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Reality with Smarandachely Denied Axiom

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Abstract: Usually, one applies mathematics to hold on the reality of matters in the universe, i.e., mathematical reality and a few peoples firmly believe that classical mathematics could handle this things with no needs on its extension. However, it is not the case because contradictions exist everywhere but classical mathematics must be logically consistent without contradiction in the eyes of human beings. The fatal flaw in this view is it's priori assumption that the universe is uniform, and then can be characterized by homogeneous characters with classical mathematics such as differential equations. However, the universe including its matters are not uniform, even being messy. This fact implies that one should extend classical mathematics to an enveloping one for understanding matters in the universe and bearing maybe with contradictions, i.e., such systems in mathematics including with Smarandachely denied axioms. Certainly, an axiom is said Smarandachely denied if the axiom behaves differently, i.e., validated and invalided, or only invalidated but in at least two distinct ways in a system S. Such a system S is said Smarandace system. The main purpose of this paper is to introduce the Smarandachely denied axiom, show its contribution to reality and explain the role of its equivalent form, the Smarandache multispace for extending classical mathematics, i.e., mathematical combinatorics for understanding matters because each matter always inherits a topological structure by its nature in the universe.

Key Words: Reality, mathematical reality, CC conjecture, mathematical universe hypothesis, Smarandachely denied axiom, Smarandache system, Smarandache multispace, mathematical combinatorics.

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§1. Introduction

Usually, all matter in the universe are colorful, maybe with mystery and complex mechanism to the human eyes no matter it is living or not. We understand matters by the reality for promoting the survival and development of humans ourselves in harmony with nature. However, we are embarrassed hardly know their true face unless their surface characters before humans and even so, it could be also a false vision or hallucination, just feelings of humans. Then, *what is the reality of a matter*? The word *reality* of a matter $\mathcal{T}$ is its state as it actually exist, including everything that is and has been, no matter it is observable or comprehensible by humans. For

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humans ourselves, a natural question is *could we really hold on the reality of matters in the universe?* It should be noted that the answer is different in the scientific and the religious. For examples, *all matters are illusion of humans* claimed by Sakyamuni in his famous *Diamond Sutra* and *the universal truth can be restated but the restated truth is not the universal one* asserted by Laozi in his *Tao Te Ching*. However, nearly all scientists firmly believe that we can open the cover enveloped on matters, find their true face and then, hold on the natural laws of universe. Here, we do not discuss who's right or who's wrong. Even if we can really open the cover on matters in the universe, there is also a question on reality, i.e., *how to characterize the reality of matters?* The answer is nothing else but the science or particularly, the mathematical sciences.

We usually understand a matter by its characters or the system S of characters by knowing the state $x_v(t)$ on time t for elements $v \in S$. However, we can only observe the state, estimate and calculate the change rate of its elements on time t. Thus, we can not obtain directly the state x_v for $v \in S$ but a system of differential equations. We have to solve the differential equation system for hold on the states $x_v(t), v \in S$.

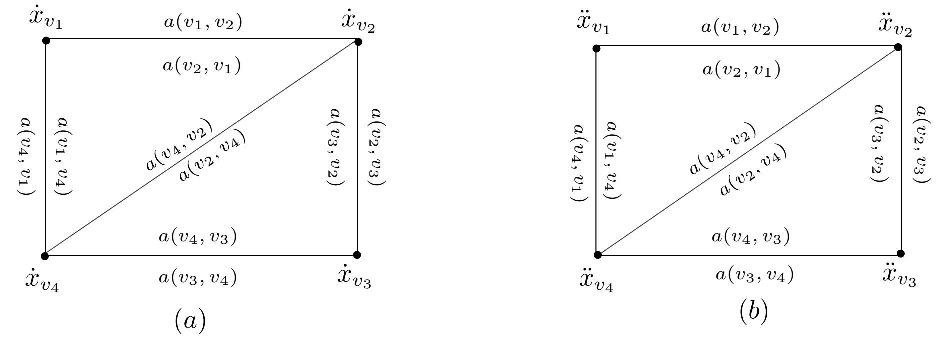

Figure 1. System States

For example, the differential equations of a 4-element system $S' = \{v_1, v_2, v_3, v_4\}$ shown in Figure 1(a) and (b) are respectively

$$\begin{cases} \dot{x}_{v_1} = F_1\left(a\left(v_2, v_1\right), a\left(v_4, v_1\right)\right) \\ \dot{x}_{v_2} = F_2\left(a\left(v_1, v_2\right), a\left(v_3, v_2\right), a\left(v_4, v_2\right)\right) \\ \dot{x}_{v_3} = F_3\left(a\left(v_2, v_3\right), a\left(v_4, v_3\right)\right) \\ \dot{x}_{v_4} = F_4\left(a\left(v_1, v_4\right), a\left(v_3, v_4\right), a\left(v_2, v_4\right)\right) \end{cases} \quad \text{or} \quad \begin{cases} \ddot{x}_{v_1} = H_1\left(a\left(v_2, v_1\right), a\left(v_4, v_1\right)\right) \\ \ddot{x}_{v_2} = H_2\left(a\left(v_1, v_2\right), a\left(v_3, v_2\right), a\left(v_4, v_2\right)\right) \\ \ddot{x}_{v_3} = H_3\left(a\left(v_2, v_3\right), a\left(v_4, v_3\right)\right) \\ \ddot{x}_{v_4} = H_4\left(a\left(v_1, v_4\right), a\left(v_3, v_4\right), a\left(v_2, v_4\right)\right) \end{cases} \quad (1)$$

and then, we subjectively equate the state of $S'(t)$ with the solutions $x_{v_1}(t), x_{v_2}(t), x_{v_3}(t), x_{v_4}(t)$ of differential equations (1) without even knowing it maybe not true, where, x_{v_i} is the character of element v_i of the matter S, $a\left(v_i, v_j\right)$ is the action of element v_i on v_j, F_i, H_i are action functions for integers $1 \leq i, j \leq 4$, and $\dot{x}, \ddot{x}$ denote the first or second differentials of x on time t. It seems that a solution of differential equations (1) is a state of system S' because they are in causality. However, *is the converse is true also, i.e., any state of $S'(t)$ can be characterized by $x_{v_i}(t), 1 \leq i \leq 4$?* The answer is inconclusive because the solution $x_{v_i}(t), 1 \leq i \leq 4$ is the state of $S'(t)$ if and only if equations (1) are solvable and there is a bijection $\phi : S'(t) \leftrightarrow \{x_{v_1}(t), x_{v_2}(t), x_{v_3}(t), x_{v_4}(t)\}$, but we can not conclude so unless subjectively in mind on classical mathematics. Certainly, a mathematical system should be logically consistent without contradiction but lots of humans misunderstand this criterion, excluded contradictory systems in mathematics, which results in the limitation of mathematics on reality of matters. It should be noted that the most important thing is not excluded contradictions but how to let them coexist peacefully in mathematics for extending the limitation of classical mathematics and establish an envelope mathematics, in which classical mathematics only be its parts for understanding matters in the universe. For this objective, the Smarandachely denied axiom is a such one presented by F.Smarandache on geometry in 1969 following ([37],[40-41]).

Axiom 1.1 *An axiom is said Smarandachely denied if in the same space the axiom behaves differently, i.e., validated and invalided, or only invalidated but in at least two distinct ways.*

By Axiom 1.1, there are Smarandachely conceptions on geometry following.

