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Research Article

Single-Valued Neutro Hyper BCK-Subalgebras

M. Hamidi [1] and F. Smarandache [2]

[1]Department of Mathematics, University of Payame Noor, Tehran, Iran
[2]Mathematics & Science, University of New Mexico, Gallup Campus, Gallup, USA

Correspondence should be addressed to M. Hamidi; m.hamidi@pnu.ac.ir

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The purpose of this paper is to introduce the notation of single-valued neutrosophic hyper BCK-subalgebras and a novel concept of neutro hyper BCK-algebras as a generalization and alternative of hyper BCK-algebras, that have a larger applicable field. In order to realize the article's goals, we construct single-valued neutrosophic hyper BCK-subalgebras and neutro hyper BCK-algebras on a given nonempty set. The result of the research is the generalization of single-valued neutrosophic BCK-subalgebras and neutro BCK-algebras to single-valued neutrosophic hyper BCK-subalgebras and neutro hyper BCK-algebras, respectively. Also, some results are obtained between extended (extendable) single-valued neutrosophic BCK-subalgebras and single-valued neutrosophic hyper BCK-subalgebras via fundamental relation. The paper includes implications for the development of single-valued neutrosophic BCK-subalgebras and neutro BCK-algebras and for modelling the uncertainty problems by single-valued neutrosophic hyper BCK-subalgebras and neutro hyper BCK-algebras. The new conception of single-valued neutrosophic hyper BCK-subalgebras and neutro hyper BCK-algebras was given for the first time in this paper. We find a method that can apply these concepts in some complex networks.

1. Introduction

The theory of logical (hyper) algebra is related to the study of certain propositional calculi and tries to solve logical problems using (hyper) algebraic methods. Jun et al. [1] has introduced a logical (hyper) algebra named hyper BCK-algebras as development of BCK-algebras, which were initiated by Imai and Iseki [2] in 1966 as a generalization of the concept of set-theoretic difference and propositional calculus. The theory of neutrosophic set as an extension of classical set and (intuitionistic) fuzzy set [3], and interval-valued (intuitionistic) fuzzy set, is introduced by Smarandache for the first time in 1998 [4] and mentioned second time in 2005 [5]. This concept handles problems involving imprecise, indeterminacy, and inconsistent data and describes an important role in the modelling of unsure hypernetworks in all sciences. Recently, due to the importance of these subjects, by combining the neutrosophic sets and (hyper) BCK-algebras, some researchers worked in more branches of neutrosophic (hyper) BCK-algebras such as MBJ-neutrosophic hyper BCK-ideals in hyper BCK-algebras, an approach to BMBJ-neutrosophic hyper BCK-ideals of hyper BCK-algebras, structures on doubt neutrosophic ideals of (BCK/BCI)-algebras under (S, T)-norms, BMBJ-neutrosophic subalgebras in (BCI/BCK)-algebras, MBJ-neutrosophic ideals of (BCK/BCI)-algebras, implicative neutrosophic quadruple BCK-algebras and ideals, neutrosophic hyper BCK-ideals, implicative neutrosophic quadruple BCK-algebras and ideals, bipolar-valued fuzzy soft hyper BCK ideals in hyper BCK-algebras, single-valued neutrosophic ideals in Sostak's sense, and multipolar intuitionistic fuzzy hyper BCK-ideals in hyper BCK-algebras [6–16]. Recently, a novel concept of neutrosophy theory titled neutro (hyper) algebra as development of classical (hyper) algebra and partial (hyper) algebra is introduced by Smarandache [17].

A neutro (hyper) algebra is a system that has at least one neutro (hyper) operation or one neutro axiom (axiom that is true for some elements, indeterminate for other elements, and false for the other elements), while a partial (hyper) algebra is a (hyper) algebra that has at least one partial (hyper) operation, and all its axioms are classical (i.e., axioms true for all elements). Smarandache proved that a neutron (hyper) algebra is a generalization of a partial (hyper) algebra and showed that neutro (hyper) algebras are not partial (hyper) algebras, necessarily. Hamidi and Smarandache [18] introduced the concept of neutro BCK-subalgebras as a generalization of BCK-algebras and presented main results in neutro BCK-subalgebras as an extension of BCK-algebras structures and their applications. In addition, the concept of neutro (hyper) algebra is studied in different branches such as neutro algebra structures and neutro (hyper) graph [19, 20].

Regarding these points, one of the aims of this paper is to introduce the concept of single-valued neutrosophic hyper BCK-subalgebras and extendable single-valued neutrosophic BCK-subalgebras and generalize the notion of single-valued neutrosophic hyper BCK-subalgebras by considering the notion of single-valued neutrosophic BCK-subalgebras. Also, we want to establish the relationship between single-valued neutrosophic BCK-algebras and single-valued neutrosophic hyper BCK − algebras. So a strongly regular relation is applied on any hyper BCK-algebras using the concept of single-valued neutrosophic hyper BCK-subalgebras, and a quotient hyper BCK-algebras (BCK − algebras) can be obtained. The main aim of this study is to introduce the notation of neutro hyper BCK-algebras as a generalization of neutro BCK-algebras in regard to single-valued neutrosophic hyper BCK-subalgebras. In the study of neutro hyper BCK-algebra, despite having key mathematical tools, there are some limitations. The union of two neutro hyper BCK-algebra is not necessarily a neutro hyper BCK-algebra so the class of neutro hyper BCK-algebra is not closed under any given algebraic operation. In addition, neutro hyper BCK-algebras are different with (intuitionistic fuzzy) hyper BCK-algebras and single-valued neutrosophic hyper BCK-algebras so could not generalize the capabilities of (intuitionistic fuzzy) single-valued neutrosophic hyper BCK-algebras to neutro hyper BCK-algebras.

2. Preliminaries

Definition 1 (see [2]) Let $X \neq \varnothing$. Then a universal algebra $(X, \vartheta, 0)$ of type $(2, 0)$ is called a BCK-algebra if, for all, $x, y, z \in X$:

(BCI − 1) $((x\varrho y)\varrho(x\varrho z))\varrho(z\varrho y) = 0$,

(BCI − 2) $(x\varrho(x\varrho y))\varrho y = 0$,

(BCI − 3) $x\varrho x = 0$,

(BCI − 4) $x\varrho y = 0$ and $y\varrho x = 0$ imply $x = y$,

(BCK − 5) $0\varrho x = 0$, where $\varrho(x, y)$ is denoted by $x\varrho y$.

Definition 2 (see [1]). Let $X \neq \varnothing$ and $P^*(X) = \{Y \mid \varnothing \neq Y \subseteq X\}$. Then for a map $\vartheta\colon X^2 \longrightarrow P^*(X)$, a hyperalgebraic system $(X, \vartheta, 0)$ is called a hyper BCK-algebra if, for all, $x, y, z \in X$:

$(H1)\,(x\vartheta z)\vartheta(y\vartheta z) \ll x\vartheta y$,

$(H2)\,(x\vartheta y)\vartheta z = (x\vartheta z)\vartheta y$,

$(H3)\,x\vartheta X \ll x$,

$(H4)\,x \ll y$ and $y \ll x$ imply $x = y$,

where $x \ll y$ is defined by $0 \in x\vartheta y$, $\forall A, B \subseteq H$, $A \ll B \Longleftrightarrow \forall a \in A\, \exists b \in B$ s.t $a \ll b$,

$(A\vartheta B) = \cup_{a\in A, b\in B}(a\vartheta b)$, and $\vartheta(x, y)$ is denoted by $x\vartheta y$.

We will call X is a weak commutative hyper BCK-algebra if $\forall x, y \in X$, $(x\vartheta(x\vartheta y)) \cap (y\vartheta(y\vartheta x)) \neq \varnothing$ [21].

Theorem 1 (see [1]). *Let $(X, \vartheta, 0)$ be a hyper BCK-algebra. Then $\forall x, y, z \in X$ and $A, B \subseteq X$:*

(i) $(0\vartheta 0) = 0$, $0 \ll x$, $(0\vartheta x) = 0$, $x \in (x\vartheta 0)$ *and* $A \ll 0 \Rightarrow A = 0$

(ii) $x \ll x$, $x\vartheta y \ll x$ *and* $y \ll z$ *implies that* $x\vartheta z \ll x\vartheta y$

(iii) $A\vartheta B \ll A$, $A \ll A$ *and* $A \subseteq B$ *implies* $A \ll B$

Definition 3 (see [22]). Let $(X, \vartheta, 0)$ be a hyper BCK-algebra. A fuzzy set $\mu\colon X \longrightarrow [0, 1]$ is called a fuzzy hyper BCK-subalgebra if $\forall x, y \in X$, $\wedge(\mu(x\vartheta y)) \geq T_{\min}(\mu(x), \mu(y))$.

Definition 4 (see [5]). Let V be a universal set. A neutrosophic subset (NS) X in V is an object having the following form: $X = \{(x, T_X(x), I_X(x), F_X(x)) \mid x \in V\}$, or $X\colon V \longrightarrow [0, 1] \times [0, 1] \times [0, 1]$, which is characterized by a truth-membership function T_X, an indeterminacy-membership function I_X, and a falsity-membership function F_X. There is no restriction on the sum of $T_X(x), I_X(x)$, and $F_X(x)$.

