neutrosophic numbers. The problem is converted into an equal crisp multi-objective linear programming problem (MOLPP) with the help of variable transformation technique and a ranking function. Fuzzy goal programming is used to solve the MOLPP that has been obtained. Finally, the usefulness of the proposed technique is established using two mathematical models.

Keywords: Optimization; Neutrosophic set; Triangular neutrosophic number; Score function; multi-objective linear fractional programming problem; fuzzy goal programming; Decision making

1 Introduction

A multi-objective programming problem (MOPP) is a mathematical programming problem that has numerous conflicting objectives and is constrained by a set of rules. Assignment difficulties, inventory problems, supply chain management, transportation challenges, problems occurring in manufacturing units, portfolio optimization, and other domains of optimization commonly comprise numerous objective functions that are generally contradictory in nature. The main difficulty in solving an MOPP lies in the fact that it is not always possible to find a single solution which can simultaneously optimize each objective function. So here arises the concept of a compromise optimal solution. Over the past years, researchers have worked a lot over different multi-objective optimization techniques. Linear fractional programming problem (LFPP) is a special type of mathematical optimization where the objective functions are considered as a ratio of two linear functions and the constraints are considered as linear functions. Much of the time at least one proportion of capacities like obligation/value proportion in corporate preparation, stock/deals, creation amount/representative underway preparation, and cost brought about/patient, nurture required/patient in clinic arranging, understudy/cost in college arranging and understudy affirmations and so forth dependent upon certain imperatives must be enhanced.
Genuine numerical improvement issues are especially difficult as the accessible data isn't right and 100% of the time. Vulnerabilities exist in different structures in financial, modern and social frameworks like inadequate and equivocal framework information, vulnerabilities in event of occasions and semantic ambiguity which emerge

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from numerous ways, including mistakes of estimation, deficient information articulation, furthermore the subjectivity and inclination of human judgment, and so on In such manner, the hypothesis of fuzzy set was presented by Zadeh [1], which assisted with displaying the vulnerability somewhat, through the level of belongingness (enrollment capacities) for the component into the attainable arrangement set. Fuzzy programming approach for MOPP was introduced by Zimmermann [2] in which participation capacities address the negligible assessment of every evenhanded. In fuzzy programming the point of the chiefs is to accomplish the most extreme worth of the participation capacity of every levelheaded. With time, different augmentations of fluffy programming have been grown, for example, fuzzy stretch programming, fuzzy stochastic programming, fuzzy objective programming, and so forth and have been applied effectively to get the arrangement of MOPP as indicated by the idea of the issue.

Several scientists have developed several ways to settle LFPP in both classical and fuzzy logic over time [3-9]. It's called fuzzy LFPP when the LFPP's parameters are regarded fuzzy numbers (FLFPP). For solving FLFPP with triangular fuzzy integers, Li and Chen [10] devised a fuzzy programming technique. Luhandjula [11] proposed a language strategy for solving multi-objective linear fractional programming problems with many objectives (MOLFPP). Pop and Stancu Minasian [12] devised a method for evaluating the fully fuzzy LFPP that uses triangular fuzzy integers to represent both the parameters and variables. Osmam et al. [13] used intuitive methods to solve a staggered multi-objective partial programming problem with fuzzy boundaries. Das et al. [14] used a positioning mechanism between two three-sided fuzzy numbers to handle the FLFPP.

Goal programming (GP) is a technique commonly used to tackle multi-objective optimization problems. GP model is based on the concept of distance where the decision maker (DM) is keen on an answer that limits the outright deviation between the accomplishment level of the goal and its objective worth. Fuzzy goal programming (FGP) is an extension of GP where the fuzziness in the goals are treated using membership functions. A fuzzy goal programming (FGP) technique to tackle MOLFPP was developed by [15]. FGP method dependent on Taylor series was utilized by [16] to assess MOLFPP. Dey et al. [17, 18] utilized FGP calculations to tackle bi-level MOLFPP and staggered partial programming issues individually. MOLPP using Charnes and Cooper's [19] variable translation technique. By reducing the negative deviational variables, FGP is used to solve the MOLPP. For MOLPP, three FGP models have been constructed, and the Euclidean distance function may be used to identify which model offers the best outcome for a given situation. Two instances were used to show the suggested method, and the findings were compared to those of alternative methods.

The flaw with fuzzy sets is that they only consider the truth membership function of each element. To solve this issue, Atanassov [20] presented intuitionistic fuzzy sets, which considered both the falsity and truth membership functions. In real-life decision-making problems, both fuzzy sets and intuitionistic fuzzy sets are unable to deal with indeterminacy. In a decision-making setting, indeterminacy is quite significant. Smarandache [21] developed neutrosophic sets for dealing with uncertainty in terms of independent truth, falsity, and indeterminacy membership functions. Wang et al. [22] proposed the concept of a single valued neutrosophic set (SVNS) to solve practical difficulties. SVNS is used to solve a wide range of decision-making problems, including multi-criteria decision-making difficulties [23, 24], educational challenges [25], social problems, and so on. In a multi-attribute group decision-making technique, Ye [26] studied ranking of neutrosophic numbers (NNs) based on possibility degree. [27, 28] examine the linear programming issue with a single target and many objectives in a neutrosophic setting. Wang et al. [29] developed a method for solving MOLPP with triangular neutrosophic numbers. Das and Edalatpanah [30] devised a triangular neutrosophic number integer programming problem. Das et al. [31] suggested a dual simplex approach and ranking function to solve NLFP with triangular neutrosophic numbers. Abdel Basset et al. [32-33] also suggested a method for solving LFPP using triangular neutrosophic numbers as parameters. Many authors have developed neuromas approaches for solving MOLFP problems (for instance, Borza and Rambely, [34]; Valipour and Yaghoobi [35]; and Pirouz and Gaudioso [36]). Many authors investigated various optimization techniques for goal programming problems [37-47]

The cost of the objective functions, technological coefficients, and resources are all treated as triangular neutrosophic numbers in this study, which leads to a new method for solving MONLFPP. With the help of the ranking function, the MONLFPP is turned into an equivalent crisp MOLFPP, which is then translated into a *Novelties of the proposed method.*

The linear membership function is used in this study, and it is simple to utilise in actual issues because it is defined merely by the upper and lower tolerance limits. The ranking function is used to transform neutrosophic numbers into equivalent crisp numbers, which is an essential topic because it is directly related to the separation measure between two neutrosophic numbers. There is currently no approach for solving multi-objective neutrosophic linear

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fractional programming problems that we are aware of (MONLFPP). As a result, we have attempted to develop a method to overcome this problem in this post.