Definition 1.2([16],[37]) *A Smarandache geometry is such a geometry that has at least one Smarandachely denied axiom.*

A conception closely related to Smarandache geometry is the Smarandache multispace defined in the following, which seems to be a generalization of Smarandache geometry but equivalent to Smarandache geometry by a geometrical view.

Definition 1.3([17],[37]) *Let* $(\Sigma_1;\mathcal{R}_1)$, $(\Sigma_2;\mathcal{R}_2)$, $\cdots$, $(\Sigma_m;\mathcal{R}_m)$ *be* m *mathematical spaces, different two by two, i.e., for any two spaces* $(\Sigma_i;\mathcal{R}_i)$ *and* $(\Sigma_j;\mathcal{R}_j)$, $\Sigma_i \neq \Sigma_j$ *or* $\Sigma_i = \Sigma_j$ *but* $\mathcal{R}_i \neq \mathcal{R}_j$. *A Smarandache multispace* $\widetilde{\Sigma}$ *is a union* $\bigcup\limits_{i=1}^{m} \Sigma_i$ *with rules* $\widetilde{\mathcal{R}} = \bigcup\limits_{i=1}^{m} \mathcal{R}_i$ *on* $\widetilde{\Sigma}$, *i.e., the union of rules* $\mathcal{R}_i$ *on* Σ_i *for integers* $1 \leq i \leq m$, *denoted by* $\left(\widetilde{\Sigma};\widetilde{\mathcal{R}}\right)$.

The Smarandache multispace inherits a topological structure G^L with a generalization, i.e., continuity flow consisting of the element in mathematical combinatorics.

Definition 1.4([11-12]) *For an integer* $m \geq 1$, *let* $\left(\widetilde{\Sigma};\widetilde{\mathcal{R}}\right)$ *be a Smarandache multispace consisting of* m *mathematical systems* $(\Sigma_1;\mathcal{R}_1)$, $(\Sigma_2;\mathcal{R}_2)$, $\cdots$, $(\Sigma_m;\mathcal{R}_m)$. *An inherited topological structure* $G^L\left[\widetilde{\Sigma};\widetilde{\mathcal{R}}\right]$ *of* $\left(\widetilde{\Sigma};\widetilde{\mathcal{R}}\right)$ *is a labeled topological graph defined following:*

$$V\left(G^L\left[\widetilde{\Sigma};\widetilde{\mathcal{R}}\right]\right) = \{\Sigma_1, \Sigma_2, \cdots, \Sigma_m\},$$

$$E\left(G^L\left[\widetilde{\Sigma};\widetilde{\mathcal{R}}\right]\right) = \{(\Sigma_i, \Sigma_j)\,|\Sigma_i \bigcap \Sigma_j \neq \emptyset,\ 1 \leq i \neq j \leq m\} \text{ with labeling}$$

$$L:\ \Sigma_i \to L(\Sigma_i) = \Sigma_i \quad \text{and} \quad L:\ (\Sigma_i, \Sigma_j) \to L(\Sigma_i, \Sigma_j) = \Sigma_i \bigcap \Sigma_j$$

for integers $1 \leq i \neq j \leq m$, *such as those shown in Figure* 2 *for the case of* $m = 4$ *and* $G \simeq K_4$.

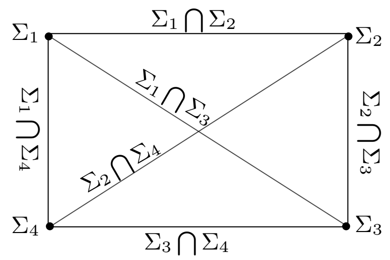

Figure 2. Graphs inherited in a Smarandache multispace

Now, *what is the contribution of Axiom* 1.1 *in extending of classical mathematics and what is its role with the reality?* The main purpose of this paper is to introduce the Smarandachely denied axiom, survey its contribution to geometry, and then from Smarandache multispace to mathematical combinatorics for extending classical mathematics to mathematical combinatorics for understanding the reality of matters in the universe because each matter always inherits a topological structure by its nature.

For terminologies and notations not mentioned here, we follow reference [4] for algebra, [5] for topological graphs, [16-18] and [37-38] for Smarandache geometry, multispaces and Smarandache systems.

§2. Smarandachely Denied Axiom to Geometry

Notice that the Smarandachely denied axiom is originally presented on geometry, which enables one to generalize geometry to Smarandache geometry concluding classical geometry as its parts. In a Smarandache geometry, the points, lines, planes, spaces, triangles, $\cdots$ are respectively called s-points, s-lines, s-planes, s-spaces, s-triangles, $\cdots$ in order to distinguish them from those in classical geometry. Although it is defined by Definition 1.2, an elementary but natural question is shown in the following.

Question 2.1 *Are there non-trivial Smarandache geometry constraint in logic?*

The answer is certainly *Yes*! For example, the axiom system of Euclidean geometry consists of 5 axioms: ① There is a straight line between any two points; ② A finite straight line can produce a infinite straight line continuously; ③ Any point and a distance can describe a circle; ④ All right angles are equal to one another; ⑤ If a straight line falling on two straight lines make the interior angles on the same side less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles. Then, we are easily construct a Smarandache geometry following.

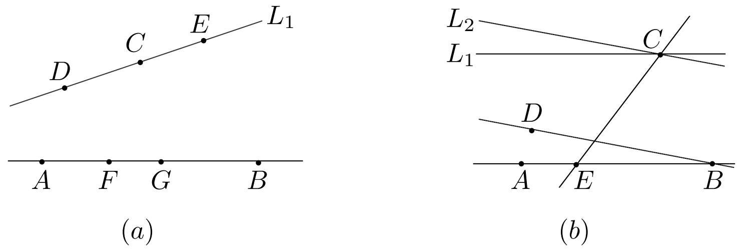

Figure 3. A non-trivial Smarandache geometry

Let $\mathbb{R}^2$ be a Euclidean plane and let A, B, C be three non-collinear points in $\mathbb{R}^2$. We define s-points $\mathcal{P}_s$ to be all usual Euclidean points in $\mathbb{R}^2$ and s-lines $\mathcal{L}_s$ to be any Euclidean line passing through one and only one of points A, B, C. We show the pair $\{P_s; L_s\}$ consists of a Smarandache geometry: 1) The axiom ① in Euclidean geometry is now replaced by *one s-line* or *no s-line* because through any two distinct s-points D, E collinear with one of A, B, C there is one s-line passing through them but through any two distinct s-points F, G lying on AB or non-collinear with one of A, B, C, there are no s-lines passing through them such as those shown in Figure 3(a); 2) The axiom ⑤ is now replaced by *one parallel* or *no parallel* because if we let L_1, L_2 be two s-lines passing through C with L_1 parallel but L_2 not parallel to AB in the Euclidean sense, then through any s-point D not lying on AB there are no s-lines parallel to L_1 but there is one s-lines parallel to L_2 if one of DB,DA or DC happens parallel to L_2. Otherwise, there are no s-lines passing through D parallel to L_2, see Figure 3(b) for details.

More examples of Smarandache geometry can be found in [3], [6-7], [10] and [40-41]. Certainly, the quantitative characterization on the real face of matters lead to the neutrosophic logic by Smarandachely denied axiom, which contributes the introduction of the degree of negation or partial negation of an axiom and, more general, of a scientific or humanistic proposition (theorem, lemma, etc.) in any field. It works somehow like the negation in fuzzy logic with a degree of truth, a degree of falsehood and a degree of truth, i.e. neither truth nor falsehood but unknown, ambiguous, indeterminate (see [40-41] for details).