3. Single-Valued Neutrosophic Hyper BCK-Subalgebras

In this section, the concept of single-valued neutrosophic hyper BCK-subalgebras will be considered as a generalization of single-valued neutrosophic BCK-subalgebras, and some of its properties will be investigated. We will also prove that single-valued neutrosophic hyper BCK-subalgebras and single-valued neutrosophic BCK-subalgebras are related, and single-valued neutrosophic hyper BCK-subalgebras and single-valued neutrosophic BCK-subalgebras can be constructed from single-valued neutrosophic hyper BCK-subalgebras via a fundamental relation. We will define the concept of extendable single-valued neutrosophic BCK-subalgebras and will show that any infinite set is an extended single-valued neutrosophic BCK-subalgebra.

Throughout this section, we denote hyper BCK-algebra $(X, \vartheta, 0)$ by X. From now on, for all, $x, y \in [0, 1]$, $T_{\min}(x, y) = \min\{x, y\}$ and $S_{\max}(x, y) = \max\{x, y\}$ are considered as triangular norm and triangular conorm, respectively. In the following definition, the notation of single-valued neutrosophic hyper BCK-subalgebra of any given nonempty is defined.

Definition 5. A single-valued neutrosophic set $A = (T_A, I_A, F_A)$ in an X is called a single-valued neutrosophic hyper BCK-subalgebra of X, if

(i) $\wedge (T_A(x \vartheta y)) \geq T_{\min}(T_A(x), T_A(y))$

(ii) $\vee (I_A(x \vartheta y)) \leq S_{\max}(I_A(x), I_A(y))$

(iii) $\vee (F_A(x \vartheta y)) \leq S_{\max}(F_A(x), F_A(y))$

The importance of the following theorems is to determine the role and the effect of truth-membership function T_A, indeterminacy-membership function I_A, and falsity-membership function F_A on the element $0 \in A$.

Theorem 2. *Let $A = (T_A, I_A, F_A)$ be a single-valued neutrosophic hyper BCK-subalgebra of X. Then*

(i) $T_A(0) \geq T_A(x)$

(ii) $\wedge (T_A(x \vartheta 0)) = T_A(x)$

(iii) $\wedge (T_A(0 \vartheta x)) = T_A(0)$

Proof

(i) Let $x \in X$. Since $0 \in x \vartheta x$, we get that $T_A(0) \geq \wedge (T_A(x \vartheta x)) \geq T_{\min}(T_A(x), T_A(x)) = T_A(x)$.

(ii) Let $x \in X$. Since $x \in x \vartheta 0$, we get that $T_A(x) \geq \wedge (T_A(x \vartheta 0)) \geq T_{\min}(T_A(x), T_A(0)) = T_A(x)$. So $\wedge (T_A(x \vartheta 0)) = T_A(x)$.

(iii) Immediate by Theorem 1. □

Theorem 3. *Let $A = (T_A, I_A, F_A)$ be a single-valued neutrosophic hyper BCK-subalgebra of X. Then*

(i) $I_A(0) \leq I_A(x)$

(ii) $\vee (I_A(x \vartheta 0)) = I_A(x)$

(iii) $\vee (I_A(0 \vartheta x)) = I_A(0)$

Proof

(i) Let $x \in X$. Since $0 \in x \vartheta x$, we get that $I_A(0) \leq \vee (I_A(x \vartheta x)) \leq S_{\max}(I_A(x), I_A(x)) = I_A(x)$.

(ii) Let $x \in X$. Since $x \in x \vartheta 0$, we get that $I_A(x) \leq \vee (I_A(x \vartheta 0)) \leq S_{\max}(I_A(x), I_A(0)) = I_A(x)$. So $\vee (I_A(x \vartheta 0)) = I_A(x)$.

(iii) Immediate by Theorem 1. □

Corollary 1. *Let $A = (T_A, I_A, F_A)$ be a single-valued neutrosophic hyper BCK-subalgebra of X. Then*

(i) $F_A(0) \leq F_A(x)$

(ii) $\vee (F_A(x \vartheta 0)) = F_A(x)$

(iii) $\vee (F_A(0 \vartheta x)) = F_A(0)$

(iv) $T_{\min}(T_A(x), I_A(0), F_A(0)) \leq T_{\min}(T_A(0), I_A(x), F_A(x))$

In the following theorem, we construct single-valued neutrosophic subset on any nonempty set.

Theorem 4. *Let $0 \notin X \neq \emptyset$. Then there exist a hyperoperation "ϑ," a single-valued neutrosophic subset $A = (T_A, I_A, F_A)$ of $X' = X \cup \{0\}$ such that $(X', \vartheta, 0)$ is a hyper BCK-algebra and A is a single-valued neutrosophic hyper BCK-subalgebra of X'.*

Proof. Let $x, y \in X'$. Define "ϑ" on X' by $x \vartheta y = \begin{cases} 0, & \text{if } x = 0, \\ \{0, x\}, & \text{if } x = y, x \neq 0, \\ x, & \text{otherwise} \end{cases}$. Clearly, $(X', \vartheta, 0)$ is a hyper BCK-algebra. Now, it is easy to see that every single-valued neutrosophic set $A = (T_A, I_A, F_A)$ that $T_A(0) = 1$, $I_A(0) = F_A(0) = 0$ is a single-valued neutrosophic hyper BCK-subalgebra of X'.

Let $\mathrm{SVN}h = \{A = (T_A, I_A, F_A) \mid A \text{ is a single–valued neutrosophic hyper BCK – subalgebra of } X\}$, whence X is a hyper BCK-algebra and $|X| \geq 1$. □

Corollary 2. *Let $X \neq \emptyset$. Then X can be extended to a hyper BCK-algebra that $|\mathrm{SVN}h| = |\mathbb{R}|$.*

Proof. Let $X = \{x\}$. Then (X, ϑ, x) is a hyper BCK-algebra such that $x \vartheta x = \{x\}$. Then for a single-valued neutrosophic set, $A = (T_A, I_A, F_A)$ by $T_A(x) = I_A(x) = F_A(x) = \alpha$ is a single-valued neutrosophic hyper BCK-subalgebra of X, where $\alpha \in [0, 1]$. If $|X| \geq 2$; then by Theorem 4, we can construct at least a hyper BCK-subalgebra on X. Now, $\forall \alpha \in [0, 1]$ define $A = (T_{A_\alpha}, I_{A_\alpha}, F_{A_\alpha})$ by

$$T_{A_\alpha}(x) = \begin{cases} 1, & \text{if } x = 0, \\ \alpha, & \text{if } x \neq 0, \end{cases}$$
$$I_{A_\alpha}(x) = \begin{cases} 0, & \text{if } x = 0, \\ \alpha, & \text{if } x \neq 0, \end{cases} \quad (1)$$
$$F_{A_\alpha}(x) = \begin{cases} 0, & \text{if } x = 0, \\ \alpha, & \text{if } x \neq 0. \end{cases}$$

Obviously, $A = (T_{A_\alpha}, I_{A_\alpha}, F_{A_\alpha})$ a single-valued neutrosophic hyper BCK-subalgebra of X and so $|\mathrm{SVN}h| = |[0, 1]|$.

Let X be a hyper BCK-algebra, $A = (T_A, I_A, F_A)$ a single-valued neutrosophic hyper BCK-subalgebra of X and $\alpha, \beta, \gamma \in [0, 1]$. Define $T_A^\alpha = \{x \in X \mid T_A(x) \geq \alpha\}$, $I_A^\beta = \{x \in X \mid I_A(x) \leq \beta\}$, $F_A^\gamma = \{x \in X \mid F_A(x) \leq \gamma\}$, and $A^{(\alpha,\beta,\gamma)} = \{x \in X \mid T_A(x) \geq \alpha, I_A(x) \leq \beta, F_A(x) \leq \gamma\}$.

Considering the relation between single-valued neutrosophic hyper BCK-subalgebras and (fuzzy) hyper BCK-subalgebra is the main aim of the following results via the level subsets. □

Theorem 5. *Let $A = (T_A, I_A, F_A)$ be a single-valued neutrosophic hyper BCK-subalgebra of X. Then*

(i) $0 \in A^{(\alpha,\beta,\gamma)} = T_A^\alpha \cap I_A^\beta \cap F_A^\gamma$

(ii) $A^{(\alpha,\beta,\gamma)}$ *is a hyper BCK-subalgebra of X*

(iii) If $0 \leq \alpha \leq \alpha' \leq 1$, then $T_A^{\alpha'} \subseteq T_A^\alpha$, $I_A^{\alpha'} \supseteq I_A^\alpha$ and $F_A^{\alpha'} \supseteq F_A^\alpha$

Proof

(i) Clearly, $A^{(\alpha,\beta,\gamma)} = A^\alpha \cap A^\beta \cap A^\gamma$ and by Theorems 2 and 3, and Corollary 1, we get that $0 \in A^{(\alpha,\beta,\gamma)}$.

(ii) Let $x, y \in T_A^{\alpha}$. Then $T_{\min}(T_A(x), T_A(y)) \geq \alpha$. Now, for any, $z \in x \vartheta y, T_A(z) \geq \inf(T_A(x \vartheta y)) \geq T_{\min}(T_A(x), T_A(y)) \geq \alpha$. Hence, $z \in T_A^{\alpha}$, and so $x \vartheta y \subseteq T_A^{\alpha}$. In similar a way, $x, y \in I_A^{\beta} \cap F_A^{\gamma}$ implies that $x \vartheta y \subseteq (I_A^{\beta} \cap F_A^{\gamma})$. Then $A^{(\alpha,\beta,\gamma)}$ is a hyper BCK-subalgebra of X.