The following is how the paper is organised: In the second section, a preliminary discussion is presented; in the third section, the LFPP with Charnes and Cooper's variable transformation method is described; in the fourth section, multi-objective LFPP is presented; in the fifth section, the FGP method for solving multi-objective programming problems is presented; and in the sixth section, MONLFPP is presented. In the seventh section, we show our proposed strategy for solving MONLFPP. To demonstrate the applicability of the suggested strategy, the eighth section includes a numerical example and an example from a real-world scenario. The ninth portion of the study concludes with concluding remarks.

2 Preliminaries

The terms neutrosophic set, single valued neutrosophic set (SVNS), neutrosophic numbers, triangular neutrosophic numbers, and operations on triangular neutrosophic numbers are all defined in this section.

Definition 1: [37] The universal set is denoted by X and $x \in X$. A neutrosophic set N_1 in X is described by three membership functions, namely, a truth membership function, an indeterminacy membership function and a falsity membership function denoted by $T^1{}_{N_1}(x), I^1{}_{N_1}(x), F^1{}_{N_1}(x)$ respectively where $T^1{}_{N_1}(x), I^1{}_{N_1}(x)$ and $F^1{}_{N_1}(x)$ are real standard or real non-standard subsets of $]^-0,1^+[$. We can write as follows:

$T^1{}_{N_1}(x): X \to]^-0,1^+[$, $I^1{}_{N_1}(x): X \to]^-0,1^+[$, $F^1{}_{N_1}(x): X \to]^-0,1^+[$ where $^-0 \le \sup T^1{}_{N_1}(x) + \sup I^1{}_{N_1}(x) + \sup F^1{}_{N_1}(x) \le 3^+$.

Definition 2: [48] A single valued neutrosophic set (SVNS) N_1 is defined over the universal set X in the form $N_1 = \left\{ \left\langle x, T^1{}_{N_1}(x), I^1{}_{N_1}(x), F^1{}_{N_1}(x) \right\rangle : x \in X \right\}$, where $T^1{}_{N1}(x): X \to [0,1]$, $I^1{}_{N_1}(x): X \to [0,1]$, $F^1{}_{N_1}(x): X \to [0,1]$ and $0 \le T^1{}_{N_1}(x) + I^1{}_{N_1}(x) + F^1{}_{N_1}(x) \le 3 \ \forall x \in X$. In a general way, an SVNS can be written as $N_1 = (n_{11}, n_{22}, n_{33})$, where $n_{11}, n_{22}, n_{33} \in [0,1]$ and $n_{11} + n_{22} + n_{33} \le 3$.

Definition 3: [43] A triangular neutrosophic number N_1 is a special kind of neutrosophic set on the set of real numbers R and is defined as $N_1 = \left\langle (a_1, a_2, a_3); \alpha_{N_1}, \beta_{N_1}, \eta_{N_1} \right\rangle$ where $\alpha_{N_1}, \beta_{N_1}, \eta_{N_1} \in [0,1]$. The truth, indeterminacy and falsity membership functions of N_1 are defined in the following way:

$$T_{N_1}(x) = \begin{cases} \dfrac{(x-a_1)\alpha_{N_1}}{(a_2-a_1)} & if \ a_1 \le x \le a_2, \\ \alpha_N & if \ x = a_2, \\ \dfrac{(a_3-x)\alpha_{N_1}}{(a_3-a_2)} & if \ a_2 \le x \le a_3, \\ 0 & otherwise. \end{cases}$$

$$I_{N_1}(x) = \begin{cases} \dfrac{(a_2 - x + \beta_{N_1}(x-a_1))}{(a_2-a_1)} & if \ a_1 \le x \le a_2, \\ \beta_{N_1} & if \ x = a_2, \\ \dfrac{(x - a_2 + \beta_{N_1}(a_3-x))}{(a_3-a_2)} & if \ a_2 \le x \le a_3, \\ 1 & otherwise. \end{cases}$$

$$F_{N_1}(x) = \begin{cases} \dfrac{(a_2 - x + \eta_{N_1}(x-a_1))}{(a_2-a_1)} & if \ a_1 \le x \le a_2, \\ \eta_{N_1} & if \ x = a_2, \\ \dfrac{(x - a_2 + \eta_{N_1}(a_3-x))}{(a_3-a_2)} & if \ a_2 \le x \le a_3, \\ 1 & otherwise. \end{cases}$$

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where $0 \le T_{N_1}(x) + I_{N_1}(x) + F_{N_1}(x) \le 3 \;\; \forall x \in N_1$. Again, if $a_1 \ge 0$ and at least $a_3 > 0$ then N_1 is called a positive triangular neutrosophic number and can be denoted as $N_1 > 0$. Again if $a_3 \le 0$ and at least $a_1 < 0$ then N_1 is called a negative triangular neutrosophic number and can be denoted as $N_1 < 0$.

Definition 4: [48] Let $N_{11} = \left\langle (a_1, a_2, a_3); \alpha^1_{N_1}, \beta^1_{N_1}, \eta^1_{N_1} \right\rangle$ and $N_{22} = \left\langle (b_1, b_2, b_3); \alpha^2_{N_1}, \beta^2_{N_1}, \eta^2_{N_1} \right\rangle$ be two triangular neutrosophic numbers. Then the arithmetic operations can be defined on them as follows:

$$N_{11} + N_{21} = \left\langle (a_1 + b_1, a_2 + b_2, a_3 + b_3); \alpha^1_{N_1} \wedge \alpha^2_{N_1}, \beta^1_{N_1} \vee \beta^2_{N_1}, \eta^1_{N_1} \vee \eta^2_{N_1} \right\rangle$$

$$N_{11} - N_{22} = \left\langle (a_1 - b_1, a_2 - b_2, a_3 - b_3); \alpha^1_{N_1} \wedge \alpha^2_{N_1}, \beta^1_{N_1} \vee \beta^2_{N_1}, \eta^1_{N_1} \vee \eta^2_{N_1} \right\rangle$$

