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Some Results on 4-Total Difference Cordial Graphs

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Abstract: Let G be a graph. Let $f : V(G) \to \{0, 1, 2, \cdots, k-1\}$ be a map where $k \in \mathbb{N}$ and $k > 1$. For each edge uv, assign the label $|f(u) - f(v)|$. f is called k-total difference cordial labeling of G if $|t_{df}(i) - t_{df}(j)| \leq 1$, $i, j \in \{0, 1, 2, \cdots, k-1\}$ where $t_{df}(x)$ denotes the total number of vertices and the edges labeled with x. A graph with admits a k-total difference cordial labeling is called k-total difference cordial graphs.

Key Words: Difference cordial labeling, Smarandachely difference cordial labeling, star, path, cycle, bistar, crown, comb.

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§1. Introduction

We consider here finite, simple and undirected graphs only. Ponraj etl., has been introduced the concept of k-total difference cordial graph in [4]. In [4,5], 3-total difference cordial labeling path , complete graph,comb ,armed crown, crown , wheel, star etc have been investigate and also we prove that every graph is a subgraph of a connected k-total difference cordial graphs in .In this paper we investigate 4-total difference of cordial labeling of some graphs like star, path, cycle, bistar, crown, comb, etc.

§2. K-Total Difference Cordial Labeling

Definition 2.1 *Let G be a graph. Let $f : V(G) \to \{0, 1, 2, \cdots, k-1\}$ be a function where $k \in \mathbb{N}$ and $k > 1$. For each edge uv, assign the label $|f(u) - f(v)|$. f is called k-total difference cordial labeling of G if $|t_{df}(i) - t_{df}(j)| \leq 1$, $i, j \in \{0, 1, 2, \cdots, k-1\}$ where $t_{df}(x)$ denotes the total number of vertices and the edges labelled with x. A graph with a k-total difference cordial labeling is called k-total difference cordial graph. Otherwise, if there is a pair $\{i, j\} \subset \{0, 1, 2, \cdots, k-1\}$ such that $|t_{df}(i) - t_{df}(j)| > 1$,such a labeling is called a Smarandachely k-total difference cordial labeling of G.*

Remark 2.2([6]) 2-total difference cordial graph is 2-total product cordial graph.

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§3. Preliminaries

Definition 3.1 *The corona of G_1 with G_2,$G_1 \odot G_2$ is the graph obtained by taking one copy of G_2 and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2.*

Definition 3.2 *Armed crown AC_n is the graph obtained from the cycle $C_n : u_1u_2 \cdots u_nu_1$ with $V(AC_n) = V(C_n) \bigcup \{v_i, w_i : 1 \leq i \leq n\}$ and $E(AC_n) = E(C_n) \bigcup \{u_iv_i, v_iw_i : 1 \leq i \leq n\}$.*

Definition 3.3 *An edge $x = uv$ of G is said to be subdivided if it is replaced by the edges uw and wv where w is a vertex not in $V(G)$.If every edge of G is subdivided,the resulting graph is called the subdivision graph $S(G)$.*

Definition 3.4 *Jelly fish graphs $J(m, n)$ obtained from a cycle $C_4 : uxvyu$ by joining x and y with an edge and appending m pendent edges to u and n pendent edges to v.*

Definition 3.5 *Triangular snake T_n is obtained from the path $P_n : u_1u_2 \cdots u_n$ with $V(T_n) = V(P_n) \bigcup \{v_i : 1 \leq i \leq n-1\}$ and $E(T_n) = E(P_n) = \bigcup \{u_iv_i, u_{i+1}v_i : (1 \leq i \leq n-1)\}$.*

Definition 3.6 *Double Triangular snake $D(T_n)$ is obtained from the Path $P_n : u_1u_2 \cdots u_n$ with $V(D(T_n)) = V(P_n) \bigcup \{v_i, w_i : 1 \leq i \leq n-1\}$ and $E(D(T_n)) = E(P_n) \bigcup \{u_iv_i, u_iw_i : 1 \leq i \leq n-1\} \bigcup \{v_iu_{i+1}, w_iu_{i+1} : 1 \leq i \leq n-1\}$.*

