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Disentangling Smarandache Multispace and Multisystem with Information Decoding

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Abstract: Certainly, a Smarandache multispace or multisystem $\widetilde{S}$ is a union of m distinct spaces or systems $S_1, S_2, \cdots, S_m$ which is an appropriate model on things T in the universe because of the limitation of humans ourselves and a thing T is complex, even overlap with other things. However, nearly all observation data on T is a multiple one $\widetilde{S}$ which implies S_i and S_j are entangled if $S_i \bigcap S_j \neq \emptyset$, we have to disentangle S_i from S_j, $1 \leq i \neq j \leq m$ for hold on the reality of thing T. Thus, disentangling a multi-space $\widetilde{S}$ to self-enclosed spaces or systems $S_1, S_2, \cdots, S_m$ is interesting, also valuable in hold on the reality of things in the universe. The main purpose of this paper is to discuss the disentangling ways on a Smarandache multispace or multisystem if we assume that each self-enclosed space or system of $S_i, 1 \leq i \leq m$ is endowed with mathematical elements such as those of topological, geometrical, algebraic structures or generally, each space or system of S_i has a character χ_i different from others for integers $1 \leq i \leq m$. As it happens, this problem is equivalent to Schrödinger's cat of quantum mechanics in the case of $m = 2$, which are extensively applied in quantum teleportation for preparation, distribution and measurement of the entangled pairs of particles and prospecting us to design a general key carrier on the Smarandachely entangling pairs in commutation.

Key Words: Entanglement, disentangling, Smarandache multispace, Smarandache multisystem, Smarandachely entangling pair, mathematical combinatorics, collapse mapping, Schrödinger's cat, quantum communication, key carrier, information decoding.

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§1. Introduction

Usually, a thing T is complex, even overlap with other things in the universe and it has many characters showing in front of humans such as those of the color, smell, density, states, solubility, still or moving of physical characters; the acidity, alkalinity, oxidizability, reducibility, thermal stability of chemical characters, also a dead or living body with growth, reproduction and habitat of biological characters. Then, *how do we understand the thing* T? Notice that a character of thing T maybe integral or partial, also conditional, the answer is nothing else but

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the union or combination of all known characters of T, i.e., the *Smarandache multispace* or *Smarandache multisystem*! For example, let $\mu_1, \mu_2, \cdots, \mu_n$ be its known and $\nu_i, i \geq 1$ unknown characters at time t. Then, the reality of thing T should be the union

$$T = \left(\bigcup_{i=1}^{n}\{\mu_i\}\right) \bigcup \left(\bigcup_{k\geq 1}\{\nu_k\}\right) \tag{1.1}$$

of characters in logic, where a character in (1.1) should be existed, existing or will existing whether or not they are observable or understand by humans. However, T is understood by

$$T[t] = \bigcup_{i=1}^{n}\{\mu_i\} \tag{1.2}$$

for humans at time t, only an approximation on the reality of T, which implies that to hold on the reality of T is a gradual process, little by little. Even so, the Smarandache multispace or multisystem appeared in (1.1) or (1.2) is the basis for systematically understanding a thing T.

Then, *what is a Smarandache multispace or multisystem?* Formally by mathematics, a *Smarandache multispace* or *multisystem* is defined in the following on spaces or systems known by humans.

Definition 1.1([18, 32, 34]) *Let $(\Sigma_1; \mathcal{R}_1)$, $(\Sigma_2; \mathcal{R}_2)$, $\cdots$, $(\Sigma_m; \mathcal{R}_m)$ be m mathematical spaces or systems, different two by two, i.e., for any two spaces or systems $(\Sigma_i; \mathcal{R}_i)$ and $(\Sigma_j; \mathcal{R}_j)$, $\Sigma_i \neq \Sigma_j$ or $\Sigma_i = \Sigma_j$ but $\mathcal{R}_i \neq \mathcal{R}_j$. Then, a Smarandache multispace or multisystem $\widetilde{\Sigma}$ is a union $\bigcup\limits_{i=1}^{m}\Sigma_i$ with rules $\widetilde{\mathcal{R}} = \bigcup\limits_{i=1}^{m}\mathcal{R}_i$ on $\widetilde{\Sigma}$, i.e., the union of rules $\mathcal{R}_i$ on Σ_i for integers $1 \leq i \leq m$, denoted by $\left(\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right)$.*

Certainly, two spaces or systems S_i and S_j are entangled if $S_i \bigcap S_j \neq \emptyset, 1 \leq i \neq j \leq m$. Notice that any matter inherits a topological structure G^L of 1-dimension by the theory of matter composition. This conclusion also holds on a Smarandache multispace or multisystem $\widetilde{S}$ determined by the definition following, which consists of the element in mathematical combinatorics on the reality of thing in the universe([13], [21]-[29]).

Definition 1.2([18 - 21],) *For an integer $m \geq 1$, let $\left(\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right)$ be a Smarandache multispace or system consisting of m mathematical spaces or systems $(\Sigma_1; \mathcal{R}_1)$, $(\Sigma_2; \mathcal{R}_2)$, $\cdots$, $(\Sigma_m; \mathcal{R}_m)$. An inherited topological structure $G^L\left[\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right]$ of $\left(\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right)$ is a labeled topological graph defined following:*

$$V\left(G^L\left[\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right]\right) = \{\Sigma_1, \Sigma_2, \cdots, \Sigma_m\},$$
$$E\left(G^L\left[\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right]\right) = \{(\Sigma_i, \Sigma_j)\,|\Sigma_i \bigcap \Sigma_j \neq \emptyset,\ 1 \leq i \neq j \leq m\} \textit{ with labeling}$$
$$L:\ \Sigma_i \to L(\Sigma_i) = \Sigma_i \quad \textit{and} \quad L:\ (\Sigma_i, \Sigma_j) \to L(\Sigma_i, \Sigma_j) = \Sigma_i \bigcap \Sigma_j$$

for integers $1 \leq i \neq j \leq m$.

Notice that a mathematical space or system $(\Sigma_i; \mathcal{R}_i), 1 \leq i \leq m$ is self-closed by definition and generally, the appearance $\widetilde{S}$ of a thing T is multilateral in front of humans, usually out of order, we need to find which self-space or self-system it belong to. This is the disentangling problem on a Smarandache multispace or multisystem $\widetilde{S}$. In fact, it is more important for understanding a thing T in the universe, which advances us to establish the *collapse mappings* $\phi : \widetilde{S} \to S_i$, i.e., disentangle $\widetilde{S}$ to character or self-closed spaces or systems S_i, $1 \leq i \leq m$ for systematically understanding T, and the case of $m = 2$ happens to be a famous problem in quantum mechanics, i.e., the Schrödinger's cat or quantum entanglement which is the foundation of quantum communication ([3], [5]) because any kind of encryption codes in communication is essentially an application of Smarandache multisystems. This fact leads to the possible applications of disentangling a Smarandache multispace or multisystem to communication, particularly, a general model to quantum communication.