(iii) Immediate. □

Corollary 3. *Let $A = (T_A, I_A, F_A)$ be a single-valued neutrosophic hyper BCK-subalgebra of X. If $0 \leq \alpha \leq \alpha' \leq 1$, then $A^{(\alpha',\alpha,\alpha)}$ is a hyper BCK-subalgebra of $A^{(\alpha,\alpha',\alpha')}$.*

Let X be a hyper BCK-algebra, S be a hyper BCK-subalgebra of X and $\alpha, \alpha', \beta, \beta', \gamma, \gamma' \in [0,1]$. Define

$$T_A^{[\alpha,\alpha']}(x) = \begin{cases} \alpha', & \text{if } x \in S, \\ \alpha, & \text{if } x \notin S, \end{cases}$$
$$I_A^{[\beta,\beta']}(x) = \begin{cases} \beta', & \text{if } x \in S, \\ \beta, & \text{if } x \notin S, \end{cases} \quad (2)$$
$$F_A^{[\gamma,\gamma']}(x) = \begin{cases} \gamma', & \text{if } x \in S, \\ \gamma, & \text{if } x \notin S. \end{cases}$$

Thus, we have the following theorem.

Theorem 6. *Let X be a hyper BCK-algebra and S be a hyper BCK-subalgebra of X. Then*

(i) $T_A^{[\alpha,\alpha']}$ is a fuzzy hyper BCK-subalgebra of X
(ii) $I_A^{[\beta,\beta']}$ is a fuzzy hyper BCK-subalgebra of X
(iii) $F_A^{[\gamma,\gamma']}$ is a fuzzy hyper BCK-subalgebra of X
(iv) $A = (T_A^{[\alpha,\alpha']}, I_A^{[\beta,\beta']}, F_A^{[\gamma,\gamma']})$ is a single-valued neutrosophic hyper BCK-subalgebra of X

Proof

(i) Let $x, y \in X$. If $x, y \in S$, since S is a hyper subalgebra of X, we get that $x \vartheta y \subseteq S$ and so

$$\wedge T_A^{[\alpha,\alpha']}(x \vartheta y) \geq \wedge T_A^{[\alpha,\alpha']}(S) = \alpha' \geq T_{\min}\left(T_A^{[\alpha,\alpha']}(x), T_A^{[\alpha,\alpha']}(y)\right). \quad (3)$$

If ($x \in S$ and $y \notin S$) or ($x \notin S$ and $y \in S$) or ($x \notin S$ and $y \notin S$), then $\wedge T_A^{[\alpha,\alpha']}(x \vartheta y) \in \{\alpha, \alpha'\}$. Thus, $\wedge T_A^{[\alpha,\alpha']}(x \vartheta y) \geq T_{\min}(T_A^{[\alpha,\alpha']}(x), T_A^{[\alpha,\alpha']}(y))$, and so $T_A^{[\alpha,\alpha']}$ is a fuzzy hyper BCK-subalgebra of X.

(ii) and (iii) They are similar to (i).

(iv) Let $x, y \in X$. If $x, y \in S$, since S is a hyper BCK-subalgebra of X, we get that $x \vartheta y \subseteq S$, and so $\vee I_A^{[\beta,\beta']}(x \vartheta y) \leq \vee I_A^{[\beta,\beta']}(S) = \alpha' \leq S_{\max}(I_A^{[\beta,\beta']}(x), I_A^{[\beta,\beta']}(y))$. If ($x \in S$ and $y \notin S$) or ($x \notin S$ and $y \in S$) or ($x \notin S$ and $y \notin S$), then $\vee I_A^{[\beta,\beta']}(x \vartheta y) \in \{\beta, \beta'\}$. Thus, $\vee T_A^{[\beta,\beta']}(x \vartheta y) \leq S_{\max}(I_A^{[\beta,\beta']}(x), I_A^{[\beta,\beta']}(y))$. In a similar way, we can see that $\vee F_A^{[\gamma,\gamma']}(x \vartheta y) \leq S_{\max}(F_A^{[\gamma,\gamma']}(x), F_A^{[\gamma,\gamma']}(y))$ an by item (i), $A = (T_A^{[\alpha,\alpha']}, I_A^{[\beta,\beta']}, F_A\ [\gamma,\gamma'])$ is a single-valued neutrosophic hyper BCK-subalgebra of X.

Let X be a hyper BCK-algebra and $x, y \in X$. Then $x\beta y \Longleftrightarrow \exists n \in \mathbb{N}$, $(a_1, \ldots, a_n) \in X^n$ and $\exists u \in \vartheta(a_1, \ldots, a_n)$such that$\{x, y\} \subseteq u$. The relation β is a reflexive and symmetric relation but not transitive relation. Let $C(\beta)$ be the transitive closure of β (the smallest transitive relation such that contains β). Borzooei et al. in [21], proved that for any given weak commutative hyper BCK-algebra X, $C(\beta)$ is a strongly regular relation on X, and $((X/C(\beta)), \varrho, \overline{0})$ is a BCK-algebra, where $C(\beta)(x)\varrho C(\beta)(y) = C(\beta)(x \vartheta y)$ and $\overline{0} = C(\beta)(0)$.

Considering the relation between single-valued neutrosophic hyper BCK-subalgebras and single-valued neutrosophic BCK-subalgebras has very important, especially in extension of single-valued neutrosophic BCK-subalgebras. So we prove the following theorems and corollaries. □

Theorem 7. *Let X be a weak commutative hyper BCK-subalgebra and $A = (T_A, I_A, F_A)$ be a single-valued neutrosophic hyper BCK-subalgebra of X. Then there exists a single-valued neutrosophic set $\overline{A} = (\overline{T_A}, \overline{I_A}, \overline{F_A})$ of BCK-algebra $((X/C(\beta)), \vartheta, \overline{0})$ that $\forall x, y \in X$,*

(i) $\overline{T_A}(C(\beta)(0)) \geq \overline{T_A}(C(\beta)(x))$
(ii) if $yC(\beta)x$, then $\overline{T_A}(C(\beta)(x)) = \overline{T_A}(C(\beta)(y))$
(iii) $\overline{I_A}(C(\beta)(0)) \leq \overline{I_A}(C(\beta)(x))$
(iv) if $yC(\beta)x$, then $\overline{I_A}(C(\beta)(x)) = \overline{I_A}(C(\beta)(y))$
(v) $\overline{F_A}(C(\beta)(0)) \leq \overline{F_A}(C(\beta)(x))$
(vi) if $yC(\beta)x$, then $\overline{F_A}(C(\beta)(x)) = \overline{F_A}(C(\beta)(y))$

Proof. Let $x, y, t \in X$. Then on $(X/C(\beta))$, define $\overline{T_A}(C(\beta)(t)) = \begin{cases} T_A(0), & \text{if } 0 \in C(\beta)(x), \\ \wedge_{tC(\beta)x} T_A(x), & \text{otherwise}, \end{cases}$, $\overline{I_A}(C(\beta)(t)) = \begin{cases} I_A(0), & \text{if } 0 \in C(\beta)(x), \\ \vee_{tC(\beta)x} I_A(x), & \text{otherwise}, \end{cases}$, and $\overline{F_A}(C(\beta)(t)) = \begin{cases} F_A(0), & \text{if } 0 \in C(\beta)(x), \\ \vee_{tC(\beta)x} F_A(x), & \text{otherwise}, \end{cases}$. Using Theorems 2 and 3, we get that:

(i) $\overline{T_A}(C(\beta)(0)) = T_A(0) \geq \wedge_{t'C(\beta)x} T_A(t') = \overline{T_A}(C(\beta)(x))$

(ii) Since $xC(\beta)y$ and $C(\beta)$ is transitive, we get that $\overline{T_A}(C(\beta)(x)) = \wedge_{tC(\beta)x} T_A(t) \geq \wedge_{tC(\beta)y} T_A(t) = \overline{T_A}(C(\beta)(y))$

(iii) $\overline{I_A}(C(\beta)(0)) = I_A(0) \leq \vee_{t'C(\beta)x} I_A(t') = \overline{I_A}(C(\beta)(x))$

(iv) Since $xC(\beta)y$ and $C(\beta)$ is transitive, we get that $\overline{I_A}(C(\beta)(x)) = \vee_{tC(\beta)x} I_A(t) = \vee_{tC(\beta)y} I_A(t) = \overline{I_A}(C(\beta)(y))$

(v) and (vi) They are similar to (iii) and (iv), respectively. □

Theorem 8. *Let X be a weak commutative hyper BCK-subalgebra and $A = (T_A, I_A, F_A)$ be a single-valued neutrosophic hyper BCK-subalgebra of X. Then there exists a single-valued neutrosophic subset $\overline{A} = (\overline{T_A}, \overline{I_A}, \overline{F_A})$ of BCK-algebra $((X/C(\beta)), \vartheta, \overline{0})$ that $\forall x, y \in X$:*

(i) There exists $t \in x \vartheta y$ such that $\overline{T_A}(C(\beta)\ (x \vartheta y)) = T_A(t)$

(ii) *There exists* $t\prime \in x\vartheta y$ *such that* $\overline{I_A}(C(\beta)(x\vartheta y)) = I_A(t)$

(iii) *There exists* $t\prime\prime \in x\vartheta y$ *such that* $\overline{F_A}(C(\beta)(x\vartheta y)) = F_A(t)$

Proof

(i) Let $x, y \in X$. Applying Theorem 7,

$$\begin{aligned}&\overline{T_A}(C(\beta)(x)\varrho C(\beta)(y)) = \overline{T_A}(C(\beta)(x\vartheta y))\\&= \overline{T_A}\{C(\beta)(m) \mid m \in x\vartheta y\} = \bigwedge_{\substack{sC(\beta)m\\ m\in x\vartheta y}} T_A(s).\end{aligned} \tag{4}$$

Now, since $sC(\beta)m$ and $m \in x\vartheta y$, then $s \in x\vartheta y$, and so there exists $t \in x\vartheta y$ such that $T_A(t) = \wedge_{\substack{sC(\beta)m\\ m\in x\vartheta y}} T_A(s)$.

