Multi-fuzzy hypergroups

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Abstract. In this paper, the concept of multi-fuzzy hyperstructure and multi-fuzzy hyperoperation are presented and developed. This concept generalized the concept of fuzzy hypergroup based on fuzzy space to the context of multi-fuzzy hypergroup based on multi-fuzzy space.
Keywords: multi-fuzzy space, multi-fuzzy hyperoperation, multi-fuzzy hypergroup and multi-fuzzy hyperstructure.

1. Introduction

Fuzzy set theory was pioneered by Zadeh [16], where the fuzzy set A of a universe of discourse U has only a basic component a membership function $A : U \longrightarrow [0, 1]$. In which, it is generalized classical set theory from the integer 0 and 1 to the interval $[0, 1]$ for the membership degree of objects. The idea of the concept of Atanassov's intuitionistic fuzzy set (AIFS) was introduced by Atanassov [3]. Atanassov's intuitionistic fuzzy set has been found to be greatly functional in dealing with vagueness. Atanassov achieved his concept by adding the non-membership term to the definition of fuzzy set that was given by Zadeh [15], while the fuzzy set has only basic component a membership function Sebastian and Ramakrishnan ([12, 13, 14]) generalized fuzzy sets to multi-fuzzy sets in terms of multi-dimensional membership functions and multi-level fuzziness and introduce multi-fuzzy subgroup.

Algebraic hyperstructures were introduced by Marty in [13] as a suitable generalisation of classical algebraic structures. In classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set. Formally, if H is a nonempty set and $\rho^*(H)$ is the set of all nonempty subsets of H then we consider maps of the following type:

$$f_i : H \times H \longrightarrow \rho^*(H),$$

where $i = 1, 2, \ldots, n$ and n is a positive integer. The maps f_i are called (*binary*) hyperoperations.

A *hypergroup* H is a *hyperstructure* $(H, \circ)$ satisfying

(1) $x \circ (y \circ z) = (x \circ y) \circ z$ for all $x, y, z \in H$ (called semihypergroup),

(2) $\{x\} \circ H = H \circ \{x\}$ for all $x \in H$.

The concept of H_v-structures ([15]) constitute a generalization of the well-known algebraic hyperstructures (hypergroup, hyperring, hypermodule). Actually some axioms concerning the above hyperstructures such as the associative law, the distributive law and so on are replaced by their corresponding weak axioms. The study of fuzzy algebraic structures started with the introduction of the concept of fuzzy subgroup of an ordinary group by Rosenfeld ([11]). Davvaz ([15]) applied the concept of fuzzy sets to the theory of algebraic hyperstructures and defined fuzzy sub-hypergroup (respectively $H_v$subgroup) of a hypergroup (respectively H_v-group) which is a generalization of the concept of Rosenfeld's fuzzy subgroup of a group.

The relations between fuzzy sets and hyperstructures have been already considered by Corsini, Davvaz, Leoreanu, Feng and others ([28-50]). A recent book ([41]) contains a wealth of applications. By this book, Corsini and Leoreanu ([37]) presented some of the numerous applications of algebraic hyperstructures, especially those from the last fifteen years, to the following subjects: geometry, hypergraphs, binary relations, lattices, fuzzy sets and rough sets, automata, cryptography, codes, median algebras, relation algebras, artificial intelligence and probabilities.

Fathi ([13]) defined fuzzy hypergroup (fuzzy H_v-group) and fuzzy subhypergroup (fuzzy H_v-subgroup). which is a generalization of the concept of Dib's fuzzy group of the fuzzy spaces ([7]). The notion of multi-spaces was introduced by Smarandache in 1969 ([19]). Hila and Davvaz ([9]) the conception of multi-hypergroup space is a generalization of the algebraic hypergroup. By combining the above Smarandache multi-spaces with hypergroups in hyperstructure theory, a new kind of algebraic hyperstructure called multi-hypergroup space.

In this work, the notion of multi-fuzzy space is used to define multi-fuzzy hyperstructure and multi- fuzzy hyperoperation. Also, the concept of multi-fuzzy hypergroup (multi-fuzzy H_v-group) is introduced and some interesting results are derived.

2. Preliminaries

In this section, we recall the definitions and related results which are needed in this work.

Definition 2.1 ([16]). *A fuzzy set A in a universe of discourse U is characterized by a membership function $\mu_A(x)$ that takes values in the interval* $[0, 1]$.

Definition 2.2 ([3]). *An intuitionistic fuzzy set A in a non-empty set U (a universe of discourse) is an object having the form:*

$$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in U\},$$

where the functions $\mu_A(x) : U \longrightarrow [0,1]$ *and* $\gamma_A(x) : U \longrightarrow [0,1]$, *denote the degree of membership and degree of non-membership of each element* $x \in U$ *to the set* A *respectively, and* $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ *for all* $x \in U$.