The main purpose of this paper is to discuss the disentangling problem of Smarandache multispace or multisystem by the assumption that each self-enclosed space $S_i, 1 \leq i \leq m$ is endowed with a mathematical structure such as those of topological, geometrical, algebraic structures or generally, each space or system S_i has a character χ_i different from others for integers $1 \leq i \leq m$ and its possible application to information encoding and decoding in communication. Certainly, we have known the application of quantum entanglement in quantum teleportation for preparation, distribution and measurement of the entangled pairs of particles. However, a general prospects on communication is the application of entangling Smarandache multispace or multisystem. For this prospection, applying model is suggested in this paper.

For terminologies and notations not mentioned here, we follow reference [1] and [30] for topology, [2] for algebra, [3] and [5] for quantum teleportation, [6], [19] and [32] for Smarandache geometry, [20] for combinatorial manifolds, [18], [33] for Smarandache multispaces and multisystems and [31] for elementary particles.

§2. Schrödinger's Cat with Entangling Pair

2.1.Schrödinger's cat. The first motivation of Schrödinger's cat was as a paradox on the explaining of instantaneous collapse for the strange nature of quantum superpositions in the macro world and then, a reasonable interpretation on this paradox is the Everetts multi-world interpretation (MWI), which maybe the first time for understanding a thing by multispaces.

Figure 1. Schrödinger's Cat

[**Schrödinger's Cat**] In this paradox, Schrodinger placed a cat in a box along with a radioactive substance, a hammer and Geiger counter and a vial of poison. When the radioactive substance kept in the box decays, the Geiger counter will detect it and will trigger the hammer to release the poison such as those shown in Figure 1. This will subsequently kill the cat. It is not possible to predict when radioactive decay will happen since it is a random process. An observer will not know if the cat is dead or alive until the box is opened. The cats fate is tied to whether the radioactive substance has decayed or not and the cat would be, as claimed by Schrodinger that the cat's "*living and dead* $\cdots$ *in equal parts*" until the box is opened to observe the cat.

The really weird matter on the Schrödinger's cat is that whether the answer is "*living*" or "*dead*" is incomplete, both of them face possible an incorrect ending. However, whether the cat is living or dead is only certainty if the box is opened once, which presents a false impression that the cat's life dependent on the observation of humans. Objectively, *how could a cat's living or dead depend on human observation?* The living or dead of the cat is certain in the nature but it is just because of that one lost a piece of cat information from the close to opening of the box, which results in establishing not the causal relationship on the cat's life. *Why do such ambiguous answers exist?* Because the cat information is incomplete, the fragment from closing the box to opening its lid is lost and there are no logical agreement causal relationship can be established on the cat's life. In this case, the best way is to set a camera inside the box, observe the cat's activity at any time, establish a causal relationship and then to answer the question on the cat being living or dead.

Then, *How to describe the life state of Schrodinger's cat in the box ?* Let $\mathbf{L}$ and $\mathbf{D}$ be respectively the living and dead state of the cat in the box. Then, the Schrodinger cat's state can be expressed as $\mathbf{L}+\mathbf{D}$, which may be a living state, i.e., there is a mapping $\alpha:\mathbf{L}+\mathbf{D}\rightarrow\mathbf{L}$ and may also be a dead state mapping $\delta:\mathbf{L}+\mathbf{D}\rightarrow\mathbf{D}$. The question lies in how one knows the Schrodinger's cat is living or dead in here. For humans, to determine the life of schrodinger's cat requires lifting the lid of the box to really see if the cat is living or dead. Thus, the state of the cat's $\mathbf{L}+\mathbf{D}$ can not be seen, one can only find the cat is living or dead when open the lid of the box. *why the* $\mathbf{L}+\mathbf{D}$ *into* $\mathbf{L}$ *or* $\mathbf{D}$ *for an instant?* In order to give a logically consistent explanation, Bohr et al. proposed the *state collapse hypothesis* on the cat's life, i.e. $\mathbf{L}+\mathbf{D}\rightarrow\mathbf{L}$ or $\mathbf{L}+\mathbf{D}\rightarrow\mathbf{D}$ depending on human observation, it is $\mathbf{L}+\mathbf{D}$ when not observed but collapse to $\mathbf{L}$ or $\mathbf{D}$ when observed because it is knowing the cat's life by humans. Thus, it is necessary to examine what the cat state $\mathbf{L}+\mathbf{D}$ is. Generally, it can be interpreted as the sum of two vectors, the superposition of the cat's living and dead states, and furthermore, it can be viewed as the state of a living being, not only the Schrodinger's cat in the box.

Notice that if we define an axioms A: "*the cat is living*" and B: "*the cat is dead*", then the axiom A or B both generate a space $\mathbf{L}$ and $\mathbf{D}$, namely $\mathbf{L}+\mathbf{D}$ is nothing else but a Smarandache multispace $\widetilde{S}$ of $m=2$ with self-closed spaces $\mathbf{L}$ and $\mathbf{D}$, i.e., $\widetilde{S}=\mathbf{L}\bigcup\mathbf{D}$ in the multi-worlds interpretation $\mathbf{L}+\mathbf{D}$ of Schrodinger cat. The cat state $\mathbf{L}+\mathbf{D}$ can be decomposed according to axiom A and B. Notice also that the living state with the dead state are mutually exclusive in the eyes of humans. Thus, there must be $\mathbf{L}\bigcap\mathbf{D}=\emptyset$, which implies the state $\mathbf{L}+\mathbf{D}$ is a special kind of vector addition, i.e., direct sum and the state $\mathbf{L}+\mathbf{D}$ can be expressed by $\mathbf{L}\bigoplus\mathbf{D}$. In this case, there are only 2 self-closed spaces, inherited a topological structure $K_2^L[\mathbf{L}\bigoplus\mathbf{D}]$ of order 2 and the collapse mappings α and δ are also exclusive, i.e., if α appears then δ can not be seen, or in other words, if one is positive then another must be negative in observing.

2.2.Entangling pair. The living state $\mathbf{L}$ and dead state $\mathbf{D}$ of the Schrödinger's cat is in entangling in the multispace $\mathbf{L}\bigoplus\mathbf{D}$, i.e., if one appear then another would be not occur in observing or in other words, we know their states if one state of the pair is determined. Such a pair has the entanglement property, observed in microscopic particles and first discussed by

Einstein A., B. Podolsky and N. Rosen in 1935 ([4], usually called EPR paper).