$$N_{11} N_{22} = \begin{cases} \left\langle (a_1 b_1, a_2 b_2, a_3 b_3); \alpha^1_{N_1} \wedge \alpha^2_{N_1}, \beta^1_{N_1} \vee \beta^2_{N_1}, \eta^1_{N_1} \vee \eta^2_{N_1} \right\rangle \; (a_3 > 0, b_3 > 0) \\ \left\langle (a_1 b_3, a_2 b_2, a_3 b_1); \alpha^1_{N_1} \wedge \alpha^2_{N_1}, \beta^1_{N_1} \vee \beta^2_{N_1}, \eta^1_{N_1} \vee \eta^2_{N_1} \right\rangle \; (a_3 < 0, b_3 > 0) \\ \left\langle (a_3 b_3, a_2 b_2, a_1 b_1); \alpha^1_{N_1} \wedge \alpha^2_{N_1}, \beta^1_{N_1} \vee \beta^2_{N_1}, \eta^1_{N_1} \vee \eta^2_{N_1} \right\rangle \; (a_3 < 0, b_3 < 0) \end{cases}$$

$$\kappa N_{11} = \begin{cases} \left\langle (\kappa a_1, \kappa a_2, \kappa a_3); \alpha^1_{N_1}, \beta^1_{N_1}, \eta^1_{N_1} \right\rangle \; (\kappa > 0) \\ \left\langle (\kappa a_3, \kappa a_2, \kappa a_1); \alpha^1_{N_1}, \beta^1_{N_1}, \eta^1_{N_1} \right\rangle \; (\kappa < 0) \end{cases}$$

$$N_{11}^{-1} = \left\langle \left(\frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1} \right); \alpha^1_{N_1}, \beta^1_{N_1}, \eta^1_{N_1} \right\rangle \; (N_{11} \ne 0)$$

$$\frac{N_{11}}{N_{22}} = \begin{cases} \left\langle \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1} \right); \alpha^1_{N_1} \wedge \alpha^2_{N_1}, \beta^1_{N_1} \vee \beta^2_{N_1}, \eta^1_{N_1} \vee \eta^2_{N_1} \right\rangle \; (a_3 > 0, b_3 > 0) \\ \left\langle \left(\frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1} \right); \alpha^1_{N_1} \wedge \alpha^2_{N_1}, \beta^1_{N_1} \vee \beta^2_{N_1}, \eta^1_{N_1} \vee \eta^2_{N_1} \right\rangle \; (a_3 < 0, b_3 > 0) \\ \left\langle \left(\frac{a_3}{b_1}, \frac{a_2}{b_2}, \frac{a_1}{b_3} \right); \alpha^1_{N_1} \wedge \alpha^2_{N_1}, \beta^1_{N_1} \vee \beta^2_{N_1}, \eta^1_{N_1} \vee \eta^2_{N_1} \right\rangle \; (a_3 < 0, b_3 < 0) \end{cases}$$

Definition 5: [49] Ranking of neutrosophic numbers is a very important concept as it is closely related to the separation measure between two neutrosophic numbers. Let N_{11} and N_{22} be two triangular neutrosophic numbers and $\Re(.)$ be a ranking function on the set of all triangular neutrosophic numbers. Then the following holds:

(i) $N_{11} \le N_{22}$ if and only if $\Re(N_{11}) \le \Re(N_{22})$.

(ii) $N_{11} \ge N_{22}$ if and only if $\Re(N_{11}) \ge \Re(N_{22})$.

(iii) $N_{11} = N_{22}$ if and only if $\Re(N_{11}) = \Re(N_{22})$.

(iv) $\min(N_{11}, N_{22}) = N_{11}$, if $N_{11} \le N_{22}$ *or* $N_{22} \ge N_{11}$.

Definition 6: [50] For the triangular neutrosophic number $N_1 = \left\langle (a_1, a_2, a_3); \alpha_{N_1}, \beta_{N_1}, \eta_{N_1} \right\rangle$, the ranking function $\Re(N_1)$ is defined in the following way:

$$\Re(N_1) = \frac{a_1 + a_2 + a_3}{9} \left(\alpha_{N_1} + (1 - \beta_{N_1}) + (1 - \eta_{N_1}) \right).$$

3. Linear fractional programming problem

A general form of LFPP is portrayed in the following way:

$$\max Z(x) = \frac{\sum_{j=1}^{s} a_j x_j + b}{\sum_{j=1}^{s} c_j x_j + d} = \frac{N(x)}{D(x)}$$

Subject to (1)

$$x \in X = \left\{ x \in R^s : \sum_{j=1}^{s} p_{lj} x_j \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} q_l ;\; x \ge 0,\; l = 1, 2, ..., r \right\}.$$

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$N(x)$ and $D(x)$ represent the numerator and denominator of $Z(x)$ respectively. All the resources, cost coefficients and technological coefficients are assumed to be real numbers. The denominator $D(x)$ may be equal to zero for some values of X. Such cases are avoided by considering that either $\left\{\sum_{j=1}^{s} p_{lj}x_j \begin{pmatrix}\le\\=\\\ge\end{pmatrix} q_l;\ x\ge 0,\ l=1,2,...,r \Rightarrow D(x)>0\right\}$ or $\left\{\sum_{j=1}^{s} p_{lj}x_j \begin{pmatrix}\le\\=\\\ge\end{pmatrix} q_l;\ x\ge 0,\ l=1,2,...,r \Rightarrow D(x)<0\right\}$.

For our convenience, it is assumed that

$$\left\{\sum_{j=1}^{s} p_{lj}x_j \begin{pmatrix}\le\\=\\\ge\end{pmatrix} q_l;\ x\ge 0,\ l=1,2,...,r \Rightarrow D(x)>0\right\}. \tag{2}$$

Definition 3.1. [51] Two mathematical programming problem (i) $\max\ G(x)$, subject to $x\in S$, (ii) $\max\ H(x)$, subject to $x\in T$ are said to be equivalent iff there exists a one - one mapping f from the feasible set of (i), onto the feasible set of (ii), satisfying the condition $G(x)=H(f(x))$ for all $x\in S$.

