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# Solving multi-objective linear fractional transportation problem under neutrosophic environment

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## Abstract

We are discussing multi-objective linear fractional transportation problem (MOLFTP) under neutrosophic environment in the present paper. In proposed methodology, we characterize each objective of our MOLFTP in three different ways such that truth membership; indeterminacy membership and falsity membership functions. Proposed methodology gives us best neutrosophic compromise programming model under the given membership functions. In the given approach, we convert MOLFTP model into the non-linear integer programming model using proposed approach and solve it using MATLAB. To support the theory a numerical example is also discussed.

***Subject Classification:*** *90C10, 90C29, 90C32, 90C30.*

***Keywords:*** *Integer programming, Multi-objective linear fractional transportation problem (MOLFTP), Neutrosophic compromise programming, Non-linear programming (NLP).*

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## 1. Introduction

The way of transportation is defined as a particular movement of object, humans and animals, from one place to another place. Starting research on the transportation problem is mainly focusing on single objective with different variations. In recent optimization problems, analysts often deal with multi-objective transportation problems. The purpose multi-objective optimization is to find the best compromise solution among all the defined and conflicting objectives. Transportation distribution with fractional objective can be formulated as multi-objective problem to identify the true benefits and give better opportunity to understand the uncertain future problems.

A fuzzy programming approach to the multi-objective solid transportation problem and obtain an optimal compromise solution using efficient solutions as well as a linear membership function were discussed by Bit, Biswal and Alam [3]. In Li and Lai [5], they show that preference of decision-maker's must be taken to calculate the best compromise optimal solution. Joshi and Gupta [4] proposed new model for finding solution of MOLFTP. Using this model, the decision-maker can consistently produce results consistent with the expectation that a higher priority goal may have higher levels of satisfaction.

Recently, the broader forms of FST and IFS are neutrosophic collections proposed by Smarandache [8]. The degree of truth, uncertainty, and falsehood are between [0, 1]. This is different from intuitionistic fuzzy sets, where the uncertainty involved in intuitionistic fuzzy sets depends on the correlation and the degree of separation; there is uncertainty, in other words, uncertainty does not depend on right and wrong values.

A compromise programming problem is designed for multi-objective linear transportation problem (MOLTP) by Rijk-Allah, Hassanien, and Elhoseny [7]. Our approach is the extension of the article [7] for solving the MOLFTP in neutrosophic environment.

Abdelfattah [10] proposed that when solving the neutrosophic linear programming model in the linear programming model, the coefficients should be fixed to a specific value in advance, and all the coefficients are represented by triangular neutrosophic numbers. Khurana [11] put forward two cases. The first case is to maintain reserve inventory at the source in an emergency, thereby limiting the total flow to a known specified level. The second case is that the additional demand of the market forces some factories to increase production, thereby increasing the total flow, to fully satisfy customer needs.

A new approach for solving MOLFTP under neutrosophic environment is discussed in this paper. This article is prepared as follows: the MOLFTP formulation in section 2 outlines; Definition and basic concepts are discussed in Section 3; In Section 4 the neutrosophic methodology for fractional transportation problem discussed; In section 5 numerical example discussed in support of theory for the MOLFTP problem; In the last section conclusions are discussed.

## 2. Multi-objective linear fractional transportation problem (MOLFTP)

Let $Z_k, k = 1,2,3,\ldots,K$ objectives to be minimized. Mathematical formulation of MOLFTP is as follows:

$$P^1: \quad \text{Min } Z_k(x_{ij}) = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^k x_{ij}}, \quad k = 1,2,3,\ldots,K$$

Subject to

$$\begin{aligned} &\sum_{i=1}^{m} x_{ij} = a_i, \quad j = 1,2,3,\ldots,n \\ &\sum_{j=1}^{n} x_{ij} = b_j, \quad i = 1,2,3,\ldots,m \\ &x_{ij} \geq 0 \quad i = 1,2,3,\ldots,m \text{ and } j = 1,2,3,\ldots,n \end{aligned} \tag{1}$$

We assume that $a_i \geq 0 \forall i,\ b_j \geq 0 \forall j$ and $c_{ij}^k, d_{ij}^k \geq 0 \forall i,j$ and $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. Where $a_i, i = 1,2,3,\ldots,m,$ and $b_j, j = 1,2,3,\ldots,n,$ are corresponding supply and demand points. The costfunction and profit function are denoted by $c_{ij}^k$ and $d_{ij}^k$ respectively (our objective function is the ratio of cost and profit function). Variable $x_{ij}$ denotes the number of units to be transported from $i^{th}$ origin to $j^{th}$ destination.