As a particular case, the Euclidean, Lobachevsky-Bolyai-Gauss and Riemannian geometries may be united altogether by the Smarandache geometry in the same space because it can be partially Euclidean and partially non-Euclidean, and it seems connecting with the *relativity theory* and *parallel universes* because it includes the Riemannian geometry in a subspace but more generalized. H.Iseri [6-7] constructed the Smarandache 2-manifolds by using equilateral triangular disks on Euclidean plane $\mathbb{R}^2$, which can be come true by paper models in $\mathbb{R}^3$ for elliptic, Euclidean and hyperbolic cases and in paper [10], L.Mao advanced a new method for constructed Smarandache 2-manifold by combinatorial maps. Generally, it should be noted that Smarandache n-manifold for $n \geq 2$, i.e. combinatorial n-manifold and a differential theory on such manifolds were constructed by L.Mao in papers [11]. For $n = 1$, i.e., a curve in differential geometry is called *Smarandache curve* if it holds with Smarandachely denied axiom, which has been extensively researched and many researching, such as those of papers [2],[9],[36],[42]-[47], [50] were published in the *International Journal of Mathematical Combinatorics* after it suggested by L.Mao for the authors of [44]. In fact, nearly all geometries in classical mathematics such as those of Riemann geometry, Finsler geometry, Weyl geometry and Kahler geometry are particular cases of Smarandache geometry.

§3. Smarandachely Denied Axiom to Mathematical Systems

Although the Smarandachely denied axiom is originally to geometry for generalizing the 5th axiom of Euclidean geometry. Its notion can be generalized further to a generalized form on all mathematical systems by replacing the word *space* with *mathematical system* following.

Axiom 3.1(Generalized Smarandachely denied axiom) *An axiom is said generalized Smarandachely denied if in the same mathematical system the axiom behaves differently, i.e., validated and invalided, or only invalidated but in at least two distinct ways and then, a Smarandache system is such a mathematical system that has at least one Smarandachely denied axiom.*

For example, a Lie group G in classical mathematics is a Smarandachely denied system because an element $a \in G$ is both a point on manifold G, also an element in group G. Thus, if we let Axiom I be an axiom that points have no neighborhood $\mathcal{C}$ on G, Axiom II be that

a^{-1} is not exist for $\forall a \in G$ and Axiom III to be that the group operations $(a,b) \rightarrow a \cdot b$ or $a \rightarrow a^{-1}$ are not C^{∞}-mapping. Then, Axioms I, II and III are invalidated in a Lie group, which implies G is a Smarandache system and in general, the result following can be verified.

Proposition 3.2 *Let* $(\mathcal{S}_1,\mathcal{R}_1),(\mathcal{S}_2,\mathcal{R}_2),\cdots,(\mathcal{S}_n,\mathcal{R}_n)$ *be* n *systems in classical mathematics with* $\mathcal{S}_i \neq \mathcal{S}_j$ *or* $\mathcal{S}_i = \mathcal{S}_j$ *but* $\mathcal{R}_i \neq \mathcal{R}_j$ *for an integers* $n \geq 2$*, where* $\mathcal{S}_i$ *is a set,* $\mathcal{R}_i \subset \mathcal{S}_i \times \mathcal{S}_i$ *for integers* $1 \leq i \leq n$*. Then, the union*

$$(\Sigma;\Pi) = (\mathcal{S}_1 \cup \mathcal{S}_2 \cup \cdots \cup \mathcal{S}_n; \mathcal{R}_1 \cup \mathcal{R}_2 \cup \cdots \cup \mathcal{R}_n)$$

is a Smarandache system.

Proof Similarly to the case of Lie group, define Axiom i to be $(a,b) \notin \mathcal{R}_i$ for $\forall a,b \in \mathcal{S}_i$ in $(\Sigma;\Pi)$ if $(a,b) \in \mathcal{R}_i$ in $(\mathcal{S}_i,\mathcal{R}_i)$ for integers $1 \leq i \leq n$. Then, each Axiom i is invalidated in $(\Sigma;\Pi)$, i.e., invalidated in at least two distinct ways. □

Notice that the Smarandache system $\left(\widetilde{\mathcal{S}};\widetilde{\mathcal{R}}\right)$ in Proposition 3.2 is in fact a Smarandache multispace by Definition 1.3, appearing not only in mathematics but also in physics, for instance the unmatter composed of particles and anti-particles [39], and generally, the generalized Smarandachely denied axiom is equivalent to Smarandache multispace.

Proposition 3.3 *A mathematical system* (Σ,Π) *is generalized Smarandachely denied if and only if it is a Smarandache multispace.*

Proof The proof on necessity of the result is similar to the decomposition shown in [22], divided into two cases following.

Case 1. There is an axiom $\mathscr{A}$ in (Σ,Π) that behaves both validated and invalided. Define

$$\Sigma_1 = \{x \in \Sigma \text{ hold with Axiom } \mathscr{A}\}, \quad \Sigma_2 = \{y \in \Sigma \text{ hold not with Axiom } \mathscr{A}\}.$$

Then, $\Sigma = \Sigma_1 \bigcup \Sigma_2$, i.e., (Σ,Π) is a Smarandache multispace.

Case 2. There is an axiom in $\mathscr{A}$ in (Σ,Π) that behaves invalidated but in distinct ways $W_1, W_2, \cdots, W_s, s \geq 2$. Define $\Sigma_i = \{x \in \Sigma \text{ hold not with Axiom } \mathscr{A} \text{ in way } W_i\}$ for integers $1 \leq i \leq s$ and $\Sigma_0 = \Sigma \setminus \bigcup\limits_{i=1}^{s} \Sigma_i$, where Σ_0 maybe an empty set. Then

$$\Sigma = \left(\bigcup_{i=1}^{n} \Sigma_i\right) \bigcup \Sigma_0 \tag{3}$$

is a Smarandache multispace.

The proof on sufficiency of the result is similar to the proof of Proposition 3.2. Let (Σ,Π) be a Smarandache multispace with $\Sigma = \Sigma_1 \bigcup \Sigma_2 \bigcup \cdots \bigcup \Sigma_n$, $\Pi = \mathcal{R}_1 \bigcup \mathcal{R}_2 \bigcup \cdots \bigcup \mathcal{R}_n$ and $(\Sigma_i;\mathcal{R}_i)$ being a mathematical space. Define Axiom $\mathcal{A}_i = \{x \notin \Sigma_i \text{ if } x \in \Sigma_i\}$ for integers $1 \leq i \leq n$. Then, each axiom $\mathcal{A}_i$ in Σ behaves both validated, invalided, and also invalidated in n distinct ways. □

Notice that there are many achievements of Smarandache multispaces in extending classical mathematics with applying to physics. For example, multigroups, multirings, multifields, multialgebra and multistructure,$\cdots$, etc. discussed in [1], [14]-[15], [17] and [48]-[49].

§4. **Smarandachely Denied Axiom to Reality**

4.1. Mathematical Reality. Proposition 3.3 enables one to discuss the reality of matters in the universe by generalized Smarandachely denied axiom. Particularly, the reality discussed by differential equations in physics, i.e., the mathematical reality is the reality on a matter or not. The conclusion following is a little surprising for the usual view on classical mathematics.