The LFPP (1), along with the condition (2), is transformed into an LPP with an additional variable using Charnes and Cooper's [20] technique in the following way:

Assuming $\dfrac{1}{D(x)}=k$ and $x_j k = y_j$, the LFPP (1) can be rewritten as:

$$\max\ Z(y,k)=\sum_{j=1}^{s} a_j y_j + bk$$

Subject to (3)

$$\sum_{j=1}^{s} c_j y_j + dk = 1$$

$$\sum_{j=1}^{s} p_{lj} y_j - q_l k \begin{pmatrix}\le\\=\\\ge\end{pmatrix} 0$$

$$y_j \ge 0,\ k>0$$

Theorem 3.1 Assuming condition (2), the LFPP (1) is equivalent to the linear programming problem (3).

Consider the following problem:

$$\max\ Z(y,k)=\sum_{j=1}^{s} a_j y_j + bk$$

Subject to (4)

$$\sum_{j=1}^{s} c_j y_j + dk \le 1$$

$$\sum_{j=1}^{s} p_{lj} y_j - q_l k \begin{pmatrix}\le\\=\\\ge\end{pmatrix} 0$$

$$y_j \ge 0,\ k>0$$

Problem (4) is obtained from problem (3) by replacing the equality constraint $\sum_{j=1}^{s} c_j y_j + dk = 1$ by $\sum_{j=1}^{s} c_j y_j + dk \le 1$.

Theorem 3.2 [51] Let $N(\zeta)\ge 0$ for some $\zeta\in X$, if (1) reaches a (global) maximum at $x=x^*$, then (4) reaches a (global) maximum at a point $(k,y)=(k^*,y^*)$, where $\dfrac{y^*}{k^*}=x^*$ and the objective functions at these points are equal.

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Theorem 3.3 [50] If LFPP (1) is a standard concave-convex programming problem which reaches a (global) maximum at a point $x = x^*$, then the corresponding transformed problem (4) attains the same maximum value at a point $(k, y) = (k^*, y^*)$, where $\frac{y^*}{k^*} = x^*$. Also (4) has a concave objective function and a convex feasible set.

Assume that

$$\max \ Z(x) = \frac{N(x)}{D(x)}$$

Subject to (5)

$$x \in X = \left\{ x \in R^s : Px \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} q, x \geq 0 \right\}$$

where $N(x)$ is concave and negative for each $x \in X$ and $D(x)$ is concave and positive on X, then we have

$$\max_{x \in X} \frac{N(x)}{D(x)} \Leftrightarrow \min_{x \in X} \frac{-N(x)}{D(x)} \Leftrightarrow \max_{x \in X} \frac{D(x)}{-N(x)}$$

Since $N(x)$ is concave and negative, so $-N(x)$ is convex and positive. So the above problem (5) gets transformed into the standard concave-convex programming problem which can be easily converted into an LPP as follows:

$$\max kD(y/k)$$

Subject to (6)

$$-kN(y/k) \leq 1$$

$$P(y/k) - q \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} 0$$

$$k > 0, y \geq 0$$

4.Multi-objective linear fractional programming problem

A classical MOLFPP is written in the following way:

$$\max Z_i(x) = \frac{\sum_{j=1}^{s} a_{ij} x_j + b_i}{\sum_{j=1}^{s} c_{ij} x_j + d_i} = \frac{N_i(x)}{D_i(x)} \ , \ i = 1, 2, ..., L$$

Subject to (7)

$$x \in X = \left\{ x \in R^s : \sum_{j=1}^{s} p_{lj} x_j \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} q_l ; \ x \geq 0, \ l = 1, 2, ..., r \right\}.$$

Here, $Z_i \ (i = 1, 2, ..., L)$ denotes the i^{th} objective function and L denotes the total number of objectives. $X = (x_1, x_2, ..., x_s)$ is the decision vector.

The process of converting MOLFPP problem into multi-objective linear programming problem (MOLPP) is discussed below.

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Let I denotes the index set $I=\{i:N_i(x)\geq 0 \;\; for\, some\, x\in X\}$ and $I^C=\{i:N_i(x)<0 \;\; for\, some\, x\in X\}$ where $I\cup I^C=\{1,2,...,L\}$. Let $D(x)$ be positive on X where X is non-empty and bounded. Let k be the least value of $1\Big/\left(\sum_{j=1}^{s}c_{ij}x_j+d_i\right)$ for $i\in I$ and the least value of $1\Big/-\left(\sum_{j=1}^{s}a_{ij}x_j+b_i\right)$ is k for $i\in I^C$. So we can write

$\bigcap_{i\in I}\dfrac{1}{\left(\sum_{j=1}^{s}c_{ij}x_j+d_i\right)}=k$ and $\bigcap_{i\in I^C}\dfrac{-1}{\left(\sum_{j=1}^{s}a_{ij}x_j+b_i\right)}=k$

which is equivalent to

$\dfrac{1}{\left(\sum_{j=1}^{s}c_{ij}x_j+d_i\right)}\geq k$ for $i\in I$ and $\dfrac{-1}{\left(\sum_{j=1}^{s}a_{ij}x_j+b_i\right)}\geq k,$ for $i\in I^C$.

With the help of the transformation $y=kx(k>0),$ definition 3.1, and theorems 3.1, 3.2 and 3.3, the MOLFPP (7) can be converted into the following:

$$\max G_i(y,k)=\begin{cases}kN_i(y/k), & for\ i\in I;\\ kD_i(y/k), & for\ i\in I^C\end{cases}$$

Subject to (8)

$$kD_i(y/k)\leq 1,\ \ for\ i\in I$$
$$-kN_i(y/k)\leq 1,\ \ for\ i\in I^C$$
$$P(y/k)-q\leq 0$$
$$k,y\geq 0.$$

5. Fuzzy goal programming

The MOLPP is solved with the help of FGP and the process is described below. Let a general MOLPP be formulated as follows:

$$\max Z(x)=[Z_1(x),Z_2(x),...,Z_L(x)]$$

Subject to (9)

$$x\in X=\left\{x\in R^s:Px\begin{pmatrix}\leq\\=\\\geq\end{pmatrix}Q;\ x\geq 0\right\}.$$

$Z_i(x)$ is linear function for each $i=1,2,...,L.$ All the parameters are assumed to be real numbers.

Each objective function of the obtained MOLPP (9) is solved individually to obtain their respective maximum and minimum values.

