Hindawi
Journal of Mathematics
Volume 2024, Article ID 5692756, 17 pages
https://doi.org/10.1155/2024/5692756

Research Article

Theory and Application of Interval-Valued Neutrosophic Line Graphs

Keneni Abera Tola , V. N. Srinivasa Rao Repalle , and Mamo Abebe Ashebo

Department of Mathematics, College of Natural and Computational Sciences, Wollega University, Nekemte, Ethiopia

Correspondence should be addressed to Keneni Abera Tola; keneniabera460@gmail.com

Received 17 March 2023; Revised 29 February 2024; Accepted 7 March 2024; Published 19 March 2024

Academic Editor: G. Muhiuddin

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Neutrosophic graphs are used to model inconsistent information and imprecise data about any real-life problem. It is regarded as a generalization of intuitionistic fuzzy graphs. Since interval-valued neutrosophic sets are more accurate, compatible, and flexible than single neutrosophic sets, interval-valued neutrosophic graphs (IVNGs) were defined. The interval-valued neutrosophic graph is a fundamental issue in graph theory that has wide applications in the real world. Also, problems may arise when partial ignorance exists in the datasets of membership [0, 1], and then, the concept of IVNG is crucial to represent the problems. Line graphs of neutrosophic graphs are significant due to their ability to represent and analyze uncertain or indeterminate information about edge relationships and complex networks in graphs. However, there is a research gap on the line graph of interval-valued neutrosophic graphs. In this paper, we introduce the theory of an interval-valued neutrosophic line graph (IVNLG) and its application. In line with that, some mathematical properties such as weak vertex isomorphism, weak edge isomorphism, effective edge, and other properties of IVNLGs are proposed. In addition, we defined the vertex degree of IVNLG with some properties, and by presenting several theorems and propositions, the relationship between fuzzy graph extensions and IVNLGs was explored. Finally, an overview of the algorithm used to solve the problems and the practical application of the introduced graphs were provided.

1. Introduction

Graph theory is a mathematical discipline that deals with mathematical representations of the links between objects. Not all systems described by science and technology can accommodate complex processes and occurrences. For situations like these, mathematical models have been created to handle different kinds of systems with uncertainty-containing components. In 1965, Zadeh presented fuzzy sets by giving membership grades to each object of the interval set [1]. Based on Zadeh's fuzzy relations, Kauffman [2] proposed fuzzy graphs. Later on, Rosenfeld [3] discussed the fuzzy analogy of many graph-theoretic notions. After this, researchers started to introduce many classes of fuzzy graphs, and they have brought remarkable advances to impressive applications of fuzzy theory.

However, linguistic terms are very important in decision-making theory, such as data mining, multiattribute decision-making (MADM) problems, and a novel type of linguistic information form to facilitate decision-makers in evaluating online learning platforms in a comprehensive manner, and the determination of decision-makers' weights is a key step prior to the aggregation of individual assessment information into a collective result [4, 5].

The membership function was insufficient to explain exactly the complexity of object features, leading to the suggestion of a nonmembership function with fuzzy sets (FS). The extension of FS, which is called intuitionistic fuzzy sets (IFS), was introduced [6–8]. Later, the notion of m-polar interval-valued intuitionistic fuzzy graphs was introduced [9]. Also, several types of arcs in the interval-valued intuitionistic (S, T)-fuzzy graphs and their properties were studied [10, 11]. Due to the dynamic nature of certain problems that cannot be addressed by FS and IFS, Smarandache [12] introduced neutrosophic sets (NS). He added another component, the indeterminacy degree, to the definition of IFS. The notion of SVNSs, which has multiple applications in entropy measure, decision-making, index distance, and similarity measure, can be independently expressed as a truth-membership function (*TMF*), an indeterminacy membership function (*IMF*), and a nonmembership or falsity function (*FMF*) by adhering to the definition of NS [13]. In any case, from a philosophical perspective, the membership, indeterminate, and nonmembership values are independent with respect to one another, and the TMF, IMF, and FMF values are in the interval of [0, 1] presented in [14]. For that reason, sometimes it happens that the membership, indeterminate, and nonmembership values cannot be measured as a point, but they can be measured as an interval. Considering this, later, an IVNS and its properties were introduced [15, 16]. In comparison to an SVNS, an IVNS provides a more accurate and flexible description of graphs. An IVNS is a generalization of SVNSs, and it has many applications in decision-making [17].

Besides the fact that single-valued neutrosophic graphs (SVNG) were proposed [18], graph theory is a basic idea in modern mathematics. Graphs are used as a mathematical tool to visually represent and evaluate social networks after all of this has been considered. Consequently, neutrosophic graphs with interval values were studied by Broumi et al. [19]. However, Akram and Shahzadi [20] provided a different definition of SVNG because this definition goes against the concepts of complement and join characteristics. He also presented the concept of interval-valued neutrosophic competition graphs [21]. An IVNG and some of its functions were discussed in light of the revised definition of SVNSs. They also noticed that IVNG may be altered to take on a regular structure [22]. Connectivity concepts are the key to graph clustering and networks, and they are the most important concept in the entire graph theory [23].

In a network, vertices hold significance due to their connections with other vertices, while edges in a line graph can be applied with vertices' attributes [24]. Typically, the structure of a line graph *L(G)* is more complex than that of the corresponding graph *G*. Many other researchers studied different classes of *L(G)*, such as classical line graphs [25], fuzzy line graphs [26], interval-valued fuzzy line graphs (IVFLGs) [27], intuitionistic fuzzy line graphs (IFLGs) [28], and the *L(G)* of IVIFG [29].

Subsequently, SVNGs were explored by researchers and used to tackle a variety of real-world modeling and optimization issues. Also, the definition and mathematical properties of SVNLG were derived from the single-valued neutrosophic graph [30]. They provided both necessary and sufficient criteria for SVNG and its corresponding SVNLG to be isomorphic. Also, neutrosophic vague line graphs were investigated [31]. Isomorphic properties of those graphs were also initiated. The reader should read articles [19, 32–34] to understand the fundamental ideas of the line graph and its properties.

The interval-valued neutrosophic graph is an elementary graph theory problem with numerous real-world applications. Nevertheless, no other academics have yet to introduce the IVNLG theory. In this study, we introduced the theory and application of interval-valued neutrosophic line graphs and described some of their properties. One of the motives of this research was to apply the concepts introduced to real-life problems. Finally, a procedure to drive IVNLG from a connected simple NG and its application are presented.

This work's framework is organized as follows: In Section 1, we provide a basic overview of fuzzy graphs (FGs), IFG, neutrosophic graphs (NGs), and the line graphs that correspond to each of these concepts. In Section 2, we cover the foundational mathematical ideas that will be applied to the research. A comprehensive definition and appropriate examples of IVNLG are provided in Section 3. Some basic IVNLG properties are presented in Section 4, along with some propositions. In Section 5, a real-world application for a decision-making problem was designed using IVNLG. Finally, further research work related to the research paper is discussed in the conclusion.

2. Preliminaries

Here, we have used standard definitions, terminologies, and results from the rest of the article.

Definition 1 (see [35]). An ordered triple $G = (V, \sigma, \mu)$ is called *FG* where $V = \{v_1, v_2, \cdots, v_n\}$ such that $\sigma: V \longrightarrow [0, 1]$, a fuzzy relation μ on σ is $\mu: V \times V \longrightarrow [0,1]$ such that $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$, for all $u, v \in V$.

Definition 2 (see [36]). The graph $G = (V, E)$ is an *IFG* if the following conditions are satisfied:

(a) Function $\sigma_1: V \longrightarrow [0, 1]$ is TMF of the vertex set of G, and $\gamma_1: V \longrightarrow [0, 1]$ is the FMF of vertex set of G and $0 \leq \sigma_1(v) + \gamma_1(v) \leq 1, \forall v \in V$.

(b) The function $\sigma_2: V \times V \longrightarrow [0, 1]$ is *TMF* of the edge set of G, and $\gamma_2: V \times V \longrightarrow [0, 1]$ is *FMF* of the edge set of G such that $\sigma_2(v_i v_j) \leq \sigma_1(v_i) \wedge \sigma_1(v_j)$ & $\gamma_2(v_i v_j) \leq \gamma_1(v_j) \vee \gamma_1(vj)$ with the condition

$$0 \leq \sigma_{2\left(v_{iv_j}\right)} + \gamma_2\left(v_i v_j\right) \leq 1, \quad \forall v_i v_j \in E. \tag{1}$$

Definition 3. For a nonempty undirected graph $G = (V, E)$ with n-vertices, consider $S_i = \left\{u\_i, x_{i_1}, \cdots, x_{k_i}\right\}, 1 \leq i \leq n, 1 \leq j \leq k_i$, vertex u_i are end vertices of edge $x_{ij} \in E$. Then, $P(E) = (S, \Lambda)$ is said to be the intersection graph such that $S = \{S_i\}$ is a node set of the graph $P(E)$ and an edge of the graph $P(E)$ is $\Lambda = \left\{S_i S_j \mid S_i \cap S_j \neq \varnothing, S_i, S_j \in S, i \neq j\right\}$.

Definition 4. Let $P(E) = (S, \Lambda)$ be the intersection graph of $G = (V, E)$. Then, $L(G)$ of G can be derived by definition of the intersection graph $P(E)$. This implies $L(G) = (Z, W)$ is a line (edge) graph where $Z = \left\{\{u_x, v_x\} \cup \{x\}: x = (u_x, v_x\right.$ $\in E, u_x, \ v_x \in V, W = \left\{S_x S_y \mid x \neq y \& S_x \cap S_y \neq \varnothing\right\}$ and $S_x = \{u_x, v_x\} \cup \{x\}$, where $x \in E$.

Definition 5 (see [37]). Let A be a subset of a universal set X. Then, $A = \{(t_A(x), i_A(x), f_A(x)): x \in X\}$ is called the neutrosophic set such that $t_A: X \longrightarrow]0^-, 1^+[, i_A \longrightarrow]0^-, 1^+[\& f_A: V \longrightarrow]0^-, 1^+[$ are TMF, IMF, and FMF, respectively, with the condition

$$0^- \leq t_A(x) + i_A(x) + f_A(x) \leq 3^+. \quad (2)$$

where the functions TMF, IMF, and FMF of A will be real standard or nonstandard subsets of $]0^-, 1^+[$.

Due to the fact that it is difficult to use the above definition of NS in real-life situations, Wang et al. introduced the idea of SVNS having TMF, IMF, and FMF values in the range of [0, 1], which will be used in scientific and engineering applications [12].

