# Reliability and Importance  Discounting
# of Neutrosophic Masses

Florentin Smarandache

°›¯°☙ In this paper, we introduce for the first time the discounting of a neutrosophic mass in terms of reliability and respectively the importance of the source.

We show that reliability and importance discounts commute when dealing with classical masses.

**Introduction** Let $\Phi = \{\Phi_1, \Phi_2, \dots, \Phi_n\}$ be the frame of discernment, where $n \geq 2$, and the set of **focal elements**:

$$F = \{A_1, A_2, \dots, A_m\}, \text{ for } m \geq 1, F \subset G^\Phi. \quad (1)$$

Let $G^\Phi = (\Phi, \cup, \cap, \mathcal{C})$ be the **fusion space**.

A **neutrosophic mass** is defined as follows:

$$m_n: G \rightarrow [0, 1]^3$$

for any $x \in G$, $m_n(x) = (t(x), i(x), f(x)),$  (2)

where    $t(x) = $ believe that $x$ will occur (truth);

$i(x) = $ indeterminacy about occurence;

and $f(x) = $ believe that $x$ will not occur (falsity).

Simply, we say in neutrosophic logic:

$t(x) =$ believe in $x$;

$i(x) =$ believe in neut$(x)$

[the neutral of $x$, i.e. neither $x$ nor anti$(x)$];

and $f(x) =$ believe in anti$(x)$ [the opposite of $x$].

Of course, $t(x),\ i(x),\ f(x) \in [0,1]$, and

$$\sum_{x\in G}[t(x) + i(x) + f(x)] = 1, (3)$$

while

$$m_n(\phi) = (0,0,0). (4)$$

It is possible that according to some parameters (or data) a source is able to predict the believe in a hypothesis $x$ to occur, while according to other parameters (or other data) the same source may be able to find the believe in $x$ not occuring, and upon a third category of parameters (or data) the source may find some indeterminacy (ambiguity) about hypothesis occurence.

An element $x \in G$ is called 𝖼𝗈𝗌̈ if

$$n_m(x) \neq (0,0,0), (5)$$

i.e. $t(x) > 0$ or $i(x) > 0$ or $f(x) > 0$.

Any 𝗈𝗌̃⁻⁻𝐘𝗌̆𝐥 𝐦𝗌̆⁻⁻:

$$m : G^\phi \to [0,1] (6)$$

can be simply written as a neutrosophic mass as:

$$m(A) = (m(A), 0, 0). (7)$$

## ˘ ¥œ±ª°¥ £š Neu°®«¯«¬¤¥ᵈ š¯¯ due °« Rel¥› ¥¥µof °¤¡

## ' «±®op

Let $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ be the reliability coefficient of the source, $\alpha \in [0,1]^3$.

Then, for any $x \in G^\theta \setminus \{\theta, I_t\}$,

where $\theta$ = the empty set

and $I_t$ = total ignorance,

$$m_n(x)_a = \big(\alpha_1 t(x), \alpha_2 i(x), \alpha_3 f(x)\big), \text{ (8)}$$

and

$$m_n(I_t)_\alpha = \Bigg( t(I_t) + (1 - \alpha_1) \sum_{x \in G^\theta \setminus \{\phi, I_t\}} t(x),$$

$$i(I_t) + (1 - \alpha_2) \sum_{x \in G^\theta \setminus \{\phi, I_t\}} i(x), f(I_t) + (1 - \alpha_3) \sum_{x \in G^\theta \setminus \{\phi, I_t\}} f(x) \Bigg)$$

$$\text{(9)},$$

and, of course,

$$m_n(\phi)_\alpha = (0, 0, 0).$$

The missing mass of each element $x$, for $x \neq \phi, x \neq I_t$, is transferred to the mass of the total ignorance in the following way:

$$t(x) - \alpha_1 t(x) = (1 - \alpha_1) \cdot t(x) \text{ is transferred to } t(I_t), \text{ (10)}$$

$$i(x) - \alpha_2 i(x) = (1 - \alpha_2) \cdot i(x) \text{ is transferred to } i(I_t), \text{ (11)}$$

and $f(x) - \alpha_3 f(x) = (1 - \alpha_3) \cdot f(x)$ is transferred to $f(I_t)$. (12)