## 3. Definition and basic concepts

3.1. **Pareto optimal solution:**If the value of objective function cannot be increased without reducing one or more other objective values, the feasible solution is called the Pareto optimal solution.

3.2. **Compromise solution:**Considering all the standards contained in the multi-objective, the compromise (eclectic) solution is a solution that decision makers take precedence over all other solutions.

3.3. **Concept of membership function**: Membership function (MF) is an important part of fuzzy approach (FA). This allows the fuzzy approach (FA) to assess situations that are unclear and uncertain. The main purpose of a membership function is to do something as the member of the fuzzy set along with human personal and subjective perception. Choosing the right membership function is an important step in building the right fuzzy set application. In many cases, to avoid nonlinearity, linear membership functions are used.

3.4. **Ideal solution:** the ideal solution of the problem is that where each objective function achieving its optimum min (In case of minimization problem).

3.5. **Anti-ideal solution:** the anti ideal solution of the problem is that where each objective function achieving its optimum max (In case of minimization problem).

3.6. **Neutrosophic set:** we characterized a neutrosophic set using three membership functions: truth, indeterminacy and falsity $(T_N(X), I_N(X)$ and $F_N(X))$. Where $X$ is set of objects or points. In the neutrosophic set $T_N(x), I_N(x)$ and $F_N(x)$ belong to $(0^-, 1^+)$. Then the neutrosophic set is defined as follows:

$$N = \{\langle x, T_N(x), I_N(x), F_N(x)\rangle \mid x \in X\}$$

## 4. Neutrosophic procedure for MOFTP

This section describes a new approach to MOLFTP based on neutrophic sets. Numerical methods influence the proposed method by extending the concept of Zimmermann. The proposed neutrosophic approach gives a new insight to deal with uncertainty prevailing in probes, aimed at minimizing truth (satisfaction), falsifiability (dissatisfaction) and maximum degree uncertainty (Somewhat Satisfaction) of Neutrosophic Decisions.

The fuzzy decision defined by [2] is $D = G \cap C$ where three different type of fuzzy set defines namely:fuzzy decisions (D) fuzzy goals (G) and fuzzy constructions (C). Consequently, the neutrosophic decision set $D_N$ defined as follows:

$$D_N = \left(\bigcap_{k=1}^{K} G_k\right)\left(\bigcap_{i=1}^{m} C_i\right) = \left(x, T_D(x), I_D(x), F_D(x)\right)$$

Where

$$T_D(x) = \min \begin{Bmatrix} T_{G_1}(x), T_{G_2}(x), \ldots, T_{G_K}(x) \\ T_{C_1}(x), T_{C_2}(x), \ldots, T_{C_m}(x) \end{Bmatrix} \text{ for all } x \in X$$

$$I_D(x) = \min \begin{Bmatrix} I_{G_1}(x), I_{G_2}(x), \ldots, I_{G_K}(x) \\ I_{C_1}(x), I_{C_2}(x), \ldots, I_{C_m}(x) \end{Bmatrix} \text{ for all } x \in X$$

$$F_D(x) = \min \begin{Bmatrix} F_{G_1}(x), F_{G_2}(x), \ldots, F_{G_K}(x) \\ F_{C_1}(x), F_{C_2}(x), \ldots, F_{C_m}(x) \end{Bmatrix} \text{ for all } x \in X$$

Where $T_D(x), I_D(x)$ and $I_D(x)$ shows the membership functions related to true, uncertainty and false-co-working in the neutrosophic decision set $D_N$. To design the MOLFTP membership function, we calculate the lower and upper bound for each objectives ($L_k$ and $U_k$). In this process we are solving all the K objectives individually and obtain the $K$ solution. By using these solution we found the boundary for each and every objective.

