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Smarandache Soft Semigroups and their Properties

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Abstract – In this paper, the notions of smarandache soft semigroups (SS-semigroups) introduced for the first time. An SS-semigroup (F,A) is basically a parameterized collection of subsemigroups which has atlest a proper soft subgroup of (F,A). Some new type of SS-semigroup is also presented here such as smarandache weak commutative semigroup, smarandache weak cyclic semigroup, smarandache hyper subsemigroup etc. Some of their related properties and other notions have been discussed with sufficient amount of examples.

Keywords – Smarndache semigroup, soft set, soft semigroup, smarandache soft semigroup.

1 Introduction

Raul [26] introduced in 1998, the notions of Smarandache semigroup in the article "Smaradache Algebraic Structures". Smarandache semigroup is analogous to the smarandache group. F. Smarandache in [30] first introduced the theory of Smarandach algebraic structures in a paper "Special Algebraic Structures". The Smarandache semigroups exhibit characteristics and features of both groups and semigroups simultaneously. The Smarandache semigroups are a class of innovative and conceptually a new structure in nature. The concept of Smarandache algebraic structures almost exist in every algebraic structure such as Smarandache groupoid which are discussed in [17], Smarandache rings [20], Smarandache semirings, semifields, semivector spaces [18], Smarandache loops [19] etc. Kandassamy have written several books on Smarandache algebraic structures and their related theory.

Molodtsov [25] initiated the theory of soft sets in 1995. Soft set theory is a mathematical tool which is free from parameterization inadequacy, syndrome of fuzzy set theory, rough set theory, probability theory and so on. This theory has been applications in many fields

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such as smoothness of functions, game theory, operation research, Riemann integration, Perron integration, and probability etc. Recently soft set theory gain much attention of the researchers since its introduction. There are a lot of soft algebraic structures introduced in soft set theory successfully. H. Aktas and N. Cagmann [1] introduced soft groups, soft semigroups [15]. The work which is based on several operations of soft sets discussed in [12,13]. Some properties and related algebra may be found in [14]. Some other concepts and notions together with fuzzy set and rough set were studied in [23,24]. Some useful study about soft neutrosophic algebraic structures have been discussed in [3,4,5,6,7,8,9,10,11,28,29,31].

The organization of this paper is below. In first section, some basic concepts and notions about smarandache semigroups, soft sets, and soft semigroups are presented. In the next section Smarandache soft semigroup shortly SS-semigroups is introduced. In this section some related theory and characterization is also presented with illustrative examples. In the further section, Smarandache hyper soft semigroup is studied with some of their core properties.

2 Basic Concepts

In this section, fundamental concepts about Smarandache semigroups, soft sets, and soft semigroups is presented with some of their basic properties.

2.1 Smarandache Semigroups

Definition 2.1.1: A smarandache semigroup is define to be a semigroup S such that a proper subset of S is a group with respect to the same induced operation. A smarandache semigroup S is denoted by S-semigroup.

Definition 2.1.2: Let S be a smarandache semigroup. If every proper subset A in S which is a group is commutative, then S is said to be a smarandache commutative semigroup.
If S has atleast one proper subgroup, then S is called a weak smarandache commutative semigroup.

Definition 2.1.3: Let S be a smarandache semigroup. If every proper subset A of S is a cyclic group, then S is said to be a smarandache cyclic semigroup.
If S has atleast one proper cyclic subgroup, then S is called a weak smarandache commutative semigroup.

Definition 2.1.4: Let S be a smarandache semigroup. A proper subset A of S is called a samarandache subsemigroup if A itself is a smarandache semigroup under the operation of S.

Definition 2.1.5: Let S be a smarandache semigroup. If A be a proper subset of S which subsemigroup of S and A contains the largest group of S.Then A is called a smarandache hyper subsemigroup.

2.2 Soft Sets

Throughout this subsection U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U, and $A, B \subset E$. Molodtsov defined the soft set in the following manner:

Definition 2.2.1: A pair (F, A) is called a soft set over U where F is a mapping given by $F : A \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $a \in A$, $F(\mathrm{a})$ may be considered as the set of a-elements of the soft set (F, A), or as the set of a-approximate elements of the soft set.

Definition 2.2.2: For two soft sets (F, A) and (H, B) over U, (F, A) is called a soft subset of (H, B) if

1. $A \subseteq B$ and
2. $F(a) \subseteq H(a)$, for all $x \in A$

This relationship is denoted by $(F, A) \subset (H, B)$. Similarly (F, A) is called a soft superset of (H, B) if (H, B) is a soft subset of (F, A) which is denoted by $(F, A) \supset (H, B)$.

Definition 2.2.3: Two soft sets (F, A) and (H, B) over U are called soft equal if (F, A) is a soft subset of (H, B) and (H, B) is a soft subset of (F, A).

Definition 2.2.4: Let (F, A) and (K, B) be two soft sets over a common universe U such that $A \cap B \neq \phi$. Then their restricted intersection is denoted by $(F, A) \cap_R (K, B) = (H, C)$ where (H, C) is defined as $H(c) = F(c) \cap \mathrm{K}(c)$ for all $c \in C = A \cap B$.