ˇ¥œ±ᵃ°¥£š "¡±°®‹¯«¬¤¥ɰ! š¯s ¥±e °« °¤¡ ℟©¬«®šᵃce «f °¤¡ '«±®ɰᴇ

Let $\beta \in [0, 1]$ be the importance coefficient of the source. This discounting can be done in several ways.

a.  For any $x \in G^\theta \setminus \{\phi\}$,

$$m_n(x)_{\beta_1} = \big(\beta \cdot t(x), i(x), f(x) + (1 - \beta) \cdot t(x)\big), (13)$$

which means that $t(x)$, the believe in $x$, is diminished to $\beta \cdot t(x)$, and the missing mass, $t(x) - \beta \cdot t(x) = (1 - \beta) \cdot t(x)$, is transferred to the believe in $anti(x)$.

b.  Another way:

For any $x \in G^\theta \setminus \{\phi\}$,

$$m_n(x)_{\beta_2} = \big(\beta \cdot t(x), i(x) + (1 - \beta) \cdot t(x), f(x)\big), (14)$$

which means that $t(x)$, the believe in $x$, is similarly diminished to $\beta \cdot t(x)$, and the missing mass $(1 - \beta) \cdot t(x)$ is now transferred to the believe in $neut(x)$.

c.  The third way is the most general, putting together the first and second ways.

For any $x \in G^\theta \setminus \{\phi\}$,

$$m_n(x)_{\beta_3} = \big(\beta \cdot t(x), i(x) + (1 - \beta) \cdot t(x) \cdot \gamma, f(x) + (1 - \beta) \cdot t(x) \cdot$$
$$(1 - \gamma)\big), (15)$$

where $\gamma \in [0, 1]$ is a parameter that splits the missing mass $(1 - \beta) \cdot t(x)$ a part to $i(x)$ and the other part to $f(x)$.

For $\gamma = 0$, one gets the first way of distribution, and when $\gamma = 1$, one gets the second way of distribution.

### ˇ¥œ±ª°¥£«f &¡ ˙Ӿ˃ ¥¥y šªῩӀ©¬«®tance «f ' «±®ces ¥n fi¡ ª¡ ®˃˙ ˇ« Not Co©©±°¡

### a. Reliability first, Importance second.

For any $x \in G^\theta \setminus \{\phi, I_t\}$, one has after reliability α discounting, where

$$\alpha = (\alpha_1, \alpha_2, \alpha_3):$$

$$m_n(x)_\alpha = \left( \alpha_1 \cdot t(x), \alpha_2 \cdot t(x), \alpha_3 \cdot f(x) \right), (16)$$

and

$$m_n(I_t)_\alpha = \left( t(I_t) + (1 - \alpha_1) \cdot \sum_{x \in G^\theta \setminus \{\phi, I_t\}} t(x), i(I_t) + (1 - \alpha_2) \right.$$

$$\left. \cdot \sum_{x \in G^\theta \setminus \{\phi, I_t\}} i(x), f(I_t) + (1 - \alpha_3) \cdot \sum_{x \in G^\theta \setminus \{\phi, I_t\}} f(x) \right)$$

$$\stackrel{\text{def}}{=} \left( T_{I_t}, I_{I_t}, F_{I_t} \right).$$

$$(17)$$

Now we do the importance β discounting method, the third importance discounting way which is the most general:

$$m_n(x)_{\alpha\beta_3} = \left( \beta\alpha_1 t(x), \alpha_2 i(x) + (1 - \beta)\alpha_1 t(x)\gamma, \alpha_3 f(x) \right.$$

$$\left. + (1 - \beta)\alpha_1 t(x)(1 - \gamma) \right)$$

$$(18)$$

and

$$m_n(I_t)_{\alpha\beta_3} = \left( \beta \cdot T_{I_t}, I_{I_t} + (1-\beta)T_{I_t} \cdot \gamma, F_{I_t} + (1-\beta)T_{I_t}(1-\gamma) \right). \ (19)$$