Let $Z_i^B=\max_{x\in X}(\tilde{Z}_i(x))$ and $Z_i^W=\min_{x\in X}(\tilde{Z}_i(x))\,;i=1,2,...,L$ represent the best and the worst value respectively of $\tilde{Z}_i(x)$.

5.1 Formulation of fuzzy goals:

Each crisp objective function is given an ambition level, resulting in a fuzzy goal. As a result, for issues where the objective function is to be maximized or minimized, the fuzzy goals can be formed as:

$\tilde{Z}_i(x)\gtrsim Z_i^B$ and $\tilde{Z}_i(x)\lesssim Z_i^W \qquad i=1,2,...,L$

As stated by Zimmerman [39], fuzziness is connected with aspiration levels and is referred to as "basically less than" and "essentially more than" i.e. $\lesssim$ and $\gtrsim$.

5.2 Construction of membership functions:

The linear membership function corresponding to the i^{th} fuzzy objective goal is described as:

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$$\mu_i(x)=\begin{cases}1 & \text{if } \tilde{Z}_i(x)\geq Z_i^B,\\ \dfrac{\tilde{Z}_i(x)-Z_i^W}{Z_i^B-Z_i^W} & \text{if } Z_i^W\leq \tilde{Z}_i(x)\leq Z_i^B,\\ 0 & \text{if } \tilde{Z}_i(x)\leq Z_i^W.\end{cases} \quad (10)$$

Here, Z_i^B and Z_i^W represent the upper tolerance and lower tolerance limit respectively associated with the i^{th} objective goal.

The objective function $\tilde{Z}_i(x),\ (i=1,2,...,L)$ is presumed to possess continuous partial derivatives of order (s +1) or less over the feasible region X. So the membership function $\mu_i(x)$ associated with the objective function $\tilde{Z}_i(x)$ possesses similar properties in the feasible area X.

The problem (9) therefore gets converted into the following form:

$$\max \mu_i(x),\ i=1,2,...,L$$

Subject to (11)

$$x\in X$$

5.3 Fuzzy goal programming (FGP) formulation

Since the highest attainable value for a membership function is one, so for the membership function defined in (10), the flexible membership goal along with the aspiration level one can be written as:

$$\mu_i(x)+D_i^- - D_i^+ = 1,\ i=1,2,...,L \quad (12)$$

Here D_i^+ and D_i^- refers to the positive and negative deviational variables respectively.

In maximization type objectives, since the objectives cannot pass over the ideal solution, positive deviation should be zero. So (12) can be rewritten as

$$\mu_i(x)+D_i^- = 1,\ i=1,2,...,L$$

Therefore to obtain compromise solution of MOLFPP, different FGP models can be formulated as follows:

Model (I):

$$\min \lambda$$

Subject to (13)

$$x\in X$$
$$\mu_i(x)+D_i^- = 1$$
$$\lambda\geq D_i^-$$
$$D_i^-\leq 1$$
$$x, D_i^-\geq 0;\ i=1,2,...,L$$

Model (II):

$$\min\ \varpi=\sum_{i=1}^{L} D_i^-$$

Subject to (14)

$$x\in X$$
$$\mu_i(x)+D_i^- = 1$$
$$0\leq D_i^-\leq 1$$
$$x, D_i^-\geq 0;\quad i=1,2,...,L$$

Model (III):

$$\min\ \delta=\sum_{i=1}^{L}\kappa_i D_i^-$$

Subject to (15)

$$x\in X$$
$$\mu_i(x)+D_i^- = 1$$
$$0\leq D_i^-\leq 1$$
$$x, D_i^-\geq 0;\quad i=1,2,...,L$$

Here κ_i represents the importance or relative weight of the objective function $\tilde{Z}_i(x)$ when compared with other objectives. The value of κ_i is set by the decision maker according to his/her preference in the decision making

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situation. The values of κ_i are so chosen to fulfil the condition $\sum_{i=1}^{L}\kappa_i = 1$. Models I, II and III are single objective linear programming problem which can easily be evaluated through conventional methods.

5.4 Choice of optimal compromise solution:

Since models I, II and III can yield different decision vectors for the same MOLFPP, there should be an appropriate deciding metric to choose the model which gives better result. Here, distance functions play an important role to select the optimal compromise solution. We use Euclidean distance function [46] to choose from the three FGP models I, II and III. The definition of the Euclidean distance function is given as:

$$E_2 = \left[\sum_{i=1}^{L}[1-\mu_i(x)]^2\right]^{1/2} \tag{16}$$

where $\mu_i(x)$ denotes the obtained membership value of $Z_i(x)$. The solution which is associated with minimal E_2 would be accepted as the optimal compromise solution.

6.Multi-objective neutrosophic linear fractional programming problem (MONLFPP)

In this section, we present the formulation of MONLFPP where the technological coefficients, resources and the cost of the objective functions are considered as triangular neutrosophic numbers.

MONLFPP can be formulated as follows:

$$\max \tilde{Z}_i(x) = \frac{\sum_{j=1}^{s}\tilde{a}_{ij}x_j + \tilde{b}_i}{\sum_{j=1}^{s}\tilde{c}_{ij}x_j + \tilde{d}_i}, \quad i = 1,2,\ldots,L$$

Subject to (17)

$$x \in X = \left\{\sum_{j=1}^{s}\tilde{p}_{lj}x_j \begin{pmatrix}\le\\=\\\ge\end{pmatrix}\tilde{q}_l;\ x_j \ge 0,\ l = 1,2,\ldots,r\right\}.$$

where $\tilde{a}_{ij}, \tilde{b}_i, \tilde{c}_{ij}, \tilde{d}_i, \tilde{p}_{lj}, \tilde{q}_l$ are triangular neutrosophic numbers for each $i = 1,2,\ldots,L$; $j = 1,2,\ldots,s$ and $l = 1,2,\ldots,r$. So the problem (17) can be rewritten in the following way:

$$\max \tilde{Z}_i(x) = \frac{\sum_{j=1}^{s}\left(a_{ij1}, a_{ij2}, a_{ij3}; \alpha_{ij}^{a}, \beta_{ij}^{a}, \eta_{ij}^{a}\right)x_j + \left(b_{i1}, b_{i2}, b_{i3}; \alpha_i^{b}, \beta_i^{b}, \eta_i^{b}\right)}{\sum_{j=1}^{s}\left(c_{ij1}, c_{ij2}, c_{ij3}; \alpha_{ij}^{c}, \beta_{ij}^{c}, \eta_{ij}^{c}\right)x_j + \left(d_{i1}, d_{i2}, d_{i3}; \alpha_i^{d}, \beta_i^{d}, \eta_i^{d}\right)}, \quad i = 1,2,\ldots,L$$

