SOME EQUIVALENT CONDITIONS OF SMARANDACHE - SOFT NEUTROSOPHIC-NEAR RING

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ABSTRACT. In this paper, we introduced Samarandache-2-algebraic structure of Soft Neutrosophic-Near ring namely Smarandache-Soft Neutrosophic-Near ring. A Samarandache-2-algebraic structure on a set N means a weak algebraic structure S_1 on N such that there exist a proper subset M of N, Which is embedded with a stronger algebraic structure S_2, stronger algebraic structure means satisfying more axioms, that is $S_1 \ll S_2$, by proper subset one can understand a subset different from the empty set, from the unit element if any, from the Whole set. We define Smarandache - Soft Neutrosophic - Near ring and obtain the some of it characterization through sub algebraic structures of near - ring. For basic concept of near - ring we refer to G.Pilz [3] and for soft neutrosophic algebraic structures we refer to Muhammed Shabir, MumtazAli, Munazza Naz, and Florentin Smarandache[4,5].

1. INTRODUCTION

In order that, New notions are introduced in algebra to better study the congruence in number theory by Florentin smarandache[2]. By <proper subset> of a set A we consider a set P included in A, and different from A, different from empty set, and from the unit element in A - if any they rank the algebraic structures using an order relationship:

They say that the algebraic structures $S_1 \ll S_2$ if: both are defined on the same set; all S_1 laws are also S_2 laws; all axioms of an S_1 law are accomplished by the corresponding S_2 law; S_2 law accomplish strictly more axioms that S_1 laws, or S_2 has more laws than S_1.

For example: Semi group$\ll$ Monoid $\ll$ group $\ll$ ring $\ll$ field, or Semi group $\ll$ to commutative semi group, ring $\ll$ unitary ring etc. They define a general special structure to be a structure SM on a set A, different from a structure SN, such that a proper subset of A is an structure, where SM $\ll$ SN.

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2. Preliminaries

Definition 2.1. Let $\langle N \cup I \rangle$ be a neutrosophic near - ring and (F, A) be a soft set over $\langle N \cup I \rangle$.Then (F, A) is called soft neutrosophic near - ring if and only if $F(a)$ is a neutrosophic sub near - ring of $\langle N \cup I \rangle$ for all $a \in A$.

Definition 2.2. Let $K(I) = \langle K \cup I \rangle$ be a neutrosophic near - field and let (F, A) be a soft set over $K(I)$. Then (F, A) is said to be soft neutrosophic near - field if and only if $F(a)$ is a neutrosophic sub near - field of $K(I)$ for all $a \in A$.

Definition 2.3. Let (F, A) be a soft neutrosophic near - ring over $\langle N \cup I \rangle$.Then (F, A) is called soft neutrosophic right near - ring. Then it satisfies for all $F(a), F(b), F(c)$ in (F, A) such that $F(a).(F(b) + F(c)) = F(a).F(b) + F(a).F(c)$

Definition 2.4. Let (F, A) be a soft neutrosophic near - ring over $\langle N \cup I \rangle$. Then (F, A) is called soft neutrosophic left near - ring. Then it satisfies for all $F(a), F(b), F(c)$ in (F, A) such that $(F(a) + F(b)).F(c) = F(a).F(c) + F(b).F(c)$

Remark. Throughout this paper we consider soft neutrosophic right near - ring only.

Definition 2.5. Let (F, A) be a soft Neutrosophic near - ring over $\langle N \cup I \rangle$ with more than one element. Then the non - zero elements of (F, A) form a group under multiplication if and only if for every $F(a) \neq 0$ in (F, A), there exist a unique $F(b)$ in (F, A) such that $F(a)F(b)F(a) = F(a)$.

Definition 2.6. Let (F, A) be a soft neurtosophic zerosymmetric near - ring over $\langle N \cup I \rangle$, which contains a distributive element $F(a_1) \neq 0$. Then (F, A) is a near - field if and only if for each $F(a) \neq 0$ in (F, A), $(F, A)F(a) = (F, A)$.