Proposition 4.1 *If $\mathscr{R}_M$, $\mathscr{R}$ are respectively the mathematical reality or the reality of a matter in the universe. Then $\mathscr{R}_M \subseteq \mathscr{R}$ and furthermore, there are many examples hold with $\mathscr{R}_M \neq \mathscr{R}$.*

Proof The relation $\mathscr{R}_M \subseteq \mathscr{R}$ is obvious. For the inequality $\mathscr{R}_M \neq \mathscr{R}$, many cases show the mathematical reality is not the reality, even the ridiculous on reality of a matter in sometimes, for instance the equations (1) are non-solvable, i.e., we can not obtain the state of system $S'(t)$. Even the equations (1) are solvable, we can not conclude their solutions describing the state of system S'. For example, let H_1, H_2, H_3, H_4 and H_1', H_2', H_3', H_4' be two groups of running horses constraint with running on respectively 4 straight lines

$$① \begin{cases} x+y &= \ \ 2 \\ x+y &= -2 \\ x-y &= -2 \\ x-y &= \ \ 2 \end{cases} \qquad \text{or} \qquad ② \begin{cases} x=y \\ x+y=2 \\ x=1 \\ y=1 \end{cases}$$

on the Euclidean plane $\mathbb{R}^2$ such as those shown in Figure $4(a), (b)$ and (c).

Figure 4. Four horses on the plane

Clearly, the first system is non-solvable because $x + y = -2$ is contradictious to $x + y = 2$, and so that for the equation $x - y = -2$ to $x - y = 2$ but the second system is solvable with $(x, y) = (1, 1)$. *Could we conclude that the running states of horses H_1', H_2', H_3', H_4' are a point $(1, 1)$, i.e., staying on the point $(1, 1)$ and H_1, H_2, H_3, H_4 are nothing*? The answer is certainly not because all of the horses are running on the Euclidean plane $\mathbb{R}^2$. However, we know nothing on the state by the solution of the two equation systems because the solvability of systems ① or ② only implies the orbits intersection, not the running state of the group of horses. □

Then, *what is wrong in the example of the two groups of horses?* The wrong appears with the assumption that the solution of ① or ② characterizes the running state of horses

H_1', H_2', H_3', H_4' or H_1, H_2, H_3, H_4. Certainly, each equation in systems ① or ② really characterizes the running state of one horse but we can not equate the running states of the four horses with the solution of ① and ② because they are different in objective, and maybe contradictory. Generally, there is contradiction maybe if we characterize a group of matters within a same space. Indeed, we can eliminate the contradiction by characterizing them with different variables of spaces, for instance in parallel spaces one by one. However, they are really a system with relations on its elements, we should know their global state of the system, not isolated one on its elements in the Euclidean plane $\mathbb{R}^2$. This case implies also that the non-solvable systems of equations characterize matters also if each of them was established on the characters of matters in the universe but the solution is not the state of the matter when it consists of characters more than 2, which implies the classical mathematics is incomplete for understanding the reality of matters, particularly, the biological systems in the universe.

Notice that a cosmologist, Max Tegmark proposed a hypothesis on the universe once, called the *mathematical universe hypothesis* following, spread widely in the scientific community.

Hypothesis 4.2(Max Tegmark,[47]) *The physical universe is not merely described by mathematics but a mathematical structure, i.e.,* $\mathscr{R}_M = \mathscr{R}$.

Certainly, the mathematical universe hypothesis is essentially a duplication of the *Theory of Everything*. However, Proposition 4.1 concludes classical mathematics is incomplete for understanding matters in the unverse, which implies that the incorrectness of the mathematical universe hypothesis, or in other words, the first step to making this hypothesis work should be the extending of classical mathematics, i.e., including contradictions, let them peaceful coexistence in mathematics, and then it maybe set up on the universe.

4.2. Non-Solvable Equation System. Certainly, each linear equation $ax + by = c$ with $ab \neq 0$ is in fact a point set $L_{ax+by=c} = \{(x,y)|ax + by = c\}$ in $\mathbb{R}^2$, such as those shown in Figure 4(a) and (c) for the linear systems ① and ② with

$$L_{x+y=2}\bigcap L_{x+y=-2}\bigcap L_{x-y=2}\bigcap L_{x-y=-2} = \emptyset, \qquad L_{x=y}\bigcap L_{x+y=2}\bigcap L_{x=1}\bigcap L_{y=1} = \{(1,1)\}$$

in the Euclidean plane $\mathbb{R}^2$. Generally, the solution manifold of an equation

$$\mathscr{F}(x_1, x_2, \cdots, x_n, y) = 0,\ n \geq 1 \tag{4}$$

is defined to be an n-manifold $S_{\mathscr{F}} = (x_1, x_2, \cdots, x_n, y(x_1, x_2, \cdots, x_n)) \subset \mathbb{R}^{n+1}$ if it is solvable. Otherwise, $\emptyset$ in geometry and a system

$$(ES_m)\quad \begin{cases} \mathscr{F}_1(x_1, x_2, \cdots, x_n, y) = 0 \\ \mathscr{F}_2(x_1, x_2, \cdots, x_n, y) = 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \mathscr{F}_m(x_1, x_2, \cdots, x_n, y) = 0 \end{cases} \tag{5}$$

of equations with initial values $\mathscr{F}_i(0),\ 1 \leq i \leq m$ in Euclidean space $\mathbb{R}^{n+1}$ is solvable or not dependent on $\bigcap\limits_{i=1}^{m} S_{\mathscr{F}_i} \neq \emptyset$ or $= \emptyset$ in it geometrical meaning, where $S_{\mathscr{F}_i} \neq \emptyset$ for integers

$1 \leq i \leq m$.

Then, *what is the reality of a matter $\mathcal{T}$?* Generally, let $\mu_1, \mu_2, \cdots, \mu_n$ be known and $\nu_i, i \geq 1$ unknown characters at time t for a matter $\mathcal{T}$. Then, $\mathcal{T}$ should be understood by

$$\mathcal{T} = \left(\bigcup_{i=1}^{n}\{\mu_i\}\right) \bigcup \left(\bigcup_{k\geq 1}\{\nu_k\}\right) \tag{6}$$

in logic but with an approximation $\mathcal{T}^\circ = \bigcup\limits_{i=1}^{n}\{\mu_i\}$ for $\mathcal{T}$ by humans at time t, which is nothing else but the Smarandache system or multispace by Proposition 3.3. The example of 4 horses run in the plane shows that applying the solution of equations (5), i.e., $\bigcap\limits_{i=1}^{m} S_{\mathscr{F}_i}$ to the state of a system S maybe cause a ridiculous conclusion, particularly in the case of the non-solvable. In fact, the state of a matter $\mathcal{T}$ described by the system equation (5) is not $\bigcap\limits_{i=1}^{m} S_{\mathscr{F}_i}$ but the equality (6), i.e., Smarandache multispace. We should extend the conception of solution of equations (5).

Definition 4.3([20]) *The $\vee$-solvable, $\wedge$-solvable and non-solvable spaces of equations* (5) *are defined respectively by*

$$\bigcup_{i=1}^{m} S_{\mathscr{F}_i}, \quad \bigcap_{i=1}^{m} S_{\mathscr{F}_i} \quad \textit{and} \quad \bigcup_{i=1}^{m} S_{\mathscr{F}_i} - \bigcap_{i=1}^{m} S_{\mathscr{F}_i}.$$

What is the importance of the $\vee$-solvable and $\wedge$-solvable space? Clearly, the $\vee$-solvable space of (5) shown the state of the system characterized by (5). For example, the state of the four horses should be the $\vee$-solvable space $L_{x+y=2} \bigcup L_{x+y=-2} \bigcup L_{x-y=2} \bigcup L_{x-y=-2}$, not the $\wedge$-solvable space $L_{x+y=2} \bigcap L_{x+y=-2} \bigcap L_{x-y=2} \bigcap L_{x-y=-2}$, the usual solution for system ①, and should be the $\vee$-solvable space $L_{x=y} \bigcup L_{x+y=2} \bigcup L_{x=1} \bigcup L_{y=1}$, not the $\wedge$-solvable space $L_{x=y} \bigcap L_{x+y=2} \bigcap L_{x=1} \bigcap L_{y=1}$, the usual solution for system ②. *And under what conditions does the $\wedge$-solvable space is the same of the $\vee$-solvable space?* If we understand matters in the universe by systems S, the answer of this question dependents on the unit elements in S, i.e., the $\wedge$-solvable space is the same of the $\vee$-solvable space only if the system of equations is characterizing an unit element of S or view the behavior of S as a particle because the solution of equations is pointing to the state of the element or the particle. Otherwise, the solution of equations (5) would be pointing to unit elements more than 2, maybe contradictory without solutions in the usual meaning.