(ii) Let $x, y \in X$. Then

$$\begin{aligned}&\overline{I_A}(C(\beta)(x)\varrho C(\beta)(y)) = \overline{I_A}(C(\beta)(x\vartheta y))\\&= \overline{I_A}\{C(\beta)(n) \mid n \in x\vartheta y\} = \bigvee_{\substack{tC(\beta)n\\ n\in x\vartheta y}} I_A(t).\end{aligned} \tag{5}$$

Now, since $tC(\beta)n$ and $n \in x\vartheta y$, then $t \in x\vartheta y$, and so there exists $t' \in x\vartheta y$ such that $I_A(t') = \wedge_{\substack{tC(\beta)n\\ n\in x\vartheta y}} I_A(t)$.

(iii) It is similar to item (ii).

Some categorical properties of single-valued neutrosophic BCK-subalgebras is investigated in the following theorem based on the categorical properties of single-valued neutrosophic hyper BCK-subalgebras. □

Theorem 9. *Let X be a weak commutative hyper BCK-algebra and $A = (T_A, I_A, F_A)$ be a single-valued neutrosophic hyper BCK-subalgebra of X. Then there exists a single-valued neutrosophic BCK-subalgebra $B = (T_B, I_B, F_B)$ of $((X/C(\beta)), F_B, C(\beta)(0))$ that $((T_B\vartheta\pi) \le T_A$, $(I_B\vartheta\pi) \ge I_A$ and $(I_B\vartheta F_B) \ge F_A)$ or the following diagrams are quasi commutative:*

$$X \longrightarrow^{T_A} [0\ \ 1]_\pi \downarrow \nearrow_{T_B} \frac{X}{C(\beta)},\ X \longrightarrow^{I_A} [0\ \ 1]_\pi \downarrow \nearrow_{I_B} \frac{X}{C(\beta)},\ X \longrightarrow^{F_A} [0\ \ 1]_\pi \downarrow \nearrow_{F_B} \frac{X}{C(\beta)}. \tag{6}$$

Proof. Choice $T_B = \overline{T_A}$, $I_B = \overline{I_A}$ and $F_B = \overline{F_A}$. Then by Theorem 7, (i) $\forall x \in X$,

$$\begin{aligned}&T_B(C(\beta)(0)) \ge T_B(C(\beta)(x)),\\&I_B(C(\beta)(0)) \le I_B(C(\beta)(x)),\\&F_B(C(\beta)(0)) \le F_B(C(\beta)(x)).\end{aligned} \tag{7}$$

(ii) By Theorem 8, $\forall x, y \in X$; there exists $\{t, t', t''\} \subseteq x\vartheta y$ that

$$\begin{aligned}&T_B(C(\beta)(x\vartheta y)) = T_A(t),\\&I_B(C(\beta)(x\vartheta y)) = I_A(t\prime),\\&F_B(C(\beta)(x\vartheta y)) = F_A(t\prime\prime).\end{aligned} \tag{8}$$

So

$$\begin{aligned}T_B(C(\beta)(x)\varrho C(\beta)(y)) &= T_B(C(\beta)(x\vartheta y)) = T_A(t) \ge \wedge\left(T_A(x\vartheta y)\right)\\&\ge T_{\min}(T_A(x), T_A(y)) \ge T_{\min}(T_B(C(\beta)(x)), T_B(C(\beta)(y))),\\I_B(C(\beta)(x)\varrho C(\beta)(y)) &= I_B(C(\beta)(x\vartheta y)) = I_A(t\prime) \le \vee\left(I_A(x\vartheta y)\right)\\&\le S_{\max}(I_A(x), I_A(y)) \le S_{\max}(I_B(C(\beta)(x)), I_B(C(\beta)(y))),\\F_B(C(\beta)(x)\varrho C(\beta)(y)) &= F_B(C(\beta)(x\vartheta y)) = F_A(t\prime\prime) \le \vee\left(F_A(x\vartheta y)\right)\\&\le S_{\max}(F_A(x), F_A(y)) \le S_{\max}(F_B(C(\beta)(x)), F_B(C(\beta)(y))).\end{aligned} \tag{9}$$

Therefore, $B = (T_B, I_B, F_B)$ is a single-valued neutrosophic BCK-subalgebra of $(X/C(\beta))$, $(T_B\vartheta\pi) \le T_A$, $(I_B\vartheta\pi) \ge I_A$, and $(I_B\vartheta\pi) \ge F_A$.

Based on the fundamental relation, we can obtain the single-valued neutrosophic BCK-subalgebras, and single-valued neutrosophic BCK-subalgebras are derived from some single-valued neutrosophic hyper BCK-subalgebras. In this regard, it is important that single-valued neutrosophic BCK-subalgebras are derived from single-valued neutrosophic hyper BCK-subalgebra with minimal order. So the concepts of (extended) extendable single-valued neutrosophic BCK-subalgebra are introduced as follows. □

Definition 6

(i) Let $(X, \varrho, 0)$ be a BCK-algebra and $(Y, \vartheta, 0)$ be a hyper BCK-algebra. We say that the BCK-algebra X is derived from the hyper BCK-algebra Y if X is isomorphic to a nontrivial quotient of Y $(X \cong (Y/C(\beta)))$.

(ii) A single-valued neutrosophic BCK-subalgebra $A = (T_A, I_A, F_A)$ of X is called an extendable single-valued neutrosophic BCK-subalgebra, if there exist a hyper BCK-algebra $(Y, \vartheta, 0)$, a single-valued neutrosophic hyper BCK-subalgebra $B = (T_B, I_B, F_B)$ of Y, and $n \in \mathbb{N}$ such that $|(X, \vartheta, A)| = |(Y, \vartheta, B)| - n$, and BCK-algebra X is derived of hyper BCK-algebra Y. If $X = Y$ and almost everywhere $(T_A, I_A, F_A) = (T_B, I_B, F_B)$ $((T_A, I_A, F_A) = (T_B, I_B, F_B)$a.e that means $|\{x; T_A(x) \neq T_B(x), I_A(x) \neq I_B(x), F_A(x) \neq F_B(x)\}| = 1)$, we will say that it is an extended single-valued neutrosophic BCK-subalgebra.

The following example introduces an extendable single-valued neutrosophic BCK-subalgebra.

Example 1. Let $X = \{-1, -2, -3, -4\}$. Then $A = (T_A, I_A, F_A)$ is a single-valued neutrosophic BCK-subalgebra of BCK-algebra $(X, \vartheta, -1)$ (see Table 1).

Now, set $Y = \{0, -1, -2, -3, -4\} = X \cup \{0\}$. Then $B = (T_B, I_B, F_B)$ is a single-valued neutrosophic hyper BCK-subalgebra of $(Y, \vartheta, 0)$ (see Table 2).

Clearly, $(Y/C(\beta)) \cong X$, $|Y| = |X| + 1$, and so $A = (T_A, I_A, F_A)$ is an extendable single-valued neutrosophic BCK-subalgebra of $(X, \vartheta, -1)$.

In the following theorem, we try to generate BCK-algebras based on single-valued neutrosophic hyper BCK-subalgebras.

Theorem 10. *Let $(X, \vartheta, 0)$ be a hyper BCK-algebra, $A = (T_A, I_A, F_A)$ be a single-valued neutrosophic hyper BCK-subalgebra of X, and $\overline{X} = \{(T_A(x), I_A(x), F_A(x)) \mid x \in X\}$. If A is one to one map, then:*

(i) There exists a hyperoperation "ϑ'" on $\overline{X}$ such that $(\overline{X}, \vartheta', (T_A(0), I_A(0), F_A(0)))$ is a hyper BCK-algebra

(ii) There exists a single-valued neutrosophic hyper BCK-subalgebra $\overline{A} = (\overline{T_A}, \overline{I_A}, \overline{F_A})$ of $\overline{X}$ related to $A = (T_A, I_A, F_A)$

(iii) There exists an operation "ϱ" (related to ϑ) on $\overline{X}$ that $(\overline{X}, \varrho, (T_A(0), I_A(0), F_A(0)))$ is a BCK-algebra

Proof

(i) Let $x, y \in X$. Define a hyperoperation ϑ' on $\overline{X}$, by

$$(T_A(x), I_A(x), F_A(x))\, \vartheta'\, (T_A(y), I_A(y), F_A(y)) = (T_A(x \vartheta y), I_A(x \vartheta y), F_A(x \vartheta y)). \tag{10}$$

It can be easily seen that $(T_A(x), I_A(x), F_A(x)) \ll' (T_A(y), I_A(y), F_A(y)) \Longleftrightarrow x \ll y$. It is easy to see that $(\overline{X}, \vartheta', (T_A(0), I_A(0), F_A(0)))$ is a hyper BCK-algebra.

