ROUGH NEUTROSOPHIC SOFT SETS

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Abstract:

Soft set(SS) theory is a mathematical tool deals with parametric data which are imprecise in nature.It is a generalization of fuzzy set theory. On the other hand Rough set(RS) theory and Neutrosophic set (NS)theory both rising as powerful tool to handle uncertain, incomplete, inconsistent and imprecise information in an effective manner.Actually Neutrosophic set is a generalization of intuitionistic fuzzy set.Earlier Neutrosophic soft set(NSS) was established by combining the concept of Soft set and Neutrosophic set .In this paper , using the concept of Rough set,Neutrosophic set and soft set a hybrid structure Rough neutrosophic soft sets(RNSS) is developed.Some properties and operations on RNSS are introduced.

Keywords: Rough set,Neutrosophic set, Neutrosophic soft set,Rough neutrosophic soft set.

1.Introduction: In 1965[15] L.A.Zadeh introduced the concept of fuzzy set which is termed as an extension of classical set or crisp set in which every element has degree of membership.It is the most successful theoretical approach to vagueness.Unlike classical set theory, fuzzy set theory is described with an aid of membership function where the membership value of every element belongs to the unit interval [0,1] so that it is used in wide range of domains.Many mathematicians and researchers work tirelessly on fuzzy set theory in different areas and able to extend this concept by developing some other theories such as vague set[4],L-fuzzy set[5],Rough set[13], intuitionistic fuzzy set[1],interval-valued fuzzy set [6],interval-valued intuitionistic fuzzy set [2]etc.But all these theories have their own limitations and it is due to the lack of parametrization in a data.Thats why soft set theory was introduced by Molodtsov[12] in 1999 to handle parametric data so that we can express the uncertain problems in more general form.Soft set is progressing more rapidly which leads to the introduction of fuzzy soft set[8],intuitionistic fuzzy soft set[9],interval-valued fuzzy soft set[3],interval-valued intuitionistic fuzzy soft set[7],rough soft set[11] etc.

In 1982, another mathematical tool which is known as rough set is introduced by computer scientist Z.Pawlak[13].There are two basic elements in rough set theory,crisp set and equivalence relation,which constitute the mathematical basis of rough set.Upper and lower approximation operators are based on equivalence relation.It is a powerful tool to deal with incompleteness.It gives information of hidden data.

F.Smarandache[14] introduced the concept of neutrosophic set which is a generalization of intuitionistic fuzzy set.It is described by three functions: a membership function,indetermining function and a non-membership function that are independently related to each other.It is a mathematical tool for handling problem involving imprecise,indeterminacy and inconsistent data.Combining neutrosophic set with soft sets ,neutrosophic soft set[10] is introduced by P.K.Maji.

Neutrosophic soft set and rough sets are two different terms,none contradict the other.The main objective of this study is to introduce a new hybrid structure called rough neutrosophic soft sets.The significance of introducing hybrid set structure is that the computation technique based on any one of these structure alone will not always yield the best results but a fusion of two or more of tem can often give better results.

2.Preliminaries:

In this section we recall some basic definations and examples which are relevant to this work.

Definition 2.1 [12] Let U be an initial universe and E be a set of parameters. Let $P(U)$ denotes the power set of U and $A \subseteq E$. Then the pair (F, A) is called a *soft set* over U, where F is a mapping given by $F : A \rightarrow P(U)$.

Example 2.1.1 Suppose that $U = \{h_1, h_2, h_3, h_4, h_5, h_6, h_7\}$ is a universe consisting of seven houses and $A = \{e_1, e_2, e_3, e_4\} \subseteq E$ is a set of parameters considered by the decision makers where e_1, e_2, e_3 and e_4 represent the parameters 'beautiful', 'modern', 'cheap', 'in green surroundings' respectively. Now, we consider a soft set (F, A) which describes the 'attractiveness of the house' that Mr. X is going to buy. In this case, to define the soft set (F, A) means to point out beautiful houses, modern houses and so on. Consider the mapping F given by 'houses (.)' where (.) is to be filled by one of the parameters $e_i \in A$. For instance, $F(e_1)$ means 'houses (beautiful)' and the functional value is the set consisting of all the beautiful houses in U.