It should be noted the entangled situation is not only appearing in the microscopic but also in the macroscopic such as the Schrödinger's cat. Certainly, there are many such pairs in classical mechanics. For example, let A and B be respectively two elastic balls with mass of M and m in a vacuum, moving backward along a straight line with velocity of V and v after the positive collision at the origin of O (see Figure 2 for details). In this case, if the velocity of A and B after collision are v_1' and v_2' respectively, then according to the conservation law of momentum

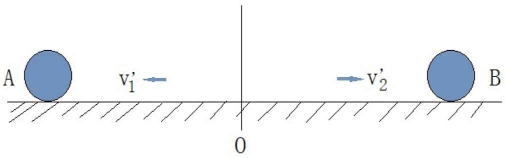

Figure 2. An elastic collision

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

in classical mechanics, we immediately get the velocity

$$v_2' = \frac{m_1(v_1 - v_1') + m_2v_2}{m_2}$$

of ball B, namely, only one speed of balls A, B needs to measure after the collision, then we know the speed of the other. However, we can not measure exactly both its position and its momentum for a microscopic particle at the same time, asserted in the uncertainty principle of quantum mechanics. Even so, there are also the entanglement property. For example, whenever one of the two separated entangled particles A and B is measured, as long as the spin direction of A is upward then the spin direction of B must be downward and conversely, if the spin direction of A is downward then the spin direction of B must be upward, such as those shown in Figure 3, i.e., the microscopic particles A and B consist of an entangling pair.

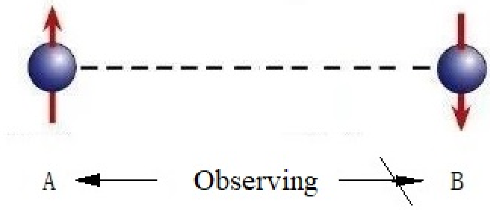

Figure 3. Entanglement particles

Generally, let S and S' be two self-closed spaces or systems. If there are known mappings $f: S \to S'$ with $f(S) \supseteq S'$ and $f': S' \to S$ with $f'(S') \supseteq S$, then S and S' are called an *entangling pair*, which implies that one of S and S' is known then another is determined. The application of mathematical results immediately enables us getting conclusions on entangling pairs following by definition.

Theorem 2.1 *Let S and S' be two sets with onto mapping $f: S \to S'$ and $f': S' \to S$. Then, S and S' are entangling. Particularly, if f is $1-1$, S and S' are entangling.*

Proof Notice that an onto mapping implies that $f(S) \supseteq S'$ and $f'(S') \supseteq S$. Therefore, S and S' are entangling by definition. □

By applying mathematical results, we can deduce many entangling pairs by Theorem 2.1.

Corollary 2.2 *Two homeomorphic spaces, isomorphic spaces, isomorphic groups, rings, fields* *or isomorphic algebraic systems, isomorphic vector spaces, isomorphic function or functional spaces, isomorphic operator spaces S and S' are entangling.*

Notice that a microscopic particle has only two directions on the spin, i.e., upward or downward. If we denote the upward direction by $+1$ and the downward direction by 0, then the entangling particles A and B implies the mapping $f : \{A, B\} \to \{0, +1\}$ posses the property that $f(A) \neq f(B)$, i.e, the exclusive property. In fact, there are many cases of binary logic in daily life. For example, a human has a pair of socks with one red and one white. On a dark night, after putting on socks he went to the street lamp and looked at the red sock on his left foot. Then, he does not need to look at his right foot again because he can deduce that it is a white sock in the binary logic case. Thus, we can generally get entangling pairs by binary logic on sets following.

Theorem 2.3 *Let S be a set with a one-valued mapping $f : S \to \{0, +1\}$ and $A = \{x \in S | f(x) = 1\}$, $B = \{x \in S | f(x) = 0\}$. Then, A and B are entangling.*

Proof Notice that f is a one-valued mapping on S, i.e., for any element $x \in S$, $f(x) = 1$ or $f(x) = 0$ and there are no elements $y \in S$ such that $f(y) = 0$ also with $f(y) = 1$. Thus, $A \bigcup B = S$ and $A \bigcap B = \emptyset$. Whence, $A = S \setminus B$ and $B = S \setminus A$. We therefore know that A and B are entangling. $\square$

§3. Disentangling Smarandache Multispace or Multisystem

Let $\widetilde{S}$ be Smarandache multispace or multisystem on a thing T in the universe. The understanding process of humans on T is gradually by holding on characters of thing T. Thus, if we view T as a set of elements, then a character can be viewed as a self-closed space or system consisting of a few elements in T. Consequently, this process is essentially a disentangling process on $\widetilde{S}$ in logic. In fact, if each self-closed space or system is endowed with a mathematical structure, the disentangling process can be carried out immediately.

3.1.Algebraic Structure. An algebraic system $(\mathcal{A}; \circ)$ is a self-closed system under the operation $\circ$, i.e., for any $a, b \in \mathcal{A}$, $a \circ b \in \mathcal{A}$. Now, if a *Smarandache multisystem* $\widetilde{A}$ is the union of algebraic systems $(A_i; O_i)$ with $1 \leq i \leq m$ and operation sets $O_i = \{\circ_{ik}, 1 \leq k \leq s_i\}$, we can disentangling $\widetilde{S}$ to algebraic systems by the ruler that *for* $\forall a, b \in \widetilde{A}$, $a, b \in A_i$ *if and only if* $a \circ_{ik} b \in A_i$ *for* $\circ_{ik} \in O_i$ with a programme following:

STEP 1.1. For an integer $i, 1 \leq i \leq m$, let a be an element of $\widetilde{A}$ with definition on $a \circ_{ik} b$ for operation $\circ_{ik} \in O_i$, integer $k, 1 \leq k \leq s_k$ and some elements $b \in \widetilde{A}$;

STEP 1.2. Choose any element $x \in \widetilde{A}$, calculate $a \circ_{ik} x, \circ_{ik} \in O_i$ for integers $1 \leq k \leq s_i$;

STEP 1.3. If $a \circ_{ik} x$ is defined on $\widetilde{A}$ then let $a, x, a \circ_{ik} x \in A_i$. Otherwise, $x \notin A_i$ and if $\widetilde{A} \setminus \{x\} = \emptyset$, then turn to STEP 1.4; if $\widetilde{A} \setminus \{x\} \neq \emptyset$, come back to STEP 1.1 by replacing a with an element of A_i and $\widetilde{A}$ with $\widetilde{A} \setminus \{x\}$;

STEP 1.4. The programming terminated if $\forall x \in \widetilde{A}$ chosen in STEP 1.2.