Definition 6 (see [20]). An SVN-graph is a pair of function $G = (A, B)$ where $A = (t_1, i_1, f_1)$ is the subset of V and $B = (t_2, i_2, f_2)$ is the subset of E with the conditions:

(a) The functions $t_1: V \longrightarrow [0, 1], i_1: V \longrightarrow [0, 1]$ and $f_1: V \longrightarrow [0, 1]$ are TMF, IMF, and FMF of the vertex set of $u \in V$, respectively, such that $0 \leq t_1(u) + i_1(u) + f_1(u) \leq 3, \forall u \in V$

(b) The function $t_2: V \times V \longrightarrow [0, 1], i_2: V \times V \longrightarrow [0, 1]$ and $f_2: V \times V \longrightarrow [0, 1]$ represent the TMF, IMF, and FMF of edge E, respectively, with the condition $t_2(u_iu_j) \leq t_1(u_i) \wedge t_1(u_j), i_2(u_iu_j) \leq i_1(u_i) \wedge i_1(u_j)$ and $f_2(u_iu_j) \leq f_1(u_i) \vee f_1(u_j)$ such that $0 \leq t_2(u_iu_j) + i_2(u_iu_j) + f_2(v_iv_j) \leq 3, \forall v_iv_j \in E$

Definition 7. Let $G = (A_1, B_1)$ be an NG with $A_1 = (t_{A_1}, i_{A_1}, f_{A_1})$ and $B_1 = (t_{B_1}, i_{B_1}, f_{B_1})$ be SVNS on V and E, respectively. Then, the intersection graph $(S, \Lambda) = (A_2, B_2)$ of SVNG G, where

(i) $t_{A2}(S_i) = t_{A_1}(v_i),\ i_{A2}(S_i) = i_{A_1}(v_i)$, and $f_{A2}(S_i) = f_{A_1}(v_i)$,for all $S_i, S_j \in S$

(ii) $t_{B2}(S_iS_j) = t_{B_1}(v_iv_j,\ i_{B2}(S_iS_j) = i_{B_1}(v_iv_j$, and $f_{B2}(S_iS_j) = f_{B_1}(v_iv_j$, for all $S_iS_j \in \Lambda$

where $A_2 = (t_{A_2}, i_{A_2}, f_{A_2})$ and $B_2 = (t_{B_2}, i_{B_2}, f_{B_2})$.

Definition 8 (see [30]). Consider $L(G) = (Z, W)$ of $G^* = (V, E)$. Then, an associated $L(G)$ of an SVNG$G = (A_1, B_1)$ is a pair $L(G) = (A_2, B_2)$ where $A_2 = (t_{A_2}, i_{A_2}, f_{A_2})$ and $B_2 = (t_{B_2}, i_{B_2}, f_{B_2})$ represent the SVNS*s* with Z and W, respectively, so that

(i) $t_{A2}(S_x) = t_{B1}(x) = t_{B1}(u_xv_x),\ i_{A2}(S_x) = i_{B1}(x) = i_{B1}(u_xv_x),\ f_{A2}(S_x) = f_{B1}(x) = f_{B1}(u_xv_x)$, for all $S_x \in Z$

(ii) $t_{B2}(S_xS_y) = t_{B1}(x) \wedge t_{B1}(y),\ i_{B2}(S_xS_y) = i_{B1}(x) \wedge i_{B1}(y),\ t_{B2}(S_xS_y) = t_{B1}(x) \vee t_{B1}(y)$, for all $S_xS_y \in W$

Definition 9 (see [28]). Suppose there are two SVNGs $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$, then the mapping $\varphi: V_1 \longrightarrow V_2$ is a homomorphism of $\varphi: G_1 \longrightarrow G_2$ such that

$$\begin{aligned} t_{A1}(u_i) &\leq t_{A2}(\varphi(u_i)), \\ i_{A1}(u_i) &\leq i_{A2}(\varphi(u_i)), \\ f_{A1}(u_i) &\leq f_2(\varphi(u_i)), \end{aligned} \quad (3)$$

and also,

$$\begin{aligned} t_{B1}(u_iu_j) &\leq t_{B2}(\varphi(u_i)\varphi(u_j)), \\ i_{B1}(u_iu_j) &\leq i_{B2}(\varphi(u_i)\varphi(u_j)), \\ f_{B1}(u_iu_j) &\leq f_{B2}(\varphi(u_i)\varphi(u_j)), \end{aligned} \quad (4)$$

where $u_i \in V_1, u_iu_j \in E_1$.

Definition 10 (see [22]). Let $X \neq \emptyset$ set of points (objects) and $A \subseteq X$. Then, we define an *IVNS* of A as follows:

$$A = \{[t_A^-(u), t_A^+(u)], [i_A^-(u), i_A^+(u)], [f_A^-(u), f_A^+(u)]: u \in X\}, \quad (5)$$

where $t_A^-(u)$, $t_A^+(u)$,$i_A^-(u)$, $i_A^+(u)$, $f_A^-(u)$, and $f_A^+(u)$ are subsets of X with $t_A^-(u) \leq t_A^+(u)$,$i_A^-(u) \leq i_A^+(u)$, and $f_A^-(u) \leq f_A^+(u)$ for all $u \in X$.

Throughout this article, we used the modified definition of IVNG which is introduced by Mohammed Akram and Nasir [22].

Definition 11. Let $X \neq \emptyset$ be a set of objects, then $G = (A, B)$ is said to be IVNG, where $A = ([t_A^-(u), t_A^+(u)], [i_A^-(u), i_A^+(u)], [f_A^-(u), f_A^+(u)])$ and $B = ([t_B^-(u), t_B^+(u)], [i_B^-(u), i_B^+(u)], [f_B^-(u), f_B^+(u)])$ are an IVN relation, which satisfies the following conditions:

$$\begin{aligned} &t_B^-(u_iu_j) \leq t_A^-(u_i) \wedge t_A^-(u_j), t_B^+(u_iu_j) \leq t_A^+(u_i) \wedge t_A^+(u_j), \\ &i_B^-(u_iu_j) \leq i_A^-(u_i) \wedge i_A^-(u_j), i_B^+(u_iu_j) \leq i_A^+(u_i) \wedge i_A^+(u_j), \\ &f_B^-(u_iu_j) \leq f(u_i) \vee f_A^-(u_j), f_B^+(u_iu_j) \leq f_A^+(u_i) \vee f_A^+(u_j), \end{aligned} \quad (6)$$

where $0 \leq t_B^+(u_iu_j) + i_B^+(u_iu_j) + ft_B^+(u_iu_j) \leq 3$ for all $u_iu_j \in E$.

Definition 12. An IVNG$G = (A, B)$ with no parallel edges or self-loops is called simple IVNG.

Definition 13. Given an IVNG $G = (A, B)$, then, for $u_i \in V$, the degree u is denoted by $d(u_i)$ and given by

$$d(u_i) = ([d_t^-(u_i), d_t^+(u_i)], [d_i^-(u_i), d_i^+(u_i)], [d_f^-(u_i), d_f^+(u_i)]), \quad (7)$$

where

$$\begin{aligned}
\left[d_t^-(u_i), d_t^+(u_i)\right] &= \left[\sum_{u_i \neq u_j} t_B^-(u_i, u_j), \sum_{u_i \neq u_j} t_B^+(u_i, u_j)\right],\\
\left[d_i^-(u_i), d_i^+(u_i)\right] &= \left[\sum_{u_i \neq u_j} i_B^-(u_i, u_j), \sum_{u_i \neq u_j} i_B^+(u_i, u_j)\right],\\
\left[d_f^-(u_i), d_f^+(u_i)\right] &= \left[\sum_{u_i \neq u_j} [zwj] f_B^-(u_i, u_j), \sum_{u_i \neq u_j} [zwj] f_B^+(u_i, u_j)\right], \text{for all } u_i, u_j \in V.
\end{aligned} \tag{8}$$

Definition 14 (see [38]). An IVNG G is strong IVNG if and only if all of the following holds:

$$\begin{aligned}
&t_B^-(u_iu_j) = t_A^-(u_i) \wedge t_A^-(u_j), t_B^+(u_iu_j) = t_A^+(u_i) \wedge t_A^+(u_j),\\
&i_B^-(u_iu_j) = i_A^-(u_i) \wedge i_A^-(u_j), i_B^+(u_iu_j) = i_A^+(u_i) \wedge i_A^+(u_j),\\
&f_B^-(u_iu_j) = f_A^-(u_i) \vee f_A^-(u_j) \text{ and } f_B^+(u_iu_j) = f_A^+(u_i) \vee f_A^+(u_j), \quad \forall u_iu_j \in E.
\end{aligned} \tag{9}$$

Definition 15. An IVNG G is complete IVNG if a graph G satisfies the following properties:

$$\begin{aligned}
&t_B^-(u_iu_j) = t_A^-(u_i) \wedge t_A^-(u_j), t_B^+(u_iu_j) = t_A^+(u_i) \wedge t_A^+(u_j),\\
&i_B^-(u_iu_j) = i_A^-(u_i) \wedge i_A^-(u_j), i_B^+(u_iu_j) = i_A^+(u_i) \wedge i_A^+(u_j),\\
&f_B^-(u_iu_j) = f_A^-(u_i) \vee f_A^-(u_j) \text{ and } f_B^+(u_iu_j) = f_A^+(u_i) \vee f_A^+(u_i), \quad \forall u_i, u_j \in V.
\end{aligned} \tag{10}$$

Thus, the neighborhood of the vertex u_i in IVNG is denoted by $N(u_i)$, and it is defined by

$$N(u_i) = \{u_j \in V\colon u_j \text{ is a neighbor of } u_i\}. \tag{11}$$

Definition 16. An edge $e = (v_i, v_j)$ of an IVNG G is said to be an effective edge if $t_B^-(u_i, u_j) = t_A^-(u_i) \wedge t_A^-(u_j)$, $t_B^+(u_i, u_j) = t_A^+(u_i) \wedge t_A^+(u_j)$, $i_B^-(u_i, u_j) = i_A^-(u_i) \wedge i_A^-(u_j)$, $i_B^+(u_i, u_j) = i_A^+(u_i) \wedge i_A^+(u_j)$, $f_B^-(u_i, u_j) = f_A^-(u_i) \vee f_A^-(u_j)$, and $f_B^+(u_i, u_j) = f_A^+(u_i) \vee f_A^+(u_j)$ for all $(u_i, u_j) \in E$. For instance, the vertex u_i is a neighbor of u_j, and u_j is neighbor of u_i.

3. Interval-Valued Neutrosophic Line Graph

We have here introduced an IVNLG for undirected IVNG and some mathematical properties of undirected IVNG with examples. We only considered IVNG without self-loops and parallel edges.

Definition 17. Let $P(E) = (S, \Lambda)$ be an intersection graph of $G^* = (V, E)$. Let $G = (A_1, B_1)$ be an IVNG of G^*. Then, an IVN-intersection graph $P(G) = (A_2, B_2)$ of $P(E)$ is defined as follows:

(a) A_2 and B_2 are IVNSs of S and Λ, respectively,

(b) $t_{A_2}^+(S_i) = t_{A_1}^+(v_i)$,
$i_{A_2}^-(S_i) = i_{A_1}^-(v_i)$,
$i_{A_2}^+(S_i) = i_{A_1}^+(v_i)$,
$f_{A_2}^-(S_i) = f_{A_1}^-(v_i)$,
$f_{A_2}^+(S_i) = f_{A_1}^+(v_i)$, for all $S_i, S_j \in S$.