(ii) Let $x \in X$. Define $\overline{A}(A(x)) = A(x)$. Clearly, $\overline{A} = (\overline{T_A}, \overline{I_A}, \overline{F_A})$ is a single-valued neutrosophic hyper BCK-subalgebra of $(\overline{X}, \vartheta')$.

(iii) Assume $x, y \in X$. Define an operation ϱ on $\overline{X}$ by

$$(T_A(x), I_A(x), F_A(x))\varrho(T_A(y), I_A(y), F_A(y)) = \begin{cases} (T_A(x), I_A(x), F_A(x)), & \text{if } y = 0, \\ (\vee T_A(x \vartheta y), \wedge I_A(x \vartheta y), \wedge F_A(x \vartheta y)) & \text{otherwise.} \end{cases} \tag{11}$$

We just prove BCI-4. Let $x, y \in X$ and

$$\begin{aligned} &(T_A(x), I_A(x), F_A(x))\varrho(T_A(y), I_A(y), F_A(y)) \\ &\quad = (T_A(x), I_A(x), F_A(x))\varrho(T_A(y), I_A(y), F_A(y)) \\ &\quad = (T_A(0), I_A(0), F_A(0)). \end{aligned} \tag{12}$$

Since A is a one to one map, $0 \in x \vartheta y$ and $0 \in y \vartheta x$. It follows that $(T_A(x), I_A(x), F_A(x)) = (T_A(y), I_A(y), F_A(y))$. It is easy to see that BCI-1, BCI-2, BCI-3, and BCK-5 are valid, and so $(\overline{X}, \varrho, (T_A(0), I_A(0), F_A(0)))$ is a BCK-algebra. □

Corollary 4. *Let $(\overline{X}, \vartheta, (T_A(0), I_A(0), F_A(0)))$ be a hyper BCK-algebra and $A = (T_A, I_A, F_A)$ be a single-valued neutrosophic hyper BCK-subalgebra of $\overline{X}$. Then there exists a binary operation "ϱ" on $\overline{X}$, such that $(\overline{X}, \varrho, (T_A(0), I_A(0), F_A(0)))$ is a BCK-algebra.*

In the following theorem, we try to generate hyper BCK-algebras based on single-valued neutrosophic hyper BCK-subalgebras.

Theorem 11. *Let X be a nonempty set, $0 \notin X$ and $X' = X \cup \{0\}$. Then there exist a hyperoperation "ϑ" on X', a hyperoperation "ϑ'" on X', a binary operation "ϱ" on X', a single-valued neutrosophic subset $A = (T_A, I_A, F_A)$ of X', and a single-valued neutrosophic subset $B = (T_B, I_B, F_B)$ of X' that:*

(i) $(X', \vartheta, 0)$ is a hyper BCK-algebra, and $A = (T_A, I_A, F_A)$ is a single-valued neutrosophic hyper BCK-subalgebra of X'

Table 1

ϱ	-1	-2	-3	-4
-1	-1	-1	-1	-1
-2	-2	-1	-2	-2
-3	-3	-3	-1	-3
-4	-4	-4	-4	-1
	-1	-2	-3	-4
T_A	1	0.2	0.4	0.6
I_A	0.1	0.3	0.7	0.9
F_A	0.05	0.25	0.45	0.65

Table 2

ϑ	0	-1	-2	-3	-4
e	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
-1	$\{-1\}$	$\{0,-1\}$	$\{0,-1\}$	$\{e,-1\}$	$\{0,-1\}$
-2	$\{-2\}$	$\{-2\}$	$\{0,-1\}$	$\{-2\}$	$\{-2\}$
-3	$\{-3\}$	$\{-3\}$	$\{-3\}$	$\{0,-1\}$	$\{-3\}$
-4	$\{-4\}$	$\{-4\}$	$\{-4\}$	$\{-4\}$	$\{0,-1\}$
	0	-1	-2	-3	-4
T_B	1	1	0.2	0.4	0.6
I_B	0.1	0.1	0.3	0.7	0.9
F_B	0.05	0.05	0.25	0.45	0.65

(ii) $(\overline{X'}, \vartheta', (T_A(0), I_A(0), F_A(0)))$ *is a hyper BCK-algebra, and* $A = (T_A, I_A, F_A)$ *is a single-valued neutrosophic hyper BCK-subalgebra of* $\overline{X'}$

(iii) $(\overline{X'}, \varrho, (T_A(0), I_A(0), F_A(0)))$ *is a BCK-algebra, and* $B = (T_B, I_B, F_B)$ *is a single-valued neutrosophic BCK-subalgebra of* $\overline{X'}$

(iv) $|X'| = |\overline{X'}| + 1$

Proof. Let $|X| \geq 2$ and $b \in X$ be fixed. For any $x, y \in X'$, define a binary hyperoperation ϑ on X' as follows:

$$x \vartheta y = \begin{cases} 0, & \text{if } x = 0, \\ \{0, b\}, & \text{if } x = y \text{ and } x \neq 0, \\ \{b\}, & \text{if } x = b \text{ and } y = 0, \\ \{0, b\}, & \text{if } x = b \text{ and } y \neq 0, \\ x, & \text{otherwise.} \end{cases} \tag{13}$$

Now, we show that $(X', \vartheta, 0)$ is a hyper BCK-algebra. We just check that conditions (H1) and (H2) are valid.

(H1): Let $x, y, z \in X'$. If $x = 0$, then $(x \vartheta z) \vartheta (y \vartheta z) = \{0\} \vartheta (y \vartheta z) = \{0\} \ll x \vartheta y$. If $x = b$, then $(x \vartheta z) \vartheta (y \vartheta z) \subseteq \{0, b\} \vartheta (y \vartheta z) \subseteq \{0, b\} \ll x \vartheta y$. If $x \notin \{0, b\}$, we consider the following cases:

Case 1: $x = y \neq z$. Then $(x \vartheta z) \vartheta (y \vartheta z) = x \vartheta y = x \vartheta x = \{0, b\} \ll \{0, b\} = x \vartheta y$.

Case 2: $x = z \neq y$. Then $(x \vartheta z) \vartheta (y \vartheta z) = \{0, b\} \vartheta (y \vartheta z) = \{0, b\} \ll x = x \vartheta y$.

Case 3: $y = z \neq x$. Then $(x \vartheta z) \vartheta (y \vartheta z) \subseteq x \vartheta \{0, b\} = \{0, b\} \ll x = x \vartheta y$.

Case 4: $x \neq y \neq z$. Then $(x \vartheta z) \vartheta (y \vartheta z) = x \vartheta y = x \ll x = x \vartheta y$.

Case 5: $x = y = z$. Then $(x \vartheta z) \vartheta (y \vartheta z) = \{0, b\} \ll \{0, b\} = x \vartheta y$.

(H2): Let $x, y, z \in X$. The proof of $(x \vartheta y) \vartheta z = (x \vartheta z) \vartheta y$ is similar to that of (H1), and then it is easy to see that $(X', \vartheta, 0)$ is a hyper BCK-algebra. Consider a single-valued neutrosophic subset $A = (T_A, I_A, F_A)$ of X' such that $T_A(0) = T_A(b) = 1$, $I_A(0) = I_A(b) = F_A(0) = F_A(b) = 0$; by equation (2) and some modifications, we get that

$$\begin{aligned} \wedge (T_A(x \vartheta y)) &\geq T_{\min}(T_A(x), T_A(y)), \\ \vee (I_A(x \vartheta y)) &\leq S_{\max}(I_A(x), I_A(y)), \\ \vee (F_A(x \vartheta y)) &\leq S_{\max}(F_A(x), F_A(y)). \end{aligned} \tag{14}$$

Hence, $A = (T_A, I_A, F_A)$ is a single-valued neutrosophic hyper BCK-subalgebra of $(\overline{X'}, \vartheta, 0)$. Now, $\forall x, y \in X$; define a hyperoperation ϑ' on $\overline{X'}$ by

$$\begin{aligned} A(x) \vartheta' A(y) &= (T_A(x), I_A(x), F_A(x)) \vartheta' (T_A(y), I_A(y), F_A(y)) \\ &= (T_A(x \vartheta y), I_A(x \vartheta y), F_A(x \vartheta y)). \end{aligned} \tag{15}$$

Define a single-valued neutrosophic subset $B = (T_B, I_B, F_B)$ of $\overline{X'}$ by

$$B(A(x)) = A(x), \\ \text{or } (T_B(T_A(x)), I_B(I_A(x)), F_B(F_A(x))) = (T_A(x), I_A(x), F_A(x)), \tag{16}$$

and an operation ϱ on $\overline{X'}$ by

$$\begin{aligned} &(T_A(x), I_A(x), F_A(x)) \varrho (T_A(y), I_A(y), F_A(y)) \\ &= (\vee (T_A(x) \vartheta' T_A(y)), \wedge (I_A(x) \vartheta' I_A(y)), \wedge (F_A(x) \vartheta' F_A(y))). \end{aligned} \tag{17}$$