Clearly, if $a \circ_{ik} x \in A_i$ then there must be $x \circ_{ik} a \in A_i$ if $x \circ_{ik} a$ is defined in $\widetilde{A}$ by this programme. Furthermore, The next result convinces us the disentangling of an algebraic Smarandache multisystem $\widetilde{A}$.

Theorem 3.1 *For any integer $1 \leq i \leq m$, A_i is maximally a self-closed algebraic system of $\widetilde{A}$ by STEP 1.1- STEP 1.4, which establishes the collapse mapping $\phi : \widetilde{A} \to A_i$ for integers $1 \leq i \leq m$.*

Proof By STEP 1.1- STEP 1.4, $A_i \subset \widetilde{A}$ is self-closed for integers $1 \leq i \leq m$. Otherwise, if there exist elements $x, y \in A_i$ with definition $x \circ_{ik} y$ for an integer $1 \leq k \leq s_i$ on $\widetilde{A}$ but $x \circ_i y \notin A_i$, it contradicts to STEP 1.3 with $x \in A_i$. And then, A_i is maximal because if there is an element $x \in \widetilde{A}$ with definition of $a \circ_{ik} x$ for an integer $1 \leq k \leq s_i$ and some elements $a \in \widetilde{A}$ there must be $x \in A_i$ by STEP 1.2. Whence, A_i is maximally a self-closed system. □

Notice that $\widetilde{A}$ is an algebraic Smarandache multisystem in Theorem 3.1, which enables us to get immediately the collapse mapping on the *Smarandache mutigroup, multiring, multifield* and *vector multispace* ([9], [11-12]) following.

Corollary 3.2 *Let $\left(\widetilde{G}; \circ_i, 1 \leq i \leq m\right)$ be a Smarandache multigroup. Then, the collapse mapping $\phi : \widetilde{G} \to G_i$ can be established by STEP 1.1- STEP 1.4 with operation $\circ_i$ of the group G_i, $1 \leq i \leq m$.*

Particularly, if $\widetilde{G}$ is finitely Abelian, i.e., $\left|\widetilde{G}\right| < \infty$ and $a \circ_i b = b \circ_i a$ for $\forall a, b \in \widetilde{G}$ and integers $1 \leq i \leq m$, then the collapse mapping can be not only on groups G_i but also on its cyclic groups with

$$\phi : \widetilde{G} \to G_i, 1 \leq i \leq m \ \ \text{and} \ \ \phi_{ij} : \widetilde{G} \to \langle a_{ij} \rangle$$

where, $a_{ij} \in G_i, 1 \leq j \leq s$ with a direct product decomposition of group G_i by $G_i = \langle a_{i1} \rangle \otimes \langle a_{i2} \rangle \otimes \cdots \otimes \langle a_{is} \rangle$.

Corollary 3.3 *Let $\left(\widetilde{R}; +_i, \cdot_i, 1 \leq i \leq m\right)$ be a Smarandace multiring. Then, the collapse mapping $\phi : \left(\widetilde{R}; +_i, \cdot_i, 1 \leq i \leq m\right) \to (R_i; +_i, \cdot_i$ can be established by STEP 1.1- STEP 1.4 with operations $+_i, \cdot_i$ of the ring $(R_i; +_i, \cdot_i)$ for integers $1 \leq i \leq m$. Particularly, the collapse mapping $\phi : \widetilde{R} \to R_i$ can be established by STEP 1.1- STEP 1.4 for Smarandache multifields.*

Corollary 3.4 *Let $(\widetilde{V}; \widetilde{F})$ be a vector Smarandache multispace with a vector set $\widetilde{V} = V_1 \bigcup V_i \bigcup \cdots \bigcup V_m$, an operation set $O(\widetilde{V}) = \{(\cdot +_i, \cdot_i) \mid 1 \leq i \leq m\}$ and a Smarandache multifiled $\widetilde{F} = F_1 \bigcup F_2 \bigcup \cdots \bigcup F_m$ with a double operation $O(\widetilde{F}) = \{(+_i, \times_i) \mid 1 \leq i \leq k\}$. Then, the collapse mapping $\phi : (\widetilde{V}; \widetilde{F}) \to (V_i; F_i)$ can be established by STEP 1.1- STEP 1.4 with operations $+_i, \cdot_i$ of the vector space $(V_i; F_i)$ for integers $1 \leq i \leq m$.*

3.2.Geomertical Structure. For an integer $n \geq 1$, a manifold M is a locally Euclidean space of dimension n, i.e., for $\forall x \in M$ there is a neighborhood $U(x)$ homeomorphic to $\mathbb{R}^n$. Now, let $\widetilde{M}$ be a connected *Smarandache multimanifold*, i.e., the union of manifolds $M_i, 1 \leq i \leq m < \infty$ with dimensions $\dim M_i = n_i, 1 \leq i \leq m$ which is connected. Then, we can disentangling $\widetilde{M}$ by the ruler that *if $x \in M_i$ with a neighborhood $U(x)$ homeomorphic to $\mathbb{R}^{n_i}$ for an integer $1 \leq i \leq m$ and $y \in U(x)$, then $y \in M_i$* with a programme following:

STEP 2.1. Let x be a point of $\widetilde{M}$ with a neighborhood $U(x)$ homeomorphic to $\mathbb{R}^{n_i}$ with $1 \leq i \leq m$ and $y \in \widetilde{M}$;

STEP 2.2. If $y \in U(x)$ then let $y \in M_i$. Otherwise, $y \notin M_i$ and if $\widetilde{\mathcal{T}} \setminus \{x, y\} = \emptyset$, then turn to STEP 2.3; if $\widetilde{\mathcal{T}} \setminus \{x, y\} \neq \emptyset$, come back to STEP 2.1 by replacing x with an element of M_i and $\widetilde{M}$ with $\widetilde{M} \setminus \{x, y\}$;

STEP 2.3. The programming terminated if $\forall x \in \widetilde{A}$ chosen in STEP 2.1.

Clearly, if $y \in \mathcal{T}_i$ then there must be $x \in \mathcal{T}_i$ also in this programme. We have the following result on the disentangling topological Smarandache multispaces.