### b. Importance first, Reliability second.

For any $x \in G^\theta \setminus \{\phi, I_t\}$, one has after importance β discounting (third way):

$$m_n(x)_{\beta_3} = \left( \beta \cdot t(x), i(x) + (1-\beta)t(x)\gamma, f(x) + (1-\beta)t(x)(1-\gamma) \right) \ (20)$$

and

$$m_n(I_t)_{\beta_3} = \left( \beta \cdot t(I_{I_t}), i(I_{I_t}) + (1-\beta)t(I_t)\gamma, \ f(I_t) + (1-\beta)t(I_t)(1-\gamma) \right).$$
$$(21)$$

Now we do the reliability $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ discounting, and one gets:

$$m_n(x)_{\beta_3\alpha} = \left( \alpha_1 \cdot \beta \cdot t(x), \alpha_2 \cdot i(x) + \alpha_2(1-\beta)t(x)\gamma, \alpha_3 \cdot f(x) + \alpha_3 \cdot \right.$$
$$\left. (1-\beta)t(x)(1-\gamma) \right) \ (22)$$

and

$$m_n(I_t)_{\beta_3\alpha} = \left( \alpha_1 \cdot \beta \cdot t(I_t), \alpha_2 \cdot i(I_t) + \alpha_2(1-\beta)t(I_t)\gamma, \alpha_3 \cdot f(I_t) + \right.$$
$$\left. \alpha_3(1-\beta)t(I_t)(1-\gamma) \right). \ (23)$$

### &emark.

We see that (a) and (b) are in general different, so reliability of sources does not commute with the importance of sources.

Šš®¥e˘šr ˛šse ³ ¤en &¡˙¥›¥¥y šªd Ł©¬«®šªœ˘¥œ±ª°¥g «Ċ ! š¯¯¡¯Co©©±°¡

Let's consider a classical mass

$$m: G^\theta \to [0, 1] \ (24)$$

and the focal set $F \subset G^\theta$,

$$F = \{A_1, A_2, \ldots, A_m\}, m \geq 1, \ (25)$$

and of course $m(A_i) > 0$, for $1 \leq i \leq m$.

Suppose $m(A_i) = a_i \in (0,1]. \ (26)$

### a. Reliability first, Importance second.

Let $\alpha \in [0, 1]$ be the reliability coefficient of $m \ (\cdot)$.

For $x \in G^\theta \setminus \{\phi, I_t\}$, one has

$$m(x)_\alpha = \alpha \cdot m(x), \ (27)$$

$$\text{and } m(I_t) = \alpha \cdot m(I_t) + 1 - \alpha. \ (28)$$

Let $\beta \in [0, 1]$ be the importance coefficient of $m \ (\cdot)$.

Then, for $x \in G^\theta \setminus \{\phi, I_t\}$,

$$m(x)_{\alpha\beta} = \big(\beta\alpha m(x), \alpha m(x) - \beta\alpha m(x)\big) = \alpha \cdot m(x) \cdot (\beta, 1 - \beta), \ (29)$$

considering only two components: believe that $x$ occurs and, respectively, believe that $x$ does not occur.