§4. Main Results

Theorem 4.1 *Any star $K_{1,n}$ is 4-total difference cordial.*

Proof Let $V(K_{1,n}) = \{u, u_i : 1 \leq i \leq n\}$ and $E(K_1, n) = \{uu_i : 1 \leq i \leq n\}$.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4r$,$r \in N$. Assign the label 1 to the central vertex. Next assign the label 0 to the vertices $u_1, u_2, \ldots, u_{2r}$ and assign the label 3 to the remaining vertices.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4r + 1$,$r \in N$. Assign the label 1 to the central vertex u .We now move to the pendent vertices.Assign the label 0 to the vertices $u_1, u_2, \cdots, u_{2r}$ and assign the label 3 to the next remaining vertices $u_{2r+1}, u_{2r+2}, \cdots, u_{4r}$ and u_{4r+1}.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4r + 2$,$r \in N$.In this case assign the label 0 to the vertices $u_1, u_2, \cdots, u_{2r}$ and u_{2r+1}.Next assign the label 3 to the vertices $u_{2r+2}, u_{2r+3}, \cdots, u_{4r+2}$.Finally assign 1 to the central vertex u.

Case 4. $n \equiv 3 \pmod 4$.

As in case (3) assign the label to u, $u_1, u_2, \cdots, u_{n-1}$. Next assign the label 3 to the vertex

u_n.

Table 1 given below establish that this vertex labeling pattern is a 4-total difference cordial labeling.

Values of n	$t_{df}(0)$	$t_{df}(1)$	$t_{df}(2)$	$t_{df}(3)$
$n \equiv 0 \pmod 4$	$2r$	$2r+1$	$2r$	$2r$
$n \equiv 1 \pmod 4$	$2r$	$2r+1$	$2r+1$	$2r+1$
$n \equiv 2 \pmod 4$	$2r+1$	$2r+2$	$2r+1$	$2r+1$
$n \equiv 3 \pmod 4$	$2r+1$	$2r+2$	$2r+2$	$2r+2$

Table 1

A 4-total difference cordial labeling of $K_{1,n}(n = 1, 2, 3)$ is given in Table 2.

Values of n	u	u_1	u_2	u_3
1	1	3		
2	1	0	3	
3	1	0	3	3

Table 2

This completes the proof. □

Theorem 4.2 *The path P_n is 4-total difference cordial for all values of n.*

Proof Let P_n be the path $u_1, u_2, \cdots, u_n$.

Case 1. $n \equiv 0 \pmod 4$ $n > 3$.

Let $n = 4r$, $r \in \mathbb{N}$, Assign the labels $3, 1, 1$ and 3 respectively to the vertices u_1,u_2,u_3,u_4.Next assign the labels $3, 1, 1$ and 3 to the next 4 vertices u_5,u_6,u_7,u_8 respectively. Proceeding like this until we reach the vertex u_n. That is in this process the last 4 vertices u_{n-3},u_{n-2}, u_{n-1} and u_n receive the labels $3, 1, 1$ and 3.

Case 2. $n \equiv 1 \pmod 4$ $n > 3$.

Let $n = 4r + 1$,$r \in \mathbb{N}$. As in Case 1, assign the label to the vertices $u_1, u_2, \cdots, u_{n-1}$. Next assign the label 3 to the vertex u_n.

Case 3. $n \equiv 2 \pmod 4$, $n > 3$.

Let $n = 4r + 2$,$r \in \mathbb{N}$. Assign the label to the vertices $u_1, u_2, \cdots, u_{n-1}$ as in Case 2. Next assign the label 1 to the vertices u_n.

Case 4. $n \equiv 3 \pmod 4$, $n > 3$.

Let $n = 4r + 3$,$r \in \mathbb{N}$. Assign the label to the vertices $u_1, u_2, \cdots, u_{n-1}$ as in Case 3. Next assign the label 1 to the vertex u_n. This vertex labels is a 4-total difference cordial labels follows from Table 3 for $n > 3$.