Theorem 3.5 *For any integer $1 \leq i \leq m$, M_i is maximally a manifold of dimension n_i of $\widetilde{M}$ by STEP 2.1- STEP 2.3, which establishes the collapse mapping $\phi : \widetilde{M} \to M_i$ for integers $1 \leq i \leq m$.*

Proof By STEP 2.1-2.3, M_i is clearly a manifold of dimension n_i by definition. For its maximality, if there is a point $y \in \widetilde{M}$ but $y \notin M_i$ with $y \in U(x)$ of a neighborhood of $x \in M_i$ homeomorphic to $\mathbb{R}^{n_i}$, then there must be $y \in M_i$ by STEP 2.2, this programme will not be terminated, a contradiction. Thus, M_i is maximally a dimensional n_i manifold of $\widetilde{M}$. $\square$

Notice that the Smarandache multimanifold $\widetilde{M}$ is called a *finitely combinatorial manifold* in [14] and [20-21], which can be characterized by vertex-edge labeled graphs inherited in $\widetilde{M}$. Furthermore, if the Smarandache multimanifold is differentiable, i.e., a *differentiable combinatorial manifold* $\widetilde{M}$ ([14]), a similar programme can be also established and get a conclusion following.

Theorem 3.6 *Let $\widetilde{M}$ be a differentiable combinatorial manifold consisting of differentiable manifolds $M_i, 1 \leq i \leq m$ of dimension $n_i, 1 \leq i \leq m$, respectively. Then, the collapse mapping $\phi : \widetilde{M} \to M_i$ for integers $1 \leq i \leq m$ can be established.*

Particularly, if all manifold $M_i, 1 \leq i \leq m$ are respectively Euclidean spaces $\mathbb{R}^{n_i}$ for integers $1 \leq i \leq m$, such a Smarandache multispace $\widetilde{M}$ is the *combinatorially Euclidean space* in this case ([14]). We get a conclusion by Theorem 3.5 following.

Corollary 3.7 *Let $\widetilde{E}$ be a combinatorial Euclidean space of $\mathbb{R}^{n_i}, 1 \leq i \leq m$. Then, the collapse mapping $\phi : \widetilde{E} \to \mathbb{R}^{n_i}$ can be established by STEP 2.1- STEP 2.3 for integers $1 \leq i \leq m$.*

Notice that a *metric Smarandache multispace* is the union $\widetilde{\mathcal{S}}$ of spaces $\mathcal{S}_i$ with a metric ρ_i for integers $1 \leq i \leq m$ which is connected. Then, we can disentangling $\widetilde{\mathcal{S}}$ by the ruler that *for $\forall x, y \in \widetilde{\mathcal{S}}$ if $\rho_i(x, y)$ is defined in $\widetilde{\mathcal{S}}$ then $x, y \in \mathcal{S}_i$ for an integer* $1 \leq i \leq m$ with a programme following:

STEP 3.1. Let x, y be points of $\widetilde{\mathcal{S}}$ and i an integer with $1 \leq i \leq m$;

STEP 3.2. If $\rho_i(x, y)$ is defined in $\widetilde{\mathcal{S}}$ then let $y \in \mathcal{S}_i$. Otherwise, $y \notin \mathcal{T}_i$ and if $\widetilde{\mathcal{T}} \setminus \{x, y\} = \emptyset$, then turn to STEP 3.3; if $\widetilde{\mathcal{T}} \setminus \{x, y\} \neq \emptyset$, come back to STEP 3.1 by replacing x with an element of $\mathcal{S}_i$ and $\widetilde{\mathcal{S}}$ with $\widetilde{\mathcal{S}} \setminus \{x, y\}$;

STEP 3.3. The programming terminated if $\forall x, y \in \widetilde{\mathcal{S}}$ chosen in STEP 3.1.

Clearly, by definition if $y \in \mathcal{S}_i$ in STEP 3.2, then there must be $x \in \mathcal{T}_i$ by STEP 3.1-STEP 3.3 and a conclusion on disentangling the metric Smarandache multispaces $\widetilde{\mathcal{S}}$ following.

Theorem 3.8 *Let $\widetilde{\mathcal{S}}$ be a metric Smarandache multispace of metrics $\mathcal{S}_i, 1 \leq i \leq m$. Then, for any integer $1 \leq i \leq m$, $\mathcal{S}_i$ is maximally a metric space of $\widetilde{M}$ by STEP 3.1- STEP 3.3, which establishes the collapse mapping $\phi : \widetilde{\mathcal{S}} \to \mathcal{S}_i$ for integers $1 \leq i \leq m$.*

Proof The proof is similar to that of Theorem 3.1. □

As we known, a topological group $(G; \circ)$ is a Smarandache multispace $G \cup G$ in the case of $m = 2$, endowed both with the topological and group properties. By definition, a topological group is a Hausdorff topological space G together with an algebraic group structure on $(G; \circ)$, namely, ① the group multiplication $\circ :\ (a, b) \to a \circ b$ of $G \times G \to G$ is continuous; ② the group inversion $g \to g^{-1}$ of $G \to G$ is continuous. That is, the identity mapping $1_G : G \to G$ is both a collapse mapping of the topological group $(G; \circ)$ to its topological space G and algebraic group $(G; \circ)$. Similarly, a *topological Smarandache multigroup* $(\widetilde{A}; \mathscr{O})$ is an algebraic Smarandache multisystem $(\widetilde{A}; \mathscr{O})$ with $\widetilde{A} = H_1 \cup H_2 \cup \cdots \cup H_m$ and $\mathscr{O} = \{\circ_i; 1 \leq i \leq m\}$ hold with conditions: ① $(H_i; \circ_i)$ is a group for each integer i, $1 \leq i \leq m$, namely, $(H, \mathscr{O})$ is a Smarandache multigroup; ② $\widetilde{A}$ is itself a connected topological Smarandache multispace; ③ the mapping $(a, b) \to a \circ b^{-1}$ is continuous for $\forall a, b \in H_i$ and $\forall \circ \in \mathcal{O}_i$, $1 \leq i \leq m$.

For example, let $\mathbb{R}^{n_i}$ be Euclidean spaces of dimension n_i with an additive operation $+_i$ for integers $1 \leq i \leq m$ and scalar multiplication $\cdot$ determined by

$$\begin{aligned}
&(\lambda_1 \cdot x_1, \lambda_2 \cdot x_2, \cdots, \lambda_{n_i} \cdot x_{n_i}) +_i (\zeta_1 \cdot y_1, \zeta_2 \cdot y_2, \cdots, \zeta_{n_i} \cdot y_{n_i}) \\
&= (\lambda_1 \cdot x_1 + \zeta_1 \cdot y_1, \lambda_2 \cdot x_2 + \zeta_2 \cdot y_2, \cdots, \lambda_{n_i} \cdot x_{n_i} + \zeta_{n_i} \cdot y_{n_i})
\end{aligned}$$

for $\forall \lambda_l, \zeta_l \in \mathbb{R}$, where $1 \leq \lambda_l, \zeta_l \leq n_i$. Then, each $\mathbb{R}^{n_i}$ is a continuous group under $+_i$. Whence, the algebraic Smarandache multisystem $(\widetilde{A}; \mathscr{O})$ is a topological multigroup by definition, where $\mathscr{O} = \{+_i; 1 \leq i \leq m\}$. Particularly, if $m = 1$, i.e., an n-dimensional Euclidean space $\mathbb{R}^n$ with the vector additive $+$ and multiplication $\cdot$ is nothing else but a topological group.