(c) $t_{B_2}^-(S_iS_j) = t_{B_1}^-(v_iv_j)$,
$t_{B_2}^+(S_iS_j) = t_{B_1}^+(v_iv_j)$,
$i_{B_2}^-(S_iS_j) = i_{B_1}^-(v_iv_j)$,
$i_{B_2}^+(S_iS_j) = i_{B_1}^+(v_iv_j)$,
$f_{B2}^-(S_iS_j) = f_{B_1}^-(v_iv_j)$,
$f_{B2}^+(S_iS_j) = f_{B_1}^+(v_iv_j)$, for all $S_iS_j \in \Lambda$.

Therefore, any IVNG of $P(E)$ is called an IVN-intersection graph.

Definition 18. Consider the line graph $L(G^*) = (Z, W)$ of the graph $G^* = (V, E)$ and let $G = (A_1, B_1)$ be an IVNG of G^*. Then, we define an IVNLG of G as $L(G) = (A_2, B_2)$ where $A_2 = ([t_{A_2}^-, t_{A_2}^+], [i_{A_2}^-, i_{A_2}^+], [f_{A_2}^-, [f_{A_2}^+])$ is an IVNS on Z and $B = ([t_{B_2}^-, t_{B_2}^+], [i_{B_2}^-, i_{B_2}^+], [f_{B_2}^-, [f_{B_2}^+])$:

(i) The vertex of an IVNLG of G is computed as

$$t^{-}_{A_2}(S_e) = t^{-}_{B_1}(e) = t^{-}_{B_1}(u_e v_e), t^{+}_{A_2}(S_e) = t^{+}_{B_1}(e) = t^{+}_{B_1}(u_e v_e),$$
$$i^{-}_{A_2}(S_e) = i^{-}_{B_1}(e) = i^{-}_{B_1}(u_e v_e), i^{+}_{A_2}(S_e) = i^{+}_{B_1}(e) = i^{+}_{B_1}(u_e v_e), \quad (12)$$
$$f^{-}_{A_2}(S_e) = f^{-}_{B_1}(e) = f^{-}_{B_1}(u_e v_e), f^{+}_{A_2}(S_e) = f^{+}_{B_1}(e) = f^{+}_{B_1}(u_e v_e), \quad \forall S_e \in Z.$$

(ii) The edge of an IVNLG of G is computed as

$$\begin{aligned} t^{-}_{B_2}\left(S_{e_i}S_{e_j}\right) &= t^{-}_{B_1}(e_i) \wedge t^{-}_{B_1}(e_j), \\ t^{+}_{B_2}\left(S_{e_i}S_{e_j}\right) &= t^{+}_{B_1}(e_i) \wedge t^{+}_{B_1}(e_j), \\ i^{-}_{B_2}\left(S_{e_i}S_{e_j}\right) &= i^{-}_{B_1}(e_i) \wedge i^{-}_{B_1}(e_j), \\ i^{+}_{B_2}\left(S_{e_i}S_{e_j}\right) &= i^{+}_{B_1}(e_i) \wedge i^{+}_{B_1}(e_j), \\ f^{-}_{B_2}\left(S_{e_i}S_{e_j}\right) &= f^{-}_{B_1}(e_i) \vee f^{-}_{B_1}(e_j), \\ f^{+}_{B_2}\left(S_{e_i}S_{e_j}\right) &= f^{+}_{B_1}(e_i) \vee f^{+}_{B_1}(e_j), \text{for all } S_{e_i}S_{e_j} \in W. \end{aligned} \quad (13)$$

Example 1. Consider an IVNG $G = (A_1, B_1)$ as shown in Figure 1 where the vector set of a graph G is $A_1 = \{u_1, u_2, u_3, u_4, u_5\}$ and $B_1 = \{u_1u_2, u_1u_5, u_2u_3, u_2u_4, u_2u_5, u_3u_5, u_4u_5\}$ is the edge set on A_1, as shown in Tables 1 and 2.

From above Figure 1, we can drive a line graph as follows: Consider (S, Λ) is the intersection graph of G, that is, $S = \{S_{e_1} = v_1v_2, S_{e_2} = v_1v_5, S_{e_3} = v_2v_5, S_{e_4} = v_2v_3, S_{e_5} = v_2v_4, S_{e_6} = v_3v_5, S_{e_7} = v_4v_5\}$. Then, by the definition of the line graph, $L(G) = (A_2, B_2)$ can be obtained by routine computation; the vertex set of IVNLG G is as follows:

$$\begin{aligned}
t_{A_2}\left(S_{e_1}\right) &= \left[t^{-}_{B_1}(e_1), t^{+}_{B_1}(e_1)\right] = [0.1, 0.2], \\
t_{A_2}\left(S_{e_2}\right) &= \left[t^{-}_{B_1}(e_2), t^{+}_{B_1}(e_2)\right] = [0.1, 0.6], \\
t_{A_2}\left(S_{e_3}\right) &= \left[t^{-}_{B_1}(e_3), t^{+}_{B_1}(e_3)\right] = [0.1, 0.2], \\
t_{A_2}\left(S_{e_4}\right) &= \left[t^{-}_{B_1}(e_4), t^{+}_{B_1}(e_4)\right] = [0.1, 0.2], \\
t_{A_2}\left(S_{e_5}\right) &= \left[t^{-}_{B_1}(e_5), t^{+}_{B_1}(e_5)\right] = [0.0, 0.2], \\
t_{A_2}\left(S_{e_6}\right) &= \left[t^{-}_{B_1}(e_6), t^{+}_{B_1}(e_6)\right] = [0.1, 0.3], \\
t_{A_2}\left(S_{e_7}\right) &= \left[t^{-}_{B_1}(e_7), t^{+}_{B_1}(e_7)\right] = [0.1, 0.4], \\
i_{A_2}\left(S_{e_1}\right) &= \left[i^{-}_{B_1}(e_1), i^{+}_{B_1}(e_1)\right] = [0.2, 0.3], \\
i_{A_2}\left(S_{e_2}\right) &= \left[i^{-}_{B_1}(e_2), i^{+}_{B_1}(e_2)\right] = [0.2, 0.3], \\
i_{A_2}\left(S_{e_3}\right) &= \left[i^{-}_{B_1}(e_3), i^{+}_{B_1}(e_3)\right] = [0.3, 0.5], \\
i_{A_2}\left(S_{e_4}\right) &= \left[i^{-}_{B_1}(e_4), i^{+}_{B_1}(e_4)\right] = [0.0, 0.5], \\
i_{A_2}\left(S_{e_5}\right) &= \left[i^{-}_{B_1}(e_5), i^{+}_{B_1}(e_5)\right] = [0.3, 0.7], \\
i_{A_2}\left(S_{e_6}\right) &= \left[i^{-}_{B_1}(e_6), i^{+}_{B_1}(e_6)\right] = [0.0, 0.5], \\
i_{A_2}\left(S_{e_7}\right) &= \left[i^{-}_{B_1}(e_7), i^{+}_{B_1}(e_7)\right] = [0.4, 0.5], \\
f_{A_2}\left(S_{e_1}\right) &= \left[f^{-}_{B_1}(e_1), f^{+}_{B_1}(e_1)\right] = [0.1, 0.2], \\
f_{A_2}\left(S_{e_2}\right) &= \left[f^{-}_{B_1}(e_2), f^{+}_{B_1}(e_2)\right] = [0.1, 0.6], \\
f_{A_2}\left(S_{e_3}\right) &= \left[f^{-}_{B_1}(e_3), f^{+}_{B_1}(e_3)\right] = [0.1, 0.2], \\
f_{A_2}\left(S_{e_4}\right) &= \left[f^{-}_{B_1}(e_4), f^{+}_{B_1}(e_4)\right] = [0.1, 0.2], \\
f_{A_2}\left(S_{e_5}\right) &= \left[f^{-}_{B_1}(e_5), f^{+}_{B_1}(e_5)\right] = [0.0, 0.2], \\
f_{A_2}\left(S_{e_6}\right) &= \left[f^{-}_{B_1}(e_6), f^{+}_{B_1}(e_6)\right] = [0.1, 0.3], \\
f_{A_2}\left(S_{e_7}\right) &= \left[f^{-}_{B_1}(e_7), f^{+}_{B_1}(e_7)\right] = [0.1, 0.4].
\end{aligned} \quad (14)$$

Using the definition of the line graph, an edge set of IVNLG G is as follows: so that an IVNLG G is shown in Figure 2.