Values of n	$t_{df}(0)$	$t_{df}(1)$	$t_{df}(2)$	$t_{df}(3)$
$n \equiv 0 \pmod 4$	$2r-1$	$2r$	$2r$	$2r$
$n \equiv 1 \pmod 4$	$2r$	$2r$	$2r$	$2r+1$
$n \equiv 2 \pmod 4$	$2r$	$2r+1$	$2r+1$	$2r+1$
$n \equiv 3 \pmod 4$	$2r+1$	$2r+2$	$2r+1$	$2r+1$

Table 3

A 4-total difference cordial labeling of $P_n (n = 1, 2, 3)$ is given in Table 4.

Values of n	u_1	u_2	u_3
1	0		
2	0	2	
3	0	2	3

Table 4

This completes the proof. □

Theorem 4.3 *The cycle C_n is 4-total difference cordial if $n \equiv 0, 1, 3 \pmod 4$*

Proof Let C_n be the cycle $u_1u_2 \cdots u_nu_1$. Assign the label to the vertices $u_1, u_2, \cdots, u_n$ as in Theorem 4.2. Table 5 given below shows that this labeling of C_n is a 4-total difference cordial.

Values of n	$t_{df}(0)$	$t_{df}(1)$	$t_{df}(2)$	$t_{df}(3)$
$n \equiv 0 \pmod 4$	$2r$	$2r$	$2r$	$2r$
$n \equiv 1 \pmod 4$	$2r$	$2r$	$2r$	$2r+1$
$n \equiv 3 \pmod 4$	$2r+1$	$2r+2$	$2r+1$	$2r+1$

Table 5

This completes the proof. □

Theorem 4.4 *The bistar $B_{n,n}$ is 4-total different cordial for all n.*

Proof Let $V(B_{n,n}) = \{u, v, u_i, v_i : 1 \le i \le n\}$ and $E(B_{n,n}) = \{uu_i, vv_i, uv : (1 \le i \le n)\}$. Clearly $B_{n,n}$ has $2n + 2$ vertices and $2n + 1$ edges. Assign the label 1 to the central vertices u and v. Assign the label 3 to the vertices $u_1, u_2, \cdots, u_n$ and v_1. We now assign the label 1 to the vertices $v_2, v_3, \cdots, v_n$. Clearly $t_{df}(0) = n$, $t_{df}(1) = t_{df}(2) = t_{df}(3) = n + 1$. Therefore f is a 4-total difference cordial labeling. □

Theorem 4.5 *The crown $C_n \odot K_1$ is 4-total difference cordial labeling for all values of n.*

Proof Let C_n be the cycle $u_1u_2 \cdots u_nu_1$ .Let $V(C_n \odot K_1)V(C_n) \bigcup \{v_i : 1 \le i \le n\}$ and $E(C_n \odot K_1) = E(C_n) \bigcup \{u_iv_i : 1 \le i \le n\}$. Assign the label 1 to the cycle vertices $u_1, u_2, \cdots, u_n$. Next we move to the pendent vertices v_i. Assign the label 3 to all pendent vertices $v_1, v_2, \cdots, v_n$.

Clearly $t_{df}(0) = t_{df}(1) = t_{df}(2) = t_{df}(3) = n$. Hence f is a 4-total difference cordial labeling. □

Corollary 4.1 *All combs are* 4*-total difference cordial labeling.*

Proof Clearly the vertex labeling in theoeam 4.5 is also a 4-total difference cordial labeling of $P_n \odot K_1$. □

Theorem 4.6 *The armed crown AC_n is* 4*-total difference cordial for all n.*

Proof Clearly AC_n has 3 vertices and $3n$ edges. Let the vertex set and edge set as in Definition 3.2. Assign the label 1 to the all the cycle vertices $u_1, u_2, \cdots, u_n$. Next we assign the label 3 to the vertices $v_1, v_2, \cdots, v_n$.

