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Operators on Single-Valued Neutrosophic Oversets, Neutrosophic Undersets, and Neutrosophic Offsets

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Abstract. We have defined *Neutrosophic Over-/Under-/Off-Set and Logic* for the first time in 1995 and published in 2007. During 1995-2016 we presented them to various national and international conferences and seminars. These new notions are totally different from other sets/logics/probabilities.

We extended the neutrosophic set respectively to *Neutrosophic Overset* {when some neutrosophic component is > 1}, to *Neutrosophic Underset* {when some neutrosophic component is < 0}, and to *Neutrosophic Offset* {when some neutrosophic components are off the interval [0, 1], i.e. some neutrosophic component > 1 and other neutrosophic component < 0}.

This is no surprise since our real-world has numerous examples and applications of over-/under-/off-neutrosophic components.

Keywords. neutrosophic overset, neutrosophic underset, neutrosophic offset, neutrosophic over logic, neutrosophic under logic, neutrosophic off logic, neutrosophic over probability, neutrosophic under probability, neutrosophic off probability, over membership (membership degree > 1), under membership (membership degree < 0), offmembership (membership degree off the interval [0, 1]).

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1. Introduction

In the classical set and logic theories, in the fuzzy set and logic, and in intuitionistic fuzzy set and logic, the degree of membership and degree of non-membership have to belong to, or be included in, the interval [0, 1]. Similarly, in the classical probability and in imprecise probability the probability of an event has to belong to, or respectively be included in, the interval [0, 1].

Yet, we have observed and presented to many conferences and seminars around the globe and published {see [1]-[8]} that in our real world there are many cases when the degree of membership is greater than 1. The set, which has elements whose membership is over 1,we called it *Overset*.

Even worst, we observed elements whose membership with respect to a set is under 0, and we called it *Underset.*

In general, a set that has elements whose membership is above 1 and elements whose membership is below 0, we called it *Offset* (i.e. there are elements whose memberships are off (over and under) the interval [0, 1]).

2. Example of overmembershipand under membership

In a given company a full-time employer works 40 hours per week. Let's consider the last week period.

Helen worked part-time, only 30 hours, and the other 10 hours she was absent without payment; hence, her membership degree was 30/40 = 0.75 < 1.

John worked full-time, 40 hours, so he had the membership degree 40/40 = 1, with respect to this company.

But George worked overtime 5 hours, so his membership degree was (40+5)/40 = 45/40 = 1.125 > 1. Thus, we need to make distinction between employees who work overtime, and those who work full-time or part-time. That's why we need to associate a degree of membership strictly greater than 1 to the overtime workers.

Now, another employee, Jane, was absent without pay for the whole week, so her degree of membership was 0/40 = 0.

Yet, Richard, who was also hired as a full-time, not only didn't come to work last week at all (0 worked hours), but he produced, by accidentally starting a devastating fire, much damage to the company, which was estimated at a value half of his salary (i.e. as he would have gotten for working 20 hours that week). Therefore, his membership degree has to be less that Jane's (since Jane produced no damage). Whence, Richard's degree of membership, with respect to this company, was - 20/40 = - 0.50 < 0.

Consequently, we need to make distinction between employees who produce damage, and those who produce profit, or produce neither damage no profit to the company.

Therefore, the membership degrees > 1 and < 0 are real in our world, so we have to take them into consideration.

Then, similarly, the Neutrosophic Logic/Measure/Probability/Statistics etc. were extended to respectively *Neutrosophic Over-/Unde-r/Off-Logic, -Measure, -Probability, -Statistics* etc. (Smarandache, 2007).

3. Definition of single-valued neutrosophic overset

Let U be a universe of discourse and the neutrosophic set $A_1 \subset U$.

Let T(x), I(x), F(x) be the functions that describe the degrees of membership, indeterminate-membership, and nonmembership respectively, of a generic element $x \in U$, with respect to the neutrosophic set A_1:

$T(x), I(x), F(x) : U \rightarrow [0, \Omega]$

where $0 < 1 < \Omega$, and Ω is called overlimit.

A Single-Valued Neutrosophic Overset A_1 is defined as:

$A_1 = \{(x, <T(x), I(x), F(x)>), x \in U\}$, such that there exists at least one element in A_1 that has at least one neutrosophic component that is > 1, and no element has neutrosophic components that are < 0.

For **example**: $A_1 = \{(x_1, <1.3, 0.5, 0.1>), (x_2, <0.2, 1.1, 0.2>)\}$, since $T(x_1) = 1.3 > 1$, $I(x_2) = 1.1 > 0$, and no neutrosophic component is < 0.