Let $F(e_1) = \{h_5, h_7\}$

$F(e_2) = \{h_1, h_4, h_6, h_7\}$

$F(e_3) = \{h_1, h_3\}$

$F(e_4) = \{h_2, h_4, h_5\}$.

Tabular representation of the soft set (F, A) is given by

	h_1	h_2	h_3	h_4	h_5	h_6	h_7
e_1	0	0	0	0	1	0	1
e_2	1	0	0	1	0	1	1
e_3	1	0	1	0	0	0	0
e_4	0	1	0	1	1	0	0

Definition 2.2 [14] Let X be an universe of discourse,with a generic element in X denoted by x,the neutrosophic(NS) set is an object having the form

$$A=\left\{\left\langle x:\mu_A\left(x\right),\nu_A\left(x\right),\omega_A\left(x\right)\right\rangle,x\in X\right\},$$

where the functions $\mu,\nu,\omega:X\to\left]^-0,1^+\right[$ define respectively the degree of membership (or Truth),the degree of indeterminacy,and the degree of non-membership(or Falsehood) of the element $x\in X$ to the set A with the condition

$$^-0\le\mu_A\left(x\right)+\nu_A\left(x\right)+\omega_A\left(x\right)\le 3^+$$

From a philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $\left]^-0,1^+\right[$ .So,instead of $\left]^-0,1^+\right[$ we need to take the interval [0,1] for technical applications,because $\left]^-0,1^+\right[$ will be difficult to apply in the real applications such as in scientific and engineering problems.

Example 2.2.1 Assume that the universe of discourse $U=\left\{x_1,x_2,x_3\right\}$, where x_1 characterizes the capability, x_2 characterizes the trustworthiness and x_3 indicates the prices of the objects.It may be further assumed that the values of x_1, x_2 and x_3 are in [0,1] and they are obtained from some questionnaires of some experts.The experts may impose their opinion in three components viz. te degree of goodness,the degree of indeterminacy and that of poorness to explain the characteristics of the objects.Suppose A is a neutrosophic set(NS) of U ,such that

$$A=\left\{\langle x_1,\left(0.3,0.5,0.6\right)\rangle,\langle x_2,\left(0.3,0.2,0.3\right)\rangle,\langle x_3,\left(0.3,0.5,0.6\right)\rangle\right\},$$

where the degree of goodness of capability is 0.3,degree of indeterminacy of capability is 0.5 and degree of falsity of capability is 0.6 etc.

Definition 2.3 [13] Let U be any non-empty set.Suppose R is an equivalence relation over U .For any non-null subset X of U ,the sets

$$A_1\left(x\right)=\left\{x:\left[x\right]_R\subseteq X\right\}\text{ and }A_2\left(x\right)=\left\{x:\left[x\right]_R\cap X\ne\varnothing\right\}$$

are called the lower approximation and upper approximation ,respectively of X , where the pair $S=\left(U,R\right)$ is called an approximation space.This equivalent relation R is called indiscernibility relation .

The pair $A\left(x\right)=\left(A_1\left(x\right),A_2\left(x\right)\right)$ is called the rough set of X in Here $\left[x\right]_R$ denotes the equivalence class of R containing x .

Definition 2.4 [13] Let $A=\left(A_1,A_2\right)$ and $B=\left(B_1,B_2\right)$ be two rough sets in the approximation space $S=\left(U,R\right)$. Then,

$A \cup B = \left(A_1 \cup B_1, A_2 \cup B_2\right),$

$A \cap B = \left(A_1 \cap B_1, A_2 \cap B_2\right),$

$\mathrm{A} \subseteq B$ if $A \cap B = A,$

$\sim A = \left\{U - A_2, U - A_1\right\}$

Definition 2.5 [10] Let U be an initial universe set and E be a set of parameters.Consider $A \subset E$.Let $P(U)$ denotes the set of all neutrosophic sets of U.The collection (F, A) is termed to be the soft neutrosophic set over U,where F is a mapping given by $F : A \to P(U)$.