$U_k = \max\{F_k(x)\}$ and $L_k = \min\{F_k(x)\}$.

$U_k^T = U_k$, $L_k^T = L_k$ for truth membership

$U_k^F = U_k^T$, $L_k^F = L_k^T + t_k(U_k^T - L_k^T)$, $t_k \in (0,1)$ for falsity membership

$U_k^I = L_k^T + s_k(U_k^T - L_k^T)$, $L_k^I = L_k^T$, $s_k \in (0,1)$ for indeterminacy membership

So the membership function are as follows:

$$T_k(Z_k(x)) = \begin{cases} 1 & \text{if } Z_k(x) < L_k^T \\ 1 - \frac{Z_k(x) - L_k^T}{U_k^T - L_k^T} & \text{if } L_k^T \le Z_k(x) \le U_k^T \\ 0 & \text{if } Z_k(x) > U_k^T \end{cases}$$

$$I_k(Z_k(x)) = \begin{cases} 1 & \text{if } Z_k(x) < L_k^I \\ 1 - \frac{Z_k(x) - L_k^I}{U_k^I - L_k^I} & \text{if } L_k^I \le Z_k(x) \le U_k^I \\ 0 & \text{if } Z_k(x) > U_k^I \end{cases}$$

$$F_k\left(Z_k\left(x\right)\right) = \begin{cases} 1 & \text{if } Z_k(x) < L_k^F \\ 1 - \frac{Z_k(x) - L_k^F}{U_k^F - L_k^F} & \text{if } L_k^F \le Z_k(x) \le U_k^F \\ 0 & \text{if } Z_k(x) > U_k^F \end{cases}$$

In each membership function $U_k^{(\cdot)} \neq L_k^{(\cdot)}$. If they are equal then corresponding membership function value will be taken to one. Then eutrosophic optimization problem of MOLFTP can be started as follows:

$$P^2 \qquad \begin{aligned} &Max\ \min_{k=1,2,\dots,K} \quad T_k(Z_k(x)) \\ &Min\ \max_{k=1,2,\dots,K} \quad F_k(Z_k(x)) \\ &Max\ \min_{k=1,2,\dots,K} \quad I_k(Z_k(x)) \end{aligned}$$

s.t.

$$\begin{aligned} &\sum\nolimits_{i=1}^{m} x_{ij} = a_{i,} \qquad j = 1,2,3,\dots,n \\ &\sum\nolimits_{j=1}^{n} x_{ij} = b_j, \qquad i = 1,2,3,\dots,m \\ &x_{ij} \geq 0 \quad i = 1,2,3,\dots,m \ \text{ and } \quad j = 1,2,3,\dots,n \end{aligned}$$

Above problem can be transformed into the following problem under neutrosophic environment

$$P^3 \qquad \begin{aligned} &Max\,\alpha \\ &Max\,\gamma \\ &Min\,\beta \end{aligned}$$

s.t.

$$\begin{aligned} &T_{Z_k} \geq \alpha, \quad I_{Z_k} \geq \gamma, \quad F_{Z_k} \leq \beta \\ &\sum\nolimits_{i=1}^{m} x_{ij} = a_{i,} \qquad j = 1,2,3,\dots,n \\ &\sum\nolimits_{j=1}^{n} x_{ij} = b_j, \qquad i = 1,2,3,\dots,m \\ &x_{ij} \geq 0 \quad i = 1,2,3,\dots,m \ \text{ and } \quad j = 1,2,3,\dots,n \\ &\alpha \geq \gamma,\ \alpha \geq \beta,\ \alpha + \gamma + \beta \leq 3,\ \alpha, \gamma, \beta \in [0,1] \end{aligned}$$

$$P^4 \qquad Max\ \ \alpha - \beta + \gamma$$

s.t.