Also $O_2 = \{(a, <0.3, -0.1, 1.1>)\}$, since $I(a) = - 0.1 < 0$ and $F(a) = 1.1 > 1$.

4. Definition of single-valued neutrosophic underset

Let U be a universe of discourse and the neutrosophic set $A_2 \subset U$.
Let T(x), I(x), F(x) be the functions that describe the degrees of membership, indeterminate-membership, and nonmembership respectively, of a generic element $x \in U$, with respect to the neutrosophic set A_2:

$T(x), I(x), F(x) : U \rightarrow [\Psi, 1]$

where $\Psi < 0 < 1$, and Ψ is called underlimit.
A Single-Valued NeutrosophicUndersetA$_2$is defined as:
$A_2 = \{(x, <T(x), I(x), F(x)>), x \in U\}$,
such that there exists at least one element in A_2 that has at least one neutrosophic component that is < 0, and no element has neutrosophic components that are > 1.
For **example**: $A_2 = \{(x_1, <-0.4, 0.5, 0.3>), (x_2, <0.2, 0.5, -0.2>)\}$, since $T(x_1) = -0.4 < 0$, $F(x_2) = -0.2 < 0$, and no neutrosophic component is > 1.

5. Definition of single-valued neutrosophic offset

Let U be a universe of discourse and the neutrosophic set $A_3 \subset U$.
Let T(x), I(x), F(x) be the functions that describe the degrees of membership, indeterminate-membership, and nonmembership respectively, of a generic element $x \in U$, with respect to the set A_3:

$T(x), I(x), F(x) : U \rightarrow [\Psi, \Omega]$

where $\Psi < 0 < 1 < \Omega$, and Ψ is called underlimit, while Ω is called overlimit.
A Single-Valued Neutrosophic OffsetA$_3$is defined as:
$A_3 = \{(x, <T(x), I(x), F(x)>), x \in U\}$,
such that there exist some elements in A_3 that have at least one neutrosophic component that is > 1, and at least another neutrosophic component that is < 0.
For **examples**: $A_3 = \{(x_1, <1.2, 0.4, 0.1>), (x_2, <0.2, 0.3, -0.7>)\}$, since $T(x_1) = 1.2 > 1$ and $F(x_2) = -0.7 < 0$.
Also, $B_3 = \{(a, <0.3, -0.1, 1.1>)\}$, since $I(a) = - 0.1 < 0$ and $F(a) = 1.1 > 1$.

6. Neutrosophic overset / underset / offset operators

Let U be a universe of discourse and $A = \{(x, <T_A(x), I_A(x), F_A(x)>), x \in U\}$ and
and $B = \{(x, <T_B(x), I_B(x), F_B(x)>), x \in U\}$ be two single-valued neutrosophic oversets / undersets / offsets.

$T_A(x), I_A(x), F_A(x), T_B(x), I_B(x), F_B(x): U \rightarrow [\Psi, \Omega]$

where $\Psi \leq 0 < 1 \leq \Omega$, and Ψ is called underlimit, while Ω is called overlimit.
We take the inequality sign ≤ instead of < on both extremes above, in order to comprise all three cases: overset {when $\Psi = 0$, and $1 < \Omega$}, underset {when $\Psi < 0$, and $1 = \Omega$}, and offset{when $\Psi < 0$, and $1 < \Omega$}.

Neutrosophic Overset / Underset / Offset Union.
Then $A \cup B = \{(x, <\max\{T_A(x), T_B(x)\}, \min\{I_A(x), I_B(x)\}, \min\{F_A(x), F_B(x)\}>), x \in U\}$

Neutrosophic Overset / Underset / Offset Intersection.
Then $A \cap B = \{(x, <\min\{T_A(x), T_B(x)\}, \max\{I_A(x), I_B(x)\}, \max\{F_A(x), F_B(x)\}>), x \in U\}$

Neutrosophic Overset / Underset / Offset Complement.
The complement of the neutrosophic set A is
$C(A) = \{(x, <F_A(x), \Psi + \Omega - I_A(x), T_A(x)>), x \in U\}$.

7. Conclusion

The membershipdegrees over 1 (overmembership), or below 0 (under membership) are part of our real world, so they deserve more study in the future. The neutrosophic overset / underset / off set together with neutrosophic overlogic / underlogic / off logic and especially neutrosophic over probability / under probability / and off probability have many applications in technology, social science, economics and so on that the readers may be interested in exploring.

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