By Definition 1.4, the $\vee$-solvable space of (5) inherits a topological structure $G^L\left[\widetilde{S}_{\mathscr{F}}\right]$ with vertex set $V\left(G^L\left[\widetilde{S}_{\mathscr{F}}\right]\right)$ and edge set $V\left(G^L\left[\widetilde{S}_{\mathscr{F}}\right]\right)$ respectively defined by

$V\left(G^L\left[\widetilde{S}_{\mathscr{F}}\right]\right) = \{S_{\mathscr{F}_i}; 1 \leq i \leq m\}$;

$E\left(G^L\left[\widetilde{S}_{\mathscr{F}}\right]\right) = \left\{\left(S_{\mathscr{F}_i}, S_{\mathscr{F}_j}\right) \text{ if } S_{\mathscr{F}_i} \bigcap S_{\mathscr{F}_j} \neq \emptyset, 1 \leq i \neq j \leq m\right\}$ and labelling

$$L : S_{\mathscr{F}_i} \to S_{\mathscr{F}_i} \text{ and } L : \left(S_{\mathscr{F}_i}, S_{\mathscr{F}_j}\right) \to S_{\mathscr{F}_i} \bigcap S_{\mathscr{F}_j},$$

where $1 \leq i \neq j \leq m$. For example, the topological structures of the equation systems ① and ② are respectively shown in Figure 5(a) and (b) with points A, B, C, D and P shown in Figure 4(a) and (c).

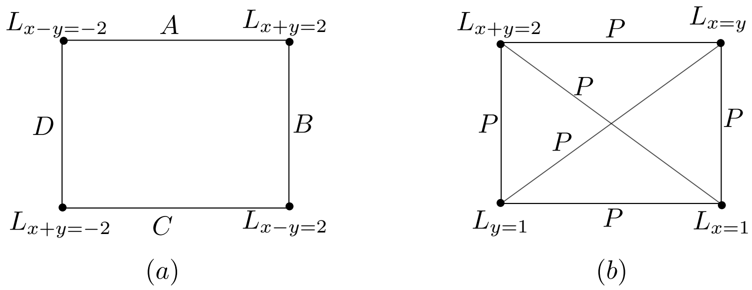

Figure 5. The inherited graphs of the running horses

Thus, the state of a system S characterized by equations (5) can be characterized by its inherits topological structure $G^L\left[\widetilde{S}_{\mathscr{F}}\right]$ on time t, i.e., *G-solution of equations* of (5) and obtain a general conclusion that *all equations characterizing the state of a system S must have a G-solution whether they are solvable or not*, which enables one to get the state $G^L\left[\widetilde{S}_{\mathscr{F}}\right](t)$ of the system S on times, no longer dependent on the solvability of the state equations of system S. For details on G-solutions of non-solvable systems of linear equations, ordinary differential equations and partial differential equations, the reader is referred to the references [19-21] and [24].

4.3. Stability. One advantage of the G-solution on equations of a system S is that we can generally define the stability of S by $G^L\left[\widetilde{S}_{\mathscr{F}}\right](t)$, not dependent on its usual solvability. In classical mathematics, a system of equations is called stable or asymptotically stable if for all solutions $Y(t)$ of the equations with $|Y(t_0) - X(t_0)| < \delta(\varepsilon)$ exists for all $t \geq t_0$, then $|Y(t) - X(t)| < \varepsilon$ for $\forall\varepsilon > 0$ or furthermore, $\lim\limits_{t\to 0} |Y(t) - X(t)| = 0$. However, if the equations are non-solvable, the classical theory of stability is failed to apply. Then *how can we hold on the stability of system characterized by equations* (5)*, maybe non-solvable?* For a system S characterized by (5), we can generalize the stability of the usual to the ω-stable by G-solution of equations (5), not dependent on its usual solvability.

Definition 4.4([20-21]) *Let $G^L_1\left[\widetilde{S}_{\mathscr{F}}\right](t)$ and $G^L_2\left[\widetilde{S}_{\mathscr{F}}\right](t)$ be two G-solutions of equations* (5) *and let $\omega : G^L\left[\widetilde{S}_{\mathscr{F}}\right](t) \to \mathbb{R}$ be an index function. Then, the G-solution of equations* (5) *is said to be ω-stable if there exists a number $\delta(\varepsilon)$ for any number $\varepsilon > 0$ such that*

$$\left|\omega\left(G^L_1\left[\widetilde{S}_{\mathscr{F}}\right](t)\right) - \omega\left(G^L_2\left[\widetilde{S}_{\mathscr{F}}\right](t)\right)\right| < \varepsilon$$

for $t \geq t_0$ or furthermore, asymptotically ω-stable if

$$\lim_{t\to\infty}\left|\omega\left(G^L_1\left[\widetilde{S}_{\mathscr{F}}\right](t)\right) - \omega\left(G^L_2\left[\widetilde{S}_{\mathscr{F}}\right](t)\right)\right| = 0$$

if the initial values hold with

$$\left|\omega\left(G^L_1\left[\widetilde{S}_{\mathscr{F}}\right](t_0)\right) - \omega\left(G^L_2\left[\widetilde{S}_{\mathscr{F}}\right](t_0)\right)\right| < \delta(\varepsilon).$$

If the index function ω is linear, we can further introduce the sum-stability of systems characterized by equations (5).

Definition 4.5([20-22], [24],[26]) *A G-solution is said to be sum-stable or asymptotically sum-stable if all solutions $\mathbf{x}_i(t), 1 \leq i \leq m$ of equations of* (5) *exists for $t \geq t_0$ and*

$$\left\| \sum_{i=1}^{m} \mathbf{x}_i(t) - \sum_{i=1^m} \mathbf{y}_i(t) \right\| < \varepsilon,$$

or furthermore,

$$\lim_{t \to t_0} \left\| \sum_{i=1}^{m} \mathbf{x}_i(t) - \sum_{i=1^m} \mathbf{y}_i(t) \right\| = 0$$

if $\|\mathbf{x}_i(t_0) - \mathbf{y}_i(t_0)\| < \varepsilon$, where, $\varepsilon > 0$ is a real number and $\|\mathbf{X}\|$ denotes the norm of vector $\mathbf{X}$.

The sum-stability of ordinary differential equations and partial differential equations are researched in references [17] and [20]. For example, the following result on the sum-stability of linear ordinary differential equations was obtained.