The next conclusion on the collapse mapping $\phi : (\widetilde{A}; \mathscr{O}) \to (H_i; \circ_i)$ can be obtained by STEP 1.1-STEP 1.4 similar to that of Theorems 3.1.

Theorem 3.9 *Let $(\widetilde{A}; \mathscr{O})$ be a topological Smarandache multispace of topological groups $(H_i; \circ_i)$, $1 \leq i \leq m$. Then, for any integer $1 \leq i \leq m$, $(H_i; \circ_i)$ is maximally a topological group of $\widetilde{A}$ by STEP 1.1- STEP 1.4, which establishes the collapse mapping $(\widetilde{A}; \mathscr{O}) \to (H_i; \circ_i)$ for integers $1 \leq i \leq m$.*

Certainly, Theorems 3.1-3.9 show that the collapse mapping on an algebraic or geometrical Smarandache multispace or multisystem can be established by the structure inherited in the spaces or systems, which proposes a question on the collapse mapping of a Smarandache multispace or multisystem naturally, i.e., *could we find a unified form on collapse mappings of a Smarandache multispace or multisystem in mathematics?* The answer is positive! Generally, let $\widetilde{S}$ be the union of spaces or systems $S_1, S_2, \cdots, S_m$, i.e., a Smarandache multispace of multisystem. If we view that each S_i of space or system is independent. Then, the Smarandache multispace or multisystem $\widetilde{S}$ can be represented by a tensor product $\widetilde{S} = S_1 \otimes S_2 \otimes \cdots \otimes S_m$ formally and view S_i to be

$$S_i \simeq 1_{S_1} \otimes 1_{S_2} \otimes \cdots \otimes S_i \otimes \cdots \otimes S_m, \quad (3.1)$$

where 1_{S_k} denotes the unit or origin of S_k for integers $1 \leq k \leq m$. In this case, we can present a unified form for collapse mapping.

By definition, a projection π_i is determined by $\pi_i : \widetilde{S} \to S_i$, i.e., $\pi_i : s_1 \otimes s_2 \otimes \cdots \otimes s_m \to s_i$, where $s_i \in S_i$ for integers $1 \leq i \leq m$. Thus, the collapse mapping $\phi : \widetilde{S} \to S_i$ can be represented by $\pi_i, 1 \leq i \leq m$. Furthermore, for an integer $1 \leq i \leq m$ define an identity projection

$$1_{\pi_i}(x) = \begin{cases} x, & \text{if } x \in S_i \\ 1_{S_k}, & \text{if } x \notin S_i \text{ but } x \in S_k, k \neq i. \end{cases}$$

Then, we can get the unified form of collapse mapping of a Smarandache multispace or multisystem following.

Theorem 3.10 *Let $\widetilde{S} = S_1 \otimes S_2 \otimes \cdots \otimes S_m$ be a Smaradache multispace or multisystem with convention* (3.1). *Then, all collapse mappings can be represented by projections*

$$\pi_i : S_1 \otimes S_2 \otimes \cdots \otimes S_m \to S_i, \quad 1 \leq i \leq m \quad (3.2)$$

and particularly, the identity projections 1_{π_i} for integers $1 \leq i \leq m$.

3.3.Character Observing. A more general question on collapse mapping of Smarandache multispace or multisystem is on the understanding model, i.e., *how to hold on the collapse mapping of Smarandache multispace or multisystem* (1.1) *or* (1.2)*?* For answering this question we consider the case of Schrödinger's cat again. According to the interpretation of Bohr et al., the collapse of the Schrödinger's cat happened in the observing of a human opening the lid of the box to hold on the living or dead of the cat. It is at this time that the superposition state of the cat's state, maybe living or dead instantly collapsed to a determined situation of "*living*" or "*dead*" that a human could understanding or in the words of Smarandache multispace or multisystem, the collapse mapping $\phi : \mathbf{L} + \mathbf{D} \to \mathbf{L}$ or $\mathbf{D}$ appears instantly at the time of a human lifting the lid of the box and observing the cat's living or dead inside the box. And then, *what time happens that a Smarandache multispace or multisytsem disentangles in the understanding things T in the universe?* It happens at the time that a thing T is understood by a character.

Certainly, we have known a Smarandache mutispace or multisystem $\widetilde{S}$ can be disentangled by its inside mathematical structure in Subsections 3.1-3.2. However, all the mathematical structures inside $\widetilde{S}$ are only a hypothesis by humans for simulating its behavior observed. We do not know if there really is one even though there is a mathematical universe hypothesis claims that our external physical reality is a mathematical structure proposed by Max Tegmark [35] in 2003. It can not be verified ([25]) because it is essentially a special case of the *Theory of*

Everything. That is, although the reality of thing T is determined by a Smarandache multispace or multisystem (1.1) or an approximation (1.2) we can not conclude T inherits itself unless endowed a mathematics on it by humans. Thus, we can not assume the spaces or systems determined in (1.1) or (1.2) by characters $\mu_i, 1 \leq i \leq n$ or $\nu_k, k \geq 1$ are self-closed mathematical spaces or systems. In this case, *how to we get the entangling mapping of the Smarandache multispace of multisystem on T?* The answer lies in how to understand the characters of thing T even though it maybe not posses a mathematical structure.

If a thing T is characterized by (1.1) or (1.2) whether or not it has a mathematical structure, *what does the characters $\{\mu_i; 1 \leq i \leq n\}$ and $\{\nu_k, k \geq 1\}$ means?* Certainly, we can view each of them as a parameter or feature. However, if we equate thing T with a Smarandache multispace or multisystem $\widetilde{T}$ consisting of elements, i.e., $\widetilde{T} = \{a_\lambda | \lambda \in \Lambda\}$, where Λ denotes an index set associated with elements in T and the character of an element a_λ is $\chi(a_\lambda)$, then each character μ_i or ν_k is essentially in classifying elements of T into subsets $\{\mu_i\}$ or $\{\nu_k\}$, i.e.,

$$\{\mu_i\} = \{a_\lambda \in T | \chi(a_\lambda) = \mu_i\}, \quad \{\nu_k\} = \{a_\lambda \in T | \chi(a_\lambda) = \nu_k\}, \tag{3.3}$$

namely, $\{\mu_i\}$ and $\{\nu_k\}$ are respectively the sets consisting of elements in T with the same character μ_i or ν_k for integers $1 \leq i \leq n$ and $k \geq 1$. In this case, the collapse mappings on T are nothing else but determined by characters

$$\mu_i : T \to \{\mu_i\} \quad \text{and} \quad \nu_k : T \to \{\nu_k\} \tag{3.4}$$

and similar to the case of Schrödinger's cat, each of them happens in observing character μ_i or ν_k of humans for an integer $1 \leq i \leq n$ or $k \geq 1$. Certainly, $\{\mu_i\}, 1 \leq i \leq n$, $\{\nu_k\}, k \geq 1$ do not have the exclusive property if $n \geq 2$, different from the case of the Schrödinger's cat in general.