$$\begin{aligned} &\sum\nolimits_{i=1}^{m} x_{ij} = a_{i,} \qquad j = 1,2,3,\dots,n \\ &\sum\nolimits_{j=1}^{n} x_{ij} = b_j, \qquad i = 1,2,3,\dots,m \end{aligned}$$

$$
\begin{aligned}
&Z_k(x)+(U_k^T-L_k^T)\alpha \le U_k^T,\\
&Z_k(x)+(U_k^I-L_k^I)\alpha \le U_k^I,\\
&Z_k(x)-(U_k^F-L_k^F)\alpha \le U_k^F,\\
&\alpha \ge \gamma,\quad \alpha \ge \beta,\quad \alpha+\gamma+\beta \le 3,\\
&\alpha,\gamma,\beta \in [0,1],\quad k=1,2,\dots,K\\
&x_{ij} \ge 0 \quad i=1,2,3,\dots,m \quad \text{and} \quad j=1,2,3,\dots,n
\end{aligned}
$$

## 5. Numerical example:

Consider the following three-objective MOLFTP (table 1) for demonstration of the proposed model

$$
\begin{aligned}
\text{Min}\quad Z_1 = {}&(5*x_{11}+2*x_{12}+3*x_{13}+7*x_{14}+16*x_{21}+8*x_{22}+9*x_{23}\\
&+10*x_{24}+12*x_{31}+9*x_{32}+14*x_{33}+13*x_{34})/(9*x_{11}\\
&+5*x_{12}+9*x_{13}+2*x_{14}+8*x_{21}+13*x_{22}+7*x_{23}+3*x_{24}\\
&+9*x_{31}+10*x_{32}+6*x_{33}+6*x_{34})
\end{aligned}
$$

$$
\begin{aligned}
\text{Min}\quad Z_2 = {}&(6*x_{11}+3*x_{12}+9*x_{13}+9*x_{14}+2*x_{21}+9*x_{22}+2*x_{23}\\
&+6*x_{24}+5*x_{31}+12*x_{32}+8*x_{33}+8*x_{34})/(12*x_{11}+7*x_{12}\\
&+8*x_{13}+15*x_{14}+6*x_{21}+8*x_{22}+2*x_{23}+5*x_{24}+8*x_{31}\\
&+11*x_{32}+7*x_{33}+7*x_{34})
\end{aligned}
$$

**Table 1**

**Cost matrix for the MOLFTP with three objective**

| | | $D_1$ | | | $D_2$ | | | $D_3$ | | | **$D_4$** | | | **$a_i$** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **objectives** | | I | II | III | I | II | III | I | II | III | I | II | III | |
| **$O_1$** | $c_{ij}^k$ | 5 | 6 | 8 | 2 | 3 | 7 | 3 | 9 | 5 | 7 | 9 | 12 | 7 |
| | $d_{ij}^k$ | 9 | 12 | 7 | 5 | 7 | 2 | 9 | 8 | 7 | 2 | 15 | 6 | |
| **$O_2$** | $c_{ij}^k$ | 16 | 2 | 9 | 8 | 9 | 5 | 9 | 2 | 3 | 10 | 6 | 13 | 9 |
| | $d_{ij}^k$ | 8 | 6 | 9 | 13 | 8 | 5 | 7 | 2 | 5 | 3 | 5 | 9 | |
| **$O_3$** | $c_{ij}^k$ | 12 | 5 | 11 | 9 | 2 | 13 | 14 | 8 | 8 | 13 | 8 | 4 | 18 |
| | $d_{ij}^k$ | 9 | 8 | 2 | 10 | 11 | 7 | 6 | 7 | 12 | 6 | 7 | 8 | |
| **$b_j$** | | 5 | | | 8 | | | 7 | | | 14 | | | 34 |

$$\text{Min} \quad Z_3 = (8*x_{11} + 7*x_{12} + 5*x_{13} + 12*x_{14} + 9*x_{21} + 5*x_{22} + 3*x_{23} + 13*x_{24} + 11*x_{31} + 13*x_{32} + 8*x_{33} + 4*x_{34}) / (7*x_{11} + 2*x_{12} + 7*x_{13} + 6*x_{14} + 9*x_{21} + 5*x_{22} + 5*x_{23} + 9*x_{24} + 2*x_{31} + 7*x_{32} + 12*x_{33} + 8*x_{34})$$

Subject to

$$\begin{aligned}
x_{11} + x_{12} + x_{13} + x_{14} &= 7 \\
x_{21} + x_{22} + x_{23} + x_{24} &= 9 \\
x_{31} + x_{32} + x_{33} + x_{34} &= 18 \\
x_{11} + x_{21} + x_{31} &= 5 \\
x_{12} + x_{22} + x_{32} &= 8 \\
x_{13} + x_{23} + x_{33} &= 7 \\
x_{14} + x_{24} + x_{34} &= 14
\end{aligned}$$

$x_{ij} \geq 0$ and integer $\forall i, j$.