Theorem 4.5([20-21]) *A G-solution of system* (5) *of linear homogenous differential equations is asymptotically sum-stable if and only if* $\mathrm{Re}\alpha_i < 0$ *for* $\overline{\beta}_i(t)e^{\alpha_i t} \in S_{\mathscr{F}_i}$, $1 \leq i \leq m$ *of linear basis, where $\overline{\beta}_i(t)$ is a polynomial of degree less than $k-1$ on t if α_i is a k-fold root of characteristic equation of the $\mathscr{F}_i(x_1, x_2, \cdots, x_n, y) = 0$.*

§5. Mathematical Combinatorics

The mathematical combinatorics is such a subject that applying the combinatorial notion, i.e. CC Conjecture in [13] to all other mathematics and all other sciences for understanding the reality of things in the universe, happens to share the same view of Smarandache's notion, particularly, the Smarandache multispace by a combinatorial view.

5.1. CC Conjecture. Notice that the Smarandache multispace can be viewed as a combinatorial theory because it discuss the combination of spaces not only one as in classical mathematics. Essentially, the CC conjecture is also a notion for extending classical mathematics presented by imitating the 2-cell partition on surface initially following.

Conjecture 5.1([13],16) *Any mathematical science can be reconstructed from or made by combinationization.*

It should be noted that this conjecture claims that we can select finite combinatorial rulers and axioms to reconstruct or make generalization for mathematical sciences likewise Euclidean geometry, abstract groups, vector space or rings such that the mathematics as a combinatorial generalization of the classical one, and we can make the combinationization of different branches in mathematics and extend them over topological structures because the classical mathematics can be viewed as a mathematics over a topological point in space.

Why is the CC conjecture important for extending classical mathematics? Because of the limitation of humans ourselves in understanding manner, i.e., only partially or locally understanding likewise the philosophical implications in the fable of the blind men and an elephant, we have to combine all the partially or locally understanding on matters in the universe, and because the universe is a combinatorial one in human eyes, we should have such a mathematics going with the understanding manner, i.e., mathematical combinatorics. Then, *do we really need a proof on the CC conjecture?* No, not need! Because we have known all matters in the universe are combined of elementary particles, and all livings are combined of cells and genes. If we approve the scientific understanding on matters in the universe is by mathematics, then we should extend classical mathematics to a combinatorial one catering to the understanding manner, i.e., mathematical combinatorics because we can then understand matters form the partial or local to the global by mathematics. Certainly, there are many achievements for extending classical mathematics by CC conjecture after it was presented in 2006. For example, combinatorial Euclidean space, combinatorial Riemannian submanifolds, combinatorial principal fiber bundles and combinatorial gravitational fields, $\cdots$, etc. are researched. For the contribution of CC conjecture to classical mathematics, the reader is refereed to [18], which was motivated by the combinatorial notion, particularly, the CC conjecture.

What is the relation of the CC conjecture with Smarandachely denied axiom? By Proposition 3.3, the generalized Smarandachely denied axiom is equivalent to the Smarandache systems on mathematical science and the CC conjecture is a notion on mathematical systems over topological structures, they are essentially equal but the direction of CC conjecture is a little clearer on extending classical mathematics because we already have the theory on topological graphs which is essentially the combinationization of 2-dimensional manifolds ([5], [16], [18]) and we can apply all achievements in the classical to mathematical combinatorics on elements, i.e., continuity flows, i.e., extending elements in classical mathematics over topological structures.

5.2. Continuity Flow. For understanding matters in the universe, classical mathematics provides a quantitative analysis on their appearance in front of humans with various hypothesis on interaction of units. However, if one observer could shrinks his body smaller as it needs, he will enters the interior space of the observed matter and observes the matter in a microcosmic level. Then, *what will he see?* He will find the observed matter is nothing else but a piece of net in the space such as the neural net of brain partially shown in Figure 6. This case implies also the necessity of mathematical combinatorics, i.e., catering for science in a microcosmic level.

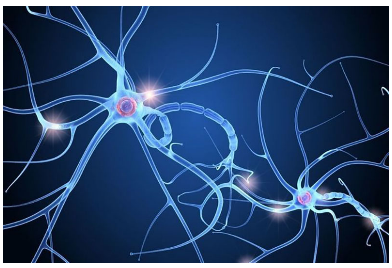

Figure 6. Neural net

Notice that such a topological structure G^L is inherited in a matter or its characterizing by Definition 1.4 on Smarandache multispace, also alluded by the traditional Chinese medicine which applies 12 meridians to heal a patient, characterizes the state of a human, i.e., an inherited topological structure in a living body of human. However, the topological structure G^L defined in Definition 1.4 has no direction on its edge, i.e., the actions of Σ_i on Σ_j and Σ_i on Σ_i both are equal but the flows in the 12 meridians of a human body all have directions and generally, all actions should have directions in the nature. Whence, we should generalized the topological structure in Definition 1.4 from no to with directions on edges, constraint with the conservation laws on all its vertices as flows in nature, which leads to the continuity flows.

Definition 5.2([28]) *A continuity flow* $\left(\overrightarrow{G}; L, A\right)$ *is an oriented embedded graph* $\overrightarrow{G}$ *in a topological space* $\mathscr{S}$ *associated with a mapping* $L : v \to L(v),\ (v,u) \to L(v,u)$, 2 *end-operators* $A^+_{vu} : L(v,u) \to L^{A^+_{vu}}(v,u)$ *and* $A^+_{uv} : L(u,v) \to L^{A^+_{uv}}(u,v)$ *on a Banach space* $\mathscr{B}$ *over field* $\mathscr{F}$

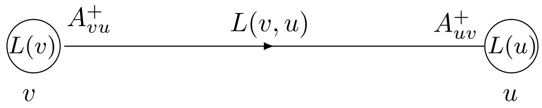

Figure 7

with $L(v,u) = -L(u,v)$ *and* $A^+_{vu}(-L(v,u)) = -L^{A^+_{vu}}(v,u)$ *for* $\forall (v,u) \in E\left(\overrightarrow{G}\right)$ *holding with continuity equation*

$$\sum_{u \in N_G(v)} L^{A^+_{vu}}(v,u) = L(v) \quad for \quad \forall v \in V\left(\overrightarrow{G}\right),$$

where $L(v)$ *is the surplus flow on vertex* v.

Particularly, if $L(v) = \dot{x}_v$, *constants* $\mathbf{v}_v, v \in V\left(\overrightarrow{G}\right)$, *the continuity flow* $\left(\overrightarrow{G}; L, A\right)$ *is respectively said to be a complex flow, an action flow, and* $\overrightarrow{G}$*-flow if* $A = \mathbf{1}_{\mathscr{V}}$, *where,* $\dot{x}_v = dx_v/dt$, x_v *is a variable on vertex* v *and* $\mathbf{v}$ *is a vector in* $\mathscr{V}$ *for* $\forall v \in E\left(\overrightarrow{G}\right)$.

For a given graph family $\mathscr{G} = \{\overrightarrow{G}_1, \overrightarrow{G}_2, \cdots, \overrightarrow{G}_m\}$, a Banach space $\mathscr{B}$ and a field $\mathscr{F}$, we denote by $\mathscr{G}_{\mathscr{B}}$ all continuity flows generated by $\overrightarrow{G} \in \mathscr{G}$ with Banach space $\mathscr{B}$, field $\mathscr{F}$ and abbreviate a continuity flow $\left(\overrightarrow{G}; L, A\right)$ to $\overrightarrow{G}^L$ in the context. Notice that a continuity flow replaced the Banach space $\mathscr{B}$ and field $\mathscr{F}$ by number field $\mathbb{R}$ with end-operators $1_{\mathbb{R}}$ on the ends of edges in $\overrightarrow{G}$, then a continuity flow $\left(\overrightarrow{G}; L, A\right)$ is nothing else but the usual network N discussed in graph theory, and the complex flow is the complex network discussed on complex system in this case. Until today, the $\overrightarrow{G}$-flows and action flows are extensively studied in [22], [26-28] and [30], and for characterizing the livings, the harmonic flow is introduced [24] by replacing vectors $\mathbf{v} \in \mathscr{B}$ on edges in $\overrightarrow{G}$ by complex vectors $\mathbf{v} - i\mathbf{v}$, which is an abstraction and also, a generalization of the 12 meridians in traditional Chinese medicine.