Example 2.5.1 Let U be the set of houses under consideration and E is the set of parameters.Each parameter is a neutrosophic word or sentence involving neutrosophic words.Consider E ={beautiful,wooden,costly,very costly,moderate,green surroundings,in good repair,in bad repair,cheap,expensive}.In this case,to define a neutrosophic soft set means to point out beautiful houses,wooden houses,houses in the green surroundings and so on.Suppose that,there are five houses in the universe U given by, $U = \{h_1, h_2, h_3, h_4, h_5\}$ and the set of parameters $A = \{e_1, \mathrm{e}_2, \mathrm{e}_3, \mathrm{e}_4\}$,where, e_1 stands for the parameter 'beautiful', e_2 stands for the parameter 'wooden', e_3 stands for the parameter 'costly' and the parameter e_4 stands for 'moderate'.Suppose that

$$F(beautiful) = \left\{\langle h_1, 0.5, 0.6, 0.3\rangle, \langle h_2, 0.4, 0.7, 0.6\rangle, \langle h_3, 0.6, 0.2, 0.3\rangle, \langle h_4, 0.7, 0.3, 0.2\rangle, \langle h_5, 0.8, 0.2, 0.3\rangle\right\},$$

$$F(wooden) = \left\{\langle h_1, 0.6, 0.3, 0.5\rangle, \langle h_2, 0.7, 0.4, 0.3\rangle, \langle h_3, 0.8, 0.1, 0.2\rangle, \langle h_4, 0.7, 0.1, 0.3\rangle, \langle h_5, 0.8, 0.3, 0.6\rangle\right\},$$

$$F(\cos tly) = \left\{\langle h_1, 0.7, 0.4, 0.3\rangle, \langle h_2, 0.6, 0.7, 0.2\rangle, \langle h_3, 0.7, 0.2, 0.5\rangle, \langle h_4, 0.5, 0.2, 0.6\rangle, \langle h_5, 0.7, 0.3, 0.4\rangle\right\},$$

$$F(\mathrm{mod}\,erate) = \left\{\langle h_1, 0.8, 0.6, 0.4\rangle, \langle h_2, 0.7, 0.9, 0.6\rangle, \langle h_3, 0.7, 0.6, 0.4\rangle, \langle h_4, 0.7, 0.8, 0.6\rangle, \langle h_5, 0.9, 0.5, 0.7\rangle\right\}.$$

The neutrosophic soft set(NSS) (F, E) is a parametrized family $\left\{F(e_i), i = 1, 2, ...10\right\}$ of all neutrosophic sets of U and describes a collection of approximation of an object.The mapping F here is 'houses(.)',where dot(.) is to be filled up by a parameter $e \in E$.Therefore, $F(e_1)$ means 'houses(beautiful)' whose functional-value is the neutrosophic set

$$\left\{\langle h_1, 0.5, 0.6, 0.3\rangle, \langle h_2, 0.4, 0.7, 0.6\rangle, \langle h_3, 0.6, 0.2, 0.3\rangle, \langle h_4, 0.7, 0.3, 0.2\rangle, \langle h_5, 0.8, 0.2, 0.3\rangle\right\}.$$

Thus we can view the neutrosophic soft set (NSS) (F, A) as a collection of approximation as below:

$$(F,A)=\begin{cases} beautiful\ houses=\{\langle h_1,0.5,0.6,0.3\rangle,\langle h_2,0.4,0.7,0.6\rangle,\langle h_3,0.6,0.2,0.3\rangle,\langle h_4,0.7,0.3,0.2\rangle,\langle h_5,0.8,0.2,0.3\rangle\}, \\ wooden\ houses=\{\langle h_1,0.6,0.3,0.5\rangle,\langle h_2,0.7,0.4,0.3\rangle,\langle h_3,0.8,0.1,0.2\rangle,\langle h_4,0.7,0.1,0.3\rangle,\langle h_5,0.8,0.3,0.6\rangle\}, \\ costly\ houses=\{\langle h_1,0.7,0.4,0.3\rangle,\langle h_2,0.6,0.7,0.2\rangle,\langle h_3,0.7,0.2,0.5\rangle,\langle h_4,0.5,0.2,0.6\rangle,\langle h_5,0.7,0.3,0.4\rangle\}, \\ moderate\ houses=\{\langle h_1,0.8,0.6,0.4\rangle,\langle h_2,0.7,0.9,0.6\rangle,\langle h_3,0.7,0.6,0.4\rangle,\langle h_4,0.7,0.8,0.6\rangle,\langle h_5,0.9,0.5,0.7\rangle\} \end{cases},$$

where each approximatin has two parts:(i) a predicate p, and (ii) an approximate value-set v(or simply to be called value-set v).