Definition 2.2.5: The extended intersection of two soft sets (F, A) and (K, B) over a common universe U is the soft set (H, C), where $C = A \cup B$, and for all $c \in C$, $H(c)$ is defined as

$$H(c) = \begin{cases} F(c) & \text{if c} \in A - B, \\ G(c) & \text{if c} \in B - A, \\ F(c) \cap G(c) & \text{if c} \in A \cap B. \end{cases}$$

We write $(F, A) \cap_{\varepsilon} (K, B) = (H, C)$.

Definition 2.2.6: The restricted union of two soft sets (F, A) and (K, B) over a common universe U is the soft set (H, C), where $C = A \cup B$, and for all $c \in C$, $H(c)$ is defined as $H(c) = F(c) \cup G(c)$ for all $c \in C$. We write it as $(F, A) \cup_R (K, B) = (H, C)$.

Definition 2.2.7: The extended union of two soft sets (F, A) and (K, B) over a common universe U is the soft set (H, C), where $C = A \cup B$, and for all $c \in C$, $H(c)$ is defined as

$$H(c) = \begin{cases} F(c) & \text{if c} \in A - B, \\ G(c) & \text{if c} \in B - A, \\ F(c) \cup G(c) & \text{if c} \in A \cap B. \end{cases}$$

We write $(F, A) \cup_{\varepsilon} (K, B) = (H, C)$.

Definition 2.2.8: A soft set (F, A) over S is called a soft semigroup over S if $(F, A)\hat{\circ}(F, A) \subseteq (F, A)$.

It is easy to see that a soft set (F, A) over S is a soft semigroup if and only if $\phi \neq F(a)$ is a subsemigroup of S for all $a \in A$.

3 Smarandache Soft Semigroups

In this section we define smarandache soft semigroups and give some of their properties with sufficient amount of examples.

Definition 3.1: Let S be a semigroups and (F, A) be a soft semigroup over S. Then (F, A) is called a smarandache soft semigroup over U if a proper soft subset (G, B) of (F, A) is a soft group under the operation of S. We denote a smarandache soft semigroup by SS-semigroup.

A smarandache soft semigroup is a parameterized collection of smarandache subsemigroups of S.

Example 3.2: Let $\mathbb{Z}_{12} = \{0,1,2,3,...,11\}$ be the semigroup under multiplication modulo 12. Let $A = \{a_1, a_2, a_3, a_4, a_5, a_6\}$ be a set of parameters. Let (F, A) be a soft semigroup over $\mathbb{Z}_{12}$, where

$F(a_1) = \{1,3,5,9\}, \mathrm{F}(a_2) = \{1,4,7,8\}$,
$F(a_3) = \{1,5,7,11\}$, $F(a_4) = \{3,4,8,9\}$,
$F(a_5) = \{1,3,9,11\}$, $F(a_6) = \{1,4,5,8\}$.

Let $B = \{a_1, a_2, a_4, a_5\} \subset A$. Then (G, B) is a soft subgroup of (F, A) over U, where

$G(a_1) = \{1,5\}, G(a_2) = \{4,8\}$,
$G(a_4) = \{3,9\}, G(a_5) = \{1,11\}$.

Thus clearly (F, A) is a smarandache semigroup over $\mathbb{Z}_{12}$.

Proposition 3.3: If S is a smarandache semigroup, then (F, A) is also a smarandache soft semigroup over S.

Proposition 3.4: The extended union of two SS-semigroups (F,A) and (G,B) over S is a SS-semigroup over S.

Proposition 3.5: The extended intersection of two SS-semigroups (F,A) and (G,B) over S is a SS-semigroup over S.

Proposition 3.6: The restricted union of two SS-semigroups (F,A) and (G,B) over S is a SS-semigroup over S.

Proposition 3.7: The restricted intersection of two SS-semigroups (F,A) and (G,B) over S is a SS-semigroup over S.

Proposition 3.8: The AND operation of two SS-semigroups (F,A) and (G,B) over S is a SS-semigroup over S.

Proposition 3.9: The OR operation of two SS-semigroups (F,A) and (G,B) over S is a SS-semigroup over S.

Definition 3.10: Let (F,A) be a SS-semigroup over a semigroup S. Then (F,A) is called a commutative SS-semigroup if each proper soft subset (G,B) of (F,A) is a commutative group.

Definition 3.11: Let (F,A) be a SS-semigroup over a semigroup S. Then (F,A) is called a weakly commutative SS-semigroup if atleast one proper soft subset (G,B) in (F,A) is a commutative group.

Proposition 3.12: If S is a commutative S-semigroup, then (F,A) over S is also a commutative SS-semigroup.

Definition 3.13: Let (F,A) be a SS-semigroup over a semigroup S. Then (F,A) is called a cyclic SS-semigroup if each proper soft subset (G,B) of (F,A) is a cyclic subgroup.

Proposition 3.14: Let (F,A) and (H,B) be two strong soft groups over a semigroup S. Then

1. $(F,A)\cap_R (H,B)$ is a strong soft group over S.
2. $(F,A)\cap_E (H,B)$ is a strong soft group over S.