There is a natural question on the continuity flow, i.e., *why does it labels vertices and edges by vectors, not as the usual numbers in continuity flow?* Because the state of units of matters is diversity and multiple directions evolving in the universe, we can characterize its evolution by vectors in multi-dimensions, and *why let vertices constraint with conservation laws?* Because a matter is only a kind form of energy which holds with the conservation law in nature. Whence, the continuity flow $\left(\overrightarrow{G}; L, A\right)$ is essentially a generalization of Smarandache multi-space in the microcosmic level combined with the energy flow's character.

5.3. Mathematical Element. Certainly, a continuity flow $\left(\overrightarrow{G}; L, A\right)$ is a digraph embedded in a topological space $\mathcal{S}$ by the view of combinatorics, i.e., a structure in space such as those shown in Figure 6. We can research it on vertex, edges or a cluster of vertices, hold on its locally characters as the usual in mathematics. However, *is it really a mathematical element itself* *as the usual number, vector, matrix, $\cdots$ with operations such as the addition, multiplication, differential or integral?* The answer is affirmative, i.e., it can be really viewed as a mathematical element.

Clearly, the continuity flow $\left(\overrightarrow{G};L,A\right)$ is a vector if there is only one vertex in $\overrightarrow{G}$, which consists of the elements in linear algebra. *Could we view a continuity flow as a vector and then establish mathematics on continuity flows?* The answer is yes with operations addition $+$ and multiplication $\cdot$ defined following.

$$\overrightarrow{G}^L+\overrightarrow{G'}^{L'} = \left(\overrightarrow{G}\setminus\overrightarrow{G'}\right)^L\bigcup\left(\overrightarrow{G}\bigcap\overrightarrow{G'}\right)^{L+L'}\bigcup\left(\overrightarrow{G'}\setminus\overrightarrow{G}\right)^{L'}, \quad (7)$$

$$\overrightarrow{G}^L\cdot\overrightarrow{G'}^{L'} = \left(\overrightarrow{G}\setminus\overrightarrow{G'}\right)^L\bigcup\left(\overrightarrow{G}\bigcap\overrightarrow{G'}\right)^{L\cdot L'}\bigcup\left(\overrightarrow{G'}\setminus\overrightarrow{G}\right)^{L'}, \quad (8)$$

$$\lambda\cdot\overrightarrow{G}^L = \overrightarrow{G}^{\lambda\cdot L}, \quad (9)$$

where $\lambda\in\mathscr{F}$ and $L:(v,u)\rightarrow L(v,u)\in\mathscr{B}, L':(v,u)\rightarrow L'(v,u)\in\mathscr{B}$ for $\forall(v,u)\in E\left(\overrightarrow{G}\right)$ or $E\left(\overrightarrow{G'}\right)$ such that

$$L+L':(v,u)\rightarrow\left(L(v,u)+L'(v,u)\right),$$
$$L\ \cdot\ L':(v,u)\rightarrow\left(L(v,u)\cdot L'(v,u)\right),$$
$$\lambda\cdot L(v,u)=\lambda\cdot L(v,u)$$

with substituting end-operator $A:(v,u)\rightarrow A^+_{vu}(v,u)+(A')^+_{vu}(v,u)$ or $A:(v,u)\rightarrow A^+_{vu}(v,u)\cdot(A')^+_{vu}(v,u)$ for $(v,u)\in E\left(\overrightarrow{G}\bigcap\overrightarrow{G'}\right)$ in $\overrightarrow{G}^L+\overrightarrow{G'}^{L'}$ or $\overrightarrow{G}^L\cdot\overrightarrow{G'}^{L'}$. Then, we can define the usual element in mathematics. For example, the *sum* and *product*

$$a_1\overrightarrow{G}_1^{L_1}+a_2\overrightarrow{G}_2^{L_2}+\cdots+a_n\overrightarrow{G}_n^{L_n} = \left(\bigcup_{i=1}^n G_i\right)^{a_1L_1+a_2L_2+\cdots+a_nL_n},$$

$$\left(a_1\overrightarrow{G}_1^{L_1}\right)\cdot\left(a_2\overrightarrow{G}_2^{L_2}\right)\cdots\left(a_n\overrightarrow{G}_n^{L_n}\right) = \left(\bigcup_{i=1}^n G_i\right)^{a_1L_1\cdot a_2L_2\cdots a_nL_n}$$

and the *polynomial*

$$a_0+a_1\overrightarrow{G}^L+a_2\overrightarrow{G}^{L^2}+\cdots+a_n\overrightarrow{G}^{L^n}=\overrightarrow{G}^{a_0+a_1L+a_2L^2+\cdots+a_nL^n}$$

with units $\mathbf{O}$ and $\mathbf{I}$ in $(\mathscr{G}_{\mathscr{B}};+)$, respectively and $(\mathscr{G}_{\mathscr{B}};\cdot)$ such that

$$\mathbf{O}+\overrightarrow{G}^L=\overrightarrow{G}^L+\mathbf{O}=\overrightarrow{G}^L,\ \ \mathbf{I}\cdot\overrightarrow{G}^L=\overrightarrow{G}^L\cdot\mathbf{I}=\overrightarrow{G}^L$$

and inverse flows $-\overrightarrow{G}^L$, $\overrightarrow{G}^{L^{-1}}$. We then get

Theorem 5.3([33-34]) *Let $\mathscr{G}$ be a graph family with Banach space $\mathscr{B}$ and field $\mathscr{F}$. Then, $(\mathscr{G}_{\mathscr{B};+,\cdot})$ is a linear space with operations* $(7)-(9)$.

Furthermore, we introduce metric on $\mathscr{G}_{\mathscr{B}}$ if $\mathscr{B}$ is a *normed space* following.

Definition 5.4([26-27]) *Let $(\mathscr{B};+,\cdot)$ be a normed space over field $\mathscr{F}$ with norm $\|\mathbf{v}\|$, $\mathbf{v}\in\mathscr{B}$ and $\overrightarrow{G}^L\in\mathscr{G}_{\mathscr{B}}$. The norm of $\overrightarrow{G}^L$ is defined by*

$$\left\|\overrightarrow{G}^L\right\| = \sum_{(v,u)\in E\left(\overrightarrow{G}\right)} \|L(v,u)\|,$$

i.e., the norm $\|\ \|$ is a mapping with $\|\ \|:\mathscr{G}^t_{\mathscr{B}}\to\mathbb{R}^+$.

Then, we get the conclusion following.