The tabular representation of the neutrosophic soft set (F,A) is as follows:

$$\begin{bmatrix} U & beautiful & wooden & costly & moderate \\ h_1 & (0.5,0.6,0.3) & (0.6,0.3,0.5) & (0.7,0.4,0.3) & (0.8,0.6,0.4) \\ h_2 & (0.4,0.7,0.6) & (0.7,0.4,0.3) & (0.6,0.7,0.2) & (0.7,0.9,0.6) \\ h_3 & (0.6,0.2,0.3) & (0.8,0.1,0.2) & (0.7,0.2,0.5) & (0.7,0.6,0.4) \\ h_4 & (0.7,0.3,0.2) & (0.7,0.1,0.3) & (0.5,0.2,0.6) & (0.7,0.8,0.6) \\ h_5 & (0.8,0.2,0.3) & (0.8,0.3,0.6) & (0.7,0.3,0.4) & (0.9,0.5,0.7) \end{bmatrix}$$

3. Rough neutrosophic soft sets:

Here we introduce the concept of rough neutrosophic soft sets by combining both rough sets and neutrosophic soft sets and perform some operations viz. union,intersection,inclusion and equality over them.

Definition 3.1

Let U be a non-empty universe set, E be a set of parameters and R be an equivalence relation on U .Considering $A\subset E$ .Let $P(U)$ denotes the set of all neutrosophic sets of U .The collection (F,A) is termed to be the neutrosophic soft set over U where $F:A\to P(U)$ with membership function μ_F ,indeterminacy function ν_F and non-membership function ω_F .The lower and upper approximation of F in the approximation (U,R) denoted by $N_*(F)$ and $N^*(F)$ are respectively and they are defined as follows:

$$N_*(F)=\cup\left\{\left\{e_i,\{\left\langle x,\mu_{N_*(F)}(x),\nu_{N_*(F)}(x),\omega_{N_*(F)}(x)\right\rangle\}:[x]_R\subseteq X,x\in U\right\}\right\},$$

$$N^*(F)=\cup\left\{\left\{e_i,\{\left\langle x,\mu_{N^*(F)}(x),\nu_{N^*(F)}(x),\omega_{N^*(F)}(x)\right\rangle\}:[x]_R\cap X\neq\varnothing,x\in U\right\}\right\},$$

$$\mu_{N_*(F)}(x) = \underset{y\in[x]_R}{\wedge} \mu_F(y), \nu_{N_*(F)}(x) = \underset{y\in[x]_R}{\vee} \nu_F(y), \omega_{N_*(F)}(x) = \underset{y\in[x]_R}{\vee} \omega_F(y)$$

and $\mu_{N^*(F)}(x) = \underset{y\in[x]_R}{\vee} \mu_F(y), \nu_{N^*(F)}(x) = \underset{y\in[x]_R}{\wedge} \nu_F(y), \omega_{N^*(F)}(x) = \underset{y\in[x]_R}{\wedge} \omega_F(y)$

provided $0 \le \mu_{N^*(F)}(x) + \nu_{N^*(F)}(x) + \omega_{N^*(F)}(x) \le 3$ and $0 \le \mu_{N_*(F)}(x) + \nu_{N_*(F)}(x) + \omega_{N_*(F)}(x) \le 3$

where the symbols $\wedge$ and $\vee$ used to denote minimum and maximum operators respectively and the pair $(N_*(F), N^*(F))$ is called the rough neutrosophic soft set in (U, R).

If $N_*(F) = N^*(F)$ then $\mu_{N_*(F)} = \mu_{N^*(F)}, \nu_{N_*(F)} = \nu_{N^*(F)}$ and $\omega_{N_*(F)} = \omega_{N^*(F)}$ .So it is no more a rough neutrosophic soft set i.e. it is called a definable neutrosophic soft set.