$$
\begin{aligned}
t_{B_2}\left(S_{e_1}S_{e_2}\right) &= [t^-_{B1}(e_1) \wedge t^-_{B1}(e_2), t^+_{B1}(e_1) \wedge t^+_{B1}(e_2)] = [0.1, 0.2],\\
t_{B_2}\left(S_{e_1}S_{e_3}\right) &= [t^-_{B1}(e_1) \wedge t^-_{B1}(e_3), t^+_{B1}(e_1) \wedge t^+_{B1}(e_3)] = [0.1, 0.2],\\
t_{B_2}\left(S_{e_1}S_{e_5}\right) &= [t^-_{B1}(e_1) \wedge t^-_{B1}(e_5), t^+_{B1}(e_1) \wedge t^+_{B1}(e_5)] = [0.0, 0.2],\\
t_{B_2}\left(S_{e_1}S_{e_4}\right) &= [t^-_{B1}(e_1) \wedge t^-_{B1}(e_4), t^+_{B1}(e_1) \wedge t^+_{B1}(e_4)] = [0.1, 0.2],\\
t_{B_2}\left(S_{e_2}S_{e_3}\right) &= [t^-_{B1}(e_2) \wedge t^-_{B1}(e_3), t^+_{B1}(e_2) \wedge t^+_{B1}(e_3)] = [0.1, 0.2],\\
t_{B_2}\left(S_{e_2}S_{e_6}\right) &= [t^-_{B1}(e_2) \wedge t^-_{B1}(e_6), t^+_{B1}(e_2) \wedge t^+_{B1}(e_6)] = [0.1, 0.2],\\
t_{B_2}\left(S_{e_2}S_{e_7}\right) &= [t^-_{B1}(e_2) \wedge t^-_{B1}(e_7), t^+_{B1}(e_2) \wedge t^+_{B1}(e_7)] = [0.1, 0.4],\\
t_{B_2}\left(S_{e_3}S_{e_4}\right) &= [t^-_{B1}(e_3) \wedge t^-_{B1}(e_4), t^+_{B1}(e_3) \wedge t^+_{B1}(e_4)] = [0.1, 0.2],\\
t_{B_2}\left(S_{e_3}S_{e_5}\right) &= [t^-_{B1}(e_3) \wedge t^-_{B1}(e_5), t^+_{B1}(e_3) \wedge t^+_{B1}(e_5)] = [0.0, 0.2],\\
t_{B_2}\left(S_{e_3}S_{e_6}\right) &= [t^-_{B1}(e_3) \wedge t^-_{B1}(e_6), t^+_{B1}(e_3) \wedge t^+_{B1}(e_6)] = [0.1, 0.2],\\
t_{B_2}\left(S_{e_3}S_{e_7}\right) &= [t^-_{B1}(e_3) \wedge t^-_{B1}(e_7), t^+_{B1}(e_3) \wedge t^+_{B1}(e_7)] = [0.1, 0.2],\\
t_{B_2}\left(S_{e_4}S_{e_5}\right) &= [t^-_{B1}(e_4) \wedge t^-_{B1}(e_5), t^+_{B1}(e_4) \wedge t^+_{B1}(e_5)] = [0.1, 0.2],\\
t_{B_2}\left(S_{e_6}S_{e_7}\right) &= [t^-_{B1}(e_6) \wedge t^-_{B1}(e_7), t^+_{B1}(e_6) \wedge t^+_{B1}(e_7)] = [0.1, 0.3],\\
t_{B_2}\left(S_{e_5}S_{e_7}\right) &= [t^-_{B1}(e_5) \wedge t^-_{B1}(e_7), t^+_{B1}(e_5) \wedge t^+_{B1}(e_7)] = [0.0, 0.2].\\
i_{B_2}\left(S_{e_1}S_{e_2}\right) &= [i^+_{B1}(e_1) \wedge i^+_{B1}(e_2), i^+_{B1}(e_1) \wedge i^+_{B1}(e_2)] = [0.2, 0.3],\\
i_{B_2}\left(S_{e_1}S_{e_2}\right) &= [i^+_{B1}(e_1) \wedge i^+_{B1}(e_2), i^+_{B1}(e_1) \wedge i^+_{B1}(e_2)] = [0.2, 0.3],\\
i_{B_2}\left(S_{e_1}S_{e_2}\right) &= [i^+_{B1}(e_1) \wedge i^+_{B1}(e_2), i^+_{B1}(e_1) \wedge i^+_{B1}(e_2)] = [0.2, 0.3],\\
i_{B_2}\left(S_{e_1}S_{e_3}\right) &= [i^+_{B1}(e_1) \wedge i^+_{B1}(e_3), i^+_{B1}(e_1) \wedge i^+_{B1}(e_3)] = [0.2, 0.3],\\
\left(S_{e_1}S_{e_4}\right) &= [i^+_{B1}(e_1) \wedge i^+_{B1}(e_4), i^+_{B1}(e_1) \wedge i^+_{B1}(e_4)] = [0.0, 0.3],\\
i_{B_2}\left(S_{e_1}S_{e_5}\right) &= [i^+_{B1}(e_1) \wedge i^+_{B1}(e_5), i^+_{B1}(e_1) \wedge i^+_{B1}(e_5)] = [0.2, 0.3],\\
i_{B_2}\left(S_{e_2}S_{e_3}\right) &= [i^+_{B1}(e_2) \wedge i^+_{B1}(e_3), i^+_{B1}(e_2) \wedge i^+_{B1}(e_3)] = [0.2, 0.3],\\
i_{B_2}\left(S_{e_2}S_{e_6}\right) &= [i^+_{B1}(e_2) \wedge i^+_{B1}(e_6), i^+_{B1}(e_2) \wedge i^+_{B1}(e_6)] = [0.0, 0.3],\\
i_{B_2}\left(S_{e_2}S_{e_7}\right) &= [i^+_{B1}(e_2) \wedge i^+_{B1}(e_7), i^+_{B1}(e_2) \wedge i^+_{B1}(e_7)] = [0.2, 0.3],\\
i_{B_2}\left(S_{e_3}S_{e_4}\right) &= [i^+_{B1}(e_3) \wedge i^+_{B1}(e_4), i^+_{B1}(e_3) \wedge i^+_{B1}(e_4)] = [0.0, 0.5],\\
i_{B_2}\left(S_{e_3}S_{e_5}\right) &= [i^+_{B1}(e_3) \wedge i^+_{B1}(e_5), i^+_{B1}(e_3) \wedge i^+_{B1}(e_5)] = [0.3, 0.5],\\
i_{B_2}\left(S_{e_3}S_{e_6}\right) &= [i^+_{B1}(e_3) \wedge i^+_{B1}(e_6), i^+_{B1}(e_3) \wedge i^+_{B1}(e_6)] = [0.0, 0.5],\\
i_{B_2}\left(S_{e_3}S_{e_7}\right) &= [i^+_{B1}(e_3) \wedge i^+_{B1}(e_7), i^+_{B1}(e_3) \wedge i^+_{B1}(e_7)] = [0.3, 0.5],\\
i_{B_2}\left(S_{e_4}S_{e_5}\right) &= [i^+_{B1}(e_4) \wedge i^+_{B1}(e_5), i^+_{B1}(e_4) \wedge i^+_{B1}(e_5)] = [0.0, 0.5],\\
i_{B_2}\left(S_{e_6}S_{e_7}\right) &= [i^+_{B1}(e_6) \wedge i^+_{B1}(e_7), i^+_{B1}(e_6) \wedge i^+_{B1}(e_7)] = [0.1, 0.3],\\
i_{B_2}\left(S_{e_5}S_{e_7}\right) &= [i^+_{B1}(e_5) \wedge i^+_{B1}(e_7), i^+_{B1}(e_5) \wedge i^+_{B1}(e_7)] = [0.0, 0.5].\\
f_{B_2}\left(S_{e_1}S_{e_2}\right) &= [f^-_{B1}(e_1) \vee f^-_{B1}(e_2), f^+_{B1}(e_1) \vee f^+_{B1}(e_2)] = [0.1, 0.6]\\
f_{B_2}\left(S_{e_1}S_{e_3}\right) &= [f^-_{B1}(e_1) \vee f^-_{B1}(e_3), f^+_{B1}(e_1) \vee f^+_{B1}(e_3)] = [0.1, 0.2],\\
f_{B_2}\left(S_{e_1}S_{e_4}\right) &= [f^-_{B1}(e_1) \vee f^-_{B1}(e_4), f^+_{B1}(e_1) \vee f^+_{B1}(e_4)] = [0.1, 0.2],\\
f_{B_2}\left(S_{e_1}S_{e_5}\right) &= [f^-_{B1}(e_1) \vee f^-_{B1}(e_5), f^+_{B1}(e_1) \vee f^+_{B1}(e_5)] = [0.1, 0.2],\\
f_{B_2}\left(S_{e_2}S_{e_3}\right) &= [f^-_{B1}(e_2) \vee f^-_{B1}(e_3), f^+_{B1}(e_2) \vee f^+_{B1}(e_3)] = [0.1, 0.6],\\
f_{B_2}\left(S_{e_2}S_{e_6}\right) &= [f^-_{B1}(e_2) \vee f^-_{B1}(e_6), f^+_{B1}(e_2) \vee f^+_{B1}(e_6)] = [0.1, 0.6],\\
f_{B_2}\left(S_{e_2}S_{e_7}\right) &= [f^-_{B1}(e_2) \vee f^-_{B1}(e_7), f^+_{B1}(e_2) \vee f^+_{B1}(e_7)] = [0.1, 0.6],\\
f_{B_2}\left(S_{e_3}S_{e_4}\right) &= [f^-_{B1}(e_3) \vee f^-_{B1}(e_4), f^+_{B1}(e_3) \vee f^+_{B1}(e_4)] = [0.1, 0.2],\\
f_{B_2}\left(S_{e_3}S_{e_5}\right) &= [f^-_{B1}(e_3) \vee f^-_{B1}(e_5), f^+_{B1}(e_3) \vee f^+_{B1}(e_5)] = [0.1, 0.2],\\
f_{B_2}\left(S_{e_3}S_{e_6}\right) &= [f^-_{B1}(e_3) \vee f^-_{B1}(e_6), f^+_{B1}(e_3) \vee f^+_{B1}(e_6)] = [0.1, 0.3],\\
f_{B_2}\left(S_{e_3}S_{e_7}\right) &= [f^-_{B1}(e_3) \vee f^-_{B1}(e_7), f^+_{B1}(e_3) \vee f^+_{B1}(e_7)] = [0.1, 0.4],\\
f_{B_2}\left(S_{e_4}S_{e_5}\right) &= [f^-_{B1}(e_4) \vee f^-_{B1}(e_5), f^+_{B1}(e_4) \vee f^+_{B1}(e_5)] = [0.0, 0.5],\\
f_{B_2}\left(S_{e_6}S_{e_7}\right) &= [f^-_{B1}(e_6) \vee f^-_{B1}(e_7), f^+_{B1}(e_6) \vee f^+_{B1}(e_7)] = [0.1, 0.3],\\
f_{B_2}\left(S_{e_5}S_{e_7}\right) &= [f^-_{B1}(e_5) \vee f^-_{B1}(e_7), f^+_{B1}(e_5) \vee f^+_{B1}(e_7)] = [0.0, 0.5].
\end{aligned}
\tag{15}
$$

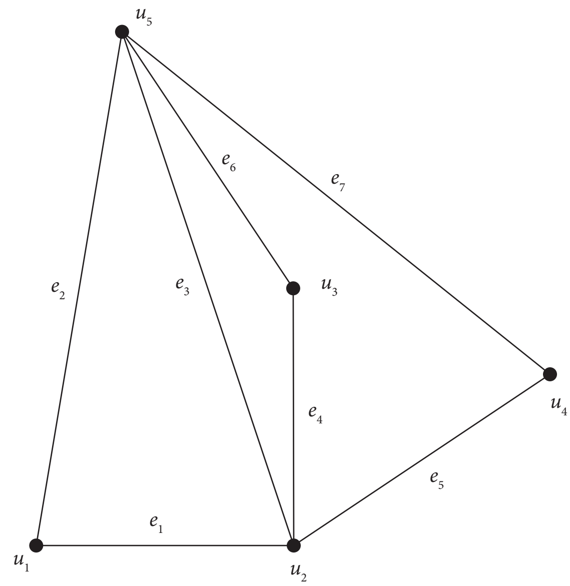

Figure 1: IVNG G.