Definition 2.7. Let (F, A) be a finite soft neutrosophic zero symmetric near - ring that contains a distributive element $F(w) \neq 0$ and for each $F(x) \neq 0$ in (F, A) there exist $F(y)$ in (F, A) such that $F(y)F(x) \neq 0$ then (F, A) is a soft neutrosophic near - field if and only if (F, A) has no proper left ideal.

Definition 2.8. Let (F, A) be a soft neurtosophic near - ring over $\langle N \cup I \rangle$, Then (F, A) is called soft neutrosophic zerosymmetric near - ring over $\langle N \cup I \rangle$.If $F(n)0 = 0$ for all $F(n)$ in (F, A).

Definition 2.9. Let (F, A) be a soft neurtosophic near - ring over $\langle N \cup I \rangle$.An element $F(e)$ in a soft neutrosophic near - ring (F, A) over $\langle N \cup I \rangle$ is called idempotent. If $F(e^2) = F(e)$.

Definition 2.10. Let (F, A) be a soft neurtosophic near - ring over $\langle N \cup I \rangle$.An element $F(b)$ in (F, A) is called distributive if $F(b)(F(a_1) + F(a_2)) = F(b)F(a_1) + F(b)F(a_2)$ for all $F(a_1), F(a_2)$ in (F, A).

Definition 2.11. Let (F, A) be a soft neurtosophic near - ring over $\langle N \cup I \rangle$. A soft neutrosophic subgroup (H, A) of (F, A) is called (F, A) - subgroup if $(F, A)(H, A) \subset (H, A)$.

Definition 2.12. Let (F, A) be a soft neurtosophic near - ring over $\langle N \cup I \rangle$ is called regular if for each $F(a)$ in (F, A). There exist $F(x)$ in (F, A) such that $F(a)F(x)F(a) = F(a)$.

Now we have introduced our basic concept, called SMARANDACHE - SOFT NEUTROSOPHIC - NEAR RING.

Definition 2.13. A soft neutrosophic - near ring is said to be smarandache - soft neutrosophic - near ring, if a proper subset of it is a soft neutrosophic - near field with respect to the same induced operations.

3. Equivalent Conditions

Theorem 3.1. *Let (F, A) soft neurosophic near - ring over $\langle N \cup I \rangle$. Then (F, A) smarandache- soft neutrosophic - near ring if and only if there exist a proper non - empty subset (H, A) of (F, A) satisfies the following conditions:*

(i) For all $H(a), H(b)$ in (H, A) such that $H(a) - H(b)$ in (H, A).

(ii) For all $H(a), H(b)$ in (H, A) such that $H(a)[H(b)^{-1}]$ in (H, A)

(iii) For all $H(a), H(b), H(c)$ in (H, A) such that $H(a).(H(b) + H(c)) = H(a).H(b) + H(a).H(c)$.

Proof. PART-I :*(i)* we assume that (H, A) be a non-empty proper subset of (F, A) such that $H(a), H(b)$ in $(H, A) \Rightarrow H(a) - H(b)$ in (H, A).

To prove that (H, A) is a abelian group under $'+'$. Since $(H, A) \neq \phi$, there exist an element $H(a)$ in (H, A).Hence $H(a) - H(a)$ in (H, A).
Thus $H(e)$ in (H, A). Also since $H(e), H(a)$ in $(H, A), H(e) - H(a)$ in (H, A).
Hence $-H(a)$ in (H, A).
Now, let $H(a), H(b)$ in (H, A).
Then $H(a), -H(b)$ in (H, A).
Hence $H(a) - (-H(b)) = H(a)H(b)$ in (H, A).
Thus (H, A) is closed under $'+'$.
Therefore (H, A) is an abelian soft neutrosophic group.

(ii) we assume that (H, A) be a non-empty proper subset of (H, A) such that $H(a), H(b)$ in $(H, A) \Rightarrow H(a)H(b)^{-1}$ in (H, A).
To prove that (H, A) is a abelian group under $'\cdot'$.
Since $(H, A) \neq \phi$, there exist an element $H(a)$ in (H, A).
Hence $H(a)H(a)^{-1}$ in (H, A).
Thus $H(e)$ in (H, A).
Also since $H(e), H(a)$ in $(H, A), H(e)H(a)^{-1}$ in (H, A).
Hence $H(a)^{-1}$ in (H, A).
Now, let $H(a), H(b)$ in (H, A).
Then $H(a), H(b)^{-1}$ in (H, A).
Hence $H(a)(H(b)^{-1})^{-1} = H(a)H(b)$ in (H, A).Thus (H, A) is closed under $'\cdot'$.
Therefore (H, A) is an abelian soft neutrosophic group.
Evidently by *(i)* and *(ii)*, *(iii)* is satisfied.