It can be easily seen that $(T_A(x), I_A(x), F_A(x)) \ll' (T_A(y), I_A(y), F_A(y)) \Longleftrightarrow x \ll y$, $(\overline{X'}, \vartheta', (T_A(0), I_A(0), F_A(0)))$ is a hyper BCK-algebra, $A = (T_A(x), I_A(x), F_A(x))$ is a single-valued neutrosophic hyper BCK-subalgebra of $\overline{X'}$, $(\overline{X'}, \vartheta, (T_A(0), I_A(0), F_A(0)))$ is a BCK-algebra, and $B = (T_B(x), I_B(x), F_B(x))$ is a single-valued neutrosophic BCK-subalgebra of $\overline{X'}$, and since $T_A(0) = T_A(b) = 1$, $I_A(0) = I_A(b) = F_A(0) = F_A(b) = 0$, we get that $|X'| = |\overline{X'}| + 1$. ☐

Corollary 5. *Each nonempty set can be constructed to an extendable single-valued neutrosophic BCK-subalgebra.*

4. Neutro Hyper BCK-Algebras

Smarandache in [17] introduced the concept of neutro hyper operation. An n-ary (for integer $n \geq 1$) hyperoperation $\vartheta: X^n \longrightarrow P(Y)$ is called a neutro hyper operation if it has n-plets in X^n for which the hyperoperation is well-defined $\vartheta(a_1, a_2, \ldots, a_n) \in P(Y)$ (degree of truth (T)), n-plets in X^n for which the hyperoperation is indeterminate (degree of indeterminacy (I)), and n-plets in X^n for which the hyperoperation is outer-defined $\vartheta(a_1, a_2, \ldots, a_n) \notin P(Y)$ (degree of falsehood (F)), where $T, I, F \in [0, 1]$, with

$(T, I, F) \neq (1, 0, 0)$ that represents the n-ary (total) hyper operation and $(T, I, F) \neq (0, 0, 1)$ that represents the n-ary anti hyper operation.

In this section, we introduce a novel concept of neutro hyper BCK-algebras as a generalization of neutro BCK-algebras and analyze their properties. The main motivation of the concept of neutro hyper BCK-algebra is a generalization of neutro BCK-algebra, which is defined as follows.

Definition 7. Let $X \neq \varnothing$ and $P^*(X) = \{Y \mid \varnothing \neq Y \subseteq X\}$. Then for a map $\vartheta: X^2 \longrightarrow P^*(X)$, a hyperalgebraic system $(X, \vartheta, 0)$ is called a neutro hyper BCK-algebra if it satisfies in the following neutro axioms:

(H1) ($\exists x, y, z \in X$ that $(x \vartheta z) \vartheta (y \vartheta z) \ll x \vartheta y$) and ($\exists x', y', z' \in X$ that $(x' \vartheta z') \vartheta (y' \vartheta z') \not\ll x' \vartheta y'$ or indeterminate)

(H2) ($\exists x, y, z \in X$ that $(x \vartheta y) \vartheta z = (x \vartheta z) \vartheta y$) and ($\exists x', y', z' \in X$ that $(x' \vartheta y') \vartheta z \prime \neq (x' \vartheta z') \vartheta y'$ or indeterminate)

(H3) ($\exists x \in X$ that $x \vartheta X \ll x$) and ($\exists x' \in X$ that $x' \vartheta X \not\ll x'$ or indeterminate)

(H4) ($\exists x, y \in X$ that if $x \ll y$ and $y \ll x$ imply $x = y$) and ($\exists x', y' \in X$ that if $x' \ll y'$ and $y' \ll x'$ imply $x' \neq y'$ or indeterminate),

where $a \ll b$ is defined by $0 \in a \vartheta b$, and $\forall A, B \subseteq H$, $A \ll B \Longleftrightarrow \forall a \in A\, \exists b \in B$ s.t $a \ll b$

If $(X, \vartheta, 0)$ is a neutro hyperalgebra and satisfies in condition (H1) to (H4), then we will call it is a neutro hyper BCK-algebra of type 4 (i.e., it satisfies 4 neutro axioms).

Investigation of partial order relation on neutro hyper BCK-algebra plays a main role in Hass diagram, so we have the following results.

Theorem 12. *Let $(X, \vartheta, 0)$ be a neutro hyper BCK-algebra, $x, y, z \in X$ and $A, B, C \subseteq X$. Then*

(i) $\exists x, y \in X$ such that $(x \vartheta y) \ll x$

(ii) $\exists x, y \in X$ such that $(x \vartheta y) \not\ll x$

(iii) $\exists x \in X$ such that $x \ll x$

(iv) $\exists x \in X$ such that $x \not\ll x$

(v) $\exists A, B \subseteq X$ such that $A \ll A$

(vi) $\exists A, B \subseteq X$ such that $A \not\ll A$

Proof. We prove only the item (ii), and other items are similar to it. Since $(X, \vartheta, 0)$ is a neutro hyper BCK-algebra, there exists $x \in X$such that $(x \vartheta X) \not\ll X$. It follows that there exist $a, y \in X$ such that $a \in x \vartheta y$ and $a \not\ll x$. Hence, $(x \vartheta y) \not\ll x$. □

Theorem 13. *Let $(X, \vartheta, 0)$ be a neutro hyper BCK-algebra, $x, y, z \in X$ and $A, B, C \subseteq X$. Then*

(i) if $A \ll B$, then $(A \cup C) \ll (B \cup C)$

(ii) if $A \not\ll B$, then $(A \cup C) \not\ll (B \cup C)$

Proof

(i) Let $a \in A$ be arbitrary. Since $A \ll B$, there exists $b \in B$ such that $a \ll b$. Hence, for $a \in (A \cup C)$, there exists $b \in (B \cup C)$ such that $a \ll b$ and so $(A \cup C) \ll (B \cup C)$.

(ii) Since $A \not\ll B$, there exists $a \in A$ such that for all, $b \in B$, we have $a \not\ll b$. Hence, there exists $a \in (A \cup C)$ such that for all, $b \in (B \cup C)$, we get that $a \not\ll b$ and so $(A \cup C) \not\ll (B \cup C)$. □

Example 2. (i) Every neutro BCK-algebra $(X, \vartheta, 0)$ is a neutro hyper BCK-algebra. Since, for all, $x, y \in X$, can define a hyperoperation ϑ on X by $x \vartheta y = \{x \varrho y\}$.

(ii) Consider $\mathbb{N}^* = \{0, 1, 2, 3, \ldots\}$. Define

$$x \vartheta y = \begin{cases} \{0, x\} & \text{if } x \leq y \\ 0 & (x, y) = (2, 3) \text{ or } (x, y) = (3, 2) \\ 2 & x = y = 1 \text{ or } (x, y) = (0, 1) \\ x & \text{otherwise} \end{cases}.$$

Clearly, $(\mathbb{N}^*, \vartheta, 0)$ is a neutro hyper BCK-algebra.

The following theorem shows that neutro hyper BCK-algebras are the generalization of hyper BCK-algebras.

Theorem 14. *Every hyper BCK-algebra can be extended to a neutro hyper BCK-algebra.*

Proof. Let $(X, \vartheta, 0)$ be a hyper BCK-algebra and $\alpha \notin X$. For all, $x, y \in X \cup \{\alpha\}$, define ϑ_α on $X \cup \{\alpha\}$ by $x \vartheta_\alpha y = x \vartheta y$, where, $x, y \in X$ and whence $\alpha \in \{x, y\}$, define $x \vartheta_\alpha y$ is indeterminate or $x \vartheta_\alpha y \in X \cup \{\alpha\}$.

We show that how to construct neutro hyper BCK-algebras from BCK-algebras. □

Example 3. Let $X = \{0, 1, 2, 3, 4\}$ and consider Table 3.

Then

(i) If $a = 0$, then $(X, \vartheta_1, 0)$ is a neutro hyper BCK-algebra and if $a = 1$, then $(X \backslash \{3, 4, 5\}, \vartheta_1, 0)$ is a hyper BCK-algebra

(ii) $(X, \vartheta_2, 0)$ is a neutro hyper BCK-algebra and $(X \backslash \{4, 5\}, \vartheta_2, 0)$ is a hyper BCK-algebra

(iii) If $s = z = 0, w = 3$, then $(X, \vartheta_3, 0)$ is a neutro hyper BCK-algebra, and for $s = 1$, $z = 3$, $(X \backslash \{5\}, \vartheta_3, 0)$ is a hyper BCK-algebra. If $s = z = 0$, $w = \sqrt{2}$, then $(X, \vartheta_3, 0)$ is a neutro hyper BCK-algebra of type 4

The importance of the following theorem is to construct of neutro hyper BCK-algebra from any given nonempty set.

Theorem 15. *Let $0 \notin X \neq \varnothing$. Then there exists a hyperoperation "ϑ" on $X' = X \cup \{0\}$ such that $(X', \vartheta, 0)$ is a neutro hyper BCK-algebra.*

Proof. Let $0 \notin X \neq \varnothing$. Using Theorem 4, there exist a hyperoperation "ϑ" on $X' = X \cup \{0\}$ such that $(X', \vartheta, 0)$ is a hyper BCK-algebra. Now, apply Theorem 14; there exist a hyperoperation "ϑ'" on $X' = X \cup \{0\}$ such that $(X', \vartheta', 0)$ is a neutro hyper BCK-algebra.

TABLE 3: Neutro hyper BCK-algebras.