Example 3.1.1

Let $U = \{h_1, h_2, h_3, h_4, h_5\}$ be a set of houses under consideration , $X \subset U$ where $X = \{h_1, h_4, h_5\}$ and R be an equivalence relation and its partition of U is,

$$U / R = \{\{h_1, h_4\}, \{h_2, h_3\}, \{h_5\}\}$$

Let E be a set of parameter where $E = \{e_1, e_2, e_3, e_4\}$ .Then (F, E) is called the neutrosophic soft set and $F : E \to P(U)$ where $P(U)$ denotes the set of all neutrosophic sets.

Now we consider the neutrosophic soft set as:

$$N(F) = \left\{ \begin{array}{l} \{e_1, \langle h_1, 0.5, 0.6, 0.3\rangle, \langle h_2, 0.4, 0.7, 0.6\rangle, \langle h_3, 0.6, 0.2, 0.3\rangle, \langle h_4, 0.7, 0.3, 0.2\rangle, \langle h_5, 0.8, 0.2, 0.3\rangle\}, \\ \{e_2, \langle h_1, 0.6, 0.3, 0.5\rangle, \langle h_2, 0.7, 0.4, 0.3\rangle, \langle h_3, 0.8, 0.1, 0.2\rangle, \langle h_4, 0.7, 0.1, 0.3\rangle, \langle h_5, 0.8, 0.3, 0.6\rangle\}, \\ \{e_3, \langle h_1, 0.7, 0.4, 0.3\rangle, \langle h_2, 0.6, 0.7, 0.2\rangle, \langle h_3, 0.7, 0.2, 0.5\rangle, \langle h_4, 0.5, 0.2, 0.6\rangle, \langle h_5, 0.7, 0.3, 0.4\rangle\}, \\ \{e_4, \langle h_1, 0.8, 0.6, 0.4\rangle, \langle h_2, 0.7, 0.9, 0.6\rangle, \langle h_3, 0.7, 0.6, 0.4\rangle, \langle h_4, 0.7, 0.8, 0.6\rangle, \langle h_5, 0.9, 0.5, 0.7\rangle\} \end{array} \right\},$$

Thus the lower and the upper rough neutrosophic soft set corresponding to X is given by,

$$N_*(F) = \left\{ \begin{array}{l} \{e_1, \langle h_1, 0.5, 0.6, 0.3\rangle, \langle h_4, 0.5, 0.6, 0.3\rangle, \langle h_5, 0.8, 0.2, 0.3\rangle\}, \\ \{e_2, \langle h_1, 0.6, 0.3, 0.5\rangle, \langle h_4, 0.6, 0.3, 0.5\rangle, \langle h_5, 0.8, 0.3, 0.6\rangle\}, \\ \{e_3, \langle h_1, 0.5, 0.4, 0.6\rangle, \langle h_4, 0.5, 0.4, 0.6\rangle, \langle h_5, 0.7, 0.3, 0.4\rangle\}, \\ \{e_4, \langle h_1, 0.7, 0.8, 0.6\rangle, \langle h_4, 0.7, 0.8, 0.6\rangle, \langle h_5, 0.9, 0.5, 0.7\rangle\} \end{array} \right\},$$

and $N^*(F) = \left\{ \begin{array}{l} \{e_1, \langle h_1, 0.7, 0.3, 0.2\rangle, \langle h_4, 0.7, 0.3, 0.2\rangle, \langle h_5, 0.8, 0.2, 0.3\rangle\}, \\ \{e_2, \langle h_1, 0.7, 0.1, 0.3\rangle, \langle h_4, 0.7, 0.1, 0.3\rangle, \langle h_5, 0.8, 0.3, 0.6\rangle\}, \\ \{e_3, \langle h_1, 0.7, 0.2, 0.3\rangle, \langle h_4, 0.7, 0.2, 0.3\rangle, \langle h_5, 0.7, 0.3, 0.4\rangle\}, \\ \{e_4, \langle h_1, 0.8, 0.6, 0.4\rangle, \langle h_4, 0.8, 0.6, 0.4\rangle, \langle h_5, 0.9, 0.5, 0.7\rangle\} \end{array} \right\},$

From the above results we can write $N_*(F) \neq N^*(F)$.