Further on,

Florentin Smarandache

Neutrosophic Theory and Its Applications. Collected Papers, I

$$m(I_t)_{\alpha\beta} = (\beta\alpha m(I_t) + \beta - \beta\alpha, \alpha m(I_t) + 1 - \alpha - \beta\alpha m(I_t) - \beta + \beta\alpha) =$$

$$[\alpha m(I_t) + 1 - \alpha] \cdot (\beta, 1 - \beta). \, (30)$$

### b. *Importance first, Reliability second.*

For $x \in G^\theta \setminus \{\phi, I_t\}$, one has

$$m(x)_\beta = \big(\beta \cdot m(x), m(x) - \beta \cdot m(x)\big) = m(x) \cdot (\beta, 1 - \beta), \, (31)$$

and $m(I_t)_\beta = \big(\beta m(I_t), m(I_t) - \beta m(I_t)\big) = m(I_t) \cdot (\beta, 1 - \beta). \, (32)$

Then, for the reliability discounting scaler α one has:

$$m(x)_{\beta\alpha} = \alpha m(x)(\beta, 1 - \beta) = \big(\alpha m(x)\beta, \alpha m(x) - \alpha\beta m(m)\big) \, (33)$$

and $m(I_t)_{\beta\alpha} = \alpha \cdot m(I_t)(\beta, 1 - \beta) + (1 - \alpha)(\beta, 1 - \beta) = [\alpha m(I_t) + 1 - \alpha] \cdot$
$(\beta, 1 - \beta) = (\alpha m(I_t)\beta, \alpha m(I_t) - \alpha m(I_t)\beta) + (\beta - \alpha\beta, 1 - \alpha - \beta + \alpha\beta) =$
$(\alpha\beta m(I_t) + \beta - \alpha\beta, \alpha m(I_t) - \alpha\beta m(I_t) + 1 - \alpha - \beta - \alpha\beta). \, (34)$

Hence (a) and (b) are equal in this case.

**1 ´š©¬¨¡¯**

## 1. Classical mass.

The following classical is given on $\theta = \{A, B\}$ :

|     | A   | B   | AUB |
| --- | --- | --- | --- |
| $m$ | 0.4 | 0.5 | 0.1 |

$$(35)$$

Let $\alpha = 0.8$ be the reliability coefficient and $\beta = 0.7$ be the importance coefficient.

### a. Reliability first, Importance second.

|                | A              | B              | AUB            |
| -------------- | -------------- | -------------- | -------------- |
| $m_\alpha$     | 0.32           | 0.40           | 0.28           |
| $m_{\alpha\beta}$ | $(0.224, 0.096)$ | $(0.280, 0.120)$ | $(0.196, 0.084)$ |

$$(36)$$

We have computed in the following way:

$$m_\alpha(A) = 0.8m(A) = 0.8(0.4) = 0.32, (37)$$

$$m_\alpha(B) = 0.8m(B) = 0.8(0.5) = 0.40, (38)$$

$$m_\alpha(AUB) = 0.8(AUB) + 1 - 0.8 = 0.8(0.1) + 0.2 = 0.28, (39)$$

and

$$m_{\alpha\beta}(B) = \big(0.7m_\alpha(A), m_\alpha(A) - 0.7m_\alpha(A)\big) = \big(0.7(0.32), 0.32 - 0.7(0.32)\big) = (0.224, 0.096), (40)$$

$$m_{\alpha\beta}(B) = \big(0.7m_\alpha(B), m_\alpha(B) - 0.7m_\alpha(B)\big) = \big(0.7(0.40), 0.40 -$$
$$0.7(0.40)\big) = (0.280, 0.120), (41)$$

$$m_{\alpha\beta}(AUB) = \big(0.7m_\alpha(AUB), m_\alpha(AUB) - 0.7m_\alpha(AUB)\big) =$$
$$\big(0.7(0.28), 0.28 - 0.7(0.28)\big) = (0.196, 0.084). (42)$$