ϑ_1	0	1	2	3	4	5
0	0	0	0	0	2	0
1	1	0	a	2	4	3
2	2	2	0, 2	0	2	0
3	3	0	1	2	4	5
4	1	4	2	1	4	3
5	0	4	0	1	4	0
ϑ_2	0	1	2	3	4	5
0	0	0	0	0	2	0
1	1	0, 1	0	0, 1	4	5
2	2	2	0	2	5	0
3	3	3	3	0	0	0
4	2	1	2	4	1	2
5	5	0	4	0	0	x
ϑ_3	0	1	2	3	4	5
0	0	0	0	0	0	5
1	1	0, 2	1	1	s	0
2	2	0, 2	0, 2	0, 2	0, 2	3
3	3	3	3	0, 2	z	0
4	4	4	4	4	0, 2	1
5	2	0	2	2	2	w

Let $(X_1, \vartheta_1, 0_1)$ and $(X_2, \vartheta_2, 0_2)$ be two neutro hyper BCK-algebras. Define ϑ on $X_1 \times X_2$ by $(x, y)\vartheta(x', y') = (x\vartheta_1 x', y\vartheta_2 y')$, where $(x, y), (x', y') \in X_1 \times X_2$ and say that $(x, y) \ll (x', y') \Longleftrightarrow (0_1, 0_2) \in (x, y)\vartheta(x', y')$. The following theorem investigates the properties of partial order relation on product of Neutro hyper BCK algebras. □

Theorem 16. *Let $(X_1, \vartheta_1, 0_1)$ and $(X_2, \vartheta_2, 0_2)$ be two neutro hyper BCK-algebras. Then*

(i) $\forall (x, y), (x', y') \in X_1 \times X_2$, $(x, y) \ll (x', y') \Longleftrightarrow (x \not\ll_1 x')$ and $(y \not\ll_2 y')$

(ii) $\forall (x, y), (x', y') \in X_1 \times X_2$, $(x, y) \not\ll (x', y') \Longleftrightarrow (x \ll_1 x')$ or $(y \ll_2 y')$

(iii) $\exists (x, y), (x', y') \in X_1 \times X_2$, $(0_1, 0_2) \in ((x, y)\vartheta(x', y'))\vartheta(x, y)$

(iv) $\exists (x, y), (x', y') \in X_1 \times X_2$, $(0_1, 0_2) \notin ((x, y)\vartheta(x', y'))\vartheta(x, y)$

Proof

(i) Immediate

(ii) Let $(x, y), (x', y') \in X_1 \times X_2$. Then $(0_1, 0_2) \in (x, y)\vartheta(x', y')$, if and only if $(0_1, 0_2) \in (x\vartheta_1 x', y\vartheta_2 y')$, if and if only $0_1 \notin x\vartheta x'$ or $0_2 \notin y\vartheta y'$, and if and only if $(x \ll_1 x')$ or $(y \ll_2 y')$

(iii) Since $(X_1, \vartheta_1, 0_1)$ and $(X_2, \vartheta_2, 0_2)$ be two neutro hyper BCK-algebras, there exist $x, y \in X_1, x', y' \in X_2$ such that $0_1 \in (x\vartheta y)\vartheta x$ and $0_2 \in (x'\vartheta y')\vartheta x'$. It follows that $\exists (x, y), (x', y') \in X_1 \times X_2$, $(0_1, 0_2) \in ((x, y)\vartheta(x', y'))\vartheta(x, y)$

(iv) Since $(X_1, \vartheta_1, 0_1)$ and $(X_2, \vartheta_2, 0_2)$ be two neutro hyper BCK-algebras, there exist $x, y \in X_1, x', y' \in X_2$ such that $0_1 \notin (x\vartheta y)\vartheta x$ and $0_2 \notin (x'\vartheta y')\vartheta x'$. It follows that $\exists (x, y), (x', y') \in X_1 \times X_2$, $(0_1, 0_2) \notin ((x, y)\vartheta(x', y'))\vartheta(x, y)$

We need to extend neutro hyper BCK-algebras to a larger class of neutro hyper BCK-algebras, so we apply the notation of product on neutro hyper BCK-algebras as follows. □

Theorem 17. *Let $(X_1, \vartheta_1, 0_1)$ and $(X_2, \vartheta_2, 0_2)$ be two neutro hyper BCK-algebras. Then $(X_1 \times X_2, \vartheta, (0_1, 0_2))$ is a neutro hyper BCK-algebra.*

Proof. We prove only the item (H4), and other items by Theorem 16 are valid. Since $(X_1, \vartheta_1, 0_1)$ and $(X_2, \vartheta_2, 0_2)$ are neutro hyper BCK-algebras, there exist $(x_1, x_2), (y_1, y_2), (x_1', x_2'), (y_1', y_2') \in X_1 \times X_2$ that if $(x_1 \ll_1 y_1, y_1 \ll_1 x_1)$, then $x_1 = y_1$, and if $(x_2 \ll_2 y_2, y_2 \ll_2 x_2)$, then $x_2 = y_2$. Also, if $(x_1' \ll_1 y_1', y_1' \ll_1 x_1')$, then $x_1 \neq y_1$, and if $(x_2' \ll_2 y_2', y_2' \ll_2 x_2')$, then $x_2 \neq y_2$. By (i), it follows that there exist $(x_1, x_2), (y_1, y_2), (x_1', x_2'), (y_1', y_2') \in X_1 \times X_2$ that if $(x_1, x_2) \ll (y_1, y_2)$, $(y_1, y_2) \ll (x_1, x_2)$, we have $(x_1, x_2) = (y_1, y_2)$, and if $(x_1', x_2') \ll (y_1', y_2')$, $(y_1', y_2') \ll (x_1', x_2')$, we have $(x_1', x_2') \neq (y_1', y_2')$.

Let $(X_1, \vartheta_1, 0_1)$ and $(X_2, \vartheta_2, 0_2)$ be hyper BCK-algebras, where $X_1 \cap X_2 = \varnothing$. For some $x, y \in X$, define a hyperoperations ϑ_t, ϑ_s as follows:

$$x\vartheta_t y = \begin{cases} (x\vartheta_1 y)\backslash\{0_1\}, & \text{if } x, y \in X_1\backslash X_2, \\ x\vartheta_2 y, & \text{if } x, y \in X_2\backslash X_1, \\ t, & \text{if } x \in X_1,\ y \in X_2, \\ 0_2, & \text{if } x \in X_2,\ y \in X_1, \end{cases}$$
$$x\vartheta_s y = \begin{cases} x\vartheta_1 y, & \text{if } x, y \in X_1\backslash X_2, \\ (x\vartheta_2 y)\backslash\{0_2\}, & \text{if } x, y \in X_2\backslash X_1, \\ s, & \text{if } x \in X_1,\ y \in X_2, \\ 0_1, & \text{if } x \in X_2,\ y \in X_1, \end{cases} \tag{18}$$

and $0_1\vartheta_t 0_1 = 0_1$ $\vartheta_t 0_2 = 0_2\vartheta_t 0_1 = 0_1$, $0_1\vartheta_s 0_2 = 0_2\vartheta_s 0_1 = 0_2\vartheta_s 0_2 = 0_2$, where $0_2 \neq t \in X_2$, $0_1 \neq s \in X_1$. Thus, we have the following theorem.

We want to extend neutro hyper BCK-algebras to a larger class of neutro hyper BCK-algebras, so we apply the notation of union on neutro hyper BCK-algebras as follows. □

Theorem 18. *Let $(X_1, \vartheta_1, 0_1)$ and $(X_2, \vartheta_2, 0_2)$ be hyper BCK-algebras, where $X_1 \cap X_2 = \varnothing$ and $X = X_1 \cup X_2$. Then*

(i) For all, $A \subseteq X_1, A \not\ll \{0_1, t\}$

(ii) For all, $A \subseteq X_1, A \not\ll 0_2$

(iii) For all, $A \subseteq X_1, A \not\ll A$, and for all, $B \subseteq X_2, B \not\ll B$

(iv) For all, $A \subseteq X_2, A \not\ll \{0_2, s\}$

(v) For all, $A \subseteq X_2, A \not\ll 0_1$

Proof

(i) Let $A \subseteq X_1$. Then $A\vartheta_t 0_1 = \cup_{a\in A}(a\vartheta_t 0_1) = \cup_{a\in A}((a\vartheta 0_1)\backslash\{0_1\})$. It follows that $0_1 \notin A\vartheta_t 0_1$, so $A \not\ll \{0_1\}$. In addition, $A\,\vartheta_t t = \cup_{a\in A}(a\,\vartheta_t t) = \{t\}$ and $0_1 \notin t\,\vartheta_t 0_1$. It follows that $0_1 \notin A\,\vartheta_t 0_1$, so $A \not\ll \{t\}$.

(ii) Let $A \subseteq X_1$. Then $A\,\vartheta_t 0_2 = \cup_{a\in A}(a\,\vartheta_t 0_t) = \{t\}$ and $0_1 \notin t\,\vartheta_t 0_2$. It follows that $0_1 \notin A\,\vartheta_t 0_1$, so $A \not\ll \{0_2\}$. In addition, $A\,\vartheta_t t = \cup_{a\in A}(a\,\vartheta_t t) = \{t\}$ and $0_1 \notin t\,\vartheta_t 0_1$. It follows that $0_1 \notin A\,\vartheta_t 0_1$, so $A \not\ll \{t\}$.