Definition 2.3 ([13]). *Let* k *be a positive integer, A multi-fuzzy set* A *in* U *is a set of ordered sequences* $A = \{u/(\mu_1(u), \mu_2(u), \ldots, \mu_i(u), \ldots, \mu_k(u)) : u \in U\}$, *where* $\mu_i \in \rho(U)$, $i = 1, 2, 3, \ldots, k$. *The function* $\mu_A = (\mu_1, \mu_2, \ldots, \mu_k)$ *is called the multi-membership function of multi-fuzzy set* A, k *is called the dimension of* A. *The set of all multi-fuzzy sets of dimension* k *in* U *is denoted by* $M^kFS(U)$. *A multi-fuzzy set of dimension 1 is a Zadeh's fuzzy set and a multi-fuzzy set of dimension 2 with* $\mu_1(u) + \mu_2(u) \leq 1$ *is an Atanassov's intuitionistic fuzzy set.*

Definition 2.4 ([7]). *A fuzzy space* $(X, I = [0,1])$ *is the set of all ordered pairs* (x, I), $x \in X$

$$(X, I) = \{(x, I) : x \in X\},$$

where $(x, I) = \{(x, r) : r \in I\}$. *The ordered pair* (x, I) *is called a fuzzy element in the fuzzy space* (X, I).

Definition 2.5 ([13]). *If* G *is an ordinary group having the ordinary binary operation* $\circ$ *and* A *is a multi-fuzzy subset of* G, *then* A *is called a multi-fuzzy subgroup of* G *if and only if*

(1) $A(x \circ y) \geq \min\{A(x), A(y)\}$,

(2) $A(x^{-1}) \geq A(x)$, *for all* $x, y \in G$.

Note. *A multi-fuzzy subgroup of dimension 1 is a Rosenfeld's fuzzy subgroup and a multi-fuzzy subgroup of dimension 2 is the Zhan's intuitionistic fuzzy subgroup.*

Definition 2.6 ([2]). *Let* X *be a non-empty set. A multi-fuzzy space* $(X, I_i^X = [0,1])$ *is the set of all ordered sequences* (x, I_i^X), $x \in X$; *where* $i \in \mathbb{N}$, *that is,* $(X, I_i^X) = \{(x, I_i^X) : x \in X\}$, *where* $(x, I_i^X) = \{(x, r_i) : r_i \in I_i^X\}$. *The ordered sequences* (x, I_i^X) *is called a multi-fuzzy element in the multi-fuzzy space* (X, I_i^X). *Therefore, the multi-fuzzy space is an (ordinary) set of ordered sequences. In each ordered sequences the first component indicates the (ordinary) element and the second component indicates a set of possible multi-membership values.*

Note. *A multi-fuzzy space of dimension 1 is a Dib's fuzzy group and a multi-fuzzy space of dimension 2 is the Fathi's intuitionistic fuzzy group.*

Definition 2.7 ([2]). *A multi-fuzzy binary operation* $\underline{F} = (F(x,y), f_{(xy)_i})$ *on the multi-fuzzy space* (X, I_i^X) *is a multi-fuzzy function from* $(X, I_i^X) \times (X, I_i^X) \longrightarrow (X, I_i^X)$ *with multi-comembership functions* $f_{(xy)_i}$ *that satisfy:*

(*i*) $f_{(xy)_i}(r_i, s_i) \neq 0$ *if* $r_i \neq 0$, $s_i \neq 0$, *where* $i \in \mathbb{N}$,

(*ii*) $f_{(xy)_i}(r_i, s_i)$ *are onto, that is,* $f_{(xy)_i}(I_i^X \times I_i^X) = I_i^X$, $x, y \in X$.

Definition 2.8 ([2]). *A multi-fuzzy algebraic system* $((X, I_i^X), \underline{F})$ *is called a multi-fuzzy group if and only if for every* $(x, I_i^X), (y, I_i^X), (z, I_i^X) \in (X, I_i^X)$ *the following conditions are satisfied:*

(i) Associative:

$$((x, I_i^X)\underline{F}(y, I_i^X))\underline{F}(z, I_i^X) = (x, I_i^X)\underline{F}((y, I_i^X)\underline{F}(z, I_i^X)),$$

that is

$$((xFy)Fz, I_i^X) = (xF(yFz), I_i^X).$$

(ii) Existence of a multi-fuzzy identity element (e, I_i^X) *for which*

$$(x, I_i^X)\underline{F}(e, I_i^X) = (e, I_i^X)\underline{F}(x, I_i^X) = (x, I_i^X),$$

that is

$$(xFe, I_i^X) = (eFx, I_i^X) = (x, I_i^X).$$

(iii) Every multi-fuzzy element (x, I_i^X) *has an inverse* (x^{-1}, I_i^X) *such that*

$$(x, I_i^X)\underline{F}(x^{-1}, I_i^X) = (x^{-1}, I_i^X)\underline{F}(x, I_i^X) = (e, I_i^X).$$