But we find some examples in real world where we can show that $N_*(F) = N^*(F)$.In those cases rough neutrosophic soft sets reduced as definable neutrosophic soft set.

Definition 3.2 If $N(F) = (N_*(F), N^*(F))$is a rough neutrosophic soft set in (U, R),the rough complement of $N(F)$is the rough neutrosophic soft set denoted by $\sim N(F) = \left(N_*(F)^c, N^*(F)^c\right)$,where $N_*(F)^c, N^*(F)^c$ are the complements of neutrosophic soft sets $N_*(F)$ and $N^*(F)$ respectively.

Then,

$$N_*(F)^c = \left\{\left\{e_i, \left\langle x, \omega_{N_*(F)}(x), 1-\nu_{N_*(F)}(x), \mu_{N_*(F)}(x)\right\rangle\right\} : x \in U\right\}$$

and $N^*(F)^c = \left\{\left\{e_i, \left\langle x, \omega_{N^*(F)}(x), 1-\nu_{N^*(F)}(x), \mu_{N^*(F)}(x)\right\rangle\right\} : x \in U\right\}$

Definition 3.3 If $N(F_1)$ and $N(F_2)$are two rough neutrosophic soft sets of the neutrosophic soft sets F_1 and F_2respectively in U ,then we have the following results:

(i) $N(F_1) = N(F_2)$iff $N_*(F_1) = N_*(F_2)$ and $N^*(F_1) = N^*(F_2)$

(ii) $N(F_1) \subseteq N(F_2)$iff $N_*(F_1) \subseteq N_*(F_2)$ and $N^*(F_1) \subseteq N^*(F_2)$

(iii) $N(F_1) \cup N(F_2) = \left\langle N_*(F_1) \cup N_*(F_2), N^*(F_1) \cup N^*(F_2) \right\rangle$

(iv) $N(F_1) \cap N(F_2) = \left\langle N_*(F_1) \cap N_*(F_2), N^*(F_1) \cap N^*(F_2) \right\rangle$

(v) $N(F_1) + N(F_2) = \left\langle N_*(F_1) + N_*(F_2), N^*(F_1) + N^*(F_2) \right\rangle$

(vi) $N(F_1).N(F_2)=\langle N_*(F_1).N_*(F_2), N^*(F_1).N^*(F_2)\rangle$

Proposition 3.4 If N, M, L are rough neutrosophic soft set in (U, R), then the following propositions are straightforward:

(i) $\sim N(\sim N)=N$

(ii) $N\cup M=M\cup N, N\cap M=M\cap N$

(iii) $(N\cup M)\cup L=N\cup(M\cup L),(N\cap M)\cap L=N\cap(M\cap L)$

(iv) $(N\cup M)\cap L=(\mathrm{L}\cap M)\cup(N\cap L),(N\cap M)\cup L=(N\cup L)\cap(\mathrm{M}\cup L)$

Proposition 3.5 (De Morgan's Laws)

(i) $\sim(N(F_1)\cup N(F_2))=(\sim N(F_1))\cap(\sim N(F_2))$

(ii) $\sim(N(F_1)\cap N(F_2))=(\sim N(F_1))\cup(\sim N(F_2))$

Proof: (i) $\sim(N(F_1)\cup N(F_2))=\sim\langle\{N_*(F_1)\cup N_*(F_2)\},\{N^*(F_1)\cup N^*(F_2)\}\rangle$

$$=\langle\sim\{N_*(F_1)\cup N_*(F_2)\},\sim\{N^*(F_1)\cup N^*(F_2)\}\rangle$$

$$=\langle\{N_*(F_1)\cup N_*(F_2)\}^c,\{N^*(F_1)\cup N^*(F_2)\}^c\rangle$$

$$=\langle\sim\{N_*(F_1)\cap N_*(F_2)\},\sim\{N^*(F_1)\cap N^*(F_2)\}\rangle$$

$$=(\sim N(F_1))\cap(\sim N(F_2))$$

(ii) Similar to the proof of (i)

Proposition 3.6 If F_1 and F_2 are two neutrosophic sets in U such that $F_1\subseteq F_2$, then $N(F_1)\subseteq N(F_2)$