Solve the following problem as single objective to get individual solution as $X^1 = (0, 0, 7, 0, 0, 8, 0, 1, 5, 0, 0, 13)$, $X^2 = (0, 0, 0, 7, 2, 0, 7, 0, 3, 8, 0, 7)$ and $X^3 = (5, 0, 2, 0, 0, 8, 1, 0, 0, 0, 4, 14)$

Calculate the objectives using the solutions obtained and then set a limit for each objective:

$$Z_1(X^1) = 1.105802,\ Z_2(X^1) = 1.054688,\ Z_3(X^1) = 0.994898,$$

$$Z_1(X^2) = 1.504386,\ Z_2(X^2) = 0.849315,\ Z_3(X^2) = 1.352113 \text{ and}$$

$$Z_1(X^3) = 1.212766,\ Z_2(X^3) = 0.992537,\ Z_3(X^3) = 0.712598,$$

i.e., $1.105802 \leq Z_1 \leq 1.504386$; $0.849315 \leq Z_2 \leq 1.054688$ and $0.712598 \leq Z_3 \leq 1.352113$ by using the neutrosophic theory we get

For $Z_1$:

$$\begin{aligned}
&U_{Z_1}^T = 1.504386, L_{Z_1}^T = 1.105802 \\
&U_{Z_1}^F = U_{Z_1}^T = 1.504386, L_{Z_1}^F = L_{Z_1}^T + t_1 = 1.105802 + t_1 \\
&U_{Z_1}^F = L_{Z_1}^T + s_1 == 1.105802 + s_1, L_{Z_1}^I = L_{Z_1}^T = 1.105802
\end{aligned}$$

$$T_1(Z_1(x)) = \begin{cases} 1 & \text{if } Z_1(x) < 1.105802 \\ 1 - \frac{Z_1(x) - 1.105802}{1.504386 - 1.105802} & \text{if } 1.105802 \leq Z_1(x) \leq 1.504386 \\ 0 & \text{if } Z_1(x) > 1.504386 \end{cases}$$

$$I_1(Z_1(x)) = \begin{cases} 1 & \text{if } Z_1(x) < 1.105802 \\ 1 - \frac{Z_1(x) - 1.105802}{s_1} & \text{if } 1.105802 \le Z_1(x) \le 1.105802 + s_1 \\ 0 & \text{if } Z_1(x) > 1.105802 + s_1 \end{cases}$$

$$F_1(Z_1(x)) = \begin{cases} 1 & \text{if } Z_1(x) > 1.504386 \\ 1 - \frac{1.504386 - Z_1(x)}{1.504386 - 1.105802 - t_1} & \text{if } 1.105802 + t_1 \le Z_1(x) \le 1.504386 \\ 0 & \text{if } Z_1(x) < 1.105802 + t_1 \end{cases}$$

For $Z_2$:

$$U_{Z_2}^T = 1.054688,\ L_{Z_2}^T = 0.849315$$
$$U_{Z_2}^F = U_{Z_2}^T = 1.054688,\ L_{Z_2}^F = L_{Z_2}^T + t_2 = 0.849315 + t_2$$
$$U_{Z_2}^F = L_{Z_2}^T + s_2 = 0.849315 + s_2,\ L_{Z_2}^I = L_{Z_2}^T = 0.849315$$

$$T_2(Z_2(x)) = \begin{cases} 1 & \text{if } Z_2(x) < 1.105802 \\ 1 - \frac{Z_2(x) - 0.849315}{1.054688 - 0.849315} & \text{if } 0.849315 \le Z_2(x) \le 1.054688 \\ 0 & \text{if } Z_2(x) > 1.054688 \end{cases}$$