Theorem 2.1 ([2])**.** *Associated to each multi-fuzzy group* $((X, I_i^X), \underline{F})$ *where* $\underline{F} = (F, f_{(xy)_i})$ *is an intuitionistic fuzzy group* $((X, I, I), \boldsymbol{F})$ *which is isomorphic to the multi-fuzzy group* $((X, I_i^X), \underline{F})$ *by the correspondence* $(X, I_i^X) \longleftrightarrow (x, I, I)$.

Definition 2.9 ([21])**.** *Let* $\langle (H, I), \diamond \rangle$ *be a fuzzy hypergroup (fuzzy* H_v*-group) and let* $U = \{(x, u_x) : x \in U_0\}$ *be a fuzzy subspace of fuzzy space* (H, I)*. Then* $(U, \diamond)$ *is called a fuzzy subhypergroup (fuzzy* H_v*-subgroup) of the fuzzy hypergroup* $\langle (H, I), \diamond \rangle$ *if* $\diamond$ *is closed on the fuzzy subspace* U *and* $(U, \diamond)$ *satisfies the conditions of hypergroup (*H_v*-group).*

Theorem 2.2 ([21])**.** *For each fuzzy hypergroup (fuzzy* H_v*-group) there is an associated ordinary hypergroup (*H_v*-group)* (H, Δ) *which is isomorphic to the fuzzy hypergroup (fuzzy* H_v*-group)* $\langle (H, I), \diamond \rangle$ *by the correspondence* $(x, I) \longleftrightarrow x$.

Definition 2.10 ([20])**.** *An intuitionistic fuzzy hypergroup is an intuitionistic fuzzy hyperstructure* $\langle (H, I, I), \underline{F} \rangle$ *satisfying the following axioms:*

(1) $((x, I, I)\underline{F}(y, I, I))\underline{F}(z, I, I) = (x, I, I)\underline{F}((y, I, I)\underline{F}(z, I, I))$, *for all* (x, I, I), (y, I, I), $(z, I, I) \in (H, I, I)$.

(2) $(x, I, I)\underline{F}(H, I, I) = (H, I, I)\underline{F}(x, I, I) = (H, I, I)$, *for all* $(x, I, I) \in (H, I, I)$.

Theorem 2.3 ([20])**.** *For each intuitionistic fuzzy hypergroup (intuitionistic fuzzy* H_v*-group) there are two fuzzy hypergroups (fuzzy* H_v*-group)* $\langle (H, I), F' \rangle$ *and* $\langle (H, I), F' \rangle$ *(such that* $F = (F, \underline{f}_{xy})$ *and* $F' = (F, 1 - \bar{f}_{xy}))$ *which are isomorphic to the intuitionistic fuzzy hypergroup (intuitionistic fuzzy* H_v*-group)* $\langle (H, I, I), \underline{F} \rangle$ *by the correspondence* $(x, I, I) \longleftrightarrow (x, I)$, $x \in H$.

3. Multi-fuzzy hypergroup

In this work, multi-fuzzy hypergroups are defined a correspondence relation between these multi-fuzzy notions and fuzzy hypergroups is given.

Definition 3.1. *Let (H, I_i^X) be a non-empty multi-fuzzy space. A multi-fuzzy hyperstructure (multi-fuzzy hypergroupoid), denoted by $((H, I_i^X), \underline{F} = (F, f_{(xy)_i}))$ is a multi-fuzzy space together with a multi-fuzzy function having onto co-membership functions (referred as a multi-fuzzy hyperoperation) $\underline{F} : (H, I_i^X) \times (H, I_i^X) \longrightarrow \rho^*((H, I_i^X))$, where $\rho^*((H, I_i^X))$ denotes the set of all non-empty multi-fuzzy subspaces of multi-fuzzy spaces (H, I_i^X) and $\underline{F} = (F, f_{(xy)_i})$ with $F : H \times H \longrightarrow \rho^*(H)$ and $f_{(xy)_i} : I_i^X \times I_i^X \longrightarrow I_i^X$.*

A multi-fuzzy hyperoperation $\underline{F} = (F, f_{(xy)_i})$ on (H, I_i^X) is said to be *uniform* if the associated co-membership functions $f_{(xy)_i}$ are identical, that is, $f_{(xy)_i} = f_i$ for all $x, y \in H$. A uniform multi-fuzzy hyperstructure $\langle (H, I), \underline{F} \rangle$ is a multi-fuzzy hyperstructure $\langle (H, I_i^X), \underline{F} \rangle$ with uniform multi-fuzzy hyperoperation.