(i) $N(F_1\cup F_2)\supseteq N(F_1)\cup N(F_2)$

(ii) $N(F_1\cap F_2)\subseteq N(F_1)\cap N(F_2)$

Proof : $\mu_{N_*(F_1\cup F_2)}(x)=\inf\left\{\mu_{(F_1\cup F_2)}(x):x\in X_i\right\}$

$$=\inf\left(\max\left\{\mu_{F_1}(x),\mu_{F_2}(x)\right\}:x\in X_i\right)$$

$$\geq\max\left\{\inf\left\{\mu_{F_1}(x):x\in X_i\right\},\inf\left\{\mu_{F_2}(x):x\in X_i\right\}\right\}$$

$$=\max\left\{\mu_{N_*(F_1)}(x_i),\mu_{N_*(F_2)}(x_i)\right\}$$

$$=(\mu_{N_*(F_1)}\cup\mu_{N_*(F_2)})(x_i)$$

Similarly, $\nu_{N_*(F_1\cup F_2)}(x)\leq(\nu_{N_*(F_1)}\cup\nu_{N_*(F_2)})(x_i)$

$$\omega_{N_*(F_1\cup F_2)}(x)\leq(\omega_{N_*(F_1)}\cup\omega_{N_*(F_2)})(x_i)$$

Thus, $N_*(F_1\cup F_2)\supseteq N_*(F_1)\cup N_*(F_2)$

We can also see that

$$N^*(F_1\cup F_2)=N^*(F_1)\cup N^*(F_2)$$

Hence, $N(F_1\cup F_2)\supseteq N(F_1)\cup N(F_2)$

(ii) The proof of (ii) is similar to the proof of (i)

Proposition 3.7

(i) $N_*(F)=\sim N^*(\sim F)$

(ii) $N^*(F)=\sim N_*(\sim F)$

(iii) $N_*(F)\subseteq N^*(F)$

Proof: By definition,

$$F=\left\{e_i,\{\left\langle x,\mu_{N_*(F)}(x),\nu_{N_*(F)}(x),\omega_{N_*(F)}(x)\right\rangle\}:x\in X\right\}$$

$$\sim F=\left\{e_i,\{\left\langle x,\omega_{N_*(F)}(x),1-\nu_{N_*(F)}(x),\mu_{N_*(F)}(x)\right\rangle\}:x\in X\right\}$$

$$N^*(\sim F)=\cup\left\{e_i,\{\left\langle x,\omega_{N^*(F)}(x),1-\nu_{N^*(F)}(x),\mu_{N^*(F)}(x)\right\rangle\}:[x]_R\cap X\neq\emptyset,x\in U\right\}$$

$$\sim N^*(\sim F) = \cup\left\{e_i, \{\left\langle x, \mu_{N^*(F)}(x), 1-(1-\nu_{N^*(F)}(x)), \omega_{N^*(F)}(x), \mu_{N^*(F)}(x)\right\rangle\} : [x]_R \subseteq X, x \in U\right\}$$

$$= N_*(F)$$

(ii) The proof is similar to the proof of (i)

(iii) For any $y \in N_*(F)$, we have

$$\mu_{N_*(F)}(x) = \wedge_{y\in[x]_R} \mu_F(y) \le \vee_{y\in[x]_R} \mu_F(y), \nu_{N_*(F)}(x) = \vee_{y\in[x]_R} \nu_F(y) \ge \wedge_{y\in[x]_R} \nu_F(y)$$

and $\omega_{N_*(F)}(x) = \vee_{y\in[x]_R} \omega_F(y) \ge \wedge_{y\in[x]_R} \omega_F(y)$

Thus, $N_*(F) \subseteq N^*(F)$

4. Conclusion: In this work we have introduced the notion of rough neutrosophic soft sets by using equivalence relation.We have also studied some basic operations on them and proved some properties.The concept of rough neutrosophic soft sets is a combination of three different theories which are rough sets,neutrosophic sets and soft sets.Soft set theory mainly concerned with parametric data, while neutrosophic set theory deals with indeterminate and consistent information and rough set theory is with incompleteness.so rough neutrosophic soft sets can be utilized for dealing with parametrization,indeterminacy and incompleteness.

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