Therefore $((H, A), \cdot, +)$ is a soft neutrosophic field.
Since every soft neutrosophic field is a soft neutrosophic near-field.
Therefore (H, A) is soft neutrosophic near-field.
Then by definition, (F, A) is smarandache - soft neutrosophic - near ring.

PART-II : Conversely, we assume that (F, A) is smarandache- soft neutrosophic - near ring.
By definition, there exist a proper subset (H, A) is soft neutrosophic near-field.
Therefore in (H, A), the conditions are trivially hold. □

Theorem 3.2. *Let (F,A) be a soft neurtosophic near-ring over $\langle N \cup I \rangle$. Where N is a near-ring. Then (F,A) is a smarandache- soft neutrosophic near - ring.if and only if for every H(a) $\neq$ 0 in (H,A),there exist a unique H(b) in (H,A) such that H(a)H(b)H(a) =H(a), where (H,A) is a soft neutrosophic near-ring,which is a proper subset of (F,A).*

Proof. Part-I: we assume that for every $H(a) \neq 0$ in (H, A). There exist a unique $H(b)$ in (H, A) such that $H(a)H(b)H(a) = H(a)$.
Now to claim that (H, A) is a Soft neutrosophic near-field. Let $H(a) \neq 0$ and $H(b) \neq 0$ in (H, A) then $H(a)H(b) \neq 0$.
For, if not there exist $H(x)$ in (H, A), such that $H(b)H(x)H(b) = H(b)$
Now $H(b)H(x - a)H(b) = H(b)H(x)H(b) = H(b)$
By uniqueness $H(x) - H(a) = H(x)$. Hence $H(a) = 0$, contradiction.
Hence (H, A) is without zero divisors. (H, A) is zerosymmetric,

Since for any $H(n)$ in $(H, A), (H(n)0)(0)(H(n)0) = H(n)0$ and
$(H(n)0)(H(n)0)(H(n)0) = H(n)0$. Hence by uniqueness $H(n)0 = 0$.
Given $H(a) \neq 0$ in (H, A), there exist a unique $H(b)$ in (H, A)
such that $H(a)H(b)H(a) = H(a)$.
So $H(a)(H(b)H(a)H(b))H(a) = (H(a)H(b)H(a))H(b)H(a)$
$= H(a)H(b)H(a) = H(a)$. By uniqueness $H(b)H(a)H(b) = H(b)$.
Note $H(a)H(b), H(b)H(a)$ are non-zero idempotents.
Let $H(e), H(f)$ be any two non-zero idempotents. Then $H(e)H(f) \neq 0$ and there exist $H(x)$ in (H, A) such that $(H(e)H(f))H(x)(H(e)H(f)) = H(e)H(f)$ and $H(x)(H(e)H(f))H(x) = H(x)$.
Let $H(y) = H(x)H(e)$,then $(H(e)H(f))(H(y))(H(e)H(f)) = H(e)H(f)$.
Hence by uniqueness $H(x) = H(y) = H(x)H(e)$.

Similarly if $H(y) = H(f)H(x)$.
We have $H(x) = H(y) = H(f)H(x)$
So $H(x^2) = (H(x)H(e))((H(f)H(x)) = H(x)((H(e)H(f))H(x) = H(x)$.
Hence $H(x^3) = H(x)$ and $H(x)((H(e)H(f))H(x) = H(x)$.
By uniqueness $H(x) = H(e)H(f)$, which is an idempotent.
So $(H(e)H(f))H(e)(H(e)H(f)) = H(e)H(f)$ and $(H(e)H(f))H(f)(H(e)H(f)) =$