Subject to (18)

$$x \in X = \left\{\sum_{j=1}^{s}\left(p_{lj1}, p_{lj2}, p_{lj3}; \alpha_{lj}^{p}, \beta_{lj}^{p}, \eta_{lj}^{p}\right)x_j \begin{pmatrix}\le\\=\\\ge\end{pmatrix}\left(q_{l1}, q_{l2}, q_{l3}; \alpha_l^{q}, \beta_l^{q}, \eta_l^{q}\right);\ x_j \ge 0,\ l = 1,2,\ldots,r\right\}.$$

where $\alpha, \beta, \eta \in [0,1]$ and denotes truth, indeterminacy and falsity membership values of each triangular neutrosophic number.

7.Proposed method to solve MONLFPP using fuzzy goal programming

Using a ranking function, we propose a new method for solving MONLFPP. The steps that make up the suggested approach are as follows.

Step 1: Step 1: We consider the MONLFPP problem (18), which has triangular neutrosophic numbers as parameters.

Step 2: Using the ranking function described in definition 6, each triangular neutrosophic number is translated into a corresponding number on the real line, problem (18) is converted into an analogous crisp MOLFPP. As a result, problem (18) is transformed into the following:

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$$\max \tilde{Z}_i(x) = \frac{\sum_{j=1}^{s} \Re\left(a_{ij1}, a_{ij2}, a_{ij3}; \alpha_{ij}^a, \beta_{ij}^a, \eta_{ij}^a\right) x_j + \Re\left(b_{i1}, b_{i2}, b_{i3}; \alpha_i^b, \beta_i^b, \eta_i^b\right)}{\sum_{j=1}^{s} \Re\left(c_{ij1}, c_{ij2}, c_{ij3}; \alpha_{ij}^c, \beta_{ij}^c, \eta_{ij}^c\right) x_j + \Re\left(d_{i1}, d_{i2}, d_{i3}; \alpha_i^d, \beta_i^d, \eta_i^d\right)}, \ i = 1, 2, \ldots, L$$

Subject to (19)

$$x \in X = \left\{ \sum_{j=1}^{s} \Re\left(p_{lj1}, p_{lj2}, p_{lj3}; \alpha_{lj}^p, \beta_{lj}^p, \eta_{lj}^p\right) x_j \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} \Re\left(q_{l1}, q_{l2}, q_{l3}; \alpha_l^q, \beta_l^q, \eta_l^q\right);\ x_j \geq 0,\ l = 1, 2, \ldots, r \right\}.$$

Step 3: The MOLFPP (19) is converted into an equivalent MOLPP using the procedure given in section 4.
Step 4: The obtained MOLPP is solved using FGP as presented in section 5. The best FGP model is chosen with the help of Euclidean distance function.

8.Numerical examples

We recall the problem from [44] and formulate it into an MOPP by adding an additional objective function.
Example 1:
A corporation manufactures three types of products: A, B, and C, each having a profit of \$8, \$7, and \$9 per unit. Each of the following products costs roughly \$8.90, \$9.90, and \$6.90, respectively. Due to the estimated time of the manufacturing process, a fixed cost of roughly 1.5 dollars is assumed to be added to the cost function. Assume that the cost of raw materials required to manufacture each unit of product A, B, and C is roughly \$4, \$3, and \$5, respectively, and that supply is limited to about 28 dollars. Daily man-hours available for manufacturing products A, B, and C are approximately 5 hours, 3 hours, and 3 hours, respectively, and total man-hours available are around 20 hours. In addition, it takes approximately 4 hours, 8 hours, and 6 hours to manufacture one unit of product A, B, and C, respectively, plus an additional 2 hours for set up. Calculate how many of each of the products A, B, and C should be produced in order to maximize total profit. The company's management intends to maximize the profit-to-investment ratio as well as the profit-to-time ratio.
Let x_1, x_2 and x_3 units of products A, B and C respectively are to be produced. So the problem can be formulated as:

$$\max \tilde{Z}_1(x) = \frac{\tilde{8}x_1 + \tilde{7}x_2 + \tilde{9}x_3}{\tilde{8}x_1 + \tilde{9}x_2 + \tilde{6}x_3 + 1.\tilde{5}}$$

$$\max \tilde{Z}_2(x) = \frac{\tilde{8}x_1 + \tilde{7}x_2 + \tilde{9}x_3}{\tilde{4}x_1 + \tilde{8}x_2 + \tilde{6}x_3 + \tilde{2}}$$

Subject to (20)

$$\tilde{4}x_1 + \tilde{3}x_2 + \tilde{5}x_3 \leq 2\tilde{8}$$

$$\tilde{5}x_1 + \tilde{3}x_2 + \tilde{3}x_3 \leq 2\tilde{0}$$

$$x_1, x_2, x_3 \geq 0$$

The parameters are predicted by the decision makers as follows:

$\tilde{8} = (7,8,9;0.5,0.8,0.3)$, $\tilde{7} = (6,7,8;0.2,0.6,0.5)$, $\tilde{9} = (8,9,10;0.8,0.1,0.4)$, $\tilde{6} = (4,6,8;0.75,0.25,0.1)$,

$1.\tilde{5} = (1,1.5,2;0.75,0.5,0.25)$, $\tilde{4} = (3,4,5;0.4,0.6,0.5)$, $\tilde{3} = (2,3,4;1,0.25,0.3)$, $\tilde{5} = (4,5,6;0.3,0.4,0.8)$,

$2\tilde{8} = (25,28,30;0.4,0.25,0.6)$, $\tilde{2} = (1,2,3.5;0.7,0.3,0.2)$, $2\tilde{0} = (18,20,22;0.9,0.2.0.6)$

Using the ranking function given in definition 6, the MONLFPP (20) gets transformed into a crisp MOLFPP as follows:

$$\max Z_1(x) = \frac{3.73x_1 + 2.56x_2 + 6.9x_3}{3.73x_1 + 6.9x_2 + 4.8x_3 + 1}$$

$$\max Z_2(x) = \frac{3.73x_1 + 2.56x_2 + 6.9x_3}{1.73x_1 + 3.73x_2 + 4.8x_3 + 1.58}$$

Subject to (21)

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$1.73x_1 + 2.45x_2 + 1.83x_3 \le 14.29$

$1.83x_1 + 2.45x_2 + 2.45x_3 \le 14$

$x_1, x_2, x_3 \ge 0$

Solving each objective function individually using Charnes and Cooper's variable transformation method we obtain their best and worst values respectively, which are presented in table 1.