Case 1. n is even.

In this case assign the label 3 to the pendent vertices $w_1 w_2 \cdots w_{\frac{n}{2}}$ and 1 to the remaining pendent vertices $w_{\frac{n}{2}+1}, w_{\frac{n}{2}+2}, \cdots, w_n$.

Case 2. n is odd.

Assign the label 3 to the vertices $w_1, w_2, \cdots, w_{\frac{n+1}{2}}$ and 1 to the vertices $w_{\frac{n+3}{2}}, w_{\frac{n+5}{2}}, \cdots, w_n$. The table 6 given below establish that this vertex labeling pattern is a 4-total difference cordial labeling.

Values of n	$t_{df}(0)$	$t_{df}(1)$	$t_{df}(2)$	$t_{df}(3)$
n is even	$\frac{3n}{2}$	$\frac{3n}{2}$	$\frac{3n}{2}$	$\frac{3n}{2}$
n is odd	$\frac{3n+1}{2}$	$\frac{3n-1}{2}$	$\frac{3n-1}{2}$	$\frac{3n+1}{2}$

Table 6

This completes the proof. □

Theorem 4.7 *The double triangular snake DT_n is* 4*-total difference cordial for all n.*

Proof Let the vertex set and edge set as in Definition 3.6.

Case 1. $n \equiv 0 \pmod 3$.

Assign the labels $3, 2, 3$ to the path vertices u_1, u_2, u_3. Next assign the labels $3, 2, 3$ to the next 3 vertices u_4, u_5, u_6 respectively. Proceeding like this until we reach the vertices u_n. That is in the process the last three vertices u_{n-2}, u_{n-1}, u_n receive the label $3, 2, 3$. Next assign the label 0 to the vertices $v_1, v_2, \cdots, v_n$ and assign the label 2 to the vertices $w_1, w_2, \cdots, w_n$.

Case 2. $n \equiv 1 \pmod 3$.

In this case assign the labels to the vertices $u_i, (1 \leq i \leq n-1)$,$v_i, w_i, (1 \leq i \leq n-1)$ as in Case 1. Next assign the labels $3, 0, 2$ respectively to the vetices u_n, v_{n-1}, w_n.

Case 3. $n \equiv 2 \pmod 3$.

As in Case 2 assign the labels to the vertices $u_1, u_2, \cdots, u_{n-1}$,$v_1, v_2, \cdots v_{n-2}$ and $w_1, w_2, \cdots w_{n-2}$.

Finally assign the label $2, 0$ and 2 to the vertices u_n,v_{n-1} and w_{n-1}. Table 7 given below establish that this labeling scheme is a 4-total difference cordial labeling of DT_n.

Nature of n	$t_{df}(0)$	$t_{df}(1)$	$t_{df}(2)$	$t_{df}(3)$
$n \equiv 0 \pmod 3$	$2n-2$	$2n-2$	$2n-1$	$2n-2$
$n \equiv 1 \pmod 3$	$2n-2$	$2n-2$	$2n-2$	$2n-1$
$n \equiv 2 \pmod 3$	$2n-2$	$2n-2$	$2n-1$	$2n-2$

Table 7

This completes the proof. □

Example 4.1 A 4-total difference cordial labeling of $D(T_6)$ is shown in Figure 1.