$H(e)H(f)$. Hence $H(e) = H(f)$.
So (H, A) contains only one non-zero idempotent say $H(e)$.
We have shown that $(H, A)^*$ is closed. For any $H(a)$ in $(H, A)^*$, there exist $H(b)$ in (H, A) such that $H(a)H(b)H(a) = H(a)$.
Since $H(b)H(a)$ is a non-zero idempotent $H(a)H(e) = H(a)$.
Hence $H(e)$ is right identity for $(H, A)^*$.
Since $H(a)H(b)$ is a non-zero idempotent $H(a)H(b) = H(e)$.
Hence $H(b)$ is the right inverse of $H(a)$.
Hence $((H, A)^*, \cdot)$ is a group.
So (H, A) is a soft neutrosophic near-field.
By definition, (F, A) is a smarandache-soft neutrosophic-near ring.

PART-II: we assume that (F, A) is a smarandache- soft neutrosophic- near ring. Then by definition, there exist a proper subset (H, A) of (F, A) is a soft neutrosophic near-field. Now to prove that $H(a)H(b)H(a) = H(a)$.
We have (H, A) be a soft Neutrosophic near-ring over $\langle N \cup I\rangle$ with more than one element. Then the non-zero elements of (H, A) form a group under multiplication if and only if for every $H(a) \neq 0$ in (H, A), there exist a unique $H(b)$ in (H, A) such that $H(a)H(b)H(a) = H(a)$. □

Theorem 3.3. *Let (F, A) be a soft neurtosophic near-ring over $\langle N \cup I\rangle$. Then (F, A) is a smarandache - soft neutrosophic - near ring.if and only if for each $H(a) \neq 0$ in $(H, A), (H, A)H(a) = (H, A)$ and $(H, A)0 \neq (H, A)$,where (H, A) is a soft neurosophic near-ring,which is a proper subset of (F, A), in which idempotents commute.*

Proof. PART-I:we assume that for each $H(a) \neq 0$ in $(H, A), (H, A)H(a) = (H, A)$ and $(H, A)0 \neq (H, A)$.
To prove that (H, A) is a soft neutrosophic near-field.
Let $H(a) \neq 0$ and $H(b) \neq 0$ be in (H, A).
Then $H(a)H(b) \neq 0$, If not then $(H, A)H(a) = (H, A)$,
so $(H, A)H(a)H(b) = (H, A)H(b)$.
Hence $(H, A)0 = (H, A)H(b) = (H, A)$, a contradiction.
Hence (H, A) is a without zero divisors. Given $H(a) \neq 0$ in (H, A), there exist $H(y)$ in (H, A) such that $H(y)H(a) = H(a)$.
So $(H(a) - H(a)H(y))H(a) = 0$. Hence $H(a) = H(a)H(y) = H(y)H(a)$.
There exist $H(x)$ in (H, A) such that $H(x)H(a) = H(y)$.
Now $H(a)H(x)H(a) = H(a)H(y) = H(a)$. Hence (H, A) is regular.
Let $H(b) = H(x)H(a)H(x)$.
Then $H(a)H(b)H(a)$
$= H(a)(H(x)H(a)H(x))H(a)$
$= (H(a)H(x)H(a))H(x)H(a)$
$= H(a)H(x)H(a)$
$= H(a)$. and
$H(b)H(a)H(b)$