Example 3.1. Let $H = \{-1, 1, -i, i\}$, where $i = \sqrt{-1}$. Define the multi-fuzzy hyperoperation $\underline{F} = (F, f_{(xy)_i})$ over the multi-fuzzy space (H, I_i^X) such that $F : H \times H \longrightarrow \rho^*(H)$ with $F(x, y) = xFy = \{x, y\}$ and $f_{(xy)_i} : I_i^X \times I_i^X \longrightarrow I_i^X$ with $f_{(xy)_i}(r_i, s_i) = r_i \wedge s_i$, that is,

$$(x, I_i^X)\underline{F}(y, I_i^X) = \{(x, f_{(xy)_i}(I_i^X \times I_i^X)), (y, f_{(yx)_i}(I_i^X \times I_i^X))\},$$

for all $(x, I_i^X), (y, I_i^X) \in (H, I_i^X)$. Therefore $\langle (H, I_i^X), \underline{F} \rangle$ is a uniform multi-fuzzy hyperstructure.

After introducing multi-fuzzy hyperstructure, the notion of a multi-fuzzy hypergroup is now defined.

Definition 3.2. *A multi fuzzy hypergroup is a multi-fuzzy hyperstrucure $\langle (H, I_I^X), \underline{F} \rangle$ satisfying the following axioms:*

(i) $((x, I_i^X)\underline{F}(y, I_i^X))\underline{F}(z, I_i^X) = (x, I_i^X)\underline{F}((y, I_i^X)\underline{F}(z, I_i^X))$, for all (x, I_i^X), (y, I_i^X), $(z, I_i^X) \in (H, I_i^X)$,

(ii) $(x, I_i^X)\underline{F}(H, I_i^X) = (H, I_i^X)\underline{F}(x, I_i^X) = (H, I_i^X)$, for all $(x, I_i^X) \in (H, I_i^X)$.

Definition 3.3. *A multi-fuzzy H_v-group is a multi-fuzzy hyperstructure $\langle (H, I_i^X), \underline{F} \rangle$ satisfying the following conditions:*

(i) $((x, I_i^X)\underline{F}(y, I_i^X))\underline{F}(z, I_i^X) \cap (x, I_i^X)\underline{F}((y, I_i^X)\underline{F}(z, I_i^X)) \neq \phi$, for all (x, I_i^X), (y, I_i^X),

$(z, I_i^X) \in (H, I_i^X)$,

(ii) $(x, I_i^X)\underline{F}(H, I_i^X) = (H, I_i^X)\underline{F}(x, I_i^X) = (H, I_i^X)$, for all $(x, I_i^X) \in (H, I_i^X)$.

If $\langle (H, I_i^X), \underline{F} \rangle$ satisfies only the first condition of Definition 3.3 (Defnition 3.3), then it is called a multi-*fuzzy semihypergroup* (multi-*fuzzy H_v-semigroup*).

The next theorem gives a correspondence relation between multi-fuzzy hypergroup and multi-fuzzy hypergroup.

Theorem 3.1. *For each multi-fuzzy hypergroup there is an associated intuitionistic fuzzy hypergroup* $\langle (H, I, I), F\rangle$ *which is isomorphic to the multi-fuzzy hypergroup* $\langle (H, I_i^X), \underline{F}\rangle$ *by the correspondence* $(x, I_i^X) \longleftrightarrow (x, I, I)$.

Proof of Theorem 3.1. *Consider the multi-fuzzy hypergroup* $\langle (H, I_i^X), \underline{F}\rangle$. *Now using the correspondence* $(x, I_i^X) \longleftrightarrow (x, I, I)$. *We can redefine multi-fuzzy hyperopertion* $\underline{F} = (F, f_{(xy)_i})$ *with* $\underline{F} : (X, I_i^X) \times (X, I_i^X) \longrightarrow \rho^*((H, I_i^X))$ *to be* $\underline{F'} : (X, I, I) \times (X, I, I) \longrightarrow \rho^*((H, I, I))$. *That is* $\underline{F'}$ *define an intuitionistic fuzzy hyperoertion over* (H, I, I) *is an intuitionistic fuzzy hyperopertion in sense Fathi.*

4. Conclusion

In this work, we have generalized the concept of fuzzy hypergroup based on fuzzy space to the context of multi-fuzzy hypergroup based on multi-fuzzy space and multi-fuzzy H_v-group.

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