Table 1: The vertices of IVNG G.

	u_1	u_2	u_3	u_4	u_5
$[t^-_{A_1}, t^+_{A_1}]$	[0.5, 0.6]	[0.1, 0.2]	[0.3, 0.4]	[0.2, 0.4]	[0.1, 0.8]
$[i^-_{A_1}, i^+_{A_1}]$	[0.2, 0.3]	[0.3, 0.7]	[0.0, 0.5]	[0.6, 0.7]	[0.4, 0.5]
$[f^-_{A_1}, f^+_{A_1}]$	[0.1, 0.3]	[0.4, 0.7]	[0.2, 0.5]	[0.1, 0.2]	[0.0, 0.5]

Table 2: The edges of IVNG G.

	u_1u_2	u_1u_5	u_2u_5	u_2u_3	u_2u_4	u_3u_5	u_4u_5
$[t^-_{B_1}, t^+_{B_1}]$	[0.1, 0.2]	[0.1, 0.6]	[0.1, 0.2]	[0.1, 0.2]	[0.0, 0.2]	[0.1, 0.3]	[0.1, 0.4]
$[i^-_{B_1}, i^+_{B_1}]$	[0.2, 0.3]	[0.2, 0.3]	[0.3, 0.5]	[0.0, 0.5]	[0.3, 0.7]	[0.0, 0.5]	[0.4, 0.5]
$[f^-_{B_1}, f^+_{B_1}]$	[0.3, 0.4]	[0.2, 0.5]	[0.3, 0.5]	[0.2, 0.4]	[0.2, 0.6]	[0.2, 0.5]	[0.1, 0.5]

Definition 19. An IVNG $K = (V', E')$ is said to be the subgraph of IVNG $G = (V, E)$ if $V' \subseteq V$ and $E' \subseteq E$. Accordingly,

$$t_A(u) == t'_A(u), i_A(u) = i'_A(u), f_A(u) = f'_A(u), \forall u \in V',$$
$$t_B(e'_ie'_j) = t'_B(e'_ie'_j), i_B(e'_ie'_j) = i'_B(e'_ie'_j), f_B(e'_ie'_j) = f'_B(e'_ie'_j), \forall e'_ie'_j \in E'. \quad (16)$$

Proposition 20. *Let $K = (V', E')$ be the subgraph of IVNG of $G = (V, E)$. Then, $L(K)$ is the subgraph of an IVNLG $L(G)$.*

Proof. Suppose that K is a subgraph of IVNG G. Then, we have $V' \subseteq V$ and $E' \subseteq E$. We know that $V(L(K)) \subseteq E(G)$, and it is also the subset of $V(L(G))$. Moreover, the edge set of the line graph $L(K)$ is a subset of the edge set of IVNLG G. Hence, $L(K) \subseteq L(G)$. □

Definition 21. (see [22]). A homomorphism mapping ψ: $K_1 \longrightarrow K_2$ of two IVNGs where $K_1 = (M_1, N_1)$ and $K_2 = (M_2, N_2)$ are the map ψ: $V_1 \longrightarrow V_2$ is defined as

(i) $t^-_{M_1}(v_i) \leq t^-_{M_2}(\psi(v_i)), t^+_{M_1}(v_i) \leq t^+_{M_2}(\psi(v_i))$, $i^-_{M_1}(v_i) \leq i^-_{M_2}(\psi(v_i)), i^+_{M_1}(v_i) \leq i^+_{M_2}(\psi(v_i))$, $f^-_{M_1}(v_i) \leq f^-_{M_2}(\psi(v_i))$, and $f^+_{M_1}(v_i) \leq f^+_{M_2}(\psi(v_i))$, for all $v_i \in V_1$.

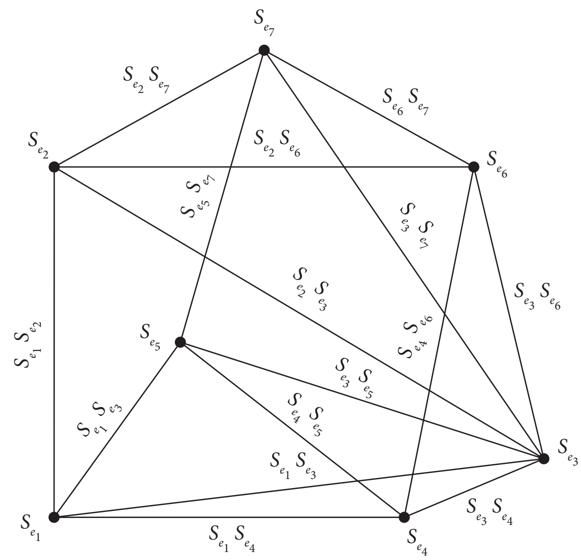

Figure 2: IVNLG of G.

(ii) $t^-_{N_1}(v_iv_j) \le t^-_{N_2}(\psi(v_i)\psi(v_j))$, $t^+_{N_1}(v_iv_j) \le t^+_{N_2}(\psi(v_i)\psi(v_j))$,
$i^-_{N_1}(v_iv_j) \le i^-_{N_2}(\psi(v_i)\psi(v_j))$, $i^+_{N_1}(v_iv_j) \le i^+_{N_2}(\psi(v_i)\psi(v_j))$,
$f^-_{N_1}(v_iv_j) \le f^-_{N_2}(\psi(v_i)\psi(v_j))$ and $f^+_{N_1}(v_iv_j) \le f^+_{N_2}(\psi(v_i)\psi(v_j))$, $\forall v_iv_j \in E_1$.

Definition 22. Let $K_1 = (M_1, N_1)$ and $K_2 = (M_2, N_2)$ are two IVNGs. The weak vertex isomorphism is a bijective homomorphism $\psi: K_1 \longrightarrow K_2$ such that

$$\begin{aligned} t_{M_1}(v_i) &= [t^-_{M_1}(v_i), t^+_{M_1}(v_i)] = [t^-_{M_2}(\psi(v_i)), t^+_{M_2}(\psi(v_i))],\\ i_{M_1}(v_i) &= [i^-_{M_1}(v_i), i^+_{M_1}(v_i)] = [i^-_{M_2}(\psi(v_i)), i^+_{M_2}(\psi(v_i))],\\ f_{N_1}(v_i) &= [f^-_{N_1}(v_i), f^+_{N_1}(v_i)] = [f^-_{N_2}(\psi(v_i)), f^+_{N_2}(\psi(v_i))]\ \forall v_i \in M_1. \end{aligned} \tag{17}$$

The weak line (edge) isomorphism of IVNG is a bijective homomorphism $\psi: K_1 \longrightarrow K_2$ if the following conditions hold:

$$\begin{aligned} t_{N_1}(v_iv_j) &= [t^-_{N_1}(v_iv_j), t^+_{N_1}(v_iv_j)] = [t^-_{M_2}(\psi(v_i)\psi(v_j)), t^+_{M_2}(\psi(v_i)\psi(v_j))],\\ i_{N_1}(v_iv_j) &= [i^-_{N_1}(v_iv_j), i^+_{N_1}(v_iv_j)] = [i^-_{M_2}(\psi(v_i)\psi(v_j)), i^+_{M_2}(\psi(v_i)\psi(v_j))],\\ f_{N_1}(v_iv_j) &= [f^-_{N_1}(v_iv_j), f^+_{N_1}(v_iv_j)] = [f^-_{M_2}(\psi(v_i)\psi(v_j)), f^+_{M_2}(\psi(v_i)\psi(v_j))]\ \forall v_iv_j \in N_1. \end{aligned} \tag{18}$$

Definition 23. If the mapping $\psi: K_1 \longrightarrow K_2$ is a bijective weak vertex and weak edge isomorphism, then we say that ψ is a weak isomorphism map of IVNGs from K_1 to K_2.

Definition 24 (see [17]). A path P in undirected IVNG $G = (A, B)$ is a sequence of distinct nodes $u_0u_1u_2 \cdots u_n$ such that $t^-_B(u_{i-1}, u_i) > 0, t^+_B(u_{i-1}, u_i) > 0, i^-_B(u_{i-1}, u_i) > 0, i^+_B(u_{i-1}, u_i) > 0, f^-_B(u_{i-1}, u_i) > 0$, and $f^+_B(u_{i-1}, u_i) > 0$, for $0 \le i \le n$. If P is a path with n-vertices, then the length of P is $n - 1$. A single node u_i may also be taken as a path with length ([0, 0], [0, 0], [0, 0]). The edges of the path are successive pairs (u_{i-1}, u_i). If $n \ge 3$ and $u_0 = u_n$, then P is referred to as a cycle.

Definition 25. If there is at least one path between each pair of nodes in an IVNG $G = (A, B)$, then IVNG G is said to be connected; otherwise, it is disconnected.

Proposition 26. *The IVNLG L(G) is connected if its original graph IVNG G is a connected graph.*

Proof. Given G is IVNG and $L(G)$ is a connected IVNLG of G, we must demonstrate that precondition. Assume G is not connected IVNG. Then, G has at least two nodes that are not connected by a path. If we choose one edge from the first component, there are no edges that are adjacent to edges in other components of G. The $L(G)$ of G is then broken and contradicting, so that G is connected. Conversely, assume that G is not a disconnected IVNG. Then, there is a path that connects each pair of nodes. Adjacent edges in G are thus neighboring nodes in $L(G)$, according to the definition of $L(G)$. As a result, each pair of nodes in $L(G)$ has a path that connects them. Hence, the proof holds. □

Proposition 27. *Consider $G^* = (V, E)$ with underlying set V and $L(G)$ is an IVNLG of G. Then, $L(G^*)$ is a line graph of G^*.*

Proof. Given an IVNG $G = (A_1, B_1)$ of G^* and $L(G) = (A_2, B_2)$ is IVNLG of $L(G^*)$, then

$$\begin{aligned} t_{A_2}(S_x) &= \left[t^-_{A_2}(S_x), t^+_{A_2}(S_x)\right] = \left[t^-_{B_1}(x), t^+_{B_1}(x)\right],\\ i_{A_2}(S_x) &= \left[i^-_{A_2}(S_x), i^+_{A_2}(S_x)\right] = \left[i^-_{B_1}(x), i^+_{B_1}(x)\right],\\ f_{A_2}(S_x) &= \left[f^-_{A_2}(S_x), f^+_{A_2}(S_x)\right] = \left[f^-_{B_1}(x), f^+_{B_1}(x)\right], \forall x \in E. \end{aligned} \tag{19}$$

This implies $S_x \in Z$ if an edge $x \in E$.

$$\begin{aligned} t_{B_2}(S_xS_y) &= \left[t^-_{B_2}(S_xS_y), t^+_{B_2}(S_xS_y)\right] = \left[t^-_{B_1}(x) \wedge t^-_{B_1}(y), t^+_{B_1}(x) \wedge t^+_{B_1}(y)\right],\\ i_{B_2}(S_xS_y) &= \left[i^-_{B_2}(S_xS_y), i^+_{B_2}(S_xS_y)\right] = \left[i^-_{B_1}(x) \wedge i^-_{B_1}(y), i^+_{B_1}(x) \wedge i^+_{B_1}(y)\right],\\ f_{B_2}(S_xS_y) &= \left[f^-_{B_2}(S_xS_y), f^+_{B_2}(S_xS_y)\right] = \left[f^-_{B_1}(x) \vee f^-_{B_1}(y), f^+_{B_1}(x) \vee f^+_{B_1}(y)\right], \forall S_xS_y \in W, \end{aligned} \tag{20}$$

where $L(G^*) = (Z, W)$. Consequently, $L(G^*)$ is a line graph of a graph G^*. □