Table 1: Best and worst solutions for each objective function

Objective function	**Best solution with solution point**	**Worst solution with solution point**
$Z_1(x)$	1.38693 at (0,0,5.714)	0.00 at (0,0,0)
$Z_2(x)$	1.92613 at (7.65,0,0)	0.00 at (0,0,0)

So it is observed that $Z_1(x), Z_2(x) \ge 0$ in the feasible region. So the MOLFPP (21) is converted to the equivalent LPP as follows:

$\max G_1(y,k) = 3.73y_1 + 2.56y_2 + 6.9y_3$

Subject to (22)

$3.73y_1 + 6.9y_2 + 4.8y_3 + k \le 1$

$1.73y_1 + 3.73y_2 + 4.8y_3 + 1.58k \le 1$

$1.73y_1 + 2.45y_2 + 1.83y_3 - 14.29k \le 0$

$1.83y_1 + 2.45y_2 + 2.45y_3 - 14k \le 0$

$y_1, y_2, y_3, k \ge 0$

Solving problem (22) we obtain the value $G_1 = 1.37039$ at $y_1 = 0.010186,\ y_2 = 0, y_3 = 0.1931, k = 0.035124.$ The corresponding values are obtained as $x_1 = 0.29,\ x_2 = 0, x_3 = 5.4976.$ Using these values, we obtain the optimal values of $Z_1(x)$ and $Z_2(x)$ as follows:

$Z_1(x) = 1.370386,\ Z_2(x) = 1.370386.$

Since there is no method in the literature which solves MONLFPP, the problem in example 1 which appears as a single objective problem in articles [32, 33], have been considered with another objective function $Z_2(x)$ along with the existing objective $Z_1(x)$. Solving the MONLFPP (20) through our proposed method, we obtain $Z_1(x) = 1.370386,\ Z_2(x) = 1.370386.$ Comparing the value of $Z_1(x)$ obtained through our proposed method with the methods of [31, 32], we have the following:

$Z_1(x)_{proposed\ method} = 1.370386 > Z_1(x)_{[31]} = 1.16$

$Z_1(x)_{proposed\ method} = 1.370386 > Z_1(x)_{[32]} = 1.078$

So it can be observed that the proposed method gives better solution compared to the methods [50- 51].

Example 2: We consider the following MONLFPP

$$\max \tilde{Z}_1(x) = \frac{\tilde{5}x_1 + \tilde{6}x_2 + \tilde{9}x_3}{\tilde{7}x_1 + \tilde{6}x_2 + \tilde{2}x_3 + \tilde{8}}$$

$$\max \tilde{Z}_2(x) = \frac{\tilde{5}x_1 + \tilde{7}x_2 + \tilde{6}x_3}{\tilde{6}x_1 + \tilde{8}x_2 + \tilde{2}x_3 + \tilde{9}}$$

Subject to (23)

$\tilde{4}x_1 + \tilde{7}x_2 + \tilde{6}x_3 \le \tilde{10}$

$\tilde{9}x_1 + \tilde{3}x_2 + \tilde{5}x_3 \le \tilde{21}$

$x_1, x_2, x_3 \ge 0$

The parameters are predicted as follows:

$\tilde{2} = (1,2,2.5;0.7,0.1,0.2),\ \tilde{3} = (2,3,4;0.6,0.2,0.3),\ \tilde{4} = (3,4,5;0.4,0.6,0.5),\ \tilde{5} = (3.5,5,6;0.8,0.3,0.4),$

$\tilde{6} = (5,6,7.5;0.5,0.7,0.3),\ \tilde{7} = (6,7,8;0.5,0.6,0.5), \tilde{8} = (6,8,9;0.6,0.4,0.2),\ \tilde{9} = (8,9,10.5;1,0.2,0.1),$

$\tilde{10} = (8,10,11.5;0.8,0.2,0.4),\ \tilde{21} = (19,21,23;0.9,0.2,0.4)$

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Using the ranking function given in definition 6, the MONLFPP (23) gets transformed into a crisp MOLFPP as follows:

$$\max Z_1(x) = \frac{3.38x_1 + 3.08x_2 + 8.25x_3}{3.266x_1 + 3.08x_2 + 1.466x_3 + 5.11}$$

$$\max Z_2(x) = \frac{3.38x_1 + 3.266x_2 + 3.08x_3}{3.08x_1 + 5.11x_2 + 1.466x_3 + 8.25}$$

Subject to (24)

$$1.733x_1 + 3.266x_2 + 3.08x_3 \le 7.21$$

$$8.25x_1 + 2.1x_2 + 3.38x_3 \le 16.1$$

$$x_1, x_2, x_3 \ge 0$$

Solving each objective function individually using Charnes and Cooper's variable transformation method we obtain their best and worst values respectively, which are presented in table 2.