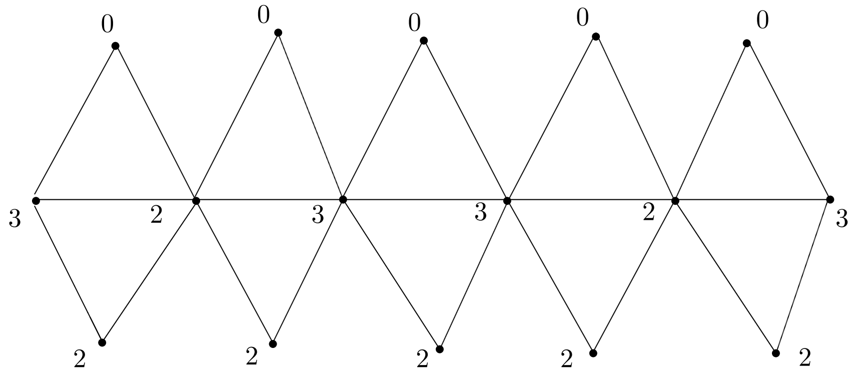

Figure 1

Theorem 4.8 *The jelly fish $J(n, n)$ is 4-total difference cordial for all n.*

Proof Let C_4 be a cycle $uxvyu$. Let $V(J(n,n))$=$V(C_4) \bigcup \{u_i, v_i : 1 \leq i \leq n\}$ and $E(J(n,n))$ $= E(C_4) \cup \{xy, xu_i, yv_i : 1 \leq i \leq n\}$. Assign the label 1 to the all cycle vertices u,x,y,v. Next we move to the pendent vertices. Assign the label 3 to the $u_1, u_2, \cdots, u_n$ and v_1,v_2. Assign the label 1 to the $v_3, v_4, \cdots, v_n$. Since $t_{df}(0) = n+3$,$t_{df}(1) = t_{df}(2) = t_{df}(3) = n+2$,f is a 4-total difference cordial labeling. □

Theorem 4.9 *The subdivision of the bistar $B_{n,n}$, $S(B_{n,n})$ is 4-total different cordial for all n.*

Proof Let $V(S(B_{n,n})) = \{u, w, v, u_i, v_i, x_i, y_i : 1 \leq i \leq n\}$ and $E(S(B_{n,n})) = \{uu_i, u_ix_i, uw,$ $wv, vv_i, v_iy_i : 1 \leq i \leq n\}$. Assign the label 1 to the vertices u,w and v.Next assign the label 3 to the vertices $u_1, u_2, \cdots, u_i$,
$x_1, x_2, \cdots, x_i$ and v_1. We now assign the label 2 to the vertices $y_1, y_2, \cdots, y_n$ and v_2. Finally assign the label 1 to the vertices $v_3, v_4, \cdots, v_n$. Since $t_{df}(0) = t_{df}(1) = t_{df}(3) = 2n+1$,$t_{df}(2) =$ $2n+2$. The labeling f is a 4-total difference cordial labeling. □

Theorem 4.10 *$P_n \odot 2K_1$ is 4-total difference cordial for all n.*

Proof Let P_n be the path $u_1, u_2, \cdots, u_n$.Let v_i,w_i be the pendent vertices adjacent to u_i $(1 \le i \le n)$. Assign the label 1 to the path vertices $u_1, u_2, \cdots, u_n$.

Case 1. n is even.

Assign the label 3 to all the vertices v_1,v_2, $\cdots$, v_n and w_1,w_2, $\cdots$, $w_{\frac{n}{2}}$. We now assign the label 1 to the vertices $w_{\frac{n}{2}+1}, w_{\frac{n}{2}+2}, \cdots, w_n$.

Case 2. n is odd.

As in Case 1 assign the label to the vertices u_i,v_i,w_i $(1 \le i \le n)$. Next assign the label 3 to the vertices u_n and assign the label 1 to the vertex w_n.

Table 8 given below establish that this vertex labeling pattern is a 4-total difference cordial labeling.

Values of n	$t_{df}(0)$	$t_{df}(1)$	$t_{df}(2)$	$t_{df}(3)$
n is even	$\frac{3n}{2}-1$	$\frac{3n}{2}$	$\frac{3n}{2}$	$\frac{3n}{2}$
n is odd	$\frac{3n-1}{2}$	$\frac{3n+1}{2}$	$\frac{3n-1}{2}$	$\frac{3n-1}{2}$