Proposition 28. *Assume that $L(G) = (A_2, B_2)$ is the IVNLG for $L(G^*)$. Then, $L(G)$ is an IVNLG of certain IVNG of $G = (A_1, B_1)$ if*

$$\begin{aligned} t_{B_2}(S_xS_y) &= \left[t^-_{B_2}(S_xS_y), t^+_{B_2}(S_xS_y)\right] = \left[t^-_{A_2}(S_x) \wedge t^-_{A_2}(S_y), t^+_{A_2}(S_x) \wedge t^+_{A_2}(S_y)\right],\\ i_{B_2}(S_xS_y) &= \left[i^-_{B_2}(S_xS_y), i^+_{B_2}(S_xS_y)\right] = \left[i^-_{A_2}(S_x) \wedge i^-_{A_2}(S_y), i^+_{A_2}(S_x) \wedge i^+_{A_2}(S_y)\right],\\ f_{B_2}(S_xS_y) &= \left[f^-_{B_2}(S_xS_y), f^+_{B_2}(S_xS_y)\right] = \left[f^-_{A_2}(S_x) \vee f^-_{A_2}(S_y), f^+_{A_2}(S_x) \vee f^+_{A_2}(S_y)\right],\\ &\forall S_x, S_y \in Z, S_xS_y \in W. \end{aligned} \tag{21}$$

Proof. Suppose (i), (ii), and (iii) are true. That implies

$$\begin{aligned} &t^-_{B_2}(S_xS_y) = t^-_{A_2}(S_x) \wedge t^-_{A_2}(S_y), t^+_{B_2}(S_xS_y) = t^+_{A_2}(S_x) \wedge t^+_{A_2}(S_y),\\ &i^-_{B_2}(S_xS_y) = i^-_{A_2}(S_x) \wedge i^-_{A_2}(S_y), i^+_{B_2}(S_xS_y) = i^+_{A_2}(S_x) \wedge i^+_{A_2}(S_y),\\ &f^-_{B_2}(S_xS_y) = f^-_{A_2}(S_x) \vee f^-_{A_2}(S_y) \text{ and } f^+_{B_2}(S_xS_y) = f^+_{A_2}(S_x) \vee f^+_{A_2}(S_y), \forall S_xS_y \in W. \end{aligned} \tag{22}$$

For each $x \in E$, we have

$$\begin{aligned}&t^-_{A_2}(S_x)=t^-_{A_1}(x), t^+_{A_2}(S_x)=t^+_{A_1}(x),\\&i^-_{A_2}(S_x)=i^-_{A_1}(x), i^+_{A_2}(S_x)=i^+_{A_1}(x),\\&f^-_{A_2}(S_x)=f^-_{A_1}(x) \text{ and } f^+_{A_2}(S_x)=f^+_{A_1}(x).\end{aligned} \tag{23}$$

Now, from (i), (ii), and (iii), we have

$$\begin{aligned}t^-_{B_2}(S_xS_y)&=\left[t^-_{B_2}(S_xS_y), t^+_{B_2}(S_xS_y)\right]\\&=\left[t^-_{A_2}(S_x)\wedge t^-_{A_2}(S_y), t^+_{A_2}(S_x)\wedge t^+_{A_2}(S_y)\right]\\&=\left[t^-_{B_1}(x)\wedge t^-_{B_1}(y), t^+_{B_1}(x)\wedge t^+_{B_1}(y)\right],\\i^-_{B_2}(S_xS_y)&=\left[i^-_{B_2}(S_xS_y), i^+_{B_2}(S_xS_y)\right]\\&=\left[i^-_{A_2}(S_x)\wedge i^-_{A_2}(S_y), i^+_{A_2}(S_x)\wedge i^+_{A_2}(S_y)\right]\\&=\left[i^-_{B_1}(x)\wedge i^-_{B_1}(y), i^+_{B_1}(x)\wedge i^+_{B_1}(y)\right],\\f^-_{B_2}(S_xS_y)&=\left[f^-_{B_2}(S_xS_y), f^+_{B_2}(S_xS_y)\right]\\&=\left[f^-_{A_2}(S_x)\vee f^-_{A_2}(S_y), f^+_{A_2}(S_x)\vee f^+_{A_2}(S_y)\right]\\&=\left[f^-_{B_1}(x)\vee f^-_{B_1}(y), f^+_{B_1}(x)\vee f^+_{B_1}(y)\right].\end{aligned} \tag{24}$$

We know that IVNS $A_1=([t^-_{A_1}, t^+_{A_1}], [i^-_{A_1}, i^+_{A_1}], [f^-_{A_1}, f^+_{A_1}])$ gives that

$$\begin{aligned}&t^-_{B_1}(v_iv_j)\le t^-_{A_1}(v_i)\wedge t^-_{A_1}(v_j). \quad t^+_{B_1}(v_iv_j)\le t^v_{A_1}(v_i)\wedge t^+_{A_1}(v_j).\\&i^-_{B_1}(v_iv_j)\le i^-_{A_1}(v_i)\wedge i^-_{A_1}(v_j). \quad i^+_{B_1}(v_iv_j)\le i^+_{A_1}(v_i)\wedge i^+_{A_1}(v_j).\\&f^-_{B_1}(v_iv_j)\le f^-_{A_1}(v_i)\vee f^-_{A_1}(v_j). \quad f^+_{B_1}(v_iv_j)\le f^+_{A_1}(v_i)\vee f^+_{A_1}(v_j),\end{aligned} \tag{25}$$

which is sufficient. The converse of this statement is obvious from the definition of IVNLG, and hence, the proof holds. □

Proposition 29. *Let G be IVNG, then its corresponding IVNLG is strong.*

Proof. It is omitted because it is obvious from the definition. □

Proposition 30. *An IVNLG is the generalization of IVIFLG.*

Proof. Assume that $L(G)=(A_2, B_2)$ is an IVNLG of $G=(V, E)$. Then, by setting the IMF and FMF values of each vertex and each edge zero, an IVNLG is transformed into the IVFLG. Therefore, the proof is completed. □

Theorem 31. *Let G be a connected IVNG path graph. Then, an IVNLG $L(G)$ is a connected path graph.*

Proof. Consider a path G is connected IVN-path graph with $|V(G)|=k$. This implies that $|E(G)|=k-1$ and that G is a P_k path graph. Since the vertex set of an IVNLG G is the same with edge set of IVNG G, it is obvious that IVNLG G is a path graph with $(k-1)$ vertices and $(k-2)$ edges, so that, $L(G)$ is a connected path graph. Again, consider $L(G)$ is a connected path graph. It implies that $d(v_i)$ can not be greater than two $\forall v_i\in L(G)$, where $d(v_i)$ is the degree of vertex v_i. Since $V(L(G))=E(G)$, every edge of G has exactly two degrees. As a result, $d(v_i)\le 2$ for every $v_i\in V(G)$, and the proof is now completed. □

4. Properties of Interval-Valued Neutrosophic Line Graphs

Definition 32. The vertex-adjacency matrix of an IVNLG G is the same as the edge-adjacency matrix of IVNG G.

Definition 33. Consider $L(G^*)=(Z, W)$ is a line graph of $G^*=(V, E)$ and $G=(A_1, B_1)$ is an IVNG. For an IVNLG $L(G)=(A_2, B_2)$ where $A_2=\{t_{A_2}, i_{A_2}, f_{A_2}\}$ and $B_2=\{t_{B_2}, i_{B_2}, f_{B_2}\}$ are IVNS on Z and W, respectively. Then, we denoted the vertex degree of $L(G)$ by $d(S_x)$ defined as$d(S_x)=([d^-_t(S_x), d^+_t(S_x)], [d^-_i(S_x), d^+_i(S_x)], [d^-_f(S_x), d^+_f(S_y)])$ where

$$
\begin{aligned}
d_t^-(S_x) &= \sum_{S_xS_y\in W} t_{B_2}^-\left(S_xS_y\right) = \sum_{x,y\in W} t_{B_1}^-(x)\wedge t_{B_1}^-(y),\\
d_t^+(S_x) &= \sum_{S_xS_y\in W} [zwj]t_{B_2}^+\left(S_xS_y\right) = \sum_{x,y\in E} [zwj]t_{B_1}^+(x)\wedge t_{B_1}^+(y),\\
d_i^-(S_x) &= \sum_{S_xS_y\in W} [zwj]i_{B_2}^-\left(S_xS_y\right) = \sum_{x,y\in E} [zwj]i_{B_1}^-(x)\wedge i_{B_1}^-(y),\\
d_i^+(S_x) &= \sum_{S_xS_y\in W} [zwj]i_{B_2}^+\left(S_xS_y\right) = \sum_{x,y\in E} [zwj]i_{B_1}^+(x)\wedge i_{B_1}^+(y),\\
d_f^-(S_x) &= \sum_{S_xS_y\in W} f_{B_2}^-\left(S_xS_y\right) = \sum_{x,y\in E} f_{B_1}^-(x)\vee f_{B_1}^-(y),\\
d_f^+(S_x) &= \sum_{S_xS_y\in W} f_{B_2}^+\left(S_xS_y\right) = \sum_{x,y\in E} [zwj]f_{B_1}^+(x)\vee f_{B_1}^+(y).
\end{aligned} \tag{26}
$$

Definition 34. Let $G = (V, E)$ be an IVNG G. Then, we have

(i) $\Delta(G) = ([\Delta_t^-(G), \Delta_t^+(G)], [\Delta_i^-(G), \Delta_i^+(G)], [\Delta_f^-(G), \Delta_f^+(G)])$ is the maximum degree of an IVN-graph G where

$$
\begin{aligned}
\Delta_t^-(G) &= \max\{d_t^-(u)\colon u\in V\}, \Delta_t^+(G) = \max\{d_t^+(u)\colon u\in \mathrm{V}\},\\
\Delta_i^-(G) &= \max\{d_i^-(u)\colon u\in V\}, \Delta_i^+(G) = \max\{d_i^+(u)\colon u\in V\},\\
\Delta_f^-(G) &= \max\left\{d_f^-(u)\colon u\in V\right\}, \Delta_f^+(\mathrm{G}) = \max\left\{d_f^+(u)\colon u\in V\right\}.
\end{aligned} \tag{27}
$$

(ii) $\varrho(G) = ([\varrho_t^-(G), \varrho_t^+(G)], [\rho_i^-(G), \varrho_i^+(G)], [\varrho_f^-(G), \varrho_f^+(G)])$ is the minimum degree of an IVN-graph G where

$$
\begin{aligned}
\varrho_t^-(G) &= \min\{d_t^-(u)\colon u\in V\}, \varrho_t^+(G) = \min\{d_t^+(u)\colon u\in V\},\\
\varrho_i^-(G) &= \min\{d_i^-(u)\colon u\in V\}, \varrho_i^+(G) = \min\{d_i^+(u)\colon u\in V\},\\
\varrho_f^-(G) &= \min\{\mathrm{d}_\mathrm{f}^-(\mathrm{u})\colon \mathrm{u}\in \mathrm{V}\}, \varrho_f^+(G) = \min\left\{d_f^+(u)\colon u\in V\right\}.
\end{aligned} \tag{28}
$$

Definition 35. Let $G = (A_1, B_1)$ be IVNG. An edge $e = uv \in \mathrm{G}$ is called an effective edge if

$$
\begin{aligned}
t_B^-(e) &= t_A^-(u)\wedge t_A^-(v). \quad t_B^+(e) = t_A^+(u)\wedge t_A^+(v),\\
i_B^-(e) &= i_A^-(u)\wedge i_A^-(v), \quad i_B^+(e) = i_A^+(u)\wedge i_A^+(v),\\
f_B^-(e) &= f_A^-(u)\vee f_A^-(v), \quad f_B^+(e) = f_A^+(u)\vee f_A^+(v)\,\text{for}\,u, v\in A_1.
\end{aligned} \tag{29}
$$

An IVNG is strong if every edge is an effective edge.