For example, let a ripple curve L of water is the composition that of L_1 and L_2 such as those shown in Figure 4.

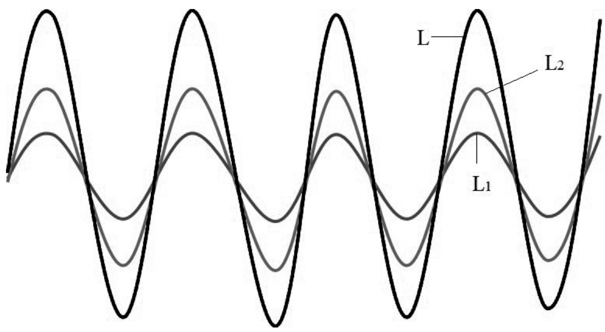

Figure 4

Then, *could one decompose L into L_1 and L_2 for hold on the collapse mapping?* The answer is positive if one knows the characters of the ripple curves of L_1 and L_2 such as those of starting point, the highest and lowest points, spacing, velocity, etc., then it is easily to get the collapse mapping $\phi : L \to L_1$ or L_2 by the characters of L_1 and L_2.

§4. Application to Information Encoding and Decoding

A transmission of information from a sender to a receiver includes information encoding, channel transmission and information decoding by a string consisting of digital numbers. Generally, let S and S' be an entangling pair. Then, one know S' if the onto mapping f is known and vice versa, know S if the onto mapping f' is known by Theorem 2.1. Thus, f, f' are keys in the information encoding and decoding if $ff' = f'f = 1_{id}$, denoted by $f' = f^{-1}$ or $f = f'^{-1}$. Usually, an information is first transformed to a digital form I and then, encode by the action of f on I to get a mixed state $f(I)$ for transmission on the channel. After received $f(I)$, the receiver decodes $f(I)$ by the action f^{-1} on $f(I)$ to know the information I.

As is known to all, a central job in the transmission of information is the encoding and decoding with the information not declassified unless the sender and the receiver. In fact, what are lots of humans value quantum entanglement in disentangling because it can provides one with a key that believed randomly for decoding in quantum teleportation. Then, *can we generalize the encoding and decoding of information by the Smarandace multisystems with disentangling in communication?* Certainly, we can generalize the usual transforming model by Smarandache multisystems.

4.1.Information Encoding and Decoding. Let $\widetilde{S}$ be a Smarandache multisystem of systems $S_1, S_2, \cdots, S_m$ with respective characters $\chi_1, \chi_2, \cdots, \chi_m$. If one or more systems of $S_1, S_2, \cdots, S_m$ are information, we can naturally view the Smarandache multisystem $\widetilde{S}$ to be a disorganized string of numbers or an encoding of the information which can be transmitted in a channel by the sender. After disentangling $\widetilde{S}$ to systems $S_1, S_2, \cdots, S_m$ by different character $\chi_1, \chi_2, \cdots, \chi_m$, the receiver knows the information I such as those shown in Figure 5, where

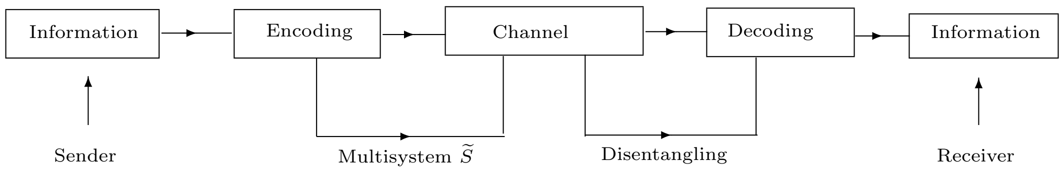

Figure 5

the type of systems $S_i, 1 \leq i \leq m$ maybe the same or district, mathematical or not, finite or infinite, determined or randomly, also be the variables as the sender and receiver wish. For example, let $S_i, 1 \leq i \leq m$ be one of finite fields

$$(\mathbb{Z}_{p_i}; +, \cdot), \qquad \left\{1, t, 2t^2, \cdots, p_i t^{p_i}\right\} \qquad \text{or} \qquad \left\{\begin{pmatrix} m \\ i \end{pmatrix} p^i q^{m-i}\right\} \tag{4.1}$$

for integers $1 \leq i \leq m$ and define the Smarandache multisystem $\widetilde{S} = S_1 \otimes S_1 \otimes \cdots \otimes S_m$ for m primes $p_1, p_2, \cdots, p_m$ and a real number $0 < p < 1$ with $p + q = 1$. Then, *how to encode and decode an information by Smarandache multisystem?* Certainly, the encoding and decoding of an information by a Smarandache multisystem are easily carried out. For example, the typical case that some systems are the transmitted information but others are all bewitching is shown in the programme following.

STEP 4.1. For a transforming Information I, choose a Smarandachely multisystem $\widetilde{S} = S_1 \cup S_2 \cup \cdots \cup S_m$ with respective characters $\chi_1, \chi_2, \cdots, \chi_m$, $m \geq 1$;

STEP 4.2. Encode information I by some systems of $S_i, 1 \leq i \leq m$;

STEP 4.3. Encode $\widetilde{S}$ by a public coding system to a digital form $I(\widetilde{S})$;

STEP 4.4. Transmit $I(\widetilde{S})$ on an opened channel;

STEP 4.5. Decode $I(\widetilde{S})$ by the public coding system to get Smarandache multisystem $\widetilde{S}$;

STEP 4.6. Disentangle $\widetilde{S}$ by characters $\chi_i, 1 \leq i \leq m$ to get systems $S_1, S_2, \cdots, S_m$.

Notice that if $m = 1$, i.e., encode information by one system S, it is the usual case in pubic. Thus, this programme includes the public case in communication. However, it applies to the secret transmitting in case of $m \geq 2$ with a property that the bigger of m or the more complex of systems $S_i, 2 \leq i \leq m$, the higher the security for transmitting of the information. In this model, all characters $\chi_i, 1 \leq i \leq m$ are keys for decoding. Certainly, we can encrypt purposely the information by applying the Smarandachely entangling pairs.