$$I_2(Z_2(x)) = \begin{cases} 1 & \text{if } Z_2(x) < 0.849315 \\ 1 - \frac{Z_2(x) - 0.849315}{s_2} & \text{if } 0.849315 \le Z_2(x) \le 0.849315 + s_2 \\ 0 & \text{if } Z_2(x) > 0.849315 + s_2 \end{cases}$$

$$F_2(Z_2(x)) = \begin{cases} 1 & \text{if } Z_2(x) > 1.054688 \\ 1 - \frac{1.054688 - Z_2(x)}{1.054688 - 0.849315 - t_2} & \text{if } 0.849315 + \mathrm{t}_2 \le Z_2(x) \le 1.054688 \\ 0 & \text{if } Z_2(x) < 0.849315 + t_2 \end{cases}$$

For $Z_3$:

$$U_{Z_3}^T = 1.352113,\ L_{Z_3}^T = 0.712598$$
$$U_{Z_3}^F = U_{Z_3}^T = 1.352113,\ L_{Z_3}^F = L_{Z_3}^T + t_3 = 0.712598 + t_3$$
$$U_{Z_3}^F = L_{Z_3}^T + s_3 = 0.712598 + s_3,\ L_{Z_3}^I = L_{Z_3}^T = 0.712598$$

$$T_3(Z_3(x)) = \begin{cases} 1 & \text{if } Z_3(x) < 0.712598 \\ 1 - \frac{Z_3(x) - 0.712598}{1.352113 - 0.712598} & \text{if } 0.712598 \le Z_3(x) \le 1.352113 \\ 0 & \text{if } Z_3(x) > 1.352113 \end{cases}$$

$$I_3(Z_3(x)) = \begin{cases} 1 & \text{if } Z_3(x) < 0.712598 \\ 1 - \frac{Z_3(x) - 0.849315}{s_3} & \text{if } 0.712598 \le Z_3(x) \le 0.712598 + s_3 \\ 0 & \text{if } Z_3(x) > 0.712598 + s_3 \end{cases}$$

$$F_3(Z_3(x)) = \begin{cases} 1 & \text{if } Z_3(x) > 1.352113 \\ 1 - \frac{1.352113 - Z_3(x)}{1.352113 - 0.849315 - t_3} & \text{if } 0.712598 + t_3 \le Z_3(x) \le 1.352113 \\ 0 & \text{if } Z_3(x) < 0.712598 + t_3 \end{cases}$$

Formation of MOLFTP neutrosophic model for the given problem:

$$Max\, \alpha - \beta + \gamma$$

Subject to:

$$\begin{aligned} x_{11} + x_{12} + x_{13} + x_{14} &= 7 \\ x_{21} + x_{22} + x_{23} + x_{24} &= 9 \\ x_{31} + x_{32} + x_{33} + x_{34} &= 18 \\ x_{11} + x_{21} + x_{31} &= 5 \\ x_{12} + x_{22} + x_{32} &= 8 \\ x_{13} + x_{23} + x_{33} &= 7 \\ x_{14} + x_{24} + x_{34} &= 14 \end{aligned}$$

$$\frac{5*x_{11} *2*x_{12} + 3*x_{13} + 7*x_{14} + 16*x_{21} + 8*x_{22} + 9*x_{23} + 10*x_{24} + 12*x_{31} + 9*x_{32} + 14*x_{33} + 13*x_{34}}{9*x_{11} + 5*x_{12} + 9*x_{13} + 2*x_{14} + 8*x_{21} + 13*x_{22} + 7*x_{23} + 3*x_{24} + 9*x_{31} + 10*x_{32} + 6*x_{33} + 6*x_{34}}$$
$$+0.398584\alpha \le 1.504386$$

$$\frac{6*x_{11} + 3*x_{12} + 9*x_{13} + 9*x_{14} + 2*x_{21} + 9*x_{22} + 2*x_{23} + 6*x_{24} + 5*x_{31} + 12*x_{32} + 8*x_{33} + 8*x_{34}}{12*x_{11} + 7*x_{12} + 8*x_{13} + 15*x_{14} + 6*x_{21} + 8*x_{22} + 2*x_{23} + 5*x_{24} + 8*x_{31} + 11*x_{32} + 7*x_{33} + 7*x_{34}}$$
$$+0.205373\alpha \le 1.054688$$