Table 8

This completes the proof. □

Theorem 4.11 *$S(P_n \odot K_1)$ is 4-total difference cordial for all n.*

Proof Let P_n be the path $u_1, u_2, \cdots u_n$.Let $V(P_n \odot K_1) = V(P_n) \cup \{v_i : 1 \le i \le n\}$ and $E(P_n \odot K_1) = \{u_i : 1 \le i \le n\}$. Let x_i be the vertex which subdivide the edge u_iu_{i+1},$\{1 \le i \le n-1\}$ and y_i be the vertex which subdivide u_iv_i : $\{1 \le i \le n\}$. Assign the label 3 to the all path vertices $u_1, u_2, \cdots, u_n$ and $x_1, x_2, \cdots, x_n$ and v_2. Next we assign the label 1 to the vertices $y_1, y_2, \ldots y_n$ and v_1. Finally we assign the label 2 to the remaining vertices $v_3, v_4, \cdots v_n$. Clearly $t_{df}(0) = t_{df}(1) = t_{df}(2) = 2n-1$,$t_{df}(3) = 2n$. Therefore, f is a 4-total difference cordial labeling of $S(P_n \odot K_1)$. □

Theorem 4.12 *$S(C_n \odot K_1)$ is 4-total difference cordial for all values of n.*

Proof Let $C_n : u_1u_2 \cdots u_nu_1$ be the cycle. Let $V(C_n \odot K_1)$=$V(C_n) \bigcup \{v_i : (1 \le i \le n)\}$ and $E(C_n \odot K_1) = E(C_n) \bigcup \{u_iv_i : 1 \le i \le n\}$. Let x_i, y_i be the vertices which subdivide the edges $u_iu_{i+1}(1 \le i \le n-1)$, $u_iv_i(1 \le i \le n)$ respectively. First we assign the label 3 to the cycle vertices $u_1, u_2, \cdots u_n$ and $x_1, x_2, \cdots x_n$. Next we assign the label 1 to the $y_1, y_2, \cdots y_n$. Finally assign the label 2 to the all pendent vertices $v_1, v_2, \cdots v_n$. Clearly $t_{df}(0) = t_{df}(1) = t_{df}(2) = t_{df}(3) = 2n$. Therefore f is a 4-total difference cordial labeling of $S(C_n \odot K_1)$. □

Theorem 4.13 *$S(AC_n)$ is 4-total difference cordial for all n.*

Proof Let the vertex set and edge set of AC_n as in definition 3.2.let x_i : $(1 \le i \le n-1)$,y_i : $(1 \le i \le n-1)$ and z_i : $(1 \le i \le n-1)$ be the vertex which subdivide the edges u_iu_{i+1} : $(1 \le i \le n-1)$,u_iv_i : $(1 \le i \le n-1)$ and v_iw_i : $(1 \le i \le n-1)$ respectively. Assign the label 3 to the vertices $u_1, u_2, \cdots, u_n$ and $x_1, x_2, \cdots, x_n$ and $w_1, w_2, \cdots, w_n$. Next assign the label 1 to the vertices $y_1, y_2, \cdots, y_n$.Then assign the label 2 to the vertices $v_1, v_2, \cdots, v_n$ and $z_1, z_2, \cdots z_n$. obviously $t_{df}(0) = t_{df}(1) = t_{df}(2) = t_{df}(3) = 3n$. Therefore f is a 4-total difference cordial labeling of $S(AC_n)$. □

Example 4.2 A 4-total difference cordial labeling of $S(AC_n)$ is shown in Figure 2.

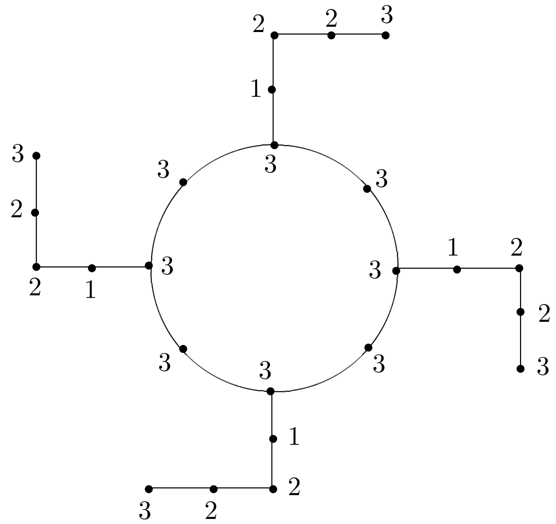

Figure 2

Theorem 4.14 $S(T_n)$ *is* 4*-total difference cordial.*

Proof Let the vertex set and edge set of T_n as in Definition 3.7. Let x_i, y_i and z_i be the vertices which subdivide the edges u_iu_{i+1},u_i, v_i and $u_{i+1}v_i$, $(1 \leq i \leq n)$.