4.2.Smarandachely Entangling Pair. Let $\widetilde{A}$ and $\widetilde{A}'$ be two Smarandache multisystems. They are called *Smarandachely entangling pair* if $\widetilde{A}$ and $\widetilde{A}'$ are entangling. In this case, there must be the known onto mappings $f : \widetilde{A} \to \widetilde{A}'$ and $f' : \widetilde{A}' \to \widetilde{A}$ holding by the sender and the receiver, respectively. Particularly, $f' = f^{-1}$ and both variable on the same Geiger counter t, i.e., $f(t)$ and $f^{-1}(t)$ beginning from an initial number $t = 0$. For example, the quantum teleportation by the pair of entangled particles A,B shown in Figure 6 is in the case.

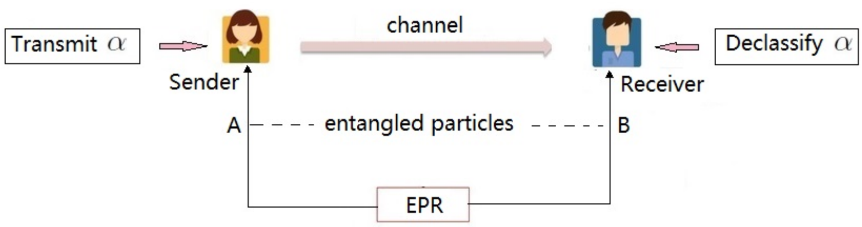

Figure 6

Then, *how to apply a Smarandachely entangling pair in an encrypting transmission of information?* A general model for the encrypting transmission by applying Smarandachely entangling pairs $\widetilde{A}, \widetilde{A}'$ is associating $\widetilde{A}$ with a transmitted information I, encoding I by $f(\widetilde{A})$ and then, the receiver decodes $f(\widetilde{A})$ by f^{-1}, such as the case of entangling particles A, B in Figure 6. By the different applying cases of entangling pairs, there are two models following.

Case 1. Apply one Smarandachely entangling pairs $\widetilde{A}, \widetilde{A}'$.

In this case, a generalized model for transmission of information I by Smarandache multisystem is shown in the following.

STEP 5.1. For a transmitted information I, choose a Smarandachely multisystem $\widetilde{A} = A_1 \cup A_2 \cup \cdots \cup A_m$ with an entangling Smarandache multisystem $\widetilde{A}'$;

STEP 5.2. Encode information I by some of systems $A_1, A_2, \cdots, A_m$ with respective characters $\chi_1, \chi_2, \cdots, \chi_m$ to get a Smarandache multisystem $\widetilde{A}$ and then, applying the entangling pair to get the Smarandache multisystem $\widetilde{A}'$;

STEP 5.3. Encode $\widetilde{A}'$ by a public coding system to a digital form $I(\widetilde{A}')$;

STEP 5.4. Transmit $I(\widetilde{A}')$ on an opened channel;

STEP 5.5. Decode $I(\widetilde{A}')$ by the public coding system to get Smarandache multisystem $\widetilde{A}'$;

STEP 5.6. Disentangle $\widetilde{A}'$ its entangling pair to get $\widetilde{A}$ and then by characters $\chi_i, 1 \leq i \leq m$ to get systems $A_1, A_2, \cdots, A_m$.

Case 2. Apply m entangling pairs $A_i, A_i', 1 \leq i \leq m$ of algebraic systems.

In this case, a generalized model for transmission of information I by Smarandache multisystem is shown in the following.

STEP 6.1. For a transmitted information I, choose an entangling pair A_i, A_i' with respective characters χ_i, χ_i' for integers $1 \leq i \leq m$;

STEP 6.2. Encode information I by some of systems $A_1, A_2, \cdots, A_m$ and then, applying the entangling pair to get the Smarandache multisystem $\widetilde{A}' = A_1' \cup A_2' \cup \cdots \cup A_m'$;

STEP 6.3. Encode $\widetilde{A}'$ by a public coding system to a digital form $I(\widetilde{A}')$;

STEP 6.4. Transmit $I(\widetilde{A}')$ on an opened channel;

STEP 6.5. Decode $I(\widetilde{A}')$ by the public coding system to get Smarandache multisystem $\widetilde{A}'$;

STEP 6.6. Disentangle $\widetilde{A}'$ by characters $\chi_i', 1 \leq i \leq m$ to get systems $A_1', A_2', \cdots, A_m'$ and then, apply the entangling pairs to get system $A_1, A_2, \cdots, A_m$.

Notice that each of systems $A_1, A_2, \cdots, A_m$ and $A_1', A_2', \cdots, A_m'$ could be constantly or variable systems in case. Particularly, if the Smarandache multisystem $\widetilde{A}$ or systems $A_1, A_2, \cdots, A_m$ are variable on $\mathbf{x}$ with known f, f^{-1}, then both of Cases 1 and 2 include the applying case of quantum entangling particles in communication by the hidden variable theory of Bohm D. and Y. Aharonov in [3]. For example, let

$$A = \left\{x_1^2(t), x_2^2(t), \cdots, x_n^2(t), \cdots\right\} \quad \text{and} \quad A' = \left\{\sqrt{x_1(t)}, \sqrt{x_2(t)}, \cdots, \sqrt{x_n(t)}, \cdots\right\}, \tag{4.2}$$

where t is determined by a Geiger counter. Then, A and A' consist of an entangling pair variable on variable t. Then, *what is the implication included in this example?* It implies that the particles in a quantum entangling pair is only an information or a key carrier if we cast off the mystery of microscopic particles and the key is in fact on hidden variables determined by observing. Thus, a general carrier for encoding and decoding of information should be designed on the Smarandachely entangling pairs and then, we can apply it to communication.

§5. Conclusion

A central topic of this paper is to disentangle Smarandache multispaces or multisystems by its mathematical structures or characters and then, generalizes the quantum entangling pairs by Smarandachely entangling pairs with possible applications to communication. In fact, the application of quantum entanglement is a hot topic in communication until today but hardly one noted its mathematical nature, bewitched by its appearance of the microscopic particles. For unraveling the mysteries of the entangling state, we discuss its general case, i.e., Smarandache multispace or multisystem and show how to disentangle a Smarandache multispace or multisystem to self-closed spaces or systems by their mathematical structures or characters, and generalize the entangling pair of particles to Smarandachely entangling pair for application of Smarandache multispace or multisystem in communication. Certainly, the application of encoding and decoding by Smarandache multispaces or multisystems needs one to design the key carrier, likewise the entangled quanta. However, we believe such a key carrier will come true in the near future by the notion.

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