### b. Importance first, Reliability second.

|              | A              | B              | AUB            |
|--------------|----------------|----------------|----------------|
| $m$          | 0.4            | 0.5            | 0.1            |
| $m_\beta$    | (0.28, 0.12)   | (0.35, 0.15)   | (0.07, 0.03)   |
| $m_{\beta\alpha}$ | (0.224, 0.096 | (0.280, 0.120) | (0.196, 0.084) |

$$(43)$$

We computed in the following way:

$$m_\beta(A) = \big(\beta m(A), (1 - \beta)m(A)\big) = \big(0.7(0.4), (1 - 0.7)(0.4)\big) =$$
$$(0.280, 0.120), (44)$$

$$m_\beta(B) = \big(\beta m(B), (1 - \beta)m(B)\big) = \big(0.7(0.5), (1 - 0.7)(0.5)\big) =$$
$$(0.35, 0.15), (45)$$

$$m_\beta(AUB) = \big(\beta m(AUB), (1 - \beta)m(AUB)\big) = \big(0.7(0.1), (1 - 0.1)(0.1)\big) =$$
$$(0.07, 0.03), (46)$$

$$\text{and } m_{\beta\alpha}(A) = \alpha m_\beta(A) = 0.8(0.28, 0.12) = \big(0.8(0.28), 0.8(0.12)\big) =$$
$$(0.224, 0.096), (47)$$

$$m_{\beta\alpha}(B) = \alpha m_\beta(B) = 0.8(0.35, 0.15) = \big(0.8(0.35), 0.8(0.15)\big) =$$
$$(0.280, 0.120), (48)$$

Florentin Smarandache

Neutrosophic Theory and Its Applications. Collected Papers, I

$$m_{\beta\alpha}(AUB) = \alpha m(AUB)(\beta, 1-\beta) + (1-\alpha)(\beta, 1-\beta) = 0.8(0.1)(0.7, 1-$$
$$0.7) + (1-0.8)(0.7, 1-0.7) = 0.08(0.7, 0.3) + 0.2(0.7, 0.3) =$$
$$(0.056, 0.024) + (0.140, 0.060) = (0.056 + 0.140, 0.024 + 0.060) =$$
$$(0.196, 0.084). \ (49)$$

Therefore reliability discount commutes with importance discount of sources when one has classical masses.

The result is interpreted this way: believe in $A$ is 0.224 and believe in $nonA$ is 0.096, believe in $B$ is 0.280 and believe in $nonB$ is 0.120, and believe in total ignorance $AUB$ is 0.196, and believe in non-ignorance is 0.084.

'š©¡ı´š©¬¨¡ ³ ¥h ˇ ¥¢®¡ªº &¡ Ÿ¥¥°®¥±°¥ª «f! š⁻⁻¡⁻ &¡ ˙š°¡Ÿ°«
Ⱡ©¬«®šªœof' «±®œ⁻

Let's consider the third way of redistribution of masses related to importance coefficient of sources. $\beta = 0.7$, but $\gamma = 0.4$, which means that 40% of $\beta$ is redistributed to $i(x)$ and 60% of $\beta$ is redistributed to $f(x)$ for each $x \in G^{\theta} \setminus \{\phi\}$; and $\alpha = 0.8$.

### a. Reliability first, Importance second.

|              | A                          | B                          | AUB                        |
| ------------ | -------------------------- | -------------------------- | -------------------------- |
| $m$          | 0.4                        | 0.5                        | 0.1                        |
| $m_{\alpha}$ | 0.32                       | 0.40                       | 0.28                       |
| $m_{\alpha\beta}$ | (0.2240, 0.0384, 0.0576) | (0.2800, 0.0480, 0.0720) | (0.1960, 0.0336, 0.0504). |

(50)

We computed $m_\alpha$ in the same way.