Theorem 5.5([28]) *Let $\mathscr{G}$ be a graph family with Banach space $\mathscr{B}$ and field $\mathscr{F}$. Then, $(\mathscr{G}_{\mathscr{B};+,\cdot})$ is a Banach space with operations $(7)-(9)$.*

An operator $f:\overrightarrow{G}_1^{L_1}\to\overrightarrow{G}_2^{L_2}$ on $\mathscr{G}_{\mathscr{B}}$ is *G-isomorphic* if it holds with conditions: ① there is an isomorphism $\varphi:\overrightarrow{G}_1\to\overrightarrow{G}_2$ of graph and ② $L_2=f\circ\varphi\circ L_1$ for $\forall(v,u)\in E\left(\overrightarrow{G}_1\right)$. Particularly, let $\varphi=\mathrm{id}_{\overrightarrow{G}}$, such an operator is determined by equation $L_2=f\circ L_1$, which enables one to define the function on continuity flows by $f\left(\overrightarrow{G}^{L[t]}\right)=\overrightarrow{G}^{f(L[t])}$, get $\lim\limits_{t\to t_0} f\left(\overrightarrow{G}^{L[t]}\right)=f\left(\overrightarrow{G}_0^{L[t_0]}\right)$ if f respect to L and L respect to t both are continuous and

$$e^{\overrightarrow{G}^{L[t]}}=\mathbf{I}+\frac{\overrightarrow{G}^{L[t]}}{1!}+\frac{\overrightarrow{G}^{2L[t]}}{2!}+\cdots+\frac{\overrightarrow{G}^{nL[t]}}{n!}+\cdots.$$

Furthermore, we generalize the differential and integral in calculus to $\mathscr{G}_{\mathscr{B}}$ by

$$\frac{df}{dt}=\lim_{\Delta t\to 0}\frac{f\left(\overrightarrow{G'}^{L'}[t+\Delta t]\right)-f\left(\overrightarrow{G}^L[t]\right)}{\overrightarrow{G'}^{L'}[t+\Delta t]-\overrightarrow{G}^L[t]}$$

if f is a G-isomorphic operator on $\mathscr{G}_{\mathscr{B}}$ with $f\left(\overrightarrow{G'}^{L'}[t+\Delta t]\right)\to f\left(\overrightarrow{G}^L[t]\right)$ if $\Delta t\to 0$ and

$$\int F\left(\overrightarrow{G}^L[t]\right)dt \ = \ f\left(\overrightarrow{G}^L[t]\right)+C \ \text{ if } \ \frac{df}{dt}\left(\overrightarrow{G}^L[t]\right)=F\left(\overrightarrow{G}^L[t]\right),$$

and then, we know formulae on differential and integral operators, i.e.,

$$\int\left(\frac{df}{dt}\left(\overrightarrow{G}^L[t]\right)\right)dt=f\left(\overrightarrow{G}^L[t]\right)+C,\quad \frac{df}{dt}\left(\int\left(f\left(\overrightarrow{G}^L[t]\right)\right)dt\right)=f\left(\overrightarrow{G}^L[t]\right)$$

and the solution

$$X[t]=e^{\int\overrightarrow{G}^{L_{c1}}dt}\cdot\left(\int\overrightarrow{G}^{L_{c0}}\cdot e^{-\int\overrightarrow{G}^{L_{c1}}dt}dt+C\right)$$

of ordinary differential equation

$$\frac{dX}{dt}=\overrightarrow{G}^{L_{c_1}}[t]\cdot X+\overrightarrow{G}^{L_{c_0}}[t]$$

in $\mathscr{G}_{\mathscr{B}}$ as the usual in calculus. Furthermore, we introduce linear functionals on $\mathscr{G}_{\mathscr{B}}$ and extend the fundamental results in functionals. For example, the Fréchet and Riesz representation theorem on linear continuous functionals following.

Theorem 5.6([22],[26],[30]) *Let* $\mathbf{T}:\overrightarrow{G}^{\mathscr{V}}\rightarrow\mathbb{C}$ *be a linear continuous functional, where* $\mathscr{V}$ *is a Hilbert space. Then there is a unique* $\overrightarrow{G}^{\widehat{L}}\in\overrightarrow{G}^{\mathscr{V}}$ *such that* $\mathbf{T}\left(\overrightarrow{G}^{L}\right)=\left\langle\overrightarrow{G}^{L},\overrightarrow{G}^{\widehat{L}}\right\rangle$ *for* $\forall\overrightarrow{G}^{L}\in\overrightarrow{G}^{\mathscr{V}}$.

And then, *could we establish a dynamics on continuity flows?* This question was asked for establishing the graph dynamics in [13] and the answer is affirmative. For example, we know a result on dynamics of continuity flows following.

Theorem 5.7([30]) *If* $L(t,\mathbf{x}(t),\dot{\mathbf{x}}(t))(v,u)$ *is a Lagrangian on edge* (v,u) *and* $\mathscr{L}\left[L(t,\mathbf{x}(t),\dot{\mathbf{x}}(t))\right]:(v,u)\rightarrow\mathscr{L}\left[L(t,\mathbf{x}(t),\dot{\mathbf{x}}(t))(v,u)\right]$ *is a differentiable functional on a continuity flow* $\overrightarrow{G}^{L}[t]$ *for* $(v,u)\in E\left(\overrightarrow{G}\right)$ *with* $[\mathscr{L},A]=\mathbf{0}$ *for* $A\in\mathscr{A}$, *then*

$$\frac{\partial\overrightarrow{G}^{\mathscr{L}}}{\partial x_i}-\frac{d}{dt}\frac{\partial\overrightarrow{G}^{\mathscr{L}}}{\partial\dot{x}_i}=\mathbf{O},\qquad 1\leq i\leq n.$$

Particularly, if the Lagrangian $\mathscr{L}\left[\overrightarrow{G}^{L}[t]\right]$ of a continuity flow $\overrightarrow{G}^{L}[t]$ is independent on (v,u), we know a conclusion on the Euler-Lagrange equations of continuity flows following.

Corollary 5.8(Euler-Lagrange) *If the Lagrangian* $\mathscr{L}\left[\overrightarrow{G}^{L}[t]\right]$ *of a continuity flow* $\overrightarrow{G}^{L}[t]$ *is independent on* (v,u), *i.e., all Lagrangians* $L(t,\mathbf{x}(t),\dot{\mathbf{x}}(t))(v,u)$, $(v,u)\in E\left(\overrightarrow{G}\right)$ *are synchronized, then the dynamic behavior of* $\overrightarrow{G}^{L}[t]$ *can be characterized by* n *equations*

$$\frac{\partial L}{\partial x_i}-\frac{d}{dt}\frac{\partial L}{\partial\dot{x}_i}=0,\qquad 1\leq i\leq n,$$

which are essentially equivalent to the Euler-Lagrange equations of bouquet $\overrightarrow{B}_1^{L}\in\overrightarrow{B}_{1\mathscr{B}}$, *i.e, dynamic equations on a particle* P.

All of the above-mentioned works shows that a continuity flow is really a mathematical element, likewise elements in classical mathematics, which can be applied to characterize and holds with the reality of matters in natural manner of philosophy. For example, [25], [33] on the structure of elementary particles. For more results on continuity flows with applications to reality of matters in the universe, the reader is refereed to references [22]-[35].

§6. Conclusion

Holding on the reality of matters is an eternal topic of humans, not only beautiful in its mathematical forms but the reality [35], which also provides us an endless resource of thought for scientific research and then, approximating the reality of matters in the universe. Although it is limited of a human life and this work will never go to the end for humans. Consequently, we need to extend our knowing constantly on matters in the universe because this process is step by step, an infinite process by the understanding paradigm (6) of humans. Then, *what we are surveyed in this paper?* We introduce Smarandachely denied axiom on space or systems for the understanding matters because they are not homogenous or equal in the eyes of humans, show its a generalized form equalizing to Smarandache multispace which is an appropriately understanding of matters by philosophy and then, explain how to extend classical mathematics to mathematical combinatorics by CC conjecture. All of the discussions firmly convince one that Smarandachely denied axiom is an important axiom or notion for extending today's science which will further pushes humans to know the truth of matters in the universe, i.e., the reality.

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