$$\frac{8*x_{11} + 7*x_{12} + 5*x_{13} + 12*x_{14} + 9*x_{21} + 5*x_{22} + 3*x_{23} + 13*x_{24} + 11*x_{31} + 13*x_{32} + 8*x_{33} + 4*x_{34}}{7*x_{11} + 2*x_{12} + 7*x_{13} + 6*x_{14} + 9*x_{21} + 5*x_{22} + 5*x_{23} + 9*x_{24} + 2*x_{31} + 7*x_{32} + 12*x_{33} + 8*x_{34}}$$
$$+0.639515\alpha \le 1.352113$$

$$\frac{5*x_{11}+2*x_{12}+3*x_{13}+7*x_{14}+16*x_{21}+8*x_{22}+9*x_{23}+10*x_{24}+12*x_{31}+9*x_{32}+14*x_{33}+13*x_{34}}{9*x_{11}+5*x_{12}+9*x_{13}+2*x_{14}+8*x_{21}+13*x_{22}+7*x_{23}+3*x_{24}+9*x_{31}+10*x_{32}+6*x_{33}+6*x_{34}}$$
$$+t_1\gamma - t_1 \le 1.105802$$

$$\frac{6*x_{11}+3*x_{12}+9*x_{13}+9*x_{14}+2*x_{21}+9*x_{22}+2*x_{23}+6*x_{24}+5*x_{31}+12*x_{32}+8*x_{33}+8*x_{34}}{12*x_{11}+7*x_{12}+8*x_{13}+15*x_{14}+6*x_{21}+8*x_{22}+2*x_{23}+5*x_{24}+8*x_{31}+11*x_{32}+7*x_{33}+7*x_{34}}$$
$$+t_2\gamma - t_2 \le 0.849315$$

$$\frac{8*x_{11}+7*x_{12}+5*x_{13}+12*x_{14}+9*x_{21}+5*x_{22}+3*x_{23}+13*x_{24}+11*x_{31}+13*x_{32}+8*x_{33}+4*x_{34}}{7*x_{11}+2*x_{12}+7*x_{13}+6*x_{14}+9*x_{21}+5*x_{22}+5*x_{23}+9*x_{24}+2*x_{31}+7*x_{32}+12*x_{33}+8*x_{34}}$$
$$+t_3\gamma - t_3 \le 0.712598$$

$$\frac{5*x_{11}+2*x_{12}+3*x_{13}+7*x_{14}+16*x_{21}+8*x_{22}+9*x_{23}+10*x_{24}+12*x_{31}+9*x_{32}+14*x_{33}+13*x_{34}}{9*x_{11}+5*x_{12}+9*x_{13}+2*x_{14}+8*x_{21}+13*x_{22}+7*x_{23}+3*x_{24}+9*x_{31}+10*x_{32}+6*x_{33}+6*x_{34}}$$
$$+(\beta-1)(1.105802+s_1)-1.504386\beta \le 0$$

$$\frac{6*x_{11}+3*x_{12}+9*x_{13}+9*x_{14}+2*x_{21}+9*x_{22}+2*x_{23}+6*x_{24}+5*x_{31}+12*x_{32}+8*x_{33}+8*x_{34}}{12*x_{11}+7*x_{12}+8*x_{13}+15*x_{14}+6*x_{21}+8*x_{22}+2*x_{23}+5*x_{24}+8*x_{31}+11*x_{32}+7*x_{33}+7*x_{34}}$$
$$+(\beta-1)(0.849315+s_2)-1.054688\beta \le 0$$

$$\frac{8*x_{11}+7*x_{12}+5*x_{13}+12*x_{14}+9*x_{21}+5*x_{22}+3*x_{23}+13*x_{24}+11*x_{31}+13*x_{32}+8*x_{33}+4*x_{34}}{7*x_{11}+2*x_{12}+7*x_{13}+6*x_{14}+9*x_{21}+5*x_{22}+5*x_{23}+9*x_{24}+2*x_{31}+7*x_{32}+12*x_{33}+8*x_{34}}$$
$$+(\beta-1)(0.712598+s_3)-1.352113\beta \le 0$$

$$\alpha \ge \gamma, \alpha \ge \beta, \alpha+\gamma+\beta \le 3, \alpha, \gamma, \beta \in [0,1],$$

$$t_1 \ge 0, \qquad s_1 \le 0.398584,$$

$$t_2 \ge 0, \qquad s_2 \le 0.205373,$$

$$t_3 \ge 0, \qquad s_3 \le 0.639515,$$

$x_{ij} \ge 0$ and integer $\forall i, j$.