But:

$$m_{\alpha\beta}(A) = \big(\beta \cdot m_\alpha(A), i_\alpha(A) + (1-\beta)m_\alpha(A) \cdot \gamma, f_\alpha(A) +$$
$$(1-\beta)m_\alpha(A)(1-\gamma)\big) = \big(0.7(0.32), 0 + (1-0.7)(0.32)(0.4), 0 +$$
$$(1-0.7)(0.32)(1-0.4)\big) = (0.2240, 0.0384, 0.0576). \quad (51)$$

Similarly for $m_{\alpha\beta}(B)$ and $m_{\alpha\beta}(AUB)$.

## b. *Importance first, Reliability second.*

|  | A | B | AUB |
|---|---|---|---|
| m | 0.4 | 0.5 | 0.1 |
| $m_\beta$ | (0.280, 0.048, 0.072) | (0.350, 0.060, 0.090) | (0.070, 0.012, 0.018) |
| $m_\beta\alpha$ | (0.2240, 0.0384, 0.0576) | (0.2800, 0.0480, 0.0720) | (0.1960, 0.0336, 0.0504). |

$$(52)$$

We computed $m_\beta(\cdot)$ in the following way:

$$m_\beta(A) = \big(\beta \cdot t(A), i(A) + (1-\beta)t(A) \cdot \gamma, f(A) + (1-\beta)t(A)(1-\gamma)\big) = \big(0.7(0.4), 0 + (1-0.7)(0.4)(0.4), 0 + (1-0.7)0.4(1-0.4)\big) = (0.280, 0.048, 0.072). \quad (53)$$

Similarly for $m_\beta(B)$ and $m_\beta(AUB)$.

To compute $m_{\beta\alpha}(\cdot)$, we take $\alpha_1 = \alpha_2 = \alpha_3 = 0.8$, (54)

in formulas (8) and (9).

$$m_{\beta\alpha}(A) = \alpha \cdot m_{\beta}(A) = 0.8(0.280, 0.048, 0.072)$$

$$= \big(0.8(0.280), 0.8(0.048), 0.8(0.072)\big)$$

$$= (0.2240, 0.0384, 0.0576). \,(55)$$

Similarly $m_{\beta\alpha}(B) = 0.8(0.350, 0.060, 0.090) =$

$(0.2800, 0.0480, 0.0720). \,(56)$

For $m_{\beta\alpha}(AUB)$ we use formula (9):

$$m_{\beta\alpha}(AUB) = \big(t_{\beta}(AUB) + (1 - \alpha)[t_{\beta}(A) + t_{\beta}(B)], \, i_{\beta}(AUB)$$

$$+ (1 - \alpha)[i_{\beta}(A) + i_{\beta}(B)],$$

$$f_{\beta}(AUB) + (1 - \alpha)[f_{\beta}(A) + f_{\beta}(B)]\big)$$

$$= (0.070 + (1 - 0.8)[0.280 + 0.350], 0.012$$

$$+ (1 - 0.8)[0.048 + 0.060], 0.018 + (1 - 0.8)[0.072 + 0.090])$$

$$= (0.1960, 0.0336, 0.0504).$$

Again, the reliability discount and importance discount commute.

### ¿«ªŒë±¯¥ª

In this paper we have defined a new way of discounting a classical and neutrosophic mass with respect to its importance. We have also defined the discounting of a neutrosophic source with respect to its reliability.

In general, the reliability discount and importance discount do not commute. But if one uses classical masses, they commute (as in Examples 1 and 2).

# °œŠª«³ ¨¡ŸÇ©¡ªº

The author would like to thank Dr. Jean Dezert for his opinions about this paper.

# &¡Ç®¡ªœ¯

1.    F. Smarandache, J. Dezert, J.-M. Tacnet, *Fusion of Sources of Evidence with Different Importances and Reliabilities*, Fusion 2010 International Conference, Edinburgh, Scotland, 26-29 July, 2010.

2.    Florentin Smarandache, *Neutrosophic Masses & Indeterminate Models. Applications to Information Fusion*, Proceedings of the International Conference on Advanced Mechatronic Systems [ICAMechS 2012], Tokyo, Japan, 18-21 September 2012.