$= (H(x)H(a)H(x))H(a)(H(x)H(a)H(x))$
$= H(x)(H(a)H(x))H(a))H(x)H(a)H(x)$
$= H(x)(H(a)H(x)H(a))H(x)$
$= H(x)H(a)H(x)$
$= H(b)$.
Now let us show that there exists only one
$H(b)$ in (H, A) satisfying $H(a)H(b)H(a) = H(a)$.
If possible let us assume $H(x)$ in (H, A) satisfying $H(a)H(x)H(a) = H(a)$.
As $H(b)H(a)$ and $H(x)H(a)$ are idempotents.
We have, $(H(x)H(a)H(b) - H(b))H(a)H(b)$
$= (H(x)H(a)H(b))(H(a)H(b)) - H(b)H(a)H(b)$
$= (H(x)H(a))(H(b)H(a))H(b) - H(b)H(a)H(b)$
$= (H(b)H(a))(H(x)H(a))H(b) - H(b)H(a)H(b)$
$= H(b)(H(a)H(x)H(a))H(b) - H(b)H(a)H(b)$
$= H(b)H(a)H(b) - H(b)H(a)H(b) = 0$
Since $H(a)H(b) \neq 0$, we have $H(x)H(a)H(b) = H(b)$.
So $(H(b) - H(x))H(a)H(b)$
$= H(b)H(a)H(b) - H(x)H(a)H(b)$
$= H(b) - H(b) = 0$.
So $H(b) = H(x)$.
Hence by theorem 2, (H, A) is a near - field. So (H, A) is a soft neutrosophic near-field. By definition, (F, A) is a smarandache - soft neutrosophic - near ring.
PART-II: We assume that (H, A) is a smarandache - soft neurtosophic - near ring. Then by, definition, there exist a proper subset (H, A) of (F, A) is a soft neutrosophic near-field. Now to prove that, for each $H(a) \neq 0$ in (H, A).
$(H, A)H(a) = (H, A)$ and $(H, A)0 \neq (H, A)$.
As given $H(a) \neq 0$ in (H, A) and for any $H(n)$ in (H, A).
$H(n) = H(n)1$
$= H(n)H(a^{-1})H(a)$
$= (H(n)H(a^{-1}))H(a)$.
Hence $H(n)$ in $(H, A)H(a)$.
So $(H, A) \subset (H, A)H(a)$.
Hence $(H, A) = (H, A)H(a)$ and clearly $(H, A)0 \neq (H, A)$. □

Definition 3.1. A soft neutrosophic subgroup (H, A) of (F, A) over $\langle N \cup I \rangle$ is called a (F, A) - subgroup if $(F, A)(H, A) \subseteq (H, A)$ and an invariant (F, A) - subgroup if in addition $(H, A)(F, A) \subseteq (H, A)$.

Theorem 3.4. *Let (F, A) be a soft neurtosophic near-ring over $\langle N \cup I \rangle$. Where N is a near-ring. Then (F, A) is a Smarandache - soft neutrosophic near - ring. if and only if (H, A) has no proper (H, A) - subgroup, where (H, A) is a soft neutrosophic near-ring which is a proper subset of (F, A) in which idempotents commute and suppose that for each $H(x) \neq 0$ in (H, A), there exist $H(y)$ in (H, A) possibly depending on $H(x)$. Such that $H(y)H(x) \neq 0$.*

Proof. PART-I: we assume that (H, A) has no proper (H, A) - subgroup.
To prove that (H, A) is a near-field.
Given for each $H(x) \neq 0$ in (H, A), there exist $H(y)$ in (H, A) possibly depending on $H(x)$ such that $H(y)H(x) \neq 0$.
For each $H(x) \neq 0$ in $(H, A), (H, A)H(x)$ is a (H, A) - subgroup of (H, A).
Since there exist $H(y)$ in (H, A). Such that $H(y)H(x) \neq 0$, $(H, A)H(x) \neq 0$.
Hence $(H, A)H(x) = (H, A)$ and clearly $(H, A)0 \neq (H, A)$.
Hence by theorem 3, (H, A) is a near-field.
By definition, (H, A) is a soft neutrosophic near-field.
Therefore (F, A) is a smarandach - soft neutrosophic - near ring.

PART-II: We assume that (F, A) is a smarandache - soft neutrosophic - near ring. Then by definition, there exist a proper subset (H, A) of (F, A) is a soft neutrosophic near-field.
Now to prove that (H, A) has no proper (H, A) - subgroup.
Since let $(G, A) \neq 0$ be a (F, A) - subgroup and let $G(a) \neq 0$ in (G, A).
Then $G(a^{-1})G(a) = 1$ in (G, A).
So for any $G(n)$ in $(G, A), G(n) = G(n)1$ in (G, A). So $(H, A) = (G, A)$. □

4. Conclusion

In this paper four equivalent conditions are obtained for a soft neutrosophic near-ring to be a Smarandache- soft neutrosophic - near ring.

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