Definition 3.15: Let (F,A) be a SS-semigroup over a semigroup S. If there exist atleast one proper soft subset (G,B) of (F,A) which is a cyclic subgroup. Then (F,A) is termed as weakly cyclic SS-semigroup.

Proposition 3.16: If S is a cyclic S-semigroup, then (F,A) over S is also a cyclic SS-semigroup.

Proposition 3.17: If S is a cyclic S-semigroup, then (F, A) over S is a commutative SS-semigroup.

Definition 3.18: Let S be a semigroup and (F, A) be a SS-semigroup. A proper soft subset (G, B) of (F, A) is said to be a smarandache soft subsemigroup if (G, B) is itself a smarandache soft semigroup over S.

Definition 3.19: Let S be a semigroup and (F, A) be a soft set over S. Then S is called a parameterized smarandache semigroup if $F(a) \subset \mathrm{S}$ such that $F(a)$ is a group under the operation of S for all $a \in A$. In this case (F, A) is called a strong soft group.

A strong soft group is a parameterized collection of the subgroups of the semigroup S.

Proposition 3.20: Let S be a semigroup and (F, A) be a soft set over S. Then S is a parameterized smarandache semigroup if (F, A) is a soft group over S.

Proof: Suppose that (F, A) is a soft group over S. This implies that each $F(a)$ is a subgroup of the semigroup S for al $a \in A$ and thus S is a parameterized smarandache semigroup.

Example 3.21: Let $\mathbb{Z}_{12} = \{0,1,2,3,...,11\}$ be the semigroup under multiplication modulo 12. Let $A = \{a_1, a_2, a_3\}$ be a set of parameters. Let (F, A) be a soft semigroup over $\mathbb{Z}_{12}$, where

$F(a_1) = \{3,9\}, \mathrm{F}(a_2) = \{1,7\}$,
$F(a_3) = \{1,5\}$.

Then $\mathbb{Z}_{12}$ is a parameterized smarandache semigroup.

Proposition 3.22: Let (F, A) and (H, B) be two strong soft groups over a semigroup S. Then

1. $(F, A) \bigcap_R (H, B)$ is a strong soft group over S.
2. $(F, A) \bigcap_E (H, B)$ is a strong soft group over S.

Remark 3.23: Let (F, A) and (H, B) be two strong soft groups over a semigroup S. Then

1. $(F, A) \bigcup_R (H, B)$ need not be strong soft group over S.
2. $(F, A) \bigcup_E (H, B)$ need not be strong soft group over S.

For this, we take the following example.

Example 3.24: Let $\mathbb{Z}_{12} = \{0,1,2,3,...,11\}$ be the semigroup under multiplication modulo 12. Let $A = \{a_1, a_2, a_3, a_4, a_5\}$ be a set of parameters. Let (F, A) be a strong soft group over $\mathbb{Z}_{12}$, where

$F(a_1) = \{1,5\}, \mathrm{F}(a_2) = \{4,8\}$,
$F(a_3) = \{7,11\}$, $F(a_4) = \{3,9\}$,
$F(a_5) = \{1,11\}$.

Let $B = \{a_1, a_2, a_7\}$. Then (H, B) is another strong soft group over $\mathbb{Z}_{12}$, where

$\mathrm{H}(a_1) = \{3,9\}, \mathrm{H}(a_2) = \{1,5\}$,
$\mathrm{H}(a_7) = \{3,9\}$.

Then clearly $C = A \cap B = \{a_1, a_2\}$. Now $F(a_1) \cap H(a_1) = \{1,3,5,9\}$ and $F(a_2) \cap H(a_2) = \{1,4,5,8\}$ are not subgroups of $\mathbb{Z}_{12}$. Thus $(F, A) \cup_R (H, B)$ is not a strong soft group over $S = \mathbb{Z}_{12}$.

One can easily show 2 with the help of examples.

4 Smarandache Hyper Soft Subsemigroups

Definition 4.1: Let (F, A) be a SS-semigroup over S and (H, B) be a SS-subsemigroup of (F, A). Then (H, B) is called a smarandache hyper soft subsemigroup if (H, B) contains a proper soft subset (K, C) such that $K(c)$ is a smarandache hyper subsemigroup of S for all $c \in B$.

Theorem 4.2: Every smarandache hyper subsemigroup is a smarandache subsemigroup.
Proof: Its obvious.

Definition 4.3: Let (F, A) be a SS-semigroup. Then (F, A) is called simple SS-semigroup if (F, A) has no smarandach hyper subsemigroup.

Theorem 4.4: If S is a simple smarandache semigroup. Then (F, A) over S is also a simple SS-semigroup.

Proof. The proof is simple.

Theorem 4.5: If S is a smarandache semigroup of prime order p. Then (F, A) is a simple SS-semigroup over S.

Conclusion

In this paper Smaradache soft semigroups are introduced. Their related properties and results are explained with many illustrative examples. This theory opens a new way for researchers to define these type of soft algebraic strucutres in almost all areas of algebra in the future.

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