Case 1. $n \equiv 0 \pmod 4)$.

Assign the label 3 to the vertices $u_1, u_2, \cdots u_n$ and $x_1, x_2, \cdots x_{n-1}$. Assign the label 1 to the vertices $y_1, y_2 \cdots y_{n-1}$. Next assign the label $2, 3, 1$ and 3 to the vertices z_1, z_2, z_3, z_4 then assign the label $2, 3, 1$ and 3 to the next 4 vertices z_5, z_6, z_7, z_8 respectively. Proceeding like this until we reach the vertices z_{n-1}. That is in the process the last four vertices are $z_{n-4}, z_{n-3}, z_{n-2}, z_{n-1}$ receive the label $2, 3, 1, 3$. Next assign the label $0, 2, 3, 2$ to the vertices $0, 2, 3, 2$ to the vertices v_1, v_2, v_3, v_4 then assign the label $0, 2, 3, 2$ to the next 4 vertices v_5, v_6, v_7, v_8 respectively. Proceeding like this until we reach the vertices v_{n-1}.That is in the process the last 4 vertices $v_{n-4}, v_{n-3}, v_{n-2}, v_{n-1}$ receive the label $0, 2, 3, 2$.

Case 2. $n \equiv 1 \pmod 4$.

As in Case 1 assign the labels to the vertices $u_i, (1 \leq i \leq n-1)$,$v_i, x_i, y_i, z_i, (1 \leq i \leq n-2)$. Next assign the labels $3, 0, 3, 1$ and 2respect to the vertices $u_n, v_{n-1}, x_{n-1}, y_{n-1}$ and z_{n-1}.

Case 3. $n \equiv 2 \pmod 4$.

In this case, assign the labels to the vertices $u_i, (1 \leq i \leq n-1)$,$v_i, x_i, y_i, z_i, (1 \leq i \leq n-2)$ as in Case 2. Finally assign the labels $3, 2, 3, 1$ and 3 to the vertices $u_n, v_{n-1}, x_{n-1}, y_{n-1}$ and

z_{n-1} respectively.

Case 4. $n \equiv 3 \pmod 4$.

As in Case 3, assign the label to $u_i, (1 \leq i \leq n-1), v_i, x_i, y_i, z_i, (1 \leq i \leq n-2)$. Next assign the labels $3, 3, 3, 1$ and 1 to the vertices $u_n, v_{n-1}, x_{n-1}, y_{n-1}$ and z_{n-1} respectively. Table 9 given below establish that this vertex labeling pattern is a 4-total difference cordial labeling.

Nature of n	$t_{df}(0)$	$t_{df}(1)$	$t_{df}(2)$	$t_{df}(3)$
$n \equiv 0 \pmod 4$	$\frac{11n-8}{4}$	$\frac{11n-8}{4}$	$\frac{11n-12}{4}$	$\frac{11n-12}{4}$
$n \equiv 1 \pmod 4$	$\frac{11n-9}{4}$	$\frac{11n-9}{4}$	$\frac{11n-5}{4}$	$\frac{11n-9}{4}$
$n \equiv 2 \pmod 4$	$\frac{11n-10}{4}$	$\frac{11n-10}{4}$	$\frac{11n-10}{4}$	$\frac{11n-10}{4}$
$n \equiv 3 \pmod 4$	$\frac{11n-9}{4}$	$\frac{11n-9}{4}$	$\frac{11n-9}{4}$	$\frac{11n-13}{4}$

Table 9

This completes the proof. □

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