Theorem 36. *Let G be an IVNG and IVNLG be the corresponding line graph of G. Then, every edge of IVNLG is an effective edge.*

Proof. From the definition of an IVNLG, the proof of this theorem is straightforward. □

5. Application

Suppose that an investor invested in four different companies, namely, food, automobile, computer, and textile companies. Investors also had employees at each company, and all four employees knew each other. Their friendship is for different purposes. Some of them are for the success of the organization. Some of them are for unknown reasons beyond the control of the manager of the company (say, political view, culture, ethnicity, language, and so on), and some employees make relationships for their own advantage only, and those who do not care about the company's mission and objectives. The investor wants to analyze the friendship of his employees between the companies to identify which relationship is for success, unknown, and failure of the organization. Also, there is uncertainty and imprecise situation of the employees in each company to perform the organization's goals. The investor conducted a survey and collected information from managers of each company about how the problem affects the performance of the organization due to the friendship of employees between the companies and within the company.

Consider employee friendship within a car company, a computer company, a food company, and a textile company as a vertex set and employee friendship between companies as a set of edges. The *NS* results from uncertainty, impreciseness, and inconsistency in the data, which is caused by the fact that the information gathered is dependent upon the manager of each company. Since *IVNSs* are more appropriate than *SVNSs*, we also use this concept. As *TMF*, *IMF*, and *FMF* values, it is evident that the employees' friendship is defined independently by *IVNSs*. An *IVNG* will be used to represent this analysis.

The degree of friendship activities of employees in the company represents the membership values of a node. Similarly, the degree of the relationship between the nodes measures the edge membership value. As a result, there are three different kinds of edge interval membership values: truth, indeterminacy, and false. Such a type of network is an example of an IVNG. Therefore, since the investor wants to analyze the relationship between edges, which means friendship between each company, transforming the given graph into a line graph is a better way to solve the problem.

In order to construct an interval-valued neutrosophic line graph for the friendship relationships between employees of different companies, a few steps can be followed:

Step 1: Define the original graph that represents the employees and their friendship relationships. Each node in the graph represents an employee, and the edges represent the friendships between them. Assign weights or values to the edges representing the strength or closeness of the friendships.

Step 2: Determine the interval values: Assign interval values to each edge in the graph to represent the membership, nonmembership, and indeterminacy associated with the friendship relationship. These interval values can be based on subjective assessments, surveys, or expert opinions. The intervals should capture the range of possibilities for the strength of the friendship, considering both the lower and upper bounds.

Step 3: Construct the interval-valued neutrosophic line graph by creating a new graph where the nodes represent the edges of the original graph. The edges in the interval-valued neutrosophic line graph represent the adjacency or connections between the edges in the original graph.

Step 4: Capture the neutrosophic aspect by accounting for the indeterminacy, ambiguity, and incomplete knowledge associated with the friendship relationships. This can be done by allowing for the existence of uncertain or ambiguous information within each interval.

Step 5: Analyze and interpret the results from the constructed interval-valued neutrosophic line graph. This can involve exploring the ranges of possibilities for the strength of the friendship relationships, identifying any uncertain or ambiguous regions, and understanding the overall patterns or trends in the graph.

5.1. Numerical Illustration. Consider an IVN-graph $G = (A_1, B_1)$ such that $A_1 = \{u_1, u_2, u_3, u_4\}$ where u_1, u_2, u_3, and u_4 represents the car company, computer company, food company, and textile company, respectively, and $B_1 = \{u_1u_2, u_1u_3, u_1u_4, u_2u_3, u_2u_4, u_3u_4\}$ is an edge relationship between companies. Then, let us consider the values of vertex and edge data collected from managers of the company, as shown in Tables 3 and 4.

It is easy to see that there are complete relationships between vertices of an IVNG G. So, we can follow the following procedure:

(1) Consider the friendship of employees within a company as a node and friendship of employees across the company as an edge, which is shown in terms of IVNG G

(2) Enter the truth, indeterminacy, and falsity membership of all employees' friendship from the collected data

(3) Drive an IVNLGG from the given IVNG using definition of the line graph

(4) Compute the degree of TMF, IMF, and FMF of all vertices of IVLG G using the following relations $d_t^-(S_{e_i}) = \sum_{S_{e_i}S_{e_j}} t_{B_2}^-(S_{e_i}S_{e_j})$, $d_t^+(S_{e_i}) = \sum_{S_{e_i}S_{e_j}} t_{B_2}^+(S_{e_i}S_{e_j})$ $d_i^-(S_{e_i}) = \sum_{S_{e_i}S_{e_j}} i_{B_2}^-(S_{e_i}S_{e_j})$, $d_i^+(S_{e_i}) = \sum_{S_{e_i}S_{e_j}} i_{B_2}^+(S_{e_i}S_{e_j})$,

Table 3: Vertex and edge values of IVNG G.

	$(t_{A_1}, i_{A_1}, f_{A_1})$
u_1	([0.3, 0.6], [0.3, 1.0], [0.1, 0.4])
u_2	([0.2, 0.7], [0.1, 0.6], [0.1, 0.9])
u_3	([0.3, 0.4], [0.4, 0.7], [0.4, 0.5])
u_4	([0.2, 0.7], [0.1, 0.6], [0.1, 0.2])

Table 4: Edge membership values of IVNG G.

	$(t_{B_1}, i_{B_1}, f_{B_1})$
$e_1 = (u_1, u_2)$	([0.2, 0.5], [0.1, 0.4], [0.1, 0.3])
$e_2 = (u_1, u_3)$	([0.2, 0.4], [0.1, 0.6], [0.2, 0.3])
$e_3 = (u_3, u_4)$	([0.1, 0.5], [0.3, 0.7], [0.3, 0.4])
$e_4 = (u_1, u_4)$	([0.2, 0.4], [0.3, 0.7], [0.2, 0.3])
$e_5 = (u_2, v4)$	([0.2, 0.4], [0.1, 0.5], [0.2, 0.3])
$e_6 = (u_2, v3)$	([0.1, 0.2], [0.1, 0.4], [0.3, 0.8])

$d^-_f(S_{e_i}) = \sum_{S_{e_i}S_{e_j}} f^-_{B_2}(S_{e_i}S_{e_j})$, $d^+_f(S_{e_i}) = \sum_{S_{e_i}S_{e_j}} f^+_{B_2}(S_{e_i}S_{e_j})$, $\forall S_{e_i}S_{e_j} \in E(L(G))$

(5) Find the maximum truth, minimum indeterminacy, and minimum falsity membership degree of IVNLG G.

(6) Take an appropriate decision based on step 4 to overcome the problem.

From above Figure 3, the IVNLG $L(G) = (A_2, B_2)$ where $A_2 = ([t^-_{A_2}, t^+_{A_2}], [i^-_{A_2}, i^+_{A_2}], [f^-_{A_2}, f^+_{A_2}])$ and $B_2 = ([t^-_{B_2}, t^+_{B_2}], [i^-_{B_2}, i^+_{B_2}], [f^-_{B_2}, f^+_{B_2}])$ are the vertex and edge set of an IVNLG G respectively as shown in Tables 5 and 6.

The vertex set and edge set of an *IVNLG* are shown in Tables 5 and 6. Figure 4 represents the line graph of the original graph. The truth-membership vertex degree of $S_e i$ in *IVNLG* is interpreted as how much the employees know each other for the success of the company, the indeterminacy membership vertex degree of $S_e i$ in *IVNLG* shows the unknown relationship of employees in the companies, and the falsity membership vertex degree of $S_e i$ in *IVNLG* represents how much the employees know each other to get illegal benefits from the company, which is harmful in organizations. Based on this, an investor wants to find the maximum TMF, minimum IMF, and minimum FMF degree between each vertex of *IVNLG* to analyze which company the employees would have good or bad relationships within the organization. As a result of Definition 19, the degree of the above line graph is calculated as follows:

$$\begin{aligned}
d^-_t(S_{e_1}) &= t^-_{B_2}(S_{e_1}S_{e_2}) + t^-_{B_2}(S_{e_1}S_{e_4}) + t^-_{B_2}(S_{e_1}S_{e_5}) + t^-_{B_2}(S_{e_1}S_{e_6}) \\
&= 0.2 + 0.2 + 0.2 + 0.1 \\
&= 0.7, \\
d^+_t(S_{e_1}) &= t^+_{B_2}(S_{e_1}S_{e_2}) + t^+_{B_2}(S_{e_1}S_{e_4}) + t^+_{B_2}(S_{e_1}S_{e_5}) + t^+_{B_2}(S_{e_1}S_{e_6}) \\
&= 0.4 + 0.4 + 0.4 + 0.2 \\
&= 1.8, \\
d^-_i(S_{e_1}) &= i^-_{B_2}(S_{e_1}S_{e_2}) + i^-_{B_2}(S_{e_1}S_{e_4}) + i^-_{B_2}(S_{e_1}S_{e_5}) + i^-_{B_2}(S_{e_1}S_{e_6}) \\
&= 0.1 + 0.1 + 0.1 + 0.1 \\
&= 0.4, \\
d^+_i(S_{e_1}) &= i^+_{B_2}(S_{e_1}S_{e_2}) + i^+_{B_2}(S_{e_1}S_{e_4}) + i^+_{B_2}(S_{e_1}S_{e_5}) + i^+_{B_2}(S_{e_1}S_{e_6}) \\
&= 0.4 + 0.4 + 0.4 + 0.2 \\
&= 1.8, \\
d^-_f(S_{e_1}) &= f^-_{B_2}(S_{e_1}S_{e_2}) + f^-_{B_2}(S_{e_1}S_{e_4}) + f^-_{B_2}(S_{e_1}S_{e_5}) + f^-_{B_2}(S_{e_1}S_{e_6}) \\
&= 0.2 + 0.2 + 0.2 + 0.3 \\
&= 0.9, \\
d^+_f(S_{e_1}) &= f^+_{B_2}(S_{e_1}S_{e_2}) + f^+_{B_2}(S_{e_1}S_{e_4}) + f^+_{B_2}(S_{e_1}S_{e_5}) + f^+_{B_2}(S_{e_1}S_{e_6}) \\
&= 0.3 + 0.3 + 0.3 + 0.4 \\
&= 1.3,
\end{aligned} \tag{30}$$

in similar computation, we can obtain the degree of each vertex:

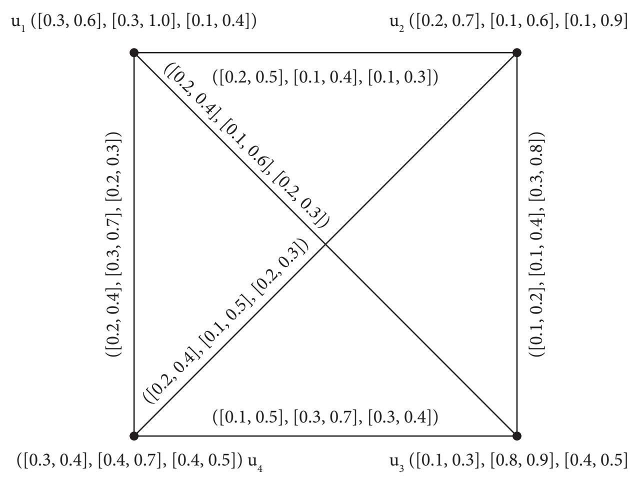

Figure 3: IVNG G.