The above non-linear integer programming problem solved using the MATLAB and solution are as follows for $x_{ij}$

$$x_{11}=5, x_{12}=0, x_{13}=1, x_{14}=1, x_{21}=0, x_{22}=3, x_{23}=6, x_{24}=0, x_{31}=0, x_{32}=5,$$
$$x_{33}=0, x_{34}=13,$$

$$t_1=1, t_2=1, t_3=1, s_1=0.2681, s_2=0.0805, s_3=0.5613,$$

$$\alpha=0.6081, \beta=0, \gamma=0.6081$$

**Table 2**

**Comparison of solution between proposed and other approaches [1,6,9]**

| Procedures for solving MOLFTP | First objective | Second objective | Third objective | $D_i^+$ | $D_i^+$ | $R$ | Rank |
|---|---|---|---|---|---|---|---|
| Ideal Solution | 1.105802 | 0.849315 | 0.712598 | 0 | 0.781042 | 1 | 1 |
| Anti Ideal solution | 1.504386 | 1.054688 | 1.352113 | 0.781042 | 0 | 0 | 6 |
| Proposed method | 1.233962 | 0.947169 | 0.982241 | 0.241397 | 0.544216 | 0.692728 | 2 |
| Abd El-Wahed [1] | 1.275924 | 0.912931 | 0.992517 | 0.333681 | 0.448998 | 0.573668 | 4 |
| Ringuest and Rinks [6] | 1.249772 | 0.952193 | 0.969213 | 0.311709 | 0.471111 | 0.601812 | 3 |
| Zangiabadi et al. [9] | 1.293221 | 0.951036 | 1.012892 | 0.368306 | 0.412801 | 0.528482 | 5 |

$Z_1 = 1.233962, Z_2 = 0.947169, Z_3 = 0.982241.$

Comparison of proposed method with other approaches (table 2) discussed using [7].

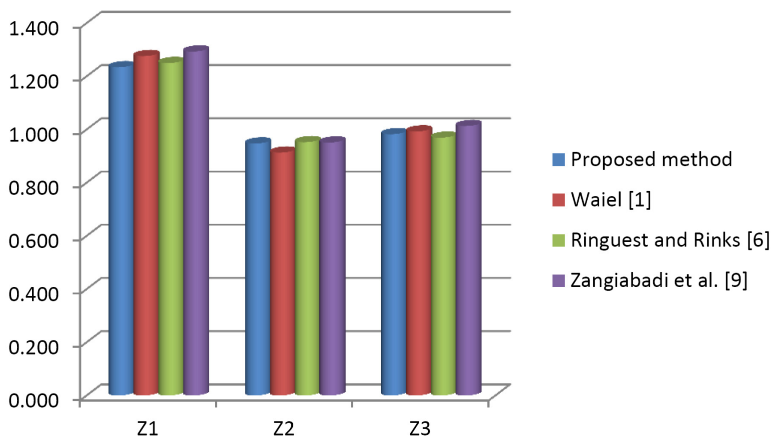


**Figure 1**

**Comparison of solution for $Z_1, Z_2, Z_3$ with different methods**

## 6. Conclusion

In this paper, we solve the MOLFTP problem in neutrosophic conditions. The methodology starts with achieving the individual minimum solution and the individual maximum and then the membership functions are constructed to the extent of truth, uncertainty, and falsehood. With the help of these conditions, we can frame neutrophic model for solving MOLFTP problem for compromise optimal solution. The efficiency of the proposed method is examined using the approach given in [7]. Proposed approach gets better results (table 2, figure 1) regarding the ranking of values with respect to other approaches [1,6,9]. This result proves the superiority of this result over existing techniques. However, to deal with uncertainty technically, the characteristics of neutrosophic need to be determined, especially when dealing with practical applications.

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