(iii) Let $A \subseteq X_1$ and $B \subseteq X_2$. Since $A\,\vartheta_t A = \cup_{a,a'\in A}(a\,\vartheta_t a') = \cup_{a,a_I\in A}((a\,\vartheta_t a')\backslash\{0_1\})$ and $B\,\vartheta_s S = \cup_{b,b'\in B}(b\,\vartheta_t b') = \cup_{b,b'\in B}((b\,\vartheta_s b')\backslash\{0_2\})$, we get that $0_1 \not\in A\,\vartheta_t A$ and $0_2 \not\in B\,\vartheta_s B$. Thus $A \not\ll A$ and $B \not\ll B$.

(iv) and (v) are similar to (i) and (ii), respectively. □

Theorem 19. *Let $(X_1, \vartheta_1, 0_1)$ and $(X_2, \vartheta_2, 0_2)$ be hyper BCK-algebras, where $X_1 \cap X_2 = \varnothing$ and $X = X_1 \cup X_2$. Then*

(i) $(X, \vartheta_t, 0_1)$ is a neutro hyper BCK-algebra

(ii) $(X, \vartheta_s, 0_2)$ is a neutro hyper BCK-algebra

Proof

(i) $\overline{(H_1:)}$ For some, $x, y, z \in X_2\backslash X_1$, $\overline{(x\,\vartheta_t z)\,\vartheta_t (y\,\vartheta_t z) \ll (x\,\vartheta_t y)}$. Since, for $x \in X_1$, $(((x\,\vartheta\,0_1)\backslash\{0_1\})\backslash\{0_1\})\,\vartheta_t 0_2 = t \neq 0_2$, we get that

$$(x\,\vartheta_t 0_1)\,\vartheta_t (0_2\,\vartheta_t 0_1) = ((x\,\vartheta\,0_1)\backslash\{0_1\})\,\vartheta_t 0_1 = ((x\,\vartheta\,0_1)\backslash\{0_1\})\backslash\{0_1\} \ll 0_2 = 0_1\,\vartheta_t 0_2. \quad (19)$$

$\overline{(H_2:)}$ For some, $x, y, z \in X_2\backslash X_1$, $\overline{(x\,\vartheta_t y)\,\vartheta_t z = (x\,\vartheta_t z)\,\vartheta_t y}$. In addition, for $x \in X_1$,

$$(x\,\vartheta_t 0_2)\,\vartheta_t 0_1 = t\,\vartheta_t 0_1 = 0_2 \neq t = ((x\,\vartheta\,0_1)\backslash\{0_1\})\,\vartheta_t 0_2 = (x\,\vartheta_t 0_1)\,\vartheta_t 0_2. \quad (20)$$

$\overline{(H_3:)}$ For some, $x \in X_2 X_1$, $x\,\vartheta_t X = x\,\vartheta\,X_2 \ll X_2 = X$. Since $t\,\vartheta_t 0_1 = 0_2$ and $(\cup_{x\in X_1}((0_1\,\vartheta\,x)\ \{0_1\}))\,\vartheta_t 0_1 = (\cup_{x\in X_1}((0_1\,\vartheta\,x)\backslash\{0_1\}))\backslash\{0_1\}$, we get that

$$0_1\,\vartheta_t X = (0_1\,\vartheta_t X_1) \cup (0_1\,\vartheta_t X_2) = \left(\underset{x\in X_1}{\cup}(0_1\,\vartheta_t x)\right) \cup \left(\underset{y\in X_2}{\cup}(0_1\,\vartheta_t y)\right) = \left(\underset{x\in X_1}{\cup}(0_1\,\vartheta\,x)\backslash\{0_1\}\right) \cup \{t\} \ll 0_1. \quad (21)$$

$\overline{(H_3:)}$ Because $0_1 \ll 0_1$ and $0_1 \in 0_1\,\vartheta_t 0_2$ and $0_1 \in 0_2\,\vartheta_t 0_1$, while $0_1 \neq 0_2$, we get the item $\overline{(H_3:)}$ is valid. Therefore, $(X, \vartheta_t, 0_1)$ is a neutro hyper BCK-algebra.

(ii) It is similar to item (i). □

4.1. Application of Neutro Hyper BCK-Algebras and Single-Valued Neutrosophic Hyper BCK-Subalgebras. In this subsection, we describe some applications of neutro hyper BCK-algebra and single-valued neutrosophic hyper BCK-subalgebra in some complex (hyper) networks.

Table 4: Neutro hyper BCK-algebra of an economic network.

ϑ	a	b	c	d	e	f
a	a	a	a	a	a	f
b	b	a, c	b	b	a	a
c	c	a, c	a, c	a, c	a, c	d
d	d	d	d	a, c	a	a
e	e	e	e	e	a, c	b
f	c	a	c	c	c	???

Table 5: Single-valued neutrosophic hyper BCK-subalgebra of a data network.

ϑ	a	b	c	d	e
a	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{b\}$	$\{a,b\}$	$\{a,b\}$	$\{e,b\}$	$\{a,b\}$
c	$\{c\}$	$\{c\}$	$\{a,b\}$	$\{c\}$	$\{c\}$
d	$\{d\}$	$\{d\}$	$\{d\}$	$\{a,b\}$	$\{d\}$
e	$\{e\}$	$\{e\}$	$\{e\}$	$\{e\}$	$\{a,b\}$
	a	b	c	d	e
T_B	1	1	0.2	0.4	0.6
I_B	0.1	0.1	0.3	0.7	0.9
F_B	0.05	0.05	0.25	0.45	0.65

Example 4 (economic network). Let $X = \{a = \text{China}, b = \text{Italy}, c = \text{Iran}, d = \text{Spain}, e = \text{Germany}, f = \text{USA}\}$ be a set of top countries, which are in an economic network. Suppose ϑ is the relations on X, which is described in Table 4, and for $x \neq y$, $x * y = D$ means that D is the set of countries that benefit from this economic partnership, whence the country x starts to country y, and for $x = y$, it means that the country x maintains its capital.

Clearly, $(X, *, \text{China})$ is a neutro hyper BCK-algebra in this model. We obtain that the USA is main source of this network; since if the USA starts to any other country, it does not benefit. In addition, if the USA starts to itself, this participation becomes indeterminate. Also, if any country starts to China, we conclude that China loss, else with USA, and if China starts to any other country, then China benefit else USA.

Example 5 (data network). Let $Y = \{a, b, c, d, e\}$ be a set of mobile sets, which are in a data network. Suppose ϑ is the relations on Y, which is described in Table 3, and for all, $x \neq, x * y = D$ means that D is a set of mobile sets that receive contents of messages that mobile set x starts to mobile set y, and for $x = y$, it means that the mobile set x retains its information. In addition, for any $y \in Y$, $T_B(y), I_B(y), F_B(y)$ are the cryptographic power, battery life, and RAM of mobile set y, respectively. Then $B = (T_B, I_B, F_B)$ is a single-valued neutrosophic hyper BCK-subalgebra of (Y, ϑ, a) in Table 5.

It is clear that if mobile set named "a" starts, then none of the devices receive the message, and if other devices start to name a mobile set "a", then this device (mobile set a) cannot receive their messages; hence, it is not suitable node in this network, since furthermore to its complex cryptography, its battery life, and RAM is weak. Also, one can see that the mobile set b is the best in this regard.

5. Conclusion

To conclude, the current paper has presented and analyzed the notion of single-valued neutrosophic hyper BCK-subalgebras and neutro hyper BCK-algebras and investigated some of their new useful properties. We defined the concept of the extended single-valued neutrosophic BCK-subalgebras and showed that for any $\alpha \in [0,1]$ and a single-valued neutrosophic subset hyper BCK-subalgebra, $A = (T_A, I_A, F_A)$, $A = (T_{A\alpha}, I_{A\alpha}, F_{A\alpha})$ is a hyper BCK-subalgebra. Through the concept of fundamental relation $C(\beta)$, we have generated the single-valued neutrosophic BCK-subalgebras from single-valued neutrosophic hyper BCK-subalgebras, so some categorical properties of single-valued neutrosophic BCK-subalgebras are investigated based on the categorical properties of single-valued neutrosophic hyper BCK-subalgebras. In addition, on any nonempty set, we have constructed at least one single-valued neutrosophic BCK-subalgebra and one extendable single-valued neutrosophic BCK-subalgebra. The concept of neutro hyper BCK-algebra as a generalization of neutro BCK-algebra is introduced in this study, and it is constructed the class of product of neutro hyper BCK-algebras and union of neutro hyper BCK-algebras via hyper BCK-algebras. In study of neutro hyper BCK-algebras, despite having key mathematical tools, there are some limitations. The union of two neutro hyper BCK-algebras is not necessarily; a neutro hyper BCK-algebras so the class of neutro hyper BCK-algebras is not closed under any given algebraic operation. In addition, neutro hyper BCK-algebras are different from single-valued neutrosophic hyper BCK-subalgebras so could not generalize the capabilities of single-valued neutrosophic hyper BCK-subalgebras to neutro hyper BCK-algebras and conversely. In final, we can apply these concepts in real world, especially in some complex (hyper) networks.

We hope that these results are helpful for further studies in single-valued neutrosophic logical algebras. In our future studies, we hope to obtain more results regarding single-valued neutrosophic (hyper) logical-subalgebras, neutro (hyper) logical-subalgebras, and their applications.

Data Availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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