Table 5: Vertex set of IVNLG G.

	$([t^-_{A_2}, t^+_{A_2}], [i^-_{A_2}, i^+_{A_2}], [f^-_{A_2}, f^+_{A_2}])$
$S_{e_1} = (u_1, u_2)$	([0.2, 0.5], [0.1, 0.4], [0.1, 0.3])
$S_{e_2} = (u_1, u_3)$	([0.2, 0.4], [0.1, 0.6], [0.2, 0.3])
$S_{e_3} = (u_3, u_4)$	([0.1, 0.5], [0.3, 0.7], [0.3, 0.4])
$S_{e_4} = (u_1, u_4)$	([0.2, 0.4], [0.3, 0.7], [0.2, 0.3])
$S_{e_5} = (u_2, u_4)$	([0.2, 0.4], [0.1, 0.5], [0.2, 0.3])
$S_{e_6} = (u_2, u_3)$	([0.1, 0.2], [0.1, 0.4], [0.3, 0.8])

$$\begin{aligned}
&d_t^-(S_{e_2}) = 0.7, \quad d_t^+(S_{e_2}) = 1.6,\\
&d_i^-(S_{e_2}) = 0.5, \quad d_i^+(S_{e_2}) = 2.0,\\
&d_f^-(S_{e_2}) = 0.9, \quad d_f^+(S_{e_2}) = 1.4,\\
&d_t^-(S_{e_3}) = 0.4, \quad d_t^+(S_{e_3}) = 1.4,\\
&d_i^-(S_{e_3}) = 0.6, \quad d_i^+(S_{e_3}) = 2.2,\\
&d_f^-(S_{e_3}) = 1.2, \quad d_f^+(S_{e_3}) = 2.0,\\
&d_t^-(S_{e_4}) = 0.6, \quad d_t^+(S_{e_4}) = 1.4,\\
&d_i^-(S_{e_4}) = 0.6, \quad d_i^+(S_{e_4}) = 2.1,\\
&d_f^-(S_{e_4}) = 1.0, \quad d_f^+(S_{e_4}) = 1.8,\\
&d_t^-(S_{e_5}) = 0.6, \quad d_t^+(S_{e_5}) = 1.4,\\
&d_i^-(S_{e_5}) = 0.4, \quad d_i^+(S_{e_5}) = 1.8,\\
&d_f^-(S_{e_5}) = 1.0, \quad d_f^+(S_{e_5}) = 1.8,\\
&d_t^-(S_{e_6}) = 0.4, \quad d_t^+(S_{e_6}) = 0.8,\\
&d_i^-(S_{e_6}) = 0.4, \quad d_i^+(S_{e_6}) = 1.4,\\
&d_f^-(S_{e_6}) = 1.2, \quad d_f^+(S_{e_6}) = 2.8.
\end{aligned} \tag{31}$$

Hence, we have

$$\begin{aligned}
&d(S_{e_1}) = ([0.7, 1.8], [0.4, 1.8], [0.9, 1.3]),\\
&d(S_{e_2}) = ([0.7, 1.6], [0.5, 2.0], [0.9, 1.4]),\\
&d(S_{e_3}) = ([0.4, 1.4], [0.6, 2.2], [1.2, 2.0]),\\
&d(S_{e_4}) = ([0.6, 1.4], [0.6, 2.1], [1.0, 1.8]),\\
&d(S_{e_5}) = ([0.6, 1.4], [0.4, 1.8], [1.0, 1.8]),\\
&d(S_{e_6}) = ([0.4, 0.8], [0.4, 1.4], [1.2, 2.8]).
\end{aligned} \tag{32}$$

Now, since the degree of TMF S_{e_1} which represents employees' relationship working in a car company and a computer company is the maximum relative to others, so the investor should encourage employee relations between a car company and a computer company. Also, S_{e_1} has the minimum degree of IMF and FMF. Similarly, by observing the degree of each vertex of IVNLG G, the investor can take a decision to attain the objective of the companies. So, in the above example, because the vertex degree of S_{e_6} is [0.4, 0.8], [0.4, 1.4], [1.2, 2.8]), which means that the truth-membership degree of S_{e_6} is minimum, the indeterminacy-membership degree of S_{e_6} is maximum, and the nonmembership degree of S_{e_6} is maximum when compared with other vertices. So, the investor should focus as much as possible on the employee's relationship S_{e6}, which is the computer company and the food company. Therefore, either disconnecting or managing employees towards the two companies is a better option to be competitive in investment by managing the employees' relationships within the company and across different organizations.

Now, since the degree of $TMF(S_{e1})$, which represents employees' relationships working in a car company and a computer company, is the highest relative to others, the investor should encourage employee relations between a car company and a computer company. Also, (S_{e1}), has the minimum degree of IMF and FMF. Similarly, by observing the degree of each vertex of IVNLG G, the investor can make a decision to attain the objective of the company. So, in the

Table 6: Edge set of IVNLG G.

	$[t^-_{B_2}, t^+_{B_2}]$	$[i^-_{B_2}, i^+_{B_2}]$	$[f^-_{B_2}, f^+_{B_2}]$
$S_{e_1}S_{e_2}$	[0.2, 0.4]	[0.1, 0.4]	[0.2, 0.3]
$S_{e_1}S_{e_4}$	[0.2, 0.4]	[0.1, 0.4]	[0.2, 0.3]
$S_{e_1}S_{e_5}$	[0.2, 0.4]	[0.1, 0.4]	[0.2, 0.3]
$S_{e_1}S_{e_6}$	[0.1, 0.2]	[0.1, 0.2]	[0.3, 0.4]
$S_{e_2}S_{e_3}$	[0.1, 0.4]	[0.1, 0.6]	[0.3, 0.4]
$S_{e_2}S_{e_4}$	[0.2, 0.4]	[0.1, 0.6]	[0.2, 0.3]
$S_{e_2}S_{e_5}$	[0.2, 0.4]	[0.1, 0.5]	[0.2, 0.3]
$S_{e_3}S_{e_4}$	[0.1, 0.4]	[0.3, 0.7]	[0.3, 0.4]
$S_{e_3}S_{e_5}$	[0.1, 0.4]	[0.1, 0.5]	[0.3, 0.4]
$S_{e_3}S_{e_6}$	[0.1, 0.2]	[0.1, 0.4]	[0.3, 0.8]
$S_{e_4}S_{e_6}$	[0.1, 0.2]	[0.1, 0.4]	[0.3, 0.8]
$S_{e_5}S_{e_6}$	[0.1, 0.2]	[0.1, 0.4]	[0.3, 0.8]

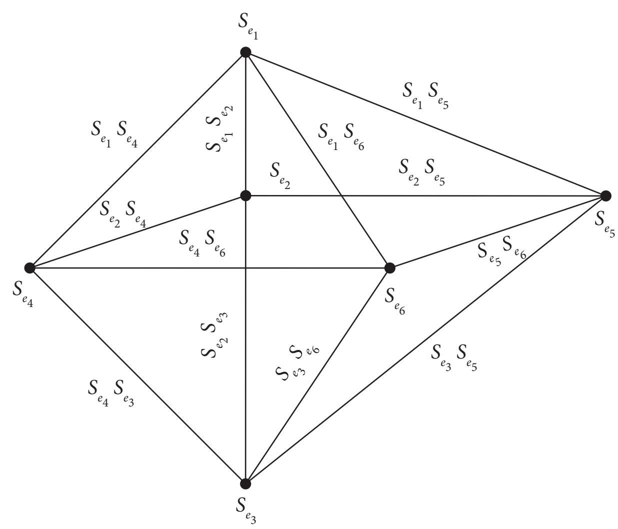

Figure 4: IVNLG G.

above example, the vertex degree of S_{e6} is [0.4, 0.8], [0.4, 1.4], [1.2, 2.8], which means that the truth-membership degree of S_{e6} is minimum, the indeterminacy-membership degree of S_{e6} is maximum, and the nonmembership degree of S_{e6} is maximum when compared with other vertices. So, the investor should focus as much as possible on the employee's relationship (S_{e6}), which is the computer company and the food company. Therefore, either disconnecting or managing employee relationships with the two companies is a better option to be competitive in investment by managing the employees' relationships within the company and across different organizations.

6. Conclusion

An interval-valued neutrosophic model is more complex than an IVIF or an IVF model. Numerous real-world systems with varying levels of precision, incompleteness, vagueness, and uncertainty can be modeled using this technique. As a result, the study concentrated on the IVNLG concept, which is crucial to real-world problems.

In this paper, our focus is to introduce both the theory and application of IVNG. These include definition, vertex degree, edge degree, isomorphic properties, and daily life applications of IVNLGs. In this regard, we explained the maximum degree and minimum degree of a vertex of the IVNLGs and their role in the art of decision-making. Many types of line graphs have already been discussed from a different perspective by other researchers, for example, classical line graphs, fuzzy line graphs, interval-valued fuzzy line graph (IVFLG), intuitionistic fuzzy line graphs (IFLG), and $L(G)$ of IVIFG. Also, the line graph of single-valued neutrosophic graphs was introduced. Interval-valued neutrosophic line graphs are the generalization of interval-valued fuzzy line graphs and interval-valued intuitionistic line graphs. In addition, weak vertex, weak line, and homomorphism properties are demonstrated. We also presented some theorems, propositions, and properties of the IVNLG. Finally, the algorithm that is used to calculate the degree of IVNLG as well as the application of IVNLG has also been discussed and illustrated by numerical examples. Based on this result, we can extend the introduced concept to several extensions of neutrosophic graphs since it is the most general form of graph today and was designed in order to capture our complex real world. We can also extend this new concept to direct neutrosophic graphs and other areas of graph theory.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

Keneni was involved in formal analysis, methodology, writing, and supervising the work. VNSRao and Mamo contributed in the conceptualization, methodology, writing, and editing the article. All the authors read and approved the final manuscript.

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