Table 2: Best and worst solutions for each objective function

Objective function	**Best solution with solution point**	**Worst solution with solution point**
$Z_1(x)$	2.26095 at (0,0,2.3409)	0.00 at (0,0,0)
$Z_2(x)$	0.639754 at (1.2897,0,1.6152)	0.00 at (0,0,0)

So it is observed that $Z_1(x), Z_2(x) \ge 0$ in the feasible region. So the MOLFPP (24) is converted to the equivalent MOLPP as follows:

$$\max G_1(y,k) = 3.38y_1 + 3.08y_2 + 8.25y_3$$

$$\max G_2(y,k) = 3.38y_1 + 3.266y_2 + 3.08y_3$$

Subject to (25)

$$3.266y_1 + 3.08y_2 + 1.466y_3 + 5.11k \le 1$$

$$3.08y_1 + 5.11y_2 + 1.466y_3 + 8.25k \le 1$$

$$1.733y_1 + 3.266y_2 + 3.08y_3 - 7.21k \le 0$$

$$8.25y_1 + 2.1y_2 + 3.38y_3 - 16.1k \le 0$$

$$y_1, y_2, y_3, k \ge 0$$

Solving each objective function of problem (25) individually we obtain the value $G_1 = 1.65322$ at $y_1 = 0,\ y_2 = 0, y_3 = 0.20039, k = 0.0856.$ The corresponding values are obtained as $x_1 = 0,\ x_2 = 0, x_3 = 2.341.$

Similarly for objective function G_2 we obtain the value $G_2 = 0.639754$ at $y_1 = 0.0884,\ y_2 = 0, y_3 = 0.1107, k = 0.0685.$ The corresponding values are obtained as $x_1 = 1.2898,\ x_2 = 0, x_3 = 1.6152.$

Also the minimum values of G_1 and G_2 are obtained as $G_1 = 0.00$ and $G_2 = 0.00.$

So the fuzzy goals are formed in the following manner:

$$G_1(y,k) \gtrsim 1.65322,\ G_2(y,k) \gtrsim 0.639754$$

Thereafter the membership functions of the MOLFPP are formulated as:

$$\mu_1(y,k) = \frac{G_1(y,k) - 0.00}{1.65322 - 0.00}$$

$$= \frac{3.38y_1 + 3.08y_2 + 8.25y_3}{1.65322}$$

$$= 2.0445y_1 + 1.863y_2 + 4.9903y_3$$

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$$\mu_2(y,k)=\frac{G_2(y,k)-0.00}{0.639754-0.00}$$
$$=\frac{3.38y_1+3.266y_2+3.08y_3}{0.639754}$$
$$=5.28328y_1+5.1051y_2+4.8143y_3$$

The FGP model (I) can be formed as:

$$\begin{aligned}
&\min \lambda\\
&\text{Subject to}\\
&2.0445y_1+1.863y_2+4.9903y_3+D_1^-=1\\
&5.28328y_1+5.1051y_2+4.8143y_3+D_2^-=1\\
&3.266y_1+3.08y_2+1.466y_3+5.11k\le 1\\
&3.08y_1+5.11y_2+1.466y_3+8.25k\le 1\\
&1.733y_1+3.266y_2+3.08y_3-7.21k\le 0\\
&8.25y_1+2.1y_2+3.38y_3-16.1k\le 0\\
&\lambda\ge D_j^-\\
&0\le D_j^-\le 1\\
&j=1,2\\
&y_1,y_2,y_3,k,D_1^-,D_2^-\ge 0
\end{aligned} \tag{26}$$

The FGP model (II) can be formed as:

$$\begin{aligned}
&\min\ \varpi=(D_1^-+D_2^-)\\
&\text{Subject to}\\
&2.0445y_1+1.863y_2+4.9903y_3+D_1^-=1\\
&5.28328y_1+5.1051y_2+4.8143y_3+D_2^-=1\\
&3.266y_1+3.08y_2+1.466y_3+5.11k\le 1\\
&3.08y_1+5.11y_2+1.466y_3+8.25k\le 1\\
&1.733y_1+3.266y_2+3.08y_3-7.21k\le 0\\
&8.25y_1+2.1y_2+3.38y_3-16.1k\le 0\\
&0\le D_j^-\le 1\\
&j=1,2\\
&y_1,y_2,y_3,k,D_1^-,D_2^-\ge 0
\end{aligned} \tag{27}$$

Taking equal preferences for both the objective functions, FGP model (III) can be formulated as:

$$\begin{aligned}
&\min\ \delta=0.5(D_1^-+D_2^-)\\
&\text{Subject to}\\
&2.0445y_1+1.863y_2+4.9903y_3+D_1^-=1\\
&5.28328y_1+5.1051y_2+4.8143y_3+D_2^-=1\\
&3.266y_1+3.08y_2+1.466y_3+5.11k\le 1\\
&3.08y_1+5.11y_2+1.466y_3+8.25k\le 1\\
&1.733y_1+3.266y_2+3.08y_3-7.21k\le 0\\
&8.25y_1+2.1y_2+3.38y_3-16.1k\le 0\\
&0\le D_j^-\le 1\\
&j=1,2\\
&y_1,y_2,y_3,k,D_1^-,D_2^-\ge 0
\end{aligned} \tag{28}$$

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The results obtained by solving the models (26), (27) and (28) are presented in Table 3.

Table 3: Results obtained by solving the three FGP models

Different methods	**Solution point** (y_1, y_2, y_3, k)	**Value of objective** G_1	**Value of objective** G_2	**Value of** μ_1	**Value of** μ_2	**Value of** E_2
FGP model I	(0.0103205, 0, 0.189919, 0.0836111)	1.6017	0.61983	0.9688	0.9688	0.04412
FGP model II	(0.000001869, 0, 0.200388, 0.085603)	1.6532	0.6172	0.9999	0.9647	0.0353
FGP model III	(0.000001869, 0, 0.200388, 0.085603)	1.6532	0.6172	0.9999	0.9647	0.0353

From table 3, it can be observed that FGP models II and III provide the same solution which is better than the solution obtained from model I. So here we can choose model II or III.

So using model II, the corresponding values of (x_1, x_2, x_3) are obtained at $(0.00002183, 0.00, 2.3409)$ and the values of $Z_1(x)$ and $Z_2(x)$ are obtained as $Z_1(x) = 2.26093,\ Z_2(x) = 0.6172$.

9. Conclusions and future works

A new method for solving MONLFPP has been developed in this study. Triangular neutrosophic numbers are used to represent the cost of objective functions, technological coefficients, and resources. With the use of a ranking function and a variable transformation approach, the MONLFPP is turned into an analogous MOLPP. After that, FGP is used to solve the MOLPP. For MOLPP, we looked at three FGP models, and the optimum model for a problem can be identified using the Euclidean distance function. Two examples have been used to exemplify the suggested strategy. Because the method takes uncertainty into consideration and can handle multi-objective programming issues, it can be used to address real-world problems in transportation, assignment, industrial production, and a variety of other disciplines.

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